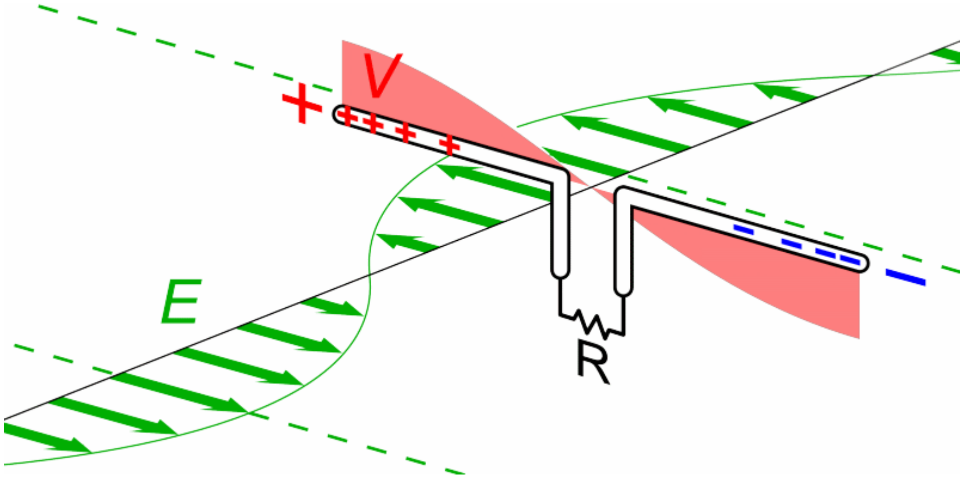
25. Antennas II



Radiation patterns

Beyond the Hertzian dipole - superposition

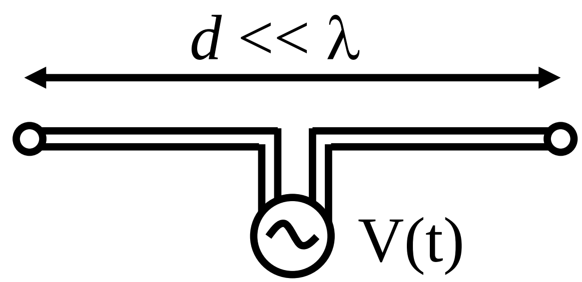
Directivity and antenna gain

More complicated antennas

Impedance matching

Reminder: Hertzian dipole

The Hertzian dipole is a linear antenna which is much shorter than the free-space wavelength:



Far field:

$$\vec{E}(r,\theta,t) = j\frac{\mu_0 \omega I_0 d}{4\pi}\sin\theta\left(\frac{e^{-jk_0 r + j\omega t}}{r}\right)\hat{\theta}$$

Radiation resistance:

$$R_{rad} = \frac{2\pi}{3} Z_0 \cdot \frac{d^2}{\lambda^2}$$

where $Z_0 \equiv \sqrt{\mu_0/\varepsilon_0} \approx 377\Omega$ is the impedance of free space.

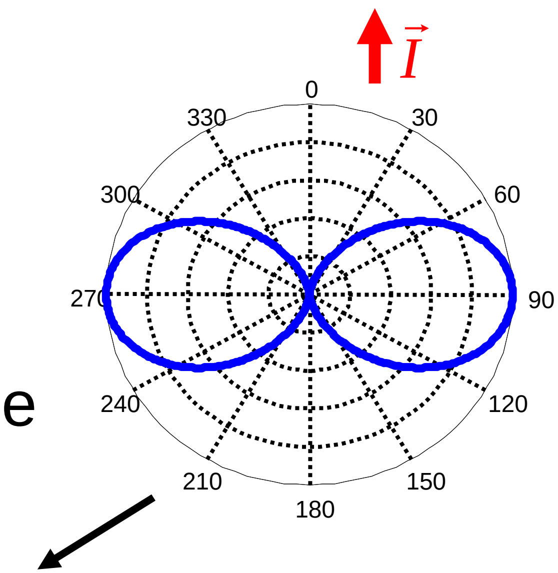
Radiation efficiency: $\eta = \dfrac{R_{rad}}{R_{rad} + R_{Ohmic}}$ (typically is small because $d << \lambda$)

Radiation patterns

Antennas do not radiate power equally in all directions. For a linear dipole, no power is radiated along the antenna's axis (θ = 0).

$$\langle S(\theta,\phi)\rangle = \frac{\mu_0 \omega^2 I_0^{\,2} d^2}{32\pi^2 c_0}\left(\frac{\sin^2\theta}{r^2}\right)\hat{r}$$

