Irving Kaplansky 1917–2006

Hyman Bass and T. Y. Lam

I liked the algebraic way of looking at things.
I'm additionally fascinated when the algebraic
method is applied to infinite objects.
—*Irving Kaplansky*

Introduction

On June 25, 2006, mathematics lost one of its leading algebraists, Irving Kaplansky. He passed away at age eighty-nine after a long illness, at the home of his son, Steven. Eight months earlier he was still doing mathematics (Diophantine equations). Steven's question, "What are you working on, Dad?" brought only, "It would take too long to explain." From a generous teacher and elegant expositor who inspired generations of students and young researchers, this utterance offers perhaps a hint of the weary burdens of his final illness.

"Kap", as Kaplansky became universally known among friends and colleagues, was not only a brilliant research mathematician and teacher, but also an accomplished musician, a distinguished institutional leader, and a devoted husband and father. The remembrances and tributes that follow are from some of the many colleagues, students, friends, and family who Kap influenced and inspired. We hope that they adequately convey the awesome breadth of Kap's life and work—as a mathematician, teacher, writer, administrator, musician, and father—that we celebrate here.

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All photos in this article, except where otherwise noted, are courtesy of Alex Kaplansky.

Hyman Bass

Some Biographical Vignettes of Kap

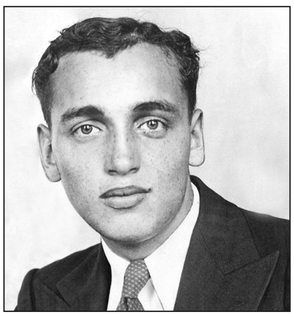
Mathematicians are conventionally measured by the depth and creativity of their contributions to research. On these grounds alone Kaplansky is a towering figure. But another, perhaps comparably important, way to contribute to the advancement of mathematics lies in the building of human capacity, in the formation of productive young researchers, through teaching, mentoring, and written exposition. In this regard, Kaplansky, with an astonishing fifty-five doctoral students (among whom I count myself), and 627 mathematical descendants, has had a singular impact on our field.

Kap was born March 22, 1917, in Toronto, the youngest of four children, shortly after his parents had emigrated to Canada from Poland. His father, having studied to be a rabbi in Poland, worked in Toronto as a tailor. His mother, with little schooling, was enterprising and built up a business, "Health Bread Bakeries", that supported (and employed) the whole family.

Family photo, approximately 1921. Irving Kaplansky (second from left, see red arrow) and his parents and siblings.

Kap showed an early and evolving talent for music, as he himself recounts [1]:

> At age four, I was taken to a Yiddish musical, Die Goldene Kala (The Golden Bride). It was a revelation to me that there could be this kind of entertainment with music. When I came home I sat down and played the show's hit song. So I was rushed off to piano lessons. After 11 years I realized there was no point in continuing; I was not going to be a pianist of any distinction.…I enjoy playing piano to this day.…God intended me to be the perfect accompanist—or better, the perfect rehearsal pianist. I play loud, I play in tune, but I don't play very well.



First Putnam Fellow, Harvard, around 1940.

Indeed, Kap became a popular accompanist and performer through much of his career. At one point, to demonstrate the virtues of a structure he discovered common to his favorite songs, he says, "I tried to show that you could [use it to] make a passable song out of such an unpromising source of thematic material as the first 14 digits of π." The resulting "Song about π" was later given lyrics by Enid Rieser and is often performed by Kap's daughter, Lucy, herself a popular folk singer-songwriter [3]. Some more personal family vignettes of Kap can be found below in Lucy's reminiscences of her father.

As a senior at the University of Toronto in 1938, Kap won the very first Putnam Competition, as did the Toronto team. This won him a fellowship to Harvard, where he earned his Ph.D. in 1941, under the direction of Saunders Mac Lane. He stayed on as a Benjamin Peirce Instructor till 1944, when Mac Lane brought him to the Applied Mathematics Group at Columbia University in 1944–45, which was doing work to support the war effort. Kap recounts, "So that year was spent largely on ordinary differential equations. I had a taste of real life and found that mathematics could actually be used for something."

From there Kap moved to the University of Chicago in the fall of 1945, where he remained till his retirement in 1984, having chaired the department during 1962–67. A year after Kap's arrival, Marshall Stone came to Chicago to direct and build up the mathematics department, ushering in what some have called "the Stone Age". Stone made four gigantic appointments—Saunders Mac Lane, Antoni Zygmund, André Weil, and Shiing-Shen Chern—followed by waves of talented young faculty and graduate students. Among the younger colleagues who greatly influenced Kap were Irving Segal, Paul Halmos, and Ed Spanier.

Kap's life style, outside his family and music, was rigorous and austere. He scheduled classes and meetings at (defiantly) early hours of the morning. Daily swimming was a lifelong practice; he loved the Lake Michigan shoreline. Lunch was lean in time as well as substance. With students he was generous and indulgent in mathematical conversation, but entertained little else.

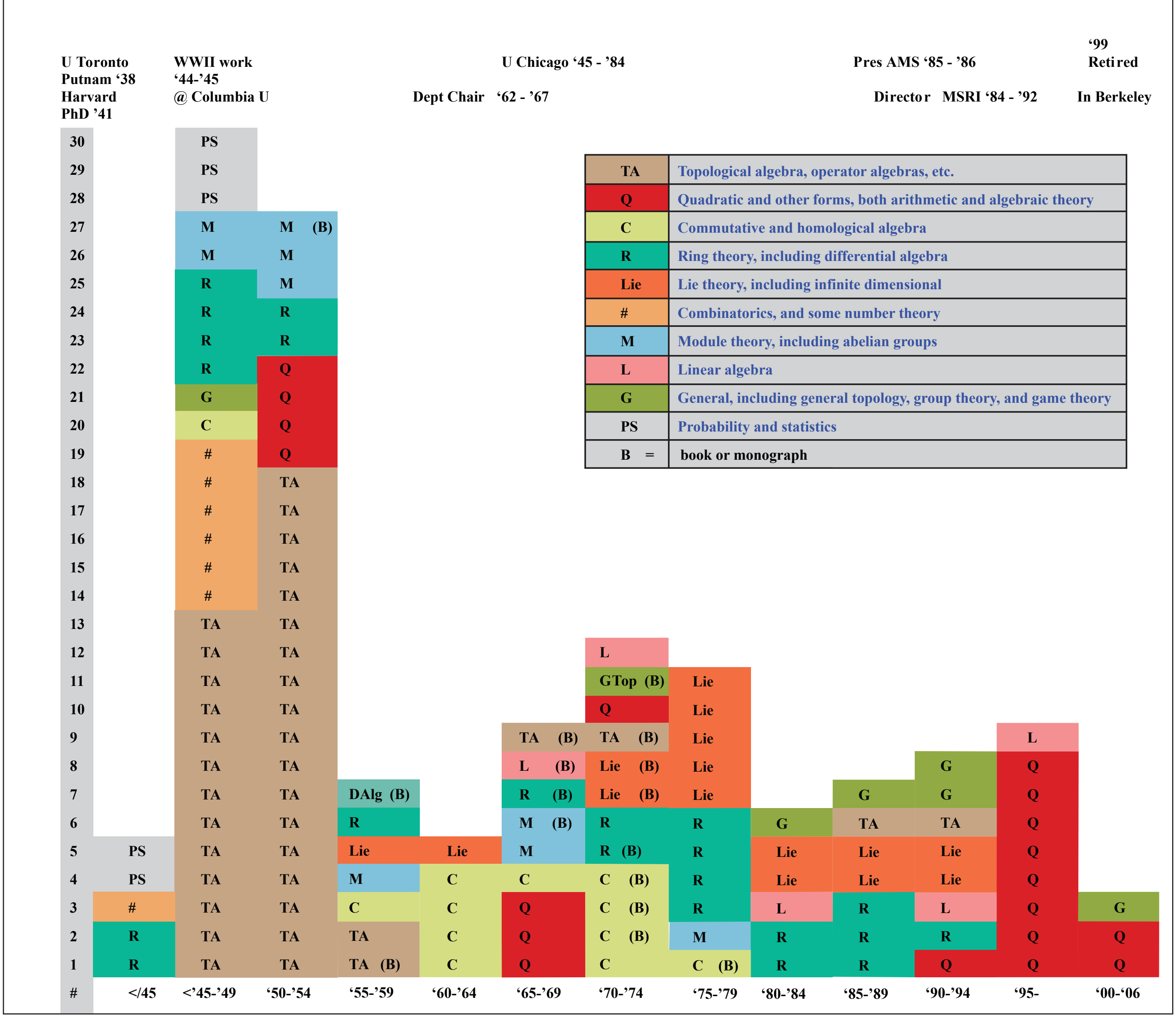
After Chicago, Kap, succeeding Shiing-Shen Chern, became the second director of the Mathematical Sciences Research Institute (MSRI) in Berkeley, 1984–1992. Also in 1984, Kap was elected president of the AMS. So we see here a career trajectory from a precocious college student to a dedicated, well established and prolific researcher, to a leader of some of the premier institutions of the profession. Along the way, Kap was honored by election to the National Academy of Sciences and to the American Academy of Arts and Sciences, and he was named an honorary member of the London Mathematical Society. In 1989 the AMS awarded him the Steele Prize, Career Award.

