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The Japanese Repo Market: Theory and Evidence

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Abstract

Repurchase Agreements (Repo) transactions are widely used as a risk-free means of raising or investing funds. Repo transactions can be categorized into the following two types: (i) general repos whose purpose is to borrow or lend funds, and (ii) special repos whose purpose is to borrow or lend securities. General repo transactions generate the linkage between the repo market and money markets including the interbank market, while special repo transactions generate the linkage between the repo market and securities markets, typically the government bond market. The objective of this paper is to examine the mechanism of the Japanese repo market from both theoretical and empirical perspectives.

First, this paper theoretically reviews a pricing mechanism of repo rates focusing on the linkage between the repo market and the government bond market. General repo rates are priced at a level close to the risk-free interest rate. However, special repo rates can be priced far below the general repo rates. Duffie (1996) and Krishnamurthy (2001) derived a mechanism by which repo rates are priced differently depending on the underlying issues. The mechanism is summarized as follows: (i) equilibrium in the repo market requires no-arbitrage profits from trading that combines repo and cash bond transactions (no-arbitrage condition), (ii) the equilibrium level of repo spreads, which are defined as the differences between general and special repo rates, is determined at the point where the supply and demand curves of the underlying bonds intersect in the repo market, and (iii) expected returns from future matched book trading are reflected in the cash prices of special bonds.

Second, the paper empirically examines the above theoretical implications using the data of repo rates and government bond prices in Japan. Our main findings are as follows: regarding the most recently issued (on-the-run) 10-year Japanese government bonds (JGBs) and the cheapest to deliver (CTD) issues, the no-arbitrage condition is significantly satisfied between the repo spread and the corresponding price premium in the cash market (cash premium), defined as the difference between the market price of special and general bonds.

Key Words: Repo Market, Government Bond Market, No-Arbitrage Condition, Repo Spread, Cash Premium, Most Recently Issued (On-the-Run) Bond, Cheapest to Deliver (CTD), Short Sales

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I. Introduction

A repo transaction is a contract that exchanges securities with high creditworthiness, typically government bonds, for funds for a fixed period of time. The securities function as collateral for raising funds and the funds function as collateral for borrowing securities. Thus, market participants recognize repo transactions as a risk-free means of raising or investing funds against securities as collateral. [1] Of these repo transactions, those for raising funds are called general repos, while those for borrowing securities are called special repos. General repo transactions generate the linkage between the repo market and money markets including the interbank market, while special repo transactions generate the linkage between the repo market and securities markets, typically the government bond market.

Regarding the linkage between the repo market and the interbank market, Griffiths and Winters (1997) found that in the US, general repo rates moved almost parallel with the uncollateralized federal funds (FF) rate. On the linkage between the repo market and the bond market, Jordan and Jordan (1997) empirically examined Duffie's (1996) theoretical insight that bond prices should reflect expected profits from future matched book trading, which takes advantage of repo spreads[2] defined as the differences between general and special repo rates. In Japan, on the other hand, Shigemi, Kato, Soejima, and Shimizu (2001) described the pricing of Japanese government bonds (JGBs) in the cash, futures, and repo markets as a consequence of the events that took place between 1998 and 1999.[3] The objective of this paper is to examine the mechanism of the Japanese repo market from both theoretical and empirical perspectives.

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[1] Legally, repo transactions in Japan differ from those in the US and Europe. In Japan, repo transactions take the form of borrowing or lending of securities against funds as collateral, whereas in the US and Europe, securities are sold or bought with an agreement to buy or sell back the same securities at a later date. Furthermore, in Japan, repo transactions across borders have been inactive due to the withholding tax levied on the interest earned on repo transactions by non-residents. However, cross-border transactions are expected to become active with the introduction of a new US-type repo system in April 2001 as a turning point. Also, the abolition of the withholding tax on non-residents was abolished in April 2002. For more details, see Kanno and Kato (2001).

[2] For matched book trading, see Box 1 in section II.

[3] Shigemi, Kato, Soejima, and Shimizu (2001) explain in detail, employing quantitative analysis and interviews with market participants, the mechanism by which a sudden change in the balance of supply and demand for JGBs caused by stresses such as the Y2K problem brought about a

First, the paper theoretically reviews the pricing mechanism of the repo rates in Japan in normal times, focusing on the linkage between the repo market and the government bond market, following the above-mentioned studies in the US. It turns out that theoretically speaking, most repo rates (general repo rates) should be priced at a level close to the risk-free interest rate. Depending on the underlying bonds, however, repo rates (special repo rates) can be priced far below general repo rates, even though the issuers are the same or equally rated in terms of credit standing. A number of market participants recognize that repo spreads are closely related to the balance between supply and demand of the underlying issues in the repo market.

Duffie (1996) and Krishnamurthy (2001) derived a mechanism by which repo rates can be priced differently depending on the underlying bonds. The mechanism is summarized as follows: (i) equilibrium in the repo market requires no-arbitrage profits from trading that combines repo and cash bond transactions (no-arbitrage condition), (ii) the equilibrium level of the repo spreads defined as the differences between general and special repo rates is determined at the point where the supply and demand curves of the underlying bonds intersect in the repo market, and (iii) expected returns from future matched book trading, which takes advantage of the differences in repo rates, are reflected in the cash prices of special bonds.

Second, the paper empirically examines the above theoretical implications using Japanese repo rate and the JGB price data. Our main findings are as follows: when we regard the most recently issued (on-the-run) 10-year JGBs and CTD issues as special, the no-arbitrage condition is significantly satisfied between the repo spread and the corresponding cash premium defined as the differences between the market prices of special and general issues.

The rest of the paper is organized as follows. Section II presents an overview of the Japanese repo market, as well as the general framework of repo transactions. Section III reviews the mechanism by which repo rates can be priced differently depending on the underlying issues, based on Duffie (1996) and Krishnamurthy (2001). Section IV conducts a simple empirical analysis of the theoretical implications reviewed in section III using data of repo rates of on-the-run 10-year JGBs and CTD issues, and the

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deterioration in the arbitrage relationships between the futures, spot, and repo markets.

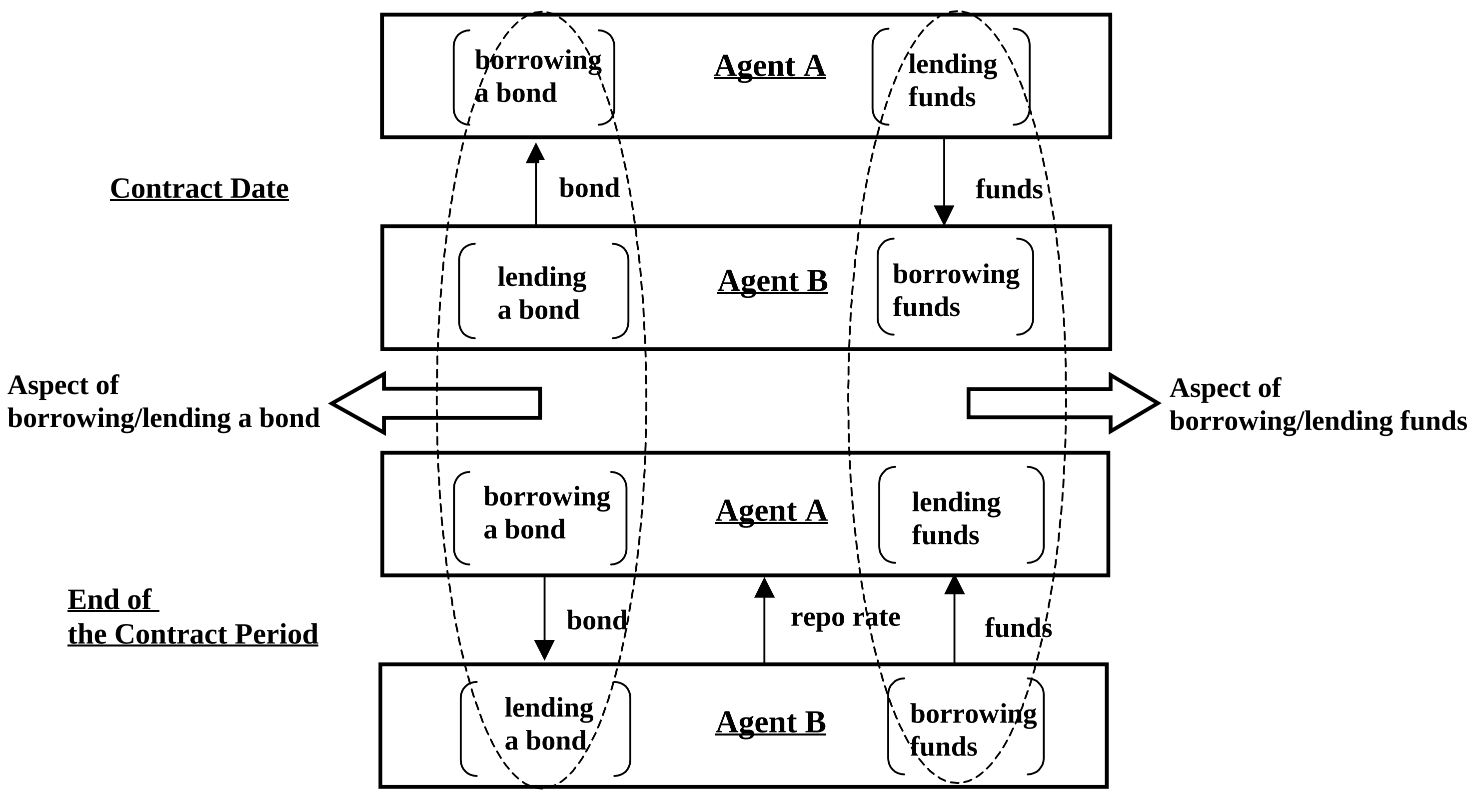
estimated cash premiums. Section V concludes by referring to the limitations of the analysis in the paper.