We've seen this picture before…



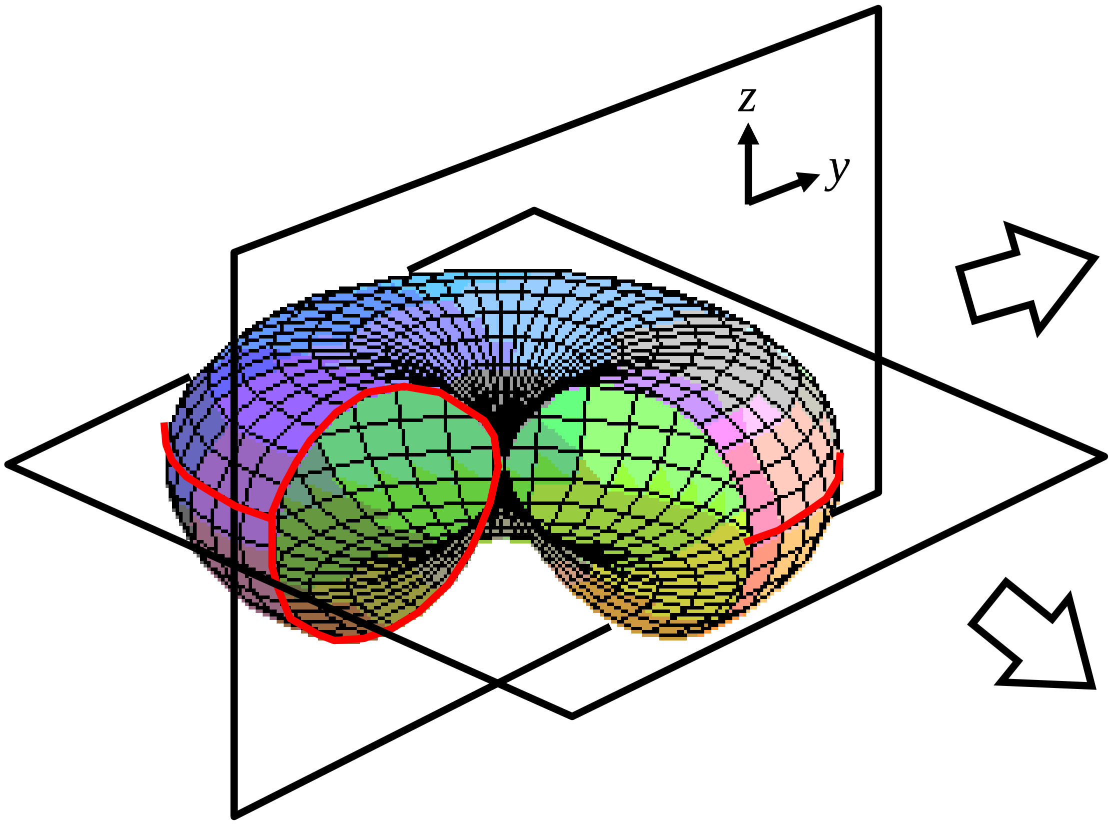
Such polar plots of far-field power vs. angle are known as 'radiation patterns'.

Note that this picture is only a 2D slice of a 3D pattern.

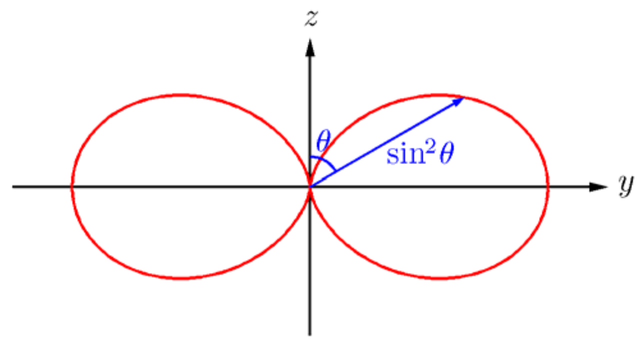
E-plane pattern: the 2D slice displaying the plane which contains the electric field vectors.

H-plane pattern: the 2D slice displaying the plane which contains the magnetic field vectors.

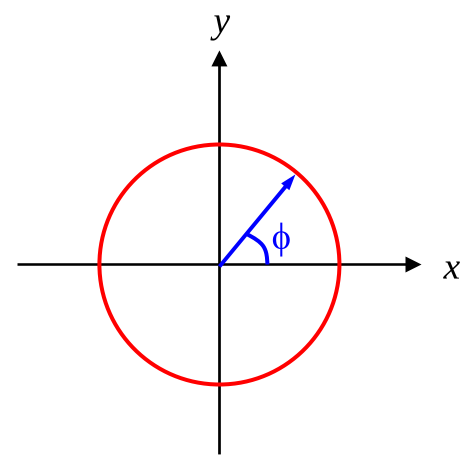
Radiation patterns – Hertzian dipole



3D cutaway view



E-plane radiation pattern



H-plane radiation pattern

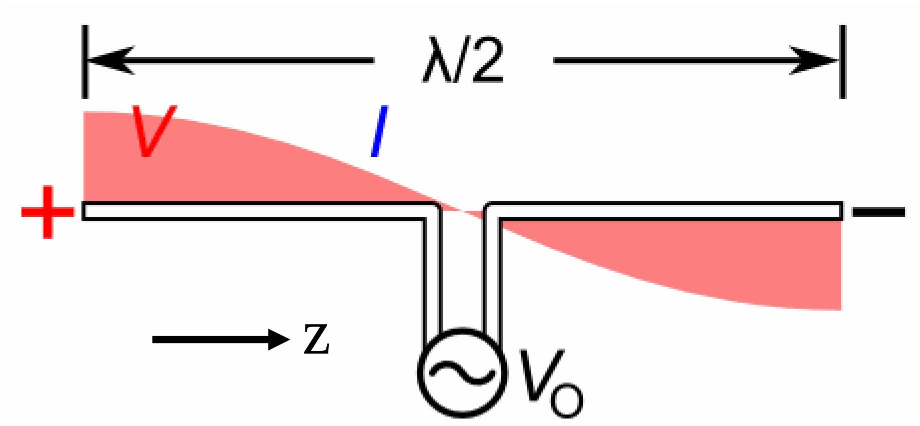
Beyond the Hertzian dipole: longer antennas

All of the results we've derived so far apply only in the situation where the antenna is short, i.e., $d << \lambda$.

That assumption allowed us to say that the current in the antenna was independent of position along the antenna, depending only on time: $I(t) = I_0 \cos(\omega t)$ ⟵ no z dependence!

For longer antennas, this is no longer true.

Example: this shows an antenna whose length is half the wavelength. Note that the current (blue) is not constant as a function of z (or of time).



