Memories of Vaughan Jones

Dietmar Bisch, David E. Evans, Robion Kirby, and Sorin Popa

Introduction

Sir Vaughan Frederick Randal Jones, who died at age 67 on September 6, 2020, was one of the most influential and inspirational mathematicians of the last four decades. His original and penetrating analysis of inclusions of von Neumann algebras led to the creation of new fields of research, while reinvigorating old ones, thereby setting off an extraordinary interplay between disparate areas of mathematics, from analysis of operator algebras, to low-dimensional topology, statistical mechanics, quantum computing, and quantum field theory. Vaughan's work had a major impact with unexpected, stunning applications, even outside of mathematics, for example to the study of knotted DNA strands and protein folding in biology. A crucial idea leading to these striking connections was his groundbreaking discovery in the early 1980s that the symmetries of a *factor* (a von Neumann algebra with trivial center), as encoded by its *subfactors*, are *quantized*. They generate "quantized groups," a completely new type of structure, endowed with a dimension function given by a *trace* and an *index* that can be nonintegral.

This article gives a panoramic view of the scientific impact and enduring legacy of Vaughan's work, as well as his personality and style of working through the contributions of colleagues and friends across mathematics and physics. Over the years, Vaughan's countless mathematical interactions forged numerous lifelong friendships, and he will be sorely missed by all.

Vaughan was born on December 31, 1952, in Gisborne on the North Island of New Zealand to parents Jim Jones and Joan Jones (née Collins) and grew up in Auckland. Between the ages of eight and twelve he was educated at the boarding school St Peter's School in Cambridge in rural North Island. Vaughan attended Auckland Grammar School until the age of sixteen and then studied Mathematics at Auckland University from 1969 to 1973. He left New Zealand in 1974 for graduate study at the University of Geneva with the intention of writing a thesis in Physics, but gradually moved in 1974–76 to work under the supervision of André Haefliger in Mathematics. It was in Switzerland where Vaughan met Martha (Wendy), who held a scholarship to study at the University of Fribourg and subsequently worked at the United Nations in Geneva. They married in 1979 and raised three children together, Bethany, Ian, and Alice.

Vaughan had several appointments in the USA until his death. However, the friendships he made during

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these formative years in New Zealand remained with him throughout his life. His love and loyalty to New Zealand would bring him back later, at least annually from 1994 on, to invigorate mathematics in his native land, often with summer schools, using his network of colleagues and friends worldwide and his scientific standing to attract other world-renowned stars to New Zealand.

In the fall of 1975, when Vaughan was switching from physics to mathematics, he met Alain Connes at a conference in Strasbourg and was very impressed. Connes had just finished his seminal work on II_1 factors, a class of von Neumann factors that have a trace with range $[0,1]$ on the lattice of their projections. In one of his papers, Connes gave a classification of periodic automorphisms of the *hyperfinite factor R*, an important II_1 factor that can be seen as the quantized version of the unit interval. Vaughan was struck by the beauty of these mathematical objects and the *continuous dimension* phenomenon, which has the remarkable feature that one can take the $t \times t$ matrix algebra $M^t := \mathbb{M}_t(M)$ over a II_1 factor M for any real number $t > 0$. He avidly studied all of the papers in this subject, from the pioneering 1936–1943 work of Murray and von Neumann, who discovered these objects, all the way to Connes's recent preprints. He gathered a list of ten possible thesis topics and travelled to Paris to show them to Connes, who went rapidly down the list, "No, no, no, maybe, no,..., good,...," and the "good" one became Vaughan's thesis. That topic was to generalize Connes's result on periodic automorphisms to arbitrary finite groups, which Vaughan did in [1]. Vaughan continued to visit Connes in Paris and then at the Institute for Advanced Study where Connes was a member in 1978–1979, with Haefliger as his formal adviser. Vaughan received his Docteur ès Sciences from the University of Geneva in 1979, and his thesis was awarded the Vacheron Constantin Prize.

Masamichi Takesaki was impressed by Vaughan's thesis and brought him to UCLA on a Hedrick assistant professorship in 1980. But after one year at UCLA, Vaughan returned to the East Coast to join his wife Wendy who was studying at Princeton. UPenn seized the opportunity and made him an offer. So during 1981–1985, Vaughan was at UPenn, first as a junior faculty member then as an associate professor, with 1984–1985 actually spent at MSRI. In 1985, he was appointed full professor at UC, Berkeley, where he remained until he retired in 2013 with the title Professor Emeritus. From 2011 on he held the Stevenson Distinguished Chair at Vanderbilt University. Vaughan was also a Distinguished Alumni Professor at the University of Auckland and Founding Director of the New Zealand Mathematics Research Institute from 1994 on. He kept in contact with Europe including spending one-year sabbaticals at the IHES during 1986–1987 and 1989–1990 and at the University of Geneva in 1993–1994 and 1998–99.

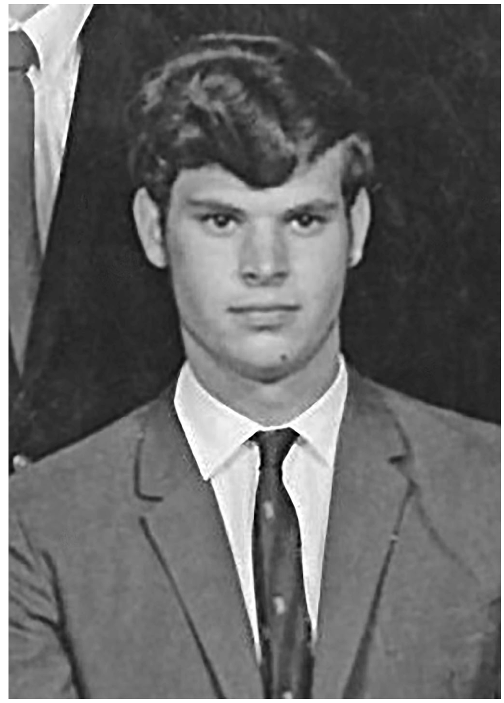

Figure 1. Auckland Grammar University Entrance Scholar 1969.

In his thesis, Vaughan developed a novel algebraic approach to the classification of actions of finite groups on II_1 factors, in which the action of the finite group was encoded by the isomorphism class of an inclusion of II_1 factors, via a crossed product construction. Soon after his thesis, this led him to consider abstract inclusions of II_1 factors, $N \subset M$, or what he later called *subfactors*, together with a natural notion of dimension of M as an N-module, that he called *the index* and denoted $[M : N]$. He noticed right away that the hyperfinite II_1 factor R contains subfactors of any index ≥ 4. This follows from the fact that $R^t \simeq R$ for any $t > 0$, a result that is due to Murray and von Neumann. He also noted that for subfactors $N \subset M$ arising from inclusions of groups $H \subset G$, the subfactor index was equal to the index $[G : H]$ of the subgroup. By early 1980, he was able to prove that the index of a subfactor $N \subset M$ can only take the values 1 and 2 when $[M : N] < 1 + \sqrt{2}$. He circulated a preprint and gave talks at conferences about these findings. The general reaction of colleagues in the field was that most certainly only the values 1, 2, 3 could occur under 4.

But by November 1981, Vaughan made the amazing discovery that the index of a subfactor can take exactly the values $\{4\cos^2(\pi/n) \mid n \geq 3\} = \{1, 2, (3+\sqrt{5})/2, 3, \dots\}$, when less than 4. Most importantly, he showed that all these

Figure 2. Vaughan playing the violin and Wendy the flute at their wedding in 1979 in Westfield, New Jersey.

values can occur as indices of subfactors of the hyperfinite factor R ([2]). The proof of the restrictions on the index, which is of stunning beauty, involves the construction of an increasing sequence of factors (a *tower*), obtained by "adding" iteratively projections (i.e., idempotents) satisfying a set of axioms which, together with the existence of the trace and its properties, provide the restrictions.

In the summer of 1982, Vaughan realized that, because of the algebraic relations they satisfy, the projections in the tower of factors provide an unexpected family of semisimple quotients of the Hecke algebras of type A_n and completely new representations of Artin's braid groups, indexed by a parameter $\lambda \in \mathbb{R}$, which are unitary exactly at values corresponding to the indices in the discrete range. During the following two years, Vaughan gradually learned of the importance of braid groups to the theory of knots, due to Alexander's theorem that any knot is a closed braid and Markov's theorem showing when two braids give rise to the same knot via "two moves." While he realized that one of the Markov moves was automatically invariant when applying the trace to the braid element in this representation in the tower of factors, it was in May 1984 that he dealt with the second Markov move, through a stroke of genius renormalization idea, that altogether gave rise to a polynomial invariant for knots and links—the *Jones polynomial*, $V_K(q)$ for an oriented link K ([4], [3]).

Once Vaughan had defined his polynomial, it was easy to see that it was not the classical Alexander polynomial, and that it could distinguish a knot from its mirror image, and then, with more work, that it solved three Tait Conjectures, century-old conjectures that concerned projections of a knot on the plane and their simplifications. Next, V_K was quickly generalized to a 2-variable polynomial, the HOMFLYPT polynomial, named after the initials of five groups who independently discovered it. Biologists immediately used the Jones polynomial to analyze knots appearing in strands of DNA.

The invariant V_K was remarkable in generating new directions for research. Most interesting for topologists was Khovanov homology, a categorification of V_K using the Kauffman bracket (itself a way of describing V_K). Khovanov homology, whose Euler characteristic is V_K, determines the unknot which V_K is not known to do and is related by a spectral sequence to knot Floer homology.

In 1988, Witten gave a physical interpretation for V_K for links (Wilson loops) in terms of Chern-Simons theory at level l that corresponds not just to V_K, but also to the HOMFLYPT polynomial. This point of view led to numerous 3-manifold quantum invariants at roots of unity using the colored V_K. Vaughan originated these spectacular developments which now form a new branch of mathematics called *Quantum Topology*.

In a parallel development which started in 1983, a connection was made with calculations by Temperley and Lieb in solvable statistical mechanics. This triggered yet another series of interactions with physics, via statistical mechanics and conformal quantum field theory. In the latter, a similar dichotomy of discrete and continuous parts occurs for the central charge in the representations of the Virasoro algebra which describes certain projective representations of the diffeomorphism group of the circle. Subfactors provide a natural framework for studying two-dimensional conformal quantum field theories. Indeed the discrete series of the central charge in the representation theory of the Virasoro algebra can be understood via conformal nets of factors, as cosets of $SU(2)$ theories. However, the power of the quantum symmetry subfactor formulation is that it permits the wondrous possibility of constructing new exotic conformal field theories beyond the known well-studied ones arising from loop groups, doubles of finite groups, or natural constructions such as cosets, with intense ongoing work.

Perhaps the deepest and most enduring of Vaughan's revolutionary work is within the theory of II_1 factors and more generally in algebras of operators on Hilbert space. II_1 factors arise naturally from groups, their actions on spaces, and unitary representations of groups. Until Vaughan's work, symmetries of a II_1 factor M were thought to be its algebra automorphisms, which under multiplication generate a group of automorphisms of M, with possible torsion, like in the case of symmetries of classical spaces. But Vaughan's work showed that symmetries of a factor M may be "quantized." Moreover, it also showed that the proper way to view a symmetry in this framework is to encode it as a subfactor $N \subset M$, or equivalently as the Hilbert $N-M$ bimodule ${}_N L^2(M)_M$ and the Jones index $[M : N]$ as the codimension of N in M. Such quantized symmetries generate a quantized group (tensor category) $\mathcal{G}_{N\subset M}$ under taking adjoints and multiplication (fusion under relative tensor product), called the *standard invariant* of $N \subset M$, with a Cayley type bipartite graph $\Gamma_{N\subset M}$. When the number of irreducible elements of $\mathcal{G}_{N\subset M}$ is finite, something that Vaughan proved to be automatic if $[M : N] < 4$, then the square norm of the graph equals the index, $\|\Gamma_{N\subset M}\|^2 = [M : N]$. The *ADE* classification of graphs of norm less than 2 as $\{2\cos(\pi/n) \mid n \geq 3\}$ offers yet another way of deriving the restrictions on the index < 4.

