MEMORIAL TRIBUTE

Remembering Jean Bourgain (1954–2018)

Introduction

Peter Sarnak and Terence Tao

Jean Bourgain was a major mathematical force throughout his entire career, from his student days to his untimely passing on December 22, 2018, at the age of 64. His profound and prolific research achievements in a remarkably broad array of fields will be studied and written about for years to come.

Remembrances and volumes in his memory have and continue to be produced around the world. The website `https://www.ias.edu/events/honoring-bourgain` contains recordings of the mathematical lectures and remembrances from the memorial "Honoring the Life and Work of Jean Bourgain" that took place at the Institute for Advanced Study in Princeton on May 30 and June 1, 2019, as well as recollections by Jean's colleagues at IAS, and letters from his family. A collection of survey articles on various aspects of Jean's mathematical impact by K. Ball, C. Demeter, C. Kenig, and T. Tao appeared in the *Bulletin of the American Mathematical Society*, vol. 58 (2021), no. 2, 155–223; we also point to the detailed tribute "Singular Adventures of Baron Bourgain in the Labyrinth of the Continuum" by A. Gamburd that appeared in the *Notices of the American Mathematical Society*, vol. 67 (2020), no. 11, 1716–1733.

This current article consists of more personal recollections of Jean by a small set of his collaborators and friends. It is impossible to do full justice to what Jean was as a mathematician and as a person, but we hope that the personal remembrances below, together with the other memorials in his honor, will at least give some glimpses of Jean's extraordinary talent and character.

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Ingrid Daubechies

My acquaintance with Jean Bourgain goes back a long way: we met in 1971, when we were both 17, sometime in the first weeks of our freshman year in college, at the Vrije Universiteit in Brussel (VUB). As is customary in many European college systems, we had to declare our major at enrollment; for Jean this was Mathematics (of course!), but mine was Physics. The VUB was then a rather small university that had just fledged as an independent institution one year earlier—before then, it had existed as the Dutch-speaking-and-teaching wing of the otherwise francophone Université Libre de Bruxelles (or ULB; in English the names of both the VUB and the ULB translate to the Free University of Brussels). The ULB was founded in the early 19th century, with the explicitly stated goal to be completely independent from state and church or any dogma, as reflected by its motto "Scientia vincere tenebras," or "Through Science conquer Darkness"; in more recent times, the VUB adopted the slogan "Redelijk Eigenzinnig" or "Reasonably strong-willed." This philosophical stance was important to Jean and me (and our parents) when the time came to pick among the many excellent college educations in Belgium.

In those early years at the VUB, students majoring in physics had many courses in common with the mathematics majors. In their first year, physics majors had more stringent courses in physics and chemistry (both of which were also subjects for math majors!) but we were excused

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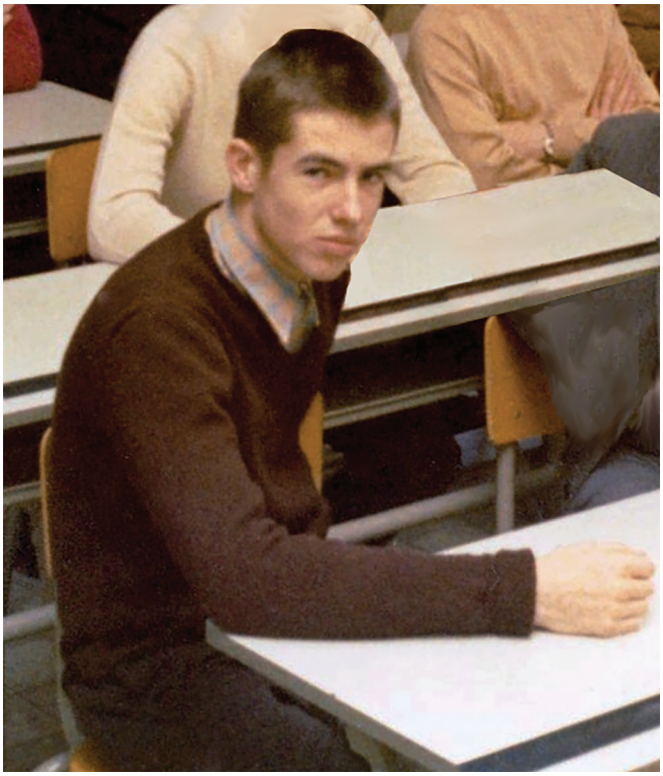

Figure 1. Jean Bourgain in spring 1971, cropped from a picture of his senior class in high school.

from projective geometry. Starting with the second year, there were a few more differences—in particular, physics majors had a beautiful course on optics, in which I learned that a (perfect) lens performs a Fourier transform, and in the lab of which we made holograms, a fantastic experience for a sophomore in 1972. This made a profound impression on me; I had been wondering about switching to a mathematics major, but this course affirmed my resolution to major in physics, a decision I never have had cause to regret (even if later in life I switched anyway and now pretend to be a mathematician).

Jean's and my courses of study diverged after the second year; nevertheless he and I shared a total of about 10 math classes over the first two academic years, from calculus and linear algebra to topology and complex analysis. As a small university, the VUB prided itself on providing a nurturing experience, with many tutorial sessions and problem seminars. As the two strongest students in mathematics, Jean and I had soon spotted each other. Jean already knew more mathematics than I did—one day an engineering student challenged us to find the curve in a pursuit problem, and he knew tricks to solve differential equations that I had never heard of. He was the first boy I met who was at least as good at math as I—and so I promptly developed a crush on him. In the weekly two-hour problem sessions held for each of our math courses, students were typically handed a sheet with problems at the start, and would then try to solve them, with help from a circulating TA. Jean and I would race each other; I earned his grudging respect by beating him a fair share of the time—and I am sure he was not letting me win! We hung out with each other a great deal the first year, talking math most of the time when we were not in classes, going to empty classrooms to use the blackboard. When I occasionally proposed that we talk about something else, or play a game, Jean would be puzzled.

After the separation of the summer (Jean's hometown was at the other end of the country, all of 70 miles away from mine!) my crush had abated, and I acquired a boyfriend among my fellow physics majors. He was a great storyteller, a deft lab partner, and a gifted piano player and composer—but he was not as good at math. Jean was taken aback by this peculiar wish of mine, choosing to spend time with someone for whom math was not the center of the universe. One incident has stuck in my mind: at the end of the Fall semester, after a written math exam that Jean and I had both finished early, we were chatting in the hallway outside the exam room, discussing how we had tackled different questions; when my boyfriend came out, wiping his brow and thanking me for coaching him beforehand, which he felt had made all the difference for him, I heard Jean mutter to himself, as he walked away—he sincerely doubted that guy had managed to deal with the math subtlety we had been discussing.

In the last two years of college, our paths crossed less often. After the final "proclamation"—the occasion where the results and degrees for the VUB '75 cohort of Mathematics and Physics majors were announced officially—all of us (not so large a group—fewer than 30 in all) decided to go out for some beers in one of the cafes on the Brussels Grand-Place. Most of us had jobs lined up; Jean and I had both gotten PhD fellowships at the VUB (in Belgium, graduate students then typically remained in the institution where they had obtained their undergraduate degree), as had some of the others (including my then-boyfriend, who recently retired from an academic career as an experimental semiconductor physicist). Yet others went on for careers in high school teaching, in various companies, or in the civil service. Although subsets of our group would reunite again at weddings and other celebrations in the years after, it was the last time we were all together. We reminisced and laughed about the previous four years; after a while, stimulated by the beers we had consumed, we started playing silly party tricks. At some point an old riddle was proposed: three friends go to a bar, and have different drinks, but decide to split the bill evenly. The total is for 25 francs; each of the three puts down a coin for 10

francs, which the waiter picks up, returning five coins of 1 franc to the table top. The friends now take each 1 franc back, and leave the remaining 2 francs on the table as tip. Wait a minute—there must be something wrong here—since they each got 1 franc back, they had effectively paid 9 francs each; together with the 2 francs tip, that makes a total of $27 + 2 = 29$. But we started with 30 francs... where has the missing franc gone? Jean had also drunk quite a few beers by then, and the stupid riddle had him stumped for several minutes, to the immense (good-natured) hilarity of everyone else. Even decades later, when some of us met, the memory of that incident could get us laughing all over again.