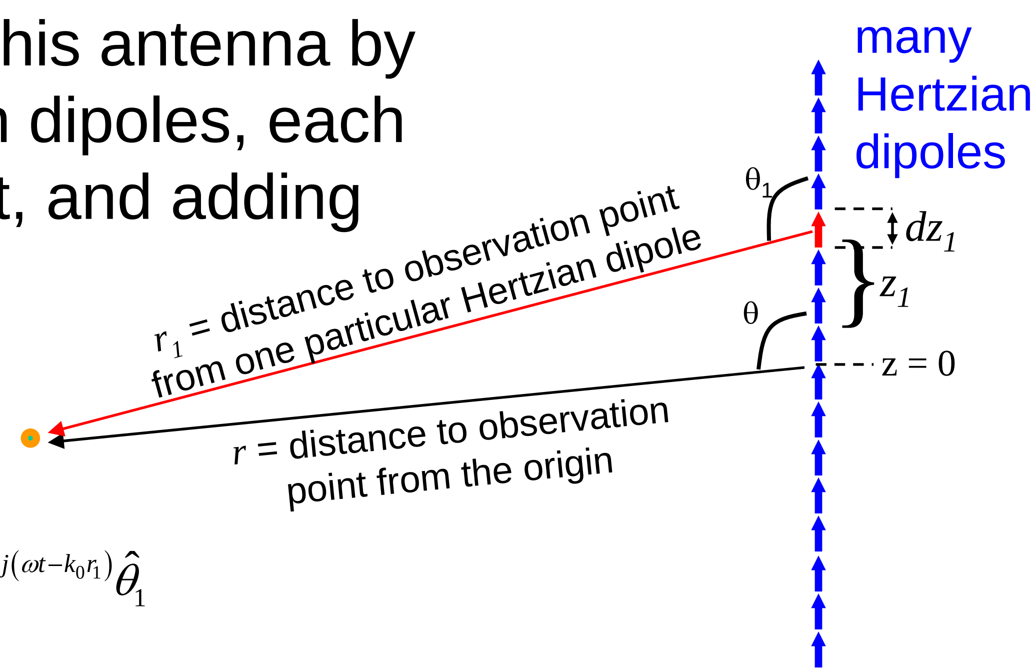
Beyond the Hertzian dipole: longer antennas

Let's consider the case of a half-wave dipole, for which the length L is half of the wavelength: $L = \lambda/2$.

As suggested by the cartoon on the last slide, we can assume that the current varies sinusoidally in both position and time:

$$I(z,t) = I_0 \cos\frac{\pi z}{L}\cos\omega t \quad \text{for } -L/2 < z < L/2$$

We find the radiation from this antenna by superposing many Hertzian dipoles, each with the appropriate current, and adding up the fields from each.



E field due to one dipole at z_1:

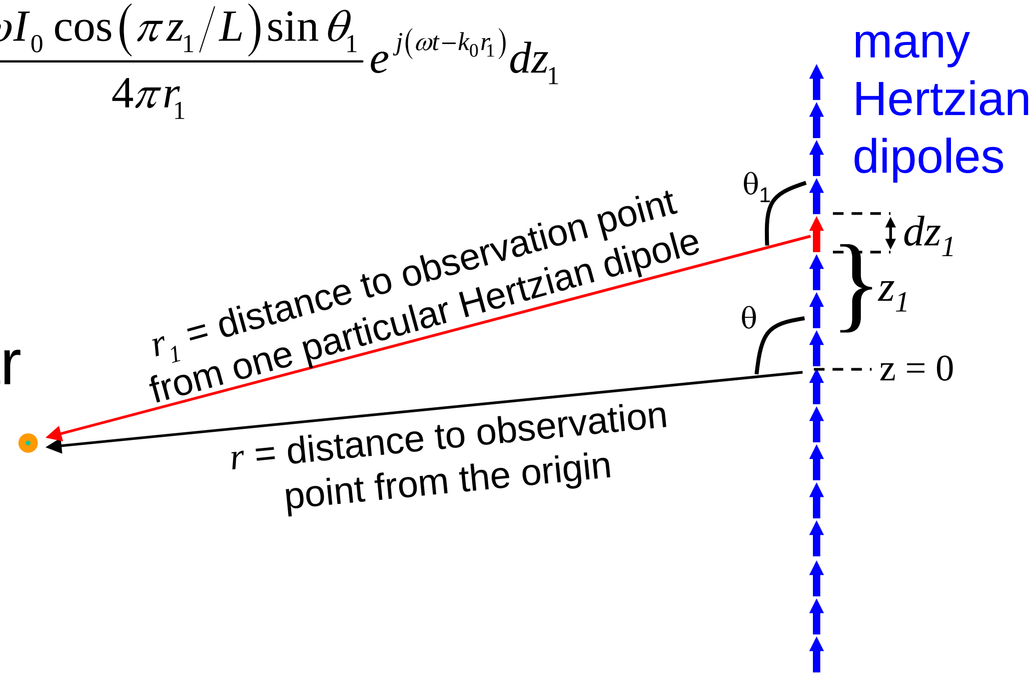
$$dE_1 = -\frac{\mu_0 \omega I_0 \cos(\pi z_1/L)\, dz_1 \sin\theta_1}{4\pi r_1} e^{j(\omega t - k_0 r_1)}\hat{\theta}_1$$

Beyond the Hertzian dipole: longer antennas

The net field is just the sum of the fields from all the dipoles:

$$E_\theta = \int dE_1 = -\int_{z_1=-L/2}^{L/2} \frac{\mu_0 \omega I_0 \cos(\pi z_1/L)\sin\theta_1}{4\pi r_1} e^{j(\omega t - k_0 r_1)} dz_1$$

Solving this integral requires approximation. If the observation point is far away from the antenna, then $\theta_1 = \theta$ and $r_1 = r$ in the denominator.



