8. The Interaction of Light and Matter: α and n

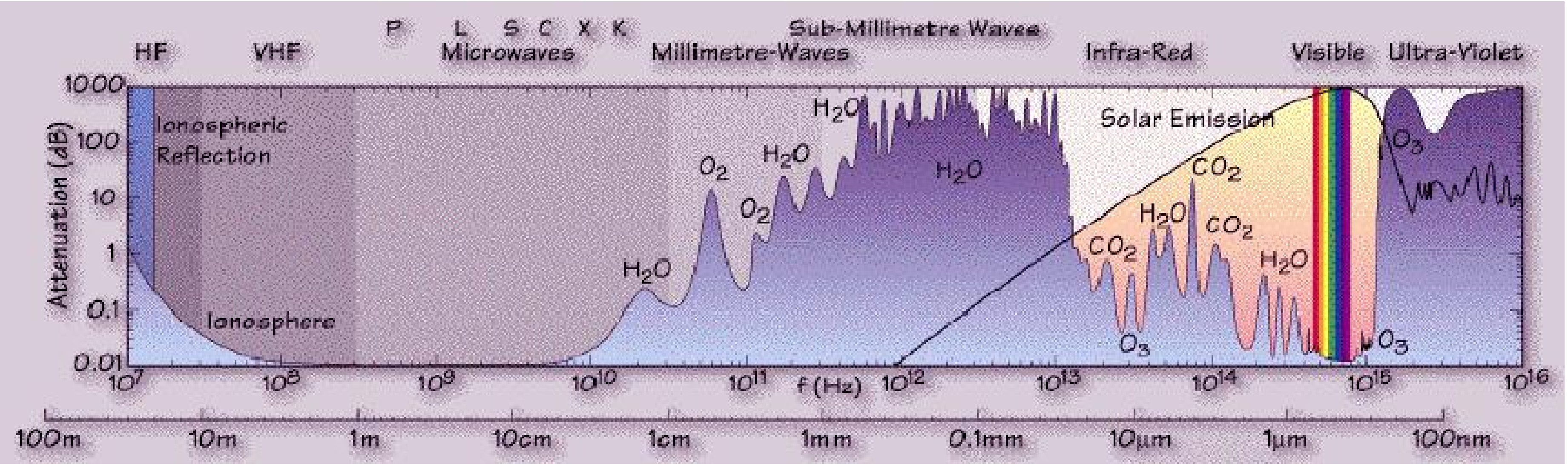
Absorption coefficient, α, and
refractive index, n.

Speed of light in a medium

Irradiance in a medium

Normal and anomalous dispersion

Examples



Absorption and refractive index

The wave equation, suitably modified using the slowly varying envelope approximation:

$$2jk\frac{\partial E_0}{\partial z} = -\mu_0\omega^2 P_0(z)$$

combined with the classical forced oscillator model for the motion of a bound electron:

$$P_0(z) = \frac{Ne^2/m_e}{2\omega_0(\omega_0 - \omega - j\Gamma)} E_0(z)$$

tells us how an EM wave propagates in a medium:

$$\boxed{E(z,t) = E_0(z=0)\cdot \mathrm{e}^{-\alpha z/2}\cdot \mathrm{e}^{jnkz - j\omega t}}$$

Absorption causes attenuation of the field with increasing z

Refractive index changes the k-vector

Note: if $\alpha = 0$ and $n = 1$, this reduces to the familiar result for waves in empty space that we derived in lecture 2: $E(z,t) = E_0(z=0)\cdot \mathrm{e}^{jkz - j\omega t}$

Refractive Index and the Speed of Light

The speed of light is ω/k. Since k becomes nk in a medium,

$$c = \omega/(nk) = (\omega/k)/n \longrightarrow \boxed{c = c_0/n}$$

where c_0 is the speed of light in vacuum.

The refractive index, n, of a medium is thus the ratio of the speed of light in vacuum to the speed of light in the medium. This is sometimes taken as an alternate definition of refractive index:

$$n \equiv c_0/c$$

The refractive index is almost always > 1, so **LIGHT SLOWS DOWN** inside materials.
(But it can be < 1. This appears to violate Relativity, but it doesn't.)

Note: often people don't write the subscript 0 on c, even when they mean c_0.

Absorption Coefficient and the Irradiance

Recall: the irradiance is proportional to the (average) square of the electric field.

Since $|E(z)| \propto \exp(-\alpha z/2)$, the irradiance decreases with increasing propagation distance:

$$I(z) \;=\; I(0)\exp(-\alpha\, z)$$

where $I(0)$ is the irradiance at $z = 0$, and $I(z)$ is the irradiance at $z > 0$.

Thus, due to absorption, a beam exponentially decreases in amplitude as it propagates through a medium.

