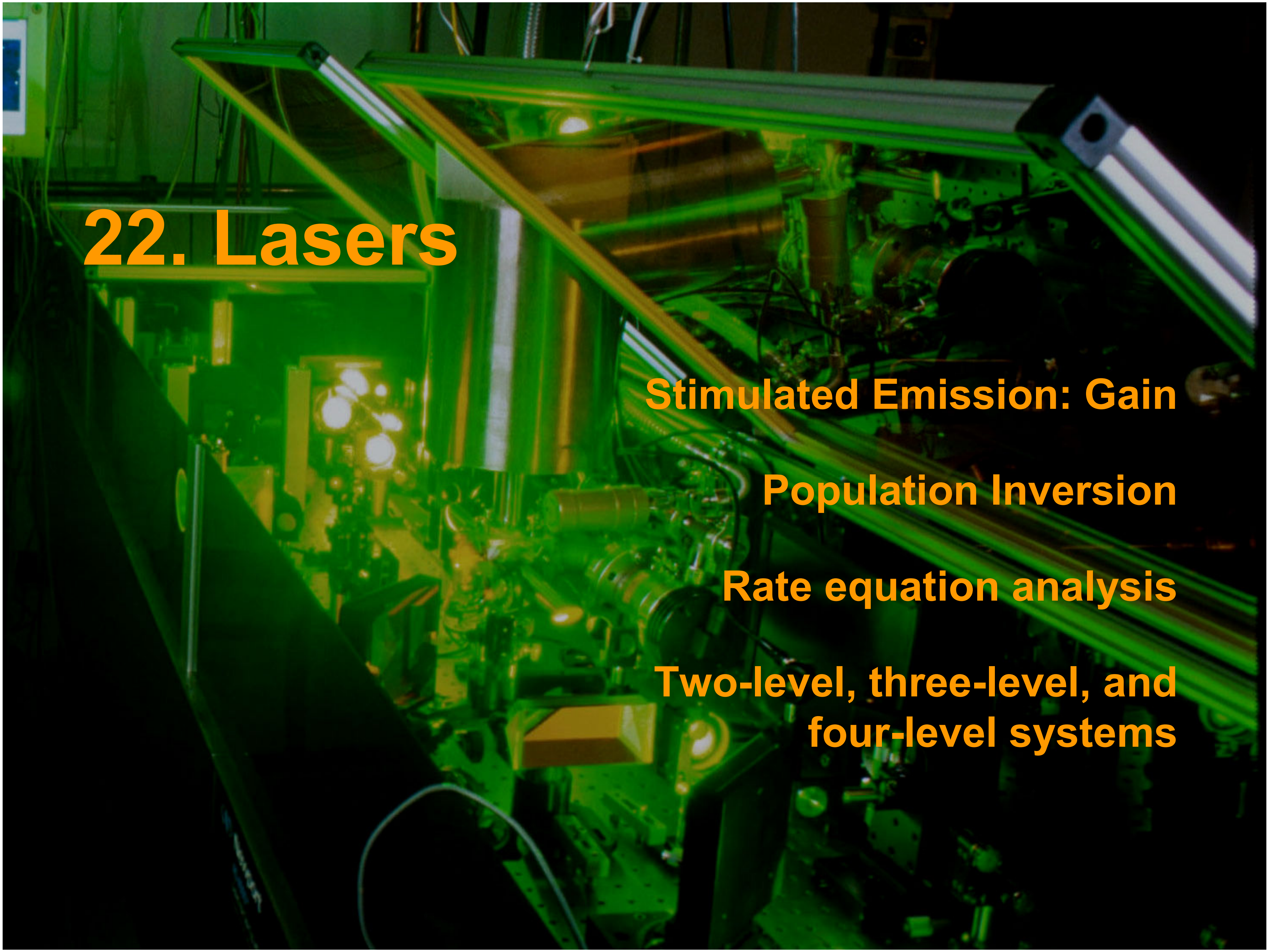


22. Lasers

Stimulated Emission: Gain

Population Inversion

Rate equation analysis

Two-level, three-level, and four-level systems

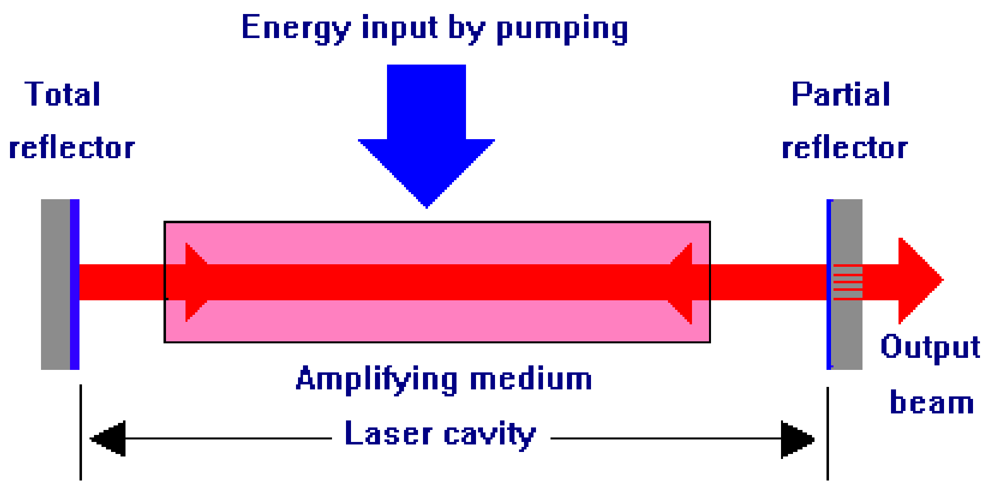
What is a laser?

