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Private International Debt with Risk of Repudiation

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Abstract: The risk of repudiation plays a central role in the size and nature of international capital flows. In this paper I address the question of whether, in a world of international capital flows with risk of default, there is a rationale for regulation of international borrowing. I model centralized arrangements of international debt where only governments borrow and lend internationally and decentralized arrangements where individuals have access to international markets and I show that a centralized setup allows more international risk sharing and higher welfare than a decentralized setup.

JEL classification: E21, F34, F41

Key words: foreign debt, risk of default, capital controls, borrowing constraints

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1 Introduction

The risk of repudiation plays a central role in the size and nature of international capital flows. There is a large literature that models this friction and attempts to address policy issues that arise from it. With few exceptions this literature models the borrowing country as a single entity, presumably a government, that makes all the borrowing and default decisions[1]. In the data, however, we do observe individuals borrowing and lending internationally, unless there are capital controls. One of the purposes of this paper is to study whether outcomes will differ if instead of a government individuals make borrowing and default decisions.

Models of centralized debt cannot study important issues like the optimal regulation of international capital flows. I model a decentralized arrangement of international debt with risk of repudiation and show that capital controls in form of governments intermediating international debt allow for more risk sharing than in an economy without a government. Furthermore, I show that in an economy without international capital controls and with impediments to domestic borrowing, it may not be welfare improving to liberalize domestic markets by removing barriers to domestic

[1] See for example Eaton and Fernandez (1995), Eaton and Gersovitz (1981) and (1984), Bulow and Rogoff (1989), Atkeson (1991), Giovanetti and Marimon (1993), Cole and P. Kehoe (1996) and Kehoe and Perri (2002). In Cole and English (1991) and (1992) individuals do make economic decisions, in particular they decide how much to borrow and lend. Still, the decision whether to expropriate foreign-owned securities is done by the government.

borrowing and lending.

Our model is decentralized in the sense that private agents within countries rather than governments decide how much to borrow abroad and whether to repay debt. We assume that individuals who default on international debt will be banned from international capital markets but can still trade in domestic markets. One can think of this as a setup applicable to a country where courts treat domestic and foreign agents differently, specifically, courts discriminate against creditors from other countries.

The literature on international bankruptcy law documents many cases of discrimination. While explicit laws against foreigners are rare,[2] discrimination is implicit most of the time. Bebchuk and Guzman (1999) note that the principle of territorialism used in bankruptcy proceedings in most countries is prone to discrimination. Territorialism means that in case of bankruptcy of a multinational firm, every country in which the firm owns assets opens separate bankruptcy proceedings. Bebchuk and Guzman note that courts tend to favor local creditors in what can be interpreted as an equilibrium in a prisoner's dilemma game where it is in each country's best interest to favor its own creditors.[3] Shifting towards a system of universalism with just

[2] Abeyratne (2001) notes that in India the Recovery of Debts Due to Banks and Financial Institutions Act allows for an expedited workout in the case of insolvent debtors but only for domestic intitutions whose claims will then be served before those from foreign creditors who have to go through the slow and inefficient court system.

[3] In the legal literature this is also called the 'grab rule.'

one single bankruptcy proceeding covering all of a firm's assets no matter in which country they may be located and equal treatment of all creditors is therefore one of the central ingredients in the international model bankruptcy law proposed by the United Nations Commission on International Trade Law (UNCITRAL).

Outside the court system there is more evidence of discrimination. In early 2002 the Argentinian government forced banks to convert dollar deposits into pesos at an unfavorable exchange rate. Some observers view this as the government (while assisting local banks) forcing subsidiaries of foreign banks into insolvency and pressuring their parent companies to recapitalize them to avoid damages to their reputation, thus effectively expropriating foreign banks.[4]

For simplicity, we make the extreme assumption in the model that contracts between domestic and foreign agents are not enforced, whereas contracts between domestic agents are. We then introduce capital controls by setting up an arrangement of centralized debt in which private agents are not allowed to borrow and lend abroad. Instead, governments act as sole intermediary for international capital flows, that is, they decide how much to borrow and lend and whether to repudiate international debt. In comparing centralized and decentralized outcomes, we derive three main propositions. First, despite the fact that agents may differ within countries, equilib-

[4] See for example IADB (2005) and also The Economist, Jan 17th: "Should I stay or should I go?" and March 28th, 2002: "Countdown to disaster."

ria in decentralized economies look as if there was a centralized borrowing constraint on the country as a whole in the sense that, at every date, either every agent is constrained from further borrowing abroad or no agent is. Second, in a decentralized economy borrowing constraints are tighter than in a model of centralized debt, leading to less international borrowing and lending. Third, since constraints are tighter in a decentralized economy, a setup where a government carries out international borrowing and lending is superior to decentralized capital flows. Because of this, when the domestic legal system gives imperfect protection to foreign creditors, there is a rationale in favor of capital controls in form of prohibiting agents to borrow internationally and giving the responsibility of international capital flows to governments.

The paper is organized as follows: Section 2 presents the environment and defines and characterizes equilibria. In section 3, I introduce capital controls of the kind mentioned above and show that this economy is equivalent to one with debt constraints as in Kehoe and Levine (1993) and (1998). Section 4 concludes the paper. All proofs can be found in the appendix.

2 Model

Time is discrete. We use a pure exchange economy with N different types of agents and M countries and restrict our attention to type-identical allocations. Each period there is one non-storable good. Subscripts denote the types of individuals and superscripts denote the country. Let $\lambda_i^j \geq 0$ denote the measure of type i people in country j.

An event ω_t is an N-vector of endowments $(\omega_{ti})_{i=1,...,N}$ drawn from a finite set Ω. A history of events s^t is a sequence $(\omega_1, ..., \omega_t)$ of events for periods one through t. The probability of event history s^t occurring is given by $\pi(s^t)$. It is not necessary to put any structure on the process of shocks, in particular, we do not need to assume a Markov process. The following notation will turn out to be helpful in the remainder of the paper: If $r \geq t$ we say that $s^r \geq s^t$ if and only if s^r and s^t coincide in the first t events.

Individuals have preferences described by:

$$U = \sum_t \sum_{s^t} \beta^t u(c(s^t))\pi(s^t)$$

where the period utility function u satisfies the usual Inada conditions, and the time preference factor β satisfies $0 < \beta < 1$.