To understand Kap's mathematical accomplishments, it is important to speak of his students as well as his publications, to distinguish and compare what these two records tell us. Kap's mathematical work is distributed across several different areas of mathematics. For purposes of surveying them, I have somewhat arbitrarily grouped them as follows, the major areas in bold font:

- **TA: Topological algebra, including operator algebras, *-algebras, locally compact rings, etc.**
- **Q: Quadratic and higher forms, both abstract and arithmetic aspects**
- **C: Commutative and homological algebra**
- R: Ring theory (noncommutative)
- **Lie: Lie theory—groups and algebras, including infinite dimensional and characteristic p**
- #: Combinatorics and number theory
- M: Module theory, including abelian groups
- L: Linear algebra
- G Miscellaneous, including general topology, group theory, game theory
- PS: Probability and statistics

In the following chronological chart I have color-coded Kap's journal articles, books, and monographs according to which of these areas they belong. The data are taken from MathSciNet. Not included are the numerous contributions to the Problem sections of the *American Mathematical Monthly*; Kap remained throughout a virtuoso problem solver and contributor.

GRAPH OF KAPLANSKY'S PUBLICATIONS



Several remarkable things stand out from this chart.

• As a fresh Ph.D during the years of WWII, Kap published, beyond his dissertation (on maximal fields with valuation), a small but interesting mix of papers on combinatorics and on probability and statistics, perhaps in part influenced by his applied work at Columbia.

• Then, in the decade 1945–54 there is an extraordinary outpouring of publications, predominantly in what we are calling topological algebra. In fact, in the four years 1948–52, Kap published thirty-two papers! Some of this may have been backlog from the war years, but it is an astonishing ensemble of cutting-edge work in this area. Kap's general inclination was to algebraically axiomatize the various structures of concern to functional analysts, in the program launched earlier by Murray and von Neumann. Dick Kadison [2] writes in some detail about this phase of Kap's work.

• Kap's work in pure noncommutative ring theory is a persistent, but relatively modest theme in his work. One of his most influential papers, on "Rings with polynomial identity", opened an important branch of noncommutative algebra. Here he proves the fundamental result that a primitive algebra with polynomial identity is finite dimensional over its center.

• Lie theory, in its many aspects, is another important strand. This includes work on the classification of simple Lie algebras in characteristic p, lecture notes on the solution of Hilbert's Fifth Problem, and work, partly in collaboration with the physicist Peter Freund, on graded Lie algebras,

Kaplansky had 55 students (1950-1978) and 627 descendents

16					Richard Tarsy '69			MSRI
15					Robert Kibler '69			
14					E. Graham Evans, Jr. '69			
13					Richard Wagner '68			
12					Richard Shaker '68			
11			Gunther Lumer '59		Howard Gorman '68			
10			Hyman Bass '59		Susanna Epp '68			
9		Jacob Feldman[1] '54	Joseph Rotman '59		Mahlon Michael Day '67			
8		Fred Wright, Jr. '53	Eben Matlis '58	Jacob Towber '64	Alphonse Buccino '67	Charles Hanna '74		
7		Oscar Litoff '53	Donald Ornstein '57	Ronald Hamelink '64	Diana Taylor '66	Daniel Anderson '74		
6		William Darsow '53	Hazleton Mirkilmozrw '57	Fred Richman '63	Wolmer Vasconcelos '66	Jacob Matijevic '73		
5		H. Arlen Brown[2] '52	Edward Posner '57	Kolumum Nagarajan '62	Dean Heller '66	Adrian Wadsworth '72		
4		Isidor Fleisher '52	George Kolettis, Jr. '57	Robert MacRae '61	Hwa Tsang '65	Peter Kohn '72		
3		Alex Rosenberg '51	Sterling Berberian '55	Edward Davis '61	Gerson Levin '65	Judith Sally '71	Harry Hutchins '78	
2		Flora Dinkines '51	Harold Widom '55	John Eagon '61	Yung-yung Kuo '65	Bruce Prekowitz '71	Michael Modica '75	
1		Chester Feldman '50	Malcolm Goldman '55	Steven Chase '60	Samuel Gedwiser '65	Stephen McAdam '70	Warren Nichols '75	
#	<'50	'50-'54	'55-'59	'60-'64	'65-'69	'70-'74	'75-'79	≥ '80

[1] Kap was actually the second advisor, the first being Irving Segal. Feldman had 36 descendants.

[2] Kap was actually the second advisor, the first being Paul Halmos. Brown had 74 descendants.

super-symmetry, and related classification problems. Peter Freund writes vividly below about their collaboration.

• Quadratic (and higher) forms: This subject, from the beginning to the end of Kap's career, was dear to his heart. This interest was first inspired by his attending L. E. Dickson's lectures in number theory and quadratic forms at Chicago in the 1940s. It was rekindled during the years of his retirement, when he turned to the arithmetic theory of such forms, partly in collaboration with William Jagy. A charming account of a significant piece of this work can be found in the contribution of Manjul Bhargava below.

• In the eyes of many mathematicians today, commutative and homological algebra is the field with which they now most associate Kaplansky's name. Yet we see that its (yellow) color occupies remarkably little of the chart of publications. How can we explain this paradox? Well, for one thing, Kap's publications in this area include several books and monographs (lecture notes), and these contain a number of new results and methods that were not elsewhere published. This also reflects the fact that Kap was generating mathematics in this rapidly evolving field more through instruction than through papers written in solitude. And so what he was producing mathematically was significantly embodied in the work of the students who were learning from him.

• We can see this phenomenon in the preceding chart of Kap's Ph.D. students, again color-coded by the areas of their dissertations.

The first thing to notice in comparing these two charts is that the "relative masses" of topological algebra and commutative algebra have been approximately reversed, of course with a time shift. In topological algebra, Kap was a pioneer and a major, intensely productive, conceptual developer of the field. In commutative and homological algebra, in contrast, the field was already in rapid motion, into which Kap boldly ventured as more of an apprentice, guiding a flock of similarly uninitiated graduate students and postdocs with him.

Homological algebra was spawned from algebraic topology. In the hands of Eilenberg, Mac Lane, Grothendieck, and others it evolved into a new branch of algebra, embracing category theory and other new constructs. Meanwhile, the Grothendieck-Serre reformulation of algebraic geometry demanded that its foundations in commutative algebra be deepened and expanded.

A basic new concept of homological algebra was that of global homological dimension, a new ring-theoretic invariant. This turned out to be uninteresting for the most investigated rings, finite dimensional (noncommutative) algebras. On the other hand, a landmark discovery (of Auslander-Buchsbaum and Serre) was that, for a commutative noetherian local ring A, the global dimension of A is finite if and only if A is regular (the algebraic expression of the geometric notion of nonsingularity). This equivalence, and Serre's homological formulation of intersection multiplicities, firmly established homological algebra as a fundamental tool of commutative algebra.

However, these developments were known mainly on a Cambridge (MA)–Paris–axis. It was in this context that Kap offered a Chicago graduate course introducing these new ideas, methods, and results, then still very much in motion. Use of these methods led to the general proof (by Auslander-Buchsbaum) of unique factorization for all regular local rings. Kap's course, and its sequels, lifted a whole generation of young researchers (myself included) into this field. This played out for Kap over the next two decades, with many students and several books to show for it.

In mathematical style, Kap was a problem solver of great virtuosity. For course goals he sought problems, and theorems of great pedigree, and probed them deeply. His main focus was on proofs (pathways), more than on theorems (destinations). He sought geodesics, and the most economic (high mileage) means to get there. Proof analysis led to double-edged kinds of generalization/axiomatization:

– A given proof yields more than claimed. The given hypotheses deliver more than the stated theorem promises.

– The hypotheses can be weakened. We can get the same results more cheaply, and so more generally.

