VOLUME XXXVI NUMBER 66

PROCEEDINGS

OF THE

Casualty Actuarial Society

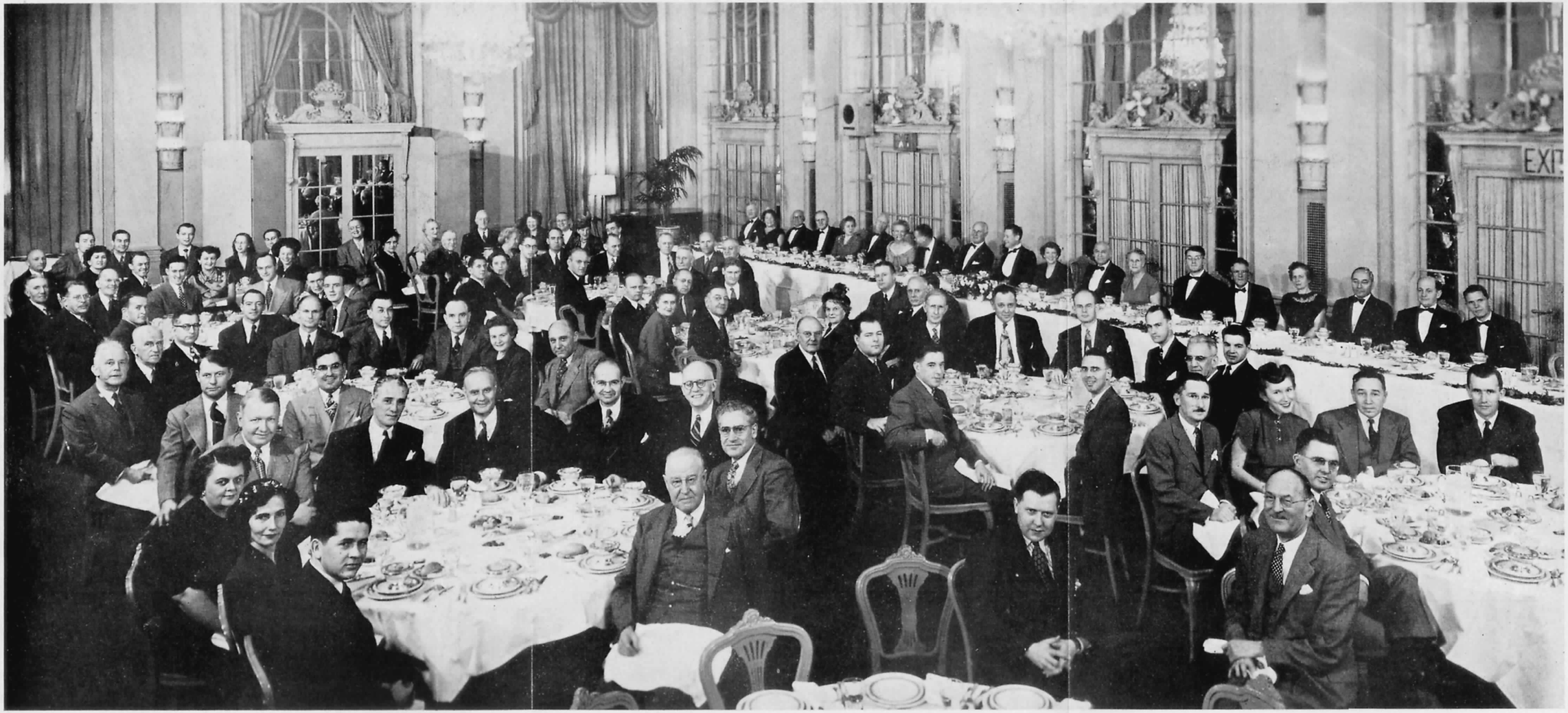
ORGANIZED 1914

NOVEMBER 18, 1949

"Full little of life and less of thought can be measured,
plumbed and weighed.
Statistics, formulas, tables, graphs, the tools of our
learned trade,
May grasp the shadows of life alone, and well if we
do not find
That after much shadow-chasing we are to its glori-
ous essence blind."

—Clarence W. Hobbs

CONTENTS OF VOLUME XXXVI



Thirty-Fifth Anniversary Dinner, November 17, 1949, Hotel Biltmore, New York

Vol. XXXVI No. 66

PROCEEDINGS

November 18, 1949

MULTIPLE LINE UNDERWRITING

PRESIDENTIAL ADDRESS BY J. M. CAHILL

Although complete multiple line underwriting powers may now be exercised in 44 states and territories, it took so long to have this accomplished that I am reminded of the opening sentences of S. D. Pinney's address[1] in 1943 when he said:

> "Twenty-five years ago, Richard Fondiller was elected Secretary-Treasurer of this Society. At approximately that same moment, I was squatting among the ruins of a little town north of Verdun, wondering if I would get turkey for Thanksgiving dinner. A quarter of a century has passed. Richard Fondiller is still Secretary-Treasurer of this Society, while I am still wondering whether I shall get turkey for Thanksgiving dinner."

Thirty-five years ago on September 16, 1914, two months prior to the organization of this Society, Insurance Commissioner Burton Mansfield of Connecticut delivered a paper[2] to the National Association of Insurance Commissioners in which he strongly advocated the abandonment of restrictions upon the classes of insurance that may be written by a company to permit multi-field writings, for both primary insurance and reinsurance writers. This paper has since become a classic and is well worth reading.

The following are the conclusions expressed by Commissioner Mansfield:

> "My conclusions may be briefly summarized as follows: The European system of classification has existed for nearly two hundred years without criticism. There has been no great stampede of companies to write at once all classes of insurance. Rather the tendency to enter all fields has been dignified, those companies which have availed themselves of the freedom of the system, being among the strongest in Europe. For a century in our own country the tendency has been toward the European system. While Massachusetts has been largely a mono-field state, Connecticut has accepted the principle of multi-field writings and New York is approaching the same position as Connecticut. So-called multi-field insurance is practically now being

[1] "What is So Peculiar About An Actuary?", *Proceedings* of the CAS, Vol. XXX, November 17, 1943, No. 60, p. 91.

[2] "Shall We Abandon The American Restrictions Upon The Classes of Insurance Written by (A) A Company Doing Direct Writings and (B) A Company Doing Reinsurance?", *Proceedings* of the National Convention of Insurance Commissioners, 45th Session, 1914, p. 147.

> practiced by casualty companies. The abandonment of the American restrictions upon the classes of insurance written, by both a company doing a direct business and a company doing reinsurance is but a step.
>
> "Just a word more. Any analysis of this question reflects our experience. We are, however, too willing many times to follow the historical precedent of this country in preference to progressing into newer fields which are admittedly more appropriate. Carlyle somewhere says that a nation's history is its true Bible. The Governor of Connecticut in commenting on this expression has remarked, 'If so it be, the last books must be better, wiser, truer than the first. There must be a new testament built upon the old—with its broader, freer, higher life.' Let us, therefore, bear this precept in mind and see if, in all fairness, we cannot use and adopt a wiser and better system of classification in the United States, even if among other changes it involves less harrowing expeditions on our part into the multitudinous details of a great business, conducted for the public good and forming a very large element in our national prosperity and growth.
>
> "Why hamper and restrict the immense insurance activity in this country by needless practices or laws? That activity should be enlarged to find its normal expansion into all reasonable fields and in all reasonable ways."

Even though this cause has been championed by many others since then, it was not until five years ago that real progress was made towards the passage of legislation that would make multiple line underwriting powers permissible everywhere. Only this year did New York enact the necessary legislation, and this has really cleared the way for the exercise of multiple line underwriting powers wherever permissible because it eliminated the road-block previously established by the Appleton rule and subsequent New York legislation that accomplished the same result.

DEFINITION

What is meant by multiple line underwriting? In this country it means the writing by an insurance company of all kinds of insurance, except life and annuities, which its charter permits. Thus, companies qualified by law to do multiple line underwriting may write casualty, surety, fire and marine coverages in the corporate name.

PRESENT STATUS

Complete multiple line underwriting powers are now permissible in 44 states and territories, and limited multiple line underwriting is permissible in the remaining eight states. Although eight states do not yet grant complete multiple line underwriting powers, it is likely that the necessary legislation will be enacted in due course. The fact that it has taken almost 35 years since Commissioner Mansfield's address to obtain quite general recognition of the desirability of multiple line underwriting powers has not been due to opposition on the part of most state supervisory authorities. On the contrary, throughout the years many insurance commissioners have given considerable support to the principle involved and have displayed strong leadership in bringing multiple line underwriting powers to fruition.

REASONS FOR MULTIPLE LINE UNDERWRITING

The following are some of the reasons advanced in favor of multiple line underwriting powers:

1. An insured may obtain the insurance protection he desires through a smaller number of policies; this will be a great convenience to the insurance buying public and to their producers.
2. More complete and attractive policy forms from the standpoint of the coverage afforded will result; for example, many fringe coverages could be included on an overall basis without materially affecting the cost of the basic insurance protection. Producers would thus have a more saleable product to present to their clients.
3. There will be a saving in expense through providing multiple coverages under a single policy rather than having to issue a multitude of separate policies; this saving will be recognized in one way or another to the benefit of the insurance buying public.
4. There will be no twilight zones of coverage for which the insured will have difficulty in obtaining insurance.
5. A lesser period of time will be required to place insurance on unusual risks.
6. There will be less litigation between companies to decide which is liable for a particular loss.
7. There will be less adverse selection against the companies for certain coverages, leading in due course to lower rate levels based on the broader spread of experience.
8. Multiple line underwriting will improve, not impair, the solvency of the companies because of the broader spread of risk. Not all kinds of insurance are profitable or unprofitable at the same time, and the further diversification should produce more stable underwriting results on an overall basis by company.
9. Subsidiary insurance corporations may be eliminated, leading to a greatly simplified corporate structure.

FINANCIAL REQUIREMENTS TO QUALIFY

Although the qualifications for multiple line underwriting powers vary by state and by type of carrier, it is of interest to note that a stock casualty and surety company with capital and surplus of $3,550,000 will qualify in New York; for a mutual casualty company the corresponding requirement is $1,925,000 surplus. Most sizeable companies may, therefore, qualify for multiple line underwriting powers without difficulty. Thus, we shall not much longer have the compartmenting of the insurance business to life, casualty and surety, fire and marine; hereafter, it will be life and all other, the latter consisting of casualty, surety, fire and marine.

ALL RISK POLICIES

Now that we are on the threshold of multiple line underwriting, the question is where do we go from here? In some circles, the reply may be "to all risk policies, of course". By an all risk form of policy is meant one where the insured would be protected against all hazards except those

specifically excluded from the contract. However, the development of comprehensive all risk policies and particularly if on a single rate basis will inevitably come slowly. It is much more likely that the trend in the foreseeable future will be in the direction of multiple line policies covering specified perils on a schedule basis. There will be some merging of coverages and rating procedures through the package approach, of course, but it would be unrealistic to expect a rapid development of either all risk policies or single rate policies. Among the reasons why this should not happen quickly are the following:

1. A statutory fire policy is prescribed in many states; law amendments will thus probably be required to make simple all risk or multiple line policy forms including fire insurance on real property permissible.
2. The premium that insureds would have to pay for complete all risk coverage would be too high to be attractive. Furthermore, many insureds, both individual and commercial, prefer to spread the expiration dates of their several policies throughout the year in order not to have all of their insurance premiums payable at one time. Installment premium payment methods will therefore be necessary.
3. In this country, ratemaking functions are performed to a considerable extent by rating organizations. There is no rating organization functioning for both casualty and fire insurance. It will be much easier for rating organizations to cooperate in the development of multiple line policies rated on a schedule basis than it would be if the same coverages were to be written on a single rate basis. For example, in the case of the automobile policy, it has been possible for years for companies to afford automobile bodily injury, automobile property damage, medical payments, automobile collision, and automobile fire, theft or comprehensive coverage on a schedule basis under one policy. This has proven to be a very satisfactory method of rating and no change to a complete single rate basis is anticipated. However, multiple line underwriting will give further encouragement to the use of common classifications, territories, effective date rules and rate revision dates for the several coverages to the extent such are practicable.
4. There will have to be more universal acceptance on the part of the insurance buying public of deductible coverage. The insurance companies cannot afford the expense involved in settling a multitude of small claims that the insured can better assume. Through the use of proper deductible provisions, the premium cost for the insurance protection really needed by the insured can be kept at a considerably lower level.
5. In America, risk situations are not so uniform and homogeneous as, say in England, and this will make underwriters more cautious in the development of policies providing multiple line coverage on a single rate basis. Furthermore, in this country it would be very difficult to obtain acceptance of policies on a 100% coinsurance basis, whereas this is the common practice in England.