Over the ensuing years, Jean and I often would run into each other; even though we were not close friends, we always had the easy relationship of people who have known each other for a long time; we typically chatted with each other in Dutch, as we had as college students, no matter where we found ourselves or how long it had been since we last spoke that language. I was glad to be at the ICM in Zurich where he was awarded his Fields Medal, and pleased to be the first to congratulate him as he walked down from the podium. At that point he was at IHES, and I was in the US, transitioning from Bell Labs to Princeton University. Not long after that, we lived for a while both in the same small town in New Jersey, half a world away from where we had first met—it is a small world! Many years later (and many awards for Jean later), it was with great pleasure that I found myself congratulating him again, this time in name of the International Mathematical Union, at the ceremony where he was awarded the 2017 Breakthrough Prize in Mathematics. He was already battling the cancer that took him away; we were all hoping he would beat it, and are sad that it was not to be.

A fellow student from our college cohort passed on the picture shown in Figure 1—in those days before smart phones, we took many fewer photos of our daily lives, and it is the only picture we found of Jean during this time. Sadly, he is not the first one of our group to disappear, but he is clearly the most distinguished and maybe also the most idiosyncratic member of our group, and we are sorry he has left us, much too young. We all, and I in particular, will continue to remember him with great affection.

Freddy Delbaen

Jean started his math studies in 1971. His father was a well-known professor at the faculty of medicine of the VUB (Vrije Universiteit Brussel, Free University of Brussels). Although the distance between his home in Ostend and the university was more than 100 km it was normal that he would study at the VUB. From colleagues I heard that there was an extremely gifted student in the first year maths. The same year there was also a gifted student in the physics program (Ingrid Daubechies). Because I was not yet lecturing in the math department, I did not immediately have contact with Jean. It was only in 1973 when Jean entered the "licence" (the forerunner of what is now called master's degree), that I had him in my probability lectures. I was just a starting professor and had to prepare my lectures and the exercises. My teaching assistant immediately noticed that Jean was not just a quick problem solver, he also found very elegant solutions. It was part of the program that students would study something on their own. I decided to ask Jean to look up the weak sequential completeness of L^1 and, related to it, the characterization of weakly compact sets—the so-called Dunford-Pettis and the Vitali-Hahn-Saks theorems. The techniques used were far above the mathematics that were available in the third year of the studies but for Jean it was a piece of cake to master the techniques and to understand them.

In the last year of the math program Jean wrote his master's thesis under my supervision. At that time I was interested in Banach space theory. One of the popular topics was the generalization of the Radon-Nikodym theorem for vector-valued measures. The theorem describes when a measure has a density. It is only valid for Banach spaces in which closed bounded convex sets have a special kind of extreme points. This property is called RNP. Among the spaces with the RNP we have separable dual Banach spaces and their subspaces. Reflexive spaces clearly satisfy the RNP and therefore weakly compact sets in general have this special kind of extreme points. Several proofs were available but a direct geometric proof was missing. In his master's thesis Jean managed to give a purely geometric proof of the fact that every weakly compact convex set in a Banach space is the closed convex hull of its strongly exposed points. At the same time he discovered that a Banach space X has the RNP if and only if every operator from X into a Banach space Y can be approximated (for the norm topology) by an operator that attains its norm on the unit ball of X (the Bishop-Phelps property). That these two classes of spaces were the same was conjectured by Diestel and Uhl in their book on vector measures.

For his PhD, Jean looked at compact sets of Baire-1 functions on separable complete metric spaces (Polish spaces). Baire-1 functions are the pointwise limit of a sequence of continuous functions and the connection with Banach space theory comes from Rosenthal's theorem. This theorem says that for a given uniformly bounded sequence of functions defined on a set, either there is a subsequence

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that converges pointwise or there is a subsequence that is equivalent to the l^1 basis (for the supremum norm on the set). Some of the results were published in the Bourgain-Fremlin-Talagrand paper. During his PhD Jean and I participated in several conferences on Banach spaces. Jean quickly got the reputation that he was easily solving difficult problems. For some people this was a reason not to mention their research problems.

One of the most spectacular stories is probably how Jean solved the $\mathcal{L}_\infty$ problem. One afternoon (Jean always came in after lunch) Jean asked what I was working on. I told him that there was a conjecture of Lindenstrauss on $\mathcal{L}_\infty$ spaces. At that time he did not know the definition and properties of these spaces so I explained to him that such a space X was the closure of the union of finite-dimensional spaces that were uniformly isomorphic to finite-dimensional l_∞ spaces. There were several conjectures about it. These spaces were introduced by Joram Lindenstrauss and one characterization goes as follows. The space X is an $\mathcal{L}_\infty$ space if and only if every compact operator defined on a subspace Y of another Banach space Z can be extended to a compact operator on Z. This Hahn-Banach type property can also be asked for weakly compact operators. But for these kinds of operators there were no characterizations and it was believed that spaces having this weakly compact extension property would be finite dimensional. This conjecture is weaker than the conjecture that $\mathcal{L}_\infty$ spaces would contain a subspace isomorphic to c_0. I was mainly interested in the weakly compact extension property where I had shown (among other things) that such a space necessarily had the Schur property, saying that weak compactness and norm compactness were the same. For many people it was counterintuitive that such a property was possible for spaces that looked like l_∞ or c_0. But there was no proof of this. After I talked about the definition of the $\mathcal{L}_\infty$ spaces and some of their properties Jean returned to his office. After 10 minutes he came back and said he could make an $\mathcal{L}_\infty$ space that would have the Schur property. The next day Jean had worked out some of the details and indeed he had found such a space. We then checked whether it had the RNP property. This took us a couple of minutes and since separable dual spaces cannot contain an infinite-dimensional $\mathcal{L}_\infty$ subspace, a conjecture on RNP spaces was solved almost for free. Later we modified the construction to get an $\mathcal{L}_\infty$ space such that every infinite-dimensional subspace of it has an infinite-dimensional subspace that is reflexive. Many researchers had worked on these $\mathcal{L}_\infty$ conjectures and Jean solved them in 10 minutes, unbelievable. When I told colleagues about the time Jean needed to solve the conjecture they politely said that I exaggerated. Of course I exaggerated, he needed more than 10 minutes, probably it was

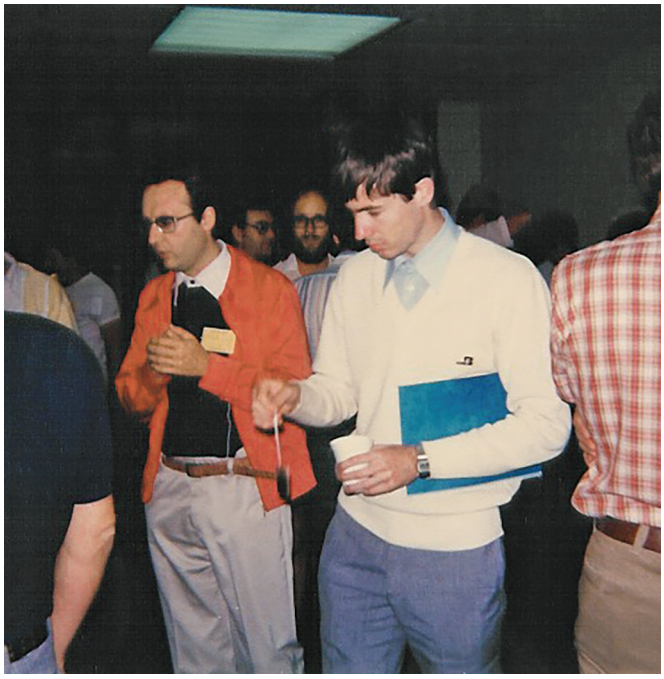

Figure 2. Freddy Delbaen and Jean, 1979 conference in Kent.

15 minutes but that includes the time to finish his coffee. How did he manage? Probably he got the inspiration from his understanding of the Schauder basis of $C[0,1]$. We had discussed this topic before and for Jean the imbedding of the finite-dimensional spaces spanned by the first basis vectors had some geometrical meaning. It was this meaning that he translated in his construction. The techniques were later used in different ways and in different problems. The paper is not the deepest or most difficult result Jean obtained but compared to the number of citations and the use of the techniques, it certainly has the highest time-efficiency value.

