Lesson Plan: Experimental Error

Background

Biophysicists and other scientists measure properties of biological and other molecules using different methods and instruments. The measurements contain errors that arise in several ways. These include errors by the scientists in making the measurements – although you may find this hard to believe, you will soon see how these can occur! – those due to the instruments, and differences in the conditions under which the measurements were made, such as temperature or humidity. We can estimate how large these errors are by having others repeat the measurements, by using different instruments or methods, or by doing the assays on different days. Because of these errors, scientists usually perform their measurements several times, rather than doing them only once. They then average the results and note the differences along with the averages. These differences from measurement to measurement are called **experimental error**. When an experiment consisting of several steps is involved, the entire experiment is repeated several times to increase the accuracy of the results.

This lesson plan shows that errors arise when people like you (or me) make measurements and that repeating a measurement can reduce the size of the experimental error. It also demonstrates that variation, for example, among individuals of a group, differs from experimental error and should be stated in a different way.

Objectives & Grade Level

Demonstrate the sources of experimental error and the effect of replicate measurements in reducing the size of the error. Show that variation exists among a group of individuals and how this should be designated. Appropriate for starting to advanced high school science students; familiarity with mathematical notation is helpful but not required.

Materials

- Ruler

Procedure

1. Using a ruler, measure the largest span of your left or right hand in centimeters (*cm*) – this is the largest distance you can reach between the tip of your smallest finger and end of your thumb (**Figure 1**). Write down your measurement, relax your hand, then measure it again. Do this again until you have a total of ten measurements.

Try not to look to see what the measurement will be before you take the reading – just stretch your hand along the ruler and then take the reading. Be sure to measure the same hand each time!

Are the results exactly the same for all ten measurements? If not, what is the largest difference between the values? Differences occur because it is hard to stretch your hand exactly the same way each time. This is also true for other types of measurements that depend on pipetting the same volume each time, exciting a specimen with the same amount of light, or applying the same amount of force to an object.

2. Calculate the average of your measured values by adding all ten measurements together and dividing by 10. This is called the **mean value**. Then calculate the **standard error of the mean** (***SEM***) of your measurements using the equations below (note that this calculation can also be done online at *http://www.endmemo.com/math/sd.php*):

$$SEM = \frac{SD}{\sqrt{n}}$$

where

- *SD* is the Standard Deviation, which can be calculated using the equation below (see **Notes 1 and 2**)

$$SD = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

- *n* is the number of measurements for both *SEM* and *SD*
- *x* is each value in a data set
- $\bar{x}$ is the mean of all values in the data set

The standard error of the mean is a measure of how much your measurements differ from one another and from the average or mean value. When you tell someone the mean value of your measurements, you should also tell them the *SEM*, together with the number of times (*n*) that you made the measurements, e.g., *mean±SEM*, *n*=10.

Approximately how large is the SEM of your measurements compared to the mean of your measurements? Is it a tenth or a twentieth of the mean value, or more?

3. Ask a classmate to measure the largest span of your left or right hand (the same hand you measured) using a different ruler. Write down the result, then repeat the measurement for a total of ten measurements, as before. Calculate the mean and *SEM* of the ten values and compare them to the *mean±SEM* values that you determined from your measurements.

Are the results the same or different for the ten measurements that your classmate made, compared to the ones that you made? As a short-cut to determine how similar the two sets of measurements are, you can determine whether your classmate's measurements overlap with your measurements. Overlap means that when you subtract the SEM from the mean or add it to the mean, your values and your classmate's values show a region in which they are the same. For example, if your measurements gave 18.5±0.7 cm and your classmate's gave 19.1±0.5 cm, the range of your values would be 17.8-19.2 cm and the range of your classmate's values would be 18.6-19.6 cm, and the values would overlap. The SEM is a measure of the 68% confidence interval, so if the two sets of measurements overlap, you can be at least 68% certain that both sets of measurements represent your true hand span.

4. If the *mean±SEM* values are not overlapping, you should repeat both sets of measurements and try to find out why they are so different. Make sure that the scales on the rulers are the same and try to stretch your hand the same way when you measure your hand span and when your classmate does it.