6. Until such time as the annual statement blank is modernized to fit multiple line underwriting, the carriers will have some difficulty in handling the accounting, reserves, etc. for contracts written on an all risk basis.

The multiple line automobile policy is an actuality since across-the-board coverage already is afforded by individual companies or by affiliated companies within one policy contract. Beyond that, the development of business on a multiple lines basis within company will undoubtedly come slowly. Particularly because of the shortage of trained personnel in each field, companies must inevitably move slowly in expanding on a multiple lines basis beyond their present spheres. By and large, they may generally be expected to observe present coverages and procedures.

PACKAGE POLICIES

Within the casualty and fire fields, the package policy has become important during the past decade. We are all familiar with the following coverages and policy forms embodying the package idea:

Automobile Comprehensive Coverage
Automobile Liability and Material Damage Policy
Comprehensive Automobile and General Liability Policies
Comprehensive Personal Liability Policy
Farmer's Comprehensive Policy
Storekeeper's Comprehensive Policy
Bankers' Blanket Bonds
Comprehensive Dishonesty, Disappearance and Destruction Policy
Extended Coverage on Fire Policy
Personal Property Floater

This approach will be further emphasized in the period ahead, not only on the schedule policy basis but also on a combination rating basis.

HOUSEHOLDER'S COMPREHENSIVE POLICY

For example, great interest has been expressed in the development of a householder's comprehensive policy form such as those written in Great Britain. Broad coverages for fire, burglary, employers' liability and public liability are written there under one contract for both the building and contents of private residences. With certain limitations, insurance is provided agains the following hazards:

1. Fire, explosion, lightning, thunderbolt, earthquake, storm, tempest, flood.
2. Riot, civil commotion, strikes, labor disturbances.
3. Bursting or overflowing of water tanks, apparatus or pipes.
4. Loss or damage caused by aircraft.
5. Impact with the building by any road vehicle, horses or cattle.
6. Burglary, housebreaking, larceny or theft.
7. Damage to mirrors.
8. Public liability.
9. Accidents to servants.
10. Damage (other than fire) for which Insured is liable as tenant.

11. Loss of rent and hotel expenses.
12. Death benefit for accidental death of Insured in the dwelling if caused by fire or burglars.

Such policy is rated by applying a percentage rate to the amount of fire insurance for buildings and contents; this rate includes the charge for all coverages regularly afforded. A number of other coverages such as glass insurance may be obtained under the same policy by paying additional premium charges.

However, for the same reasons cited earlier, it will be difficult and perhaps impracticable to develop such a policy for the entire American market on a sound basis. Nevertheless, a policy form suitable for the mass market that does not purchase insurance today which would provide limited protection against specified perils on a single rate basis is much to be desired. Such a policy would be truly beneficial to the public if it encouraged a higher proportion of the lower income families to carry insurance against the danger of major losses. It could be the source of a large volume of new business for the carriers. The reason why this market has not been reached heretofore may be that the premiums involved for individual policy forms have been too small to warrant any sales effort on the part of insurance producers or direct writing companies.

COMPOSITE RATING

The trend under package policies where the aggregate premium is sizeable will be towards composite rating whereby the equivalent of the usual tariff premium is determined by the use of a single rate applicable to a single exposure base subject to audit. This is the type of rating that many insureds prefer because they then know what their insurance will cost in simple terms as a percentage of payroll or sales, etc. This sound and practical method of rating fits in perfectly with the concept of multiple line underwriting and will unquestionably have a tremendous growth in the years ahead.

PROBLEMS CREATED

Many problems will be created as companies embark on multiple line underwriting. Among these will be the following:

1. Policy Forms. New, concise policy forms will have to be drafted to avoid cumbersome, lengthy, and possibly ambiguous forms if existing policies were merely tied together in one jacket.
2. In ratemaking, many new approaches will have to be taken and judgment will have to be utilized to a considerable extent, in the initial stages at least.
3. Accounting and statistical procedures will have to be revised, particularly where single rate rating methods are established. The annual statement blank must be completely revamped to accommodate multiple line underwriting, and this revision in a number of instances will call for amendment of state statutes prescribing the information to be reported in the annual statement.
4. Companies will have to build up their organizations, often by first acquiring experienced personnel to plan their entry into the new fields.

It will be a real problem for a casualty company entering the fire field to train a staff from top to bottom to handle its fire business properly. The same will be true of fire companies entering the casualty field. To illustrate, in the matter of claim adjustment the results will be very unsatisfactory if casualty claim adjusters trained in the handling of third party claims use the same methods and viewpoint in handling fire losses. Similarly, fire companies will have to learn that claim adjustment work in connection with third party claims is vastly different from settling property losses suffered by your own insured.

5. Common commission scales for the several kinds of insurance afforded under package policies will become essential.
6. One of the most serious problems under the expansion of multiple line underwriting will be in the matter of commissions and the self-regulation thereof to avoid commission wars. If all casualty and fire companies should embark on the appointment of a large number of general agents, the end result will be that practically every producer of any size will get general agency commissions even though he is not qualified to handle the business in the manner contemplated of general agents. This could lead to serious trouble in the industry.

INTERSTATE RATING

The principle of interstate rating must be given universal recognition if multiple line underwriting is to function in the effective manner contemplated. Now that insurance has been held by the United States Supreme Court to be commerce and state regulatory laws have been enacted to preserve the regulation of insurance by the states, it is absolutely essential that the supervisory authorities do not act in a way that impedes composite rating, etc. on an interstate basis. The insurance industry has a duty to afford the insurance protection desired by insurance buyers on as simple, complete and economical a basis as possible. The last three sentences of Commissioner Mansfield's conclusions which I quoted earlier should be read and heeded by all state supervisory authorities.

EFFECT ON CASUALTY ACTUARIAL SOCIETY

What will multiple line underwriting mean to casualty actuaries and to the Casualty Actuarial Society? Our business as we have known it will be subject to marked change in the future. Those of us without experience in the fire field, particularly in fire and inland marine ratemaking practices, will have to learn their fundamentals of ratemaking. There will be a demand from fire companies for men with casualty actuarial experience. We must enlarge our training of casualty actuaries if the supply is to come anywhere near meeting the demand.

Further, the Casualty Actuarial Society should give serious thought in the near future to the type of expansion it desires to bring men with experience in rating work for fire and allied lines into active status with this Society. I suggest that the Committee on Development give immediate attention to this subject because in the not too distant future fire and inland marine rating will be an integral part of the operations of companies that we now know

only as casualty insurance companies. It is logical that the men performing all ratemaking work on behalf of such companies be members of this Society. In this way the Society will not only keep in step with the times, but we shall make further progress in giving the leadership in actuarial science which the industry expects of us.

A DISCUSSION OF GROUP ACCIDENT AND HEALTH INSURANCE

BY

HAROLD F. LACROIX, JR.

During the past few years Casualty Insurance Companies have shown a growing interest in the Group Accident and Health field. In recognition of the resulting interest of casualty actuaries and actuarial students, it is the intent of this paper to outline a possible statistical plan for carriers newly entering the field. Because the *Proceedings* at present lack any article dealing with the principle types of Group Accident and Health coverage and with the development of this comparatively new line of insurance, a brief discussion of these subjects, with particular reference to temporary disability legislation, is also included as a prerequisite to any examination of a possible statistical plan.

I. PRINCIPAL TYPES OF COVERAGE.

Ordinarily Group Accident and Health insurance excludes coverage for disabilities resulting from work or for which the employee is entitled to Workmen's Compensation benefits. Any major exceptions to this statement will be noted in the descriptions of the various types of coverage.

The most important Group Accident and Health sublines are described in the following paragraphs.

A. *Weekly Indemnity Insurance* provides a specified weekly indemnity if the employee becomes totally disabled as a result of bodily disease, injury or pregnancy. The insurance is written with various waiting periods for accident and sickness disabilities. Four maximum indemnity paying periods are common; 13, 26, and 52 weeks, for disabilities resulting from bodily injury or disease, and 6 weeks for disabilities resulting from pregnancy. In most cases, plans are classified, first, according to the day from which benefits are payable and, second, to the maximum indemnity paying period. For example, a 1-4-26 plan would pay weekly indemnity from the first day for disabilities resulting from injuries, from the fourth day for disabilities resulting from disease, and for a maximum of 26 weeks.

Infrequently weekly indemnity coverage is written to provide benefits for occupational disabilities. Such coverage is usually furnished so as to provide benefits during the waiting period stipulated by the Workmen's Compensation Law, or to provide supplemental weekly indemnity, so that approximately the same amounts of weekly indemnity are payable from the same day of disability for both occupational and non-occupational disabilities when Workmen's Compensation benefits are combined with the supplemental Group Accident and Health coverage.

B. *Accidental Death, Dismemberment, and Loss of Sight Insurance* provides specified amounts of indemnity for loss caused by injuries sustained through accidental means. The specified amount, known as the Principal Sum, is payable for the loss of:

1. Life
2. Both hands
3. Both feet
4. One hand and one foot
5. Sight of both eyes
6. Either hand or foot and the sight of one eye.

One-half the principal sum is payable for the loss of:

1. One hand
2. One foot
3. Sight of one eye.

In only this type of Group Accident and Health insurance is coverage frequently afforded for both occupational and non-occupational accidents.

C. *Employee Hospital Expense Insurance* usually provides the following benefits if the employee becomes confined as a resident patient in a legally constituted hospital upon the recommendation of a physician as a result of a bodily disease or injury:

1. A specified daily benefit for each day of confinement for a maximum number of days (usually 31 or 70 days) during any one period of disability. This benefit is commonly called "The Daily Benefit." This Daily Benefit is sometimes sold on a reimbursement rather than the indemnity basis described here.

2. An Amount equal to the hospital charges for medical care and treatment—other than charges for board and room, nursing care and attendance by a physician—but not more than a specified number of times the Daily Benefit during any one period of disability (generally 5, 10, 15, or 20 times the Daily Benefit). These benefits are usually referred to as "Miscellaneous Benefits."

If, however, the confinement results from pregnancy, the benefit payable is commonly either:

(a) A specified Daily Benefit for each day of confinement for a maximum of 14 days, plus an amount equal to the hospital's miscellaneous charges, but not more than a specified number of times the Daily Benefit (commonly 5, 10, 15 or 20 times the Daily Benefit) or,

(b) An amount equal to the hospital's charges for both room and board, and medical care and treatment (other than charges for nursing care and attendance by a physician), but not more than 10 times the Daily Benefit.

It should be mentioned that benefits for pregnancy confinements following the termination of the insured employees insurance are paid if the pregnancy commenced while the employee's insurance is in force. This benefit, sometimes called "The Extended Term Maternity Benefit," is required by rulings of the New York and Michigan Insurance Departments, and is provided generally. This does not mean, however, that maternity benefits during the first year of an Employee Hospital Expense contract are not payable until the 10th month of the policy period, since in most instances immediate maternity coverage is provided to all employees becoming insured within one month of the contract date. For Employees becoming insured subsequent to this date a nine months waiting period before maternity benefits become payable is usually required. Practices in regard to this matter vary between companies. Occa-

sionally the pregnancy benefit is excluded from the Employee Hospital contract.