In each state history there are $M+1$ full sets of state contingent one-period bonds, one traded internationally and one type of bond traded exclusively in each of the M countries. Bond prices are $Q(s^t, \omega_{t+1})$ for international and $P^j(s^t, \omega_{t+1})$ for domestic bonds and the quantities of international and domestic bonds are denoted $f_i^j(s^t, \omega_{t+1})$, $b_i^j(s^t, \omega_{t+1})$, respectively, where $f_i^j(s^t, \omega_{t+1})$ refers to a bond traded internationally after history s^t that pays one unit next period in case event ω_{t+1} occurs, and likewise country j domestic bonds $b_i^j(s^t, \omega_{t+1})$.[5]

A consumer chooses optimal sequences for consumption and for domestic and international bonds subject to his budget and participation constraint:

$$\max_{c_i^j, b_i^j, f_i^j} \sum_t \sum_{s^t} \beta^t u(c_i^j(s^t))\pi(s^t)$$

subject to

$$\begin{aligned} \omega_i(s^t) + b_i^j(s^t) + f_i^j(s^t) &= c_i^j(s^t) + \sum_{\omega_{t+1}} Q(s^t, \omega_{t+1}) f_i^j(s^t, \omega_{t+1}) \\ &\quad + \sum_{\omega_{t+1}} P^j(s^t, \omega_{t+1}) b_i^j(s^t, \omega_{t+1}) \text{ for all } s^t \\ \sum_{r \geq t} \sum_{s^r} \beta^{r-t} u(c_i^j(s^r))\pi(s^r|s^t) &\geq V_i^j(s^t, b_i^j(s^t)) \text{ for all } s^t \qquad (1) \\ f_i^j(s^t, \omega_{t+1}), b_i^j(s^t, \omega_{t+1}) &\geq -\bar{B} \text{ for all } s^t, \omega_{t+1} \qquad (2) \\ b_i^j(s^0) &= f_i^j(s^0) = 0 \end{aligned}$$

where $V_i^j(s^t, b(s^t))$, the value after default, is determined as

$$V_i^j(s^t, b_i^j(s^t)) = \max_{c_i^j, b_i^j} \sum_{r \geq t} \sum_{s^r} \beta^{r-t} u(c_i^j(s^r))\pi(s^r|s^t)$$

subject to:

$$\begin{aligned} \omega_i(s^r) + b_i^j(s^r) &= c_i^j(s^r) + \sum_{\omega_{r+1}} P^j(s^r, \omega_{r+1}) b_i^j(s^r, \omega_{r+1}), \text{ for all } r \geq t \text{ and } s^r \qquad (3) \\ b_i^j(s^r, \omega_{t+1}) &\geq -\bar{B} \\ b_i^j(s^t) &\text{ given} \end{aligned}$$

[5] One can motivate the state-contingent nature with excusable default as in Grossman and van Huyck (1988).

The participation constraint (1) says that an individual would never want to default on international debt to trade only with his fellow countrymen afterwards.[6] Equation (2) is a No-Ponzi condition.[7]

Furthermore, an individual who considers defaulting on international debt does not take into account that together with him other fellow countrymen may default as well. Instead he assumes that nobody else defaults and, therefore, domestic bond prices will not change as he is infinitesimally small. That way, since everybody else in his country is still borrowing and lending internationally, an agent thinks he can still trade internationally via other agents in his country. Evidently, this kind of punishment is far less severe than the autarky punishment in a Kehoe-Levine economy. In fact, at this point it is not even obvious whether this is a punishment at

[6] Notice that an alternative approach to private international debt with risk of repudiation would have been to model agents having access to international capital markets but governments making default decisions. In an earlier version of the current paper, Jeske (2001) shows that equilibria can exist only if perfect risk sharing is attainable, that is, if marginal rates of substitution equalizes across all agents and countries. The intuition is that without the equalization of marginal rates of substitution and thus domestic interest rates, agents in a constrained country with higher interest rates have an arbitrage opportunity they would use ad infinitum in the absense of borrowing limits in their own optimization problem.

[7] If an individual ran a Ponzi-scheme in the international market then for all s^t : $\sum_{r\geq t}\sum_{s^r}\beta^{r-t}u(c_i^j(s^r))\pi(s^r|s^t) = \infty$ and equation (1) always holds, which is why the problem requires a No-Ponzi condition for international bonds despite the participation constraint.

all. Later in the paper we will see that exclusion from foreign borrowing indeed involves a punishment as domestic borrowing will be more expensive than international borrowing.

Let us now define and characterize equilibria in this economy:

Definition 1 *An* ***equilibrium*** *is*

1. *Allocations* $\left\{c_i^j(s^t), b_i^j(s^t, \omega_{t+1}), f_i^j(s^t, \omega_{t+1})\right\}_{s^t, \omega_{t+1}, i, j}$

2. *Bond prices* $\{Q(s^t, \omega_{t+1}), P^j(s^t, \omega_{t+1})\}_{s^t, \omega_{t+1}, j}$

such that

1. *each individual maximizes utility*

2. *Feasibility and market clearing:*

$$\begin{aligned}
\sum_{j=1}^{J}\sum_{i=1}^{I}\lambda_i^j c_i^j(s^t) &= \sum_{j=1}^{J}\sum_{i=1}^{I}\lambda_i^j \omega_i(s^t) \text{ for all } s^t \\
\sum_{i=1}^{I}\lambda_i^j b_i^j(s^t, \omega_{t+1}) &= 0 \text{ for all } s^t, \omega_{t+1}, j \\
\sum_{j=1}^{J}\sum_{i=1}^{I}\lambda_i^j f_i^j(s^t, \omega_{t+1}) &= 0 \text{ for all } s^t, \omega_{t+1}
\end{aligned}$$

Because of Walras' law one of the feasibility constraints is implied by the others together with consumers' budget constraints, but for completeness I include all feasibility constraints in the definition.

First order conditions are:

$$
\begin{aligned}
0 &= \beta^t u'(c_i^j(s^t))\pi(s^t) - \kappa_i^j(s^t) + \sum_{r\le t}\sum_{s^r}\mu_i^j(s^r)\beta^{t-r}u'(c_i^j(s^t))\pi(s^t|s^r) &(4)\\
0 &= -P^j(s^t,\omega_{t+1})\kappa_i^j(s^t) + \kappa_i^j(s^t,\omega_{t+1}) - \mu_i^j(s^t,\omega_{t+1})\frac{\partial V_i^j(s^t,\omega_{t+1},b_i^j(s^t,\omega_{t+1}))}{\partial b_i^j(s^t,\omega_{t+1})} &(5)\\
0 &= -Q(s^t,\omega_{t+1})\kappa_i^j(s^t) + \kappa_i^j(s^t,\omega_{t+1}) &(6)
\end{aligned}
$$

where κ and μ are the multipliers on the budget and participation constraints.