One can hardly overstate the importance and depth of these discoveries. This led right away to a huge number of beautiful and exciting problems, such as the classification of subfactor inclusions $N \subset M$ when M is hyperfinite, the problem of axiomatizing the objects $\mathcal{G}_{N\subset M}$ and characterizing the bipartite graphs $\Gamma_{N\subset M}$ that can occur as graphs of subfactors, and the problem of investigating what kind of quantum symmetries can "act" on a specific factor and what values of the index can occur, etc.

Many outstanding results by a large number of people have followed. Vaughan was much involved in these developments, notably finding the best way to characterize the objects $\mathcal{G}_{N\subset M}$ arising as standard invariants of subfactors as a two-dimensional diagrammatic structure of tangles called a *planar algebra* (1999). Vaughan developed planar algebras as a tool to efficiently carry out intricate computations with the standard invariant of a subfactor. It allowed for topological arguments in the analysis of subfactors and led to remarkable results in the classification programme of subfactors, including the construction of stunning "exotic" quantized symmetries, captured as planar algebras. These powerful tools were successfully used by Vaughan and some of his former students to classify all such symmetries up to index 5 (1995–2014) ([5]), which was then pushed further up to 5.25. While traditional classification attempts focused on subfactors with small indices, the planar algebra approach shifted the point of view to a generators and relations approach. Thus, singly generated planar algebras and then Yang-Baxter relation planar algebras played a key role, through which important colored variants of the Temperley-Lieb algebras were discovered as the fundamental quantized symmetries associated to intermediate subfactors.

Planar algebras, together with a quest to produce a conformal theory from subfactors, led Vaughan to a study of the Thompson groups as discrete approximations to the diffeomorphism group of the circle, and again to unexpected spin-offs for the theory of knots and links (2015–2020).

More details of all these mathematical developments will be found in a forthcoming issue of the *Bulletin* of the AMS which is dedicated to Vaughan.

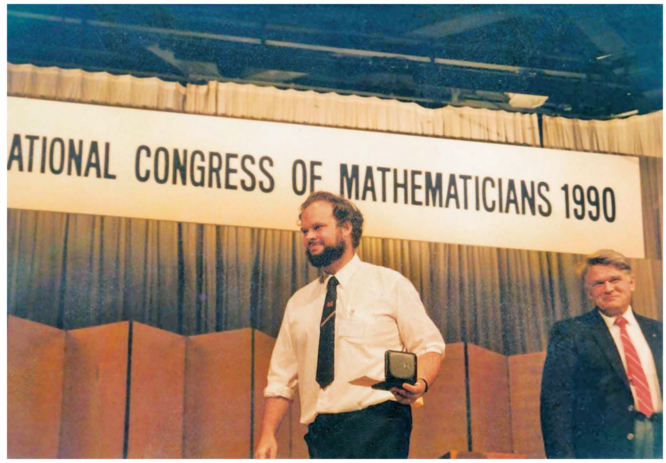

Figure 3. Vaughan receiving the Fields Medal from Ludvig Faddeev at ICM-90 in Kyoto.

Vaughan was awarded the Fields Medal in Kyoto in 1990, and was elected Fellow of the Royal Society in the same year, Honorary Fellow of the Royal Society of New Zealand Te Apārangi in 1991, member of the American Academy of Arts and Sciences in 1993 and of the US National Academy of Sciences in 1999, and foreign member of national learned academies in Australia, Denmark, Norway, and Wales. He received the Onsager Medal in 2000 from the Norwegian University of Science and Technology. In 2002, he was made a Distinguished Companion of the NZ Order of Merit DCNZM, later redesignated Knight Companion KNZM. The same year, he became an honorary member of the London Mathematical Society. The Jones Medal of the Royal Society of New Zealand Te Apārangi is named in his honor.

Vaughan had a strong commitment of service to the community. In 1994, he was the principal founder and Director of the New Zealand Mathematical Research Institute, leading summer schools and workshops in New Zealand each January. He was Vice President of the American Mathematical Society in 2004–2006, and Vice President of the International Mathematical Union in 2014–2018.

Vaughan had an unusual and very personal style of doing research. He would freely share ideas about a project and discuss initial speculations and possible applications and concrete steps for how one might obtain the final result. Vaughan was a warm and gregarious individual whose humor and humility led to the generosity and openness from which the mathematical community drew substantial benefit. Vaughan had over 30 graduate students and was a sought-after doctoral advisor. His presence at mathematical events was stimulating for all who came in contact with him. He will be dearly missed by his family and his many friends all over the world.

The following contributions are roughly in the order in which the authors met and interacted with Vaughan.

MEMORIAL TRIBUTE

Thomas Schücker

Vaughan has left us. I have no words to express our sorrow. Rather let me talk about joys, of which Vaughan had many, and he shared them with generosity. To choose from this plenitude, I use a leitmotiv that was dear to him: find harmony between distant persons or ideas or phenomena.

The physical distance between two persons can hardly be larger than the one between Auckland and Geneva. But Vaughan was not lost in translation in 1974. He quickly picked up French and made lifetime friends in Geneva. Also this is where he got to know Wendy and where Bethany was born.

Vaughan was exceptionally gifted in bringing together friends from different worlds and sparking harmony among them. The harmony between music and mathematics is not well understood, but it is broadly appreciated. In 1974 or 75, I was doing my homework as an undergraduate in the library of the Ecole de Physique in Geneva. Vaughan's office was at the extreme opposite side of the Ecole, some 70m away. Suddenly I heard him singing a passage of Berlioz' Te Deum and I went to see him. He had just finished the proof of a theorem in axiomatic quantum mechanics.

After one year Vaughan lost his office at the Ecole de Physique and was appointed assistant and PhD student of André Haefliger at the Section de Mathématiques at the other side of the Arve river. We followed André's lectures with enthusiasm, Vaughan gave the exercises and was co-examinator, I was among the students being examined. Frequently I dropped by his office and was always bewildered by the disorder on his desk. For me, it was clear that with this sense of order, Vaughan would never be a good mathematician. André kept his office in a somewhat better order, but his desk was overloaded with piles of papers. Once when Vaughan and I came to his office, we found it tidied up and Vaughan complimented André on his order. Andŕe replied with a smile: "Yes, but now I don't find anything anymore."

It is impossible to agree on a distance between von Neumann algebras and knot theory and this made his discovery of a harmony between them only more amazing. Others of Vaughan's longlasting passions came from more hands-on links: between knot theory and knot tying; between the depicted hydrodynamic paradox combined with vector addition and a far more time-consuming activity, sailing.

The harmony between physics and mathematics is well understood. It consists of local isomorphisms, for

Figure 4. Vaughan and Thomas Schücker at Bodega Head, Bodega Bay, CA, in 2006.

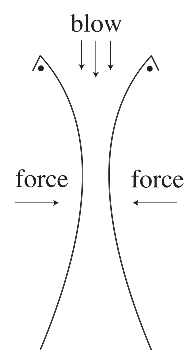

Figure 5. Illustration of the hydrodynamic paradox.

example between the trajectories of planets, satellites, comets, apples, ... and solutions of a second order ordinary differential equation due to Newton. We both cherish some of the above local isomorphisms and I owe my career to Vaughan.

It was sheer pleasure to witness Vaughan sharing joy with Wendy, to see his joys spill over to Bethany, Ian, and Alice, to their grandchildren, to my sons, to his students and colleagues, and to so many of his friends. We all miss him dearly.

Pierre de la Harpe

Vaughan Jones was a PhD student in Geneva from 1974 to 1980. He arrived in the Physics Department to work with Josef-Maria Jauch, on the mathematical foundations of quantum mechanics, but Jauch died suddenly one week after their first meeting; Jones worked in close contact with

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two colleagues of Jauch, Constantin Piron and Jean-Pierre Eckmann. He was also interested in mathematics and followed lecture courses by André Haefliger, on foliations and on de Rham theory. In the beginning of 1976, he moved to the Mathematics Department, and became a PhD student of Haefliger. He completed a thesis which provides a classification of the actions of finite groups on the hyperfinite factor of type II_1 (the case of finite cyclic groups is due to Connes). Alain Connes was an unofficial and decisive codirector. This is exemplary in several respects. Haefliger had a strong influence on Jones, but he is certainly not a specialist of von Neumann algebras; this shows the originality of the student and the open mind of the advisor.

During the first three months of his stay in Switzerland, Jones spent all of his time on a language course to learn French. "It was the coolest three months in my life" (les mois les plus sympa de ma vie), he said. He became almost bilingual. A good lesson for all students and colleagues who believe that English is enough to settle down anywhere in the academic world, forgetting that thinking together with other people and in another language is both possible and worthwhile.

Though he was a very hard worker and a first-class mathematician, Vaughan was a marvelous companion in every way. His informal style of working was encouraging exchanges of all kinds, and he was always ready to share his ideas. He was singing for many years as an excellent baritone in the University Choir. He would practice chamber music with his advisor and the Choir's director Chen Liang-Sheng. He would practice his favourite sports with many friends. He was behaving with modesty, humor, and respect for everybody, but he could also look glorious: during his thesis defense, he was dressed in a superb smoking jacket, as a king addressing his people and four modest jury members, who were André Haefliger, Alain Connes, Michel Kervaire, and me.

Colleagues and friends had the chance to see him back in Geneva on many occasions, for sabbatical periods, for shorter visits, and for memorable talks. After one on subfactors, he went to the café downstairs and talked with a PhD student who told him about braid groups, of which Artin presentations have something in common with relations written by Jones for algebras associated to subfactors; this would be an important ingredient of his work on the Jones polynomial. By an incredible coincidence, a few days after his discovery of the polynomial link invariant in June 1984, Vaughan was travelling from New York to Bucharest and stayed over in Geneva for a couple of days. So his first talk on this amazing discovery was again at the University of Geneva.

His last visit in Geneva was in May 2019, and the next one was planned for November 2020. Each meeting with him was a gift.