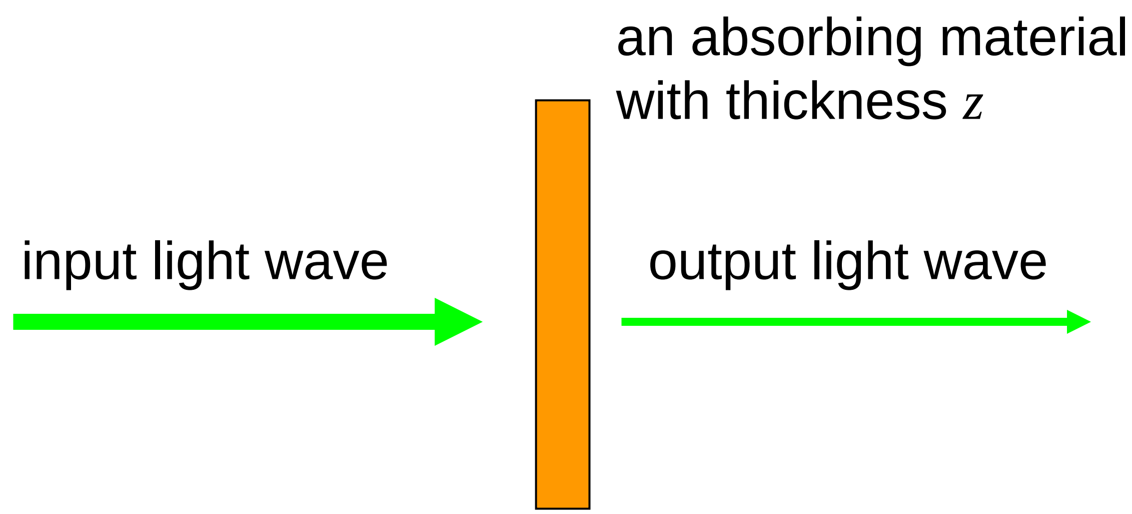
Note: α must have units of 1/length

The distance $1/\alpha$ is a rough measure of the distance light can propagate into a medium. It is the distance where the irradiance has decayed to 1/*e* of the value it had at the entrance to the medium.