LASER: Light Amplification by Stimulated Emission of Radiation

"light" could mean anything from microwaves to x-rays

Essential elements:

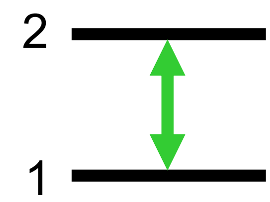
1. A laser medium - a collection of atoms, molecules, etc.
2. A pumping process - puts energy into the laser medium
3. Optical feedback - provides a mechanism for the light to interact (usually many times) with the laser medium



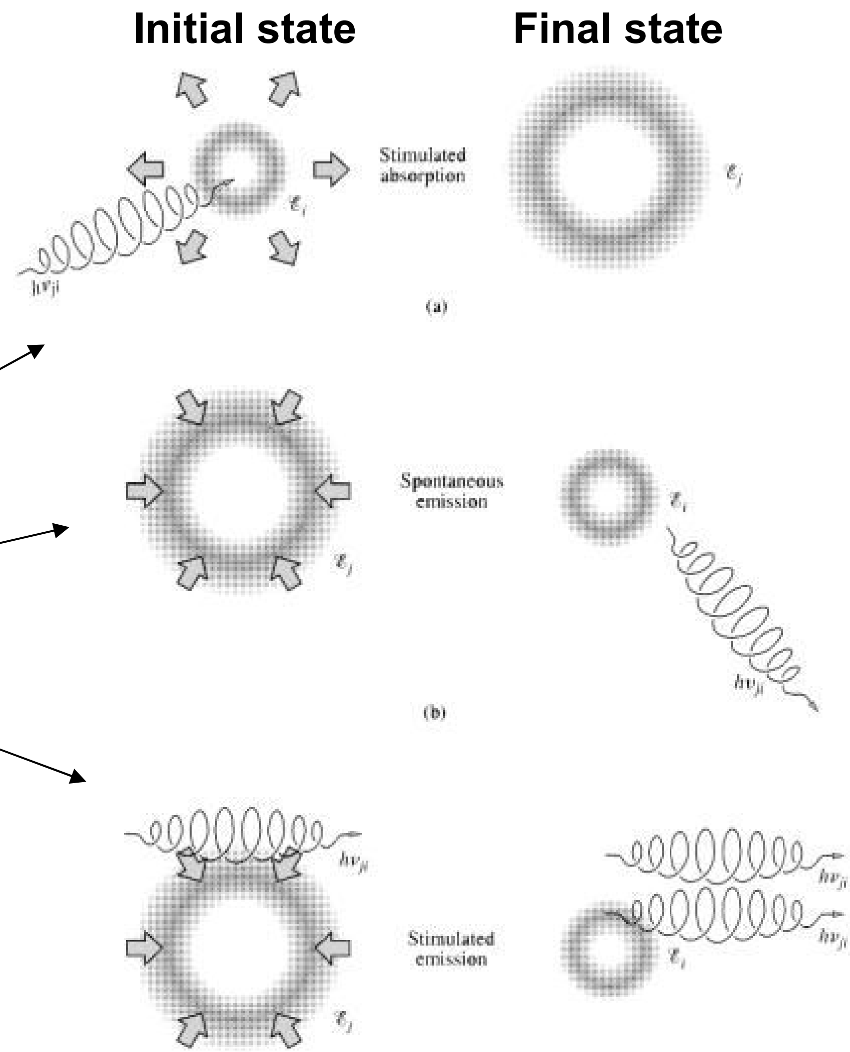
Stimulated emission causes the number of photons in the laser beam to grow.

The factor by which an input beam is amplified by a medium (during one pass through) is called the **gain** and is represented by *G*.

Reminder:
Einstein A and B coefficients



In 1916, Einstein considered the various transition rates between two energy levels (say, 1 and 2) involving light of irradiance, I.



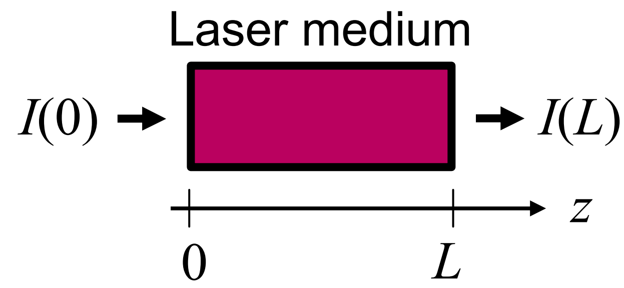
Absorption rate = $B N_1 I$

Spontaneous emission rate = $A N_2$

Stimulated emission rate = $B N_2 I$

N_1, N_2 = populations of states 1 and 2
I = irradiance of the light field
A, B = Einstein coefficients

Laser gain



Neglecting spontaneous emission:

$$\frac{dI}{dt} = \frac{c}{n}\frac{dI}{dz} \propto BN_2 I - BN_1 I$$

$$\propto B\left[N_2 - N_1\right] I$$

[Stimulated emission minus absorption]

The solution is:

Proportionality constant is called the **cross-section**

$$I(z) = I(0)\exp\left\{\sigma\left[N_2 - N_1\right]z\right\} \longrightarrow I(z) = I(0)\exp(-\alpha z)$$

There can be exponential gain or loss in irradiance. Normally, $N_2 < N_1$, and there is loss (absorption).

But if $N_2 > N_1$, there's gain, and we define the gain, G:

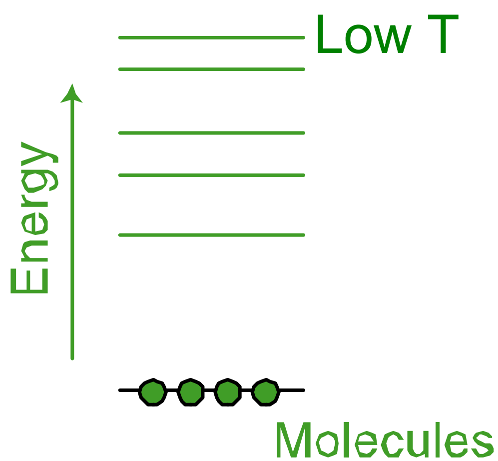
$$G \equiv \exp\left\{\sigma\left[N_2 - N_1\right]L\right\}$$

If $N_2 > N_1$: $\quad g \equiv \left[N_2 - N_1\right]\sigma$

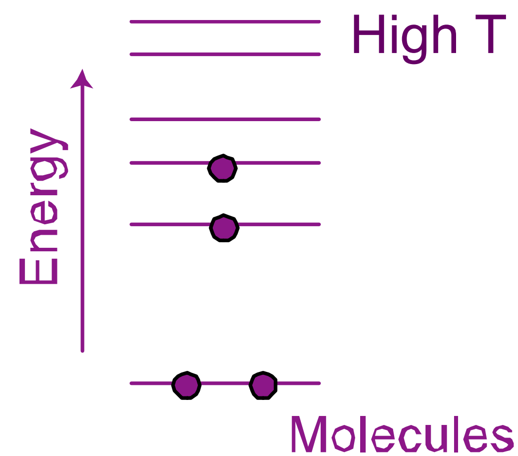
If $N_2 < N_1$: $\quad \alpha \equiv \left[N_1 - N_2\right]\sigma$

Reminder:
The Maxwell-Boltzman Distribution

In the absence of collisions, molecules tend to remain in the lowest energy state available.