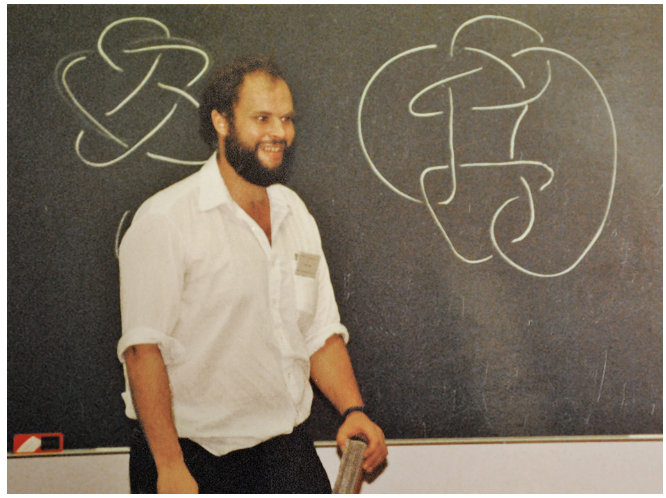

Figure 6. Vaughan at the Newton Institute, Cambridge 1993, program on *Low Dimensional Topology and Quantum Field Theory with 11-crossing Conway knot which has trivial Alexander polynomial and non-trivial Jones polynomial.*

Alain Connes

I met Vaughan Jones while he was working on his thesis under André Haefliger. Vaughan was fascinated by the fact that all infinite-dimensional subfactors N of the hyperfinite factor R are isomorphic to R itself and he undertook the study of their relative position inside R. After defining the index $[R : N]$ of N as the Murray-von Neumann dimension of R as an N-module, his first breakthrough result was that while the index can take any value larger than 4, the smaller values form the sequence $4\cos^2(\pi/n)$ for integers $n \geq 3$. In his proof, he used a "basic construction" adapting to subfactors the technique of iteration of crossed products (which was pivotal in understanding the periodic automorphisms of R) and discovered a profound link with the growing sequence of Hecke algebras associated with Coxeter systems of symmetric groups. This led him to discover new traces on the colimit of these Hecke algebras. His amazing breakthrough in the early 80s was to use these new traces together with the braid group description of the knots in three space to obtain a new polynomial invariant of knots! His discovery has triggered remarkable developments in both domains of knot theory, solving old open questions, as well as in operator algebras where the general theory of subfactors of finite index has become a central topic of research as a generalization of the concept of a finite group action. Vaughan Jones was a truly original thinker and his work will remain forever as a testimony of his genius.

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MEMORIAL TRIBUTE

Masamichi Takesaki

These are some of my personal memories of Sir Vaughan F. R. Jones, following the devastating news of his death. He was too young to be taken from us.

I first became aware of Vaughan as a rising young superstar in the fall of 1979 when I was asked to referee his thesis, "Actions of finite groups on the hyperfinite type II_1 factor," for the *Memoirs* of the AMS. His extraordinary talent became apparent as soon as I started to examine his thesis; of course there was no question that it should be accepted for publication. With the permission of the editor, I contacted Vaughan and suggested that he apply for a UCLA Hedrick Assistant Professorship, a prestigious junior position. His application was successful, and he took up the position in the fall of 1980.

After having studied his thesis, I was impatient to begin working with him and so I invited him to visit UCLA for a few weeks in the spring of 1980. When I went to the Los Angeles International Airport to pick him up, he came out of the restricted area at the airport with a huge smile on his face. As soon as we started to drive toward his accommodation, he began to talk about his ideas about subfactors and said that there was a forbidden zone below index 2 and a continuous zone above index 4, but that the interval between 2 and 4 was unknown and looked mysterious. I was deeply shocked by his claim and thought that he was on the verge of a huge discovery of an entirely new field within operator algebras. I couldn't find the words to express my surprise and stopped my car on the road side to calm myself. I told him "You are on the edge of an entirely new field of mathematics. You shouldn't miss this wonderful discovery even though it could be a tiny fact after your completion of the new theory."

I kept encouraging him to pursue his investigation of the allowed values of the indices. Unfortunately, there wasn't much progress during his stay at UCLA for the academic year of 1980/81. Vaughan moved to a junior academic position at the University of Pennsylvania for the 1981/82 academic year, allowing him to rejoin his wife, Wendy (who had been studying at Princeton while he was at UCLA), and to benefit from the strong operator algebra culture at Penn. Shortly after he moved to Penn, he determined the possible values of indices, $4\cos^2(\pi/n)$, $n \geq 3$ or in the continuous half line $[4,\infty]$, the now-famous range of Jones indices.

I invited him again to UCLA for a week in the spring of 1982, at which time he showed us an early version of his famous theory of the Jones index. His discovery attracted the attention of the algebraists and number theorists at UCLA, so his talk was packed with algebraists and functional analysts, a rare occurrence. When he explained about the Jones tower and the relations among the Jones projections, Bob Steinberg pointed out to him that these relations were similar to those for Hecke algebras. His talk left a huge impression on everybody in the Mathematics Department, UCLA.

Soon after Vaughan realized that his family of projections in the tower gave rise to new representations of the braid group. The appearance of the braid relations in subfactor theory already suggested a close relationship with knot theory, and Vaughan continued to work on this relationship. Then in the spring of 1984, he realized that he had discovered a new knot invariant, known today as the Jones polynomial.

Fortuitously, the Mathematical Sciences Research Institute (MSRI) had scheduled a year-long focus on low-dimensional topology and operator algebras for the academic year 1984/85. The choice had been deliberate; both areas had been very active in the preceding years and the newly-established MSRI wanted the program to cover as wide a range of mathematics as possible. On hearing of Vaughan's breakthrough, I passed the news on to the MSRI Deputy Director, Professor Calvin Moore; the news left him almost speechless. Vaughan's work had provided a bridge between these seemingly unrelated areas. Consequently a major focus of the 1984/85 program was for the operator algebraists and the low-dimensional topologists to become at least familiar with each other's work. The program was a huge success which, as a member of the program committee, I was extremely pleased to see.

This interaction has continued over the subsequent 35 years, becoming both broader and deeper. The lesson to be learned from this is that the truly exciting advances in mathematics, and probably also in many areas of science, are not predictable in advance, and are very often brought about by talented and committed young researchers. Vaughan and his work exemplify this lesson in the best possible way.

In addition to his mathematical prowess, Vaughan had many other interests at which he excelled, including music and sport. He brought a wonderful "joie de vivre" to everything that he did; I am very sad to have to say "Goodbye, Vaughan" and pray for the peaceful rest of his soul.

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Colin Sutherland

The news of Vaughan's death came as a huge shock to me. The world of mathematics, and in particular New Zealand mathematics, has lost one of its most outstanding

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Figure 7. Vaughan and Sorin Popa at NZMRI Summer Workshop on Operator Algebras, Nelson, NZ, 2001.

practitioners and communicators, and I and my wife have lost an exceptional friend.

Vaughan will be most remembered for his seminal work on index theory for subfactors and the subsequent interactions with knot theory. However, my first contact with him, in the late 1970s, was in relation to classifying actions of finite groups on the hyperfinite II_1 factor up to cocycle conjugacy. He recognized the correct cohomological framework for invariants used earlier by Connes (for cyclic groups), and succeeded in showing that the resulting characteristic invariant was in fact a complete invariant. This work already marked him as an outstanding talent, and provided the basis for subsequent developments by many people (including Vaughan himself), leading to the analogous result for discrete amenable groups acting on injective factors.

Throughout this work, Vaughan was an unstinting source of encouragement and inspiration. He was extremely generous with his time and always willing to contribute ideas and constructive appraisal; this generosity extended to all of the many fields to which he made critical contributions.

It also extended to his support for mathematics in New Zealand. From 1994 on, Vaughan codirected an annual Summer School designed for researchers and postgraduate students in New Zealand. A different topic and venue would be chosen each year, and Vaughan would use his influence to attract leading international researchers to deliver a series of expository lectures culminating in a discussion of at least some current problems. The main requirements were that the lectures be (mostly) accessible to nonspecialists, and that the venue have sufficient wind and water for Vaughan to be able to indulge his passion for kitesurfing. The Summer Schools have been very successful, in no small part because of Vaughan's guiding hand.

Vaughan was highly accomplished in many areas outside mathematics and kitesurfing. He was an avid sportsman, passionately supporting the All Blacks, and relishing golf and squash; the sight of him in full cry on a squash court was more than a little intimidating. He was a skilled musician, a qualified barista, and cooked wonderful potatoes au gratin; and in his younger days, he had an almost legendary capacity for beer. He could be, and often was, the "life of the party," but he also had a quiet contemplative side as I saw during a long road trip we took in 2014 from Lake Te Anau to Nelson. But most of all, Vaughan was somebody who enjoyed life, and who stimulated those in contact with him to greater enjoyment and achievement in their own lives. I shall miss his mathematical energy and inspiration, his zest for living, and his wonderful company as a dinner companion. Vale, Vaughan.

Klaus Schmidt

I first met Vaughan Jones at a workshop on Ergodic Theory in March 1980 at Les Plans-sur-Bex near Geneva. At the time, Vaughan had only just completed his PhD, but it was obvious to everyone that he was an exceptionally gifted mathematician with a wonderfully open and friendly personality to match. When I organized a symposium on von Neumann Algebras and Ergodic Theory at the Warwick Mathematics Institute in 1980/81 with leading experts from both fields, I also invited Vaughan. He gave a lecture on the indices of subfactors and mentioned that there was a gap in the range of possible indices—an early glimpse of his remarkable Index Theory. In 1986/87 David Evans organized another symposium at Warwick, this time focused on Operator Algebras, at which Vaughan presented his remarkable discoveries on subfactors, the Jones polynomial, and spectacular applications in the theory of operator algebras and low-dimensional topology.

At Warwick, Vaughan and I returned to conversations started in 1980 at Les Plans about asymptotically invariant sequences of Borel sets for type II_1 ergodic equivalence relations (Alain Connes and Benjamin Weiss subsequently coined the term *strongly ergodic* for such relations without asymptotically invariant sequences). Around 1986, I realized that such a relation R fails to be strongly ergodic

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if and only if it has a hyperfinite equivalence relation S as a "quotient." When I told Vaughan about this he immediately asked when one could split off this quotient S as a direct summand of R or, equivalently, when R is orbit equivalent to a product relation $R \times S$ with S hyperfinite. This is the case if and only if the full group of R contains nontrivial *asymptotically central* sequences. The analogous problem for factors, which had been solved by Dusa McDuff in 1970 in the type II_1 case and by Alain Connes in 1976 in the general case, leads to formally quite similar answers, but neither version of this problem implies the other one.

In 1994, I returned to Austria—to the Erwin Schrödinger Institute (ESI) in Vienna. In 1996 Vaughan agreed to become a member of the Scientific Advisory Board for five years, and he remained a friend, supporter, and regular visitor to the Institute for many years. He gave a keynote lecture on the occasion of the 10th anniversary of the ESI in 2003, and a prestigious Kurt Gödel Lecture at the Austrian Academy of Science, followed by an informal, amusing, and somewhat subversive lecture to interested school kids on the challenges and rewards of being a mathematician. In appreciation of his lectures, the Academy presented him with a ticket to a performance of *Der Rosenkavalier* at the Vienna State Opera which Vaughan, as a passionate music lover, enjoyed very much.

When the ESI was unexpectedly threatened with imminent closure in 2010, Vaughan unhesitatingly joined the protests of the international scientific community and wrote a strong letter of support which certainly helped to convince the Austrian authorities to agree to a rescue of the Institute.

The news of Vaughan's death came as a deep shock to me. In addition to my admiration for his mathematical creativity, I will always remember his warm and generous personality, his (and, of course, Wendy's) hospitality and kindness on numerous occasions, and the boundless and infectious enthusiasm with which he pursued his wide range of interests. In many ways he was larger than life, but at the same time he was very modest and unassuming. Like his many friends all over the world, I miss him very much.

Roberto Longo

In my eyes, Vaughan Jones was primarily an artist, a visionary mathematician who produced theorems like a great painter can paint a picture. Capable of incredible connections with the simplicity of the greats.

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I cannot but immediately say that Vaughan was a great friend to those who had the fortune to know him, a person of humanity and extraordinary understanding.

A few years ago, in my talk at a conference in China on the occasion of his 60th birthday, I said that I was probably the person in that room who had known Vaughan first. Vaughan intervened to confirm and said: "even before my wife!" I clearly remember how we met, fresh from graduation at a conference in Marseille: I was looking for a restaurant at the port and Vaughan approached me with empathy asking for a preprint!