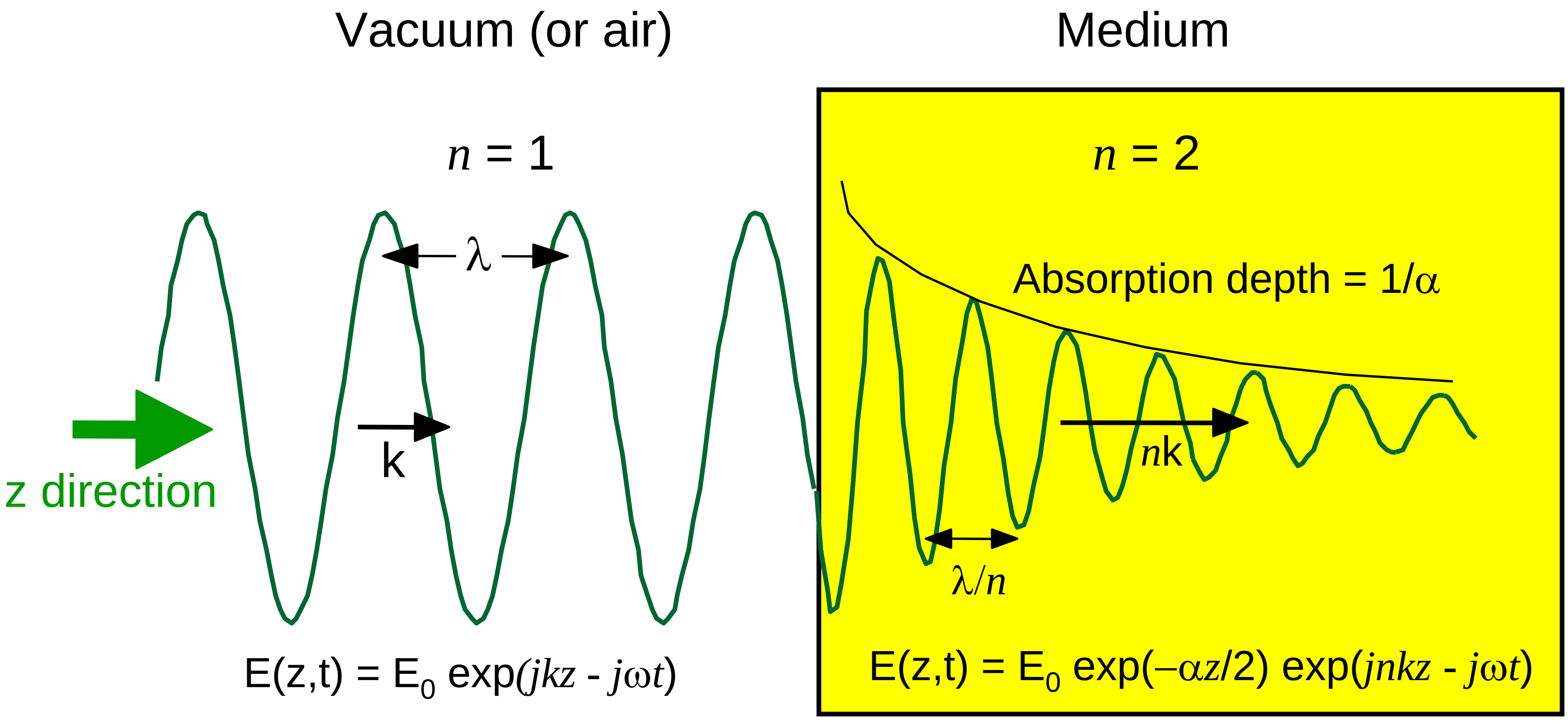
Measuring the absorption coefficient

We have: $$I(z) = I(0)\exp(-\alpha z)$$

and therefore: $$\alpha = -\frac{1}{z}\ln\left[\frac{I(z)}{I(0)}\right]$$



A light wave entering a medium

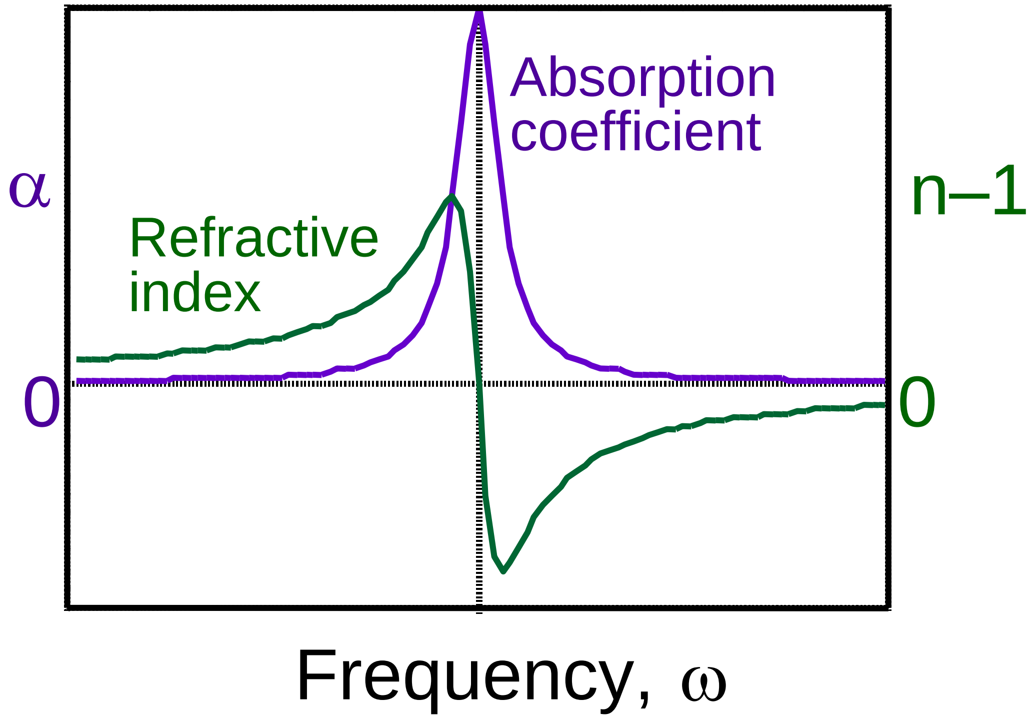


Typically, the speed of light, the wavelength, and the amplitude decrease.

Note: THE FREQUENCY DOES NOT CHANGE! Why?

α and n depend on frequency

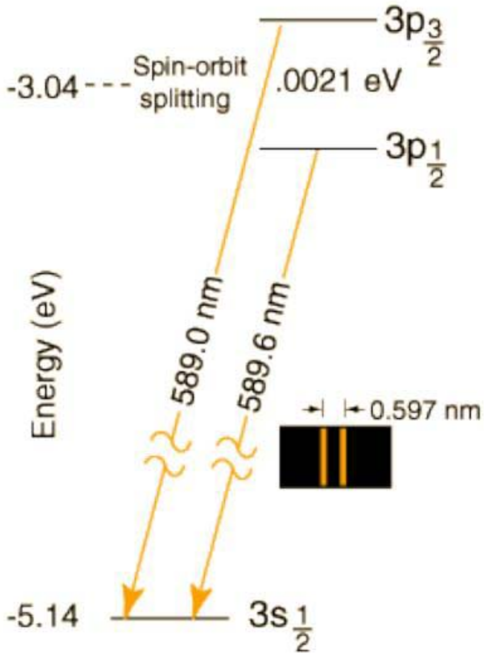
These functions are, together, a Complex Lorentzian (with some constants in front).



$$\alpha = \frac{Ne^2/m_e}{2\varepsilon_0 c_0} \cdot \frac{\omega}{\omega_0} \cdot \frac{\Gamma}{(\omega_0 - \omega)^2 + \Gamma^2} \qquad n - 1 = \frac{Ne^2/m_e}{4\varepsilon_0 \omega_0} \frac{(\omega_0 - \omega)}{(\omega_0 - \omega)^2 + \Gamma^2}$$

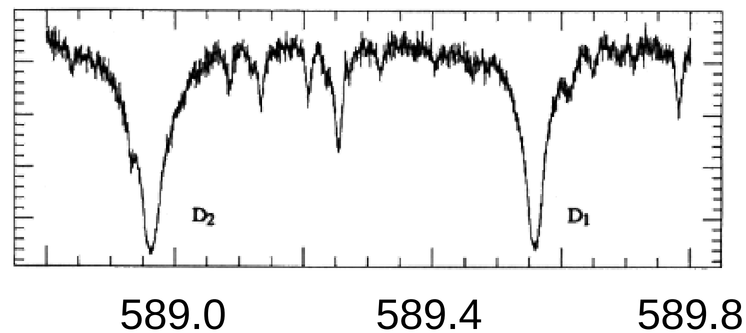
Absorption resonance: An example

Atomic sodium has two closely spaced resonances near λ_0 = 589 nm (yellow).



Intensity of the light from the sun, passing through the atmosphere

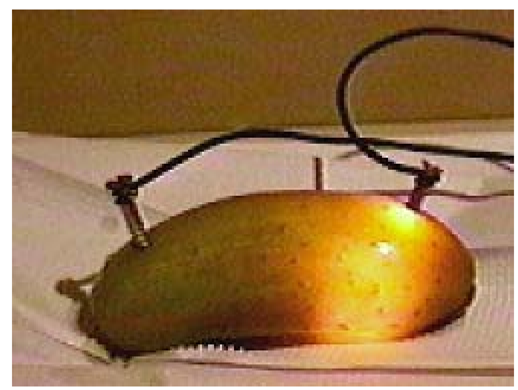
Absorption:



wavelength in nanometers

Sodium lights are yellow because of emission from these resonances.