In addition, several of the principal Group writing companies provide benefits up to 5, 10, or more times the Daily Benefit if the employee does not qualify for any of the benefits described in paragraphs one and two above and, as a result of a non-occupational accident, incurs hospital charges for emergency medical care and treatment of bodily injuries within 24 hours of the time of the accident. This benefit covers those employees who incur hospital charges without becoming resident patients and is generally described as "The Emergency Accident Benefit."

D. *Dependent Hospital Expense Insurance* provides benefits for board and room and miscellaneous charges on the same basis as employee hospitalization coverage, if hospitalization commences as a result of accident or disease. The Daily Benefit in the case of Dependent Hospital Expense Insurance is usually on a reimbursement basis. If the confinement results from pregnancy, an amount up to 10 or 15 times the Daily Benefit, depending on the policy, is available for both board and room and miscellaneous charges although this coverage may also be obtained on an ex-maternity basis. In contrast to Employee Hospital Expense Insurance the pregnancy benefit is usually provided only for pregnancies commencing while the insurance in respect to the dependent wife is in force, although immediate maternity coverage can be obtained for the initial group on the payment of an additional premium for the first year. The Extended Term Maternity Benefit described in the paragraph on Employee Hospital Expense Insurance applies generally in the case of Dependent Hospital Expense Insurance. The Emergency Accident Benefit is the same as that described for employees.

E. *Employee Surgical Expense Insurance* provides a benefit if the employee is operated upon by a duly qualified surgeon as a result of bodily disease, injury or pregnancy. The Benefit is an amount equal to the surgeon's fee, but not more than the maximum listed for the particular operation in the Schedule of Operations. Various schedules are offered, although a large volume of the business now in force provides benefits under a basic schedule which provides a maximum benefit of $150.00.

Immediate Maternity coverage is generally provided in accordance with the conditions outlined for Employee Hospital Expense Insurance. An Extended Term Maternity Benefit is also available under Employee Surgical Expense Insurance, provided the operation results from pregnancy commencing while the employee's insurance is in force. The coverage may be written on an ex-maternity basis.

Several State Medical Associations (Rhode Island, Wisconsin, Tennessee and Maine) have sponsored Surgical Plans which may be underwritten by carriers conforming to the specifications prescribed by the Medical Associations. These Surgical Plans have two main features:

(1) The Medical Associations have themselves established the fees set forth in their various surgical schedules.

(2) The fees listed in the surgical schedules of the Plans are accepted by the participating physicians in full payment for operations performed, if the annual income of the insured employee is less than a stated amount. If the annual income of the insured employee exceeds the maximum income

stipulated in the Plan, the benefit listed in the surgical schedule is accepted either in full or partial payment for the operation.

These Medical Association Plans almost always include the maternity benefit.

F. *Dependent Surgical Expense Insurance* provides the same benefits as those described in the paragraph on Employee Surgical Expense Insurance, if the Employee, on behalf of his dependent, incurs the expense of a surgical operation performed by a duly qualified surgeon as a result of bodily disease, injury or pregnancy.

Benefits for operations resulting from pregnancy are payable only when the pregnancy commenced while the insurance was in force, although immediate maternity coverage for those becoming insured within one month of the date the policy is initially written can be obtained from most carriers by the payment of an additional premium during the first policy period. The Extended Term Maternity Benefit described for Employee Surgical Expense Insurance also applies to Dependent Surgical Expense Insurance. Dependent Surgical Expense Insurance is, however, sometimes written on an ex-maternity basis.

The Medical Association Surgical Plans described in the paragraph on Employee Surgical Expense Insurance can also be written to cover the dependents of the employee.

G. *Employee or Dependent Medical Expense Insurance* provides a benefit if the employee or the employee's dependent is necessarily treated by a physician. Although at present there are many varieties of Medical Expense Plans, most of them can be put into one of three principal types:

1. The In-Hospital Plan—providing benefits during any period of total disability for which Hospital Expense Insurance is payable.

2. The Total Disability Required Plans—providing benefits during any period of *total* disability resulting from bodily disease or injury.

3. The Total Disability Not Required Plans—providing benefits during treatment.

Medical Expense Plans falling within type (1) commonly provide a benefit in amount equal to the physician's charges subject to a maximum determined by multiplying the amount provided for one day ($2.00, $3.00, or $4.00 for example) by the number of days of hospital confinement, but not more than a specified number of times the Daily Benefit (generally 31 or 70 times, so as to parallel the hospital expense coverage) for all visits during any one period of disability.

Plans falling within type (2) are commonly written with a benefit equal in amount to the physician's charges up to $3.00 for each home or hospital visit, and $2.00 for each office visit. Medical Plans of this type provide for various waiting periods. The exclusion of the first two visits in connection with treatment for both injuries and disease, for example, is common. Such plans usually limit the benefits payable during any one disability to a maximum amount ($150.00 is a maximum amount frequently used) and in some Plans the benefits are payable only when Weekly Indemnity Benefits are payable.

Type (3) Medical Plans have been developed only recently and provide benefits similar to those discussed for type (2) Medical Plans, except that the maximum amount is payable for treatment resulting from any one injury or

disease. Benefits under type (3) Medical Plans are never, of course, restricted to a period during which Weekly Indemnity Benefits are payable.

Ordinarily in all three types benefits are not payable for visits, in connection with surgical operations, post-operative care by the Surgeon, or pregnancy.

H. *Employee or Dependent Laboratory and X-Ray Examination Expense Insurance* provides a benefit if the employee or employee's dependent undergoes a laboratory or X-ray examination on order of a physician as a result of bodily disease or injury. This coverage is still on an experimental basis and practices differ widely between carriers. Most Laboratory and X-ray Expense Plans fall into one of two broad categories:

1. Scheduled plans providing benefits equal to the charge for the examination up to a maximum for the examination shown in the schedule.

2. Non-scheduled plans providing benefits equal to the charges for the examination with a stated maximum for all examinations (for example $25.00 is common).

In both types a maximum amount (frequently $25.00 or $50.00) is usually established for all examinations as a result of injuries sustained in any one accident or as a result of one disease.

II. DEVELOPMENT OF GROUP ACCIDENT AND HEALTH INSURANCE WITH PARTICULAR REFERENCE TO STATE TEMPORARY DISABILITY LAWS.

Since Annual Statement requirements did not require the separation of Group Accident and Health premiums from other Accident and Health premiums until 1942, it is difficult to trace the growth of this line of insurance prior to that year. During 1942, however, Group Accident and Health coverage written by insurance carriers, not including Blue Cross coverage produced $120,848,000.00 of Statement written premiums. In 1948 the comparable figure was $383,425,000.00.* Due perhaps principally to the fact that Life Companies could offer Group Life Insurance as well as Group Accident and Health Insurance, the large bulk of this premium volume was written by a few large Life Insurance Companies. In fact 10 Life Companies wrote over 73% of the previously mentioned premium volume for 1948.

The development of Group Accident and Health Insurance prior to World War II was on a purely voluntary basis and, as would be expected, the Employee Coverages for which the demand was greatest expanded steadily. Dependent coverages, even at present, produce only a small part of the total premium volume. During World War II, the first step was taken by a state government to make temporary disability coverage mandatory. In 1942 the Rhode Island legislature passed a Temporary Disability Benefits Law to become effective April 1, 1943, thereby initiating a new legislative trend.

This first successful attempt at temporary disability legislation produced a monopolistic state fund which was to pay benefits to those unable to work because of disability. The fund was to be administered by the Unemployment Compensation Commission.

On the termination of World War II, which had temporarily interrupted the Disability Benefits legislative trend, the legislatures of several states

*Argus Charts 1944 and 1949

became increasingly active in considering this type of legislation. On March 5, 1946, California became the second state to enact a Compulsory Temporary Disability Benefits Law, the benefit provision becoming effective December 1, 1946. New Jersey followed California's lead with similar benefits effective January 1, 1949. The State of Washington legislature passed a Temporary Disability bill to become effective June 10, 1949, but because of a referendum petition this measure will not become effective unless approved by the people in the next general election in November of 1950. The latest and most important of these legislative developments was taken in April 1949, when the New York legislature passed a temporary disability benefits law providing for compulsory Temporary Disability benefits effective July 1, 1950. As a further indication of the increasing interest in this type of legislation it is interesting to note that in 1949 bills for temporary disability legislation were introduced in Alaska, Colorado, Connecticut, Delaware, Florida, Hawaii, Maryland, Massachusetts, Minnesota, Nevada, New Mexico, Pennsylvania, Tennessee, and Wisconsin.

Except for Rhode Island the State laws thus far enacted provide for both State Funds and private plans either self-insured or underwritten by insurance carriers.

In California and New Jersey the State Funds are tax supported on the Unemployment Compensation model and the State Fund administrators supervise the private or voluntary plans. In New York every subject employer must buy insurance. Here the State Fund is a competing insurance company subject, as private insurers are, to the administration of the act by the Workmen's Compensation Board and to premium taxes and assessments.

The California, New Jersey and New York disability laws provide weekly indemnity for those employed who are unable to work because of disability for a limit of 26 weeks in the case of California and New Jersey, and 13 weeks in the case of New York. In all three laws payment of benefits begins after a 7 day waiting period and disabilities due to pregnancy are for the most part not covered. The amount of Weekly Benefits under the California and New Jersey laws is figured on an Unemployment Compensation base period concept, while the New York weekly indemnity is set at 50% of average weekly wages. All three laws provide minimum and maximum weekly indemnities within which the benefit formulas range. All three state plans provide for the payment of benefits during unemployment disabilities. In California and New Jersey these unemployment disability benefits are financed by the interest on funds withdrawn from Federal Unemployment Trust Funds and, if the benefits exceed this interest, by an assessment on private plans limited to .03 of 1% and to .02 of 1% of the taxable wages in California and New Jersey respectively. The New York Act finances unemployment disability benefits, after the initial fund is set up by taxes on the employers and employees, by assessments on Insurance Companies, self insurers and the State Insurance Fund with *no* limit on the size of the assessment. The difference in the methods of financing unemployment disability benefits, as provided for by the California and New Jersey Laws on one hand, and the New York Law on the other, is assuming greater importance every day. Available figures indicate that the California assessment for calendar year 1949 will

reach the maximum of .03 of 1%. If California had financed these benefits by an assessment without limit on the carriers and state fund without using the interest on the funds withdrawn from Federal Trust Fund monies, the assessment for the relatively prosperous year of 1949 would be considerably higher than the .03 of 1% of taxable wages which constitutes the present limit assessment. Provision for the cost of this benefit in the New York Temporary Disability premium rates, and for the treatment in experience rating of the charges to accumulate the necessary reserves for depression years constitutes a serious problem.

The California law has recently been amended so that, effective January 1, 1950, indemnity at the rate of $8.00 per day for a limit of 12 days during a period of hospitalization is payable in addition to the basic Weekly Indemnity Benefits. In the event of hospital confinement the unexpired portion of the usual 7 days waiting period for both accident and sickness disabilities is also waived during the continuance of disability.

The California law provides that coverage under the State Fund is automatic unless a private plan with more favorable benefits is elected by the employer and the employees. Employee contributions cannot be higher than the 1% they would pay the State Fund and employers must assume any balance of the cost. Employees not electing private plans are covered by the State. Thus, even if a majority of the employees of the risk elect a private plan, the minority are covered in the State Fund. The New Jersey Law goes one step forward and provides automatic State Fund coverage unless the employer and a majority of the employees elect a private plan with benefits at least equal to the State Fund benefits and contributions of employees no higher than ¾ of 1% required for the State Fund. All employees in a risk are, therefore, covered entirely in the State Fund or by a private plan. The recently enacted New York Law requires the employer to choose either a carrier or the State Fund or to self insure.

