Tricky Math, but *Trippy* Graphics: The Quixotic Search for the "3D Mandelbrot"

Sophia D. Merow

Figure 1. *Geometric Properties* invited visitors to "embark upon a cinematic audio-visual journey where the sheer beauty of mathematics, nature and architecture coincide to inspire introspection and awe."

From March through October 2021, ARTECHOUSE's experiential art center in New York City staged an exhibition perhaps trippy enough to blow even Benoit B. Mandelbrot's mind.

Billed as "an immersive audio-visual journey through fractal dimensions," *Geometric Properties* enveloped visitors in the kaleidoscopic creations of self-styled "fractal artist" Julius Horsthuis.[1] Through thoughtfully choreographed

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[1] *Fractal* math *is not Horsthuis's wheelhouse. "The actual math, including complex numbers, is still completely alien to me," he told* Vice *(see* `https://bit.ly/3Ct7hPn`*), "but I have gained a feel of their effects, and can predict them up to a certain level."*

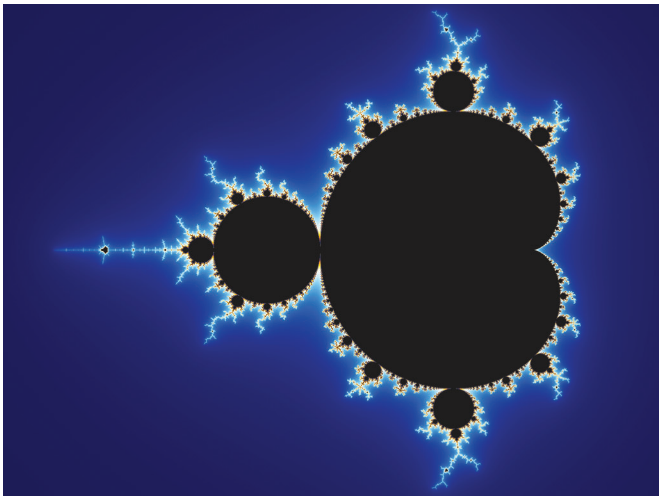

Figure 2. The Mandelbrot set is the set of complex numbers c for which the function $f_c(z) = z^2 + c$ does not diverge when iterated from $z = 0$.

visuals mirroring elements of the natural world, invoking the architecture of sacred manmade spaces, and gesturing toward a sci-fi flavored future—all set to ethereal music, some of it originally composed for planetarium shows—Horsthuis hoped to evoke in audiences feelings of connectedness, calm, and awe (see `https://bit.ly/3zl64b6`). This mesmerizing meditation on the ideas of iteration and self-similarity owed its existence to Horsthuis's vision and ARTECHOUSE's impressive projection capabilities, yes, but also to fractal enthusiasts' dogged quest for an object as infinitely intricate in three dimensions as the iconic Mandelbrot set (see Figure 2) is in two.

Computing power constrained twentieth-century attempts to visualize three-dimensional fractals, but that didn't stop mathematician, computer scientist, and "cyberpunk transreal novelist" (his words) Rudy Rucker from imagining what a three-dimensional version of the Mandelbrot set might look like. In his 1987 science-fiction short story "As Above, So Below" (see `https://bit.ly/3u4t7pG`), Rucker described a root-like object: a big sphere with a dimple in the bottom and bulbs on it, with further warts on the bulbs. Sketching a three-dimensional Mandelbrot set in prose is one thing, of course, and defining one mathematically quite another. Since there is no three-dimensional analog of the complex plane, a canonical three-dimensional Mandelbrot set does not exist.

Rucker knew this, but still he and other seekers wondered: might it be possible to construct in three dimensions something somehow Mandelbrot-esque? One strategy is to view the Mandelbrot set as the result of a geometric process and try to replicate something akin to it in three dimensions. In 2007, fractal fanatic Daniel White proposed to fellow aficionados on the `FractalForums.com` website a formula that replaced complex multiplication's rotation around a circle with rotation around angles φ and θ (see details at `https://bit.ly/3ExURrr`). The resulting

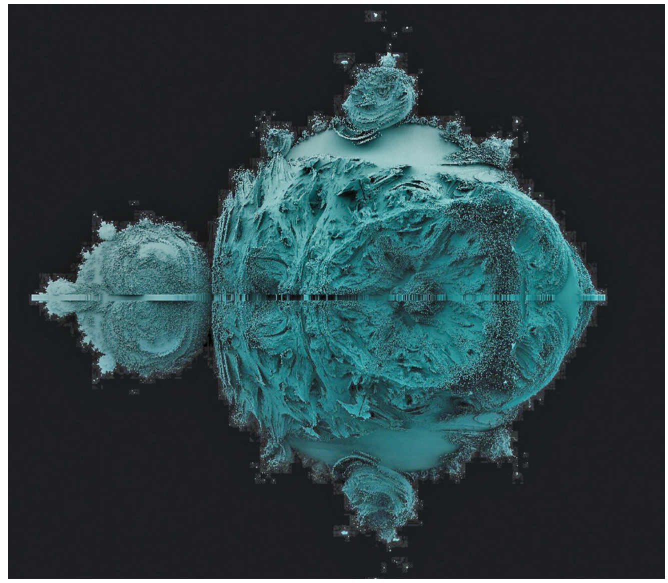

Figure 3. Creator Daniel White deemed the above "almost 3D Mandelbrot-esque."

graphic was, White admitted, "somewhat disappointing" (see Figure 3). Resembling the two-dimensional Mandelbrot spun around an axis of symmetry, it didn't quite exhibit what White was after: "exquisite detail on all axes and zoom levels."