5. If the *mean±SEM* values are overlapping, the two sets of data approximate each other sufficiently to combine together. Take the twenty measurements and calculate the mean and *SEM* of all twenty values.

Is the SEM for the twenty values less than or greater than each of the sets of ten measurements? The SEM should decrease as the number of measurements increases. This means that the average of twenty measurements is closer to the true value than the average of ten measurements.

6. Now compare the mean value of the twenty measurements that you and your classmate obtained for your left or right hand with those for others in your class. These values are likely to differ quite a lot from one another because of differences among individuals in your class – the differences are due to actual differences in hand size from one person to another, rather than errors in measurement. Remember that because each person's hand was measured twenty times, the mean values will be close to the true values and the experimental error will be greatly reduced. You can determine how much variation in hand size there is among your class members by putting the average values for left hands into one group and those for right hands into a second group, and determining the mean values for each group by adding up the values and dividing by the number in each group.

Then calculate the **standard deviation** (***SD***) of the values using the equation above (or use the online calculator at *http://www.endmemo.com/math/sd.php*). The **standard deviation** is a measure of the amount of variation due to differences among individuals, in this case, differences in hand size among you and your classmates. It is not due to errors in measurement (hopefully) and differs from the *SEM*, which *is* caused by errors in replicate measurements.

Are the SDs for the left and right hand spans of your classmates the same or different from one another? If they are different, which hand shows the greatest variation among the members of your class? That is, which hand has the larger SD?

Notes

1. The *SEM* depends on knowing the standard deviation, so the *SD* should be calculated first.

2. The *SD* can be easily calculated by most beginning high school students by filling in the table below. First, determine the mean of the ten measurements (column 1), then subtract each value from the mean (column 2) and square the difference (column 3). Sum the squared values, divide by 10 (the number of values), then take the square root of the dividend.

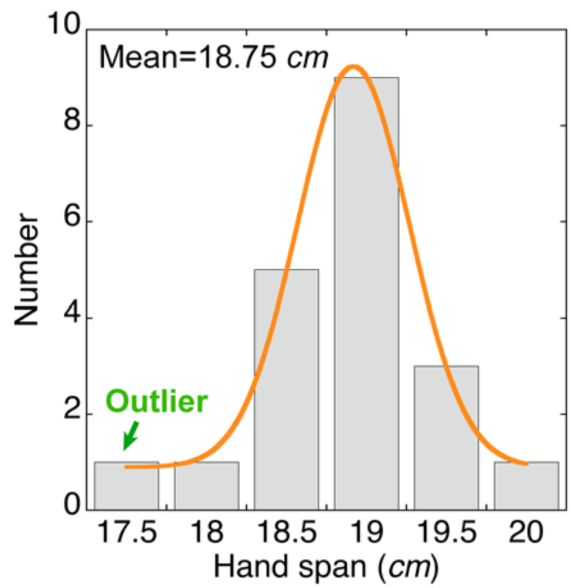
	Column 1	Column 2	Column 3	
Hand: __Left __Right	Measurements (*cm*), x	$\bar{x} - x$	$(\bar{x} - x)^2$	
Measured by: 1)				
2)				
3)				
4)				
5)				
6)				
7)				
8)				
9)				
10)				
Mean or $\bar{x}$=Total/10	$\bar{x}$=		Total or $\sum(x - \bar{x})^2$=	$\frac{\sum(x-\bar{x})^2}{10} =$

$$SD = \sqrt{\frac{\sum(x-\bar{x})^2}{10}} =$$

3. **Advanced topic 1**: another way of determining the mean value of a set of measurements is to plot the data to see if they form a **Gaussian distribution**, which is also known as a “normal distribution” or “bell-shaped curve”. First divide your measurements into groups of equal values, called *bins*. Then plot the number of measurements in each bin versus the bin value. Below is a plot showing how the measurements at the left would look when plotted in this way. Note that the peak value from the plot, which is taken as the mean value of the measurements ($\bar{x}$ =18.75 *cm*), differs from the calculated mean value ($\bar{x}$=19.1±0.1 *cm*, *n*=20). The reason for this is that one of the measurements is very different from the rest – it lies to one side of the Gaussian distribution peak and is known as an *outlier*. Outliers for sets of measurements like the ones you just did are most likely due to differences in the way your hand was stretched during the measurements (rather than differences in the actual size of your hand!). Determining the mean from the peak of the Gaussian plot is probably more accurate than the mean value calculated from all of the measurements, as it excludes outliers due to measurement errors.