But in the exponent, $r_1 = r - z_1\cos\theta$, because we have to account for phase delays accurately.

$$E_\theta \approx -\frac{\mu_0 \omega I_0}{4\pi r} \sin\theta e^{j\omega t - k_0 r} \int_{z_1=-L/2}^{L/2} \cos(\pi z_1/L) e^{j(k_0 z_1 \cos\theta)} dz_1$$

and for this antenna, we have: $k_0 = \pi/L$

The half-wave dipole antenna

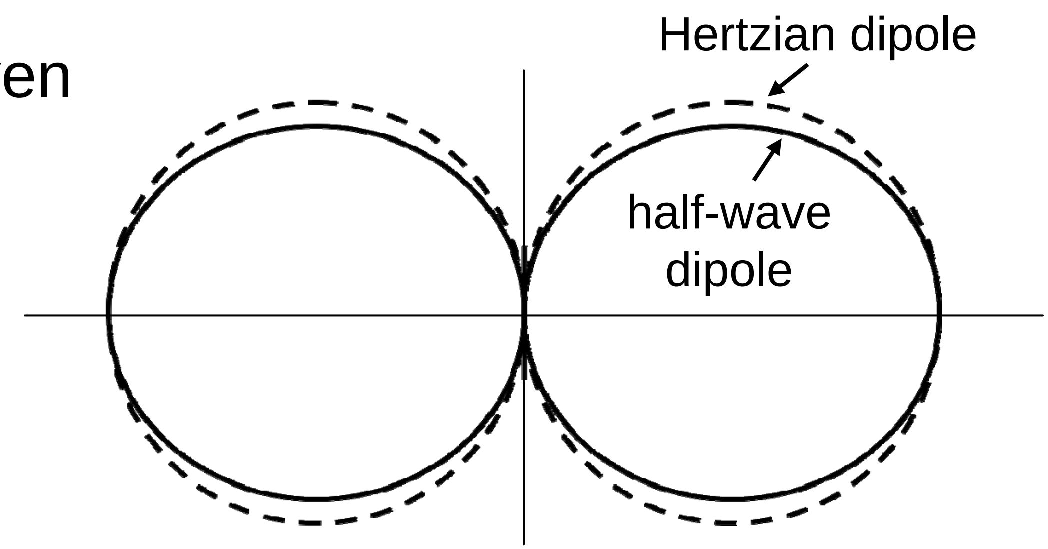
Doing the integral with those approximations gives this result:

$$\vec{E}(r,\theta,t) = -\frac{\mu_0 c_0 I_0}{2\pi r} f_{\lambda/2}(\theta) e^{j(\omega t - k_0 r)} \hat{\theta} \quad \text{where:} \quad f_{\lambda/2}(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

And, as usual, $\vec{B}(r,\theta,t)$ is perpendicular to $\vec{E}$ (and therefore points along $\hat{\phi}$), with a magnitude smaller by a factor of c_0.

The radiation pattern, given by $\left|f_{\lambda/2}(\theta)\right|^2$, is slightly narrower than that of a Hertzian dipole.

We can quantify this narrowing effect.



Directivity

"directivity" of an antenna: the ratio of the maximum radiated intensity to the average radiated intensity.

Directivity gives a measure of how strongly directional is the radiation pattern.

For a Hertzian dipole, the total radiated power is:

$$P_{total} = \frac{\pi \mu_0 c_0 I_0^{\,2}}{3} \cdot \frac{d^2}{\lambda^2}$$

The direction-averaged intensity S_{ave} is given by P_{total} divided by the area of a sphere:

$$S_{ave} = \frac{P_{total}}{4\pi r^2} = \frac{\mu_0 c_0 I_0^{\,2}}{12 r^2} \cdot \frac{d^2}{\lambda^2}$$

Directivity

Now, the angular distribution is given by the time-averaged Poynting vector:

$$\left\langle S(\theta,\phi)\right\rangle = \frac{\mu_0\omega^2 I_0{}^2 d^2}{32\pi^2 c_0}\left(\frac{\sin^2\theta}{r^2}\right)\hat{r}$$

The maximum of this function occurs at $\theta = \pi/2$:

$$S_{\max} = \frac{\mu_0\omega^2 I_0{}^2 d^2}{32\pi^2 c_0}\left(\frac{1}{r^2}\right) = \frac{\mu_0 c_0 I_0{}^2}{8r^2}\left(\frac{d^2}{\lambda^2}\right)$$

Therefore, the directivity is given by:

$$D = \frac{S_{\max}}{S_{ave}} = 1.5$$

A perfectly isotropic radiator (equal power in all directions) would have a directivity of 1. But there is no such antenna (because of the Hairy Ball Theorem).

Directivity: half-wave dipole antenna

The calculation is the same as for the Hertzian dipole.

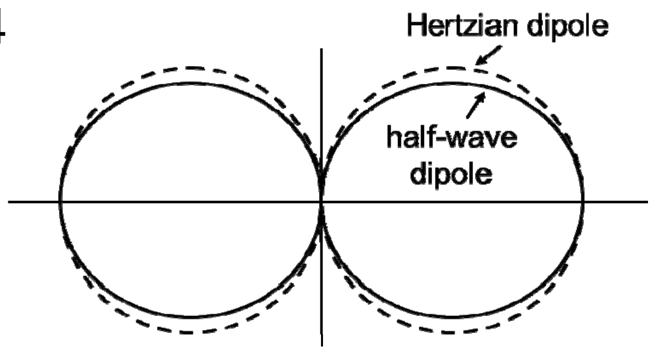
The only tricky part is that you need to compute the value of this definite integral, which must be done numerically:

$$\int_0^{\pi} \left| f_{\lambda/2}(\theta) \right|^2 \sin\theta d\theta = \int_0^{\pi} \frac{\cos^2\left[(\pi/2)\cos\theta\right]}{\sin\theta} d\theta \approx 1.219$$

It is now easy to see that the directivity is given by:

$$D = \frac{4\pi}{2\pi \times 1.219} = 1.64$$

slightly more directional than a short ($d << \lambda$) antenna.



Radiation resistance: half-wave dipole

A half-wave antenna has a radiation resistance of:

$$R_{rad} = \frac{1.219}{2\pi} Z_0 \approx 73\Omega$$

much larger than for a Hertzian dipole antenna! It is therefore a much more efficient radiator.

A steel rod of length L = 1.5 meters, radius a = 1 mm is used as an antenna for radiation at f = 100 MHz (FM radio). This frequency corresponds to λ = 3 meters, so this is a half-wave dipole.