The following proposition deals with the problem that our maximization problem is not a standard convex optimization problem, since the constraint set will not be convex in general. A way to show that first order conditions are also sufficient is to define an alternative maximization problem with the same objective function and a convex constraint set which is a superset of the original (non-convex) constraint set. We then show that a solution that solves the alternative convex problem is also affordable and individually rational in the original non-convex problem.

Proposition 2 *Denote* $p^j(s^t) = \prod_{k=0}^{t-1} P^j(s^k,\omega_{k+1})$ *the implicit date zero price of a* t *period domestic contingent bond. Then for all* s^t :

$$\sum_{s^r\ge s^t} p^j(s^r)\left[f_i^j(s^r) - \sum_{\omega_{t+1}} Q(s^r,\omega_{t+1})f_i^j(s^r,\omega_{t+1})\right] \ge 0 \tag{7}$$

implies that participation constraint (1) holds. Moreover, if (1) holds with equality and $\mu(s^t) > 0$, *then (7) holds with equality.*

Using the previous proposition we can prove the following result:

Proposition 3 *First order conditions for the consumer's problem together with a transversality condition are also sufficient.*

The previous result utilizes the fact that first order conditions are identical except for scaling the Lagrange multipliers and that replacing the participation constraint (1) in the household problem with the affine constraint (7) makes the maximization problem convex.

The next result characterizes consumption of agents with $\mu_i^j(s^t) > 0$. It states that consumption has to be such that in any history where an individual is indifferent between default and the equilibrium allocation, the consumption stream after default has to be the same as the original consumption path from the equilibrium.

Proposition 4 *In equilibrium, if $\mu_i^j(s^t) > 0$ for some i, j then*

$$c_i^{j,D}(s^r) = c_i^j(s^r) \text{ for all } r \geq t \text{ and } \pi(s^r|s^t) > 0,$$

where $c_i^{j,D}$ denotes consumption after default occurred in history s^t.

The next two propositions characterize bond prices. The first result says that the formula for $P^j(s^t, \omega_{t+1})$ just reduces to domestic marginal rate of substitution and the second shows how domestic and international bond prices are related:

Proposition 5 *In equilibrium, for all i, j :*

$$P^j(s^t, \omega_{t+1}) = \beta \frac{u'(c_i^j(s^t, \omega_{t+1}))}{u'(c_i^j(s^t))} \pi(\omega_{t+1}|s^t)$$

Proposition 6 *In equilibrium international bond prices are:*

$$Q(s^t, \omega_{t+1}) = \max_{j=1,..,M} \left\{ P^j(s^t, \omega_{t+1}) \right\}$$

Alternatively one could have said that international interest rates are the minimum of the domestic interest rates. The intuition for this result is that, since domestic markets are complete and contracts perfectly enforceable, marginal rates of substitution have to equalize across agents within one country. The international interest rate must be the minimum of all domestic interest rates. On the one hand it cannot be lower, because there would be some individuals that are not borrowing constrained, who could use the arbitrage opportunity to borrow at a low international rate and lend at a high domestic rate. On the other hand international rates will never be higher than the lowest domestic interest rate because people could use the arbitrage opportunity in the other direction. Note that nobody is ever 'lending constrained.'

The next result states that individuals in one country even if their endowments differ, behave as if there was a borrowing constraint on the country as a whole: Either everybody or nobody is constrained in any given history.

Proposition 7 *For all countries* $j = 1, .., M$ *and all histories* (s^t, ω_{t+1}), *there is a type* $i^\star$ *with* $\mu^j_{i^\star}(s^t, \omega_{t+1}) > 0$ *if and only if* $\mu^j_i(s^t, \omega_{t+1}) > 0$ *for all* $i = 1, .., N$ *and*

$$Q(s^t, \omega_{t+1}) > \beta \frac{u'(c^j_i(s^t, \omega_{t+1}))\pi(s^t, \omega_{t+1})}{u'(c^j_i(s^t))\pi(s^t)}$$

The intuition for this results is that domestically marginal rates of substitution have to equalize, because markets are complete within a country. Also note that this proposition explains how it is possible that in certain countries domestic rates can be higher than international interest rates. In this model the high domestic interest rates we observe for example in some emerging market countries are an equilibrium outcome. The arbitrage opportunity simply cannot be used by agents, because everybody is constrained from further international borrowing.

How is it possible that there is an equilibrium with international borrowing and lending at all, or put differently, why do people pay back their international debt, when defaulting still leaves them with the opportunity to trade domestically? There is an advantage from defaulting, but what will be the punishment on default? The punishment comes from the last three propositions: The domestic interest rate will be higher in exactly the periods when a person wants to borrow. Hence, the punishment for defaulting is that it becomes more expensive to insure against risk in the domestic market only. This interest rate differential always has to exist to enforce international debt, therefore, equalization of the two domestic interest rates across all dates and states can never be possible if there is international borrowing and lending. This can be summarized in the following proposition:

Proposition 8 *For all* $0 < \beta < 1$*, if equilibrium bond prices are*

$$P^j(s^t, \omega_{t+1}) = Q(s^t, \omega_{t+1}) \text{ for all } j, s^t, \omega_{t+1}$$

then

$$f_i^j\left(s^t, \omega_{t+1}\right) = 0 \textit{ for all } i, j, s^t, \omega_{t+1}$$

3 Centralized International Borrowing and Lending

For comparison purposes I model centralized international borrowing and lending and compare welfare in this economy with welfare in the decentralized economy. Suppose that there is a government in each country imposing capital controls of the following form: Only governments are allowed to trade internationally. Domestically, the government and individuals trade bonds in a complete market with perfect enforcement. Hence, governments act as intermediaries for international borrowing and lending. Each government's objective is to maximize a weighted sum of utilities of its citizens.

For simplicity I will set up an economy in which governments trade internationally and hand out lump sum transfers domestically while agents do not access any markets. It is easy to see that the same equilibrium allocation can be decentralized as a Ramsey equilibrium allocation in which the government borrows internationally with other governments and domestically with its citizens.