*Measured Values (*cm*)	Bins (*cm*)	Number
17.5, 19.2, 18.7, 18.3, 19.3, 19.4, 19.1, 20.0, 19.8, 18.6	17.5-17.9	1
19.0, 19.4, 18.9, 19.1, 19.5, 19.2, 19.5, 18.8, 19.2, 18.9	18.0-18.4	1
	18.5-18.9	5
	19.0-19.4	9
	19.5-19.9	3
Mean±*SEM*=19.1±0.1 *cm* *n*=20	20.0-20.4	1

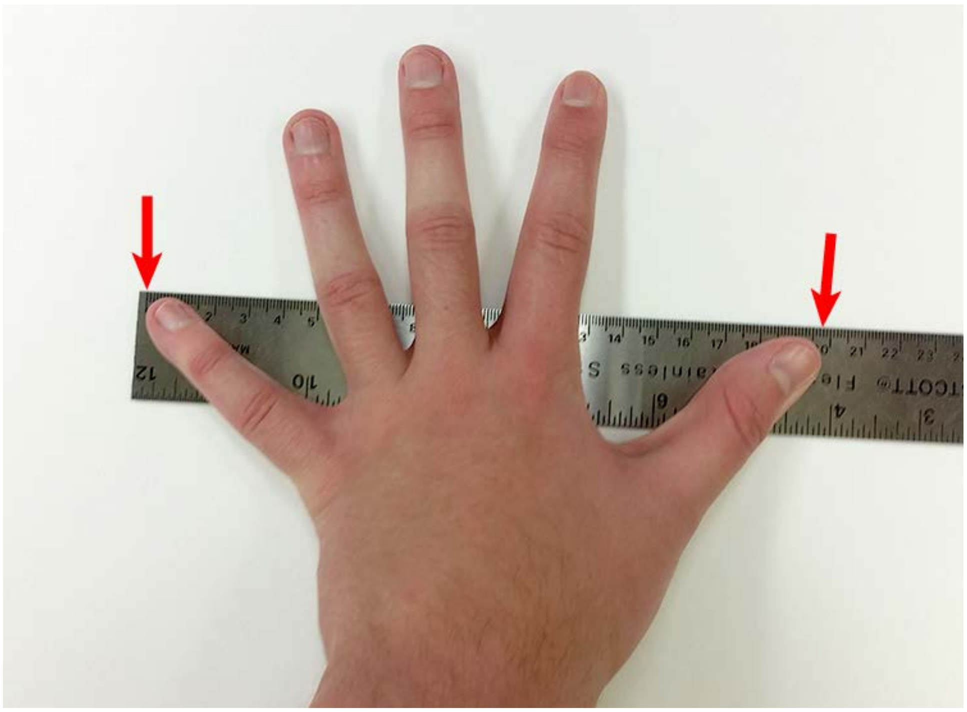
*Numbers between 178-192, corresponding to 18.5±0.7 *cm*, and 186-196, corresponding to 19.1±0.5 *cm*, were generated by a random number generator, then divided by 10 to give the “Measured Values” example above; one number was changed to create an outlier.



Values were binned, plotted and fit to a Gaussian curve, $y=0.89971 + 8.34*e^{-(x-3.8367)^2/0.98774^2}$, using *Kaleidagraph v 4.1*. The curve is shown fit to the tops of the histogram columns but goes to zero on both sides.

Figure

Figure 1 Measuring Your Hand Span with a Ruler



With a ruler, stretch your hand along the centimeter scale and measure the largest distance between the tip of your smallest finger and end of your thumb (red arrows). Repeat the measurements for a total of ten values. Be sure to measure the same hand when repeating the measurements. Then follow the steps above to determine the error in your measurements.

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