When colleagues asked how it feels to have a student like Jean, my response was always the same. Jean was my student only for a small amount of time. I learned more maths from him than he learned from me. Pełczyński once quoted Zygmund: when after five years a professor has a student who is much better than himself, he can retire. When he does not have a student who is better, he must retire. For me it became more and more difficult to follow Jean's mathematical progress. By the time I had read one of his papers, he already had finished two others. When he saw a problem he quickly understood what was the essential part of it. He had a lot of tricks (called techniques) to transform the problem to its basics and then solve it. Sometimes he needed more time to rebuild the original problem, sometimes new difficulties showed up, but most of the time he was right.

After his PhD and habilitation, Jean got more and more interested in what he considered the real topics in mathematics: number theory and nonlinear partial differential equations. I got more interested in the mathematics of insurance and in mathematical finance. Mathematically I lost contact with him. Each time I visited Princeton University, Jean and I had dinner or lunch; sometimes Ingrid organized a "Belgian" dinner. We then talked about more human things.

Many colleagues have asked me if I knew whether Jean worked on the Riemann hypothesis. I cannot answer this question, but once—when he was still in Belgium—he told me that he saw no good idea to tackle the problem and hence he did not want to concentrate on it. Nevertheless he worked on problems related to the hypothesis.

When Jean became ill, he moved to Belgium and stayed with his sister who is a pathologist. Jean's health condition was getting worse and the chemotherapies had an effect on his working abilities. My wife Rita and I visited Jean on our visits to Belgium. Of course he knew that one of these days the treatment would not be sufficient anymore but as long as he could do mathematics he could go on. When he did not answer our emails we realized we were going to lose a great mathematician and friend.

Larry Guth

I was lucky to get to work with Jean. I learned a huge amount from him and it changed the direction of my career. Most of our work was over email, and we only talked occasionally in person, but I remember some conversations vividly. In general, his manner was formal and a little conservative, but when he was excited about a piece of mathematics, his eyes would twinkle. I haven't seen this expression captured in a photograph, but it's the thing I remember most vividly from talking with him. For instance, discussing a certain analysis estimate, he might first describe a simple approach, and then he would say, "but that would be too costly," and his eyes would twinkle as he explained how to get a better estimate.

I read some of Jean's work on restriction theory when I was a postdoc. It made a big impression on me because of the mix of analysis, geometry, and combinatorics. I first met Jean in January 2010 when I visited IAS, and we started to talk about a question in restriction theory. That conversation led to our first project. In 2010–11, I spent the year at IAS, working out that project with Jean. He started working on the project while I was moving. He would write up his ideas and scan them and send them to me. By the time I arrived at IAS, he had sent me fifty-some pages of notes about our project, and they kept coming. I spent the whole year trying to catch up with the flow of these notes. In hindsight, I should probably have gone and asked him more questions, but I was shy. Those notes became our joint paper, and it took me two years to read the whole paper. This experience was a big part of my education in the field.

At Jean's memorial conference, Elon Lindenstrauss said that he was at the same time very generous and very competitive. Jean was always very generous giving me credit for contributing to our two joint papers. I also saw a little of his competitive side. Before our paper on restriction theory, the best restriction estimate proven for the sphere in $\mathbb{R}^3$ was for exponents $p > 10/3$. Our approach gave a very different proof for this same range $p > 10/3$. Jean told me that he worked "perpetuum mobile" (like a perpetual motion machine) to beat that bound. He found an intricate argument, building on our approach but with a lot more to it, giving the range $p > 3.3$. He included it in our "joint" paper.

After that, I didn't talk with Jean for several years. In 2014, Jean and Ciprian Demeter proved a major conjecture in restriction theory called the decoupling conjecture. This paper came as a complete shock to me. The proof is to some extent related to the paper that Jean and I wrote the year I was at IAS, and I didn't think those kinds of techniques could possibly prove such a result. I've spent most of my research time since then trying to understand that proof better—extending it in different directions, giving seminars about it, trying to find alternate proofs, etc.

In the summer of 2015, I learned that Jean had cancer and that he was seriously ill. He came to Boston for surgery. We emailed about meeting before the surgery, but it didn't work out.

That fall I was thinking about a problem Jean and Ciprian had raised about decoupling for the moment curve, which was connected to Vinogradov's mean value conjecture in number theory. I had an idea about the problem which I was excited to share with them. But on the other hand it was only a few months since Jean's surgery, and I wasn't sure whether he would want to hear about math. I was going to Princeton to give a talk. So the week before my talk, I decided to look on the arXiv to see whether Jean had done any work since the surgery. It turned out that, since his surgery, he had put out far more papers than I had. So I wrote to Jean and Ciprian with my thoughts about the moment curve, and Jean and I agreed to meet that Monday in Princeton before my talk.

When I met Jean in the Princeton common room, I wasn't sure how he would look after the cancer treatment. His face looked longer and he was bald. He was friendly

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and sharp, but he also seemed tired. He came to my talk, and during the talk he was writing something. After my talk, we went back to the common room and he showed me how to get a sharp decoupling estimate for the 3-dimensional moment curve. I believe he worked it out during my talk. He explained at the blackboard. He wrote on the board at high velocity, and when he got to the end of the board, he would erase it with one hand while writing more with the other hand at the same time. In this way, he filled ten boards without pausing. At the end of the argument, he said, "So I can still do a computation on the board," and I saw the twinkle in his eyes.

Svetlana Jitomirskaya

As Jean has had very few PhD students, many think his influence on other mathematicians, enormous as it is, has been largely through his work. Indeed, on top of all the ideas and techniques he introduced, simply getting through even one of Bourgain's articles can take one to a different level in math. However, in addition to the superhuman impact of his math, there is also another, less direct but very human, dimension to his impact.

For who is a true mentor? This is a person whose role in your life does not end with your receiving a PhD diploma or even the approval of tenure, and may even only start then. Mentors are people who influence us the most, not just through their work but also through their combination of high standards and targeted support, shaping what we work on and how we work on it. A true mentor treats you as if you are a better mathematician than you are, and then you actually become that better mathematician. Somebody, whose "this doesn't look entirely trivial" may mean more to you emotionally than an *Annals* acceptance. In all these ways, Jean has been a great mentor to many, often without them or even Jean himself realizing it.

My first encounter with Jean happened in late 1994, when he, a freshly awarded Fields medalist, visited UCI along with Mei-Chu and little Eric, a precocious bubbly two-year-old. I remember being struck by how the matter of most pride for Jean was clearly Eric, and not anything else. Yet he left time for math discussions, and people signed up for brief meetings with him. Definitely not me though: a recently hired assistant professor, who started out at UCI as a part-time lecturer, I felt like the lowest person in the department who had no business trying to meet with famous visitors. It was all the more a true Cinderella moment, when Jean suddenly requested to meet with me: he had seen my paper, to appear in *CMP*, and wanted to know more details. We talked for maybe 15–20 minutes, and had no further contact for the next five years. In the meantime, my UCI colleagues asked Jean to support my Sloan application the following fall, after which he also supported my promotions and likely other things that I don't even know about. So generously supporting a person one has hardly met, based essentially on one paper, is highly unusual, especially if the paper in question is not obviously prominent. Yet it was typical for Jean.

Moreover, unbeknownst to either of us, our 1994 encounter was the start of his lifelong role as my mentor. The sole fact that he liked the *CMP* paper encouraged me to keep developing the same direction. I eventually figured out the last piece of the puzzle which led to the solution of a problem that has brought me much recognition. However, the most important impact of this result for me is in Jean's saying that it had influenced his work and inspired him to enter the field of ergodic operators. His involvement has since been transformative for the field: it has revolutionized it, opened new vistas, and made it a lot more compelling for others. For me, it also marked the start of our collaboration and communication.