Suppose a utilitarian government in country j has welfare weights $\left\{\varphi_i^j\right\}_{i=1}^N$. The government's problem is then

$$G^j = \max_{c_i^j, f^j} \sum_{i=1}^{I} \varphi_i^j \sum_t \sum_{s^t} \beta^t u(c_i^j(s^t))\pi(s^t) \tag{GP}$$

such that for all s^t :

$$\begin{aligned}
\sum_{i=1}^{I} \lambda_i^j c_i^j(s^t) + \sum_{\omega_{t+1}} Q(s^t, \omega_{t+1}) f^j(s^t, \omega_{t+1}) &= \sum_{i=1}^{I} \lambda_i^j \omega_i(s^t) + f^j(s^t) && (8) \\
\sum_{i=1}^{I} \varphi_i^j \sum_{r \geq t} \sum_{s^r} \beta^{r-t} u(c_i^j(s^r)) \pi(s^r|s^t) &\geq V^j(s^t) && (9) \\
f^j(s^t, \omega_{t+1}) &\geq -\bar{B} && (10)
\end{aligned}$$

That is, the government redistributes the country's endowment plus the net borrowing and faces a participation constraint. The cutoff value or country autarky value is determined as the maximum weighted utility that can be attained if the whole country is banned from international borrowing and lending and the government can only redistribute goods because all future net inflows are set equal to zero:

$$V^j(s^t) = \max \sum_{i=1}^{I} \varphi_i^j \sum_{r} \sum_{s^r} \beta^{r-t} u(c_i^j(s^r)) \pi(s^r|s^t) \tag{11}$$

such that for all $s^r \geq s^t$

$$\sum_{i=1}^{I} \lambda_i^j c_i^j(s^r) = \sum_{i=1}^{I} \lambda_i^j \omega_i(s^r)$$

Definition 9 *An* ***equilibrium in the economy with capital controls*** *is:*

1. *An allocation* $\left\{c_i^j(s^t), f^j(s^t, \omega_{t+1})\right\}_{s^t, \omega_{t+1}, i, j}$

2. *Bond prices* $\{Q(s^t, \omega_{t+1})\}_{s^t, \omega_{t+1}}$

such that:

1. *Each government maximizes welfare.*

2. *Feasibility and market clearing:*

$$\begin{aligned}\sum_{j=1}^{J}\sum_{i=1}^{I}\lambda_i^j c_i^j(s^t) &= \sum_{j=1}^{J}\sum_{i=1}^{I}\lambda_i^j \omega_i(s^t) \text{ for all } s^t \\ \sum_{j=1}^{J} f^j(s^t,\omega_{t+1}) &= 0 \text{ for all } s^t,\omega_{t+1}\end{aligned}$$

Even without knowledge of the welfare weights we can characterize the amount of international bonds traded. If preferences are homothetic then the amount a government borrows internationally does not depend on the welfare weights:

Proposition 10 *If u is homothetic then f^j is independent of $\{\varphi_i\}_{i=1}^N$.*

Notice that in the proof of proposition 10 the optimization problem in step 1 is identical to one in a one-good Kehoe-Levine economy:

Corollary 11 *If u is homothetic then $c^j(s^t) = \sum_{i=1}^{I}\lambda_i^j c_i^j(s^t)$ are equilibrium consumption paths of a Kehoe-Levine economy with one good and J individuals that have endowments $\omega^j(s^t) = \sum_{i=1}^{I}\lambda_i^j\omega_i(s^t)$.*

In other words, we can aggregate each country into one representative agent, thanks to the complete market structure within countries.

We now notice that there is a major difference between economies with and without capital controls. Remember that in an economy without capital controls complete risk sharing can never be an equilibrium allocation, unless countries can achieve this without international borrowing and lending. In an economy with capital controls, however, we will observe complete risk-sharing if the discount factor is high enough and we put some structure on the process of endowments.[8]

Quite intuitively in an economy with capital controls more international risk-sharing and higher welfare are possible since governments can impose a more severe penalty on themselves than individuals in an economy with private international debt. For individuals default does not affect domestic interest rates whereas for the government it does. One can think of this as the government internalizing an externality.

Assume that a small open economy with private debt imposes capital controls. Then, since international bond prices did not change, the original allocation is affordable for the government and also individually rational, since participation constraints are less tight. Hence, a government can do at least as well as individuals borrowing and lending in the international market.

We can formalize this in the sequence of results below. Assume $\left\{\bar{c}_i^j(s^t)\right\}_{s^t,i}$ solves the government problem (GP). Also construct an altered government problem (GP′) where we use the following participation constraint instead of (9):

[8] An earlier version of this paper went through a numerical example. See Jeske (2001).

$$\sum_{i=1}^{I}\varphi_i^j\sum_{r\geq t}\sum_{s^r}\beta^{r-t}u(c_i^j(s^r))\pi(s^r|s^t)\geq W^j(s^t)\text{ for all }s^t \tag{12}$$

where $W^j(s^t)$ is defined as follows:

$$W^j(s^t)=\max\sum_{i=1}^{I}\varphi_i^j\sum_{r}\sum_{s^r}\beta^{r-t}u(c_i^j(s^r))\pi(s^r|s^t)$$

such that for given bond bond prices P^j and all $s^r\geq s^t$

$$\sum_{i=1}^{I}\lambda_i^j c_i^j(s^r)+\sum_{i=1}^{I}\lambda_i^j\sum_{\omega_{t+1}}P^j\left(s^r,\omega_{r+1}\right)b_i^j(s^r,\omega_{r+1})=\sum_{i=1}^{I}\lambda_i^j\omega_i(s^r)+\sum_{i=1}^{I}\lambda_i^j b_i^j(s^r) \tag{13}$$

Also assume that the history s^t bond distribution is such that $\sum_{i=1}^{I}\lambda_i^j b_i^j(s^t)=0$. One can view this as the government not taking into account the aggregate resource constraint after default and instead assuming it can keep borrowing at domestic interest rates just like individuals in the decentralized economy. This is a weaker participation constraint than in the centralized economy, because the consumption stream after default in the centralized economy satisfies (13) by setting bonds equal to zero. We therefore proved the following result:

Lemma 12 *For all j, $\left\{\varphi_i^j\right\}_{i=1}^{N}$ and s^t :*

$$V^j(s^t)\leq W^j(s^t)$$

Moreover, if $\left\{\bar{c}_i^j(s^t)\right\}_{s^t,i}$ solves (GP) and $\left\{\tilde{c}_i^j(s^t)\right\}_{s^t,i}$ solves (GP′) then

$$\sum_{i=1}^{I}\varphi_i^j\sum_{t}\sum_{s^t}\beta^t u(\bar{c}_i^j(s^t))\pi(s^t)\geq\sum_{i=1}^{I}\varphi_i^j\sum_{t}\sum_{s^t}\beta^t u(\tilde{c}_i^j(s^t))\pi(s^t)$$

To create a link between the economy with private borrowing and lending notice that for the right welfare weights the consumption path in the decentralized economy solves (GP′):