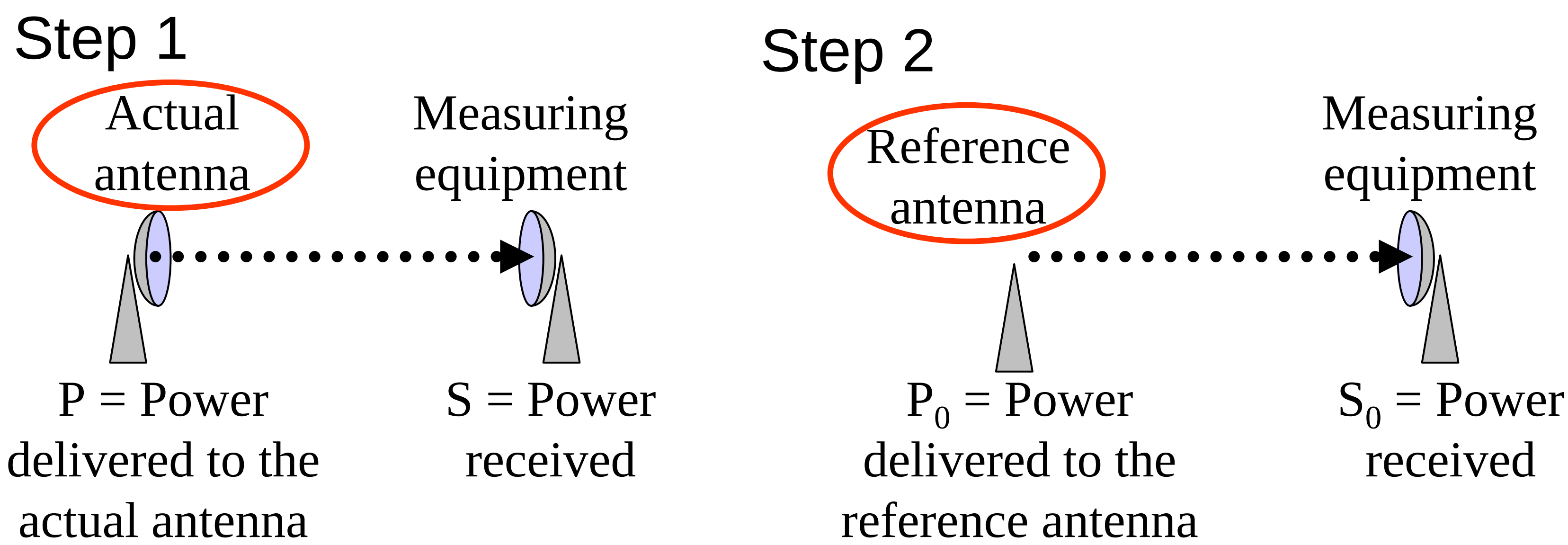
The resistance of the metal wire is given by: $R_{Ohmic} = \sqrt{\frac{\pi f \mu}{\sigma}} \frac{L}{2\pi a} = 3.4\Omega$

The radiation efficiency is therefore: $\eta = \frac{R_{rad}}{R_{rad} + R_{Ohmic}} = 0.955$

(often expressed in decibels: $\eta = -0.2$ dB)

Antenna gain

Gain: the ratio of the power required at the input of a loss-free reference antenna to the power supplied to the input of the given antenna to produce, in a given direction, the same field strength at the same distance



$$Antenna\ Gain = \left(\frac{P}{P_0}\right)_{S=S_0}$$

Antenna gain: some comments

Unless otherwise stated, gain refers to the direction of maximum radiation power.

Different options for the “reference antenna”:

- G_i “isotropic power gain” - the reference antenna is isotropic
- G_d - the reference antenna is a half-wave dipole isolated in space
- G_r - the reference antenna is linear, much shorter than one quarter of the wavelength, and normal to the surface of a perfectly conducting plane

Directivity relates to the power radiated by the antenna.
Gain relates to the power delivered to the antenna.

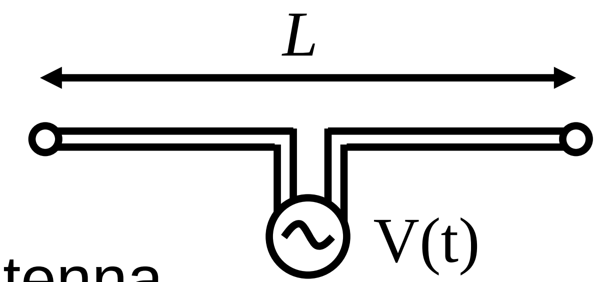
Antenna gain: some examples

Type of antenna	G_i [dB]	Angular beam width
Isotropic (hypothetical)	0	360^0x360^0
Half-wave Dipole	2	360^0x120^0
Helix (10 turn)	14	35^0x35^0
Small dish	16	30^0x30^0
Large dish	45	1^0x1^0

Note that smaller beam widths correspond to larger gain.

Other linear antennas

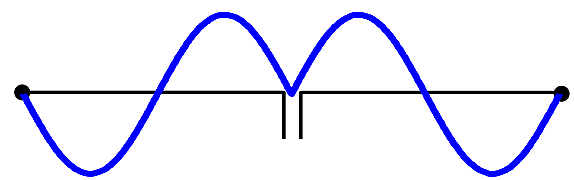
For a center-fed linear antenna of arbitrary length L, we can assume that the current:



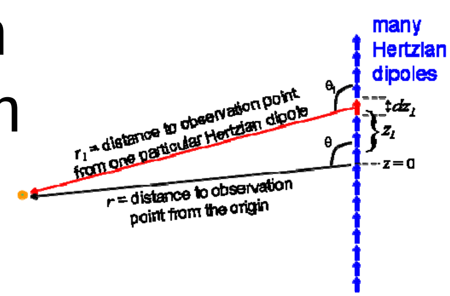
- varies sinusoidally along the length
- is symmetric about the center of the antenna
- goes to zero at both ends

$$I(z,t) = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} - |z|\right)\right]\cos(\omega t)$$

e.g., a dipole with $L = 2\lambda$ has current vs. position $I(z)$ that looks like this:



The same method of superposition of a bunch of Hertzian dipoles can be used to compute the fields from an antenna of arbitrary length.