The strength of this disposition was perhaps sometimes over-zealous, pushing toward "premature maturation" of the mathematics. But it was an effective mode of instruction, yielding powerful conceptual command of the territory covered.

As the record above indicates, and the testimonials below will affirm, Kap was a gifted teacher, mentor, and writer. Here are a few of the things he himself has said in reflection on this.

> *I like the challenge of organizing my thoughts and trying to present them in a clear and useful and interesting way. On the other hand, to see the faces light up, as they occasionally do, to even get them excited so that maybe they can do a little mathematical experimentation themselves—that's possible, on a limited scale, even in a calculus class.*
>
> Advice to students: *"Look at the first case, the easiest case that you don't understand completely. Do examples, a million examples, 'well chosen' examples, or 'lucky' ones. If the problem is worthwhile, give it a good try—months, maybe years if necessary. Aim for the less obvious, things that others have not likely proved already."*
>
> And: "*Spend some time every day learning something new that is disjoint from the problem on which you are currently working (remember that the disjointness may be temporary). And read the masters."*
>
> *When a great mathematician has mastered a subject to his satisfaction and is presenting it, that mastery comes through unmistakably, so you have an excellent chance of understanding quickly the main ideas.* [He cites as examples, Weil, Serre, Milnor, Atiyah.]
>
> *…the thing that bedevils the mathematical profession—the difficulty we have in telling the world outside mathematics what it is that mathematicians do. And for shame, for shame, right within mathematics itself, we don't tell each other properly.*

And here is a sampling of how Kap was seen by others, including some of his students:

"He was not only a fantastic mathematician but a marvelous lecturer, and he had a remarkable talent for getting the best out of students."

—Richard G. Swan

"I knew Kaplansky in his later years, and also through some of his books. Cheerful, gracious, and elegant are some of the words that come to mind when I think of him."

—Roger Howe

"The mathematical community in India is shocked to have news of the demise of Professor Irving Kaplansky. We all feel very sad at this irreparable loss. Professor Kaplansky was a source of inspiration for mathematicians around the world. He will no doubt live for all time through his mathematical contributions. We will miss his personal wit, charm and warm personality."

—I. B. S. Passi, President,
Indian Mathematical Society

"I did know about the work of Emmy Noether and it may have influenced my choice of area, algebra, although I think the teaching of Irving Kaplansky was what really inspired me."

—Vera Pless

Kaplansky's books "have one feature in common. The content is refreshing and the style of exposition is friendly, informal (but at the same time mathematically rigorous) and lucid. The author gets to the main points quickly and directly, and selects excellent examples to illustrate on the way."

—Man Keung Siu

"I learnt from his books in my youth, and would not have survived without them. Even today, I ask my students to read them, to learn the 'tricks' of the trade."

—Ravi Rao

"Kaplansky was one of my personal heroes: during my student years, I discovered his little volume on abelian groups and noticed that algebra too has stories to tell…"

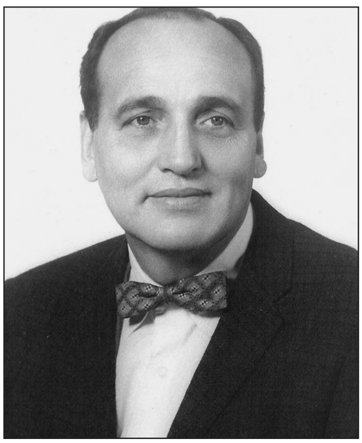
—Birge Huisgen-Zimmermann

Kap as a Thesis Advisor: "I was very young and very immature when I was Kap's student. I'm deeply indebted to Kap for putting up with me and helping me to develop in my own eccentric way. I asked Kap for a thesis problem that didn't require any background and, surprisingly, he found one with enough meat in it to allow me to get a feeling for doing research.

"It wasn't until I had my own thesis students that I realized how hard it must have been to accommodate my special needs and help me develop in my way, not in his way."

—Donald Ornstein (Kap Ph.D., 1957)

For me Kap's transition from course instructor to thesis advisor was almost imperceptible, since I had become deeply engrossed in his courses on commutative and homological algebra and questions about projective modules, an exciting territory wide open for exploration, and for which Kap had laid a solid groundwork. He did float a few other problems to me, such as the structure of certain infinite dimensional Lie algebras, whose significance I only later came to appreciate. But I didn't take that bait then. He was a generously available and stimulating advisor, often sharing promising ideas that he had not yet had time to pursue. What I remember most of that time is the brilliance of his courses, and the richness and excitement of the mathematical milieu that he had created among his many students then. This milieu powerfully amplified the many mathematical resources that Kap had to offer. I think that it is fair to say that Kap's students are an important part of his oeuvre. One could hardly have asked for a better teacher and advisor.



Chair, University of Chicago Math Department, 1962–1967.

References

[1] Donald J. Albers, Interview with Irving Kaplansky, *The College Mathematics Journal* **22** (1991), no. 2, pp. 99–117.

[2] Richard V. Kadison, Irving Kaplansky's role in mid-twentieth century functional analysis, *Notices* of the AMS (to appear).

[3] Ivars Peterson, A song about π, MathTrek, *MAA Online*, July 12, 1999.

T. Y. Lam

Kap: A Tale of Two Cities

Through Professor Hyman Bass, Kap was my mathematical grandfather. This and the fact that Kap offered me my first job as an instructor at Chicago were perhaps not statistically independent events. The time was forty years ago, when Kap was finishing his five-year term as Chicago chair. The offer was consummated by a Western Union telegram—the 1960s equivalent of email. Kap didn't ask for my C.V. (I wouldn't have known what that was); nor did he want to know my "teaching philosophy" (I had none). For my annual salary, Kap offered me US$8,000—a princely sum compared to my then T.A. stipend of US$2,000 at Columbia University. I have joked to my colleagues that I'll always remember Kap as the only person through my whole career to have ever quadrupled my salary. But in truth, a ticket to Chicago's famed Eckhart Hall for postdoctoral studies was more than anything a fledgeling algebraist could have dreamed. For this wonderful postdoctoral experience Kap afforded me through his unconditional confidence in a mathematical grandson, I have always remained grateful.

I met Kap for the first time in the fall of 1967 when I reported to work in Hyde Park. By that time, Kap had already taught for twenty-two years at the University of Chicago. Although he was Canadian by birth, Chicago had long been his adopted home and workplace: it is, appropriately, the city where our "tale" begins.

For students interested in abstract algebra, Kaplansky is virtually a household name. In graduate school, I first learned with great delight Kap's marvelous theorem on the decomposition of projective modules, and his surprisingly efficient treatment of homological dimensions, regular local rings, and UFDs. It was to take me forty more years, however, to get a fuller glimpse of the breadth and depth of Kap's total mathematical output. In these days of increasing specializations in mathematics, we can only look back in awe to Kap's trail-blazing work through an amazingly diverse array of research topics, ranging from valuation theory, topological algebra, continuous geometry, operator algebras and functional analysis, to modules and abelian groups, commutative and homological algebra, P.I. rings and general noncommutative rings, infinite-dimensional Lie algebras, Lie superalgebras (supersymmetries), as well as the theory of quadratic forms in both its algebraic and arithmetic flavors. Kap was master of them all. In between the "bigger" works, Kap's publications also sparkled with an assortment of shorter but very elegant notes, in number theory, linear algebra, combinatorics, statistics, and game theory. All of this, still, did not include the many other works recorded in "fourteen loose-leaf notebooks" (referred to in the preface of [1]) that Kap had kept for himself over the years. One cannot help but wonder how many more mathematical gems have remained hidden in those unpublished notebooks!

For me, reading one of Kap's papers has always proved to be a richly rewarding experience. There are no messy formulas or long-winded proofs; instead, the reader is treated to a smooth flow of novel mathematical ideas carefully crafted to perfection by an artisan's hand. Some authors dazzled us with their technical brilliance; Kap won you over by the pure soundness of his mathematical thought. In his publications, Kap was much more given to building new conceptual and structural frameworks, than going down single-mindedly into a path of topical specialization. This style of doing mathematics made him a direct intellectual descendant of Emmy Noether and John von Neumann. As a consequence, many of Kap's mathematical discoveries are of a fundamental nature and a broad appeal. The famous Kaplansky Density Theorem for unit balls and his important inaugural finiteness result in the theory of rings with polynomial identities are only two of the most outstanding examples.