Lemma 13 *Let* $\left\{\tilde{c}_i^j(s^t)\right\}_{s^t,i}$ *be country* j'*s equilibrium consumption stream in an economy with private international borrowing and let* $\{P^j(s^t,\omega_{t+1}),Q(s^t,\omega_{t+1})\}_{s^t,\omega_{t+1}}$ *be equilibrium bond prices. Then there are welfare weights constructed as*

$$\varphi_k^j = \frac{\lambda_k^j}{\lambda_1^j}\frac{u'(\tilde{c}_1^j(s^0))}{u'(\tilde{c}_k^j(s^0))}$$

such that $\left\{\tilde{c}_i^j(s^t)\right\}_{s^t,i}$ *solves (GP′) given bond prices.*

The previous two Lemmas prove that - keeping international interest rates fixed - under centralized borrowing a government can always do weakly better than an economy with private international borrowing. We can now prove the main result that in a non-autarkic economy the government can do strictly better:

Proposition 14 *Let* $\left\{\tilde{c}_i^j(s^t),\tilde{b}_i^j\left(s^t,\omega_{t+1}\right),\tilde{f}_i^j\left(s^t,\omega_{t+1}\right)\right\}_{s^t,i}$ *be country* j'*s equilibrium allocation in an economy with private international borrowing such that there is a* (s^t,ω_{t+1}) *with* $\sum_{i=1}^{I}\lambda_i^j\tilde{f}_i^j(s^t,\omega_{t+1})<0$*. Construct* $\left\{\varphi_i^j\right\}_{i=1}^{N}$ *as in Lemma 13 and assume that* $\left\{\bar{c}_i^j(s^t)\right\}_{s^t,i}$ *solves (GP) given those welfare weights. Then*

$$\sum_{i=1}^{I}\varphi_i^j\sum_t\sum_{s^t}\beta^t u(\bar{c}_i^j(s^t))\pi(s^t) > \sum_{i=1}^{I}\varphi_i^j\sum_t\sum_{s^t}\beta^t u(\tilde{c}_i^j(s^t))\pi(s^t)$$

This section had the seemingly unpleasant implication that prohibiting private international borrowing makes people better off because a government can internalize an externality associated with international borrowing constraints. Wright (2005) builds upon this paper to find a less radical way of attaining the constrained optimum through a system of subsidized private international borrowing. He performs this exercise for both the economy in this present paper with debt constraints of the style of Kehoe and Levine as well as Alvarez and Jermann (1998, 2000) type solvency constraints. Wright made use of the fact that default is too attractive in a world of private international borrowing. Hence, subsidizing international borrowing is another way of internalizing the externality to attain the constrained optimum.

4 Conclusion

The purpose of this paper is to specify a model rich enough to address the question of whether, due to risk of repudiation on international debt, regulations on international borrowing and lending may be welfare improving. My results indicate that if courts do not enforce contracts between foreigners and domestic agents then, it is in fact welfare improving to exclude agents from international markets and use governments to intermediate international capital flows.

This kind of capital control can actually increase the amount of borrowing and lending because a government can borrow more from abroad as the penalty for default is more severe for a government than for an individual. In that sense this model differs from the work of Cole and English (1991) and (1992), where a decrease in foreign investment was welfare improving because it lowers the probability of expropriation.

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5 Appendix

5.1 Private international debt

Drop the i, j subscripts and superscripts in the derivation of first order conditions and the proof of propositions 2 to 4 to simplify notation. Also write $s^r \leq s^t$ as a shortcut for $s^t = (s^r, \omega_{r+1}, ..., \omega_t)$. Using first order conditions of the consumer problem:

$$\kappa(s^t) = \beta^t u'(c(s^t))\pi(s^t)\left[1 + \sum_{s^r \leq s^t} \mu(s^r)\beta^{-r}\frac{\pi(s^t|s^r)}{\pi(s^t)}\right]$$

Let $\nu(s^t)$ be the Lagrange multiplier of the $V(s^t, b(s^t))$ problem. Then

$$\begin{aligned}\frac{\partial V(s^t)}{\partial b(s^t)} &= \nu(s^t) \\ &= u'(c^D(s^t))\end{aligned}$$

Solving for international bond prices yields:

$$\begin{aligned}Q(s^t, \omega_{t+1}) &= \frac{\kappa(s^t, \omega_{t+1})}{\kappa(s^t)} \\ &= \beta\frac{u'(c(s^t, \omega_{t+1}))}{u'(c(s^t))}\pi(\omega_{t+1}|s^t)\frac{1 + \sum_{s^r \leq (s^t, \omega_{t+1})} \mu(s^r)\beta^{-r}\frac{\pi((s^t, \omega_{t+1})|s^r)}{\pi(s^t, \omega_{t+1})}}{1 + \sum_{s^r \leq s^t} \mu(s^r)\beta^{-r}\frac{\pi(s^t|s^r)}{\pi(s^t)}}\end{aligned}$$

Hence, international bond prices are equal to MRS of an individual that is not borrowing constrained this period. Solving for domestic bond prices yields:

$$\begin{aligned}P(s^t, \omega_{t+1}) &= \frac{\kappa(s^t, \omega_{t+1}) - \mu(s^t, \omega_{t+1})\frac{\partial V\left((s^t, \omega_{t+1}), b(s^t, \omega_{t+1})\right)}{\partial b(s^t, \omega_{t+1})}}{\kappa(s)} \\ &= \frac{\beta^{t+1}u'(c(s^t, \omega_{t+1}))\pi(s^t, \omega_{t+1})}{\beta^t u'(c(s^t))\pi(s^t)} \times \frac{1 - A_1 + A_2}{1 + A_3} \\ &= \beta\frac{u'(c(s^t, \omega_{t+1}))}{u'(c(s^t))}\pi(\omega_{t+1}|s^t) \times \frac{1 - A_1 + A_2}{1 + A_3} \qquad (14)\end{aligned}$$

where

$$
\begin{aligned}
A_1 &= \mu(s^t,\omega_{t+1})\frac{u'(c^D(s^t,\omega_{t+1}))}{u'(c(s^t,\omega_{t+1}))}\beta^{-t-1}\frac{1}{\pi(s^t,\omega_{t+1})} \\
A_2 &= \sum_{s^r\leq(s^t,\omega_{t+1})}\mu(s^r)\beta^{-r}\frac{\pi((s^t,\omega_{t+1})\,|s^r)}{\pi(s^t,\omega_{t+1})} \\
A_3 &= \sum_{s^r\leq s^t}\mu(s^r)\beta^{-r}\frac{\pi(s^t|s^r)}{\pi(s^t)}
\end{aligned}
$$