II. Outline of Repo Transactions[4]

A. Two Aspects: Borrowing/Lending of Funds and Securities

As shown in figure 1, a repo transaction starts when agent A exchanges his or her funds for a bond held by agent B at market price on the contract date. At the end of the contract period, agent A receives back the funds plus interest, while the bond is returned to agent B. The interest rate thus paid on the funds is called the repo rate, which is defined as "the interest rate on the funds minus securities lending fee".

Figure 1: General Framework of Repo Transactions



[4] Uetsuki (1997), Maeda (1998), and Repo Trading Research (2001) provide a concise review of repo transactions.

Figure 1 reveals two aspects of repo transactions. The first is the borrowing of funds against bonds as collateral. Suppose agent A is an investor with surplus funds and agent B a bond dealer who needs to finance his bond portfolio. In this setup, the investor can lend funds in exchange for bonds with high creditworthiness as collateral. The bond dealer, on the other hand, can reduce funding costs by using his bond holdings as collateral, thereby curtailing the credit risk premium required by the investor.

The second is the borrowing of bonds against funds (cash) collateral. Suppose agent A is a bond dealer who wants to build a short position and agent B an investor with an extensive bond portfolio. In this setup, the bond dealer can cover his short position through a repo transaction by borrowing the necessary bonds against cash collateral (see Box 1). The investor, on the other hand, can effectively reduce funding costs by lending the dealer the bonds in demand.

The former transactions for lending and borrowing funds are called general repos, while the latter for lending and borrowing securities are called special repos. In general repos the underlying securities are not specified, while in special repos they are specified when the parties enter into a contract.

BOX 1: The Use of Repo Transactions

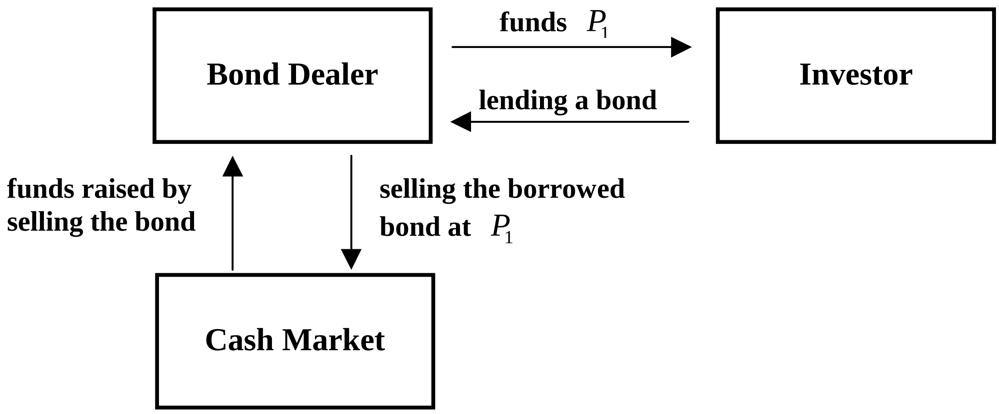
Special Repos and Short Sales

Bond dealers can build short positions efficiently using repo transactions.[5] The most typical way would be to sell short a bond in the cash market while simultaneously borrowing the same bond through a special repo for delivery of the bond. As illustrated in Box figure 1, funds raised by selling short the bond can be used to cover the funds required for settling the special repo transaction. In this way, short positions can be built without holding any initial capital and the specific bonds.[6]

[5] In fact, Keane (1996) argues that many special repo transactions stem from the need to cover short sales.

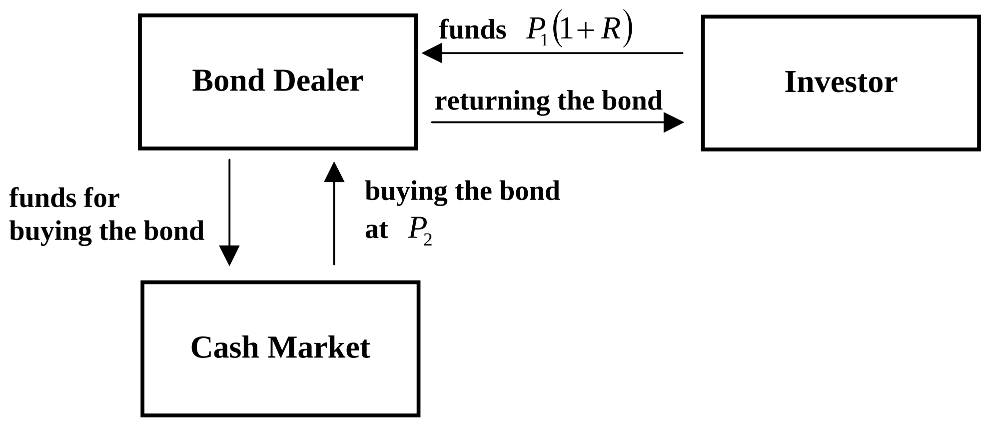
[6] For simplicity, we ignore the costs entailing the haircut for managing the risk of price fluctuations, administrative costs, and costs arising from differences in settlement timing.

Box Figure 1: Contract Date



At the end of the contract period, the bond dealer needs to buy the bond in the cash market in order to return it to the investor. The dealer can use the funds $P_1(1+R)$ (R =repo rate) paid by the investor to buy back the bond in the cash market (see Box figure 2).

Box Figure 2: End of the Contract Period



Then, why do bond dealers want to build short positions using repo transactions? The dealer's profit/loss from bond trading can be expressed as P_1 (selling price) $-P_2$ (buying price). At the same time, the dealer invests funds P_1 at the repo rate R and receives P_1R. Thus, the return for the dealer is expressed as $P_1 - P_2 + P_1R$. Note that the return is always positive when the bond price falls ($P_1 > P_2$) during the contract period as long as R takes a positive value.

In this way, when bond prices are expected to fall, speculative dealers have an incentive to build short positions with a view to increasing dealing profits. Risk averse bond dealers may also benefit from holding similar short positions as they can hedge against losses from a possible fall in the prices of their bonds.

Matched Book Trading

Investors with a rich bond portfolio can generate profits without being exposed to risks by taking advantage of various levels of repo rates. For example, the investor can lend bonds at a low repo rate (a high lending fee), and invest the funds at a higher repo rate, yielding profits equivalent to the spread between the two repo rates. These transactions using repo spreads are called "matched book" trading. While widely used in the US as a means of reducing the funding costs of portfolios, it is still in limited use in Japan.

B. Risk-free Characteristic of Repo Transactions

As stated above, in repo transactions both bonds and funds function as collateral that ensures safeness in the event of the default of either counterparty. In addition, "marking to the market", also called "margin calls," is practiced to protect against bond price volatility during the contract period. If the lender of the bonds (borrower of funds) defaults while the bond price falls during the contract period, the borrower of the bonds (the lender of funds) is not able to recover the full amount of his funds by selling the bonds put up as collateral. To avoid the loss, the system of margin calls enables traders to ask for additional provision of collateral when a shortfall occurs during the contract period, based on the calculation of the margins, that is, the shortage/excess of the value of collateral outstanding in bonds or funds. This kind of risk management method ensures a high degree of safeness for repo transactions.[7,8]

[7] See Kanno and Kato (2001) for details on various risk management methods for repo transactions.
[8] By analogy with forward transactions, the general repo rate turns out to be equivalent to the risk-free rate. This point will be explained in section III.

C. Japanese Repo Market

(i) Scale of the Market

In Japan, there has been a financing method utilizing securities, which is called *gensaki* transactions. In *gensaki* transactions, as is the case with repo transactions, funds and securities are exchanged for a fixed period of time. However, market participants demanded further refinements of the *gensaki* transactions for the following reasons: (i) the *gensaki* transactions lacked an appropriate means of risk management against price fluctuation and default during the contract period, and (ii) the settlement of government bond transactions was shifting to the rolling method in 1996.[9,10] Under these circumstances, the repo market was launched in April 1996 with a view to providing investors with a market for investing surplus funds, and bond dealers such as securities firms with a market for financing their portfolios, which includes a means of covering short positions. Initially, as of the end of December 1996, the outstanding balance of the repo market stood at eight trillion yen, which accounted for just 6% of the money markets total (see figure 2). By the end of September 2001, however, the outstanding balance increased to about 42 trillion yen, accounting for about 22%.[11]

Figures 3 (1) and (2) show the scale of the repo markets in the US, the UK, and Japan in terms of the outstanding balance itself as well as the percentage share of the outstanding balance to total government bonds outstanding, respectively. In the US, whose repo market has a long history with a vast number of participants, the outstanding balance of the market stood at about 184 trillion yen (about 1,500 billion dollars), which is more than four times the size of the Japanese market.[12] The repo market in the UK,

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[9] Under the rolling method, transactions are settled after a certain period of time from the contract date. Compared with the previous method, under which settlements were made on the 5th and 10th of each month, the rolling method is expected to reduce settlement risk by lowering the number of unsettled outstanding contracts.

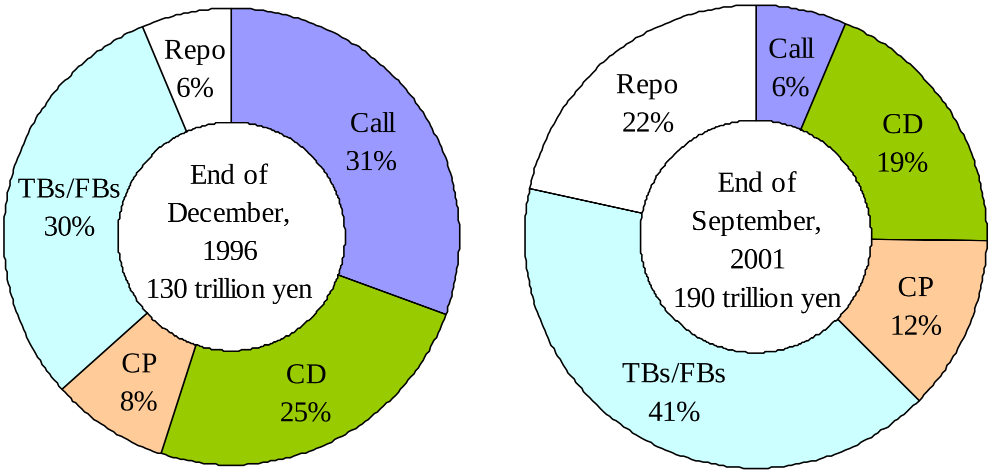
[10] For an outline and brief history of JGB borrowing/lending transactions, including the *gensaki* transactions, see Kanno and Kato (2001).