Enter White's fellow FractalForums contributor Paul Nylander. Nylander experimented with raising the z in the Mandelbrot formula to powers other than 2. When 8 seemed to generate the most visually appealing output, the so-called Mandelbulb—an innovation *New Scientist* trumpeted with its November 2009 headline "first 'true' 3D image of famous fractal" (see `https://bit.ly/2XMYsRM`)—was born (see Figure 4). This elaborately lobed orb lent its name to the software Julius Horsthuis uses to render his fractal extravaganzas: Mandelbulb3D (see `https://bit.ly/3kV6SiU`).

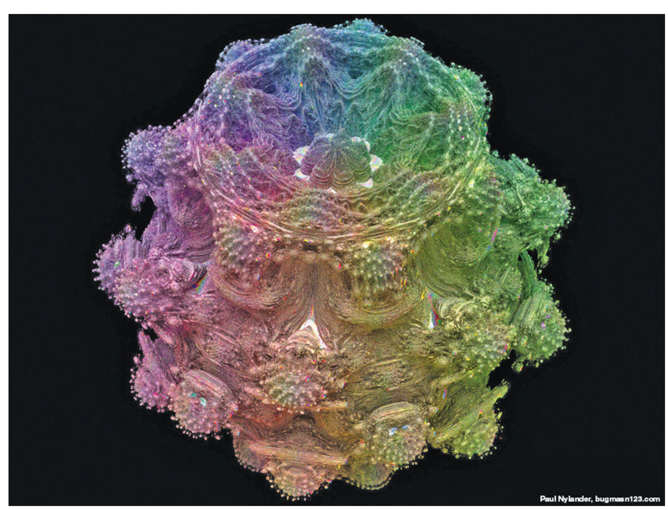

Figure 4. Paul Nylander posted this first rendering of an 8th order Mandelbulb in August 2009.

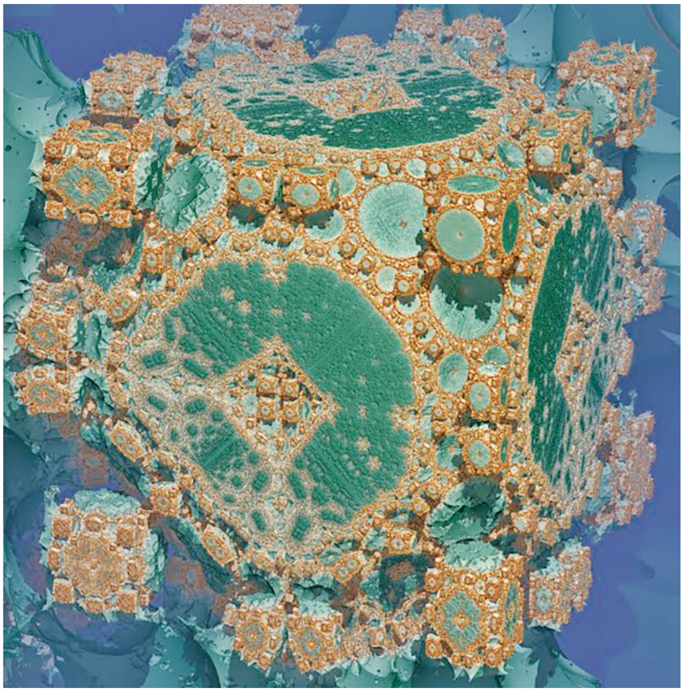

Figure 5. Do an image search for "3d fractal," says Tom Lowe, and the graphics you'll find are almost all Mandelbox variants. Left: a Mandelbox; right: a close-up of one corner of a Mandelbox.

It's a different effort to achieve something approaching Mandelbrot's marvels in three-space, however, that underlies both the current implementation of Mandelbulb3D and much of the other fractal software in use today. Computer scientist Tom Lowe framed his foray into three-dimensional fractal creation—inspired by Daniel White's online account of the Mandelbulb's development (see `https://bit.ly/3zr5yZ1`)—as a search for a formula that was continuous, made of shape-preserving transformations, and neither arbitrary nor overly complicated. In 2010 Lowe devised what he dubbed the Mandelbox (see Figure 5). One selling point: the Mandelbox[2] does not contain the long extruded or squashed shapes (see Figure 6) often cited by connoisseurs as shortcomings of the Mandelbulb and its ilk. Detail is visible everywhere and at all scales. It was a Mandelbox flythrough that first drew Horsthuis down the fractal animation rabbit hole.[3]

While the Mandelbox—which permits a diversity of variants—has birthed ample worlds for artists and hobbyists to explore, it doesn't entirely square with their conception of the Mandelbrot set's three-dimensional peer. The Mandelbox is "characterized by lines and corners more than the organic curves of a theoretical 3D Mandelbrot," says Daniel White. The real 3D Mandelbrot McCoy, he enthuses, "would exhibit incredible shapes and designs, the likes of which can barely be imagined."