Proof of proposition 2: The expression on the left hand side of (7) is simply the discounted present value of payments an individual receives from foreigners evaluated at domestic prices. If this value ever becomes negative it would be in the agent's best interest to default and to go to the domestic market in order to buy and sell the same amount of bonds he would have traded in the international market. That way he could have resources left over to increase utility, which contradicts the fact that he was utility maximizing in the first place. Proving the second part of this proposition is straightforward by looking at first order conditions of the two alternative maximization problems, the original problem and the one where we replace the participation constraint with weaker condition (7). The corresponding Lagrange multipliers are positive in the one problem if and only if they are positive in the other problem. ■

Proof of Proposition 3: The consumer's optimization problem with the participation constraint replaced by the weaker condition is now:

$$
\max_{c,b,f}\sum_t\sum_{s^t}\beta^t u(c(s^t))\pi(s^t)
$$

subject to

$$
\begin{aligned}
c(s^t) + \sum_{\omega_{t+1}} Q(s^t, \omega_{t+1}) f(s^t, \omega_{t+1}) + \sum_{\omega_{t+1}} P(s^t, \omega_{t+1}) b(s^t, \omega_{t+1}) &= \omega(s^t) + b(s^t) + f(s^t) \\
\sum_{s^r \geq s^t} p(s^r) \left[f(s^r) - \sum_{\omega_{t+1}} Q(s^r, \omega_{r+1}) f(s^r, \omega_{r+1}) \right] &\geq 0 \\
b(s^r, \omega_{r+1}) &\geq -\bar{B} \\
b(s^0) &= f(s^0) = 0
\end{aligned}
$$

Let κ and μ be the multipliers on the budget constraint and the alternative participation constraint, respectively. First order conditions are:

$$
\begin{aligned}
\kappa(s^t) &= \beta^t u(c(s^t)) \pi(s^t) \\
P(s^t, \omega_{t+1}) &= \frac{\kappa(s^t, \omega_{t+1})}{\kappa(s^t)} \\
0 &= -Q(s^t, \omega_{t+1}) \kappa(s^t) + \kappa(s^t, \omega_{t+1}) \\
&\quad + \sum_{s^r \leq (s^t, \omega_{t+1})} \mu(s^r) p(s^t) - \sum_{s^r \leq s^t} \mu(s^r) p(s^t) Q(s^t, \omega_{t+1})
\end{aligned}
$$

Then

$$
\begin{aligned}
P(s^t, \omega_{t+1}) &= \beta \frac{u(c(s^t, \omega_{t+1}))}{u(c(s^t))} \pi(\omega_{t+1} | s^t) \\
Q(s^t, \omega_{t+1}) &= \frac{\kappa(s^t, \omega_{t+1}) + \sum_{s^r \leq (s^t, \omega_{t+1})} \mu(s^r) p(s^t)}{\kappa(s^t) + \sum_{s^r \leq s^t} \mu(s^r) p(s^t) Q(s^t, \omega_{t+1})} \\
&= \beta \frac{u(c(s^t, \omega_{t+1}))}{u(c(s^t))} \pi(\omega_{t+1} | s^t) \frac{1 + \sum_{s^r \leq (s^t, \omega_{t+1})} \frac{\mu(s^r) p(s^t)}{\kappa(s^t, \omega_{t+1})}}{1 + \sum_{s^r \leq s^t} \frac{\mu(s^r) p(s^t)}{\kappa(s^t, \omega_{t+1})}}
\end{aligned}
$$

Hence, first order conditions are identical to the original maximization problem if we rescale the Lagrange multipliers on the participation constraint. ■

Proof of proposition 4: First we show that if $\{c(s^r)\}_{s^r \geq s^t}$ is affordable without default, then it will also be affordable after default. Then, since the $V_i^j(s^r, b(s^r))$ problem is a convex problem with solution $\left\{c^D(s^r)\right\}_{s^r \geq s^t}$, the two consumption streams $\{c(s^r)\}_{s^r \geq s^t}$ and $\left\{c^D(s^r)\right\}_{s^r \geq s^t}$ have to be identical, because if they were not, then a linear combination of the two would yield a higher utility than $\left\{c^D(s^r)\right\}_{s^r \geq s^t}$ after default.

The budget constraint after default in date s form rather than sequential representation is:

$$\sum_{s^r \geq s^t} p(s^r) \left[c^D(s^r) + \sum P(s^r, s_{r+1}) b(s^r, s_{r+1}) - \omega(s^r) - b(s^r)\right] \leq 0$$

Notice that since the agent is constrained in s :

$$\sum_{s^r \geq s^t} p(s^r) \left[c(s^r) + \sum P(s^r, s_{r+1}) b(s^r, s_{r+1}) - \omega(s^r) - b(s^r)\right] = 0$$

Therefore, $\{c(s^r)\}_{s^r \geq s^t}$ satisfies the budget constraint in the $V_i^j(s^r, b(s^r))$ problem. ■

Proof of Proposition 5: From the previous Proposition we know that $c_i^j(s^t) = c_i^{j,D}(s^t)$ for all histories s^t with $\mu_j^i(s^t) > 0$. Plug this into (14) and the fraction $\frac{1-A_1+A_2}{A_3}$ on the right hand side of this equation collapses to one. Then for all i, j :

$$P^j(s^t, s_{t+1}) = \beta \frac{u'(c_i^j(s^t, s_{t+1}))}{u'(c_i^j(s^t))} \pi(s_{t+1}|s^t)$$

■

Proof of Proposition 7: The formulas for bond prices imply for all i, j :

$$Q(s^t, \omega_{t+1}) = P^j(s^t, \omega_{t+1}) \frac{1 + \sum_{s^r \leq (s^t, \omega_{t+1})} \mu_i^j(s^r) \beta^{-r} \frac{\pi(s^t, \omega_{t+1} | s^r)}{\pi(s^t, \omega_{t+1})}}{1 + \sum_{s^r \leq s^t} \mu_i^j(s^r) \beta^{-r} \frac{\pi(s^t | s^r)}{\pi(s^t)}} \tag{15}$$

Then $\mu_i^j(s^t, \omega_{t+1}) > 0$ for one individual in country j implies that $Q(s^t, \omega_{t+1}) > P^j(s^t, \omega_{t+1})$ which in turn means that $\mu_i^j(s^t, \omega_{t+1})$ has to be positive for all individuals in country j. ■