The famous *Jones index* and *Jones polynomial* came only a few years after. In the mid-80s I received an envelope from Vaughan with his articles and greetings. I immediately had the feeling that there was some relationship to my work on split inclusions of von Neumann algebras, although the contexts were disjoint. I put Vaughan's index paper in my briefcase and used to read it while traveling.

Meanwhile, there was a growing interest in low-dimensional quantum phenomena and Vaughan envisaged the relevance of the subject. In particular, J. Fröhlich was looking for a conceptual understanding of the exotic, anyon statistics of low-dimensional quantum fields. In January '88, as a final comment of a seminar talk in Rome, J. Roberts said that, in low dimension, the Doplicher, Haag, and Roberts (DHR) statistics was given by a braid group representation and no analysis existed on that.

I well remember the day in April '88 that I was reading the basic DHR paper on superselection sectors and a light bulb came on: *the DHR statistical dimension is the square root of the Jones index!* Jones's index so intrinsically entered into crucial interplay with Quantum Field Theory. Such an exciting and absorbing period for me, taking also into account that my first son was to be born in a couple of weeks!

Concerning conformal QFT nets of von Neumann algebras, the Jones index is quite a powerful tool and allowed a first classification, a lucky adventure that I experienced together with Y. Kawahigashi, with contributions by F. Xu, D. Evans, and S. Carpi among others.

In 2016, the University of Rome Tor Vergata awarded Vaughan Jones the Laurea Honoris Causa in Mathematics. Vaughan gave, with his usual empathy, a simple and effective speech[1] to give non-mathematicians an idea of what mathematics is.

Last year we were co-organisers of the Operator Algebras and Applications program at the Simons Center in Stony Brook. The discussions with Vaughan there are a great memory for me, and I think for all the participants.

[1] *His speech was later published by Italian newspaper* Sole 24 Ore *and can be found on my web page* `http://www.mat.uniroma2.it/longo/`.

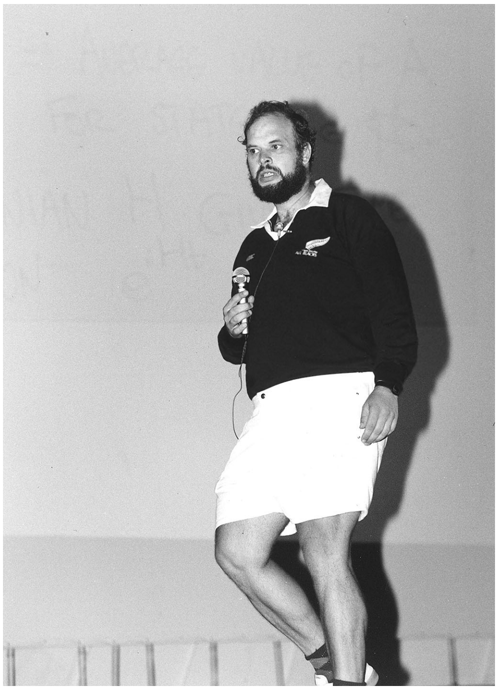

Figure 8. Vaughan during his plenary address at ICM Kyoto 1990.

Georges Skandalis

For more than 40 years, I enjoyed meeting Vaughan at a variety of occasions. I had a great time with him, both in math discussions and at a more personal level. It feels so painful to recall these happy moments knowing that they will not happen again.

We met in many places under various circumstances. I especially remember two very fortunate ones. I was incredibly lucky to be in Geneva on June 14, 1984, when Vaughan gave his first-ever talk on his polynomial for knots. I remember very precisely when unexpected objects like Hecke algebras, braid groups, Markov conditions, and knots appeared in the world of subfactors. I also recall the excitement of the audience with many questions and a vigorous discussion. Vaughan was jet-lagged, and obviously exhausted, but also so proud and happy. I am sure that we all shared the same feelings, and were so excited to be there and to see such deep and beautiful mathematics being revealed by our friend.

Later Vaughan happened to be in Paris when I defended my thèse d'Etat in 1986. A rugby scrum developed during the party with five or six people against Vaughan—who, as a true New Zealander and All Black, pushed us all the way to the other end of the room.

We frequently met over the years. Recently, we could not drink as many beers as before, but it was always a great pleasure to be with Vaughan. He has always been for me, even more than a great mathematician, a great guy, very positive and lively, very much sensitive of others. I miss you, Vaughan.

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Fred Goodman

One of the privileges of being a mathematician is that one gets to know some truly extraordinary people. After my PhD, I was able to spend several years as a postdoc at the University of Pennsylvania. My colleagues—in operator algebras and associated areas alone—included Richard Kadison, Robert Powers, Michael Fell, David Shale, Jonathan Rosenberg, and Joachim Cuntz, as well as Antony Wassermann and Hans Wenzl, who were graduate students at the time. But for me, the great bit of luck was that Vaughan Jones joined the faculty during 1982–1984, and these were exactly his *anni mirabiles* in which he created the subject of (von Neumann) subfactors with his first remarkable paper on the subject, and then started the subject of quantum invariants in topology with his discovery of the Jones link invariant. A particular bit of luck for me was that Vaughan was living in Princeton and commuting to Philadelphia, so to reduce his commuting time, it was often convenient for him to stay overnight with me. Two consequences were that Vaughan became a lifelong friend and Leffe is still my favorite beer. Vaughan's technique for producing his first theorem on discrete values of index for subfactors, as well as his link invariant, involved producing from a given subfactor $N_0 \subseteq N_1$ a whole tower of subfactors $N_i \subseteq N_{i+1}, i \geq 0$, and an associated sequence of projections $e_i : L^2(N_{i+1}) \to L^2(N_i)$. At some point, Vaughan and I discussed doing the same thing in a finite-dimensional setting and connecting this with an old theorem of Kronecker about integer matrices of small norm. We intended to write a small expository article about this. A year passed before we did anything about this, and in the meanwhile Pierre de la Harpe joined the project, and the article expanded into a book-length exposition. This was the first book on the subject of subfactors, and it seems quite a few people have found it useful, even though the subject quickly developed far beyond what was treated in the text. Vaughan had a great spirit and was a generous friend. His sudden death is a huge loss for mathematics and for his many friends.

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Arthur Jaffe

It came as an unbelievable shock to learn that Vaughan had died at the young age of 67 this past September [2020]. Not only was Vaughan a mathematical visionary, a powerful analyst, the spiritual leader of a large group of researchers who followed in his mathematical footsteps, and an extraordinary gentleman, but he was also a good personal friend. I find it incomprehensible that he will not renew the familiar Kiwi twang resonating in my brain. As far as I know, Vaughan's last public seminar was a beautiful talk on July 21, 2020, on *Applied von Neumann Algebras* that one can view on YouTube.

I first met Vaughan in the 1980s, when I recall our talking about his interest in knot theory and the structure of proteins. Vaughan was always interested in reaching out in new directions in an interesting and unusual way, including his recent interest in the Thompson group, as a possible road to an interesting quantum field theory. Vaughan often asked me, "What is ..." and two familiar themes were "reflection positivity" and "a Quon." While they are both closely related to Vaughan's ideas in planar algebra, we never got to the end of those discussions.

In 2015, Vaughan's student Zhengwei Liu came to Harvard, and afterward Vaughan became a more frequent visitor. Occasionally, Vaughan stayed at the Mary Prentiss Inn, close to where I live, and on those occasions, we would often meet in the morning for coffee at Simon's across the street. There Vaughan would joke with the barista that he could draw a better swan—and once Vaughan was challenged, only to demonstrate his superior skill with steamed milk. Vaughan's love of coffee also led to an interesting birthday present: two coffee cups from New Zealand. Coffee was not Vaughan's only non-mathematical interest; he had many, including a love for surfing that led last year to an ear infection.

Images from our most recent meeting when we were both visiting Tsinghua University, as well as other interactions, invoke happy thoughts that help transcend our loss.

John Ratcliffe

I first met Vaughan Jones in June of 1982, in Geneva, Switzerland, when I was visiting Michel Kervaire at the University of Geneva. I spent four wonderful weeks in Geneva with my wife, Susan, and our four-year old daughter Kim. During our visit, Kervaire and Vaughan asked me to serve on the PhD committee of Oscar Pino Ortiz, who was their joint student and Vaughan's first PhD student. Oscar's thesis was on relative cohomology of groups, which was a topic in Vaughan's PhD thesis, and also in my PhD thesis.

Figure 9. Vaughan at the *Subfactors in Sydney 2019* workshop wearing Bodega Bay Institute of Mathematics shirt.

During my visit, Vaughan gave a pair of remarkable talks that left a lasting impression on me. The first was on his celebrated index theorem for a subfactor of a type II factor. I remember being struck by the beauty of the theorem. The second was on representations of Artin's braid group in the algebras arising from his tower of factors construction. That the braid group should have such a representation was surprising to me, and I felt that Vaughan had discovered something important. I remember having lunch with Vaughan and Nathan Habegger, a fellow student of Haefliger, after the talk. It was a beautiful sunny day, and we were eating al fresco. Vaughan was in good spirits, as usual, and I remember telling him that his braid group representation was very interesting and important.

I was delighted when Vaughan joined our department at Vanderbilt in 2011. Vaughan became not only a valued colleague, but also a good friend to many members of our department.

When I found out that Vaughan played tennis, I invited him to join my doubles group. Occasionally, Wendy, who played tennis in college, also played in our game. Vaughan liked to team up with me to play the first set. Most of our sets were close, but we won more often than not. Sometime towards the end of our last tennis season, we were feeling our ages, and we were down 0-5 in games. Vaughan turned to me and said, "John, I think we can play better than this," and I said, "I think we can too." We proceeded to win six games straight. Vaughan was serving the next game, and I turned to him and said, "We have to win this; otherwise, we will not be able to brag about our comeback." All Vaughan did was give me a big grin, and then he served out the set. I will always remember Vaughan as kind and generous, and a winner.

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Gus Lehrer

I first met Vaughan in Sydney in about 1983. Being a representation theorist, with primary interests in algebra and topology, I initially did not think that we had much in common mathematically. This turned out to be completely wrong, and from our first meeting until his tragic early death, we had an ongoing mathematical dialogue, which centred around Hecke algebras of various sorts, as well as invariant theoretic themes. In the early days we played squash, and after taking numerous beatings at my hands, Vaughan, ever resourceful, persuaded me to play racquetball instead. Hitting the ball to the roof of a court, or at the junction of the front wall with the floor, were techniques unknown in squash, and the result was that the odds were very much evened up.

In later years, when Vaughan took up golf quite seriously, he extracted more than adequate revenge for his early beatings on the squash court. His always smiling and humorous demeanour did nothing to conceal his pleasure at showing me what a frustrating game golf can be. My most recent game with him was in Cambridge in 2018, at a very small course, during the INI conference there. I remember it fondly.

I made numerous visits to see Vaughan in Berkeley, and have very fond memories of drinking his expertly made cappucinos, particularly at his beach house in Bodega Bay. Although I did not share his passion for windsurfing and kiteboarding, I often went surfing while he was doing those activities. Vaughan also visited me and my wife Nanna in Sydney several times, sometimes with Wendy. I recall a recent Thanksgiving, where Vaughan roasted an enormous turkey at our home, which fed a huge party.