In the beginning, we talked very little about the papers we published together. Our first paper resulted from a five-minute discussion, literally on a napkin. It was as if we both knew something that felt worthwhile to write up and were amazingly on exactly the same page. In less than two weeks, as I was preparing to start working out the details, Elly Gustaffson sent me the pdf file, and Jean apologized for the rush explaining that he promised something to Milman and this was handy. Our second paper originated from an even shorter conversation, without even a napkin to aid. After Jean's lectures at UCI, he invited some of the participants to join his family on a fishing trip departing from the nearby Newport Beach. Despite Jean's bragging about huge yellowtails he was regularly scoring on similar trips, the total catch for the group that day was a single small but very spiky rockfish that my daughter then insisted on setting free. Jean still looked happy: he clearly was there for the experience. For me it was the opposite: I spent all four hours suffering from severe sea-sickness, exacerbated by the realization that I was wasting such a perfect opportunity to talk with Jean. I was able to collect my strength only when the boat almost returned to the dock, and we talked for a few minutes, again agreeing on something that could also be done. This time I insisted on writing it myself. Five months later, I finally sent Jean a draft, admitting that I had to settle for a result not as good as envisioned in our fishing trip discussion because I was stuck on not being able to remove an unnecessary technical condition. To this Jean immediately replied with "let me have a look," and by 1 a.m. the next morning sent me a lemma that took care of the issue. Once the

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paper was ready, he said he promised it to *DCDS*. I then cautiously asked whether we could try *Acta* or *Inventiones* instead. He said "That is OK with me if that's what you prefer. I told the *DCDS* guy he would get it within a week, but I could send him something else then." Our third paper, that later proved very fundamental and influential, was also promised, this time to *JSP*. Jean was very generous to journals, and often published well below where he could, which is especially true about his solo papers.

Our discussions gradually grew deeper and were not generally tied to our joint work. Jean had a remarkable ability to lift his conversation partner up, and subtly encourage them. Almost every contact with him left a profound and lasting effect on my confidence. I am not as fast as many, and was very self-conscious about that, when I was younger. This had made me especially nervous when communicating with some senior people; I often felt almost as if I was turning stupid in their presence. With Jean, the effect was miraculously the opposite. From that very first encounter in 1994, it was always easy for me to talk to him, as if his subtle encouragement had been making me instantaneously smarter. My best real-time mathematical triumph happened when I found a simple counterexample to discrete unique continuation during Jean's talk when he claimed he didn't know if it was true or not. Jean may have thought I was always like that. Little did he know it was exclusively in his presence! In general, my growing communication with him was a big matter of my internal pride, something that encouraged me almost more than anything else.

Incidentally, Jean admired those who were fast, but would say that he himself was slow. "IMO problems? Give me a week, I can solve them. But in four hours? Forget it! I have no idea how these people do it." This is from someone who only needed a few hours to produce a lemma that I had been stuck on for several months. My attempts to express my admiration of the latter fact would be met with "When you've been in this business for as long as I did, you know a trick or two."

In August 2001, during a long connection to a flight to Rio at the Miami airport, I heard from a loudspeaker: "Jean Bourgain, your party is waiting at Gate B37." Naturally, I went there and met Mei-Chu and Eric who were already getting nervous. Some more time had passed until Jean arrived, with a big smile (my thought was: "he just proved a theorem!") and calmly informed Mei-Chu that he forgot his medicine in the checked luggage. Apparently this problem can also be solved, and the medicine was retrieved. I was struck by how calm Jean was, completely undisturbed by his misadventures. We happened to be on the same overnight flight, which was not that much of a coincidence since we were both traveling to the same conference.

As far as I can tell, the entire night Jean was reading papers. He had a big pile on his tray table, and was making notes on the margins. When we arrived in Rio the next morning, I asked Jean what he was reading. "Oh, just some refereeing." This is another thing about Jean: he was a remarkable citizen of the community. He knew everything that was going on in analysis, took his leadership very seriously, and was an enormous asset to journal editors and others seeking unbiased input. He would also read and appreciate the papers of others, independently of refereeing, something few of us do. In the last few years, confined in Belgium for medical treatment, he would watch videos of presentations from all the interesting meetings. I heard of others calling Jean competitive, but it is not something I ever observed. He was interested not in himself in math, but in math itself, and greatly supported others who produced good math.

My last in-person meeting with Jean was in December 2015, when I visited him for a day in Princeton, and we started a new collaboration. He was in the middle of a devastating illness, but was very happy: math was going well. Indeed, the Vinogradov's conjecture paper was recently finished, and he was full of other plans and ideas. His optimism was inspirational. Later, even when things got really bad healthwise, he still projected strength, humor, and kindness in every conversation. It was overwhelming for me to learn a few weeks ago that the nomination for the prize I got recently was initiated by Jean in 2017, when "treatments," as he called them, left him with precious little time to enjoy what he loved. His selflessness was unreal.

Jean's support, influence, and mentorship have fundamentally changed my life, and they persist to this day in many different forms. I keep learning from and being influenced by his ideas as well as measuring my work by thinking of how he would have reacted. I try to emulate his ways when dealing with young unconfident people around me. His memory inspires me to try to be both a better mathematician and a better person. And I am just one of many people for whom Jean was a true mentor.

Alex Kontorovich

Jean Bourgain had an immeasurable effect on my life and career, and our relationship went from postdoc advisor, to collaborator (on a dozen papers over a decade), to friend.

Many have reminisced about the challenges faced when embarking on their first reading of a Bourgain paper; my initiation was not by choice, but by fire. In the fall of 2008,

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a year out of grad school, I applied to IAS for the 2009–2010 special program in Analytic Number Theory. I'd proposed to inject bilinear forms techniques into the affine sieve. Apparently Jean read my application, because the next time I visited Peter Sarnak, I was told that Jean wanted to see me. Until that point, we had exchanged pleasantries at conferences, but never talked math.

After working for a few hours with Peter (itself an exhausting event), I went to afternoon tea, and soon thereafter, Jean appeared. He saw me and said, "ah good, you're here; come," and turned and left for Simonyi Hall. I felt as if summoned to the principal's office.

As I arrived, Jean launched into a three-hour lecture at his blackboard; I didn't understand a word he said, but dutifully wrote everything down in my notebook. I left in a complete daze and spent the next three weeks trying to work out from my chicken scratch what he was getting at. Eventually I realized that Jean had just explained to me the solution of the problem I'd planned to work on for the next year. Since this comprised the bulk of my proposal, I was sure that my membership application would be denied. Instead it turned out that Jean, in his generosity, wanted to collaborate!

At first I had a hard time working with Jean. At the blackboard, I would have an idea, then think it's stupid and keep my mouth shut, only to have Jean suggest it five minutes later. I'm usually lighthearted, so was surprised to be clamming up in front of Jean. I confided this to Peter, who instantly diagnosed the issue:

"You're worried that you're not as good as him. Let me alleviate your concerns. You're NOT! **Nobody** is!" Hearing that seemed to do the trick, and henceforth ideas flowed freely and truly collaboratively.

Fortunately for me, the solution required a stubbornly technical result from the spectral theory of automorphic forms, and this was one of the very few areas Jean didn't have instantly at his fingertips. So these became our first two papers, the main theorem, and technical companion (the latter joint also with Peter).

When these were finished, I'd assumed that would be it, but as luck would have it, the ideas developed in those papers turned out to be useful in attacking a number of unrelated problems, from Zaremba's conjecture, to the local-global conjecture for Apollonian circle packings, to our "Beyond Expansion" program.

As our collaboration flourished, our *modus operandi* standardized. Once every few weeks, on the prescribed day, we met around tea time, 3:30 or so, and worked until 8:57 p.m. We then drove to Blue Point Grill on Nassau St., which closed at 9:30, arriving by 9:05 p.m. You could see the depressed expressions as the staff saw us coming, realizing they'd be working late. (If we arrived any later than 9:05, they would gleefully tell us the kitchen was already closed, and we would be relegated to Tiger Noodles next door.) We would finish a bottle of Medoc as tables all around us saw their chairs inverted. By 10:30, we would be back in Jean's office, working until 1:30 a.m., when he would drive me to Princeton Junction to catch the last train back to NYC. I would arrive home at 3 a.m., and on checking my email, discover a message from Jean. Attached would be a scan of a handwritten note solving what mere hours ago was an impenetrable obstacle.