Those of us who have had the privilege of listening to Kap all knew that he was extremely well spoken and had indeed a very special way with words. However, this gift did not always manifest itself when Kap was in social company with Chellie. It was quite clear to all his colleagues who Kap thought was the better orator in the family. Dinner parties the Kaps attended were often replete with Chellie's amusing stories about the Chicago department and its many colorful mathematical personalities, from an austere André Weil down to the more transient, sometimes bungling graduate students over the years. As Chellie recounted such funny stories with her characteristic zest and candor, Kap would listen admiringly on the side —without interruption. Only at the end of a story would he sometimes add a clarifying comment, perhaps prompted by his innate sense of mathematical precision, such as "Oh, that was 1957 summer, not fall."

Kap's extraordinary gift in oral (and written) expression was to find its perfect outlet in his teaching, in which it became Chellie's turn to play a supporting role. In the many lecture courses Kap gave at the University of Chicago in a span of thirty-nine years, he introduced generation after generation of students to higher algebra and analysis. In those courses he taught that were of an experimental nature, Kap often directly inspired his students to new avenues of investigation, and even to original mathematical discoveries at an early stage. (Schanuel's Lemma on projective resolutions, proved by Stephen Schanuel in Kap's fall 1958 Chicago course in homological algebra, was perhaps the best known example.) It was thus no accident that Chicago graduate students flocked to Kap for theses supervision. Over the years, Kap directed doctoral dissertations in almost every one of the mathematical fields in which he himself had worked. Many of Kap's fifty-five Ph.D. students from Chicago are now on the senior faculty at major universities in the U.S. Currently, the Mathematics Genealogy Project listed Kap as having 627 descendants—and counting. This is the *second* highest number of progeny produced by mathematicians in the U.S. who had their own Ph.D. degrees awarded after 1940. We leave it as an exercise for the reader to figure out who took the top honor in that category, with the not-too-useful hint that this mathematician was born a year after Kap.

Books of Irving Kaplansky

- *Infinite Abelian Groups*, 1954, 1969
- *An Introduction to Differential Algebra*, 1957, 1976
- *Introduction to Galois Theory (in Portuguese)*, 1958
- *Rings of Operators*, 1968
- *Fields and Rings*, 1969, 1972
- *Linear Algebra and Geometry. A Second Course*, 1969, 1974
- *Commutative Rings*, 1970, 1974
- *Lie Algebras and Locally Compact Groups*, 1971, 1974
- *Set Theory and Metric Spaces*, 1972, 1977
- *Matters Mathematical (with I. Herstein)*, 1978
- *Selected Papers and Other Writings*, 1995

While Kap had clearly exerted a tremendous influence on mathematics through his own research work and that of his many Ph.D. students, the books written by him were a class by themselves. The eleven books listed in the sidebar on this page traversed the whole spectrum of mathematical exposition, from the advanced to the elementary, reaching down to the introduction of mathematics to non-majors in the college. *Differential Algebra* typified Kap's broad-mindedness in book writing, as its subject matter was not in one of Kap's specialty fields. On the other hand, *Infinite Abelian Groups* introduced countless readers to the simplicity and beauty of a subject dear to Kap's heart, while *Rings of Operators* served as a capstone for his pioneering work on the use of algebraic methods in operator algebras. *Lie Algebras*, *Commutative Rings*, as well as *Fields and Rings*, all originating from Kap's graduate courses, extended his classroom teaching to the mathematical community at large, and provided a staple for the education of many a graduate student worldwide, at a time when few books covering the same materials at the introductory research level were available. In these books, Kap sometimes experimented with rather audacious approaches to his subject matters. For instance, *Commutative Rings* will probably go down on record as the only text in commutative algebra that totally dispensed with any discussion of primary ideals or artinian rings.

As much as his books are appreciated for their valuable and innovative contents, Kap's great fame as an author derived perhaps even more from his very distinctive writing style. There is one common characteristic of Kap's books: they were all short—something like 200 pages was the norm. (Even *Selected Papers* [1] had only 257 pages, by his own choice.) Kap wrote mostly in short and simple sentences, but very clearly and with great precision. He never belabored technical issues, and always kept the central ideas in the forefront with an unerring didactic sense. The polished economy of Kap's writing makes it all at once fresh, crisp, and engaging for his readers, while his mastery and insight shone on every page. The occasional witty comments and asides in his books—a famous Kaplansky trademark—are especially a constant source of pleasure for all. In retrospect, Kap was not just a first-rate author; he was truly a superb expositor and a foremost mathematical stylist of his time.

After I moved from Chicago to Berkeley, my contacts with Kap became sadly rather infrequent. So imagine my great surprise and delight, sixteen years later, when word first came out that Kap was to retire from the University of Chicago, in order to succeed Chern as the director of MSRI! In the spring of 1984, the Kaplanskys arrived and established their new abode a few blocks north of the university campus—in Berkeley, California, the second city of our tale.

The math departments at Chicago and Berkeley share much more than the "U.C." designation of the universities to which they belong. There has been a long (though never cantankerous) history of the Berkeley department recruiting its faculty from the Chicago community, starting many years ago with Kelley, Spanier, and Chern. Indeed, when Kap himself joined the U.C.B. faculty in 1984, there were at least as many as sixteen mathematicians there who had previously been, in one way or another, associated with the University of Chicago. It must have given Kap a tinge of "nostalgia" to be reunited, in such an unexpected way, with so many former graduate students, postdocs, and colleagues from his beloved Chicago department. But if anyone had speculated that, by coming West, Kap was to spend his golden years resting on his laurels, he or she could not have been more wrong. In fact, as soon as Kap arrived at Berkeley in 1984, he was to take on, unprecedentedly, two simultaneous tasks of herculean proportions: (a) to head a major mathematics research institute in the U.S., and (b) to preside over the largest mathematical society in the world—the AMS[1].

Other contributors to this memorial article are in a much better position than I to comment on Kap's accomplishments in (a) and (b) above, so I defer to them. In the following, my reminiscences on Kap's Berkeley years are more of a personal nature. From 1984 on, I certainly had more occasions than ever before to interact mathematically with Kap—discussing with him issues in quadratic forms and ring theory. Kap seemed to favor the written mode of communication (over the oral), but his letters were just as concise as his books. I still have in my prized possession an almost comical sample of Kap's terseness, in the form of a covering letter for some math notes he sent me. Written out on a standard-size 8½ by 11 MSRI letterhead, the letter consisted of twelve words: "Dear Lam: I just did a strange piece of ring theory. Kap." It was briefly—but of course unambiguously—dated: "Apr. 11 /97".

Another interaction with Kap in 1998 led to some mathematical output. In preparation for a special volume in honor of Bass's sixty-fifth birthday, I was very much hoping to commission an article from Kap. In his usual self-effacing fashion, Kap protested that he had really nothing to write about. However, after much persuasion on my part (stressing that he must write for Bass), he gave in and wrote up in his impeccable hand a short note in number theory [2]. Glad that my tactics had paid off, I worked all night to set Kap's written note in $\TeX$, and delivered a finished printout to him early the next morning. Kap was surprised; he thanked me profusely, but said that maybe he shouldn't have written his article. It was too late.

One of Kap's best known pieces of advice to young mathematicians was to "spend some time every day learning something new that is disjoint from the problem on which you are currently working,…and read the masters" [3]. Amazingly, even after reaching his seventies, Kap still took his own advice personally and literally. In all the years he was in Berkeley, Kap made it his habit to go to every Monday's Evans-MSRI talk and every Thursday's Math Department Colloquium talk. He even had a favorite seat on the left side of the front row in the colloquium room, which, in deference to him, no local Berkeley folks would try to occupy. In the years 1995–97 when I worked at MSRI, I saw Kap quite frequently at the periodicals table in the library, poring over the *Mathematical Reviews* to keep himself abreast with the latest developments in mathematics. And he read the masters too, e.g., in connection with his work on the integral theory of quadratic forms. Members of MSRI have reported sightings of Kap using a small step-ladder in the library to reach a certain big book on a high shelf, and putting the book back in the same fashion after using it (instead of leaving it stray on a table). That tome was an English translation of *Disquisitiones Arithmeticae*: the fact that even a six-foot-tall Kap needed a step-ladder to access it was perhaps still symbolic of the lofty position of the work of the twenty-year-old Carl Friedrich Gauss.

My two-years' stay at MSRI was rich with other remembrances about Kap. Undoubtedly, a highlight was Kap's eightieth birthday fest in March

[1]*Kaplansky served as AMS president-elect in 1984, and president for the 2-year period 1985–86.*

1997, which was attended by three MSRI directors and six MSRI deputy directors, as well as visiting dignitaries such as Saunders Mac Lane, Tom Lehrer, and Constance Reid. Another most memorable gathering was the holiday party in December 1996, where a relaxed and jovial Kap sang some of his signature songs for us all, accompanying himself on the piano in the MSRI atrium. His energetic, sometimes foot-stomping performance really brought down the house! It saddens me so much to think that, now that Kap is no longer with us, these heart-warming events will never be repeated again.