[11] The outstanding balance of the repo market is calculated as the average of the amounts lent and borrowed in repo transactions.

[12] In the US, repo transactions started in 1918 when the Federal Reserve launched operations utilizing discounts on Bankers' Acceptances (BA). The primary market for BA discounts was in

which was launched at roughly the same time as that in Japan, has an outstanding balance of about 22 trillion yen (about 126 billion pounds), about half the size of the Japanese market.[13] Whereas in both the US and the UK, the outstanding balance of the repo market is about 900 billion yen per one trillion yen of government bonds, in Japan it is a just 110 billion yen. This comparison may indicate that the Japanese repo market still has much room for further expansion.

Figure 2: Percentage Shares of the Short-term Money Markets

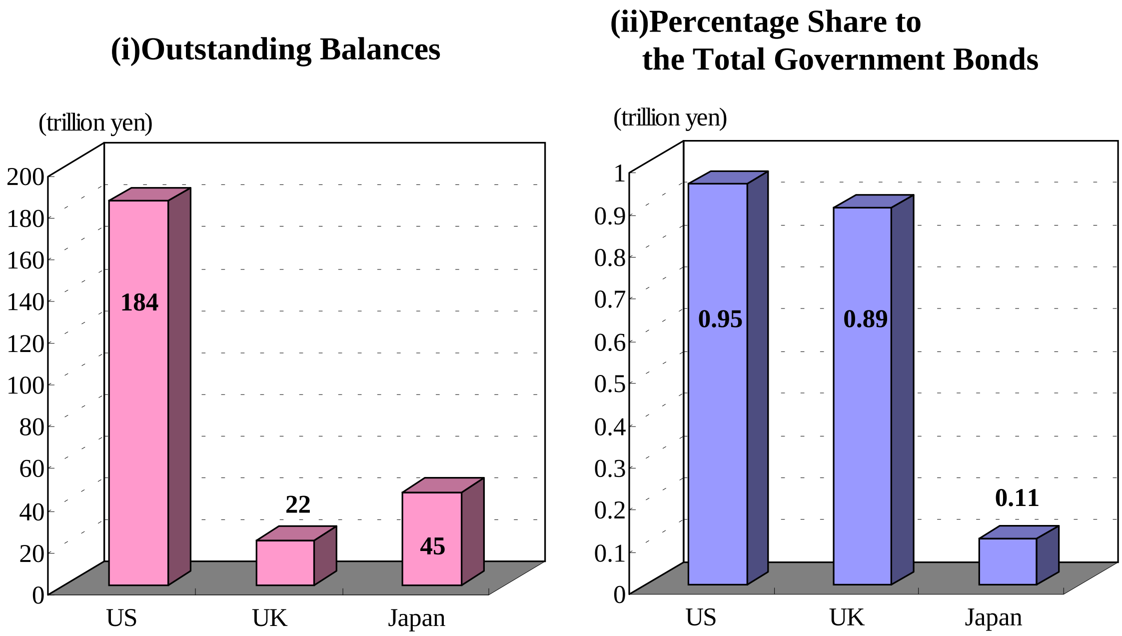


Sources: CP: Bank of Japan, Repo: Japan Securities Dealers Association, Others: Bank of Japan's *Financial and Economic Statistics Monthly*

London at that time. Given the high costs, however, efforts were being made to establish a market in the US. Against this background, the Federal Reserve bought BA discounts with resale agreements, thereby providing funds at the official discount rate. These operations were the beginning of repo transactions. See Stigum (1989) for details.

[13] In the UK, the volume of repo transactions utilizing US Treasury bonds and German government bonds has increased since the launch of the Big Bang initiatives in 1986. Repo transactions utilizing UK government bonds, however, have been virtually limited to operations by the Bank of England. The UK repo market has been opened to all market participants since January 1996. See Repo Trading Research (2001) for details.

Figure 3: Comparison of the Size of the Repo Markets



Note: The date for the US and Japan is as of the end of March, 2001. The data for the UK is as of the end of May, 2001.

Sources: US: *Federal Reserve Bulletin* for the repo data. *Flow of Funds Accounts* for the bond data. Both are published by the Board of Governors of Federal Reserve System

UK: *Monetary and Financial Statistics* (Bank of England) for the repo data. The debt Management Office for the bond data.

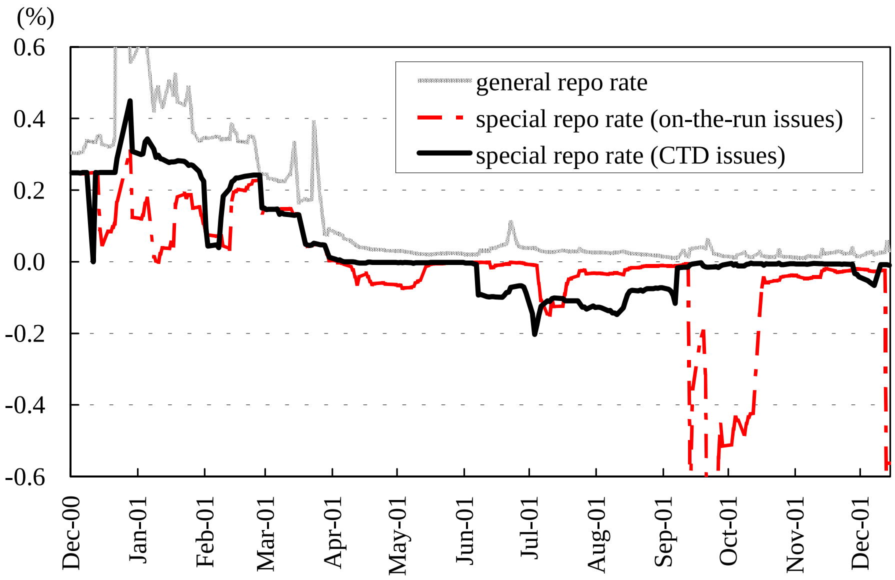
Japan: Japan Securities Dealers Association for the repo data. *Flow of Funds Accounts* (Bank of Japan) for the bond data.

(ii) Repo Rates in Japan

Special repo rates are mostly independent of, and lower than, general repo rates. Figure 4 shows overnight general repo rates and special repo rates on on-the-run issues of 10-year JGBs and the CTD issues.[14] Evidently, there is a large difference between repo rates. They sometimes take negative values depending on the underlying issues. We will briefly explain here the mechanism by which this phenomenon occurs although the bonds put up as collateral are of the same quality in terms of credit risk. A detailed examination will be made in section III.

[14] Overnight transactions are those whose settlements are made two days after the contract date, also known as spot/next transactions.

Figure 4: Repo Rates



Source: Bank of Japan

As stated earlier, the repo rate is defined as the interest rate minus lending fee. All funds are equal in quality by definition. Thus, theoretically, the interest rate should equal the average rate for uncollateralized lending like TIBOR (Tokyo Interbank Offered Rate), whether it is a general or special repo rate. With regard to lending fee, since most issues can be used for general repos, it is natural to think that fees are on the same level.

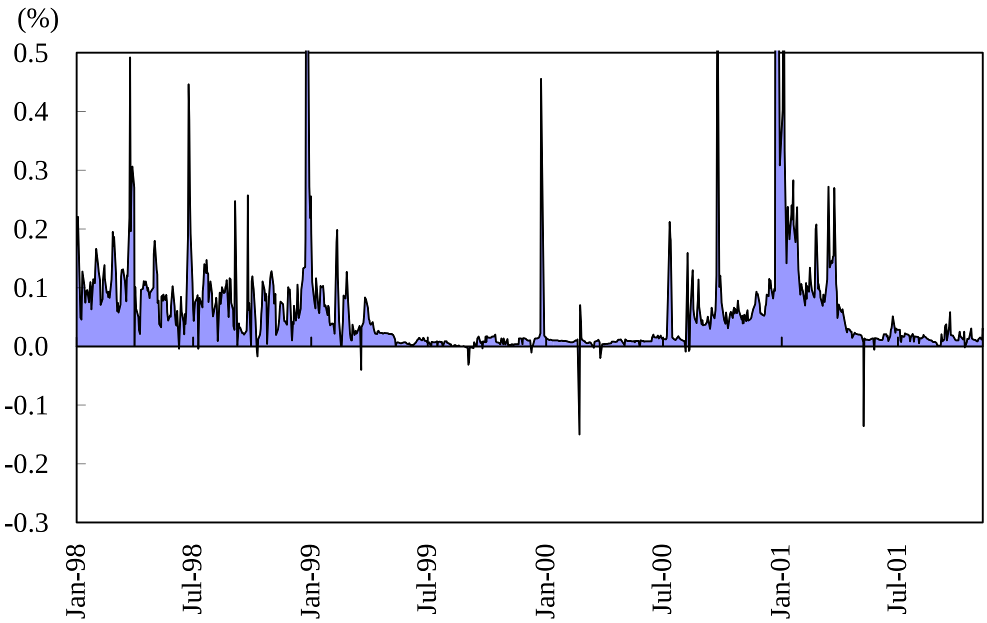
Suppose, however, that a certain issue cannot be substituted for others, resulting in its own unique supply/demand structure. If demand for this issue exceeds supply in the repo market, the market mechanism would work to raise the lending fee of this issue by (Δlending fee) above that of other general issues. As a result, the repo rate for this issue would fall by (Δlending fee) by definition, which leads to the following relationship: interest rate - (lending fee+Δlending fee) = original repo rate - Δlending fee. When the (Δlending fee) exceeds the original repo rate, the repo rate turns negative.

