

**Question 1.** *Is  $f(z) = \sin(z)$  an entire function? Why? What about  $\cos(z)$ ?*

*Solution.* By definition,

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}.$$

We know that the exponential function  $g(z) = e^z$  and any polynomial are the entire functions. The class of entire functions is closed under the composition, so  $\sin z$  and  $\cos z$  are entire as the compositions of  $e^z$  and linear functions.  $\square$