Other linear antennas

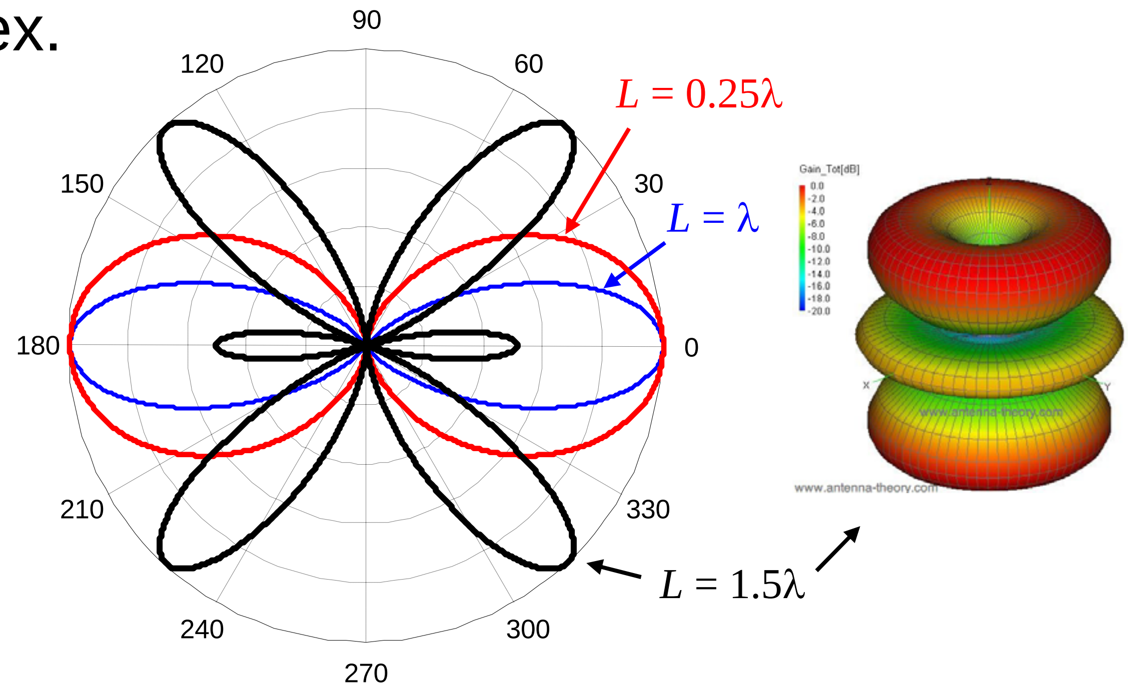
The result is: $\vec{E}(r,\theta,t) = -\frac{\mu_0 c_0 I_0}{2\pi r} f_L(\theta) e^{j(\omega t - k_0 r)} \hat{\theta}$

where the radiation pattern is given by: $f_L(\theta) = \frac{\cos\left(\frac{k_0 L}{2}\cos\theta\right) - \cos\left(\frac{k_0 L}{2}\right)}{\sin\theta}$

You might not think that a simple linear antenna could give so complicated a result. But the behavior can be quite complex.

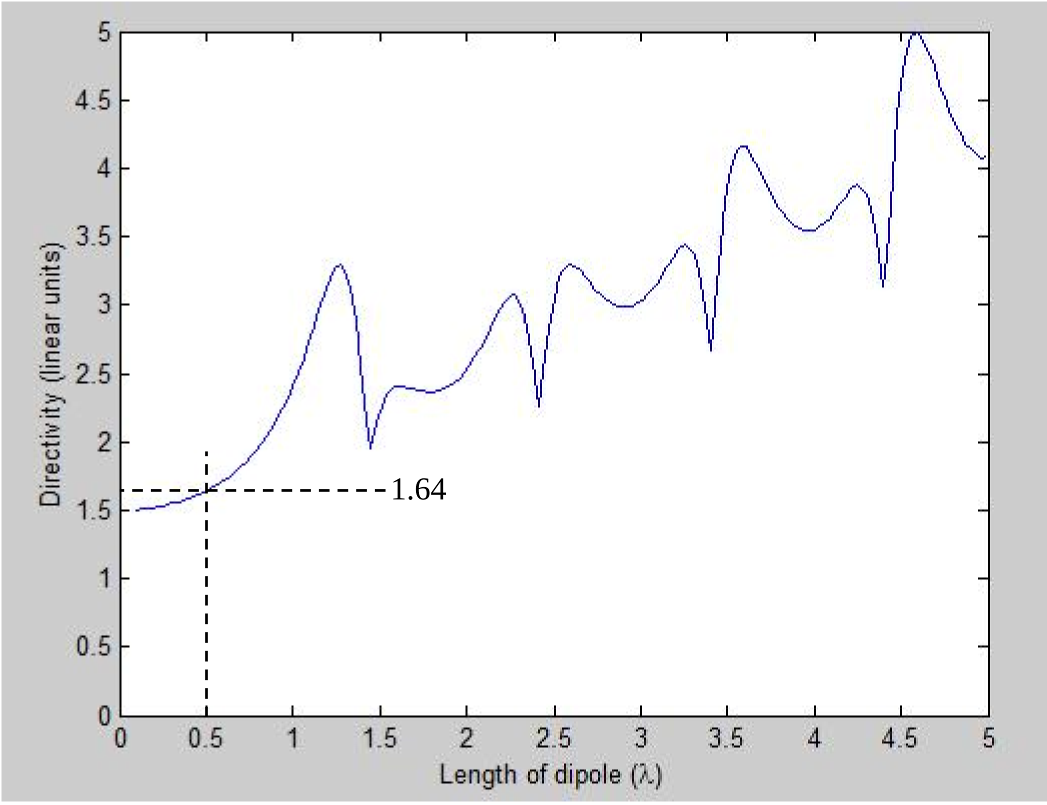
e.g., the antenna pattern for a dipole of length 1.5λ shows three lobes.