Collisions can knock a molecule into a higher-energy state. The higher the temperature, the more this happens.



In equilibrium at a temperature T, the ratio of the populations of any two states is given by:

$$\frac{N_2}{N_1} = \frac{\exp\left[-E_2 / k_B T\right]}{\exp\left[-E_1 / k_B T\right]} = \exp\left[-\Delta E / k_B T\right]$$

Since $T > 0$, we always find $N_2 < N_1$.

Populations and the cross-section σ

$$\frac{N_2}{N_1} = \exp\left[-\Delta E / k_B T\right]$$

At low temperatures, or for large values of ΔE, we find $N_2 << N_1$.

Example:

• The energy level separation is equal to the photon energy.
For visible light, e.g., λ = 600 nm: $h\nu = hc/\lambda = 3.3\times10^{-19}$ joules.

• At room temperature: $k_B T = 4.1\times10^{-21}$ joules.

$$\frac{N_2}{N_1} = \exp\left[-\Delta E / k_B T\right] = e^{-79} \sim 10^{-35}$$

So N_2 = 0, effectively.

If $N_2 > N_1$: $g \equiv \left[N_2 - N_1\right]\sigma$

If $N_2 < N_1$: $\alpha \equiv \left[N_1 - N_2\right]\sigma$

$$\sigma = \frac{\alpha}{N_1}$$

units of σ are m² (i.e., area)

σ is something like the absorption per molecule. It is the effective size (area) of a molecule as seen by an incoming photon.

Inversion

In order to achieve G > 1, stimulated emission must exceed absorption:

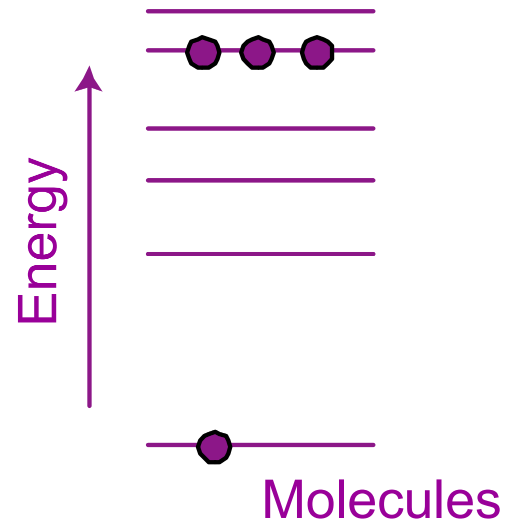
$$B\,N_2\,I \;>\; B\,N_1\,I$$

Or, equivalently,

$$N_2 \;>\; N_1$$

This condition is called **inversion**. It does not occur naturally in steady-state. It is inherently a non-equilibrium state.

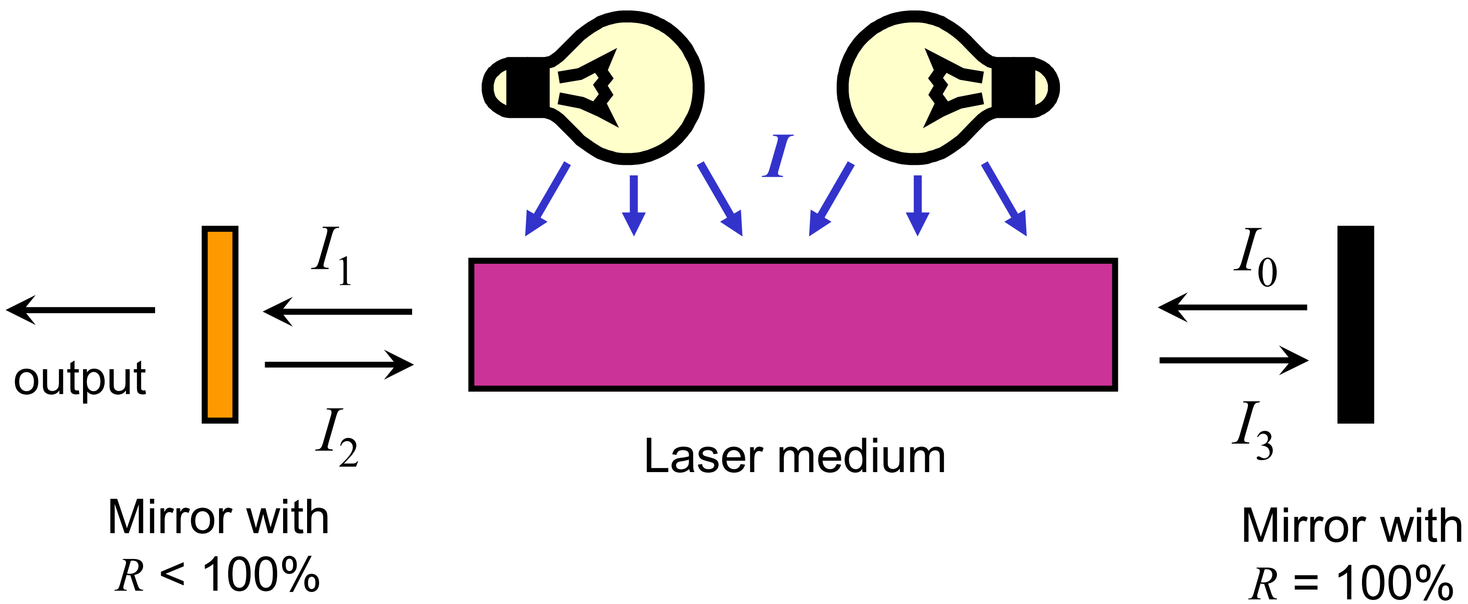
A population inversion



In order to achieve inversion, we must pump a lot of energy into the gain medium. And it needs to be the right medium.

Achieving inversion: Pumping the laser medium

Suppose we pump energy into the laser medium, using another light source with intensity I:



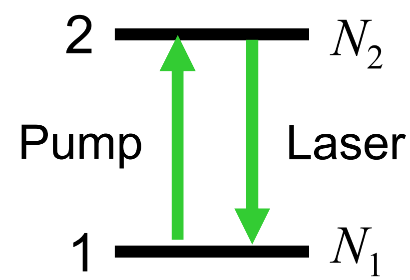
The key question for the remainder of today's lecture:

Will this intensity I be sufficient to achieve inversion, $N_2 > N_1$?