Proof of Proposition 8: This proof is similar to the one Bulow and Rogoff (1989) use. Define $g_i^j(s^t) = f_i^j(s^t) - \sum_{\omega_{t+1}} f_i^j(s^t, \omega_{t+1}) Q(s^t, \omega_{t+1})$ and to note that

$$\sum_{s^r \geq s^t} p^j(s^r) g_i^j(s^r) \geq 0 \text{ for all } s$$

In case of complete risk sharing we would get

$$P^j(s^r) = Q(s^r) \text{ for all } s^r, j$$

which together with $\sum_j \sum_i \lambda_i^j g_i^j(s^r) = 0$ implies $g_i^j(s^r) = 0$ and therefore $f_i^j(s^r) = 0$ for all histories s^r. ■

5.2 Centralized International Borrowing and Lending:

Let κ and μ be the multipliers on the government budget constraint and the participation constraint, respectively. We drop the country superscripts to simplify notation. First order conditions are:

$$\begin{aligned} \varphi_i \sum_t \sum_{s^t} \beta^t u'(c_i(s^t)) \pi(s^t) + \sum_{s^r \leq s^t} \beta^r \varphi_i \mu(s^r) u'(c_i(s^r)) \pi(s^t | s^r) &= \kappa(s^t) \lambda_i \\ \kappa(s^t, \omega_{t+1}) &= \kappa(s^t) Q(s^t, \omega_{t+1}) \end{aligned}$$

Hence,

$$
\begin{aligned}
Q(s^t, \omega_{t+1}) &= \frac{\kappa(s^t, \omega_{t+1})}{\kappa(s^t)} \\
&= \beta \frac{u'(c_i(s^t, \omega_{t+1}))}{u'(c_i(s^t))} \pi(\omega_{t+1}|s^t) \frac{1 + \sum_{s^r \leq (s^t, \omega_{t+1})} \mu(s^r) \beta^{-r} \pi(s^r)^{-1}}{1 + \sum_{s^r \leq s^t} \mu(s^r) \beta^{-r} \pi(s^r)^{-1}}
\end{aligned}
$$

and

$$
\frac{u'(c_i(s^t))}{u'(c_k(s^t))} = \frac{\lambda_i}{\lambda_k} \frac{\varphi_k}{\varphi_i}
$$

Alternatively, the government could have solved the planning problem in two steps

Step 1

$$
\max \sum_t \sum_{s^t} \beta^t u(c(s^t)) \pi(s^t) \tag{16}
$$

such that

$$
\begin{aligned}
\omega(s^t) + f(s^t) &= c(s^t) + \sum_{\omega_{t+1}} Q(s^t, \omega_{t+1}) f(s^t, \omega_{t+1}) \quad (17) \\
\sum_{s^r \geq s^t} \beta^r u(c(s^r)) \pi(s^r|s^t) &\geq \sum_{s^r \geq s^t} \beta^r u(\omega(s^r)) \pi(s^r|s^t) \quad (18)
\end{aligned}
$$

where $\omega(s^t) = \sum_{i=1}^N \lambda_i \omega_i(s^t)$.

Step 2

$$
\max \sum_{i=1}^N \varphi_i \sum_t \sum_{s^t} \beta^t u(c_i(s^t)) \pi(s^t) \tag{19}
$$

such that:

$$
\sum_{i=1}^N \lambda_i c_i(s^t) = c(s^t) \ \forall s \tag{20}
$$

First order conditions:

$$Q(s^t, \omega_{t+1}) = \beta \frac{u'(c(s^t, \omega_{t+1}))}{u'(c(s^t))} \pi(\omega_{t+1}|s^t) \frac{1 + \sum_{s^r \leq (s^t, \omega_{t+1})} \mu(s^r) \beta^{-r} \pi(s^r)^{-1}}{1 + \sum_{s^r \leq s^t} \mu(s^r) \beta^{-r} \pi(s^r)^{-1}} \quad (21)$$

$$\frac{u'(c_i(s^t))}{u'(c_k(s^t))} = \frac{\lambda_i}{\lambda_k} \frac{\varphi_k}{\varphi_i} \ \forall s, i, k \quad (22)$$

Now we can show that the government's problem and the proposed two step procedure have identical solutions if preferences are homothetic. First, let us derive the following result:

Lemma 15 *Let $U = E \sum_t \sum_{s^t} \beta^t u(c_i(s^t))$ be homothetic and let u satisfy the usual Inada conditions. Then for all $\varphi \in \left\{ x \in \mathcal{R}^N_{++} : \sum_i x_i = 1 \right\}$ and $\alpha_i > 0$ the function $h : u^{-1}(\mathcal{R}) \longrightarrow \mathcal{R}$ defined by $h(x) = \sum_{i=1}^N \varphi_i u(\alpha_i u^{-1}(x))$ is affine and strictly increasing.*

Proof. Compute the derivative of h

$$\begin{aligned} h'(x) &= \sum_i \varphi_i u'(\alpha_i u^{-1}(x)) \alpha_i \frac{1}{u'(u^{-1}(x))} \\ &= \sum_i \varphi_i \alpha_i \frac{u'(\alpha_i c)}{u'(c)} \text{ for } c = u^{-1}(x) \end{aligned}$$

Since U is homothetic

$$\frac{u'(\alpha c_1)}{u'(\alpha c_2)} = \frac{u'(c_1)}{u'(c_2)} \Rightarrow \frac{u'(\alpha_i c)}{u'(c)} \text{ constant for all } c.$$

Therefore $h'(x)$ is a positive constant. ■

Proof of Proposition 10: We have to show that the solution of the two step procedure also solves the government's problem. With homothetic preferences we get

$$c_i(s^t) = \alpha_i c(s^t)\ \forall s^t, i$$

and hence

$$\frac{u'(c(s^r))}{u'(c(s^t))} = \frac{u'(c_i(s^r))}{u'(c_i(s^t))}\ \forall s^t, s^r, i$$

where $\omega(s) = \sum_{i=1}^{N} \lambda_i \omega_i(s)$.where $\sum_i \alpha_i = 1$. (21) and (22) are then just identical to first order conditions in the original government's problem. Also the government's budget constraint is satisfied when (17) and (20) hold. All we have to show is that the participation constraint in the government's planning problem is satisfied. As shown in the previous Lemma, h is affine and strictly increasing. Therefore

$$\begin{aligned}
\sum_{s^r \geq s^t} \beta^r u(c(s^r))\pi(s^r|s^t) &\geq \sum_{s^r \geq s^t} \beta^r u(\omega(s^r))\pi(s^r|s^t) \\
\Rightarrow h\left(\sum_{s^r \geq s^t} \beta^r u(c(s^r))\pi(s^r|s^t)\right) &\geq h\left(\sum_{s^r \geq s^t} \beta^r u(\omega(s^r))\pi(s^r|s^t)\right) \\
\Rightarrow \sum_{s^r \geq s^t} \beta^r h(u(c(s^r)))\pi(s^r|s^t) &\geq \sum_{s^r \geq s^t} \beta^r h(u(\omega(s^r)))\pi(s^r|s^t) \\
\Rightarrow \sum_{i=1}^{N} \varphi_i \sum_{s^r \geq s^t} \beta^r u(c_i(s^r))\pi(s^r|s^t) &\geq \sum_{i=1}^{N} \varphi_i \sum_{s^r \geq s^t} \beta^r u(\omega_i(s^r))\pi(s^r|s^t) \\
&= D(s)
\end{aligned}$$