Note that the maximum power is not radiated at 90 degrees to the antenna axis.



Directivity of longer dipoles

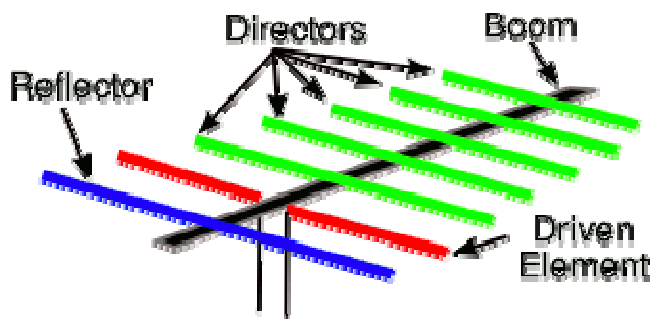
This shows the directivity of a linear antenna, as a function of its length. For L larger than about 1.25λ, the result is complicated and non-monotonic.



Antenna engineering

Much effort has been put into designing antennas with very specific radiation patterns.

A classic example: the “Yagi Uda” antenna (1926):

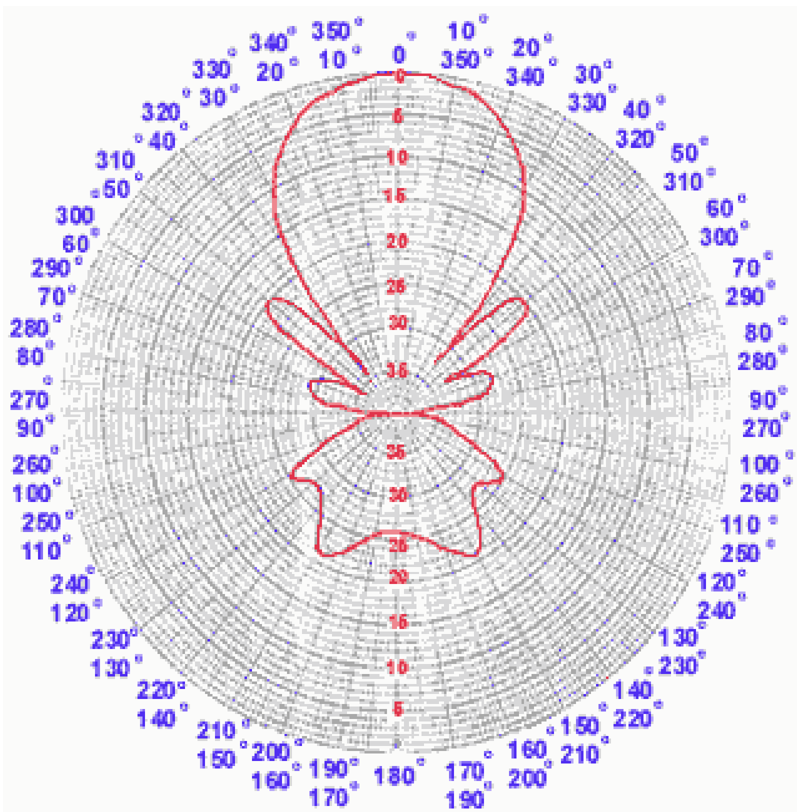


Hidetsugu Yagi & Shintaro Uda

A modern Yagi Uda antenna with 17 directors and 4 reflectors arranged in a corner-reflector pattern

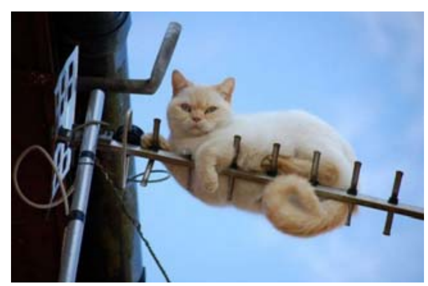
Antenna engineering

Such antennas can produce high gain and good directivity, and are often used for cellular reception in remote areas.



Radiation pattern of a 10-element Yagi-Uda antenna (log scale)

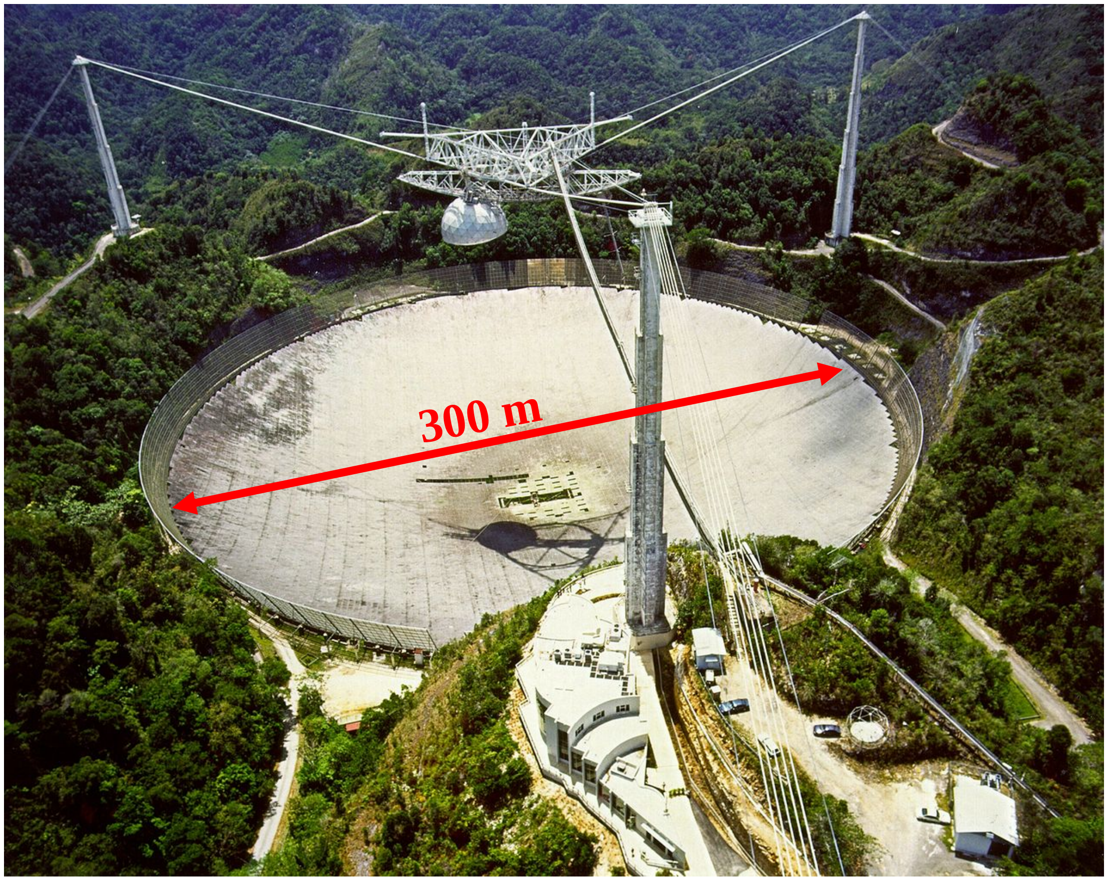
Note that the main lobe is about 100 times more intense than the next largest side lobes.