As to financing, the California benefits are entirely supported by taxes on employees (1% of the first $3,000.00 of annual wages). New Jersey divides the cost between the employees who pay ¾ of 1% of the first $3,000.00 of annual wages, and the employer who pays the remainder of the cost of the plan (for employers in the State Fund the cost will range from 1/10 of 1% to ¾ of 1% according to the experience rating formula in the law.) The New York benefits are also on a contributory basis with employees paying ½ of 1% of the first $60.00 of weekly wages and the employers paying the balance of the cost.

One result of the flat 1% tax rate for the State Fund in California is that it is unattractive for a risk which requires a higher rate to insure with an insurance company. The law contains a provision which states in effect that the writing of private plans must not result in substantial "adverse selection" against the State Fund. This has been interpreted by the administrative authorities to mean that each insurance carrier must maintain within its own business a percentage of female exposure at least as high as the percentage of female exposure statewide. In contrast, the New Jersey law makes an attempt to measure the individual risk's hazard by experience rating on a basis similar to that used in experience rating unemployment compensation although the

New Jersey Fund also initially collects 1% of taxable payroll from all risks insured by the fund. Under the New York law the State Fund as well as the private carriers may charge the rate required by the risk.

III. A POSSIBLE STATISTICAL PLAN FOR CASUALTY COMPANIES NEWLY ENTERING THE GROUP ACCIDENT AND HEALTH FIELD

For presentation purposes, this section of the paper has been divided into two parts: (First) a discussion of a possible method of producing policy year experience, and (Second) a discussion of a plan for producing calendar year experience. Both policy year and calendar year figures have proven to be very valuable in conducting the Group Accident and Health business although they have been used for essentially different purposes. The calendar year experience has ordinarily been used to watch current trends of a general nature, while the policy year experience has been utilized for more exacting and detailed research and analysis.

A. Policy Year Statistics:

It might be well to review first the various types of premium accounting generally used throughout the Group Industry, because the accuracy of the policy year exposure depends to a large extent upon the type of premium accounting used. The principal type of premium accounting used to date is sometimes called the "unit rate" method. This type of accounting seeks to obtain an accounting of each unit of exposure for each individual insured as exactly as may be practicable and produces an accurate earned premium when the unit premium rate is applied to the exposure. A second type is a variation upon the first type and might be called the "simplified unit rate" method. Here the exact exposure is figured only at the beginning of the insurance month, by the insured company rather than the carrier, and is assumed to remain constant throughout the month when computing the earned premium for the month. (A further possible variation is a method which makes an accounting of only the number of employees insured as of perhaps the beginning of the insurance month. Of course, such an accounting method is only possible in conjunction with a premium rate per employee.) A third method might be called the "taxable payroll" method, the taxable payroll being the Unemployment Compensation payroll (a payroll consisting of the first $3,000.00 of each employee's annual wages). In this case the earned premium is produced by applying a rate per $100.00 of "taxable payroll" to the taxable payroll of the risk, with the employer usually forwarding a copy of the Unemployment Compensation tax form to the carrier quarterly, so that the computation of earned premium can be verified. At present only a relatively small portion of the country wide Group Accident and Health premiums in force is on a taxable payroll basis. Recently an unknown volume of the premium written under the several existent State Disability laws was written on a taxable payroll basis. The only advantage of any importance demonstrated by the taxable payroll method of accounting is its simplicity. To more than counterbalance this feature are the many disadvantages and dangers which this system produces. It is not necessary to dwell on the possibilities of a situation where simultaneously the premium might decline and the exposure remain constant during a period of recession.

For policy year experience purposes, however, use of either of the first two premium accounting bases explained above, the "unit rate" and "simplified unit rate" methods, enables the computation of an almost exact exposure by simply dividing the earned premium of the policy year, before retroactive credits or dividends are deducted, by the annual premium rate per unit of exposure in force during the policy period. At present, computation of any sort of reliable standard exposure from earned premiums obtained from the taxable payroll method of premium accounting is decidedly experimental. This is because it is not known, at present, whether or not the wage distribution used in computing taxable payroll premium rates reflects the wage distribution of the business in force. Considerable study and research on this subject should be conducted and it is perhaps a proper subject for the attention of the Casualty Actuarial Society.

The first step in compiling the policy year experience for any plan of insurance is to produce the exposure for each risk separately by the above method and to transfer the resulting figure to a "unit report" card which shows the name of the insured, the policy number and the coverage, and also has spaces for year by year premiums, premium rates, percentages of female exposure, actual exposures and losses.

The common practice is to express exposures for the various Group Accident and Health sublines in the following units:

Subline	*Unit of exposure*
Weekly Indemnity	$10.00 of Weekly Indemnity
Employee Hospital Expense	$1.00 of Daily Benefit
Dependent Hospital Expense	$1.00 of Daily Benefit
Employee Surgical Expense	Basic Surgical Schedule (for example $150.00 maximum Surgical Schedule)
Dependent Surgical Expense	Basic Surgical Schedule (for example $150.00 maximum Surgical Schedule)
Employee Medical Expense	$3.00 Home or Hospital, $2.00 Office benefit for all except In-Hospital plans where the unit of exposure is $1.00 Daily Benefit
Dependent Medical Expense	$3.00 Home or Hospital, $2.00 Office benefit for all except In-Hospital plans where the unit of exposure is $1.00 Daily Benefit
Employee Laboratory and X-Ray Expense	The Employee
Dependent Laboratory and X-Ray Expense	The Employee.

The principal problem in compiling the policy year experience seems to be the accumulation of claim data. Depending on how much information is desired, and to what extent it is desirable to break down the losses, the necessary

claim information can either be recorded from closed claim files or from claim drafts. One large Group insurer has found it advantageous to keep the drafts as simple as possible and to record detailed claim information, such as is needed for policy year experience, from closed claim files.

If claim data was recorded from closed claim files, it is advantageous to record the following data, all of which is not strictly necessary for policy year experience but which is valuable for independent studies, such as continuation, average duration, and special premium rate studies, to name just a few:

Policy number
Claim identification number
Employee or Dependent coverage
Subline under which claim is paid
Sex of claimant
Age of claimant
State of principal employment of claimant
Policy month and year
Disability month and year
Waiting Period (days)
Limit of Indemnity Paying Period (weeks)
Limit of Hospital Benefit Period (days)
Limit Miscellaneous Benefits ($)
Limit Surgical Schedule ($)
Cause of Disability
Number of Days for which benefits paid
Amount of Payment.

Regardless of the source of the claim information for policy year experience, the claims for the mature policy year can be accumulated by risk separately for each subline. It is also recommended by the writer that a division of the claims to maternity and other than maternity disabilities be made for all sublines with maternity coverage and that hospital claims, in addition, be split to Daily Benefit and Miscellaneous Benefit payments. It is also advantageous, in the case of Dependent coverages, to split the losses to claims incurred by wives and by children. Such an accumulation of losses on an individual risk basis can be accomplished easily and economically by utilizing mechanical transfer-post machines which post the accumulated losses by policy year split as desired to individual risk loss cards. Also a risk loss card affords a ready reference from which to obtain losses for both prospective and retrospective experience rating purposes, if experience rating is to be accomplished prior to the maturity of the policy year, as is generally the practice in the Group Industry at present.

From the risk loss card, the incurred claims of each risk for the policy year under consideration for each subline (separated to maternity and other than maternity disabilities) can be posted to the unit reports.

The next step is to record the policy year data from the unit report on punch cards, so that the experience can be tabulated. At least the following

information ought to be recorded on the policy year experience punch card from the unit report, so that maximum value can be obtained from the experience:

Policy number

Subline

Waiting Period (days)—for Weekly Indemnity or Medical Expense Coverage only

Limit Indemnity Paying Period (weeks)—for Weekly Indemnity or some Medical Expense Coverages

Limit Hospital Benefit Paying Period (days)—for Hospital Expense or type (1) Medical Expense coverage

Miscellaneous Benefit Multiple—for Hospital Expense coverage only

Full or Ex-Maternity Coverage

Code to describe Surgical or Laboratory and X-Ray Schedule

Code to describe Maternity Coverage

Indemnity or Reimbursement basis of paying benefits

State

Units of Exposure

Percentage of female exposure

Incurred Losses—For Dependent coverages, the claims should be split to claims incurred by wives and by children; for Hospital Expense coverage the losses should be split to Daily Benefits and Miscellaneous Benefits

Cause of Disability Code—Other than maternity

Maternity

Policy month and year.

The next step is the tabulation, presentation, and utilization of this basic data. The principal and most important function of the policy year experience is that it provides a test of the adequacy of the pure premium underlying the current manual rates. Group Accident and Health basic premium rates are generally quoted for an all male risk (in practice an all male risk is considered to be any risk with less than 11% female exposure). For other than all male risks, the basic premium rate is roughly loaded by the percentage of female exposure to the total exposure of the risk. Therefore, an integral part of testing the pure premium underlying the manual premium rates is to check also the female loading practice. This female loading procedure assumes that the claim cost per unit of female exposure is twice that per unit of male exposure. By tabulating the actual exposures and losses by percentage of female exposure, an all male exposure for risks in each percentage of female exposure can be obtained by increasing the actual exposure by an amount equal to the ratio of female to total exposure times the actual exposure. In other words, an all male exposure for each percentage of female exposure can be obtained by doubling the female exposures. Thus, a pure premium can be obtained for risks falling within each percentage of female exposure on an actual and on an all male basis. If the assumption underlying the female loading procedure is correct, the all male pure premiums for risks falling within each percentage of

female exposure should be practically equal. In practice, rather than tabulate the experience for each percentage of female exposure, brackets of female exposure are used (0%, 1% but less than 11%, 11% but less than 21%, etc.).

If, on examination of the pure premiums on both an actual and all male exposure basis, the assumption underlying the female loading procedure (that the claim cost per unit of female exposure is twice that per unit of male exposure) is found to be incorrect, various statistical procedures can be utilized to obtain an all male claim cost reflecting a more accurate assumption as to female morbidity. For example fitting a "least squares" line or a second degree curve to the data are common methods of determining an all male pure premium. The decision as to which of these two methods would be used depends on whether or not female morbidity is assumed to be proportional to the percentage of female exposure.

Different assumptions, however, as to female morbidity rates must be made if the experience is that of a plan on an ex-maternity basis (female morbidity equal to 150% of male morbidity is a frequent assumption which at present has not been tested too thoroughly). In the case of Dependent coverage, the percentage of female exposure has, of course, no bearing on the pure premiums and as a result the pure premiums are shown on an actual basis.

It is common practice to express the pure premiums on a monthly rather than an anual basis for easy comparison with the manual premium rates which are almost universally shown on a monthly basis. Direct comparison with current manual rates can be obtained by simply loading the monthly pure premiums by the percentage of gross premium needed by each company for expenses.

B. Calendar Year Statistics:

The method of producing calendar year statistics outlined in this section is that of one of the large Group writing companies and is only one of the various methods in use in the industry at present. This carrier compiles its calendar year experience by producing for each Group Accident and Health subline a loss ratio on an "incurred losses–earned premium" basis. The incurred losses for the calendar period under consideration can be produced in several ways. One method which can be used incorporates the principle of the so-called "notice average" basis of determining claim reserves (described in Chapter IX page 251 of "Casualty Insurance Principles" by Thomas F. Tarbell). Another possible procedure for obtaining incurred losses is based upon the maintenance of an exhibit of the development, month by month, of the paid losses for each accident month. The paid losses for each accident month of the experience period being considered can be projected to an ultimate basis by using the experience of past calendar years. The valuation date should be at least three months subsequent to the last accident month of the experience period if accuracy is to be expected. It might be well to mention that the month by month development of accident-month paid losses for past calendar years can at times be misleading. The occurrence of epidemics, for example, can invalidate to some degree the indications of past experience in the development of losses, especially in regard to weekly indemnity experience. This and other variable factors should be observed and considered when projecting the claims to an ultimate basis.