The electric pickle:

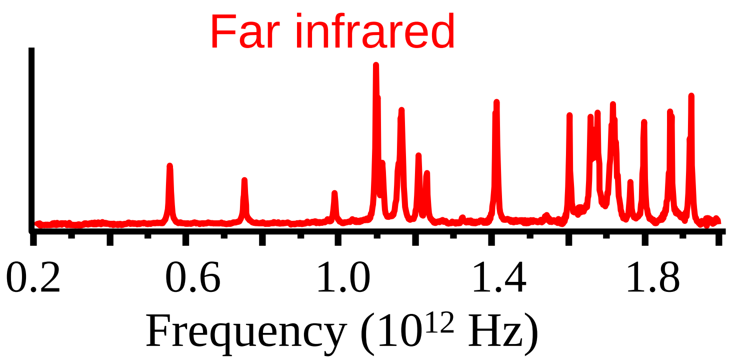


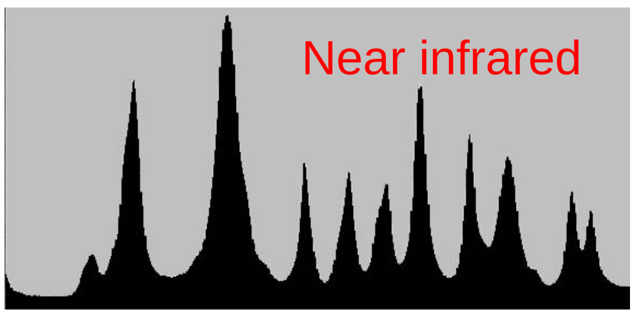
(I did NOT advise you to try this at home…)

Molecules have more electronic states than atoms

As a result, their absorption spectra are more complex.

Even a simple molecule like H_2O has a very complex spectrum, with many absorption lines.

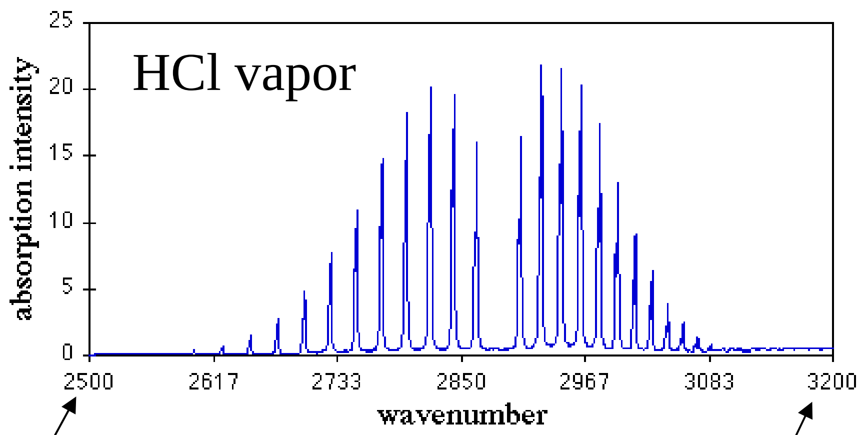




925 nm

926 nm

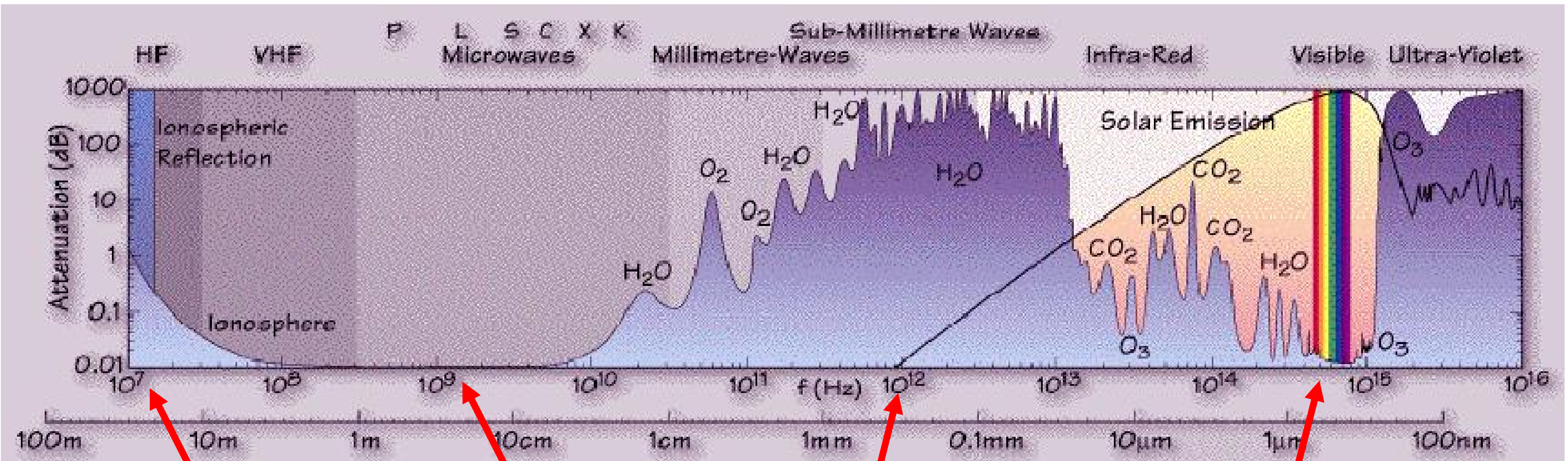
Sometimes, the symmetry of the molecule results in a simpler spectrum:



λ = 4 μm

λ = 3.13 μm

Absorption coefficient of the atmosphere



radio

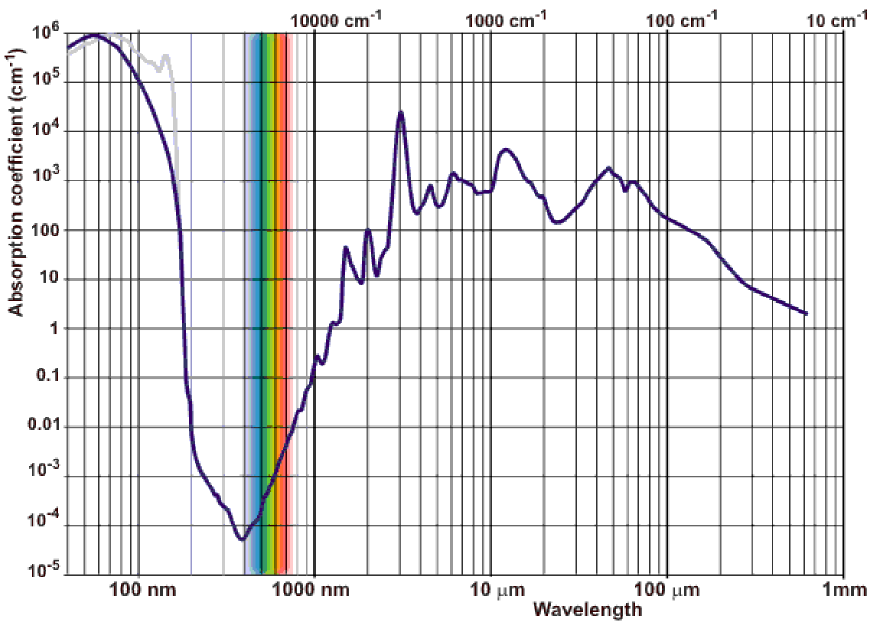
microwaves

THz radiation

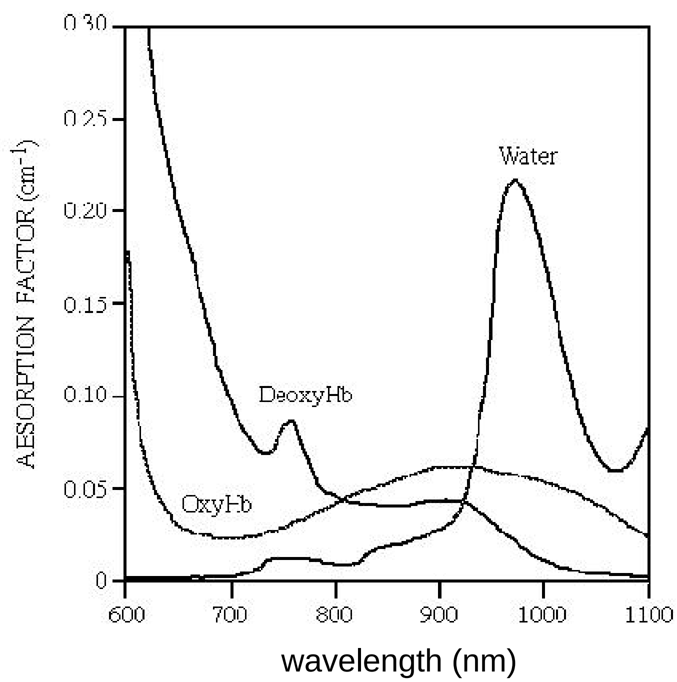
visible light

Solids and liquids often have very broad absorption lines

Liquid water



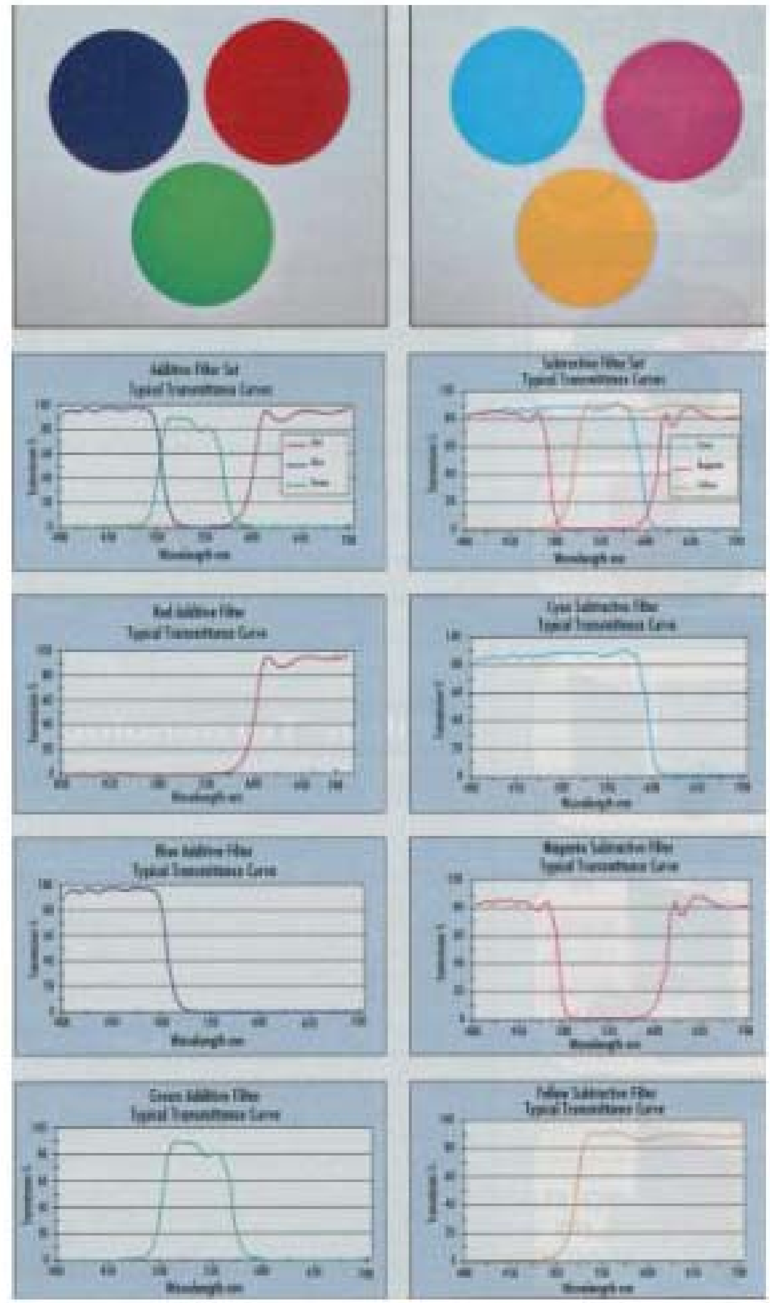
Components of blood



Absorbing glass filters

A wide range of absorbing glass filters allow us to manipulate the frequencies contained in a light beam.

Materials don't do everything you might like, but there's an incredible variety.



Refractive Index and the permittivity ε

We have just seen that $n = c_0/c$

But we also know from the wave equation that the speed of light is given by:

$$c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$ in empty space

$$c = \frac{1}{\sqrt{\varepsilon \mu}}$$ in a medium with ε, μ

Nothing we've said has anything to do with μ, so we can take $\mu = \mu_0$.

$$n = \frac{c_0}{c} = \frac{\sqrt{\varepsilon \mu_0}}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