Similar to the general repo rate, the special repo rate is defined as "interest minus the lending fee for a specific issue". Thus, the repo spread is equivalent to "the lending fee for a specific issue minus that for general issues", which is equal to the above-mentioned (Δlending fee). When the repo spread is positive, the lending fee for this specific issue rises compares with that for general issues due to the tight supply/demand condition for that issue.

Figure 5 shows the spreads between the overnight general repo and major uncollateralized lending rates. As stated earlier, although general repo transactions have a risk-free characteristic, they are almost always higher than uncollateralized call rates, often staying above euro-yen rates. This observed fact stands out when compared with the relationship between US general repo and FF rates. As Stigum (1989) points out, in the US, the general repo rate moves almost in parallel, a few basis points below the uncollateralized FF rate. For example, from January 1998 to August 2001, the spread between the general repo and uncollateralized call rate (FF rate for the US) averaged 0.05% in Japan, compared with minus 0.07% in the US.[15]

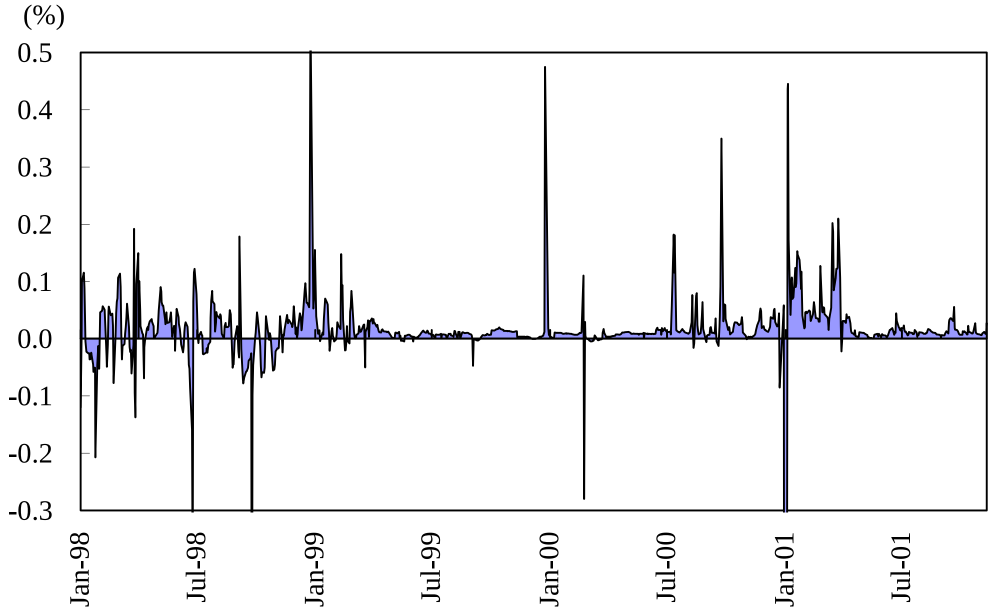
Figure 5: Spreads between the General Repo Rate and the Uncollateralized Rate

(i) Spread between the General Repo Rate and the Uncollateralized Call Rate



[15] At present, no consensus has been reached on why risk-free repo rates are higher than uncollateralized rates. We attempt to find an answer to this question in the Appendix, focusing on the period after January 2001, when the RTGS (real-time gross settlement) was introduced.

(ii) Spread between the General Repo Rate and the Euro-yen Rate



Source: Bank of Japan

III. Theoretical Review

Duffie (1996) and Krishnamurthy (2001) theoretically derived the mechanism by which repo rates can be priced differently depending on the issue in perfectly-competitive equilibrium. In this section, we review the mechanism to understand the working of the repo market.

A. General Repo Rate

As shown in figure 6, a general repo transaction can also be thought of as the combined position of a short sale of a bond at price P on the contract date and a forward contract that receives P x (1+general repo rate R) at the end of the contract period.

In equilibrium, the expected return from the above transaction, $E[F - P(1+R)]$, should be zero, where E denotes the expectation operator. This leads to the following equation :

$$F = P(1+R). \tag{1}$$

That is, the general repo rate should be equivalent to the risk-free interest rate.[16]

Figure 6: Forward and Repo Transactions

Contract Date	End of the Contract Period
Contracted Price P	Contracted Price $P \times (1 + \text{Repo Rate } R)$
$\|$	$\|$
Cash Price P	Forward Price F

B. Special Repo Rate

(i) No-Arbitrage Condition

Now let us review how special repo rates are priced based on Duffie (1996) and Krishnamurthy (2001). The preceding discussion shows that the general repo rate should be equal to the risk-free interest rate. On the other hand, as shown in section II, the special issues have their own special repo rates, which are lower than general repo rates. For equilibrium to be reached where repo spreads are present, the profits expected from arbitrage trading using repo spreads need to be zero. As explained earlier, an arbitrage position can be built without holding any initial capital or bond issues by combining a repo or reverse repo contract with an outright cash bond transaction. Therefore, examining the zero-profit condition from these transactions (non-arbitrage condition) will provide a clue to the level of equilibrium repo spreads.

For simplicity, we consider a two-period model without any costs incurred by uncertainty with respect to inventory arising from mismatching in the timing of buying and selling, as well as asymmetric information. We assume that settlement of

[16] Equation (1) means that the period rate of return, computed from futures and spot prices, is equal to the general repo rate. Market participants actually conduct arbitrage trading, taking advantage of this relationship, which combines (i) the futures and spot sales/purchases of CTD issues and (ii) repo transactions. Repo Trading Research (2001) provides a simple example of this arbitrage trading. The rate of period return is also called "implied repo rate". In a normal situation, the implied repo rates should be equal to repo rates on CTD issues as a result of arbitrage trading. In reality, however, this is not always the case because of inherent delivery risk when the CTD issue changes. See Shigemi, Kato, Soejima, and Shimizu (2000) for details.

transactions in bonds and funds takes place once a day. There are two bond issues, S and G, priced at P_S and P_G, respectively. The former is used as collateral for a special repo and the latter for a general repo. The respective repo rates are R_S and R_G, respectively. There are a large number of risk neutral bond dealers in the market, who know for certain that the bond prices, $P_S > P_G$ on date $t = 1$, will converge to P_{con} on date $t = 2$.

To build an arbitrage position, a bond dealer will sell short the issue S and use the funds raised to borrow the same issue in the special repo market. Simultaneously, the dealer will buy issue G to hedge interest rate risk using the funds raised through a general repo. As a result, profit/loss π can be expressed as

$$\begin{aligned}\pi &= -[P_{con} - P_S] + [P_{con} - P_G] - P_G R_G + P_S R_S \\ &= P_S - P_G - P_G R_G + P_S R_S\end{aligned}. \tag{2}$$

The first and second terms of the right hand side of equation (2) represent the capital gains on issues S and G. Equilibrium requires profit from this arbitrage trading to be zero. Thus, the following no-arbitrage condition should hold:

$$\frac{P_S}{P_G} = \frac{1 + R_G}{1 + R_S}. \tag{3}$$

Equation (4) below is a logarithmic version of equation (3). The left hand side of equation (4) represents the cash premium, which measures the extent to which S is evaluated higher than G and the right hand side represents the repo spread:

$$\frac{P_S - P_G}{P_G} = R_G - R_S. \tag{4}$$

Equation (4) shows that profit from the repo spread is fully offset by the higher market price of issue S that was purchased to build the arbitrage position. It also implies that, in equilibrium, repo rates adjust themselves to eliminate any profits expected from the arbitrage position taking advantage of the difference in bond prices. The above argument suggests that the parity encompassing the repo and bond markets should hold, just like the parity linking the foreign exchange rate and interest rate differential between any two countries. Also, in equilibrium, the following condition (5) is always satisfied:

$$R_S \leq R_G . \tag{5}$$

To prove condition (5), all we have to do is to prove that $R_S > R_G$ contradicts the notion of equilibrium. If $R_S > R_G$ holds, issue S borrowed through a reverse repo can be used in a general repo. This transaction enables a dealer to make arbitrage profits indefinitely without holding any initial capital, and thus equilibrium is never reached. Therefore, in equilibrium, condition (5) needs to be satisfied.[17]

(ii) Derivation of Supply and Demand Curves

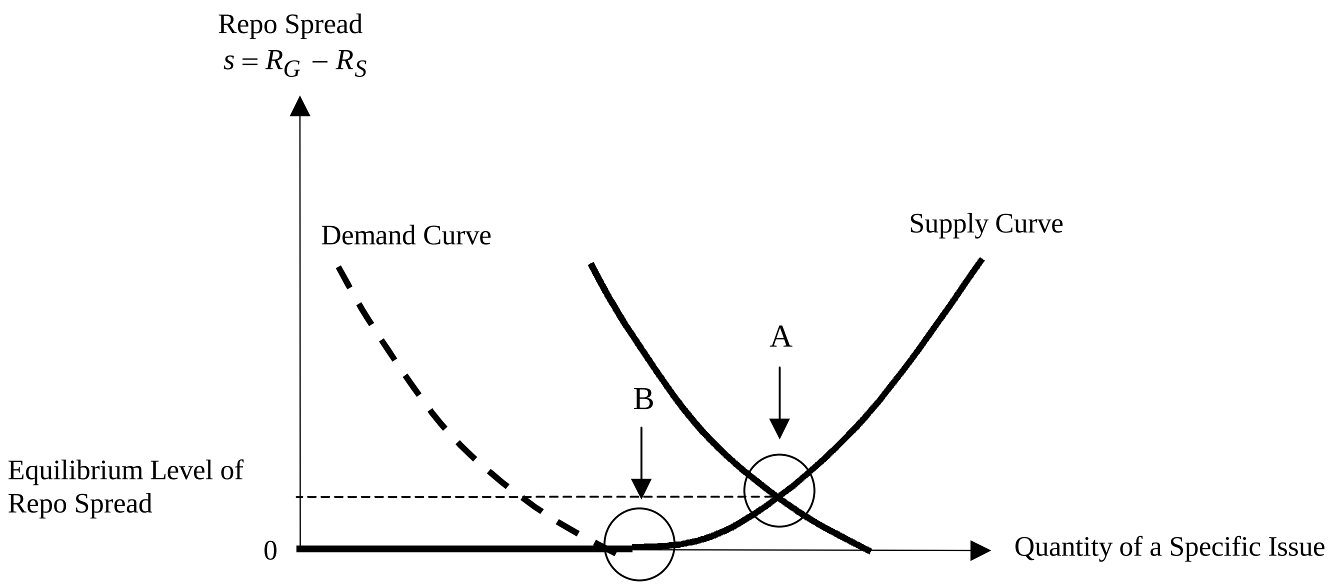
The above explanation enables us to understand that a repo spread and the no-arbitrage condition can coexist in the form of parity between repo spread and cash premium. However, this no-arbitrage condition alone does not determine the equilibrium level of repo spread and cash premium. Since a repo spread is a kind of asset price, its equilibrium will be achieved at a level where demand and supply for the issue in the repo market meet. Let us take a look at figure 7 to examine this point.