■

Proof of Lemma 12: As noted in the main text, the constraint set of the V^j problem is a subset of the constraint set of the W^j problem because in equation (13) we can always set $b_i^j(s^r, \omega_{r+1}) = 0$ for all i, s^r, ω_{r+1}. This proves the first half of the result. Using the same argument in the other direction, that is, noting that the constraint set of (GP′) is a subset of the constraint set of (GP), proves the second half. ■

Proof of Lemma 13: Write down first order conditions in the altered government problem (GP′)

$$
\begin{aligned}
0 &= \varphi_i^j \beta^t u'(c_i^j(s^t))\pi(s^t) - \lambda_i^j \kappa\left(s^t\right) + \varphi_i^j \sum_{r\leq t}\sum_{s^r} \mu\left(s^r\right)\beta^{t-r}u'(c_i^j(s^t))\pi\left(s^t|s^r\right) && (23)\\
0 &= -Q(s^t,\omega_{t+1})\kappa\left(s^t\right) + \kappa\left(s^t,\omega_{t+1}\right) && (24)\\
0 &= \sum_{i=1}^{I}\lambda_i^j c_i^j(s^t) + \sum_{\omega_{t+1}} Q(s^t,\omega_{t+1}) f^j(s^t,\omega_{t+1}) - \sum_{i=1}^{I}\lambda_i^j \omega_i(s^t) - f^j(s^t) && (25)\\
0 &\geq \sum_{i=1}^{I}\varphi_i^j \sum_{r\geq t}\sum_{s^r}\beta^{r-t}u(c_i^j(s^r))\pi(s^r|s^t) - W^j(s^t) && (26)
\end{aligned}
$$

Set the welfare weights

$$
\varphi_k^j = \frac{\lambda_k^j}{\lambda_1^j}\frac{u'(\tilde{c}_1^j(s^0))}{u'(\tilde{c}_k^j(s^0))} = \frac{\lambda_k^j}{\lambda_1^j}\frac{u'(\tilde{c}_1^j(s^t))}{u'(\tilde{c}_k^j(s^t))}\forall s^t
$$

noticing that in the economy with private international borrowing, marginal rates of substitution equalize across types and states. Now notice that the $\tilde{c}$ equilibrium allocation satisfies the first order conditions of the (GP′) problem: After rescaling Lagrange multipliers, equation (23) is identical to (4) given the welfare weights we chose and equation (24) is identical to (5). Equation (25) holds because $\sum_{i=1}^{I}\lambda_i^j \tilde{b}_i^j(s^t,\omega_{t+1}) = 0$ for equilibrium bonds $\tilde{b}$ in the decentralized economy.

What remains to show is that the participation constraint (26) holds. Just as in proposition 2, take the detour over the linear participations constraint. In equilibrium in country j, for all i, s^t :

$$\sum_{s^r \geq s^t} p^j(s^r)\left[\tilde{f}_i^j(s^r) - \sum_{\omega_{t+1}} Q(s^r, \omega_{t+1})\tilde{f}_i^j(s^r, \omega_{t+1})\right] \geq 0$$

therefore

$$\sum_{s^r \geq s^t} p^j(s^r)\left[\bar{f}(s^r) - \sum_{\omega_{t+1}} Q(s^r, \omega_{t+1})\bar{f}(s^r, \omega_{t+1})\right] \geq 0 \tag{27}$$

where $\bar{f}(s^r) = \sum_{i=1}^{I} \lambda_i^j \tilde{f}_i^j(s^r)$. Using the same argument as in the proof of proposition 2 the constraint (27) implies that first order condition (26) is satisfied. To summarize, consumption and bond holdings $\left\{\tilde{c}_i^j(s^t), \bar{f}(s^r)\right\}_{s^t,i}$ satisfy the first order conditions of the (GP′) problem (under appropriate choice of welfare weights). First order conditions in the (GP′) are sufficient using the same argument as in proposition 3. ■

Proof of Proposition 14: By assumption the country as a whole borrows in history s^t to pay back in (s^t, ω_{t+1}). This implies there must be a history (s^r, ω_{r+1}) with $s^r \geq s^t$ such that $Q(s^r, \omega_{r+1}) > P^j(s^r, \omega_{r+1})$ to enforce repayment of the bond, which implies $\mu_i^j(s^r, \omega_{r+1}) > 0$ for all i (utilizing proposition 7) and $\tilde{f}_i^j(s^r, \omega_{r+1}) < 0$ for all i. This implies that from history (s^r, ω_{r+1}) onward the $\tilde{c}$ allocation must be non-autarkic, in other words $\sum_{i=1}^{N} \lambda_i^j \tilde{c}_i^j(s) = \sum_{i=1}^{N} \lambda_i^j \omega_i^j(s)$ for all $s \geq (s^r, \omega_{r+1})$ is not possible. This is because the $\tilde{c}$ allocation also solves the (GP′) problem and $\tilde{f}_i^j(s^r, \omega_{r+1}) < 0$ together with $\sum_{i=1}^{N} \lambda_i^j \tilde{c}_i^j(s) = \sum_{i=1}^{N} \lambda_i^j \omega_i^j(s)$ for all $s \geq (s^r, \omega_{r+1})$

would imply that the social planner in the (GP′) problem could increase welfare by defaulting in history (s^r, ω_{r+1}).

Next, recall proposition 4. The result obviously extends to the problem (GP′), that is, in case of a binding participation constraint the equilibrium $\tilde{c}$ consumption stream from (s^r, ω_{r+1}) is identical to the stream after default in the $W^j(s^r, \omega_{r+1})$ problem. Since the allocation after (s^r, ω_{r+1}) is non-autarkic, the continuation values are such that $W^j(s^r, \omega_{r+1}) > V^j(s^r, \omega_{r+1})$ since the objective function is strictly concave. A social planner in the original centralized problem (GP) could therefore relax the participation constraint in history (s^r, ω_{r+1}) that had a strictly positive Lagrange multiplier. ■