The answer depends on the laser medium's energy level configuration.

Rate equations for a two-level system

Earlier we neglected spontaneous emission. Let's look again, and be a bit more careful.



Rate equations for the population densities of the two states:

Absorption — Stimulated emission — Spontaneous emission

$$\frac{dN_2}{dt} = BI(N_1 - N_2) - AN_2$$

Pump intensity: I

$$\frac{dN_1}{dt} = BI(N_2 - N_1) + AN_2$$

If the total number of molecules is N:

$$N \equiv N_1 + N_2$$

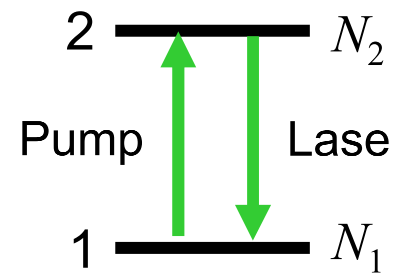
$$\Delta N \equiv N_1 - N_2$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + 2AN_2$$

$$2N_2 = (N_1 + N_2) - (N_1 - N_2) = N - \Delta N$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

How does the population difference depend on pump intensity?



$$\frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

In steady-state the time derivative is zero: $0 = -2BI\Delta N + AN - A\Delta N$

Solve for ΔN: $(A + 2BI)\Delta N = AN$

$$\Rightarrow \quad \Delta N = AN/(A+2BI) = \frac{N}{1+\dfrac{2B}{A}I}$$

$$\Rightarrow \boxed{\Delta N = \frac{N}{1+2I/I_{sat}}}$$

where: $I_{sat} = A/B$

I_{sat} is called the **saturation intensity**, a unique parameter for any gain medium.

What is the saturation intensity?

$$I_{sat} = A / B$$

A is the excited-state relaxation rate due to spontaneous emission: $1/\tau$

B is the absorption cross-section, σ, divided by the energy per photon, $\hbar\omega$: $\sigma / \hbar\omega$

Both σ and τ depend on the frequency of the light. And they are different for each molecule.

$$I_{sat} = \frac{\hbar\omega}{\sigma\tau}$$

$\hbar\omega$ ~10^{-19} J for visible/near IR light

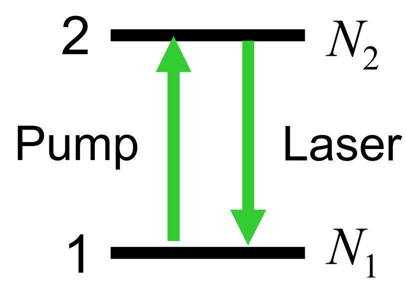
τ ~10^{-12} to 10^{-8} s for molecules

σ ~10^{-20} to 10^{-16} cm^2 for molecules (on resonance)

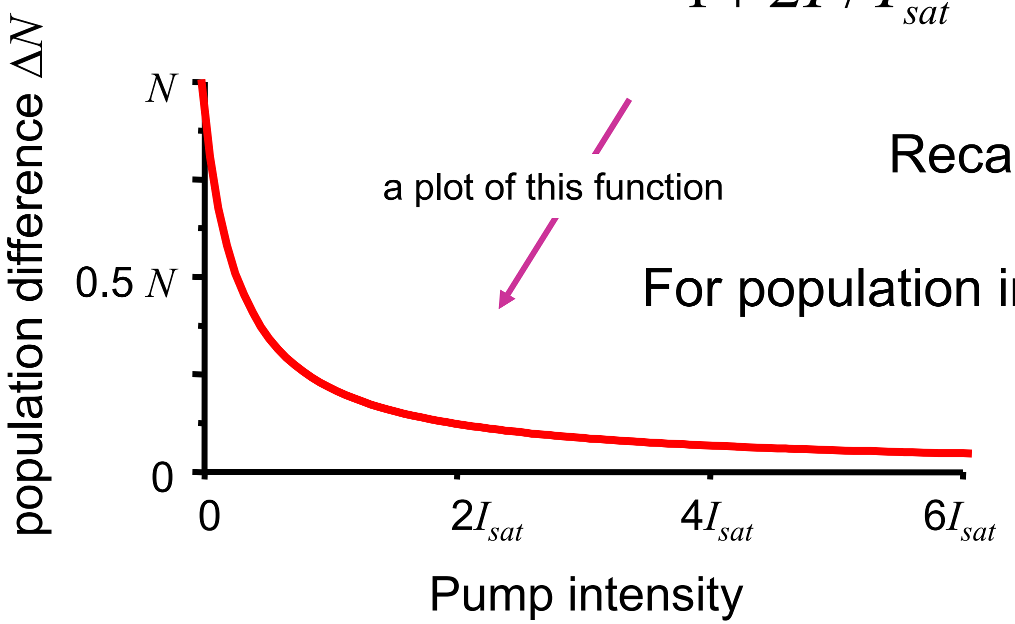
10^5 to 10^{13} W/cm^2

The saturation intensity plays a key role in laser theory. It is the intensity which corresponds to one photon incident on each molecule, within its cross-section σ, per recovery time τ.

Why inversion is impossible in a two-level system



Population difference $\Delta N = \dfrac{N}{1+2I/I_{sat}}$



Recall that $\Delta N \equiv N_1 - N_2$

For population inversion, we require $\Delta N < 0$

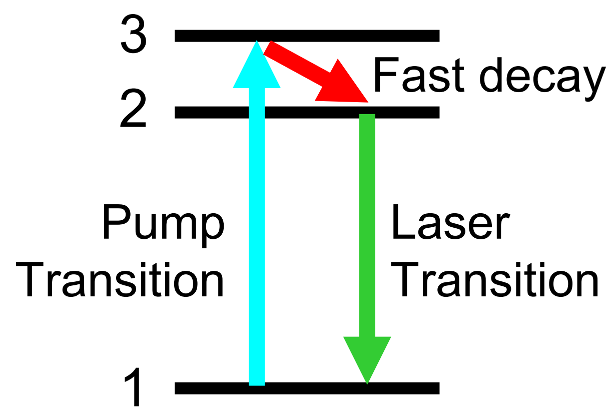
ΔN is always positive, no matter how hard we pump on the system!

It's impossible to achieve a steady-state inversion in a two-level system!