First, the supply curve is written as upward sloping. The logic behind this is as follows. Let us assume a case where an investor with issues on special trades in the repo market. We assume for simplicity that the investor is a price taker. The investor can raise funds at a lower special repo rate of R_S and at the same time invest them at a higher general repo rate of R_G. Then, the investor's rate of return per issue will be equal to the repo spread, or the investor can lower portfolio financing costs by the repo

[17] Let us confirm that when condition (5) is satisfied in equilibrium, a positive arbitrage profit is not necessarily ensured. When trying to make a profit from the spread $R_G - R_S$, an issue borrowed through a general repo cannot be put up as collateral in a special repo. Thus, there is a need to purchase a special issue in the cash market. In this case, even if a general repo issue was sold short to hedge against interest rate risk, no-arbitrage condition (4) ensures a zero profit. If the position were unhedged, the transactions would no longer be arbitrage trading, but speculative trading, which

spread. Also, assuming that these transactions are cost-free, the investor with special issues will have an incentive to supply the entire portion of special issues to the repo market. In this case, the supply curve will level out at a certain point in the repo spread.[18]

Figure 7: Determination of Repo Spread



In reality, however, various transaction costs arising from accounting and institutional factors are incurred. Thus, not all special issues will be supplied to the repo market. To elaborate on this point, let us examine a case where transaction costs are incurred and marginal cost increases with transaction volume. These transaction costs may include such costs as back-office operations and delivery costs.

Let us now consider the optimal choice for investors. As long as the repo spread, that is, marginal profit, is larger than marginal cost, investors will supply issues to the special repo market and will continue to do so until marginal cost rises to the level of the repo spread. Therefore, the supply curve should be upward-sloping.

Next, let us turn to the shape of the demand curve. Bond dealers are the major

does not guarantee a positive profit.

[18] To be precise, the supply curve will flatten out at a certain level of the repo spread, ending at the upper limit of supply of the special issue.

agents borrowing collateral issues in the special repo market. As shown in equation (4), in equilibrium the repo spread is fully offset by the cash premium. This means that, regardless of the size of the repo spread, bond dealers' net profits should be zero. Therefore, optimal behavior by dealers alone will not create a downward sloping demand curve. Here, we should pay attention to the dealers' position combining reverse repos and short sales in the cash market. As explained in section II, the main reason for bond dealers to raise specific issues through special repos is to cover their short positions in the cash market. In other words, behind dealers' demand for repos lie investors who would like to buy the issues from dealers. Since the no-arbitrage condition holds, these investors are paying an additional cost, equal to the repo spread, to acquire these specific issues.[19]

Why do investors place greater value on specific issues than on others? For example, life insurance companies, major institutional investors in Japan, tend to prefer issues with high current yields and coupons. Also, in the JGB futures market fictitious issues with 10 years to maturity and a coupon rate of 6% are traded. The settlement price on the due date is determined by the futures price multiplied by the conversion factor (CF). The seller of the futures naturally prefers to deliver the cheapest (CTD) issues that will produce the smallest net basis when the above settlement price is subtracted from the contract price. Thus, there is an incentive for sellers of futures to hold CTD issues in advance. Furthermore, investors generally like to trade issues with high market liquidity. BIS (1999a) defines a liquid market as "a market where participants can rapidly execute large-scale transactions with a small impact on prices". Factors causing differences in liquidity may include whether the issue is on-the-run, and whether the issue is being reopened, consolidating with others. In the JGB market, trading is heavily concentrated on on-the-run issues and marketability soon becomes extremely thin as they are incorporated in investors' portfolios as "buy-and-hold" issues.

Now, let us assume that the benefit from holding a certain issue diminishes as the

[19] The repo spread may reflect the convenience yield, which represents the convenience of holding actual commodities rather than futures positions in the same commodities. For example, if a commodity falls in short supply temporarily, it will be of great convenience to actually have the commodity in question. If the commodity is a raw material, there is great convenience in being able to continue production. See Hull (2000) for details.

size of the holding increases: the larger the benefit stemming from an issue, the greater the decline in benefit from the addition of the same issue (diminishing marginal utility). If this is the case, a decrease in the repo spread means a decline in marginal cost relative to marginal utility, which will prompt investors to increase purchases. Similarly, since an increase in the repo spread means a rise in marginal cost, investors will reduce purchases to a level in line with marginal utility. As a result, the demand curve should be downward sloping.[20]

Under the supply/demand structure given above, the equilibrium point for an issue on special is at point A in figure 7, while that for a general issue is at point B.

(iii) No-Arbitrage Condition in a Multi-Period Setting

We have so far limited our attention to a two-period horizon. In reality, however, market participants operate over a multi-period horizon. Following Krishnamurthy (2001), we extend the no-arbitrage condition to a multi-period setting. As stated earlier, there is a close relationship between cash bond prices and special repo rates. Also, investors with special issues can raise funds at a low interest rate through special repos, while at the same time they invest them in general repos, earning returns equal to the repo spreads. Naturally, an investor who expects a certain issue to become special is likely to try to make profits by using such an issue in repo transactions. If a large number of market participants form similar expectations regarding the same issue, the present value of the expected repo spread will be reflected in the cash market price. Let us look at this point in more detail.

The assumptions are basically analogous to the two-period setting. Two issues exist. The two sets of cash prices and repo rates at time *t* are (i) $P_{S,t}$ and $R_{S,t}$ and (ii) $P_{G,t}$ and $R_{G,t}$, respectively. We assume $P_{S,t} > P_{G,t}$. Market participants know that the prices of both issues will converge at P_{con} at time T.

Under this setting, a risk-neutral bond dealer builds an arbitrage position over the multi-period horizon. To be more specific, at time *t*, the dealer sells short the higher-

[20] Duffie (1996) derives a downward-sloping demand curve by assuming the presence of risk averse

priced issue and covers the position through a special repo, while raising funds through a general repo to buy a lower-priced issue with a view to hedging against interest rate risk. Return π_t at time t+1 can be expressed as

$$\pi_{t+1} = \left(P_{G,t+1} - P_{G,t}\right) - \left(P_{S,t+1} - P_{S,t}\right) - P_{G,t}R_{G,t} + P_{S,t}R_{S,t} \tag{6}$$

Equilibrium would not be reached if this position continued to yield profits/losses. Therefore, under the condition that information at time t is shared by all market participants, equilibrium must be reached at the point where the expected return is zero, which is, $E_t\left[\pi_{t+1}\right] = 0$. Hence,

$$E_t\left[\left(P_{G,t+1} - P_{G,t}\right) - \left(P_{S,t+1} - P_{S,t}\right)\right] = P_{S,t}R_{S,t} - P_{G,t}R_{G,t}\,. \tag{7}$$

By assumption, market participants know that the prices of the two issues will converge at time T. Thus, equation (7) yields the following relationship (8) between present bond prices and repo rates by summing up from time 0 to T-1:

$$P_{S,0} - P_{G,0} = \sum_{t=0}^{T-1} E_0\left[P_{G,t}R_{G,t} - P_{S,t}R_{S,t}\right]. \tag{8}$$

In equation (8), $P_{S,0} > P_{G,0}$ and $R_{S,t} < R_{G,t}$ hold. Therefore, we reach the same conclusion as the two-period model that the repo spread is priced at a level that fully offsets the price differential between the underlying issues in the cash bond market. In addition, we can find another interesting insight from this equation. Equation (8) can be rewritten as follows:

$$P_{S,0} - P_{G,0} = P_{G,0}R_{G,0} - P_{S,0}R_{S,0} + \sum_{t=1}^{T-1} E_0\left[P_{G,t}R_{G,t} - P_{S,t}R_{S,t}\right]. \tag{9}$$

The left hand side of equation (9) represents cash premium at time 0. The first term on

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investors.

the right hand side represents the repo spread (valued in terms of return, and not rate of return) at time 0. The second term denotes the sum of future repo spreads expected at time 0. Put differently, the expected repo spreads are reflected in the present cash premium. In the two-period model, issues were assumed to be special for just one period. However, when a certain issue is expected to remain special until some time in the future, a repo spread ensures that the no-arbitrage condition is satisfied over the multi-period horizon. That is, a cash premium could emerge when market participants expect an issue will be special because of the high likelihood that it will become the next CTD, for example.