The use of "Statement" earned premiums, however, poses a difficult problem. Because of "adjustment premiums" and premiums which for various reasons enter the accounting records late, statement earned premiums, produced by combining the written premium and the increase or decrease in the unearned premium reserve, are frequently not a measure of the actual exposure. One method of smoothing the earned premiums so that they are a reasonable measure of exposure is to assign all premiums, regardless of charge date, to the months during which the premiums were actually earned rather than assigning all expired premium to the months in which charged. The earned premiums produced by this type of procedure must be given sufficient time to mature to a reasonably ultimate basis.

Because drafts rather than closed claims are commonly used when producing this experience, a breakdown of the experience in more detail than has been described (a loss ratio for each Group Accident and Health subline) is not feasible. If, however, closed claims are utilized to compute incurred losses, a more detailed development of the experience could be attempted.

UNIFORM ACCOUNTING
A STUDY OF REGULATION

BY

DUDLEY M. PRUITT

So many speeches have been given and so many papers written in the past few years on the subject of Uniform Accounting that this writer suffers a sense of guilt for adding these notes to the din. A review, however, of the recent *Proceedings* of the Society reveals that this current hue and cry, loud as it has seemed to us in the market place, has failed to penetrate the sound proofed walls of our actuarial ivory tower. One's immediate reaction is, of course, that this is just as well, but sober judgment tells us that Uniform Accounting is a very significant current in the broadening river of regulation which had its rise in the S.E.U.A. decision, and even actuaries must keep abreast of the stream. This paper is, therefore, written primarily for the record, but since the author is a confirmed disputant, he cannot refrain from an occasional comment of his own.

REVELATION

It seems that those who live in a chaos seldom see the chaos. It takes a more sublime eye, more detached from these earthly efforts, to recognize the basic disorder by which we order our affairs. In reviewing the *Proceedings* of the Society and other related writings the author has been interested to find rather less criticism than expected of the way in which the insurance business conducted its accounting prior to the introduction in 1945 to the New York legislature of an insurance-department-sponsored uniform accounting bill. A Current Note in Vol. *VIII*, page 340, does state, "The question of the proper allocation of administrative expense by lines of insurance is one which heretofore has received too little attention in casualty insurance." This was back in 1921-22 and the implication was that the shortcoming was being corrected forthwith. Mr. R. S. Hull, the author of that note, wrote further in the same vein in his paper "The Allocation of Administrative Expense by Lines for Casualty Insurance Companies," appearing in Vol. *IX*, page 38. Mr. H. O. Van Tuyl in commenting on Mr. Hull's paper, Vol. *IX*, page 310, states, "Heretofore, in making rates the percentages of the premium estimated to be needed for expenses have been determined on the basis of very inadequate data." The tone of anticipated improvement seemed fully justified by the impending birth of the Casualty Experience Exhibit later reported on by Mr. Van Tuyl in Vol. *X*, page 17, "A New Experience Exhibit for Casualty Insurance Companies."

Seven years later, in Vol. *XVII*, page 41, "The Theory of the Distribution of the Expenses of Casualty Insurance," Mr. F. S. Perryman writes, "However, it cannot be said that the distributions of expenses of all companies even to

lines of business is entirely satisfactory. Too many rules of thumb and premium volume pro rates appear to be used by a good many companies—not all small ones—and we should not still have these at this stage of the development of casualty insurance."

Again another seven years later, Vol. *XXIV*, page 45, in "The Distribution of Casualty Administration Expense by Line Insurance," Messrs. Thomas F. Tarbell and Harry V. Waite, the authors, quite frankly inspired by Mr. Perryman's earlier paper, still say, "Too little importance has been attached to the equitable distribution by line of insurance for this classification (general administration expenses.)"

Mr. Sydney D. Pinney, in his presidential address made before the Society May 16th, 1941, Vol. *XXVII*, page 238, implied an overstatement when he said, "Without exception, each one who has previously written in our *Proceedings* on the subject of expense allocation for casualty insurance has recognized the need for improvement in the cost accounting methods followed by the companies." It must be admitted that the submission of a paper suggesting improved methods necessarily implies a recognition of the need for improvement, but relatively few papers viewed the subject with the alarm Mr. Pinney's tone suggests.

Of all the voices crying in the wilderness Mr. Perryman's seems to have been charged with the most prophetic quality. His paper referred to above, presented before the Society November 21, 1930, is a masterly analysis of the problems of expense allocation and has undoubtedly been used as a foundation on which has been built most of the constructive thinking (and some that is less constructive) on the subject to date. In rereading his words today one is struck with the same sense of vague familiarity experienced by a habitue of Tin Pan Alley in hearing a musical composition by an old master. Some of his phrases have been unduly syncopated, there has been perhaps too much vapid repetition of his more catchy tunes, but the inspiration is manifest.

The situation, then, prior to the enactment of uniform accounting legislation was one of considerable flexibility, which weakened the prestige of the rate maker and complicated the problems of state supervision of rates.

LEGISLATION

In 1945 the New York Insurance Department sponsored a bill before the State Legislature amending the insurance law to give the Superintendent of Insurance power "to prescribe uniform methods of keeping accounts, statistical data, records and books to be observed by insurers——. He may also—— prescribe, by regulation, forms of accounts, records and memoranda to be kept——." In public hearing this bill was urged by the Superintendent because "the insurance business says there is need of uniform accounting and has been working on it for twenty-two years but still hasn't got the answer." In view of this twenty-two years that have elapsed "the day of voluntary effort is over."

This bill was vigorously opposed by the companies on the stated grounds that the companies were making great progress toward uniformity of allocation on a voluntary basis, that other states might prescribe uniformity that was not uniform with New York, and most especially that supervision of and prescribed uniformity in the methods of bookkeeping were most unnecessary

and undesirable. At the hearing the suggestion was made by Richard Wagner Esq., legislative counsel for the Association of Casualty and Surety Executives, that this third objection could be eliminated by substituting "uniform *classification* of accounts, statistical data and records," thus giving the superintendent power to prescribe the substance but not the form. Of interest is the reporter's comment on this point made in the National Underwriter for March 1, 1945: "Evidently Mr. Dineen did not regard this as a satisfactory means of accomplishing his aims, for he made no reference to it later when he rose to speak in defense of the bill."

Mr. Tarbell in 1929 and again in 1941 had written in his paper "Casualty Insurance Accounting and the Annual Statement Blank," *Proceedings* Vol. *XV*, page 141 and Vol. *XXVII*, page 294, "Methods of casualty insurance accounting are not and probably never will become standardized. Opinions differ as to the most efficient methods of compiling accounting data." In March of 1945 Mr. Tarbell's qualifications as a prophet seemed to be something less than perfect.

The opposition of the companies, however, was successful and the measure was killed in committee.

It would seem that Mr. Dineen was more impressed with Mr. Wagner's suggested modification of the bill than the National Underwriter's reporter had detected, for in 1946 he again sponsored a bill, but this time the wording was that "the superintendent shall have power———to prescribe uniform classifications of accounts to be observed, and statistics to be reported." This revised bill received no opposition from the companies, was passed by the Legislature March 19, and signed by Governor Dewey March 28, 1946.

Whether or not a law providing only for "uniform classifications of accounts ———and statistics" can rightly be called a uniform accounting law, that name, which grew out of the implications of the original bill, has prevailed. The insurance superintendent does not have power to prescribe uniform *methods* of account and Mr. Tarbell may be reinstated as a prophet for the industry.

At this point the fire insurance industry began to grow increasingly aware of the implications of this uniform accounting law. Possibly casualty men showed less concern because of their assurance that their accounting was already fairly satisfactory, the prophetic utterances herein cited to the contrary notwithstanding. But Superintendent Dineen had been rather blunt about certain fire insurance rate making deficiencies and had already begun an investigation of fire insurance practices in expense allocation, financed by special appropriations obtained from the Legislature.

Early in March, even before the bill was passed, the National Board of Fire Underwriters approached Commissioner of Insurance James M. McCormack, Jr. of Tennessee, as president of the National Association of Insurance Commissioners, with a view to promoting the consideration of uniform rules for expense allocation on a nationwide basis. Much fear was being currently expressed that with New York prescribing uniform rules other states might exercise their constitutional sovereignty by prescribing other uniform rules. Thus the industry might be required to conform to uniform rules made intolerable by a lack of uniformity. The subject was discussed by the committee on laws and legislation of the National Association of Insurance Commissioners in June 1946, with Mr. Dineen asking the committee to take no action for at least a

year in order to enable his department to study the matter further and eliminate any "bugs." No action was taken. Subsequently the Fire and Marine Committee of the National Association of Insurance Commissioners appointed a Sub-Committee on Expense which named the New York Department as its agent in the expense distribution investigation of fire insurance companies still going on. Thus the National Association of Insurance Commissioners sought to avoid the calamity envisaged.

On April 17th, 1946 Superintendent Dineen addressed the (Fire) Insurance Accountants Association on the subject, "The Development of Uniform Classifications of Accounts." In this address he outlined seven main objectives the Insurance Department hoped to accomplish by the law. It was the Department's hope, said Mr. Dineen, that the new law would

(1) Obviate the criticism that it is illogical for companies to charge uniform rates made in concert when little uniformity is employed in compiling the figures upon which the uniform rates are based;

(2) Provide a more scientific basis for the making of rates and for the regulation of rates;

(3) Aid the management of competing companies to compare both aggregate costs and the components of such costs;

(4) Enable companies to do equity among all classes in the distribution of expenses as regards such considerations as expense constants and discounts for size;

(5) Furnish superior statistical data demonstrating that price differentials are not unfair price discriminations under the Federal Trade Commission Act and the Robinson-Patman Act;

(6) Convince Congress that the performance of the states in this field measures up to the congressional policy established in the Interstate Commerce Act and Federal Power Act;

(7) Aid in convincing the public that insurance rates are fair and equitable.

It is interesting to note that five of the seven objectives given deal specifically with rate making, while the implications in item six also are primarily in the rate making field. Only item three has no rate making considerations and is, by its nature, an unsolicited by-product which could hardly be made the objective of regulatory legislation. Again, in his concluding paragraphs, Mr. Dineen repeated, "This plan should be a tool designed to improve rate making and rate regulation." In speeches and testimony prior to the passage of the bill, also, Mr. Dineen had made it clear that improvement in rate making and rate regulation was his objective. Nowhere does one find, at this time, any thought that the uniform accounting regulations would be of assistance in insurance department supervision of the financial stability and performance of companies. The aims were actuarial rather than fiscal.

INVESTIGATION

The New York Department's investigation of fire insurance accounting practices culminated in a "Preliminary Report on the Uniform Accounting Investigation," dated November 27, 1946, copies of which were made available

to the public. The Department then turned its attention to the casualty companies, publishing in December, 1947, a second report entitled "Methods of Expense Distribution in Casualty Insurance Companies." Both these reports present analyses and tabulations of company practice in expense allocation with the conclusion fairly obvious that companies have not been uniform. Upon study however, the variation became less pronounced, since usually most companies were uniform in principle, the variations being confined largely to a few companies in each instance. Much of the apparent variation sprang from true functional differences among companies which would appear as differences in allocation because the long established classifications as set up in the annual statement were a mixture of expenses by "nature" and expenses by "purpose," using Mr. Perryman's terms. To some extent the variations resulted also from differing philosophies, with some companies placing "purpose" ahead of "nature" and some "nature" ahead of "purpose." The philosophy, however, that prompted one company to allocate a part of the rent of post office boxes to salaries was novel at least.