The relative permittivity is equal to the square of the refractive index.

Reminder: Damped Forced Oscillator Solution for Light-driven atoms

When $\omega << \omega_0$, the electron vibrates 180° out of phase with the light wave.

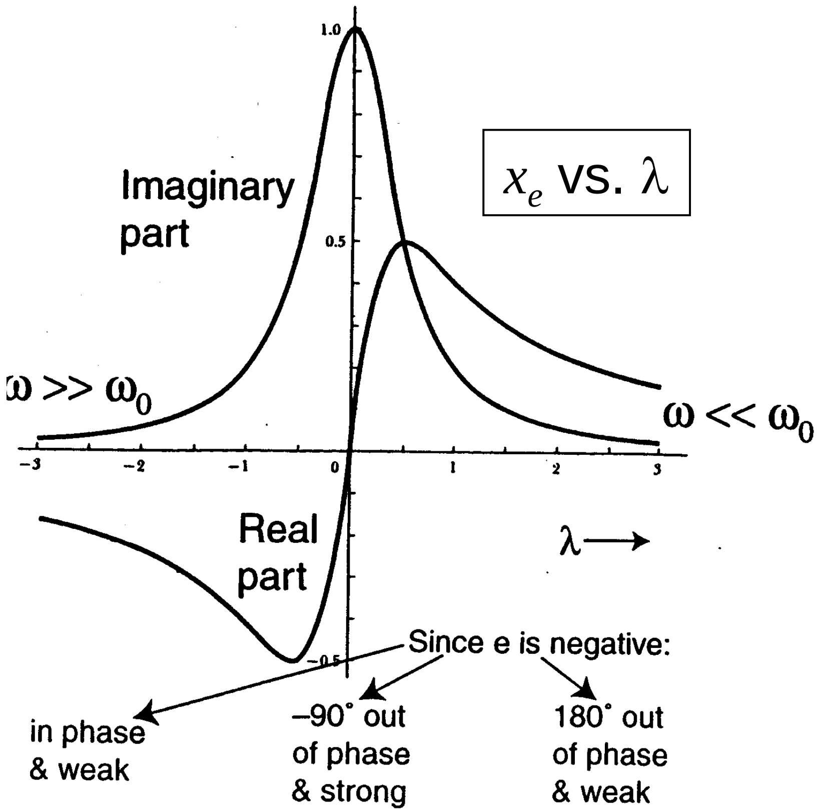
→ absorption is low, but refractive index is still important.

When $\omega = \omega_0$, the electron vibrates 90° out of phase with the light wave.

→ absorption is high and refractive index changes rapidly with frequency.

When $\omega >> \omega_0$, the electron vibrates in phase with the light wave.

→ absorption is low, but refractive index is still important.



The atoms always oscillate at the frequency of the incident light. The light is not always absorbed by the atoms, but it is always changed by its interaction with the atoms.

Refractive Index and dispersion

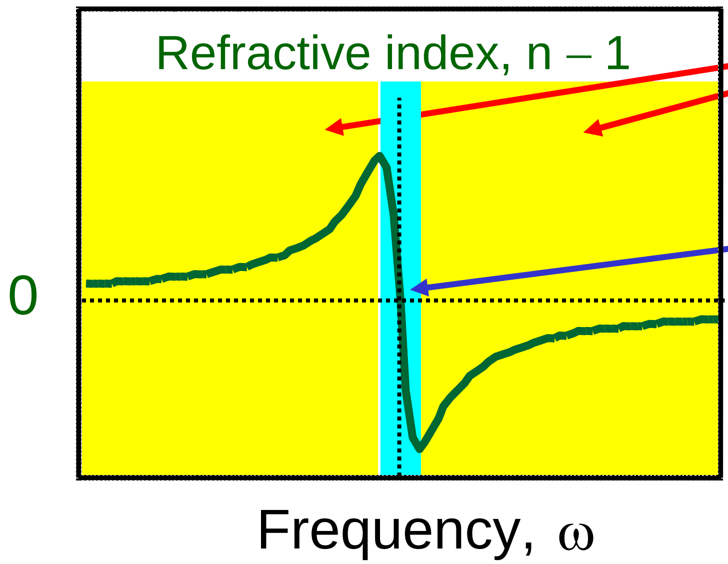
Recall that permittivity is related to the screening of an applied field:

$$E_{net} = (\varepsilon_0/\varepsilon)\, E_0$$

and that permittivity must be smaller at large ω than it is at $\omega = 0$.