The above discussion implies that the repo spread could provide a clue to future cash premium. If the term structure of the repo spread is estimated, the implied repo spread could be worked out backward by analogy with the pure expectations hypothesis. This line of logic suggests the possibility that the future cash premium of a specific issue may be estimated in advance from the implied repo spread. In reality, however, trading volume of term transactions in the Japanese repo market is so thin that future cash premium thus computed might not be reliable.[21] Furthermore, if we assume the presence of risk averse market participants, the term structure of the repo spread would include a premium arising from uncertainty. If that is the case, the implied repo spread would lead to an overvaluation in future cash premium.[22]

[21] Shigemi, Kato, Soejima, and Shimizu (2001) point out that compared with the US market, term transactions, e.g. three-month repos, are not actively traded in the Japanese market. As a result, some market participants noted that in the formation of repo rates for long-term contracts, the cut-off rate for the Bank of Japan's repo operations act as a benchmark for term instruments. See Kato (2001) for details. Also, see BIS (1999b) for the role of the repo market in monetary policy.

[22] For example, Buraschi and Menini (2001) use the US data to examine the term structure of repo spreads, rejecting the expectations hypothesis.

(iv) The Repo Market under Stress

Analysis so far has focused on a normal situation where the market is free from stress. By stress we mean a situation where one or the other preconditions for the model describing a normal situation is not satisfied or where an unexpected event causes market participants to overreact in response to the event. Various mechanisms can cause stress, which makes it difficult to generalize. To illustrate the repo market under stress, we review in Box 2 a stressful event that occurred in Japan at the year-end of 1999, triggered by a rumor about repo transactions carried over to the following year. This episode will help readers understand the limitations of the model describing a normal situation, once a stress occurs.

Box 2: Case Study of the Repo Market under Stress

According to Shigemi, Kato, Soejima, and Shimizu (2001), a rumor that "some public funds would not be engaged in repo transactions (lending bonds) over the year-end because of concerns over the Y2K problem" circulated in the repo and JGB markets around August 1999. This rumor fueled speculation that other government-related institutions would also withdraw from repo transactions over the same period. Bond dealers had built short positions on the assumption that large institutional investors such as public funds and life insurance companies would continue to lend bonds normally through repo transactions. Sparked by the rumor, they rushed to close their positions, which triggered a precipitous fall in SC repo rates, thus rapidly expanding repo spreads.

According to Duffie (1996) and Krishnamurthy (2001), repo spreads are determined by the balance between supply and demand of specific issues in the repo market. In the above case, the rumor would cause the supply curve to shift leftward (see figure 7). Applying this logic, the following interpretation is possible. The speculation that the supply of specific issues would be limited over the year-end created expectations among bond dealers that cash bond prices would shoot up. Thus, they immediately closed positions, triggering expansion in the repo spread.

There is a possibility, however, that the expansion in the repo spread was much greater than that predicted by Duffie (1996) and Krishnamurthy (2001). Shigemi, Kato, Soejima, and

Shimizu (2001) summarize the mechanism of the surge in the repo spread as follows: (i) the rapid expansion in basis prompted bond dealers, who had been conducting arbitrage trading that combines cash and futures positions (short-basis trading), to cut losses by buying back cash bonds and selling futures; the resultant further expansion in basis caused a chain reaction by which implied repo rates fell further (a surge in lending fees), leading to a sharp rise in cash prices; and (ii) some repo dealers, who had built short positions through repo transactions by lending cash bonds on a long-term basis, rapidly reversed the reversal of the positions under loss-cut rules as losses expanded due to the sharp fall in repo rates.

IV. An Empirical Analysis

As explained in section III, no-arbitrage condition (4) should hold when the repo market is in equilibrium. In other words, when a repo spread exists, the price of the issue should be higher than other issues by the repo spread. When this relationship is extended to a multi-period setting, it follows that the bond price should reflect the repo spread expected to arise in the future. In this section, we try to verify whether such a theoretical relationship holds in the Japanese repo market, basically following the methodology by Jordan and Jordan (1997).[23]

A. Data

In carrying out a precise analysis, it is necessary to collect data on two different bond issues: general and special ones with the same coupon rate and maturity. In reality, however, we cannot find such pair of issues, and thus need to estimate their theoretical prices. In this paper, we first estimated theoretical cash prices by employing the cubic spline function, and then proceeded to obtain bond premiums by comparing them with actual cash prices.[24,25] To be more specific, we derived a discount factor from the price

[23] Jordan and Jordan (1997) confirmed that there is a close relationship as in equation (4) between the repo and Treasury securities markets in the US.

[24] In this paper, we employed the cubic spline function devised by McCulloch (1975). This function is widely used since a simple linear regression can be used for estimations. Given that the discount factor is estimated from different sets of price data on bonds with different remaining maturity, we used generalized least squares (GLS) on the assumption that the standard deviation of the error term

data of general issues to compute the theoretical prices of special issues.[26] These would be effectively "theoretical prices of the bond if it had not become special." The difference between the theoretical price P_G and the actual market price P_S can be regarded as the cash premium. In this way, we computed the cash premium of on-the-run JGB and CTD issues from the discount factors estimated from the data on 10-year JGBs, excluding the following issues: on-the-run, CTD, JGB issues with issue dates close to the CTD issues, and others that market participants reported were special. We used on-the-run and CTD issues after the Bank of Japan started collecting data on repo rates.[27]

Table 1 shows the average cash premiums on on-the-run and CTD issues and the cash premiums on corresponding general issues. With a few exceptions, we found that cash premiums on special issues are larger than others.

Table 1: Estimated Cash Premiums (Average)
(i) On-the-Run Issues

	226th	227th	228th	229th	230th	229th*	231st	232nd	233rd	234th
Cash Premium	0.33 (0.01)	0.20 (0.01)	0.11 (0.00)	0.18 (0.00)	0.02 (0.00)	-0.14 (0.00)	-0.05 (0.00)	0.03 (0.00)	0.09 (0.00)	0.19 (0.02)

(ii) CTD Issues

	202nd	203rd	205th
Cash Premium	0.24 (0.00)	0.12 (-0.01)	0.17 (-0.01)

Notes: Cash premium is defined as the difference between the market price of a special issue and the theoretical price as expressed per 100 yen of par value. The figures in parentheses are the average cash premiums on the interest-bearing 10-year JGB issues during the same sample period, excluding on-the-run, CTD, and other issues that market participants reported were special. The 229th issue is a reopened one. The sample period for on-the-run issues is from the auction date to the last business date before the issuance of the next JGB issue. For the CTD issues, it is from the date when the issue became the CTD issue to the last business date before it lost the status.

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of each issue is in proportion to the duration.

[25] For JGBs, we used the closing price (3:00 p.m.) announced by Japan Bond Trading Co. Ltd.

[26] A discount factor can be understood as the price of a discount bond paying 1 yen at time-point *t* as a function of *t*.

[27] The Bank of Japan started collecting data on repo rates on December 18, 2000. We excluded the 225th issue (issued November 11, 2000) because repo spread data for the whole period during which the issue held the status as the on-the-run issue was not available. Similarly, regarding the CTD issue, we used data for issues for which repo spreads for the whole period were available.

Regarding the expected sum of repo spreads, we adopted the methodology of Jordan and Jordan (1997) and calculated equation (10) below using general and special repo rates reported by participants in Bank of Japan market operations:

$$RS_{it} \equiv \left[\sum_{\tau=t}^{T} \left(P_{G,t} R_{G,t} - P_{S,t} R_{S,t} \right) \right]_i . \tag{10}$$

In this equation, *i* denotes the index for each issue, *t* the date of observation, and T the last day when the issue holds the status of on-the-run or CTD issue. R_G and R_S denote the general and special repo rates, respectively. This equation assumes that market participants expect the issues to cease to be special on the day they lose status as on-the-run or CTD issues.[28]

B. Relationship Between Repo Spread and Cash Premium

Figure 9 depicts a cross-sectional relationship between (i) cash premiums[29], and (ii) the expected repo spreads. There appears to be a positive correlation between them.

To verify this observation statistically, we conducted a regression analysis using equation (11) below:

$$\Delta P_{i,t} = \alpha_i Dummy_i + \beta RS_{i,t} , \tag{11}$$

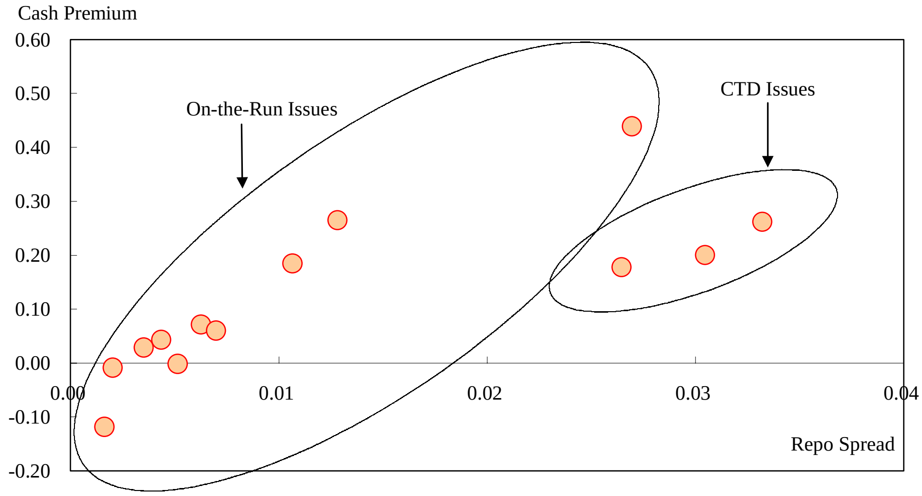
where $\Delta P_{i,t}$ denotes cash premium on issue *i* at time *t*. Note that right-hand side of equation (11) includes dummy variables, in addition to the expected repo spread, taking account of attributes of each issue.[30]

[28] The cash premium on the 229th issue was positive before it was reopened, but turned negative after being reopened. According to the model in section III, both cash premium and repo spread should fall when supply increases due to reopening. But, as shown in equation (5), in equilibrium, both cash premium and repo spread must be positive. Regarding this point, we had difficulty conducting a reliable analysis due to the limited data on reopened issues.