Many people spoke of receiving such miraculous e-faxes out of the blue in their email. But I had just spent eight hours struggling with the guy. In those hours, he was a mere human, and we were totally stuck! What black magic he did in those wee hours I will never know. His wife, Mei, and son, Eric, were unable recently to find in his office his Fields Medal; perhaps they should have been searching for this oracle!

The craziest part was that the solution was never of the form: "Here is how we should have plowed through that massive wall." It was always much more creative: "Yes, that wall may be impenetrable, but if you take three steps to the right and start digging, you'll find a hidden tunnel across." It just didn't seem fair.

On one fateful such dinner in 2014, Jean told me the biopsy came back positive. He looked me in the eyes and said that the five-year survival rate was absolute zero. I was devastated but also incredulous; here was a man full of vitality telling me his days were numbered. Jean fought valiantly and courageously, with much credit due to his extraordinary sister, Claire, a doctor in Belgium. As many have described, his mathematical output continued if not increased (if that's even possible) through all the surgeries and chemo until the end.

I am so blessed and fortunate to have been afforded the privilege of knowing and working with one of the greatest and most generous minds in the history of mathematics. I miss my friend dearly.

Elon Lindenstrauss

Jean Bourgain was an incredible mathematician and an exceptional person. I had known Jean already as a teenager, as he was a close friend and collaborator of my father. In the 80s and early 90s Jean Bourgain would come almost every year for an extended visit to Jerusalem, and he would quite often come to join us for an informal dinner. My father was tremendously impressed with him and held him in very high regard.

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Figure 3. Jean, Tomek Szankowski, and Joram Lindenstrauss, Svendborg 1984.

I got to know Bourgain from a different vantage point when I came to the IAS in Princeton as a postdoc in 1999. Bourgain, who was by then a faculty member at the IAS, was assigned to be my mentor. At least at the time, mentorship at the IAS was a fairly loose arrangement. During the two years I was at the IAS as a postdoc, Bourgain was mostly interested in topics that were further from me, such as the discrete Schrödinger equation and Anderson localization, and in my first year at the IAS we had only a few mathematical discussions. Bourgain was, however, a mathematician with very wide interests, one common interest being the interactions between ergodic theory and harmonic analysis in the quantum unique ergodicity problem, and during my second year as a postdoc we met much more and wrote a paper together on entropy of quantum limits. This paper gave me a strong initial push in a program I was thinking of at the time (and eventually was able to push through) regarding how to try to prove Arithmetic Quantum Unique Ergodicity using ergodic-theoretic tools. During my years as a postdoc at IAS he also mentioned to me the Erdős Volkmann ring conjecture about the Hausdorff dimension of Borel-subrings of $\mathbb{R}$, suggesting I try to think about it. I was quite surprised he was interested in this problem, since it did not seem in character with his main interests at the time, but this turned out to have been an excellent suggestion (that I have not pursued): two independent proofs of this conjecture, by Edgar and Miller and by Bourgain turned out to be both very important for the subsequent flourishing of the field of arithmetic combinatorics.

After leaving the IAS in 2001 I spent three years at Stanford and in New York before returning to Princeton. During this time Bourgain's interests shifted, and to my delight he now became very interested in arithmetic combinatorics problems that were much closer to what I was interested in. I started chatting with him on a regular basis about these problems, learning from him a lot. Typically, I would come by bike late at night to his office and we would chat for an hour or two. Between these chats we would exchange emails, and his emails containing some mathematical thoughts he had would often end with the words "more later." He was always very generous with his ideas and thoughts. During this time I had the opportunity to watch him do mathematics much more closely, and I collaborated with him on a couple of projects. Watching him work from up close was an awe-inspiring, if somewhat humbling, experience: he was incredibly quick and persistent, driven with an intense passion to achieve his goal. To him getting to the finish line was the most important, exactly how he got there, and whether his route was the optimal one to get to the finish line, was only secondary. I was also struck by how thorough and hard-working a mathematician he was; his fame and incredible achievements only drove him to work even harder. For instance, a few times we discussed some dynamics related question that was new to him and I would convey what I knew on the topic (typically not much), and then a few days later he would have studied the literature, read it, and digested it completely.

After I left Princeton in 2008 I continued to visit there every year, with a key attraction being the possibility of spending time discussing mathematics with Bourgain and learning more from him, sharing ideas, thoughts, hopes, and insights. Learning of his illness in 2014 was a big blow to me. It did not slow him down mathematically, at least not until the last months, and when he came back in the summer of 2015 he became as animated as always when discussing mathematics. Ever the optimist, he endured grueling treatments in good spirit with mathematics keeping him energized.

In some sense I got to know Bourgain twice: once (mostly) indirectly, from my father's stories, and then again on my own after I graduated. When I got to know him more closely, I could easily see that the superlatives and remarkable stories I heard about him from my father were precise factual descriptions of the incredible mathematician and great man that Bourgain was, though the Bourgain I met was probably a bit more mellow than the brash young mathematician my father met when he first got to know him. Bourgain was an extremely competitive man, and sometimes talked about proving a big result as "scoring." At the same time, he was also a generous person, who was particularly sensitive about the need to support young mathematicians, and went out of his way to help young mathematicians he thought needed it.

I often think of him, especially when encountering a piece of mathematics of the kind he liked, thinking how much fun it would have been to come to his office late at night when the IAS was quiet and he the most animated,

Figure 4. Jean and Vitali Milman at a conference in Vancouver, 1999.

and chat about this. He was a remarkable person, and I miss him a lot, both as a mathematician and as a friend.

Vitali Milman

I first met Jean sometime in 1982/1983 at a conference. At that time he was involved in all aspects of the classical (infinite-dimensional) Banach space theory. I noted his interest in finite-dimensional talks and suggested that I could "introduce" him to this theory. A quick ironic smile passed over his face, but he came to Israel in the 83/84 academic year, first to Jerusalem and then to Tel Aviv to see me, and we started our first discussion and later published a paper (on "distances between normed spaces"). The next academic year 84/85 we spent together at IHES, Bures-sur-Yvette (near Paris). It was an amazingly productive time, many open problems were solved and we became very close friends. Later, during our scientific cooperation throughout our lives, I was responsible, as an editor, for publishing 85 of his papers in two forms: 35 papers in the journal *GAFA* and 50 more in the serious GAFA Seminar Notes (*Israel Seminar on Geometric Aspects of Functional Analysis*).

In my talk at the conference "Honoring the Life and Work of Jean Bourgain" on May 31, 2019, I shared some memories and reflections on Jean's style of work. I will not repeat them here—one can find the video of the talk at the IAS webpage. Many of Jean's expressions mentioned there became the standard "dictionary" for my students, who remember them and spread them to the next generation of mathematicians.

Jean was a very nice and open person with the people he liked, and he had an extremely responsible and strong personality. When we (my wife and I) visited him in his sister's house in Belgium a few months after he was diagnosed with cancer, almost the first thing he said was: "Why should we be sad? Is being sad helpful? It is not, so we are not going to be sad!" Saying that he brought a bottle of very good wine, which he could not drink himself, as we learned after it was opened and served to us. He knew at that time, as did we, that the expected survival rate with his diagnosis is between half a year and one year! It was a miracle that he survived and worked (!) for four and a half years. The creator of the miracle was, of course, his sister, Professor of Medicine, Claire Bourgain. We are infinitely grateful to her.

The very limited space I have in this article allows me only to recall a couple of stories which may demonstrate Jean's character, his human side, that are not on the video of my lecture at the IAS.

Jean mostly worked during the first part of the night. This was his way to be isolated, undisturbed, having absolute quietness. However, he also needed, from time to time, an atmosphere of "white noise." For that he often walked along the Champs-Elysees. Sometimes, for the same goal, he took a metro from Bures-sur-Yvette to Paris (around 40 minutes), and immediately returned (by the way, after a while he never bought tickets for both directions; he told me that when doing so he was constantly losing the return ticket and had to buy a new one).

Jean actually liked to be in extreme situations. Once we flew together from the US to Germany to an Oberwolfach meeting. He did not yet have a driver license (which he received quite late, around the age of 33–34). Jean suggested we rent a very good car, like a Mercedes, at the airport, and wanted to be driven to Oberwolfach at the speed of 200 km per hour (which is allowed on the highways in Germany). "Can you do it?"—he asked, and I reacted "of course" (I am not proud of myself about this). So, we drove. Jean looked hypnotized at the speedometer all the way, without moving.