In the casualty report, but not in the fire report, tabulations were made of company practice in distributing expenses by lines of business. This was the type of allocation most frequently found wanting by the various contributors in the *Proceedings* from which quotations were given above. Here the report shows a rather persistent leaning toward allocation by premium volume, which of course in its rate making implications is a Procrustean treatment. The ultimate result of such a practice, if persisted in, would be to produce uniform indicated expense loadings for all lines, regardless of actual differentials. Ten of the sixty companies reporting used premium volume to allocate either in whole or in part the salaries of loss department employees, and three used it for allocating the (already) allocated loss adjusting expenses. Had the allocation of losses been treated in this report one might be led to wonder, from the evidence at hand, whether some companies would not have reported that allocation also by premium volume.

REGULATION

On March 1, 1948, the New York Insurance Department issued its first concrete proposals in the "Preliminary Draft of Operating Expense Classifications," and followed that very promptly on April 26th, 1948, with the "Preliminary Draft of the Allocation of Expenses to Companies, Expense Groups and Lines of Business."

Back in 1946, when the first preliminary report on the uniform accounting investigation was made, Shelby Cullom Davis, the then Deputy Superintendent of the New York Department, wrote in its introduction, "Since the Department is acting as agent in doing this work for the Sub-Committee on Expense of the Fire and Marine Committee (of the N.A.I.C.) any recommendations will undoubtedly come from the latter body after adoption, it is hoped and expected, by all the Insurance Commissioners." By 1948 the N.A.I.C. had dismissed the Sub-Committee on Expense and had set up in its stead a Uniform Accounting Committee.These preliminary drafts of March 1 and April 26, 1948 were "prepared by the New York Insurance Department in consultation with the Uniform Accounting Committee of the National Association of Insurance Commissioners."

On May 17, 1948, the Uniform Accounting Committee of the N.A.I.C. held a hearing in the New York Insurance Department offices in New York City. Since this was barely three weeks after the second and more controversial draft had been released, the general cry was for more time. A few of the more radical innovations were questioned by certain speakers but most of the industry representatives who spoke expressed their inability to do the subject justice because they had been unable to give it adequate study. Commissioner J. P. Gibbs of Texas, Chairman of the Committee, adjourned the hearing after asking that the industry have concrete proposals in hand for presentation before the committee another three weeks later when it should meet on June 7, in Philadelphia during the N.A.I.C. convention. Industry representatives were not at all sanguine about their ability even then to make an appropriate presentation, and so expressed themselves.

The uniform accounting provision of the New York insurance law states in part, "Any regulation or amendment thereto shall be promulgated by the superintendent at least six months before the beginning of the calendar year in which the same shall take effect." On June 1, 1948, Robert E. Dineen, Superintendent of Insurance for the State of New York, addressed a notice "to all Fire and Marine and Casualty and Surety Insurers authorized to do business in the State of New York" to the effect that a hearing would be held in New York City "on June 22nd, 1948 at 10 a.m. to consider the adoption of a regulation, effective January 1, 1949 relating to uniform classifications of expenses." A copy of the proposed regulation, designated as No. 30, was attached to the notice. The notice also stated that "if it is your intention to offer objections—it is required that you—submit—on or before June 17, 1948 a memorandum in writing, specifying in detail what your objections are and definite reasons therefor." June 17 then was the real deadline.

Thus the two "preliminary drafts" became the "proposed regulation" of the New York Department.

On June 7 the Uniform Accounting Committee of the N.A.I.C. met. As requested by Mr. Gibbs on May 17 a committee representing nearly all branches of the industry had done some feverish conferring and now presented a rather extensive report. Our prophets of the past, as might be expected, had now become the spokesmen of the present. The report was written and signed by Mr. Perryman as chairman of the industry committee but, due to his absence, was presented to the commissioners committee by Mr. Tarbell. The position taken by the industry report was that insufficient time had been given for an adequate study and testing of the regulations, that everyone was in favor of uniformity, proper uniformity, that the basic approach of the New York Department was sound, traditional, and to be expected, that such study as had been made revealed some rather serious weaknesses, which were presented as an indication of the need for further study and certainly not as a complete list of shortcomings, and that insufficient time had been given——.

The Uniform Accounting Committee of the N.A.I.C. listened very politely and took no action other than to endorse in principle the proposals made by the New York Department and to request the convention to continue the committee in order that it might give the problem further study and make a report at the December meeting.

There followed in the next two weeks some very intensive conferences between the industry committee and the New York department with a view to reconciling differences on some of the features of the proposed Regulation No. 30 that were regarded by the industry committee as objectionable.

On June 22, the New York Department held its formal hearing with all testimony taken under oath in order to provide the proper legal foundation in the event the matter were made the subject of "judicial review by any insurer or organization aggrieved thereby," as provided in the law. Material was formally placed in evidence, protagonists and antagonists spoke their pieces like well rehearsed actors and as the National Underwriter of June 24 reported, "Deputy Martineau adjourned the hearing after everyone had his say." Only Mr. Conick, speaking for the National Board of Fire Underwriters, stirred the embers when he quoted from Mr. Dineen's speech of April 17, 1946, before the Insurance Accountants Association, "But this we emphasize. If this program is to be a success, it cannot be the product of the companies alone or of the New York Department or even the joint product of both. The project is national in its scope. It calls for the cooperation of insurers throughout the length and breadth of the land in collaboration with the National Association of Insurance Commissioners."

On June 30, the Department issued its now famous Regulation No. 30 giving the force of law to the proposals with minor changes, proposals which the industry had seen for the first time on March 1 and April 26.

In December the National Association of Insurance Commissioners adopted the New York uniform accounting rules (Regulation No. 30) as instructions for completing the expense portions of the Annual Statement and the Insurance Expense Exhibit for companies doing business in New York and subject to Regulation No. 30, as to 1949 operations, and for other companies optionally as to 1949 but on a mandatory basis as to 1950.

Little opposition was expressed in public by the commissioners to this program, except for Mr. Harrington, Commissioner of Insurance for Massachusetts, who spoke rather sharply against the resolution on the not unreasonable ground that the annual statement, which is fundamentally a financial statement and for that reason subject to an early filing date, should not be burdened with requirements for rate making data. Mr. Harrington, apparently, still thought of the uniform accounting movement as actuarial in purpose and not, as some seemed to be considering it, a virtue having its own reward. Mr. Harrington quite possibly was recalling the June, 1946 meeting of the commissioners where he was one of the leaders in urging consideration for the gathering of uniform data as an adjunct to the new rating laws of the various states. At that time he praised the direction being taken by New York.

The next official step was the creation by the New York Insurance Department effective April 1, 1949, of a Uniform Accounting Bureau and the appointment of James J. Higgins as Chief of the Bureau. As stated in the Journal of Commerce, March 31, "Mr. Higgins was directly responsible for the basic theory, construction and actual wording of Regulation No. 30." Mr. Higgins was also appointed chairman of the Sub-Committee on Uniform Accounting Instructions of the Uniform Accounting Committee of the National Association of Insurance Commissioners. Thus on both the New York level and the

national level the machinery is now set for orderly processes of interpretation and revision with some assurance of coordination between the two levels. Mr. Morrill, Deputy Superintendent of the New York Department, in an address before the Insurance Accounting and Statistical Association, May 20, 1949, said in commenting on this arrangement, "Although in New York our rules have the force of law, and can be amended only through a procedure prescribed by statute, we will adapt that procedure so as to follow the actions of the N.A.I.C. rather than attempt to lead the procession."

One is led to wonder, however, whether the N.A.I.C. can contribute any actions for New York to follow in view of the wording of the first rule of procedure adopted June 26, 1949 by the Sub-Committee which reads, "All proposed amendments and revisions shall be filed with the Chairman of the Sub-Committee on or before January 1 of each year." Since the "before" cannot exist, it would seem that New Year's day will be a busy day for some.

DISPUTATION

It is not the purpose of this paper to discuss the regulations in detail. Such scope should be reserved for other authors and other papers. This record, however, would be incomplete without some report of the more important points of controversy. The New York Department accepted many of the minor suggestions made by the industry and incorporated them in its final official regulation. One such amendment is noteworthy in that the original proposal of the insurance department revealed a certain quality of supervisory cynicism which, it is to be hoped, is not deserved by the industry. In the original proposals the item "Donations to Organized Charities" was required to be included in the "Advertising" account. At the suggestion of the industry it was transferred to "Miscellaneous."

The most serious disagreement, and the area in which the department revealed the least flexibility, was in connection with the selection and the composition of expense groups.

Addition. It was felt that the definition of the expense group "Acquisition, Field Supervision and Collection Expenses" covered too much territory. As Mr. Perryman put it, "something has been added," the something being collection expenses. The old Casualty Expense Exhibit grouped only acquisition and field supervision expenses. The aim of the Insurance Department in enlarging the area was to create a group that would be susceptible of uniform treatment for all companies regardless of method of doing business, a least common denominator fitting stock, mutual, reciprocal, casualty, surety, fire or marine. Since the largest single denominator was considered to be the commission paid to the general agent which not only covered the functions of acquisition and field supervision, but also those of policy writing and collection, it was felt that such costs, even when borne directly by the companies should be included in the same group as commissions. From the point of view of functional uniformity there is little fault to find with the Department's point of view. The regulation however is not entirely consistent, since one of the items included under this group reads:

> "Miscellaneous activities of agents, brokers and producers *other than employees*, when performed by them; inspections; quoting premiums;

signing policies; examining and mailing policies; applications and daily reports; compiling figures for current accounts; correspondence and sundry bookkeeping and clerical work."

The words "other than employees," which I have italicized, seem to do violence to the cause of functional uniformity.

One of the rules of life is that after regulation must come interpretation. I quote from a bulletin released by the Insurance Accounting and Statistical Association October 1, 1949, which purported to have the approval of Mr. Higgins, chief of the New York Insurance Department Uniform Accounting Bureau:

> "As a result of these differences in methods of operation and the impracticability of setting down rules for segregating like expenses in the various companies into the same expense groups, New York's Uniform Accounting Section is now using the basis of whether the operation is paid for by commission or salary to distinguish between operations to be charged to Acquisition and General. There is one exception to this rule, however, and that is those operations outlined in the Regulation as definitely chargeable to Acquisition such as premium collection, commission payments, etc. These operations are considered as Acquisition no matter where performed or how paid for."

The difficulty lies, of course, in the fact that there is practically no limit to the functions which are performed by agents and paid for by commission. The regulation has attempted to provide for this difficulty by directing that commission paid for services other than those listed, "when such services are not duplicated or otherwise compensated by the company," shall be allocated to other expense groups. But, as noted, this attempt seems, by the nature of things doomed to failure.

Mr. Perryman, in one of the hearings, expressed doubt as to the wisdom of labelling under "Acquisition" so much that is clearly administrative in nature. That way aggravates public misunderstanding and criticism.

It has been suggested that another approach could be made to the problem by adding the concept of functional purity to that of functional uniformity. Admittedly commissions are a mixture of apples and pears and it would be in error to add unmixed apples to such a mixture. The regulation has attempted to correct the error by requiring that along with the apples when added some pears must be added also in uncertain proportion. The suggestion is that we might be able to separate the apples from the pears and add apples to apples, pears to pears.

Mr. Perryman's brief stated,

> "True Acquisition and Field Supervision Cost in the case of a stock company consists of two elements;
>
> "1. The amount of sales cost paid to the producer,
>
> "2. The securing, developing and supervising of the sales force."

The qualification, "in the case of a stock company," might well be omitted. If uniform accounting is to be primarily the servant of rate-making, and

that has been its principal justification, functional purity in the matter of acquisition expense would seem desirable.