But what it does in between can be complicated.

Since $\varepsilon/\varepsilon_0 = n^2$, similar statements can be made about the refractive index.



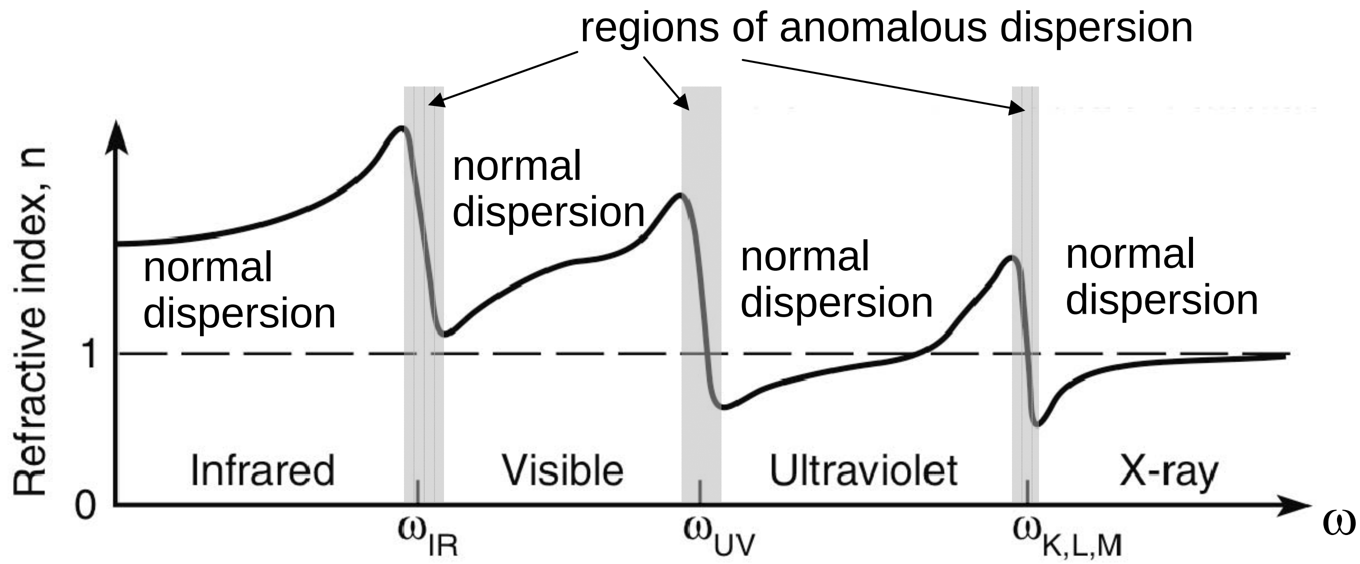
"normal dispersion" : $\dfrac{dn}{d\omega} > 0$

"anomalous dispersion" : $\dfrac{dn}{d\omega} < 0$

Anomalous dispersion often occurs at frequencies where absorption is large

Refractive Index vs. Frequency

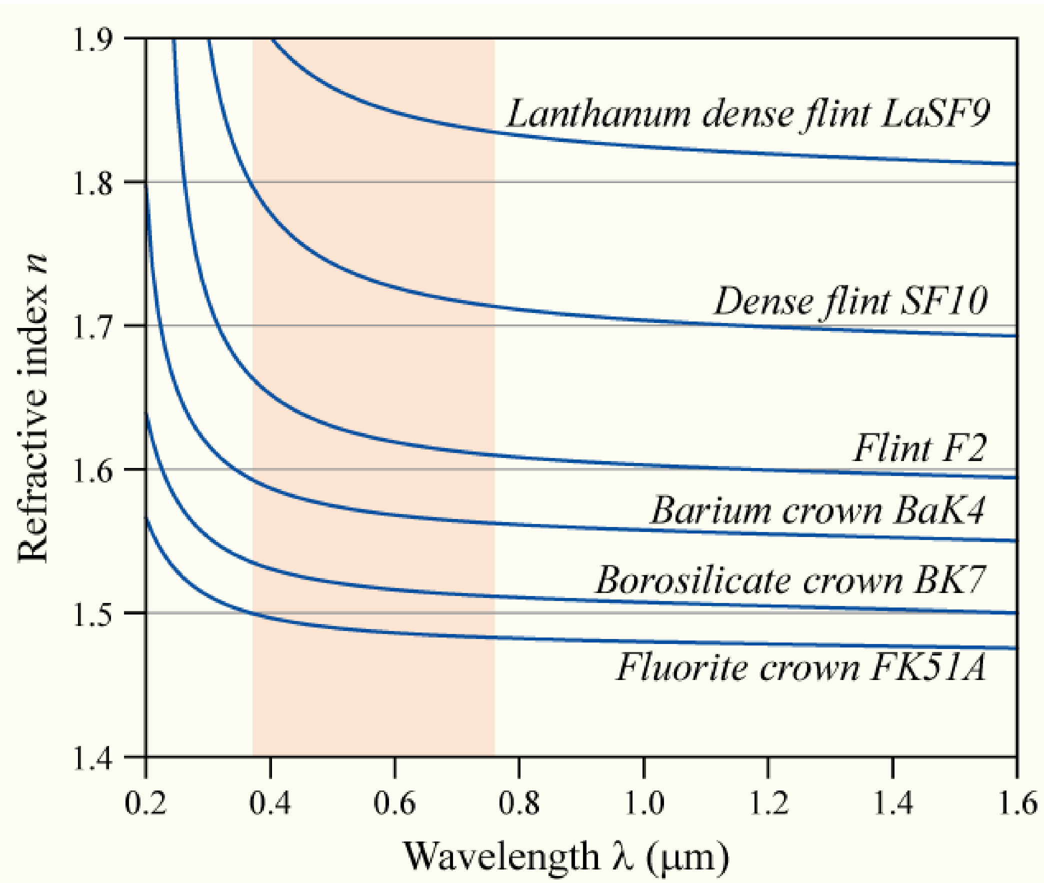
Since resonance frequencies exist in many spectral ranges, the refractive index varies in a complicated manner.