[29] For on-the-run issues, cash premiums were computed as of the issue date. For CTD issues, they were computed as of their first trading day.

[30] By definition, the period during which an issue holds the status as either the on-the-run or the

Figure 9: Relationship between Repo Spread and Cash Premium



Note: Cash premium is expressed per 100 yen of par value, while repo spread is expressed as percentage point.

Estimation results are shown in table 2. First, with regard to on-the-run issues, the expected repo spread is statistically significant at the 10% level. At the same time, most dummy variables are significant at the 5% level, suggesting that individual attributes influence the level of cash premiums.

Regarding CTD issues, as was the case for on-the-run issues, it is observable that the expected repo spread is significant at the 10% level. Also, all of the dummy variables are all significant at the 5% level.

One possible reason for the relatively low explanatory power of the repo spread for CTD issues, relative to that of on-the-run issues, could arise from the use of overnight repo rates in our analysis. Unlike repo rates for on-the-run issues, repo rates for CTD issues are often substituted by term repo rates of the closing month for futures contracts as a benchmark. Therefore, in order to analyze the relationship between CTD cash premium and repo spread, it may be more appropriate to use term instruments covering the delivery period of the futures contract rather than overnight instruments. In order to conduct a more robust analysis, we should wait until enough data has been accumulated.

CTD issue does not overlap with others. For this reason, equation (11) has two suffixes, *i* and *t*. These, however, do not represent the panel data.

Table 2: Estimation Results
(i) On-the-Run Issues

(Sample Number:191[Daily Basis])

	Coefficient	*t*-value	*p*-value
Expected Repo Spread: β	3.878	1.938	0.054
Dummy1 (α_1:226th)	0.272	6.748	0.000
Dummy2 (α_2:227th)	0.173	5.211	0.000
Dummy3 (α_3:228th)	0.096	3.386	0.001
Dummy4 (α_4:229th)	0.167	6.139	0.000
Dummy5 (α_5:230th)	0.008	0.272	0.786
Dummy6 (α_6:229th [reopened])	-0.058	-2.129	0.035
Dummy7 (α_7:231st)	-0.142	-5.346	0.000
Dummy8 (α_8:232nd)	0.024	0.867	0.387
Dummy9 (α_9:233rd)	0.090	3.457	0.001
Dummy10(α_{10}:234th)	0.134	3.262	0.001
Adjusted R^2		0.544	

(ii) CTD Issues

(Sample Number:182[Daily Basis])

	Coefficient	*t*-value	*p*-value
Expected Repo Spread: β	1.018	1.755	0.081
Dummy1(α_1:202nd)	0.226	22.21	0.000
Dummy2(α_2:203rd)	0.114	14.64	0.000
Dummy3(α_3:205th)	0.189	15.38	0.000
Adjusted R^2		0.439	

It should be noted that the estimated β s on the expected repo spread differ greatly between on-the-run and CTD issues. There are two possible reasons for this. First, we assumed that on-the-run issues are special for about one month (until the next issue is issued). Thus if the issue is special for more than one month, the estimated β might become overvalued to bridge the gap. Second, there seem to be large differences in the degree of uncertainty over the length of time during which on-the-run and CTD issues are special. The multi-period models of Duffie (1996) and Krishnamurthy (2001) are based on the assumption that risk neutral market participants have perfect foresight. If uncertainty is present while market participants are risk averse, however, a premium reflecting uncertainty might be added in the cash premium. For CTD issues, it is relatively easy to obtain information on the overall positions of futures contracts or implied repo rates, both of which can be used to forecast the length of time that the CTD issues are special. For on-the-run issues, however, it is difficult to obtain robust implied repo rates due to (i) the unpredictability of supply and demand conditions because of the absence of a when-issued (WI) market[31] in Japan, and (ii) thin market volume in futures compared with CTD issues. Consequently, uncertainty about the length of time during which the on-the-run issue is special might be reflected in the cash premium.

V. Concluding Remarks

This paper has examined the Japanese repo market, which was launched as recently as April 1996. Despite its short history, the market now constitutes a main pillar of Japanese money markets, at least in terms of scale. After reviewing the theoretical mechanism by which repo rates are priced, we conducted a simple empirical analysis. Not many studies on overseas repo markets are available, let alone on the Japanese market. We thus hope that the paper will be of some help in activating discussions on the Japanese repo market among market participants, academics, and central bankers.

Finally, we would like to add some words of caution. First. the paper disregards the effects of uncertainty. Second, it is based on the assumption of the Walrusian law of market clearing.

Regarding uncertainty, in real-life markets it is very difficult to determine when any specific issue becomes special and how long it maintains such status. In the US, it is relatively easy to identify which issue becomes special since the repo spread tends to follow a predictable pattern. In Japan, in contrast, due partly to the fact that the JGB holdings are concentrated on a handful of institutions, some issues become special unexpectedly.[32] Furthermore, regarding on-the-run JGB issues, the absence of a WI market implies that dealers have to participate in auctions without much prior information on future demand conditions. This uncertainty may give rise to inventory costs, which might add a risk premium to the repo spread.

[31] WI trading begins with the announcement of auction details, which include auction date, coupon rate, size of the issue, maturity date, interest payment date, bond code number, and whether the issue will be reopened or not. It ends on the auction or issue date. Settlement and delivery both take place on the issue date. Thus, we can regard it as a type of forward market. For details, see Soejima, Hanajiri, and Shimatani (2001).

[32] In US markets, there is a close relationship between the auction cycle and repo spreads. For details, see Keane (1996).

As for the market clearing assumption, we should note that actual repo transactions are not conducted via open markets like stock exchanges, but on an over-the-counter basis, normally on the telephone between bond dealers, securities firms, banks, and life insurance companies. In other words, market participants need to search to locate appropriate counterparties. Furthermore, since transactions are negotiable between the parties concerned, the balance of bargaining power may influence terms and conditions.

Appendix: Explaining the Gap between the General Repo Rate and Uncollateralized Interest Rates after the Introduction of RTGS

According to the simplest form of no-arbitrage theory, when general repo and uncollateralized interest rates differ, profits can be made by raising funds at the lower of the two rates and investing them at the higher rate. In equilibrium, however, there would not be any arbitrage profits, and the rates should converge. Furthermore, as the general repo rate is a collateralized rate, it should theoretically be lower than any uncollateralized interest rates, which should include credit risk premiums.

As seen in section II, however, the general repo rate is generally higher than the uncollateralized interest rate. How can we explain this phenomenon? First, it has been pointed out that since the introduction of "Real Time Gross Settlement" (RTGS) [33] in January 2001, settlement and collateral management costs for general repo transactions have increased.[34] These costs can be regarded as invariable, at least in the short run. Figure 5 in section II, however, shows that spreads between general repo and uncollateralized interest rates fluctuate over time. Thus, it seems that transaction costs alone do not provide a satisfactory explanation for the movement of the spread.

Second, some have pointed out that the gap emerges because the length of time between contract date and settlement date differs between general repo and uncollateralized transactions. The settlement of repo transactions is usually conducted

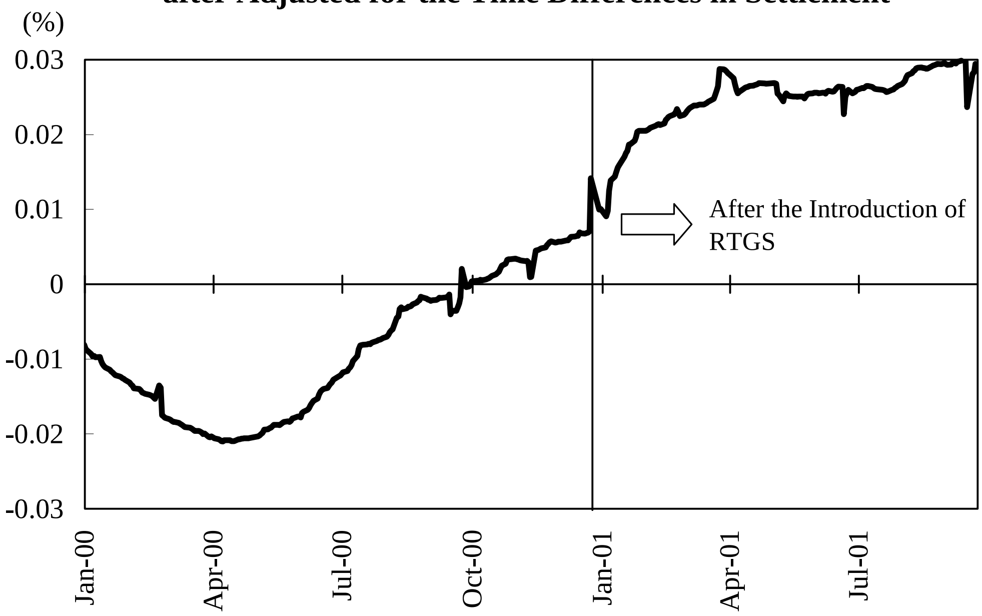
[33] The Bank of Japan's real-time gross settlement system for current account deposits and JGBs.
[34] Compared with the previous designated-time net settlement, RTGS substantially reduces systemic risk. On the other hand, it increases workloads and collateral management costs, since it involves real-time settlement and management of outstanding balances, in addition to an increase in the amount of funds and JGBs required for settlement.

two days after contract date while uncollateralized call transactions are settled on the contract date.[35] This fact suggests that investment in the general repo market against funding in the uncollateralized call market could yield the gap because of uncertainty over the availability of funds two days later. To examine this hypothesis, in figure A-1 we plotted the difference between (i) the spread between the general repo rate and the uncollateralized call rate, and (ii) the spread between the uncollateralized tomorrow/next interest rate and the uncollateralized call rate, multiplied by two.[36] Here, the settlement date of the uncollateralized tomorrow/next interest rate is contract date plus one day. The figure shows that after the introduction of RTGS, the former is constantly above the latter. It suggests that the general repo rate is higher than the uncollateralized call rate, even if the difference in the settlement timing is taken into account.