Jean hated to teach classes, but liked to give scientific talks about his very latest results. Once he told me "two months remain to the conference [to which we were going together] but I did not yet prove a theorem I plan to present. Actually, I have not yet decided what theorem I want to prove." (Of course, he eventually gave a talk about the results he obtained in the last month before the meeting). However, about regular classes, even for faculty, the situation was different. Perhaps, he just could not understand what he should explain in more detail, and what is obvious or known. One funny story gives the picture. It was the fall of 1985; Jean just started his job in IHES and also received the Doob Chair in Urbana-Champaign. By the regulations of the Chair, he had to start with some

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classes, which were a series of lectures (for faculty). He asked me to come, to serve, I believe, as moral support. His lectures were twice per week and I arrived for the first week. We had coffee before the first lecture (lectures started, as far as I recall, at noon) and Jean intensively computed something on a piece of paper. I asked: "are you computing something for your talk?". "No"—he answered—"I am computing how much I will be paid for the lecture, and when I see the number I feel easier about delivering it." I understood, and in fact found it reasonable. In two days time, Thursday, we again had coffee at the same place before his second lecture. Again Jean was computing something. I asked him "what are you computing now, preparing your lecture?" "No"—Jean answered—"I am computing how much I will be paid for the lecture. And when I see the number I feel more comfortable about going to deliver it." "But you did it already on Tuesday"—I reacted with some surprise. "Yes"—Jean said—"but I need to SEE it to be able to give a talk." I think this explains well how he felt about giving classes.

Generally, in his young years, Jean's talks were considered to be nonunderstandable. I think that he just did not make an effort to realize what the audience may not know. This worried me very much during the 84/85 years at IHES. Our joint dream was that he would impress the permanent members there to the level that he would be invited to stay at IHES at least for the next two years. In the evening before each of his talks there (and Jean gave a lot of those) we discussed it. I usually told him:

"When people at conferences don't understand how you solved problems they worked on for years, they respect you even more. However, if the people at IHES, like Gromov, Sullivan, Connes, and others, will not understand you, they will consider you to be stupid, not themselves! So, if you want to impress them, they should understand!" And Jean tried, and succeeded!

In June 1985, he called me to Kiel (I was already in Germany) and said: "Kuiper (the director of IHES) suggested me to stay in IHES." I asked with excitement: "For two years?" "NO,"—Jean answered—"permanently." Our conversation then had a very funny continuation, but this is for some other occasion.

Unfortunately, later Jean stopped making the effort to be understandable, and his talks later became again difficult to follow.

However, in 2016, his video talk on the conference "Analysis and Beyond: Celebrating Jean Bourgain's Work and Impact," his last, I think, public talk, was truly excellent, extremely well prepared and delivered. I was so happy to hear it, and his clear, short, and to the point answers to some questions from the auditorium. Only few people in the packed auditorium knew that just a day earlier he had a very heavy (scheduled) chemo session. And in such conditions he delivered what I believe was his best lecture ever! I already passed my space limit and should stop. I feel I owe Jean a much longer article about him, trying to explain, in particular, how he got a permanent offer from IHES just nine months after coming there as an almost unknown visitor, or discuss the epic with Jean's Fields Medal which was not a simple story, as most people think today. Memories of Jean are always with me, and I still don't feel he left forever. I still feel his presence and see him quite alive. There is his portrait on the wall of the living room in my apartment and I see him every day. I feel very proud (and lucky) that I had such a friend.

Gilles Pisier

École Polytechnique (Palaiseau) October 7, 1975, 10:30 a.m.: The Maurey-Schwartz seminar speaker is a mathematician from the US who, shortly before that, had done remarkable work on the Radon-Nikodym property (in short RNP) for Banach spaces, namely Charles Stegall, who settled in Austria as a professor at Linz University. This is a seminar mainly on Banach spaces (their geometry and operators on them) with primarily a regular audience including many young mathematicians, PhD students like myself, mostly in jeans and cultivating post-1968 laid back appearances. Out of the blue appears a very young (he was 21) but very straight-looking fellow carrying an attaché case and wearing a jacket who comes and sits down without talking to anyone. I approach him after the talk to try to socialize, but cutting short he says he came from Brussels to talk with the speaker. I play the go-between; of course Stegall agrees and off they go to his office to discuss one on one. After what was surely more than an hour, they come out. Jean leaves immediately and Stegall looks exhausted. Feeling a sense of responsibility as host to the visitor, I worry that he might have been bombarded with too many questions and to my surprise, he says, no, no, more like bombarded with answers. He then tells me solemnly (much to his credit, as I distinctly remember): I am absolutely sure that I just met someone exceptional. Of course I was a priori skeptical (Descartes's "doute méthodique") but gradually the whole Banach space community (including myself) discovered, before many other communities, that Jean Bourgain was indeed an exceptional mathematician, head and shoulders above anyone else in the field.

After that initial appearance, Jean came back as a speaker six times in the subsequent five years of existence

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of the Maurey-Schwartz seminar. He also gave numerous lectures for Choquet's "équipe d'analyse" at Paris VI University (now "Sorbonne Université") and later on in Orsay's harmonic analysis seminar, before joining the IHES in 1985.

Zeev Rudnick

I am here to share some brief memories of Jean. I collaborated with him over a stretch of about eight years. Here are two vignettes from our interactions.

Where is the beef? My first significant interaction with him came when I was invited to give the colloquium at the Princeton math department around 2005. I spoke about lattice points, and was surprised to see him in the audience, as he tended to ration the number of talks that he attended. After the lecture he came up to me, smiled, and said "I was hoping for more beef"... Meaning that he wanted to see the gory details. I explained that I wanted to keep the audience awake, which made him laugh. He did not share this approach. Once after a particularly demanding lecture of his, observing the audience's glazed looks, he sheepishly told me that he clearly did not have enough practice at teaching calculus, or anything else for that matter. As most of you will know, Jean's brilliance was recognized early on, and he did not have to do any real teaching for most of his career.

What is a draft? We started to work together during a two-year sabbatical that I spent at the Institute in 2008–10. Early on in my stay, I had a discussion with Peter Sarnak and quickly discovered a cute result about a restriction theorem for eigenfunctions of the Laplacian on the torus. The following day Jean showed up in my office, saying that he heard about it from Peter and wanted to see it, as he had thought of related matters. I quickly realized that he meant that he had already found the result, but was too polite to say so explicitly. In any case, we started discussing various variations of the idea, which led to a long collaboration.

When working on a project we quickly settled into a routine where we would take turns. Jean would come into the office after lunch and start chewing on the problem. Around 3 a.m. he would send me a scan of his handwritten notes. I would get those upon waking up, and after putting my kids on the school bus at Weyl Lane, would go to the office to face the challenge of understanding the notes and working on them, sometimes (not often) making my own little progress, which I would tell him about or send by email before going to sleep. I found this an exhausting process, as it was all I could do to keep up with even a small proportion of his ideas.

All too quickly, Jean would resolve the problem, cut and paste together his handwritten notes (I mean *physically* cut and paste) into a manuscript, complete with hand-drawn figures, and declare victory. These he would give to his long-time assistant, Elly Gustafsson, to type. The first time that I got such a manuscript I blanched, and trying to be diplomatic, told Jean that this was a good first draft. Jean looked at me, smiled, and said "What is a draft"? My challenge became to intercept the notes before Elly got to them, or at worst ask her for her TeX files, which I would work on until satisfied with the exposition (top down rather than bottom up). This process did not interest Jean at all, and he was quite happy to let me waste my time on polishing the paper while he turned his attention to other things.

* * * * * * * * * * * * *

Jean was a prince among men. As a mathematician, he combined sheer strength with an uncanny and enviable ability to maintain concentration for long stretches of time. He is sorely missed.

Peter Sarnak

Like many mathematicians of my generation I became aware of our contemporary Jean Bourgain, through tales by others as to his brilliance in solving problems in functional analysis. Over the years as Jean moved into other areas, some closer to ones that I have worked in, I witnessed his genius first hand and it is clear to me that such tales cannot be overstated.