This illustrates a typical distribution of resonances, with electronic resonances in the UV; vibrational and rotational resonances in the IR, and core electronic resonances occur in the x-ray region.

n increases with frequency, except in "anomalous dispersion" regions. But the overall trend is a decrease in n, as ω increases.

Refractive indices of various types of glass



<u>Note</u>

As λ increases, the index decreases. Thus: $$\frac{dn}{d\lambda} < 0$$

But: $$\frac{dn}{d\omega} = \frac{dn}{d\lambda} \cdot \frac{d\lambda}{d\omega}$$

(the chain rule)

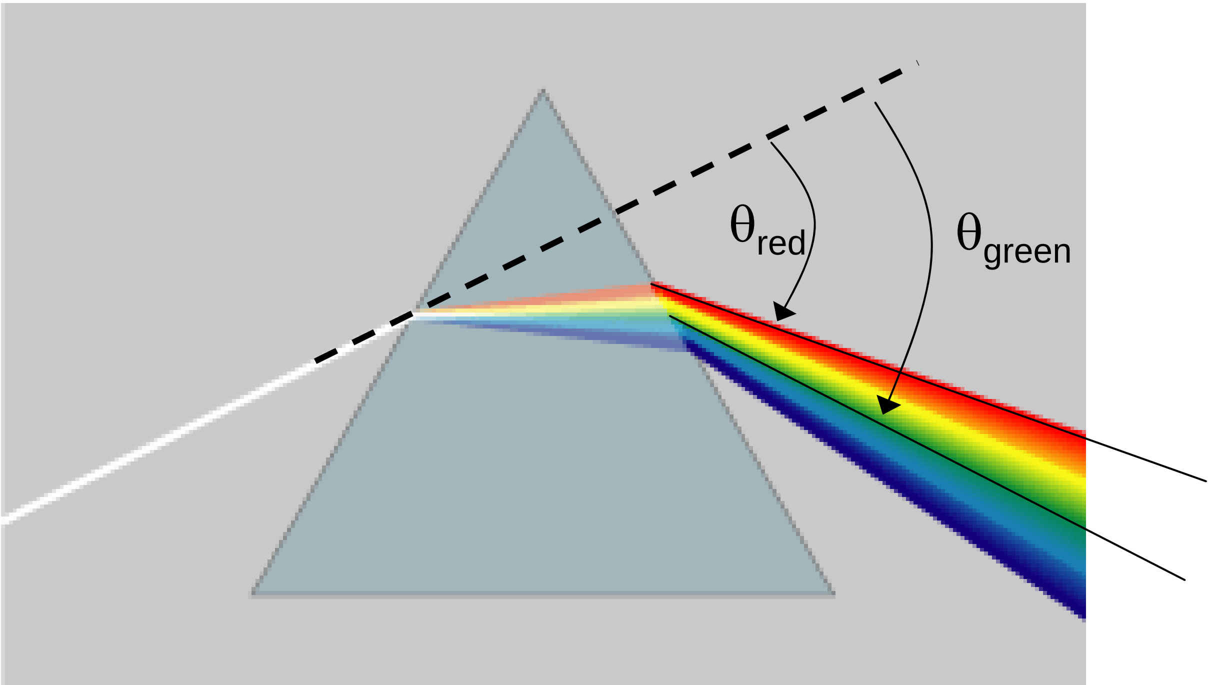
and: $$\lambda = \frac{2\pi c}{\omega}$$

(which implies that $d\lambda/d\omega < 0$)

Therefore: $$\frac{dn}{d\omega} > 0$$

⟶ <u>Normal</u> dispersion

Dispersion in action



The bend angle depends on the refractive index of the glass.

And the refractive index depends on wavelength.

If glass weren't dispersive, we would not see the rainbow in the light refracted through a prism.

The Irradiance inside a medium

In empty space, the irradiance of a wave is given by:

$$I = \tfrac{1}{2} c_0 \, \varepsilon_0 \left| \underset{\sim}{\vec{E}_0} \right|^2$$

For c_0, we substitute c_0/n. For ε_0, we substitute $\varepsilon_0 \, n^2$.

So the irradiance in the medium becomes

$$\boxed{I = \frac{n}{2} c_0 \, \varepsilon_0 \left| \underset{\sim}{\vec{E}_0} \right|^2}$$

The irradiance in a medium is larger by a factor of n, simply due to the change in speed.

This ignores changes in the irradiance that could result from other effects: absorption, reflection, scattering, etc.