The above is an attempt to interpret the gap between the general repo and the uncollateralized call rates in terms of the differences in transaction practices. Given that current monetary policy keeps the uncollateralized call rate at a record low, however, it might be natural that the two rates should diverge. Then, let us consider why the general repo and the uncollateralized rates other than the call rates, such as the euro-yen rate, do not converge. We attempt to examine the mechanism with special attention to the demand structure for funds.

[35] For details, see Bank of Japan's Financial Markets Department (2002).
[36] All used rates are overnight rates.

Figure A-1: Still High Level of the General Repo Rate after Adjusted for the Time Differences in Settlement



Note: The data is calculated as the 180-day moving average of the following formula: (general repo rate)+(uncollateralized call rate)-2× (uncollateralized tomorrow/next interest rate).
Source: Bank of Japan

In the Japanese general repo market, main lenders are trust, city, long-term credit, and foreign banks while main borrowers are securities firms. In the euro-yen market, on the other hand, both main lenders and borrowers are city, long-term credit, and foreign banks. These banks can borrow funds in the call market, where the call rate is held down by monetary policy, and lend them in both the repo and euro-yen markets. Put differently, city, long-term credit, and foreign banks function as main funds lenders to both markets, interacting with different agents in need of funds in each market. Thus, these banks can build arbitrage positions using the repo, euro, and call markets, each of which has a distinctive fund demand structure.

Next, let us take a look at the demand structure for funds in each market. The main borrowers in the euro-yen market, that is, city, long-term credit, and foreign banks, have access to various markets other than the euro-yen market. In contrast, the main borrowers in the repo market, securities firms, do not have any other major means to raise funds other than the repo market. Thus, it is extremely difficult for securities firms to search for more competitive funding means even if unfavorable rates are offered in the repo market. Their negotiating power vis-a-vis lenders is likely to be weak, which implies that the interest-rate elasticity of demand is low. Moreover, since the introduction of RTGS, the incentive to reduce settlement volume has led to an increase in direct deals, resulting in a tendency among market participants to conduct repo transactions with a limited number of known counterparties.[37] The increase in direct deals has put securities firms in a more difficult position in finding competitive lenders, possibly weakening their power to negotiate further.

Given the above setup, let us now describe the optimal behavior for city, long-term credit, and foreign banks. What we employ here is a simple microeconomic tool. Let R_i denote the interest rate, Q_i quantity, MR_i marginal return (i = 1 for the repo market and i = 2 for the euro-yen market), and MC marginal cost.[38] We assume that city, long-term credit, and foreign banks are risk neutral and try to maximize profits in each period. Their optimal strategy, as expressed in figure A-2, would be to offer the rate that satisfies the following condition:

$$MC = MR_1 = MR_2 .$$

If we let ε_1 and ε_2 ($\varepsilon_1 < \varepsilon_2$) denote the interest-rate elasticity of demand for funds in each market, then marginal return can be expressed as follows:

$$MR_1 = R_1\left(1 - \frac{1}{\varepsilon_1}\right), \qquad MR_2 = R_2\left(1 - \frac{1}{\varepsilon_2}\right).$$

Now, we obtain the following optimal condition:

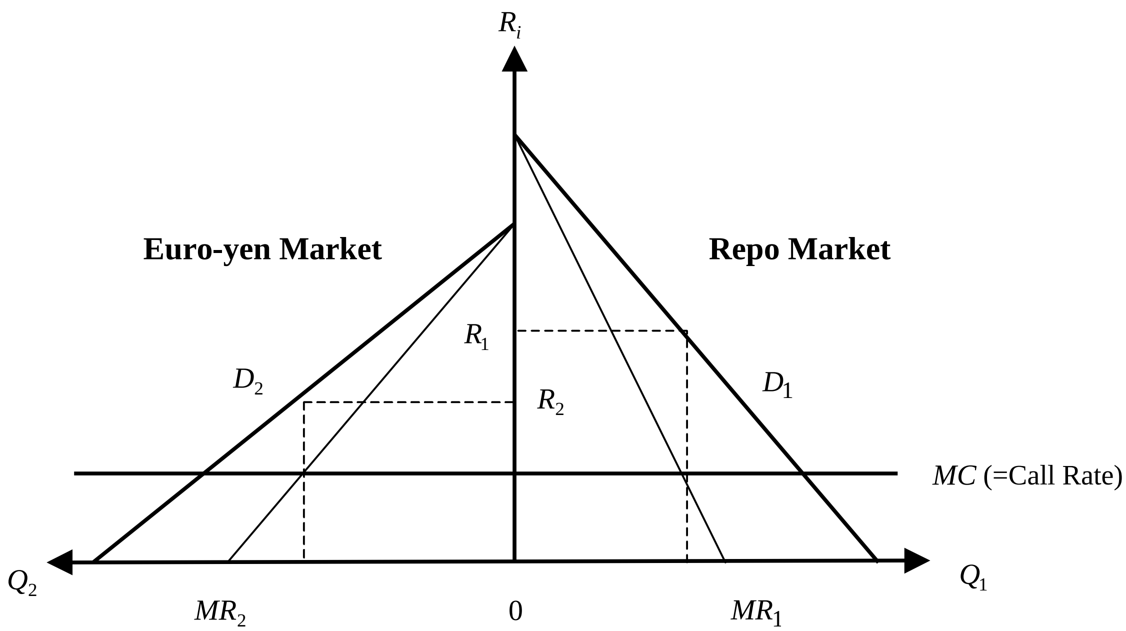
$$R_1\left(1 - \frac{1}{\varepsilon_1}\right) = R_2\left(1 - \frac{1}{\varepsilon_2}\right).$$

[37] See Bank of Japan's Financial Markets Department (2002).
[38] Lenders (city, long-term credit, and foreign banks) are the same in both markets, and thus marginal cost is set at a level equivalent to the call rate.

Thus, we find that $\varepsilon_1 > \varepsilon_2 \rightarrow R_1 < R_2$. This means that optimal behavior leads to the condition that the greater the interest-rate elasticity of demand for funds, the lower the rate that will be offered.

The above discussion raises a useful viewpoint on the question "Why don't rates converge despite active arbitrage trading between the two markets?" The argument that interest rates converge as a result of active arbitrage trading stems from the assumption of perfect competition, which does not envisage lenders with market power. If such lenders exist, lending more funds to the repo market beyond a certain point in the name of arbitrage trading turns out to be sub-optimal in that it only leads to a decline in profits for them.

Figure A-2: Interest Rate Discrimination Strategy For a Lender of Funds



References

Bank of Japan's Financial Markets Department, "A Review of the Financial Market Functioning after the Introduction of the RTGS System: Summary of Discussions with the Bank of Japan Market Operations Counterparties (Second Round)," Market Review 2002-J-1, Market Policies Group, Money and Capital Markets Division, February 2002 (in Japanese).

BIS (Bank for International Settlements), "Market Liquidity: Further Findings and Selected Policy Implications," 1999a.

BIS (Bank for International Settlements), "Implications of Repo Markets for Central Banks," 1999b.

Buraschi, A., and D. Menini, "Liquidity Risk and Special Repos: How Well do Forward Repo Spreads Price Future Specialness?" mimeo, 2001.

Duffie, D., "Special Repo Rates," *Journal of Finance,* 51, 1996, pp.493-526.

Griffiths, M., and D. Winters, "The Effect of Federal Reserve Accounting Rules on the Equilibrium Level of Overnight Repo Rates," *Journal of Business Finance and Accounting*, 24, 1997, pp.815-832.

Hull, J, *Options, Futures, and Other Derivatives*, 4th edition, Prentice Hall, 2000.

Jordan, B. D., and S. D. Jordan, "Special Repo Rates: An Empirical Analysis," *Journal of Finance*, 52, 1997, pp.2051-2072.

Kanno, H., and T. Kato, "The Introduction of New Repo in Accordance with Global Standards," Market Review 2001-J-9, Bank of Japan's Financial Markets Department, 2001 (in Japanese).

Kato, S., "BOJ's Repo Operations and Repo Rates: Analysis of Bond Loan Rate Implied in Repo Rate," Bank of Japan's Financial Markets Department Working Paper Series, 2001-J-3, 2001 (in Japanese).

Keane, F., "Repo Rate Patterns for New Treasury Notes," *Current Issues in Economics and Finance*, FRB New York, Vol. 2, No. 10, 1996.

Krishnamurthy, A., "The Bond/Old-Bond Special," mimeo, Northwestern University, 2001.

Maeda, H., "The Japanese Repo Market: Current Situation and Challenges)," Yuseikenkyu Geppo, Yuseikenkyujo, 1998 (in Japanese).

McCulloch, H. J., “The Tax Adjusted Yield Curve,” *Journal of Finance*, 30, 1975, pp.811-830.

Repo Trading Research, “An Introduction to Practical Finance: The Latest Book on Repo Transactions,” Nihonjitsugyou Shuppansha, 2001 (in Japanese).

Shigemi, Y., S. Kato, Y. Soejima, and T. Shimizu, “Market Participants' Behavior and Pricing Mechanisms in the JGB Markets: Analysis of Market Developments from the End of 1998 to 1999, Bank of Japan Financial Markets Department Working Paper Series, 01-E-1, 2001.

Soejima, Y., T. Hanajiri, and T. Shimatani, “Enhancing the Linkages between Primary and Secondary Government Bond Markets: Institutional Design of Primary Markets in G5 Countries and Some Empirical Studies,” Bank of Japan Financial Markets Department Working Paper Series, 2002-J-1, 2001 (in Japanese).

Stigum, M., *The Repo and Reverse Markets*, Irwin, 1989.

Uetsuki, M., “A Practical Introduction to Repo Transactions,” Nihonkeizai Shimbunsha, 1997 (in Japanese).