Mandelbox creator Lowe agrees that the Holy Grail remains elusive. "We haven't found anything nearly as significant as the Mandelbrot set in 3D," he concedes. Lowe has outlined candidates (see, for example, Figure 7) in his

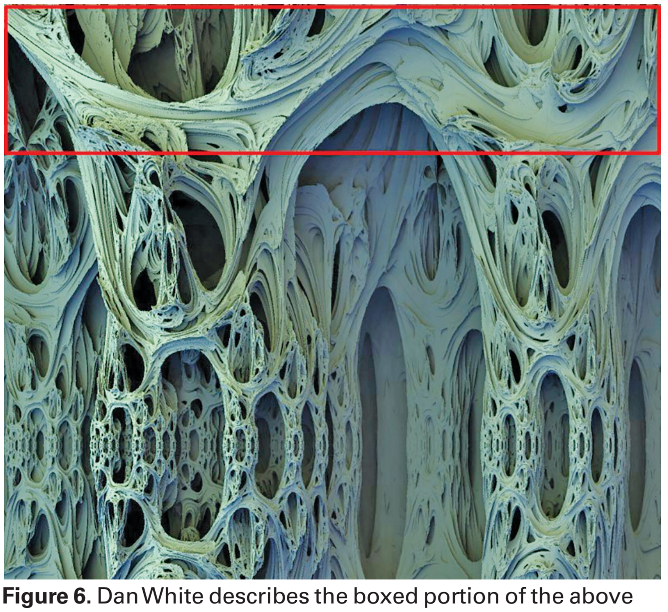

Figure 6. Dan White describes the boxed portion of the above graphic as "like taffy." Rudy Rucker calls such (in his view undesirable) features "taffy/whipped-cream/spun-glass excrescences" (`https://bit.ly/3kossff`).

[2]*The interested reader may find details of the Mandelbox's construction at* `https://bit.ly/3uuK2So` *(Lowe's website) or* `https://bit.ly/3mu2gj9`.

[3]*Of the video that hooked Horsthuis (see* `https://bit.ly/3hphxjB`*), Jesus Diaz wrote in* Gizmodo, *"This is how I imagine a trip into the brain of Hunter S. Thompson after eating a slice of Benoit Mandelbrot's brain, sautéed with a bit of pepper, olive oil, and mescal shaves"* *(*`https://bit.ly/3A2ZmaM`*)*.

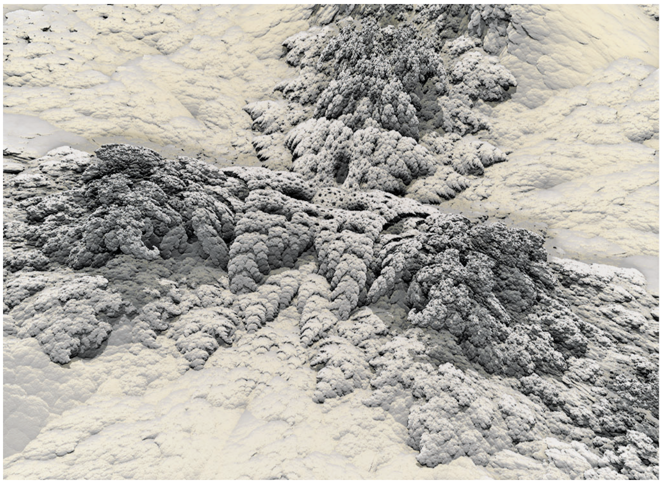

Figure 7. The 3D Möbius multiset shares more properties with the Mandelbrot set than the Mandelbox does.

recent book,[4] but he isn't aware of mathematicians seeking three-dimensional fractals that might rival Mandelbrot. They're deterred, he figures, by the nonexistence of a division algebra in three-space.

"The FractalForums site has been fun," he says, "because people there don't worry too much about such inconvenient facts, and just want to find something amazing looking."

"So 'holy grail' is an apt name," Lowe replies when, adopting language White uses, I liken looking for a three-dimensional Mandelbrot to the hunt for that fabled chalice. "It is a search for something that doesn't exist, but you see some great things along the way."

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Credits

Figure 1 is courtesy of ARTECHOUSE.
Figure 2 was created by Wolfgang Beyer with the program Ultra Fractal 3., CC BY-SA 3.0 (`http://creativecommons.org/licenses/by-sa/3.0/`), via Wikimedia Commons.
Figures 3 and 6 are courtesy of Daniel White.
Figure 4 is courtesy of Paul Nylander.
Figures 5 and 7 are courtesy of Tom Lowe.
Author photo is by Igor Tolkov.

[4]*Thomas Lowe,* Exploring Scale Symmetry, *World Scientific, 2021; see review at* `https://bit.ly/3EAgwzq`.