Why? Because absorption and stimulated emission are equally likely.

Even for an infinite pump intensity, the best we can do is $N_1 = N_2$ (i.e., ΔN = 0)

Rate equations for a three-level system



So, if we can't make a laser using two levels, what if we try it with three?

Assume we pump to a state 3 that rapidly decays to level 2.

$$\frac{dN_2}{dt} = BIN_1 - AN_2$$

$$\frac{dN_1}{dt} = -BIN_1 + AN_2$$

The total number of molecules is N:

$$N \equiv N_1 + N_2$$

Level 3 decays fast and so $N_3 \approx 0$.

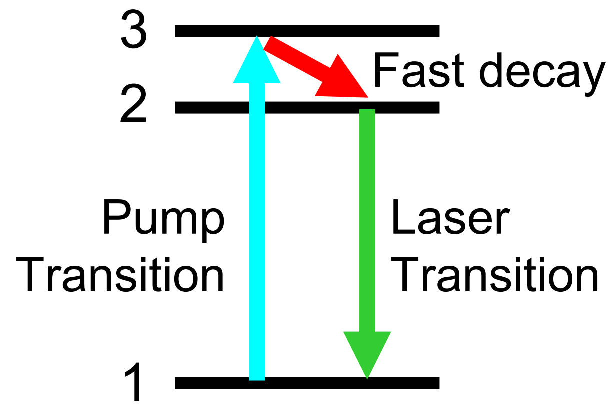
$$\Delta N \equiv N_1 - N_2$$

$$\frac{d\Delta N}{dt} = -2BIN_1 + 2AN_2$$

$$2N_2 = N - \Delta N$$

$$2N_1 = N + \Delta N$$

Why inversion is possible in a three-level system



$$\frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -BIN - BI\Delta N + AN - A\Delta N$

Solve for ΔN: $$\Delta N = N\,\frac{A - BI}{A + BI} = N\,\frac{1-(B/A)I}{1+(B/A)I}$$

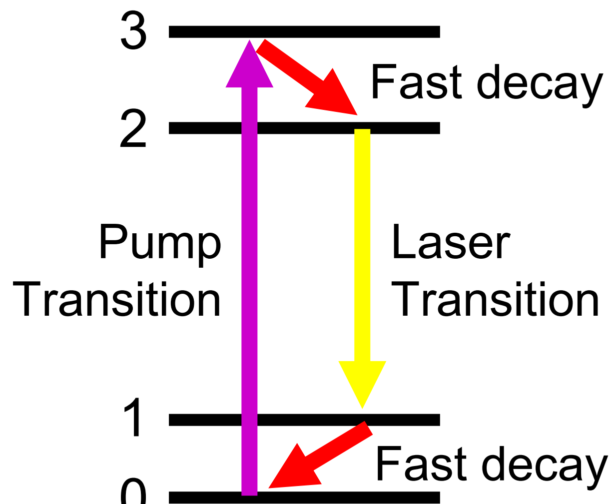
$$\Rightarrow \boxed{\Delta N = N\,\frac{1 - I/I_{sat}}{1 + I/I_{sat}}}$$

where, as before: $I_{sat} = A/B$

I_{sat} is the **saturation intensity**.

Now if $I > I_{sat}$, ΔN is negative!

Rate equations for a four-level system



Now assume the lower laser level 1 also rapidly decays to a ground level 0.

As before: $$\frac{dN_2}{dt} = BIN_0 - AN_2$$

The total number of molecules is N:

$$N \equiv N_0 + N_2$$

$$N_0 = N - N_2$$

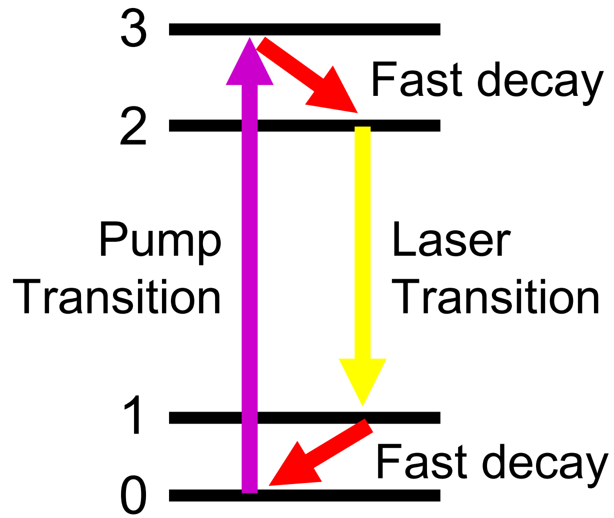
$$\frac{dN_2}{dt} = BI(N - N_2) - AN_2$$

Because $N_1 \approx 0$, $\Delta N \approx -N_2$

$$-\frac{d\Delta N}{dt} = BIN + BI\Delta N + A\Delta N$$

At steady state: $$0 = BIN + BI\Delta N + A\Delta N$$

Why inversion is easy in a four-level system



$$0 = BIN + BI\Delta N + A\Delta N$$

Solve for ΔN:

$$\Rightarrow \quad \Delta N = -BIN/(A + BI) = -N\frac{(B/A)I}{1+(B/A)I}$$

$$\Rightarrow \quad \boxed{\Delta N = -N\frac{I/I_{sat}}{1+I/I_{sat}}}$$

where: $I_{sat} = A/B$

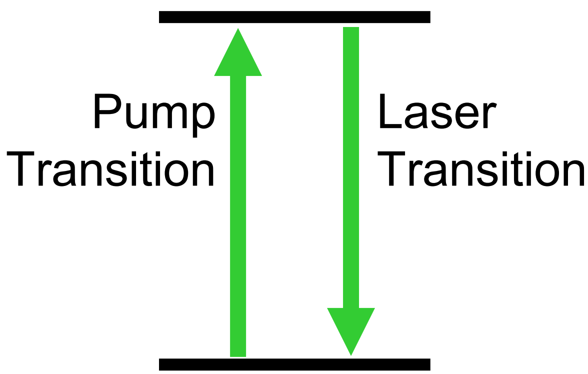
I_{sat} is the **saturation intensity**.

Now, ΔN is negative—for any non-zero value of I!