The National Association of Insurance Agents asked specifically that commissions be analysed and only the amounts paid for the direct production of business, in other words, item 1 above, be reported as commission. This position was motivated largely by a consideration of public relations and the feeling that the producer was being blamed by the public for a larger "take" than he was in fact taking for his sales efforts. Commenting on this request by the N.A.I.A. Mr. Thomas C. Morrill,Deputy Superintendent of the New York Insurance Department, said in an address before the Insurance Accountants Association, November 9, 1948, that it is "not without merit, at least in principle." It was "not made a part of Regulation No. 30 because our direct knowledge of insurance accounting limitations led us to believe that to require a more refined functional allocation in the annual statement, with its early filing date, was impractical, and that to insist on it would only force the industry into the very rule-of-thumb guesswork that we are trying to eliminate." The accountants of the industry, are naturally very thankful to the framers of the regulation that they do not have to make a more refined functional allocation of commissions. It might be noted at this point that the annual statement blank for the year ending December 31, 1949 as revised to integrate with Regulation No. 30, and required to be filed in most jurisdictions on or before March 1, 1950, does not require the functional separation of the expense group "Acquisition, etc." from "General," but groups these two along with "Taxes, Licenses and Fees" in one grouping of "Underwriting Expenses," the separate display of these groups being required only in the Insurance Expense Exhibit to be filed not later than May 15, 1950.

Subtraction. If the Department went too far in the definition of acquisition expense it did not go far enough in certain other directions, according to the casualty spokesmen. The regulation made no provision for segregating as expense groups the functions of inspection and of payroll audit, (or exposure audit, the new and more accurately descriptive name.) Segregation of the expense of these two functions is necessary for the making of not unfairly discriminatory rates. These expenses vary materially as to the lines and sublines of the casualty business, and also as to the size of individual risks.

The New York Department's position was that these functions represent relatively such small amounts of money in the aggregate that separate treatment is not warranted, that to define and segregate these functions would be difficult and impractical in many lines of insurance, and that it was unwilling to impose on all companies this item of cost analysis, with the force of law, but rather that whatever data is needed for rate making may be obtained by special call to the companies. Here one is led to wonder if the Department remembered the original rate making purpose of the uniform accounting movement. Some compromise was obtained by the rate makers in that the Insurance Expense Exhibit as finally adopted for the year ending December 31, 1949 provides a break-down of General Expenses into

Inspection expenses paid

Boards, bureaus and associations

Payroll audit expenses paid
Other general expenses paid.

A footnote, however, calls attention to the fact that "the allocations to 'inspection' and 'payroll audit' are not subject to Instructions for Uniform Classification of Expenses." In the light of this lack of authority in an authoritarian realm one can only conjecture with what ease supervising authority may challenge any new rate making departures predicated on such findings.

SUMMATION

It is perhaps too early to appraise the effect of uniform accounting regulations. The first full year of operation under them will not have been completed till December 31, 1949 and reports on that year will not be made public and consolidated till some months thereafter. But we have had now over a year to study the regulations as promulgated and the better part of a year of actual use in company offices. We have had an opportunity also to watch the attitude of regulatory authority develop during this period.

Although there has been considerable grumbling, companies have in general accepted uniform acounting as a part of the present day scene. Insurance accountants have, in fact, used it as a great diversion, the subject of endless papers, forums, and conferences, and a most welcome justification to senior executives for delayed reports and additional personnel. Companies are making an honest effort to understand and conform to the rules. Because of imperfections in the regulations certain amusing situations have arisen. The regulations, for example, have no specific account for "Insurance," on the assumption perhaps that insurance is one thing insurance companies do not need. Unfortunately for accounting sanity, insurance companies do buy insurance on occasion. The Committee on Interpretation of Regulation 30 of the Association of Casualty and Surety Accountants and Statisticians was recently asked,

> "Where should insurance premiums paid by a company on excess insurance over the insurance company's blanket bond be charged, such excess insurance to cover the company's own securities in its safe deposit box?"

The answer, which had to be given under the principle of analogous items provided for in the regulation, follows:

> "Since the coverage is for its own securities, the premium on the excess coverage increases the cost of the safe deposit box rental, and therefore, should be charged to Account 13—Rent and Rent Items."

Ridiculous as it may seem, this answer is probably the best that can be given. The regulation treats insurance as a substitute or *ersatz* commodity, one that is purchased in place of something better and must be so classified. The insurance industry would like to believe that insurance performs a function of its own and is worthy of independent classification and accounting.

Another rather interesting development in interpretation might well be noted here. As pointed out earlier in this paper, premium volume pro rates have been viewed in general with considerable suspicion by our actuarial prophets. The Uniform Accounting Regulation treats pro rating by premium volume largely as a method of last resort after all other methods have been

examined and found wanting. We have spoken of the Procrustean or leveling effect of such practices. Nevertheless the bulletin of the I.A.S.A., previously quoted, which purports to have Mr. Higgins' approval, states:

> "The New York Department considered the use of premiums as an allocation basis for the secondary lines as producing a more favorable result when all factors are considered.
> "To use an item count basis of allocating expenses to secondary lines would increase the expense rate of small premium lines such as Auto, Fire and Theft. This could have the effect of causing premium increases for these lines which might affect their saleability to the insuring public. The New York Uniform Accounting group did not want to do anything which would result in harm to the insurance business, a reason primarily responsible for their advocating premium volume as a basis of allocation."

Whether or not one agrees with the end sought, the actuarial reasoning expressed is interesting.

For some companies Regulation No. 30 has given incentive to experiment a little in cost analysis beyond its minimum requirements. If a case has been rightly made against the industry as a whole for failure to analyse its costs satisfactorily for rate making purposes, a case can be made for a like failure to analyse for management purposes. Judged by the standards of the manufacturing industries, insurance management has operated and made decisions in comparative ignorance. Aside from the analysis of premiums and losses, which has been very well developed, there has been no real science of cost accounting. A great deal of study has been made to relate losses to their proper exposures, but little has been done to discover the reasonable and proper measures of a company's exposure to expense. Although such a science is bound to develop under present day conditions, there can be little doubt that Regulation No. 30 is giving it rapid impetus, if for no other reason than that, having been forced by regulation to lay an elaborate foundation, companies can erect a useable superstructure for themselves at such slight extra cost.

From the point of view of rate making some small benefits of uniformity are beginning to accrue. In the study of costs by size of risk it is now firmly recognized that the homogeneity of data sampled from various companies under study is fairly assured because of the uniform accounting regulation, and the industry committee now doing that study has officially made its bow in this direction. It is reasonable also to assume that more facts will be available in the future in support of, and at times in opposition to, expense loadings as utilized in submitted rate filings and deviations. Hereafter we will know the facts, we will have all the peas counted that are in the pod, we will have the apples separated from the pears. Then what?

Why did Mr. Dineen feel constrained to say in his speech before the Insurance Accountants Association on April 17, 1946, "You may rest assured that the New York Department will resolutely resist any tendency to use this law to throttle initiative and enterprise in the insurance business?" Why should Mr. Malone, Insurance Commissioner of Pennsylvania and Chairman of the N.A.I.C. Uniform Accounting Committee, feel the need to state in his address before the National Association of Independent Insurers on November 18, 1948, "I think careful thought will reveal that far from promoting uniformity in

rate making, the great improvement in expense data which we have a right to expect under the new system will stimulate rate competition, based on a new confidence in expense differences revealed in operating statements?" Is it not because the danger they protest against is very real? No matter how honestly and conscientiously administered, a controlling force with regulatory authority is to play it safe. Variation means insecurity, high mountains and steep valleys are hazardous. Regulation, like erosion, exalts the valleys and lays the mountain low. Uniform accounting is a potent tool of regulation and by its name and nature it is uniform and makes for uniformity.

Mr. Morrill, Deputy Superintendent of the New York Department, is quoted by the Journal of Commerce for January 20, 1949 as saying, "Uniform Accounting is progressive, and —— will produce substantial benefits in the regulation and management of insurance carriers." You will note that Mr. Morrill does not speak of the regulation of rates but of carriers. There is a substantial difference. And does he imply that regulation itself is progressive? We hope not, for in a thoroughly regulated society insurance fills no need and disappears.

ON NON-LINEAR RETROSPECTIVE RATING

BY

CHARLES W. CROUSE

Introduction. A retrospective rating agreement is an agreement providing that the premium u ultimately to be considered as earned for a certain insurance or set of insurances, shall be determined as a function of the losses actually incurred by the insurer on account thereof, valued (in accordance with a set of prescribed rules) *after* the events causing such losses have occurred.

All of the plans for the making of such agreements which have ever been explicitly approved by any of the insurance commissioners in the United States, are plans under which u becomes a continuous function of the sum of the losses so valued, which is linear within some interval and constant outside that interval. The range of practicable possibilities under the condition that u shall be such a linear or sectionally linear function of the sum of the actual values of the losses, has been very well explored by T. O. Carlson [4] and F. S. Perryman [10].*

Now, the following question naturally arises. *What practicable retrospective rating possibilities (if any) lie beyond the domain governed by that condition?* In seeking an answer to this question, it is necessary first to determine precisely what we mean by the term "practicable retrospective rating possibility." Section B of this paper is therefore devoted to a formulation of the conditions which it is either necessary or desirable that any particular functional relationship shall satisfy in order that it may be employed in practice as a formula for the determination of u. In Section C, continuous functions linear in each interval of a set consisting of any finite number of contiguous intervals covering the entire range of possible values of the sum of incurred losses, are examined as possible retrospective rating formulae and found to leave unsatisfied at least one of the conditions which it is desirable that such formulae should fulfill. In the first paragraph of Section D, a class of non-linear functions is defined, and it is asserted that the members of that class are fundamentally better adapted to serve as retrospective rating formulae than those belonging to the sectionally linear class; and the remainder of the paper is devoted to the defense of that thesis.

* Bold face numerals in square brackets refer to the Bibliography on page 62.

A. Terminology and Notation

Let x be a one-dimentional random variable, i.e., a random variable of which each possible value is a single real number. If, for every real number x, the probability that x will not exceed x (i.e., the probability of the event: $\mathbf{x} \leqq x$) is given as the value of a real function $F(x)$, then $F(x)$ will be called the *distribution function* of x and may be regarded as defining the *probability distribution* of x. Such a function is necessarily single-valued and non-decreasing. Moreover, $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to +\infty} F(x) = 1$.

Hence the Stieltjes integral, $$\int_{-\infty}^{+\infty} dF(x) = 1. \tag{1}$$

Let $\mathbf{z} = \psi(\mathbf{x})$ represent any function of x. Then the *mathematical expectation* or *mean value* of z, which will be denoted by the symbol $\mathbf{E}(\mathbf{z})$, is defined by the equation: $$\mathbf{E}(\mathbf{z}) = \int_{-\infty}^{+\infty} \psi(x) dF(x) \tag{2}$$

and will be said to exist if the Lebesgue-Stieltjes integral on the right exists and has a finite value. If $\mathbf{z} = (\mathbf{x} - c)^k$ where k is a positive integer, then $\mathbf{E}(\mathbf{z})$ is the kth *moment of the distribution* of x *about the point* c. Obviously, for $k = 1$ and $c = 0$, $\mathbf{E}(\mathbf{z})$ becomes $\mathbf{E}(\mathbf{x})$, the mean value of x. The moments about $\mathbf{E}(\mathbf{x})$, often called the *central moments*, will be denoted by the customary symbol, $\mu_k(\mathbf{x})$.

Thus: $$\mu_k(\mathbf{x}) = \mathbf{E}\left[\{\mathbf{x} - \mathbf{E}(\mathbf{x})\}^k\right] = \int_{-\infty}^{+\infty} \{x - \mathbf{E}(\mathbf{x})\}^k dF(x) \tag{3}$$

If $F(x)$ is continuous in the interval $(-\infty, +\infty)$ and has a continuous differential coefficient, $\mathrm{F}'(x) = f(x)$, at every point of that interval, with the admissible exception of points of which any finite interval contains at most a finite number, then the probability distribution defined by $F(x)$ will be said to be of the *continuous type*, and $f(x)$ will be called the *frequency function* of x.