Vaughan loved our beach house at Werri, south of Sydney, because there was a very reliable wind which sprang up every afternoon. I recall one occasion when Vaughan went kiteboarding at Werri. I took a walk towards Kiama on the headland, and could see him very well when he started. However the wind was good and he was swept eastwards at great speed. He soon disappeared over the horizon towards San Francisco, and I could only hope for his safe return, as we had a workshop in Canberra the following week.

It is arguable that in the long run, Vaughan Jones's most significant contribution to mathematics may turn out to be the formalism of planar algebras. Their recursive nature, and their already well-documented links to many branches of geometry, algebra, number theory, and topology, as well as the growing literature concerning them, augur well for their future.

My own work with John Graham on the affine Temperley-Lieb category interacted with that of Vaughan through his work on the annular Temperley-Lieb algebra. When I first met Vaughan in the early 1980s he was very dismissive of nonsemisimple modules and algebras, because in the world of operator algebras, one always has nondegenerate traces, which imply semisimplicity. However, when I explained the above results to him (in a series of talks that he organised), and said that admission of nonsemisimplicity is an intrinsic part of cellular theory, I believe that he changed his mind, and realised that venturing into the nonsemisimple world can reveal new things about the semisimple one.

Vaughan's passing was a great shock for all who knew him. He was a larger than life figure, full of goodness. He was always a pleasure to be with. He will be greatly missed.

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Joan Birman

I want to remember here personal moments that we shared after Vaughan's discovery of his polynomial. One of them occurred two years later, as Vaughan was writing his 1986 paper for the *Annals of Mathematics*. We were talking on the telephone, Vaughan was in Berkeley and I was at home in my office in New Rochelle, New York, three hours ahead of him in time. He was telling me about calculations he had done to determine the braid index of the prime knots up to 10 crossings, using the then-standard knot tables in Dale Rolfsen's book. He did them because he wondered whether his polynomial detected braid index. I asked him how he found the time and patience to do so many calculations, and do them so carefully. He told me that his infant son Ian suffered from colic, and that he, Vaughan, would be up at night doing his best to comfort Ian. So Vaughan would walk with Ian on his shoulders, back and forth across the room, in the middle of the night, manipulating pictures of the knots in Dale's tables and jotting down his results as he did so. Those tables are in his 1986 *Annals* paper, for posterity.

In a different direction, after the discovery of the polynomial, Vaughan and I both attended a number of conferences and gatherings of mathematicians: possibly during the academic year 1984–5, when (by a lucky accident) there just happened to be "special years" in both Operator Algebras and Low Dimensional Topology at MSRI. But it could also have been when we were both at the symposium on "BRAIDS" that was held in 1986 at Santa Cruz, CA, where the grass was not cut until students had trampled it down to show (by usage) where they wanted the paths to be, an act that was in progress when we were there;

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Figure 10. Sir John Meurig Thomas, David Evans, and Vaughan with portrait of Peter Guthrie Tait, Peterhouse, Cambridge, 2017 during the Newton Institute program on *Operator Algebras: Subfactors and Applications*.

or maybe it was at Luminy, in the wind-eroded mountains overlooking the Mediterranean. In each of those places, at around 9:30 or 10:00 p.m., after a long day of intense mathematics, Vaughan was very likely to be found at the center of an admiring group of friends who were also math colleagues. He was relaxed, gay and happy, enjoying a beer with his many math friends, holding forth in fine form. He was happy. After a while the conversation became quieter, we were all tired and knew there would be more mathematics to come tomorrow. So, one or two at a time, we drifted off, to get a night's sleep and get ready for another intense day of mathematics. That's the way I want to remember my old and very dear friend Vaughan Jones.

Edward Witten

In his work on knot theory, Vaughan Jones gave an inspiring example of openness to unexpected opportunities in research. His starting point was a rather surprising theorem about subfactors of von Neumann algebras of small index, a subject that sounds miles away from braids, links, and knots. But the rich algebraic structure that arose in Jones's analysis of the subfactors turned out to be closely related to the Artin relations of the braid group and the Temperley-Lieb algebra of two-dimensional statistical mechanics. Those relationships led Jones to a remarkable series of discoveries about knots and links and their relationships to other areas of mathematics and mathematical physics, including von Neumann algebras, Hecke algebras, and statistical mechanics, leading ultimately to new insights about quantum groups, quantum field theory, and more.

From the beginning, Jones's work was related to physics in a variety of ways. At a basic level, operator algebras are important in quantum mechanics and quantum field theory; this relationship was part of von Neumann's motivation for studying the algebras that bear his name. More specifically, from the beginning, Jones's work had a variety of relationships to statistical mechanics. This began with the appearance of the Temperley-Lieb algebra in Jones's work. An additional step was taken by Louis Kauffman, who discovered a state summation model for the Jones polynomial, relating it directly to the Potts model on any finite planar graph. Jones made a number of important further discoveries relating knot theory to statistical mechanics, using the Yang-Baxter equation and constructing vertex models in which the Jones polynomial and many of its generalizations are directly computed as statistical sums on finite lattices associated with knot diagrams.

Up to a certain point, there was a simple explanation for why integrable statistical mechanics could be connected to knot theory: the Yang-Baxter equation, which underlies the important models of integrable statistical mechanics, has a striking analogy to the Reidemeister moves of knot theory. The similarity between the two is obvious from Figure 11, but so are the differences; the Yang-Baxter relation is purely two-dimensional, with no notion of whether one line is crossing "over" or "under" another, while the analogous Reidemeister move is essentially three dimensional. Moreover, in knot theory there are other Reidemeister moves, though the one depicted in the figure is arguably the main one. I do not know whether anyone before the discovery of the Jones polynomial took seriously the analogy between the Yang-Baxter equation and the corresponding Reidemeister move of knot theory, but even if one did take this analogy seriously, it would have been far from obvious how or why the structures that appear in solving the Yang-Baxter equation would be useful in constructing knot invariants.

Somewhat analogously, although knots have an obvious similarity to braids, there are also important differences. A knot can be constructed by taking the "trace" of

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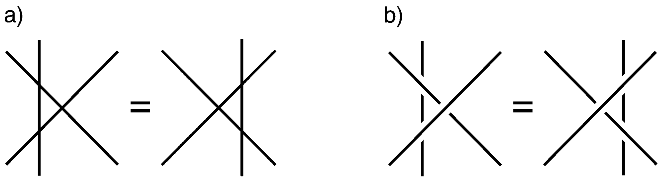

Figure 11. a) The Yang-Baxter relation, which appears in two-dimensional mathematical physics in several different ways. In one interpretation, the lines are trajectories of particles in a two-dimensional spacetime and the equation says that it does not matter in which order the particles collide. b) The corresponding Reidemeister move of knot theory. Note that here the lines pass "over" or "under" each other in three dimensions, while in the Yang-Baxter case, the particles actually collide in two dimensions.

a braid, but because the same knot can arise from many different braids, it was difficult to exploit the relation between knots and braids. Jones, however, discovered some novel representations of the braid group from which knot invariants could be constructed. The defining property of the Jones representations of the braid group is that each of the usual braid group generators has only two distinct eigenvalues in each of the Jones representations. From a suitable linear combination of the traces in these representations, Jones constructed his knot polynomial.

An essentially new relation of the Jones representations of the braid group with theoretical physics was discovered in 1988 by Akihiro Tsuchiya and Yukihiro Kanie. They showed that the Jones representations can be defined as the monodromy representations of the "conformal blocks" of a certain two-dimensional conformal field theory (the WZW model). Related to this, the Jones representations can be interpreted in terms of the nonabelian statistics of "anyons" in two-dimensional materials (that is, in thin films), with possible applications to quantum computing. For the Jones polynomial of a knot or link, the analogous statement is that it can be defined via a quantum gauge theory in three dimensions. In that theory, the classical action is the Chern-Simons invariant of a connection. Because of its multiple connections to physics, the Jones polynomial is probably much better known among physicists than anything in knot theory other than the Gauss linking number.

Michael Freedman

My earliest memory of Vaughan Jones is of a bear of a man thrashing a bike up Strawberry Canyon in the heat to lecture at MSRI. I would end up spending 30 years of my own life in one way or another woven around the Jones polynomial; for me it became a way of understanding

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Figure 12. Dietmar Bisch, Mike Freedman, and Vaughan at Vanderbilt, NCGOA Spring Institute 2019 & 34th Shanks Lecture.

quantum computation. But in the early years I did not know I needed to know about it. For Vaughan, the Jones polynomial was obviously a manifestation of physics, he knew it was related to integrable models of statistical mechanics. I also think he had no doubt of its quantum mechanical significance since it arose from the study of the trace on von Neumann algebras which he understood in terms of expectation values of observables, having been trained in a tradition that valued the perspective of physics.

By the late 80s, there was almost an embarrassment of riches as far as connections between the Jones polynomial and physics was concerned. It is now a famous story that in 1988 Michael Atiyah exercised his "administrative talent" in realizing: A) the Jones polynomial must in some way be organizing the expectation values of a quantum field theory, B) he, himself, did not have the knowledge at his finger tips to assemble the picture, and C) his colleague Ed Witten did, and was the person for the job. Witten's explanation of the Jones polynomial put it near the center of my life for the next 30 years.

In the fall of '88, I was visiting Cliff Taubes at Harvard and Cliff asked me to join the seminar he and Raoul Bott were running on Witten's mimeographed notes deriving the Jones polynomial from SU(2)-Chern-Simons (CS) gauge theory. Up to then, I had had no contact with physics and was highly suspicious. I had not heard of Witten and his notes immediately looked wrong to me. He was saying that to compute an evaluation of the V (the Jones polynomial) you should compute the trace of a holonomy integral. To my mind this appeared to be utter nonsense since holonomy varies smoothly as one varies the knot, and appeared perfectly well defined if a crossing change occurred, so it looked to me that if Witten's integral was well defined at all, it would be a homotopy, not an isotopy invariant, not a knot invariant. I wrote a letter to Witten complaining about this and other points that were obvious to him and opaque to me. He answered that the integration measure was not on functions but on distributions and I should not try to sample the distribution at the same point twice (as at an intersection point), and that the innocent trick of completing the square in the exponent (and thus introducing the Green's function) made the risky divergence near a crossing point manifest. I was converted; a physicist, by understanding what completing the square means, could see farther into topology than I. Vaughan of course was already there; for him Witten's work was a new and rich connection, but to him not unexpected.

From then on I was in Vaughan's circle, and I liked it. I wanted to use his polynomial to build a quantum computer, and Vaughan found my enthusiasm for cryogenic engineering amusing. With the enthusiasm of the converted I wanted to manipulate topological phases (manifesting the Jones polynomial via the CS connection) not just study them mathematically. Vaughan was always fun to talk to about these things (and everything else). I remember early on, before I had absorbed the concept of a "low energy effective field theory," the only place I knew of in physics where an SU(2) gauge group came up was the SU(2) associated with the weak force connected to nuclear decay. It was 1990 and the Cold War seemed to be ending, Vaughan and I had a great time over a beer wondering if we could procure a few nuclear weapons, maybe the whole arsenal, to carry out quantum computation. We didn't see exactly how to extract the Jones polynomial from a nuclear explosion, but we were young, and nothing seemed impossible. Of course, the thinking now is to work at 20 millikelvin, but in 1990 our thoughts ran toward the other end of the spectrum.