Two-, three-, and four-level systems

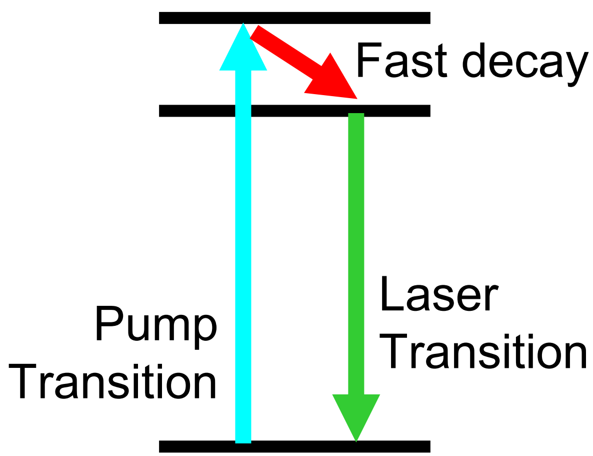
It took laser physicists a while to realize that four-level systems are best.

Two-level system



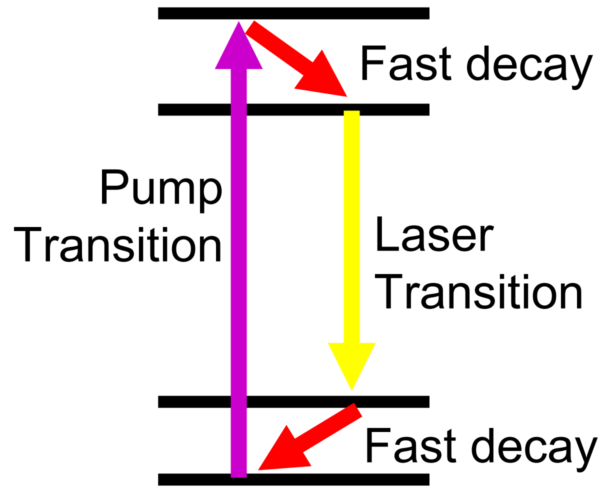
At best, you get equal populations. No lasing.

Three-level system



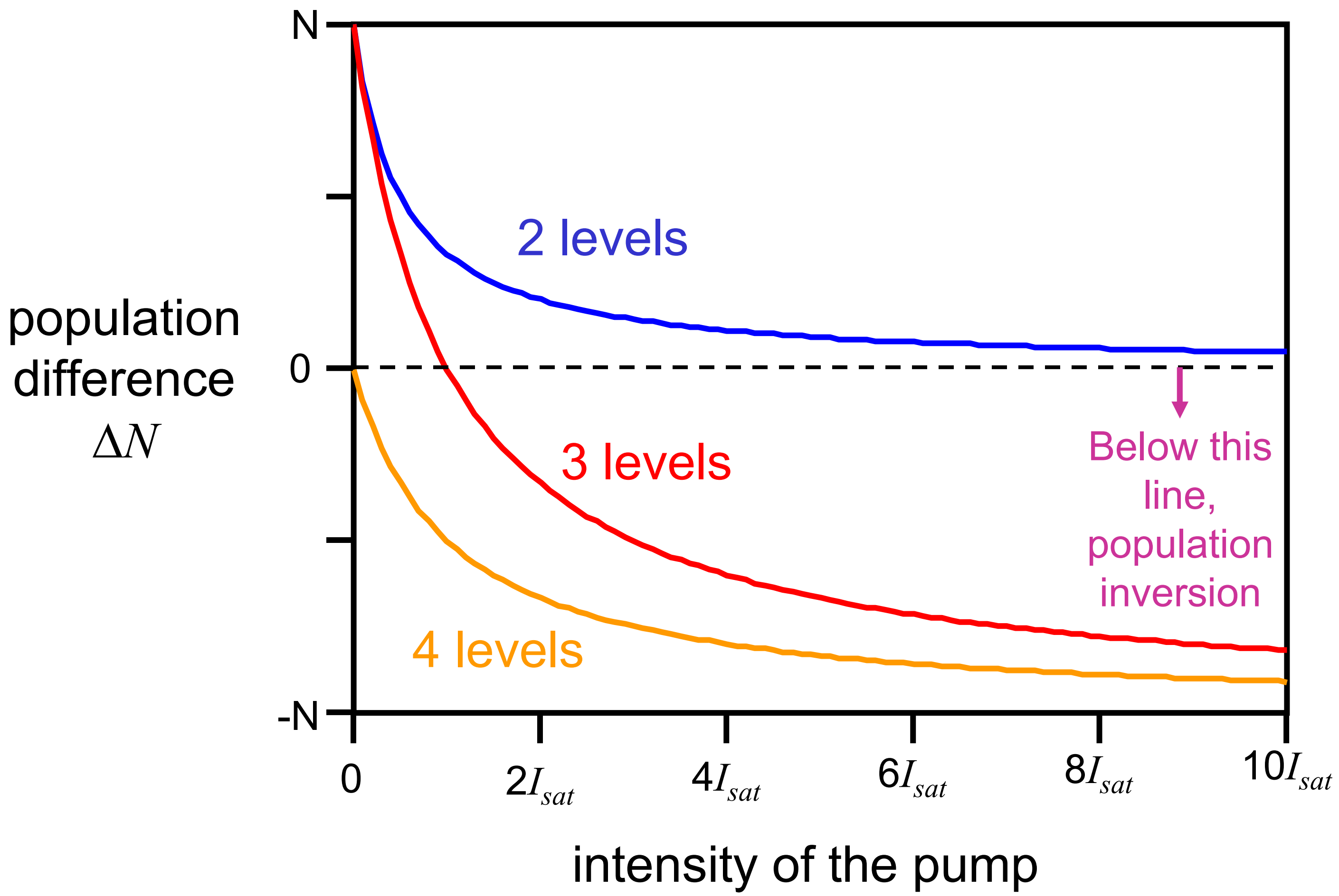
If you hit it hard, you get lasing.