I first got to know Jean thanks to Tom Wolff (whose mathematics we both admired) when we both visited Caltech in 1989. At some point Jean asked me if I had any challenging problems so that he could "score in number theory." I pointed him to an analytic conjecture of Hugh Montgomery about L^p norms of Dirichlet polynomials, which if true has striking arithmetic applications. It didn't take long (but perhaps now that I know Jean much better it was perhaps long for him) for him to get back to me; not only did he show that as stated the conjecture was false but once corrected that it implied the Kakeya conjecture—a favorite problem of Jean to which he made many profound contributions over the years. The next year I spent some time socializing with Jean at the ICM in Kyoto. Jean always spoke frankly and directly about everything and he was very disappointed about being overlooked for a Fields Medal. His analysis was that he wasn't working in fashionable enough fields, and that what he should do is to solve problems in fields which would attract the attention of committees for such prizes. The rest is history as they say, besides settling many further open problems in harmonic

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Figure 5. Jean, Eric, and Mei-Chu, Berkeley, 2009.

analysis and ergodic theory, Jean turned to problems in nonlinear dispersive partial differential equations, mathematical physics, number theory, and theoretical computer science. In all cases and with his golden touch not only did he resolve longstanding problems in these areas, but his solutions introduced new analytic tools and ideas that have become standard ones for researchers today. So while he was motivated by problem solving he became just as much of a theory builder. Looking back it is clear to me that his missing the Fields Medal in 1990 was a good thing; it resulted in his permanent enrichment and discovery of interconnections between the above fields, and also four years later was recognized with the cherished Fields Medal.

In the 90s Jean moved to the Institute for Advanced Study in Princeton. His impact was immediate and lasting, he attracted many up and coming stars to visit and to work with him, and his interests and influence broadened substantially. When I later became his colleague at the Institute we became close friends. He was the head of the School of Mathematics, a job that to the surprise of many, he excelled at. Super efficient, fair, and confident in his well thought out views he led the school from 2000 to 2018. He could not have done so without the help of his academic assistant Elly Gustafsson. Thanks to her efforts and understanding of Jean's commitment to his work, as well as that of the administrative officer Mary Jane Hayes, Jean was able to work on his research uninterrupted, head the School of Math, and commute regularly to California to be with his wife Mei-Chu Chang and their son Eric, with everything moving smoothly and ambitious goals being met continuously.

In 2015 Jean was diagnosed with pancreatic cancer. He faced it honestly and head-on and with an optimism that was one of his defining characteristics. His positivity about everything was infectious; it is what allowed him to tackle the hardest unsolved mathematical problems and it is also what allowed him to live a few more very productive years. Thanks to his sister Claire who is a leading physician in Belgium, Jean was able to get the most promising treatments and a quality of life that enabled him to devote his remaining time to work on some of his pet projects. Among these are his works with Demeter on l^2 decoupling which led to the solution of the Vinogradov mean value conjecture (joint with Demeter and Guth), and count as some of Jean's finest mathematical achievements and done under trying conditions. I remember very clearly the joy and excitement when he first presented his ideas to me as to how to settle the conjecture—there was this sparkle in his eyes that are captured by many of the photos of Jean.

Like Kolmogoroff before him, Jean was a wizard analyst, opening and closing doors in the many areas where analytic/combinatorial reasoning play a central role. He was taken from us all too soon but his mathematics will live on in textbooks and research papers. I miss him dearly.

Wilhelm Schlag

I consider it one of the greatest privileges of my mathematical life to have had the opportunity to work with Jean on problems related to Anderson localization of disordered systems. This started in 1999 and continued for about two years. Jean wrote a highly influential paper with Michael Goldstein on nonperturbative localization for one-dimensional quasi-periodic operators with positive Lyapunov exponents (*Annals of Mathematics*, 2000). Shortly thereafter Michael and I wrote a paper on Hölder continuity of the integrated density of states (*Annals*, 2001). The three of us then collaborated on localization for Schrödinger cocycles with large disorder and for the base dynamics given by the skew shift on the 2-torus. Jean was extremely interested in the skew shift dynamics at that time, and remained so ever since. The key problem of positive Lyapunov exponents for any disorder for the skew-shift remains open. The skew-shift dynamics involves the distribution of the fractional parts of $n^2\omega$ rather than $n\omega$ as in the case of the standard shift.

Jean had an uncanny intuition which allowed him to go to the heart of almost anything that he thought about, uncovering the true essence of the problem. He stated repeatedly that he preferred to develop robust methods, which held the potential to be developed much further. Yet, he often left the systematic exploration of these further applications to others. Jean enjoyed the challenge of opening

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many doors, and was very generous in letting others discover the riches that lay beyond them.

It was of course no small feat to work with Jean. On several occasions he would call me to his office at 10 p.m., where we remained until 2 a.m. I would then excuse myself so as to be able to teach the next morning. Needless to say, Jean remained and continued working for a few more hours.

In addition to the aforementioned skew-shift problem, Jean was keenly interested in the discrete Schrödinger operator on higher-dimensional lattices with quasi-periodic potentials. Under some natural genericity conditions, Jean, Goldstein, and I established localization in two dimensions for large disorders around 2001. But the method partially depended on an arithmetic argument designed to eliminate small sets of resonant dynamical parameters. Unfortunately, this piece of the technique does not generalize at all to higher dimensions. In a 2007 *GAFA* paper, Jean circumvented this obstruction by replacing it with semialgebraic arguments of the Bezout type. Once again, he added an astonishing and deep insight which proved to be robust. For example, it was of pivotal importance in work that Goldstein, Voda, and I conducted on multifrequency quasi-periodic operators on the line during the years 2016 and 2017.

I will remain forever grateful to have experienced Jean's radiant brilliance, his integrity, fairness, and generosity.

Gigliola Staffilani

I was a beginning graduate student at the University of Chicago when I first heard of Jean Bourgain. My advisor Carlos Kenig was just developing with Luis Vega and Gustavo Ponce some techniques involving oscillatory integrals and restriction of Fourier transform, in order to prove sharp existence and uniqueness results (well-posedness) for nonlinear dispersive equations. Their techniques could only be used in $\mathbb{R}^n$, and at that time the periodic case was still completely out of reach. It was Jean who shortly after presented a proof for certain Strichartz estimates, fundamental to prove well-posedness, in the periodic setting. He used analytic number theory results that in my view completely revolutionized this part of harmonic analysis and it paved the way to the connection, that we understand much better today, between solutions to certain Diophantine equations and theorems that deal with the restriction of the Fourier transform on curved surfaces—see for example the proof of the l^2 decoupling conjecture (Bourgain and Demeter) and the Vinogradov conjecture (Bourgain, Demeter, and Guth). Going back to my first indirect introduction to Jean Bourgain, it was in fact through his first papers on dispersive equations, a topic on which I decided to work on for my thesis. In 1994, while I was still a graduate student, Jean received his Fields Medal, which was also assigned for *nonlinear partial differential equations from mathematical physics,* as the citation reads. Less than two years later, in 1995–96, I had the great privilege to call Jean my mentor while I spent a year at the IAS. Everyone who had the chance to interact with Jean at the IAS I am sure realized that he was completely absorbed by his mathematics, but he was also an extremely conscientious person. I was nominally assigned to him and he took this assignment very seriously. As soon as I arrived at the IAS he called me into his office, and he described to me some work of his that according to him I could improve. During that conversation I struggled to pay attention while I was overwhelmed with a sense of inferiority and gratitude for the fact that he was spending his precious time with me. Eventually I proved the result that he had suggested and that suggestion opened the door to a line of research that I am still pursuing today. Since that first time I interacted with Jean at the IAS my admiration for him grew exponentially. Of course, as all of us did, I recognized his giant stature in mathematics: he could prove in one year more open conjectures than ten excellent mathematicians in a life time…and I am exaggerating only a little. But I grew to appreciate sides of Jean that were less known to people. He had an incredibly original and sharp sense of humor, which was made even more charming by his unequivocal French accent! Let me give you a little taste of it. During one of my visits at the IAS, Andrea Nahmod and I were sitting at a seminar by Vadim Kaloshin at the IAS who was presenting a recent result he had finished with Jean. At some point Vadim said that it took Jean a couple of days to figure out part of a certain argument that was missing…and at that point Andrea and I whispered to each other something along the lines of "I am wondering how long it would have taken us…." Jean heard, he looked at us with his disarming and charming smile and said "Maybe a couple of years"? I would have defined anybody else who had made this remark an extremely rude and self-absorbed person, but neither of us thought so at all about Jean. We knew he was absolutely right in gauging the time it would have taken us, and by the way he said it was also clear that he was being humorous.