Twenty years may have been only about a third of Kap's professional life, but I hope that Kap cherished his twenty years in Berkeley with as much fondness as he had cherished his thirty-nine years in Chicago. Those were the two cities (and universities) of his choice, for a long and very distinguished career in mathematics. In Chicago, Kap was a researcher, a chairman, a teacher, a mentor, and an author. In Berkeley, while remaining a steadfast researcher, Kap also became a scientific leader, a senior statesman, and a universal role model. In each of these roles, Kap served his profession with devotion, vigor, wisdom, and unsurpassed insight. His lifetime work has profoundly impacted twentieth century mathematics, and constituted for us an amazingly rich legacy.

On a personal level, Kap—mathematical grandpa and algebraist par excellence—will continue to occupy a special place in my heart. I shall miss his great generosity and easy grace, but thinking of Kap and his towering achievements will always enable me to approach the subject of mathematics with hope and joy.

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Richard Kadison

Letter from Richard Kadison to Section 11 (Mathematics) of the National Academy of Sciences

(Addressed to the Chair, Paul Rabinovich, July 1, 2006)

Dear Paul,

Just about ten minutes ago, I sat down to my email; I had looked at it at about 9:30 a.m.—before the sad news about Kap arrived. So, I saw the message appended below (you have it as well) only a few minutes ago. I was shocked by the news. "Sad" really doesn't begin to describe my feelings; Kap was almost as close, where I'm concerned, as a beloved parent. Of all my graduate school teachers (Stone, Zygmund, Chern, Spanier, Halmos, Segal, Weil, Graves, Hestenes, Mac Lane, Albert, etc.), and I revered each and every one of them, Kap was my favorite. A half-hour-to-hour conversation with him about mathematics generated so much excitement that I spent the rest of the day walking on a cloud. Irv was immensely popular with the graduate students; he was always ready to talk math with us and make good and useful suggestions for our work, but he was also somewhat "scary" for many of the students. His "social" behavior was even more peculiar than the "standard" behavior of dedicated mathematicians. Most of us have an exaggerated sense of the "futility" of small talk; Irv's view of that had to be described as "excessive". For example, if you met him in the hallway and stopped for a conversation with him, when the conversation was clearly over, he just walked on, turned and walked away, whatever—absolutely no decompression stage (or phrases, e.g., the currently popular, and almost always, fatuous "have a nice day"—recently inflated to "have a great day"). Handshakes? Forget it! As fast and smart and creative as he was, and all that (genuine, not affected) no-nonsense behavior of his, we loved ("worshipped" might be more accurate) him. Chatting with him in his office, after a few years, he asked me a nice question that had occurred to him, a fine blend of algebra and analysis (nilpotents of index 2 and approximation). I thought about it for fifteen minutes or so that evening and didn't see how to get started. Being busy with other things I dropped it and didn't get back to it until I met and talked to him a day or two later. He asked me if I had thought about the problem. I said that I had, but hadn't been able to get started on it, and then asked him if he really thought it

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Kap and Chellie Kaplansky on their wedding day, 1951.

was true. His response was, "When God whispers a theorem in your ear, you should listen." Now, of course I understood his "cute" way of giving me some valuable mathematical advice, but I chose to misinterpret it. When I reported this to the other graduate students, I told them the story and added that Kaplansky had finally revealed himself, and as many of us had suspected, he was God. In those very early years (end of the 1940s), Irv lived an austere life. He rented a single room in a house near the U. of Chicago campus, paid $5 a week for it, if I remember correctly, and saved almost all the rest of his salary. The word was that he was (relatively) wealthy in those days. (Of course, that could mean anything from someone with a bank balance of $100 and up and no debts, in those days and our society.) Chellie (Rochelle), Irv's wife of fifty-five years was tremendous fun, great sense of humor. I had the impression that she could wrap him around her little finger, he knew it, and he enjoyed it. She entered the picture in 1951. Some years later (about five), George Mackey was having dinner with Karen and me at our apartment in Cambridge (we were visiting MIT that year). The conversation turned to Irv. (Kap and George were great friends.) When the subject of Kap's purported wealth came up, George told of a conversation he and Chellie had had some little time back. He said that he had asked Chellie if she didn't feel that she was lucky to have married a wealthy man—to which she replied, with a (feigned) surprised smile, "Oh, that—it was only about $30,000 and I went thru that in no time!" George paused after reporting that, assumed a troubled, somber look and said, "Fair, sent a chill down my spine!" It probably helps to know that, in those days, George was still a bachelor, and lived a frugal, austere existence—completely by choice. Both Kap and Mackey were perfectly willing to spend their money when the occasion warranted it. I've had many fine meals with each of them. Kap did not eat lunch with us during our graduate student days, we took too long with it. I remember a bunch of us walking down the stairs of Eckhart Hall on our way over to lunch at the Commons. Irv came bouncing past us, evidently on the same mission. The Commons is a few hundred meters from Eckhart. We sauntered over arriving in time to see Kap emerging from the Commons, lunch over. Chellie probably slowed him down over the years. In my first years at Chicago, Irv had no discernible social life. He liked swimming in Lake Michigan during the summer and did so early each morning. Then, in 1949 he had a few quarters off and went out for a stay at UCLA. The rumors flew back to Chicago, Kap had bought himself a convertible, now drank liquor, socially, and smoked. One day it was said that the "new" Kaplansky had returned. A day later, I happened on my dear pal and fellow graduate student, Arnold Shapiro. He told me that he had just talked with Irv a few hours ago. I asked, "The new Kaplansky?" Arnold's reply was, "What new Kaplansky? It's just the old Kaplansky—with a smile on his face." Shortly after that, a few of us finished our Ph.D. requirements. As tradition had it, we invited one and all to a party. Walking around with a tray and two drinks ("highballs") on it, one primarily scotch and the other bourbon, I offered one of those drinks to Irv. His question for me was, "Which is the perfume and which is the hair tonic?" That, apparently, was the "new" Kaplansky. Of course, I could tell you so many more stories about Irv, many of them that have some mathematical significance. They are all memories I treasure. Kap is one of the very few people I've known well most of my working life of whom I can say that I have nothing but enjoyable memories.

—Kindest,
Dick

Peter G. O. Freund

Irving Kaplansky and Supersymmetry[2]

I arrived in Chicago some two decades after Irving Kaplansky, and I met Kap, as we all called him, shortly after my arrival here. We became friends later, in 1975, while collaborating on a paper on supersymmetry. Lie superalgebras, graded counterparts of ordinary Lie algebras, play a central role in string theory and other unified theories. A classification of the simple ones was of essence. I took some initial steps, but the real work started when Yitz Herstein put me in touch with Kap. At first, communication was not easy. We couldn't quite make out each other's reasoning, much as we agreed on results. It didn't take long however,

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[2]*Based on remarks at the Irving Kaplansky Memorial at MSRI, Berkeley, CA, February 23, 2007.*

to get used to the other's way of looking at things. Mathematicians and physicists think in similar ways after all, all that was needed was a dictionary. This was during the early phase of the rapprochement between mathematics and theoretical physics. After the glorious first half of the twentieth century—when the likes of Poincaré, Hilbert, Weyl, von Neumann, Élie Cartan, Emmy Noether, and others made major contributions to the then-new physical theories of general relativity and quantum mechanics, while physicists like Jordan, Dirac, Casimir, and Feynman made major contributions to mathematics—physics entered a period best described as phenomenological. During this period, some advanced complex function theory aside, very little modern mathematics was drawn on. To give you an idea, when in his celebrated "Eightfold Way" paper, Murray Gell-Mann wrote down a basis of the three-dimensional representation of the $su(3)$ Lie algebra, this was heralded by physicists as a great mathematical feat. "Imagine, he found a 3×3 generalization of the famous 2×2 Pauli matrices," is what most people said. To get there, Murray had consulted with Block and Serre!

It was in the fields of supersymmetry and gauge theory that the initial steps in modern mathematical physics were taken. This convergence of the paths of mathematics and of theoretical physics is typical of times when major new physical theories—gauge theory and string theory in this case—are being born. The earliest example of such a convergence is the creation of calculus at the birth of Newton's mechanics and of his theory of gravitation. Weyl's spectacular work on group theory under the impact of the newborn quantum mechanics is another such example.