Four-level system

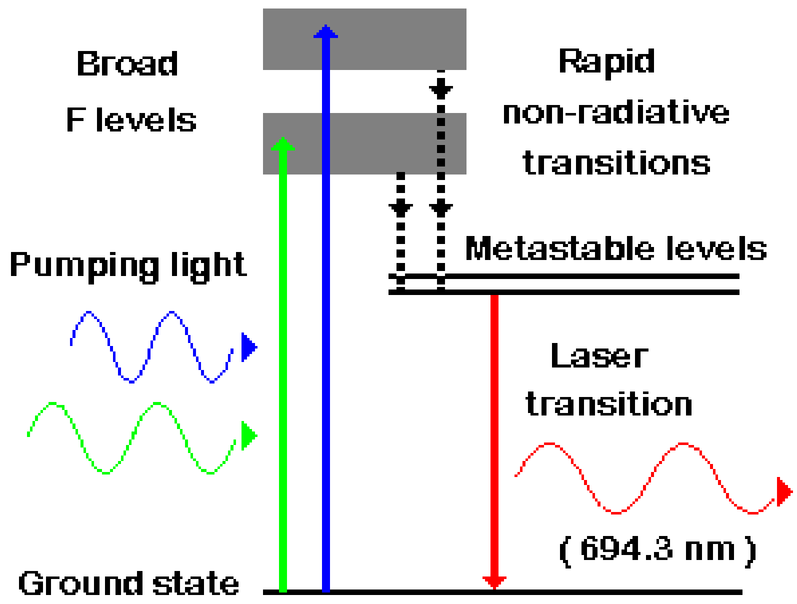


Lasing is easy!

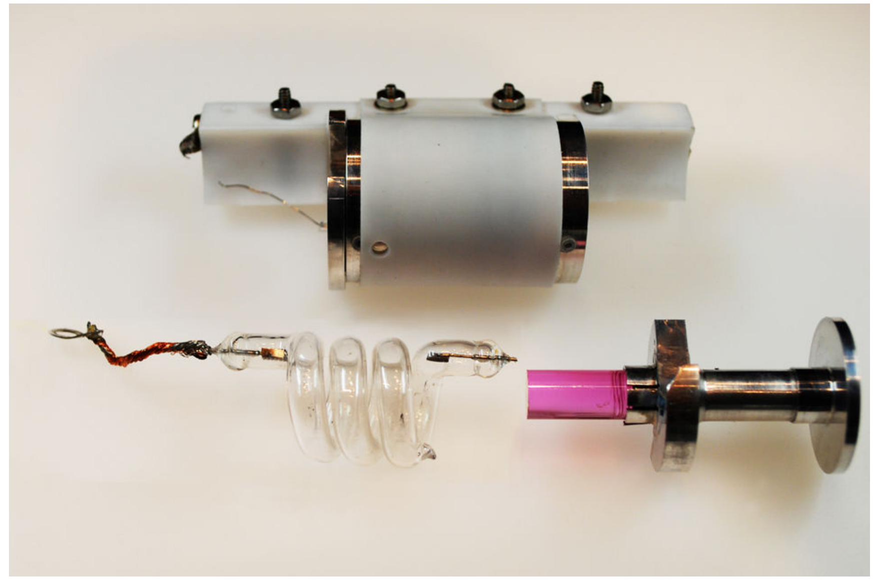
Population inversion in two-, three-, and four-level systems



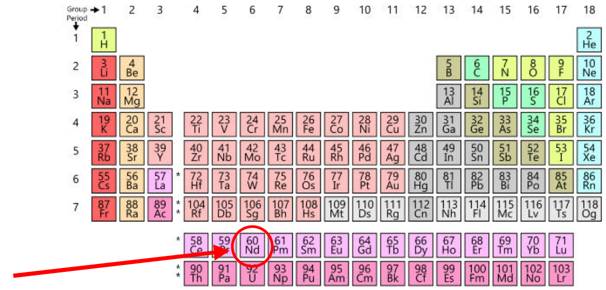
The first laser ever built was a three-level system