I had the great opportunity to spend another full year at the IAS in 2003–4. Kenig was the professor in residence, and many of the experts in dispersive equations were present. In spite of the fact that during that year my twins were turning only one, the set-up of the institute allowed me to take full advantage of the fervent mathematical activities that were filling the days. Jean's presence was

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Figure 6. Jean and Rebecca Dolinsky at a birthday party in March 2003.

crucial, and although by nature he would often shy away from crowds, when invited to social gatherings he would happily participate. I remember that for my birthday we organized a small party and we invited Jean. He showed up at the door with a huge champagne bottle and he delighted the small gathering with his humorous company. I found a photo (here included; see Figure 6) that brought back very special memories from that evening.

Unfortunately my more recent memories of Jean are not as happy. It is still impressed in my mind the moment he was giving a talk at the conference I coorganized for Carlos Kenig in September 2014 and his cell phone started ringing. It took a while for him to stop and for the audience to stop smiling. There was nothing to smile about. Jean told me later that the call was the one with which his doctor was trying to tell him he had cancer. The year after, during one of those gorgeous May days that make spring in New England a spectacle to see, some of my colleagues told me that Jean had stopped by my office. I found this strange so I emailed him. He was in Boston because a couple of days later he would go through a grueling surgery at MGH so that his cancer could at least be delayed a little. Andrea Nahmod was in town as well and we went to visit him together. I will never forget that meeting in the well-appointed lobby of one of the apartment buildings used by patients of MGH and their families. Jean looked scared, in need of support and reassurance. To distract him from the incumbent surgery we talked about math and only then he went back to being the usual Jean, the Jean who can think ten steps ahead of you even when you believe you know everything you possibly can about the problem at hand.

In 2015 I was asked to coorganize a conference for Jean Bourgain to celebrate his monumental work. Putting this event together gave me the chance to really grasp the far-reaching influence that he had in mathematics. His influence in harmonic analysis, PDE, ergodic theory, number theory, and many other fields of hard analysis is astonishing. His work in each of these fields would have made him famous, all his work together made him an absolute star. Jean could not attend the conference in person, but from Belgium he was listening to most of the talks and he even gave his own contribution remotely. That was the last time I heard his voice, his unassuming tone while presenting an amazing theorem. In fact in the last five years of his life, while he was battling an incurable disease, he proved some of the most influential mathematical results in years! If this is not amazing, what is?

Terence Tao

When I was a graduate student in Princeton, Tom Wolff came and gave a course on recent progress on the restriction and Kakeya conjectures, starting from the breakthrough work of Jean Bourgain in his now famous 1991 paper in *Geom. Funct. Anal.* I struggled with that paper for many months; it was by far the most difficult paper I had to read as a graduate student, as Jean would focus on the most essential components of an argument, treating more secondary details (such as rigorously formalising the uncertainty principle) in very brief sentences. I still have my physical copy of that paper today; it is covered with question marks and frustrated comments such as "I hate Jean Bourgain."

Eventually, though, and with the help of Eli Stein and Tom Wolff, I managed to decode the steps which had mystified me—and my impression of the paper reversed completely. I began to realise that Jean had a certain collection of tools, heuristics, and principles that he regarded as "basic," such as dyadic decomposition and the uncertainty principle, and by working "modulo" these tools (that is, by regarding any step consisting solely of an application of these tools as trivial), one could proceed much more rapidly and efficiently. By reading through Jean's papers, I was able to add these tools to my own "basic" toolkit, which then became a fundamental starting point for much of my own research. Indeed, a large fraction of my early work could be summarised as "take one of Jean's papers, understand the techniques used there, and try to improve upon the final results a bit." In time, I started looking forward to reading the latest paper of Jean. I remember being particularly impressed by his 1999 *JAMS* paper on global solutions of the energy-critical nonlinear Schrodinger equation for spherically symmetric data. It's hard to describe (especially in lay terms) the experience of reading through (and finally absorbing) the sections of this paper one by one; the best analogy I can come up with

would be watching an expert video game player nimbly navigate his or her way through increasingly difficult levels of some video game, with the end of each level (or section) culminating in a fight with a huge "boss" that was eventually dispatched using an array of special weapons that the player happened to have at hand. (I would eventually end up spending two years with four other coauthors trying to remove that spherical symmetry assumption; we did finally succeed, but it was and still is one of the most difficult projects I have been involved in.)

While I was a graduate student at Princeton, Jean worked at the Institute for Advanced Study which was just a mile away. But I never actually had the courage to set up an appointment with him (which, back then, would be more likely done in person or by phone rather than by email). I remember once actually walking to the Institute and standing outside his office door, wondering if I dared knock on it to introduce myself. (In the end I lost my nerve and walked back to the University.)

I think eventually Tom Wolff introduced the two of us to each other during one of Jean's visits to Tom at Caltech (though I had previously seen Jean give a number of lectures at various places). I had heard that in his younger years Jean had quite the competitive streak; however, when I met him, he was extremely generous with his ideas, and he had a way of condensing even the most difficult arguments to a few extremely information-dense sentences that captured the essence of the matter, which I invariably found to be particularly insightful (once I had finally managed to understand it). He still retained a certain amount of cocky self-confidence though (not to mention an extremely subtle and clever sense of humour). I remember posing to him (sometime in early 2002, I think) a problem Tom Wolff had once shared with me about trying to prove what is now known as a sum-product estimate for subsets of a finite field of prime order, and telling him that Nets Katz and I would be able to use this estimate for several applications to Kakeya-type problems. His initial reaction was to say that this estimate should easily follow from a Fourier analytic method, and promised me a proof the following morning. The next day he came up to me and admitted that the problem was more interesting than he had initially expected, and that he would continue to think about it. That was all I heard from him for several months; but one day I received a two-page fax from Jean with a beautiful handwritten proof of the sum-product estimate, which eventually became our joint paper with Nets on the subject (and the only paper I ended up writing with Jean). [Sadly, the actual fax itself has been lost despite several attempts from various parties to retrieve a copy.]

Péter Varjú

I have often been asked what it was like to be a student of Jean Bourgain. It was very good, and I feel very lucky and privileged that I could learn from him.

Jean was very generous with his ideas. I learned a lot from him and, in fact, I still continue to do so. Reading his papers is not easy, but very rewarding. Most of my best work has been inspired by something he did. Recently, I suggested to a colleague to look at a paper, where a method was discussed that we could try in our work. My colleague replied, he had seen that part of the paper; that is where the authors write "We thank Jean Bourgain for suggesting to use this idea. . . ," and then the paper suddenly becomes hard.

Jean was very generous with his time. I wrote to him once on a Tuesday in the early hours with my draft of an argument that I was working on for a couple of months, and which later became the main argument in my thesis. According to Gmail, it took him 17 minutes to get back to me suggesting we meet on Wednesday after 5 p.m. writing this would give him enough time to go over it. And indeed, I received very detailed comments from him even pointing out all my typos, which I appreciated even more knowing that he did not do this careful reading with his own papers.

Jean was very generous with problems. In December 2013, there was a very nice conference in Jerusalem that I especially remember for two things. We had the snowstorm of the century and that was the last time I met Jean in person. He told me there the following problem, which to the best of my knowledge, is still open. Is it possible to find n points in the unit square such that the $1/n$-neighborhood of any line contains no more than C of them for some absolute constant C? The motivation for this problem comes from a possible construction of spherical harmonics as a combination of Gaussian beams, which would have L^∞ norm bounded by a constant independently of the degree.

Credits

Figure 1 is courtesy of Jean-Pierre Ragaert.
Figure 2 is courtesy of Freddy Delbaen.
Figure 3 is courtesy of Elon Lindenstrauss.
Figure 4 is courtesy of Vitali Milman.
Figure 5 is courtesy of Eric Chang.
Figure 6 is courtesy of Gigliola Staffilani.

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