Being in Vaughan's circle was not all about math, science, and wild plans for the future. Vaughan led a group of vigorous young colleagues and graduate students. He was soon to happen upon his greatest invention—the perfect scientific conference: a gathering of friends (that requirement excluded no one as Vaughan's openness and generosity made him a friend to all) willing to work intensely from 8:15 to 12:30, play hard all afternoon, and start in again 6:30–9:00 p.m. The venues varied: Maui, Maui, New Zealand, Maui, New Zealand, even Bodega Bay. Vaughan was very coordinated and made first windsurfing and then kiteboarding look easy (though the latter always looked and was quite dangerous). Vaughan would always have mountains of gear on hand and the time and generosity to teach anyone, the slightest bit willing, how to do these things. I was able to learn (barely) to windsurf from him. Kiting was another matter. It is mathematically impossible to start. If you are going along it seems just possible to keep going (for a short while) but all planets must align in perfect syzygy to launch. I do not know how he did it. Vaughan liked the wind strong; 30 knots was worth canceling a lecture for. I've seen him on a broad reach in those conditions, his large, helmeted body flicked airward by some additional gust and then gracefully, or brutally, returned to the water. I've seen both. Vaughan was often hurt a bit but usually seemed more amused than concerned. I recall in July 2019 some strap or guy cut his right eye (ball), and it affected his vision to the extent that he lectured with that eye closed. Some sequence of arm injuries had reduced the range of motion in his elbows to a number of degrees < 90. I enjoy similar minor debilities from a lifetime of rock climbing so Vaughan and I would compare. His elbows were slightly worse. I think our unspoken conclusion was that this was a mark of a life well spent.

Vaughan had notebooks full of his kiting adventures. As he would travel alone around New Zealand (mostly the North Island I believe) in his van packed with kite gear, he would note the wind direction and strength, currents, time of day, surf state, and other relevant factors. The map at the front was annotated with a blizzard of pencil and ink markings documenting days of shear brave fun and narrow escapes. He would show up for his talks sun burned and caked in salt still wearing beach shorts—a dress code he would describe as the "classic tradition." He had an admirable air of fatalism which I had known before only among fine rock climbers, who knew they could and would execute competently no matter the circumstance and that, not the outcome, was the point. He certainly could have been blown to Fiji, but was not.

Last December (2019) I was at a conference with Vaughan in Auckland. There was a formal moment, a prize, I think, being awarded, likely to Witten, for he was in attendance. I don't recall the prize, but Witten gave a superb lecture on the LIGO project, a remarkable 10 orders of magnitude in energy below his own specialty. Vaughan introduced him and wore black pants. I was concerned, but on closer inspection they turned out to be only sweatpants.

Hugh Woodin

Anyone who knew Vaughan also knew of his passion for windsurfing and later for kitesurfing. In his office was a large map of the world covered with pins, marking locations where he had windsurfed or kitesurfed (with pins color coded for the two activities).

Vaughan was very methodical. The transition from windsurfing to kitesurfing involved a trainer kite with

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Figure 13. Hugh Woodin and Vaughan in Chile, March 2010.

which he had many sessions over several months. Finally the day came when he felt ready to attempt the real thing. I was his duly appointed second and off we went to Alameda beach. The wind was too light when we arrived even for kitesurfing, but after a few hours it strengthened. I in the role of second, helped launch the kite and off Vaughan went. Afterwards, Vaughan was absolutely euphoric. He rarely windsurfed again having been completely ensnared. Watching him fly across the water, always so graceful, was an enduring pleasure for me over the years.

Vaughan and I each travelled extensively and we would conspire to meet whenever possible. This led to climbing Etna in Sicily, seeing an eclipse in Aruba, dog sledding in Switzerland, and exploring the Atacama Desert in Chile. Among my fondest memories is driving all over New Zealand scouting locations for the regular January meetings of NZMRI.

When I was growing up, we would use the phrase as far away as Timbuktu to indicate an impossibly remote place. In New Zealand when Vaughan was growing up, the phrase was as far away as the Chatham (Rekohu) Islands. And so it came to pass that Vaughan decided we have to go there. This was our last exotic trip together. Vaughan, Dietmar, and I boarded a plane in Wellington in January of 2017. The first indication of the remoteness of our destination was that the interior of the plane was full of cargo. Most of the seats had been removed to make room.

The Rekohu Islands lie some 500 miles east of Christchurch. The longitude is actually 176W even though the time zone in Rekohu is 45 minutes ahead of the rest of New Zealand, instead of just over 23 hours behind. Vaughan earned pins on several exceedingly remote beaches, and the trip was a remarkable adventure. Just last fall we were in the process of choosing our next destination.

I have been spending the COVID-19 pandemic in Arizona. My brothers and I have a ranch now in the middle of Ironwood Forest National Monument. The monument surrounds Ragged Top Mountain, a spectacular volcanic core with cliffs of several hundred feet. When I turned 50 (some years ago) I decided to keep count of my ascents and Vaughan was with me on hike number one. In the days since he so suddenly passed away, I continue my isolation taking comfort in my regular ascents.

This summer has been unusually hot. Even delaying my hike until late afternoon, I would frequently be heading out when it was 110F. But there is usually a comforting wind and now when hiking up the mountain, I can hear Vaughan's voice as if lofted by that wind, and as clearly as if he was standing beside me; "All this wind and no water, what a waste!" I smile.

Nicolai Reshetikhin

This article is more personal than professional. It is a personal tribute to a good friend and a fantastic, inspiring mathematician.

Here I will not attempt to give an overview of all his work and how it influenced the research in subfactors, operator algebras, invariants of knots, and in mathematical physics. I will mostly focus on events of 1987–1989 when he played a very important role in the direction my life took after 1989.

The first time I heard the name Vaughan Jones was back in 1986 when I was still in Leningrad, USSR. There was no email, and the latest developments were delivered by mail or in person. Vladimir Turaev visited Geneva and returned in December 1986 inspired by conversations with Vaughan Jones; so Turaev gave a talk and that is how I learned about the Jones polynomial.

Soon after came Vladimir's and my work on knot polynomials and R-matrices, and my work with Kirillov, on q-6j symbols, world of shadows, etc. All this was deeply inspired by Vaughan's paper from 1985 and from unpublished notes from his talk at Atiyah's seminar about how to use solutions to the Yang-Baxter equations to produce invariants of knots. In these notes, Vaughan explained that one can construct an invariant of knots from an R-matrix that satisfies the Yang-Baxter equation and an extra property that gives the Markov trace for the braid group representation obtained from R.

Turaev and I started to work together and came up with a comprehensive construction of invariants of knots from quasitriangular Hopf algebras and in particular from

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quantized universal enveloping algebras for simple Lie algebras. At this time, the preprint "Quantum field theory and the Jones polynomial" by Ed Witten came by mail and Turaev immediately asked me if I knew a solution of the Yang-Baxter equation that would satisfy Kirby moves and thus would not only be an invariant of links, but also an invariant of 3-manifolds. We did find such a solution, and thus a 3-manifold invariant. Later Vaughan encouraged us to construct a full TQFT, that is, invariants of 3-manifolds with boundary.

I finally met Vaughan in March 1989 and conversations with him were very inspiring and informal. My personal memories from that time include a spectacular walk at point Pinole, a dinner at Vaughan's house where his family patiently dealt with my rudimentary English and many other small cheerful events. Vaughan was a generous person. Later, when I just came to Berkeley as a faculty member, and I did not know how to drive, he offered to let me use his stick-shift Honda Civic, and I was driving it for a whole year. Miraculously, the car and I survived.

Many deep developments involving algebraic structures related to invariants of knots and 3-manifolds happened during that time. One of them was the result by Moore and Seiberg connecting conformal field theory and tensor categories. Another important development was the connection to operator algebras, axiomatic quantum field theory, and conformal field theory. In our papers with Turaev and in many later developments, representation theory rooted in works of Drinfeld on quantum groups played an important role. Looking back, one can see how a large portion of these developments was inspired by Vaughan's paper from 1985 and by his other works.

Perhaps one of the most remarkable developments after Jones polynomials and after an explosion of results that followed was Khovanov's construction of a two-variable extension of the Jones polynomial of knots. The key idea in this construction is the discovery of a homology theory (Khovanov homology) which can be regarded as a categorification of the skein algebra related to the Jones polynomial. This new polynomial is a weighted Euler characteristic of it. This direction became one of the most active and important areas of research at the interface of topology and mathematical physics.

As it was mentioned at the beginning, these notes are personal and very brief. It takes more than two columns to describe in detail the areas of mathematics that were influenced by Vaughan's work. Looking back, I can certainly say that reading Vaughan's paper from 1985 and getting to know him personally were among the few important events that changed my life.

Figure 14. Ed Frenkel and Vaughan at Tarifa, Spain, in 2003.

Berkeley Students

Arnaud Brothier, Pinhas Grossman, Michael Hartglass, Zeph Landau, David Penneys, Emily Peters, Stephen Sawin, Noah Snyder, James Tener, and Dylan Thurston

Vaughan advised more than 30 PhD students over the course of his career, including more than 20 at Berkeley. He cared about the progress and development of all of his students, and had a flexible approach to advising that he adapted to meet the needs of each individual student. He formed lasting relationships with many of his students, and his guidance and friendship continued throughout our careers. The community of students that he built and cultivated provided a supporting foundation and sense of belonging as we began our own academic journeys.

For many of us, starting a PhD with one of the world's great mathematicians was an intimidating experience. Vaughan would typically start us off by posing a relatively straightforward, doable problem as a confidence-builder. Afterwards, we would move on to more open-ended projects. He was always extremely generous with his time and ideas, listening patiently as we discussed our thoughts and obstacles, and giving valuable feedback (which sometimes took years to truly appreciate). As we progressed in our research, he would take an increasingly hands-off approach, encouraging us to explore independently. There were two complementary sides to Vaughan as an advisor: the mentor/friend with whom you could have a casual conversation about mathematics (or any topic), and the professor looking at you very seriously when you were at the blackboard, asking you to justify every detail.

We learned a lot from what Vaughan taught us, and perhaps even more from the example he set. Vaughan approached mathematics the way he approached many of his other passions, such as kitesurfing and golf, with playful and unreserved joy. He chose research topics which he found fun and exciting, and had the ability to find an interesting seed of an idea and nurture it (for years!) and see what it would grow into.

He had a contagious optimism which carried us through the tougher stretches of our PhD years. He reassured us that being stuck or confused is a normal part of the mathematical process, and that arriving at mutually contradictory conclusions is exciting because it means you're about to learn something!

Vaughan built a wonderful mathematical community at Berkeley. The focal point of this community was the Friday afternoon *Subfactor Seminar*, followed by *Beer and Pizza*. The weekly seminar typically featured talks by Vaughan's students, postdocs, and visitors. Vaughan believed that a seminar where no one asked questions was a disaster; he led the way by asking persistent, probing questions, which often resulted in two-hour talks. This led to a lively environment where students could ask questions and admit to being confused, since Vaughan also did those things. At the same time, speaking in Vaughan's seminar could be a mildly terrifying experience, since we never knew ahead of time which particular detail of our talk Vaughan would be grilling us about.

Afterwards, we would meet for food and drink, and more informal mathematical discussions. In later years, the venue shifted from La Val's Pizza on the North side to Raleigh's Pub on Telegraph Avenue, but Vaughan still liked to call it *Beer and Pizza*, despite the absence of pizza. Every week Vaughan would pick up the check and stare at it as if he was doing a complicated calculation, before inevitably saying we each owed $5. Vaughan's subfactor seminar was also very welcoming to people coming from neighboring fields, many of whom had no background in operator algebras, but learned about them because it was such a wonderful seminar and community.