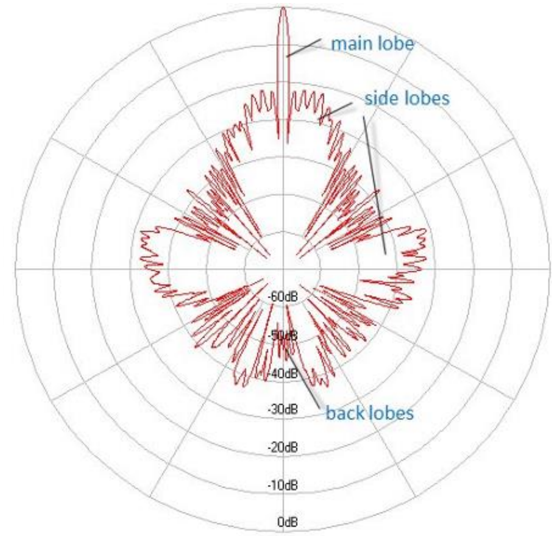
A configuration that is not recommended for high gain.

Example: A really BIG antenna

The Arecibo radio telescope (Puerto Rico)



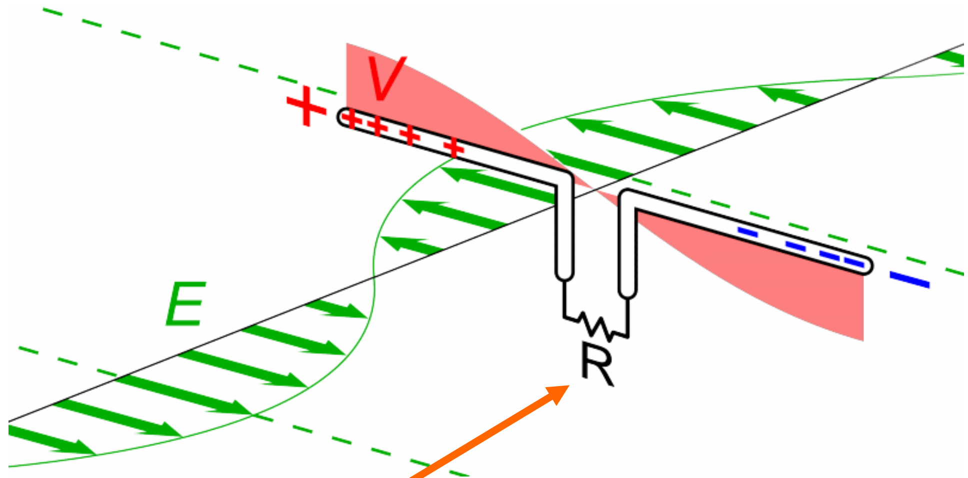
Radiation pattern (beam width: much less than 1°)



Gain: about 68 dBi (at ν = 3 GHz)

Transferring power to a load

We imagine that the voltage induced in the antenna by an incoming wave at frequency ω causes current to flow in an external circuit (shown here as just a simple resistor).



$$V_0 \cos \omega t = I_0 \cos \omega t \cdot R_{total}$$

What is the best value of the resistor to optimize power transfer to the load?

Recall that the antenna has a radiation resistance R_{rad}, which tells us how much power is dissipated by radiation. Thus $R_{total} = R_{rad} + R_{load}$.

Impedance matching

The power input to the circuit is $P_{in} = \left\langle V \cdot I \right\rangle = \dfrac{\left\langle V_0{}^2 \cos^2 \omega t \right\rangle}{R_{rad} + R_{load}}$

The power transferred to the load is:

$$P_{load} = \left\langle I^2 \right\rangle R_{load} = R_{load} \cdot \frac{\left\langle V_0{}^2 \cos^2 \omega t \right\rangle}{\left(R_{rad} + R_{load} \right)^2}$$

To maximize this, take the derivative and set it equal to zero:

$$\frac{dP_{load}}{dR_{load}} = \left\langle V_0{}^2 \cos^2 \omega t \right\rangle \frac{R_{rad} - R_{load}}{\left(R_{rad} + R_{load} \right)^3} = 0$$

The maximum power is transferred when the load resistance is equal to the antenna resistance. This is known as impedance matching.

Impedance matching

Perfect impedance matching is achieved when $R_{rad} = R_{load}$.

In this case, $P_{load} = P_{in} / 2$. Half of the power in the circuit is transferred to the load.

Where does the other half go?

Recall: there is a current flowing in the antenna.
So it must be re-radiating power.

Even in the best case, half of the power absorbed by the antenna is immediately re-radiated, without being transferred to any external circuitry.