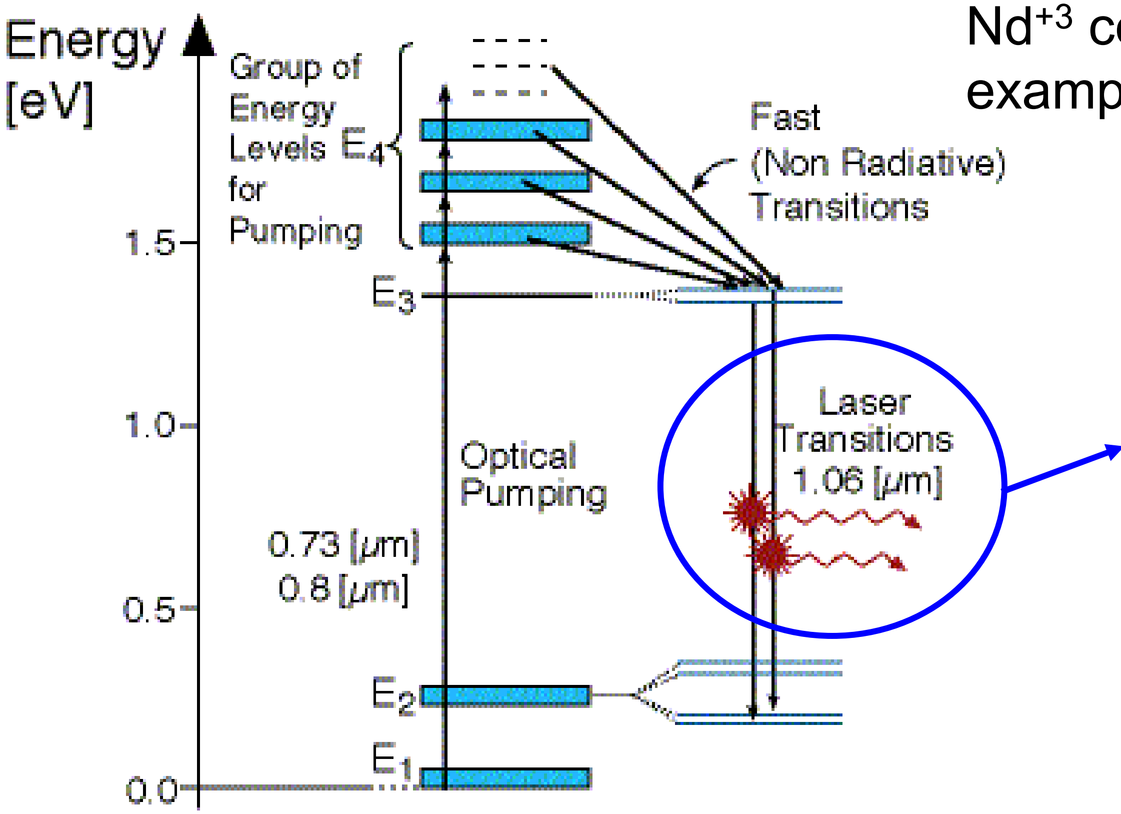
Energy levels of chromium ions in ruby



Many lasers are almost ideal 4-level systems

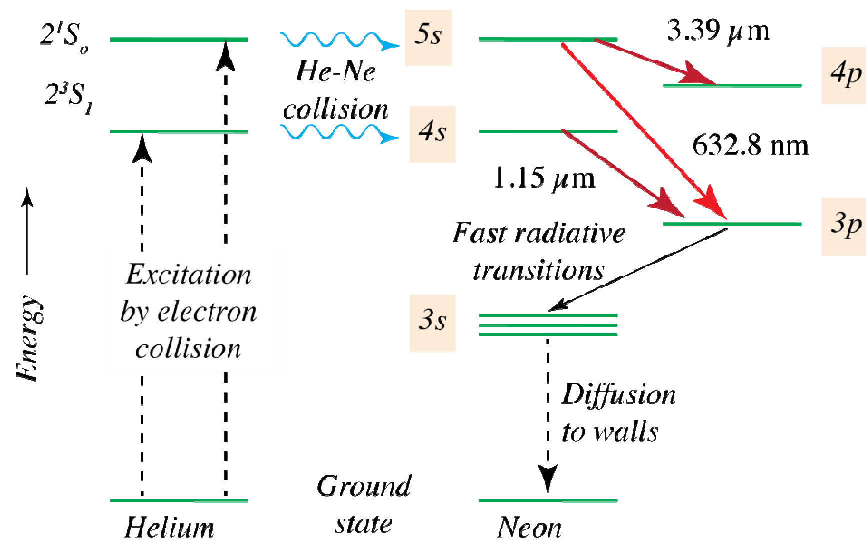


For example, the green laser we use in class is an Nd^{+3}:YAG laser. The energy levels of Nd^{+3} constitute a classic example of a four-level system.

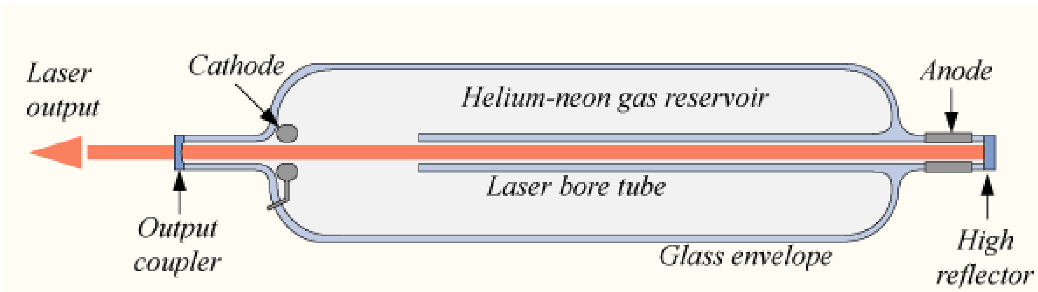


Infrared laser emission is converted to green light by a nonlinear optical process: frequency doubling! (see lecture 36)

The HeNe laser is a 4-level system



Lasing via a three-step process:
1. Ne excited by collisions with He atoms
2. lasing from 5s to 3p energy levels of Ne
3. fast decay to from 3p levels to ground state



There are nine possible 5s-3p transitions, ranging from green (543nm) to red (730nm).

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Transition Probabilities for the 5s′[1/2]₁-3p Transitions of Ne I†*

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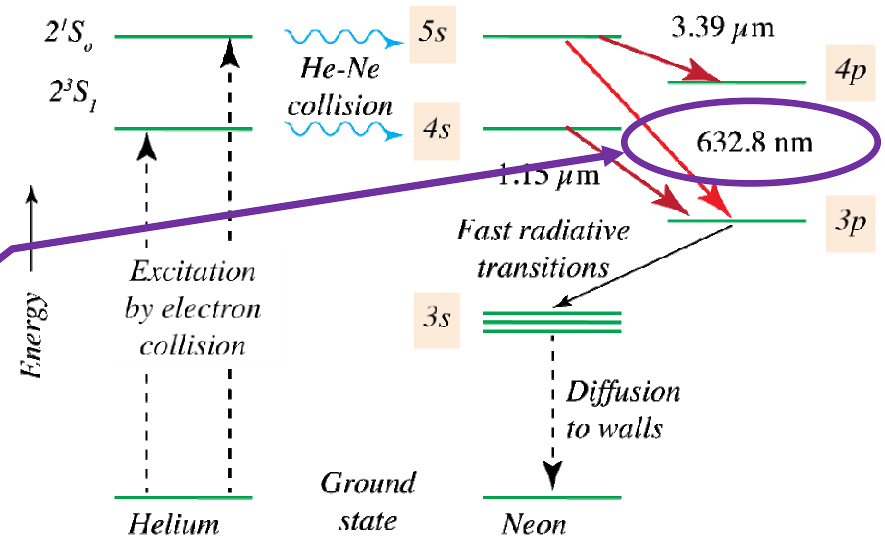
(Received 12 March 1973)

TABLE II. The absolute transition probabilities for the $3s_2$-$2p_i$ transitions of Ne I.

Transitions	λ (nm)	This expt.	Hansch and Toschek (Ref. 1)	(10^6 sec^{-1}) Bychkova *et al.* (Ref. 2)	Decomps and Dumont[a] (Ref. 12)
$3s_2$-$2p_1$	730.483	0.255	0.37		
$2p_2$	640.107	1.39			
$2p_3$	635.185	0.345	0.52	0.70	
$2p_4$	632.816	3.39	5.1	6.56	3.24
$2p_5$	629.374	0.639		1.35	
$2p_6$	611.801	0.609	0.93	1.28	
$2p_7$	604.613	0.226	0.39	0.68	
$2p_8$	593.931	0.200		0.56	
$2p_9$	forbidden				
$2p_{10}$	543.365	0.283	0.52	0.59	
Estimated errors		4%	14–21%[b]	17–24%[b]	19%

[a] This experiment was based on the Hanle-effect measurements, while the others used the same method as described in the text.
[b] These two data sets were based on one laser line at 632.8 nm, and the value of $A_{609.6} = 23.8 \times 10^6$ sec^{-1} (Ladenberg, Ref. 8) was used to convert relative values to absolute values. The error included in this value, which is 30%, is not included in the error estimations given here.

The HeNe laser is a 4-level system



Question: if there are nine different transitions here, why does the laser produce just one color?