Another critical component of Vaughan's community was the retreats he organized, usually to Maui, Lake Tahoe, or his home in Bodega Bay. These retreats featured Vaughan's kitesurf hard, do math hard attitude, where half the day would consist of kitesurfing, windsurfing, or skiing, while the other half consisted of math talks. One of Vaughan's favorite ways to keep the quality of the talks high, even when people were physically tired, was to designate an official question asker to make sure that the audience stayed engaged. Many important collaborations started at these retreats.

Vaughan was tremendously generous as a host, and would often open his house to his students even if he couldn't make it there himself.

Figure 15. Vaughan kiteboarding, Maui, 2009 and 2018.

At these retreats, and at many conferences and research programs around the world, Vaughan was a dynamic presence. He took an interest in catching up with the lives and careers of his students and colleagues. Many people were eager to talk to him about their research questions and goals, and he often took the opportunity of introducing people whom he thought might share similar or complementary interests, which led to numerous collaborations.

The community that Vaughan built continues to thrive, but it will not be the same without him.

Rodney Baxter

In the 1980s, Vaughan had realised that the Temperley-Lieb algebra, which arises in the Potts model in equilibrium statistical mechanics, is related to the problem of evaluating the Jones polynomial of a knot. I was working on various lattice models in statistical mechanics, including the Potts model, so we had an interest in common. We were also both antipodeans (he by origin, myself by adoption).

Our paths probably first crossed when Vaughan talked on this work during a conference in the Maths School at the Australian National University in Canberra in July 1989. Then in 1990 I was invited (with Barry McCoy—a fellow statistical mechanic) to the International Congress of Mathematicians in Kyoto, Japan, where Vaughan was given the Fields Medal for his work.

Our paths have crossed a number of times since: in particular he invited me to speak at one of the summer schools he organised in New Zealand, this one being at the village of Tologa Bay in the North Island in January 1996. The village was small, with a population which was largely Maori. It was a memorable meeting and I was impressed

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Figure 16. *Topics in von Neumann algebras* workshop, Banff, 2006. 1 Jesse Peterson, 2 Alin Ciuperca, 3 Zhuang Niu, 4 Dima Shlyakhtenko, 5 Ken Dykema, 6 Narutaka Ozawa, 7 Luis Santiago Moreno, 8 Uffe Haagerup, 9 Juliana Erlijman, 10 Eric Rowell, 11 Imre Tuba, 12 David Evans, 13 Adrian Ioana, 14 Masaki Izumi, 15 Yasuyuki Kawahigashi, 16 Vaughan Jones, 17 Terry Gannon, 18 George Elliott, 19 Hans Wenzl, 20 Dietmar Bisch, 21 Magdalena Musat, 22 Roman Sasyk, 23 Maria Grazia Viola, 24 Roberto Longo, 25 Martin Argerami, 26 Nicolas Monod, 27 Kenley Jung, 28 Shamindra Ghosh, 29 Holly Hauschild Mosley, 30 Stefaan Vaes, 31 Sorin Popa, 32 Fred Goodman, 33 V.S. Sunder, 34 Pinhas Grossman, 35 Pedro Massey.

by Vaughan's organization: many of the attendees were accommodated in tents in the campground, while I shared a two-bedroom unit in the motel with Peter Goddard and his wife from Cambridge, England. Ruth Lawrence was another participant. Meals were prepared and served in the community hall, and in the evening we would congregate in the hotel.

We enjoyed one another's company, and he supported my application for a visiting Miller Professorship in 1999 in the Math Department at Berkley. One day I was sitting in Vaughan's office while he was away on sabbatical when a friend of his delivered a case of Belgian beer. I was a little apprehensive, as I knew that many American universities had a strict ban on the presence of alcohol on campus—certainly in an academic's office. I had visions of suddenly being surrounded by armed police and dragged off to a campus dungeon. I had the sense not to advertise the presence of the beer and was pleased when Vaughan returned and assumed responsibility.

He would sometimes visit the ANU when he was south of the equator. In January 2016 he gave a seminar in the Maths School, and we had an enjoyable dinner with him afterwards. It came as a shock when I learnt that he had died. He, with his unconventionality and exuberant humour, will be greatly missed.

M. Izumi and Y. Kawahigashi

We both met Vaughan as graduate students, and this early experience motivated us to study subfactors and stay at Berkeley as Miller Fellows. Later we both invited him to Japan.

Though I, Yasu, had met Vaughan several times during my PhD study at UCLA, it was during his visit to Japan for ICM-90 at Kyoto when I first had real mathematical interactions with him. He gave a colloquium talk in Tokyo just before ICM-90 and everyone in Japan treated him as the next Fields medalist. It was during this short stay in Tokyo, that I decided to start working on his subfactor theory. We met on the morning of the opening ceremony of ICM-90 at Kyoto and I was amazed by his clothes, since I had never seen him dress formally before. He had a formal tie, but immediately after the ceremony, he said "I don't need this anymore," and took it off. He showed me his Fields medal, and it is my only experience of physically touching a Fields medal. A few days later at a party for him at Kyoto, he played with the medal by throwing it up in the air. I applied for a Miller Research Fellowship because of this interaction with him and with his work. He kindly supported my application, and I stayed at Berkeley from 1991 to 1992. It was my most fruitful year both mathematically and socially. I was supposed to meet him at Berkeley again in March 2020, but I was forced to cancel my flight on the day of the departure after checking in, due to the coronavirus closing MSRI. I missed this last chance to see him. His legacy will be remembered eternally by all of us.

I, Masaki, was a second year graduate student at RIMS when my advisor Araki invited Vaughan to Kyoto just before ICM-90. I remember I got excited when I attended Vaughan's seminar talk because I had just learned the background material of his talk. Fortunately, after ICM-90, I obtained a result on subfactors and wrote my master thesis on it, which led to my stay at Berkeley in 1994–1996 as a Miller Fellow. Like Yasu, I had a wonderful time in Berkeley, and learnt so many things, which became the basis of my career. Vaughan was always so generous to me—giving an encouraging comment every time, literally every time, I gave a talk, which never changed until the end. Vaughan visited Kyoto in 2015 and in 2016. When I asked him to give a talk for undergraduate students, I again witnessed his generosity: he was really enjoying the interaction despite the students' insufficient language skills. It is well known that Vaughan was a good rugby player back in

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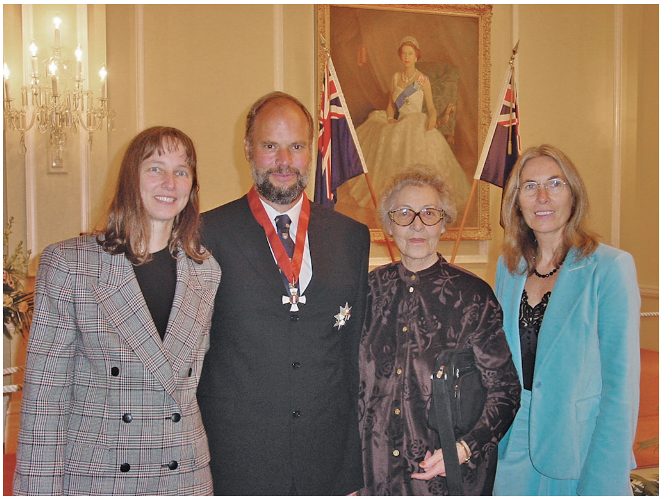

Figure 17. Wendy, Vaughan, mother Joan, and sister Tessa when receiving his knighthood in 2002 at Government House, Auckland.

New Zealand. I used to play rugby too, and we sometimes talked about rugby, but not very much until 2015 when Japan beat South Africa in the Rugby World Cup in England. Right after the match, I received email from Vaughan "Congratulations Masaki!!!!" Since then it became our custom to exchange email after the match whenever either New Zealand or Japan played in the RWC. It continued until last year when the RWC took place in Japan. My wife and I took pictures of the quarterfinal between New Zealand and Ireland, and sent them to Vaughan from the Tokyo stadium, never expecting that it would be our last email exchange.

Marston Conder and Gaven Martin

Vaughan Jones was a quintessential 'Kiwi'—the colloquial (and much loved) term for a New Zealander since the first world war. Attributes that Kiwis most identify with are a can-do attitude, down-to-earth and easy-going character, fairness, and a love of the outdoors. Those who knew Vaughan will recognise each of these traits. Kiwis usually don't consider themselves as sophisticated, risk-takers, or worldly! Vaughan wearing a rugby jersey for his Fields Medal lecture 1990 in Kyoto is typical. There are not too many images of Vaughan in anything other than informal attire—a Hawaiian shirt, shorts, and 'jandals' (of which he was a connoisseur).

Vaughan enjoyed the outdoors, adventure and sports, playing rugby in his youth, and squash and tennis for many years. Visitors to Berkeley would often be regaled at Caffe Strada with his many "near" victories at squash against the likes of Hugh Woodin and Steve Evans. He was also a committed golfer and a passionate sailor; first windsurfing, and more recently kiteboarding. It was no coincidence that our annual NZMRI conference was always held near water.

He was adventurous in other ways too. A friend recalls that when Vaughan moved from Los Angeles to the east coast of the US, in his 20s, he bought a large motorcycle (a Kawasaki 1000) and rode it across the country. He'd never owned a motorbike before, but he decided he wanted to learn how to ride one so, as is the Kiwi way, you get on and go. Vaughan was also committed to coffee-making and took a course in barista skills at the local community college. (The CEO was an old high school mate—Vaughan had friends and relatives everywhere.) His qualification from the City and Guilds of London as a trained barista was framed and hung on his kitchen wall—the only certificate hung in his house!

Vaughan spent most of his career in the United States, but he gave time generously to the University of Auckland (Distinguished Alumni Professor) and to New Zealand mathematics, offering courses and lectures to encourage and mentor younger mathematicians. And so we come to the New Zealand Mathematics Research Institute (NZMRI). His New Zealand connection was important and he used his Auckland appointment to fulfill his dream of developing New Zealand's mathematical talent by organising annual mathematical workshops. The original philosophy, which remains in place, was that high-quality mathematicians from all over the world who were also excellent expositors would present a series of lectures, giving students and local mathematicians the opportunity to learn of developments. It was also important that the workshops were held near a beach, with ample time for informal discussions ... and windsurfing. Over time these workshops have indeed attracted many of the world's top researchers.

The first workshop took place at Huia in December 1994, on a shoestring budget, and others proceeded from there in January (mid-summer), at various locations around the country. The more remote and undeveloped the venue, the more it appealed to Vaughan. One meeting in 1997 at Tolaga Bay was held in an old wool shed with corrugated iron walls, which was cold and draughty, even before the biggest cyclone in recent memory came through. Plenty of wind though! At these workshops he taught many participants the rudiments of windsurfing, which sometimes resulted in visits to the local hospital by slow learners. He was often caught looking out a window to gauge the wind and its likely direction, and looking sad on an otherwise beautiful summer day when there was no wind.

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Figure 18. Vaughan's Latte art.

At our fourth workshop it was decided to establish a more formal and long-term structure, and the NZMRI was created and became a charitable trust. We were the founding codirectors, together with Rod Downey and David Gauld, and with Vaughan as our chairperson. Our main role is to arrange themes and organisers for the workshops, but in the early days we also had the challenge of raising the money. With support from the Marsden Fund and nine years of "Centres of Research Excellence" funding for the associated New Zealand Institute of Mathematics and its Applications, and also with generous support from Vaughan himself, the NZMRI has reached a position where it now seems financially secure. Typically we have 50 participants with many students. Some invited speakers have developed strong ties with local mathematicians, resulting in exchanges of research visits, and others have even taken up permanent positions in New Zealand. Some of our students have their PhDs supervised by researchers they met at these workshops.