A few words about our joint paper [1] are in order here. In it we found all the infinite families of simple Lie superalgebras, as well as 17-, 31- and 40-dimensional exceptional ones. We also discussed real forms and explained why supersymmetry can act on 4-dimensional anti-de Sitter but not on de Sitter space, a result essential for understanding why the remarkable duality discovered by Maldacena [2] in the 1990s, is of the AdS/CFT and not of the dS/CFT type. We were convinced that we had found all simple Lie superalgebras (as we actually had), but we lacked a proof of this fact. The proof came from the powerful independent work of Victor Kac [3]. Amusingly, in his beautiful proof, Kac somehow overlooked one of the exceptional superalgebras, namely the 31-dimensional superalgebra $G(3)$, whose Bose (even) sector consists of the ordinary Lie algebra $g_2 + sl(2)$, the only simple Lie superalgebra to have an exceptional ordinary Lie algebra as one of the two constituents of its Bose sector. I said "amusingly" above because, as I learned from Kap, in the classification of ordinary simple Lie algebras, in his extremely important early work, Killing had found almost all of them, but he "somehow overlooked one," namely the exceptional 52-dimensional simple Lie algebra F_4, which remained to be discovered later by Élie Cartan. Apparently, $G(3)$ is the exceptional Lie superalgebra which carries on that curse of the ordinary exceptional Lie algebra F_4.

I mentioned the almost total lack of contact between theoretical physicists and mathematicians, when this work got going. It went so deep that in 1975 most physicists, if asked to name a great modern mathematician, would come up with Hermann Weyl, or John von Neumann, both long dead. Mathematicians had it a bit easier, for if they read the newspapers, they could at least keep track of the Nobel Prizes, whereas newspaper editors rarely treated Fields Medal awards as "news fit to print."

I recall that while standing by the state-of-the-art Xerox machine to produce some ten copies of our paper in about…half an hour's time, I asked Kap, "Who would you say, is the greatest mathematician alive?" He immediately took me to task: my question was ill-defined, did I mean algebraist, or topologist, or number-theorist, or geometer, or differential geometer, or algebraic geometer, etc.…I replied that I did not ask for a rigorous answer, but just a "gut-feeling" kind of answer. "Oh, in that case the answer is simple: André Weil," he replied, without the slightest hesitation, a reply that should not surprise anyone, who has heard today's talks. "You see," Kap went on, "We all taught courses on Lie algebras or Jordan algebras, or whatever we were working on at the time. By contrast, Weil called all the courses he ever taught simply 'mathematics' and he lived up to this title."

Kap went on to tell me about Weil's legendary first colloquium talk in Chicago. This was the first time I heard that very funny story. Weil had been recruited for the Chicago mathematics department by its chairman, Marshall Stone. With Stone sitting in the first row, Weil began his first Chicago colloquium talk with the observation, "There are three types of department chairmen. A bad chairman will only recruit faculty worse than himself, thus leading to the gradual degeneration of his department. A better chairman will settle for faculty roughly of the same caliber as himself, leading to a preservation of the quality of the department. Finally, a good chairman will only hire people better than himself, leading to a constant improvement of his department. I am very pleased to be at Chicago, which has a very good chairman." Stone laughed it off; he did not take offense.

The lack of communication between mathematicians and physicists was to end soon. By 1977, we all knew about Atiyah and Singer, and then the floodgates came down fast, to the point that an extremely close collaboration between mathematicians and physicists got started and, under the leadership of Ed Witten and others, is ongoing and bearing beautiful fruit to this day. By the way, on Kap's desk I noticed some work of his on Hopf algebras. I asked him about Hopf algebras, and got the reply, "They are of no relevance whatsoever for physics." I took his word on this, was I ever gullible. In the wake of our joint work, Kap and I became good friends. This friendship was fueled also by our shared love of music; he was a fine pianist, and I used to sing. For me, the most marvelous part of my collaboration and friendship with Kap was that for the first time I got to see up-close how a great mathematician thinks.

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Calvin C. Moore

Kap Encounters in Chicago and Berkeley

I first "met" Kap mathematically when I was a graduate student at Harvard working in functional analysis and read and studied his striking 1951 papers on C*-algebras in which he defined and explored the properties of what he called CCR and GCR algebras. His algebraic insight into these objects arising in analysis turned out to be of seminal importance and indeed were years ahead of their time. I also read about what was by then called the Kaplansky density theorem for von Neumann algebras dating from 1952 and studied his wonderful 1948 paper on groups with representations of bounded degree and its connection with polynomial identities.

I was eager to meet this algebraist whose work had been so influential in my own studies in a very different field, and I had that opportunity when I had a postdoctoral appointment at Chicago in 1960–61. However, I soon left Chicago for Berkeley, and it was many years before our paths crossed again. When we were planning a full year program at the Mathematical Sciences Research Institute (MSRI) in 1983-84 on the topic of infinite dimensional Lie algebras, Kap's broad and deep insight and understanding in algebra led us to select him as the chair of the program committee. We also recognized that his subtle and effective diplomatic skills would be essential ingredients in making this program the great success that it was.

Almost at the same time, the Board of Trustees of MSRI selected Kap to succeed Shiing-Shen Chern as director of MSRI in 1984. We served together at MSRI, he as director and I continuing as deputy director for a year before I left MSRI for an administrative post in the University of California. It was a wonderful learning and teaching experience for both of us. I learned much from Kap's wisdom and experience, and I in turn tried to convey to him what I knew about MSRI operations. I subsequently watched more from a distance, and it was clear that MSRI grew and prospered under his eight years of excellent leadership as director. He also maintained a lively research program while serving as director and for many years after stepping down. We all miss this generous and wise man of many talents.

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Susanna S. Epp and E. Graham Evans Jr.

Kap as Advisor

We are two of the fifty-five students who completed a doctorate with Kaplansky between 1950 and 1978. This is an astonishing number. Indeed during the years 1964–1969, when we were at the University of Chicago, Kap oversaw an average of three completed dissertations a year despite serving as department chair from 1962–1967. His secret, we think, was an extraordinary instinct for productive avenues of research coupled with a generous willingness to spend time working with his students. He also often encouraged students to run a seminar, with beginning students presenting background material and advanced students presenting parts of their theses.

When Evans worked with him, Kap was teaching the commutative algebra course that was published soon afterward by Allyn and Bacon. As with each course he taught, he filled it with new thoughts about the subject. For instance, at one memorable point he experimented to see how much he could deduce if he knew only that $\mathrm{Ext}^1(A,B)$ was zero. He managed to get pretty far, but eventually the proofs became unpleasantly convoluted. So he abruptly announced that henceforth, he would assume the full structure of $\mathrm{Ext}^i(A,B)$, and the next day he resumed lecturing in his usual polished fashion. This episode was atypical in that he first developed and then cut off a line of inquiry. More frequently, after commenting on

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new insights of his own, he would interject questions for students to explore and develop. In his lectures he made the role of non-zero divisors, and hence regular sequences, central in the study of commutative rings. At one point he gave an elegant proof, avoiding the usual filtration argument, that the zero divisors are a finite union of prime ideals in the case of finitely generated modules over a Noetherian ring. Then he asked Evans to try to determine what kinds of non-Noetherian rings would have the property that the zero divisors of finitely generated modules would always be a finite union of primes. One of the ideas in Kap's proof was just what Evans needed to get the work on his thesis started.

The year that Epp worked with Kap, he was not teaching a course but had gone back to a previous and recurring interest in quadratic forms. A quintessential algebraist, he was interested in exploring and expanding classical results into more abstract settings. Just as in his courses he tossed out questions for further investigation, in private sessions with his students he suggested various lines of inquiry beyond his own work. In Epp's case this meant exploring the results Kap had obtained in generalizing and extending H. Brandt's work on composition of quaternary quadratic forms and trying to determine how many of these results could be extended to general Cayley algebras.

Kap typically scheduled an early morning weekly meeting with each student under his direction. For some it was much earlier than they would have preferred, but for him it followed a daily swim. He led our efforts mostly by expressing lively interest in what we had discovered since the week before and following up with question after question. Can you prove a simpler case? Or a more general one? Can you find a counterexample? When one of us arrived disappointed one day, having discovered that a hoped-for conjecture was false, Kap said not to be discouraged, that in the search for truth negative results are as important as positive ones. He also counseled persistence in other ways, commenting that he himself had had papers rejected—a memorable statement because it seemed so improbable. Having made contributions in so many fields and having experienced the benefits of cross-fertilization, he advised being open to exploring new areas. Some of his students may have taken this advice further than he perhaps intended, ultimately working far from their original topic areas at the National Security Agency, at the Jet Propulsion Laboratory, and in K–12 mathematics education, for example.

Kap derived a great deal of pleasure from having generated 627 mathematical descendants, perhaps especially from meeting his mathematical grandchildren and great-grandchildren. When one encountered him at the MSRI bus stop one day and, not knowing what to say, commented on the weather, Kap responded with a smile, "Cut the crap. Let's talk mathematics." They did, and he became one of the many students Kap mentored long after he retired.

Joseph Rotman

Student Memories of Kap

As a graduate student at the University of Chicago, I attended many of Kaplansky's elementary courses: complex variables, group theory, set theory, point-set topology; later, I attended more advanced courses: commutative algebra, Hilbert's fifth problem, abelian groups, homological algebra. Every course, indeed, every lecture, was a delight. Courses were very well organized, as was each lecture. Results were put in perspective, their applications and importance made explicit. Humor and droll asides were frequent. Technical details were usually prepared in advance as lemmas so as not to cloud the main ideas in a proof. Hypotheses were stated clearly, with examples showing why they were necessary. The exposition was so smooth and exciting that I usually left the classroom feeling that I really understood everything. To deal with such arrogance, Kap always assigned challenging problems, which made us feel a bit more humble, but which also added to our understanding. He was a wonderful teacher, both in the short term and for the rest of my mathematical career. His taste was impeccable, his enthusiasm was contagious, and he was the model of the mathematician I would have been happy to be.

Kap was my thesis advisor. I worked in abelian groups (at the same time, he had five other advisees: two in homological algebra and three in functional analysis). He set weekly appointments for me. When I entered his office, he was usually sitting comfortably at his desk, often with his feet up on the desk. He'd greet me with "What's new?" I would then talk and scribble on the blackboard as he listened and asked questions. Once I had axiomatized a proof of his and Mackey's, enabling me to generalize their result. "How did you think of that?" he asked. I replied that that was the way he had taught me to think; he smiled.

Both of us spent a sabbatical year in London at Queen Mary College. Of course, I continued to enjoy his mathematics, but I saw another side of him as well. N. Divinsky was another sabbatical visitor (as was H. Flanders), and I was dubbed Rotmansky to go along with Kaplansky and Divinsky. Kap discovered cricket, and often went to Lord's Cricket Grounds. But Kap really loved Gilbert and Sullivan. He arranged an evening in which we performed Iolanthe. Kap was at the piano, Divinsky

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did the patter songs, Flanders was on the recorder, and I was Strephon.

There are few giants in the world, and now there is one less.

Lance Small

Kap as Teacher and Mentor

I was not a student of Kaplansky—at least, not in the sense we usually mean in mathematics. He was, however, my teacher in a number of courses, undergraduate and graduate, and was chairman of the University of Chicago math department when I was a graduate student. Kap's "style", mathematical as well as personal, shone through everywhere.

Photograph courtesy of Julius Zelmanowitz, MSRI.

Kaplansky at the keyboard.

Nowadays, most math departments offer a "bridge" course for their majors. This course is designed to ease the transition to real, upper-division mathematics from (increasingly) less rigorous calculus courses. Chicago has had such a course for years. In my day, it was Math 261; at present it has the fashionably inflated number 26100. Currently, just as it did several decades ago, the course covers "sets, relations, and functions; partially ordered sets; cardinal numbers; Zorn's lemma, well-ordering, and the axiom of choice; metric spaces; and completeness, compactness, and separability." When I took the course, Kap used notes of Ed Spanier on "Set Theory and Metric Spaces". Spanier never got around to writing these notes up as a book. Kap, however, did! *Set Theory and Metric Spaces* appeared in 1972 and continues in the AMS Chelsea series. Kaplansky's style is as appealing to current students as it was to us decades ago. I have used the book in my classes for many years. One of my recent students enjoyed the book so much that she bought it as a birthday present for her engineer father!

As chairman, Kap maintained a keen interest in graduate students and the graduate program. His sensitivity to grad student-advisor dynamics can be illustrated by the following anecdote. One afternoon at math tea, my advisor, Yitz Herstein, and I got into a "discussion" on how Kap (of Canadian origin like Yitz) pronounced "schedule". I maintained that Kap would pronounce it with an "sk" as Americans do and Yitz, of course, said that Kap would say "shedule", as Canadians and Britons do. So, Yitz and I bet a quarter. When Kap arrived at tea, Yitz and I bounded up to him and told him of our bet. Kap thought for an instant and, then, carefully pronounced "skedule" remarking that faculty shouldn't take money from students and that Yitz "should pay up." However, I only got 15 cents.

Kap's rhetorical flourishes are well known; but, sometimes they had unintended consequences. For my first job, I needed official certification that I had completed the Ph.D. A letter from the chairman would suffice. Kap wrote such a letter concluding "…and, barring catastrophe, he will receive the degree on June 11…." This was deemed insufficient by a departmental administrator at Berkeley who quoted the "barring catastrophe" remark. Kap washed his hands of it and sent me off to the Dean of Students in the Division of Physical Sciences for a "really" official letter.

Even, at the last moment, during my final oral exam, Kap's style was apparent. He asked me where would you find a commutative ring with some property or other. I started to construct the ring when he interrupted: "No, no, in what book would you look for it?" I replied, "Nagata" and was off the hook!

Kap's lessons and advice remain fresh to this day. His books and his expositions are as attractive to the current generation of students as they were to mine.

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Manjul Bhargava

Kap Across Generations

I was a graduate student at Princeton in the year 1999. And being a student of algebra, I obviously knew of Professor Kaplansky, though I knew of him more as a "legend" than as a person. His name was one that was attached to a number of great theorems, some going back to the 1940s. At the time I suspect it never occurred to me that he might be an actual person who was still doing great mathematics.

While working on my dissertation, I became interested in a classical problem from number theory relating to quadratic forms. (It was not really a problem in the "Kaplansky style", or so I thought!) The question was: When does a positive-definite integral quadratic form represent all positive integers? (For example, Lagrange's Four Squares Form $a^2+b^2+c^2+d^2$ gives such an expression—i.e.,

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every positive integer can be written as a sum of four square numbers.) This was a beautiful question of Ramanujan that Professor Conway taught me about and got me hooked on.

After working on the question for some time, I realized that some good headway could be made provided that one could understand the classification of what are known as "regular ternary forms". In particular, I needed to know: How many such regular ternary forms are there? I did some searches on MathSciNet, and soon enough found a 1997 (!) paper by W. Jagy and I. Kaplansky entitled: "There are 913 regular ternary forms".

Here was the exact answer to my question in the very title of a paper written only two years ago! It was quite exciting, and I thought to myself "Surely this is not the same Kaplansky!," but after some research I soon discovered that it was.

I emailed Jagy and Kaplansky later that week, and heard back from both almost immediately. Kap and Will (Jagy) were also both very excited that their recent work had found applications so soon. I mentioned to them that I would be in Berkeley for a few weeks that summer to learn tabla with my teacher, and Kap kindly invited me to visit MSRI while I was there.

Kap asked David Eisenbud, the director of MSRI, to give me an office for the summer, and David generously agreed. That summer turned out to be one of my most productive summers ever. I worked on mathematics during the day and played tabla by night. Rather than working in my private office, I found myself mostly working in Kap's office! We didn't really work together, but rather we worked independently and then shared what we had discovered or learned at various intervals throughout the day. Kap, Will, and I discussed and learned various mathematical topics together in what were some extremely enjoyable sessions. Kap's love, enthusiasm for, and unique view of mathematics were constantly evident and always inspiring!

In addition, I talked to Kap a lot about other things; we shared common interests not only in mathematics but also in music, making it a rather frequent topic of conversation. In the process, I also learned a great deal about Kap's amazingly regular life and his other associated charming idiosyncrasies. He brushed his teeth more often than anyone I've ever known. And no matter how exciting a particular conversation or work session was, if it was time for his daily noon swim, then there was no stopping him from running off to the pool! (The same occurred when it was time for his chosen 5:14 p.m. end-of-the-day bus from MSRI.) I found myself changing my own schedule to match his work schedule better (including waking up rather early!).