Vaughan's impact on mathematics in New Zealand has been immeasurable, significantly raising the standard and connections of mathematicians here, and it will continue to be so.

The loss of Vaughan to our community is a tragedy, but his valuable legacy remains and will thrive.

Dimitri Shlyakhtenko

I first met Vaughan (or more precisely saw him from a distance) when I arrived as a graduate student in Berkeley in 1993. Vaughan was radiating mathematics and was surrounded by a large crowd of graduate students and visitors. I was primarily interested in Voiculescu's free probability theory and was concentrating on coursework and passing exams, so despite my interest in subfactors any real meeting with Vaughan would have to wait another year or so.

After passing my qual in fall 1994 (and turning 19) it was time for me to select my PhD committee. My advisor Dan Voiculescu suggested I ask Vaughan to be on it. It was almost the end of the semester and Vaughan was besieged by undergraduate students from his calculus class. They really loved him (one said that he understands now why Vaughan got his Fields Medal—he *really* knows calculus) so his office hours were always full. I had a form for him to sign and after a few unsuccessful attempts at catching him, I learned that he was about to hold extra office hours on some day. I came there to find the door closed. I knocked and a somewhat irritated Vaughan opened the door, looked me over (determining that I am from his calculus class) and told me that, *as he explained in class*, his extra office hours were cancelled. I mumbled something about a very quick question, and he said it was all right to ask. I handed him the PhD committee form and explained what I wanted. Vaughan changed completely and we ended up having a half-hour conversation about what I am thinking of working on. Vaughan was very encouraging and nice, which is how I always remember him.

Around that time I started regularly attending his subfactor seminar, and the famous beer and pizza that followed it in the evening. Both were incredible. I learned a lot about subfactors, as well as how much beer can someone drink and still be able to derive the formula for the cubic equation on the back of a napkin. It was fascinating to see Vaughan at work during these seminars. He had an amazing mind and wonderful ways of looking at things. At around that time, he was developing a "pictorial" approach to subfactor theory, later to be known as planar algebras. It was unbelievably elegant and beautiful.

I remained in contact with Vaughan after I finished my PhD and moved to UCLA in 1998. Vaughan got interested in connections between his "subfactor pictures" and random matrix theory and free probability. We talked about it once in a while, but without making any real headway. It was not until Alice Guionnet spent a year at Berkeley in 2006–7 as a Miller Professor that the three of us managed to make some real progress. After many conversations with Alice, Vaughan came up with a way of interpreting the semicircle law pictorially as a trace on an algebra of diagrams; and all together we were able to show that the pictorial picture holds true for any planar algebra. As a consequence, we were able to reprove a foundational result of Popa showing that every planar algebra (satisfying some positivity axioms) arises from a subfactor.

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Vaughan was an amazing mathematician and an amazing person. He was one of those rare people that would find something nice to say in a difficult situation, and one whose advice you could always trust. His personality was able to gracefully combine so many seemingly irreconcilable talents, from mathematics to music, to windsurfing and kiteboarding, to barista art. He left a tremendous legacy, both in terms of his mathematical work, and a generation of mathematicians he helped to raise. He will be terribly missed.

Alice Guionnet

I heard about Vaughan long before I met him. His exceptional work preceded him as well as the rumor that he gave his talks dressed as Indiana Jones. While this was not entirely true, Vaughan did wear a beautiful New Zealand hat at the first talk I attended at Luminy in 2004, and his knowledge in knots certainly surpassed Indiana's. It was only the following year that I got to know Vaughan better, when I went to Berkeley with my family. Over the following nine months, not only did he offer to embark in a mathematical project, but he introduced my family to his wife Wendy and their kids. They invited us to their beautiful summer house in Bodega Bay, and we enjoyed our first American Thanksgiving in their company. Later, I participated in conferences in Maui and New Zealand that Vaughan initiated. There, I saw that working hard in paradise is possible. Meeting Vaughan was not only discovering a new field of mathematics but sharing a way of life.

When I met Vaughan I was working on the relation between random matrices and the enumeration of maps. Vaughan boldly suggested to use this relation in subfactors theory. With the help of Dima Shlyakhtenko, not only did we use random matrices to construct towers of factors for any possible index but, somewhat reciprocally, we used subfactor theory, more precisely Vaughan's construction of a planar algebra of a graph, to construct matrix models for the Potts models on random graphs, a construction which had not been foreseen in physics. Working with Vaughan was an amazing and unique experience. We would meet once or twice a week in one of his favorite coffee shops. To be honest, at the beginning I could hardly understand a word of what Vaughan was saying. But very kindly, week after week, he explained to me the basics of subfactor theory, drawing many planar algebras on the shop's paper napkins. I think my main contribution to the project was to keep repeating that probability theory would be quite boring if it was restricted to Dirac measures, and similarly Vaughan should consider more fancy states, like those in random matrix theory. One day, Vaughan could incorporate this little brick into his theory and the big picture emerged.

When I think about Vaughan, I remember him preparing the ropes of his kitesurf, explaining in the wind some beautiful mathematics and looking at us with his incredibly kind smile. I'll miss him badly.

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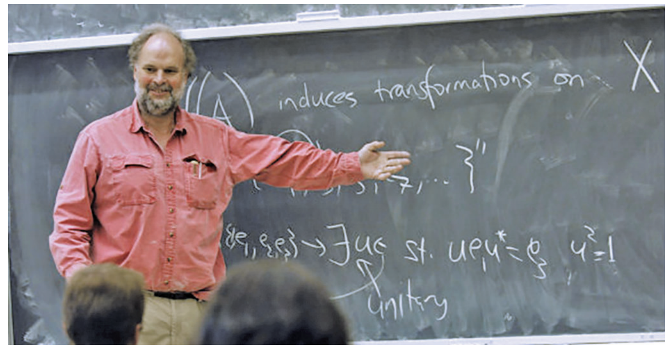

Figure 19. Vaughan teaching at Vanderbilt in 2012.

Vanderbilt Students

Sayan Das, Bin Gui, Corey Jones, Zhengwei Liu, and Yunxiang Ren

Every trip to a wonderland requires a wizard and our journey started with Vaughan. Entering his office for either weekly meetings or private conversations always was a magical experience, and shall remain a great source of inspiration and intellectual stimulation for years to come. His electrifying presence made subfactors come to life. Vaughan had a great style of advising. Vaughan would not let any statement pass without being proved, no matter how easy or difficult the proof was.

He would always ask us about our own motivations for a problem or a proposed solution, till it felt "natural" for us. "Do your own problem" was his refrain; so that we remained motivated. Vaughan had an amazing capacity for distilling out the essence of a problem by considering the easiest nontrivial example. He would lead us by asking seemingly trivial but natural questions, which turned out to reveal deeper concepts and connections the more we kept thinking about them.

Vaughan's taste in mathematics was wide-ranging, and he had an eye for beautiful mathematical ideas. A smile on Vaughan's face would definitely mean that the result and the proof were to his taste, while the dreaded frown would be the prelude for more hard work. To help us cultivate our taste in mathematics, Vaughan generously funded our trips to conferences around the world, including his favorite conference venue—Maui. Vaughan's thirst for knowledge was insatiable, and the greatest demonstrations of this were during the Subfactor Seminar. These

Figure 20. Ian, Wendy, Alice, Bethany, and Vaughan. Christmas 2004 at Squaw Valley Ski Resort, CA.

90-minute Friday seminars were the most intellectually stimulating and beautiful seminars that we have been a part of. Vaughan would continually question the speakers, no matter what the topic was. His questions elucidated the heart of the problem, and the speakers' solution. Vaughan would then buy us dinner. The friendly banter in wide-ranging topics was in sharp contrast to the earlier intellectual debate in almost all respects, save one. Vaughan was still at center stage.

Occasionally Vaughan himself would speak at the Subfactor Seminar. He would start with a simple yet beautiful idea, share memorable anecdotes, and then create a masterpiece at the end of his talk, keeping the audience mesmerized. This was also his style of teaching. "If you are interested in group actions and group representations (which you should be, no matter which field of mathematics you study), then you must be interested in von Neumann algebras." Vaughan's memorable introduction at the start of his von Neumann algebras course had us hooked, and forever since have we traversed these glades in Vaughan's footsteps. Besides mathematics, Vaughan also encouraged us to enjoy life. Under his influence, we became interested in golf and often played with him. He told us that you should always say, "this is my average shot" when you get a good hit. This reflects that he always encouraged us to seize the day and enjoy the beauty of it.

Dear Vaughan, thank you for memories of what has been, and never more will be.

Ian Jones

My older sister Bethany once told our father he was our Gandalf, or our Aslan. It moved him deeply. Like Tolkien's wizard or Lewis's lion, both of whom he introduced us to as children, Dad was not always physically present in our lives, since his career took him all over, but there was a sense of magic whenever he was around, and a mystique to his work. We have lost a father, a grandfather, a husband, a real prop and stay, and a force of nature in our family.

To say nothing of his presence in the wider world. Everywhere he went, it seemed, the doors of friends' and colleagues' homes opened for him and for us. He was our guide and our teacher through countless trips and adventures, who knew everyone everywhere and always had that opaque and inexplicable work to do at every stop.

He even looked the part. With a dark, curly-haired lion's mane of his own, set atop a strong, resilient body and framing a bold face with a ready grin and a twinkle in his eye, Dad projected confidence and character fit to be an adventure tale's mentor, source of energy, and occasional savior.

Our parents have always set an example for us on how to live life: 100% on their own terms, always ready to learn and give something new a try, undeterred by initial setbacks. An idiosyncrasy of Dad's was a steadfast refusal to read instructions for any new gadget or take lessons in any new endeavor. The memory as viewed from the shore of the Berkeley Marina of his first attempt at windsurfing, alone, getting up on his new board, pulling up a sail and falling repeatedly back into the water, still makes me chuckle. He might have learned more quickly by appealing to an expert, but figuring it out on his own was a big part of the fun—he didn't want to learn, he wanted to discover. A peculiar trait for a professional teacher to have, maybe, but a valuable lesson for me at least about being oneself and owning one's failures. The only time I can think of him capitulating and taking lessons was for golf but then, golf is really hard.

More than anything, Dad was simply a lot of fun. From family card games (he was never allowed to keep score, since he always tried to fudge the math in his favor) to camping and ski trips, his wind/kitesurfing obsession, making gourmet espresso drinks, and a slew of other activities and interests, he was so often the cynosure of the family or of any room he was in. Which explains his great gift for storytelling. Bedtime stories, whether a classic like Tolkien or Lewis, or one of a variety he came up with on his own, were a masterful performance. As the middle child,

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I often got two experiences: a wondrous first-time hearing with my older sister listening in; and my turn as observer, sometimes chiding him for telling it differently as he delighted my younger sister, Alice. We were all delighted, and lucky, to see him joyously bring back the same magic a generation later for his young grandsons.

For all his interests, though, what will remain with me most is when he was doing math. I will always remember him sitting on our front patio, puffing on a pipe or cigar, in silent thought or in serious conversation with a colleague he'd invited over, pad of paper and pen in his hands. His collaborative spirit touched so many, and his boundless enthusiasm for life taught us so much, that though his life has ended, his story has not.

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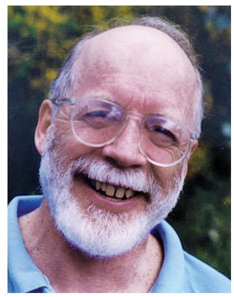
Dietmar Bisch

David E. Evans



Robion Kirby

Sorin Popa

Credits

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