The same schedule was adhered to the following few summers, as he always generously invited me back (He would write, "Looking forward to renewing our sessions!," and there were always new and exciting things to discuss; every year I looked forward to it.) Until the very last summer, when I heard the sad and devastating news. I've since always felt that it was unfair that I got to know him only toward the later years of his life. Of course, deep down I know I should be grateful that I got to meet him at all, and to have been one of the lucky ones in my generation to have had the privilege of knowing him. He was so encouraging to me always, as a person, as a musician, and most of all, as a mathematician. I will always cherish the memories of his enthusiasm, brilliance, generosity, and friendship. I will miss him very much.

David Eisenbud

Kap at MSRI

Kap was enormously influential in many fields of mathematics, through his papers, through his books, and perhaps most of all through his Ph.D. students and the many many additional students who, like myself, listened raptly to his courses. I remember well his highly entertaining and beautifully polished lectures from my student days in Chicago —whatever he taught, I signed up for the course, it being such a pleasure to listen to him. From being on the first winning team of the Putnam competition to being president of the AMS and National Academy member, his career was truly remarkable —you can find more information starting from the AMS website, `http://www.ams.org/ams/48-kaplansky.html`.

As the second director of MSRI, Kap served the Institute directly from 1984 through 1992. He greatly developed the reputation and influence of MSRI, building on the start provided by the founders, Chern, Moore, and Singer. My own first experiences at MSRI were under Kaplansky's directorship. As with everything he did, he paid attention to every detail of the operation—he boasted to me once that he personally read and signed every single letter of invitation that the Institute sent out during his eight years in office. He and his wife, Chellie, were also very present and available to the members—literally thousands will remember Kap's musical performances at the Christmas parties. Among the many marks Kap left on MSRI was the start of fundraising activity. For example Kap formed the "International Board of Friends of MSRI", and the connections made through this group are still of the utmost importance to us. Kap's first paper appeared in 1939. After stepping down as MSRI director, at seventy-five, Kap went back to full time research mathematics, and returned to number theory, one of his first loves.

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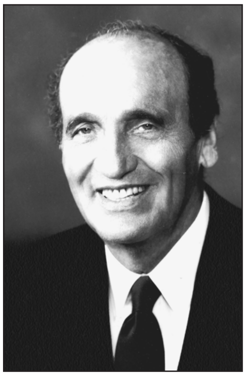

Photograph courtesy of Julius Zelmanowitz, MSRI.

AMS president, 1985–86.

Some of his most recent work, on integral quadratic forms, was published in 2003, when he was eighty-six.

Mathematically, Kap was my brother: he, the first student of Mac Lane, I, nearly the last. But he was much more an uncle to me who had been down most of the avenues that I later began to explore. He was always generous in advice, counsel, and in giving credit. I saw him nearly every day in my student days at Chicago, and again, nearly every day, over the first eight years I was MSRI director. Interacting with Kap was always a pleasure, crisp, clear, and somehow uplifting. It is one that I shall deeply miss.

John Ewing

Kap and the AMS

For more than forty years, Irving Kaplansky was active in the American Mathematical Society, and for much of that time he was a driving force. He began as associate editor of the *Bulletin* at the age of twenty-eight in 1945—the same year that he joined the faculty of the University of Chicago. Two years later, he became an editor for the *Transactions*, and ten years after that he was an editor for the *Proceedings*. He thus served as editor for the entire complement of AMS journals at the time.

In addition to his role with journals, Kap was active in the Society's governance for many years. He was elected to the Council in 1951 as a young faculty member, and later was elected to the Board of Trustees (as an older one). He was elected vice president in 1974, putting him back on the Council, and finally in 1985–86 was elected president of the AMS. All together, he served a total of ten years on the Council and seven years on the Board—a great many meetings for anyone!

The four years from 1984–87, which included his time as president elect and past president, were particularly eventful for the Society. Kap played a key role in every one of those events. The AMS hosted the 1986 International Congress, which took place in Berkeley; Kap was on the local organizing committee and oversaw many aspects of the Congress. The Society was undergoing some radical changes during this time, including its recovery from a disastrous financial situation earlier in the decade and a restructuring of Mathematical Reviews administration; again, Kap played a key role in reshaping the AMS. And it was during this period that the AMS decided to create a premier journal—the *Journal of the American Mathematical Society*. Kap was the one who championed this idea (which came from the Committee on Publications) and helped bring the journal to life by carefully choosing the first editorial board.

The most remarkable feature about Kap's service to the Society was his style. In every job he undertook—in everything he did—he was forceful and yet graceful, eloquent and yet thoughtful, energetic and yet polite. When he received the AMS Steele Prize, Career Award in 1989, the citation acknowledged that style by honoring him for "his energetic example, his enthusiastic exposition, and his overall generosity." It went on to point out that he "has made striking changes in mathematics and has inspired generations of younger mathematicians."

Kap left his mark on many parts of mathematics, but he especially left his mark on the Society.

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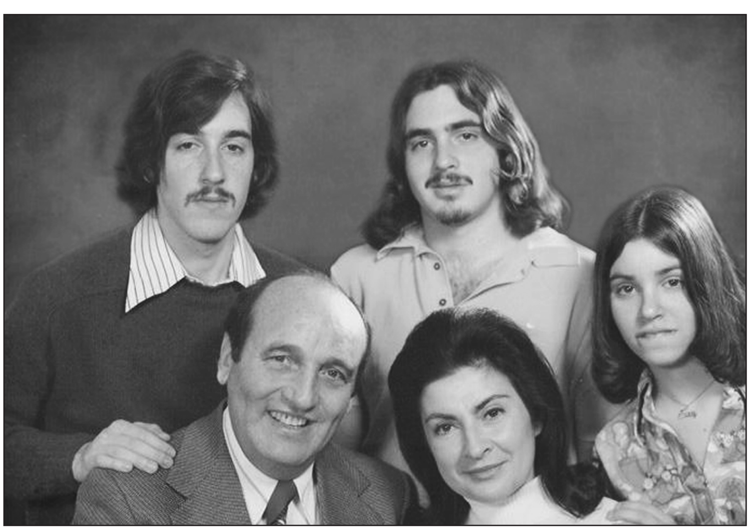

1972 family photo. Top row, left to right: Steven, Alex, and Lucy. Front: Irving and Chellie Kaplansky.

Lucy Kaplansky

Kap Was My Father

My dad, Irving Kaplansky, was a mathematician, but he was also a teacher, and he taught me many things. When I was a little girl he taught me to play checkers. In our games together he would start with half his checkers, and he'd beat me anyway. But whenever I played checkers with other kids, I demolished them. He got a huge kick out of that.

He taught me math. I would come home from school when I was in grade school and high school and he would re-teach me that day's math lesson. He was always patient and clear, and he made it all make sense. I'd go back to school the next day, and

Singer-songwriter Lucy Kaplansky lives in New York City. Her website is `www.lucykaplansky.com`

often I was the only one who would understand what was going on in math class.

I ran into a couple of my math teachers from grade school recently and they told me when they found out I was in their classes they were petrified because they knew exactly who my dad was!

My dad taught me to be organized in everything, reliable, and punctual. I think I'm the only musician I know who always shows up on time and actually does what I say I'm going to do.

He taught me that I should love what I do for a living. Throughout my childhood he would sit in his study, classical music always on the radio, doing math. Sometimes he'd look like he was doing nothing, maybe even sleeping, but he'd always say he was "thinking mathematics". He instilled in me one of the central ideas that has informed my life, that making money for money's sake was not important, that doing work you love is everything.

I asked him once why he loved math. He responded simply "it's beautiful."

He taught me that learning was fun. He especially loved learning about history and he was forever reading about and discussing history, all kinds. Because of him, I, too, love to learn about history; because of him I love to learn, period.

And perhaps most of all he taught me to love music. He was a gifted pianist, and there's a story I've heard my whole life that when he was three years old he and his family attended a Yiddish musical in Toronto, and when they got home he sat down at the piano and played the show's main song perfectly, note for note.

From as early as I can remember I would sing while he played the piano. He taught me dozens of songs from the 1930s and 1940s, as well as from Gilbert and Sullivan operettas. I still remember most of these songs.

When I was older and pursued a career as a singer-songwriter, I started performing songs that he had written; one of the most popular was "A Song About Pi". To this day it's one of my most requested songs.

When my dad was already in his eighties, my parents often went on the road with me when I was doing concerts. We'd all get in the car and stay in hotels, and he would sell my CDs for me after the show, sometimes he was even asked for autographs. And if there was a piano on stage he would accompany me on a couple of his songs. He always brought down the house. I'm so grateful we were able to share this. The last time he sat in with me onstage he was 88 years old.

I've heard from so many of my dad's students over the years what a wonderful teacher he was. I know that. He was my teacher.