For distributions of the continuous type, $F(x) = \int_{-\infty}^{x} f(x)dx$ for every real value of x, and the Lebesgue-Stieltjes integral in equation (2) is equiv-

alent to an ordinary Riemann integral, $$\int_{-\infty}^{+\infty} \psi(x)f(x)dx = \mathbf{E}(\mathbf{z}), \qquad (2')$$

provided $\psi(\mathbf{x})$ is almost everywhere * continuous and the integral on the left side of (2') is absolutely convergent.

Each random variable with which we shall have occasion to deal, will be represented by a lower-case letter in bold-face Roman type, and the same letter in italic type of ordinary face will be used as the variable in its distribution function and its frequency function. Regardless of what random variable it is to which we are referring, the letter F will be used to denote its distribution function and the letter f to denote its frequency function. Thus $F(x)$ will mean the distribution function of x; $F(y)$ will mean the distribution function of y; and these may be two quite different functions, i.e., $x = c$ and $y = c$ will *not* imply that $F(x) = F(y)$. This departure from the usual convention under which $F(y)$ means the value of $F(x)$ when x has the value y, should not be confusing, and it will avoid the introduction of a number of different letters, or of subscripts or other distinguishing marks upon the basic letters F and f.†

Now let J be any set of one or several insurances, and let $\mathbf{n}$ be the number of events (hereinafter called *loss-events*) each of which results in some loss of positive value being incurred by the insurer under one or several of the contracts belonging to J. From a purely theoretical viewpoint, it seems unnecessary to make any assumptions about J. It may include Workmen's Compensation insurances, liability insurances of one or several kinds, group life or group disability insurances, and even property insurances of certain forms. However, there are a great many sets of insurances to which it would be impracticable to apply a retrospective rating agreement; we assume that J is not one of them.

Let the $\mathbf{n}$ loss-events be simply ordered; let $\mathbf{a}_k$ be the actual value of the loss to the insurer resulting from the kth loss-event; and let

$$\mathbf{s} = \mathbf{a}_1 + \mathbf{a}_2 + \ldots\ldots + \mathbf{a}_n. \qquad (4)$$

Now, suppose that $\mathbf{u}$, the premium ultimately to be considered earned for all of the insurances included in J, is to be determined in accordance with a retrospective rating agreement as a function of $\mathbf{v}_1, \mathbf{v}_2, , , \mathbf{v}_n$, where for each positive integer $k \leq \mathbf{n}$, $\mathbf{v}_k$ is the value placed upon the loss resulting from the

* I.e., with the admissible exception of points forming a set of Lebesgue measure zero.

† A very beautiful exposition of random variables and probability distributions will be found in Cramér [5, Second Part, or 6].

kth loss-event in accordance with some prescribed set of rules. The possibility that $\mathbf{v}_k = \mathbf{a}_k$ for some or all values of k, is — of course — not excluded.

Under any of the approved plans for the making of retrospective rating agreements, the functional relationship which $\mathbf{u}$ would bear to $\mathbf{v}_1, \mathbf{v}_2, , , \mathbf{v}_n$, would be of the following form:

$$\mathbf{u} = \begin{cases} H & \text{if } \mathbf{t} \leq h \\ B + C\mathbf{t} & \text{if } h \leq \mathbf{t} \leq g \\ G & \text{if } \mathbf{t} \geq g \end{cases} \tag{5}$$

where $\mathbf{t} = \mathbf{v}_1 + \mathbf{v}_2 + \ldots\ldots + \mathbf{v}_n$; $\quad H = B + Ch$; $\quad G = B + Cg$; and B, C, h and g are parameters independent of $\mathbf{n}$, $\mathbf{a}_k$, $\mathbf{v}_k$ and every other characteristic or consequence of any particular loss-event. H is usually called the *Minimum Retrospective Premium*; G, the *Maximum Retrospective Premium*; B, the *Basic Premium*; and C, the *Loss Conversion Factor*; though in most of the plans at present in use these two latter terms refer respectively to B/τ and C/τ, where τ is a factor dependent solely upon the ratio to $\mathbf{u}$ of the sum of all taxes which will be levied directly upon $\mathbf{u}$ or upon some part of $\mathbf{u}$.

The range of possible values of H, G, h, g, B and C, and the determination of any three of them for any set of insurances when the remaining three are given, have been thoroughly discussed by Carlson and Perryman in the papers, [4] and [10], to which we have previously referred. Though, for each particular set of insurances, the class of sectionally linear functions defined by (5) includes a three-fold infinitude of members each of which represents a practicable retrospective rating possbiility, it is nevertheless a very restricted class of functions.

Therefore, let us suppose that $\mathbf{u}$ is to be some single-valued function of $\mathbf{t}$, say $\mathbf{u} = R(\mathbf{t})$, where $R(\mathbf{t})$ is not necessarily of the form (5), but

$$\mathbf{t} = \mathbf{v}_1 + \mathbf{v}_2 + \ldots\ldots + \mathbf{v}_n \tag{6}$$

and every parameter or variable other than $\mathbf{t}$ which may be involved in the determination of $\mathbf{u}$, is—as in (5)—independent of $\mathbf{n}$, $\mathbf{a}_k$, $\mathbf{v}_k$, and every other characteristic or consequence of any particular loss-event. We have then to inquire: *What conditions must be imposed upon the function* $R(\mathbf{t})$ *in order that* $\mathbf{u} = R(\mathbf{t})$ *shall be a practicable retrospective rating formula*?

These conditions are in general dependent upon the set of rules which have been prescribed for the determination of the values, $\mathbf{v}_1, \mathbf{v}_2, , , \mathbf{v}_n$. At the outset of our inquiry we shall assume those rules to be of the following form.

$$\text{For every } k\text{: } \mathbf{v}_k = \mathbf{a}_k \text{ if } \mathbf{a}_k < \lambda_k; \qquad \mathbf{v}_k = \lambda_k \text{ if } \mathbf{a}_k \geq \lambda_k; \tag{7}$$

where λ_k is a constant depending only upon characteristics of the kth loss-event other than the actual value $\mathbf{a}_k$ of the loss resulting therefrom, e.g., upon the number of persons injured in consequence thereof, upon the number or the nature of the insurances involved, or simply upon the index k. Every set of rules for determining $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ appearing in any of the retrospective rating plans which have ever been approved in any of the States, can be shown to be of the form (7) under appropriate definitions of the terms *loss* and *loss-event*. In those plans which provide that actual values of losses without limitation shall be used in the computation of $\mathbf{u}$, $\lambda_k = \infty$ and $\mathbf{v}_k = \mathbf{a}_k$ for every k, so that $\mathbf{t} = \mathbf{s}$. In subsequent sections of this paper, we shall refer to $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ as the *modified losses*.

In the course of our inquiry, we shall need to consider the following random variable in addition to those already introduced:

$\mathbf{w}$ = the sum of all expenses (other than losses) * incurred by the insurer and directly allocable to the insurances included in J, plus an appropriate portion of the insurer's expenses (such as home office rent) not directly allocable to any particular set of insurances, but rather to the totality of the insurer's business.

We assume throughout that the probability distributions of $\mathbf{s}$ and $\mathbf{w}$ are such that both $\mathbf{E}(\mathbf{s})$ and $\mathbf{E}(\mathbf{w})$ exist, for otherwise the risk covered by J could hardly be regarded as insurable. From the existence of $\mathbf{E}(\mathbf{s})$, it follows—in view of (4), (6) and (7)—that $\mathbf{E}(\mathbf{t})$ exists and $\mathbf{E}(\mathbf{t}) \leq \mathbf{E}(\mathbf{s})$. (8)

From the existence of $\mathbf{E}(\mathbf{w})$, it follows † that there is a single-valued real function $W(t)$ such that, if t' and t'' be any two real numbers for which $F(t'') - F(t') > 0$, then $\dfrac{1}{F(t'') - F(t')} \displaystyle\int_{t'}^{t''} W(t)\, dF(t)$ is the mathematical expectation of $\mathbf{w}$ under the condition $t' < \mathbf{t} \leq t''$. Hence $\mathbf{E}\{W(\mathbf{t})\} = \mathbf{E}(\mathbf{w})$. (9)

In fact, there may be an infinite class of such functions. But if $W_1(t)$ and $W_2(t)$ be any two members of that class, then $W_1(t) = W_2(t)$ for all values of t except those belonging to a set Z such that the probability that $\mathbf{t}$ will fall in Z, is zero. Although the probability that no loss whatever will be incurred

* The term *expenses (other than losses)* includes taxes other than net income taxes, and also includes all expenses for investigation and adjustment of claims except such as may be included by definition within the denotation of the term *losses*.

† Kolmogoroff [9, Fünftes Kapitel, § 4].

under J may be very small, we assume that it is not zero. Hence, zero does not belong to any such set Z. Therefore, $W(0)$ is an uniquely determined number, which is quite properly to be regarded as the mathematical expectation of $\mathbf{w}$ under the condition $\mathbf{t} = 0$. Since $\mathbf{w}$ will certainly exceed zero even though no loss whatever be incurred under J, $W(0) > 0$. Since the expenses* allocable to any particular insurance never decrease in consequence of any increase in the losses incurred thereunder, we assume that every function having the character ascribed to $W(t)$ is monotone non-decreasing throughout the domain of non-negative real numbers with the possible exception of those belonging to a set Z of the aforesaid character.

By their definitions, none of the random variables: $\mathbf{n}$, $\mathbf{a}_1$, $\mathbf{a}_2$, , , , $\mathbf{v}_1$, $\mathbf{v}_2$, , , , $\mathbf{s}$, $\mathbf{t}$, $\mathbf{w}$, can have a negative value. Therefore, their distribution functions, $F(n)$, $F(a_1)$, etc., are all identically zero for negative values of n, a_1, etc., so that every integral of the form (1), (2), (2′) or (3) may be written with 0 as its lower limit, in place of $-\infty$, without changing its value, so long as $\mathbf{x}$ is one of the random variables listed above. Moreover, since $\mathbf{t}$ cannot be negative, $R(\mathbf{t})$ need not be defined for $\mathbf{t} < 0$. All that is hereinafter said about $R(\mathbf{t})$ is to be understood as referring to $R(\mathbf{t})$ for $\mathbf{t} \geqq 0$.

It may be argued that, since $\mathbf{t}$ will be expressed as an integral multiple of the smallest current fraction of some monetary unit, $R(\mathbf{t})$ need be defined only for values of $\mathbf{t}$ which are such multiples of that fraction, e.g., integral multiples of .01 if $\mathbf{t}$ is to be expressed in dollars. However, we shall regard the domain of possible values of losses, expenses and premiums as a continuum identical with the set of all non-negative real numbers. Accordingly, $R(\mathbf{t})$ must be defined for every non-negative real value of $\mathbf{t}$.

B. Conditions of Practicability

Let t' and t'', $t' < t''$, be any pair of possible values of $\mathbf{t}$. From (4), (6) and (7), it is clear that an increase in $\mathbf{t}$ from t' to t'' can not occur except in consequence of an increase in $\mathbf{n}$ or in $\mathbf{a}_k$ for some one or several values of k. Therefore, $R(t'')$ must not be less than $R(t')$ for, if it were, the insurer in effect would be offering the insured a reward in the form of a reduction in ultimate premium for an increase either in the number of loss-events or in the actual value of the loss to the insurer resulting from some one or several of them. Thus, one of the conditions imposed upon $R(\mathbf{t})$ by the requirement of practicability, is that

$$R(\mathbf{t}) \text{ } \textit{be monotone non-decreasing.} \qquad \text{(I)}$$

But $R(t'')$ must not so greatly exceed $R(t')$ that an increase in $\mathbf{t}$ from t' to t'' would be likely to result in an increase in the insurer's underwriting profit, $\mathbf{u} - (\mathbf{s} + \mathbf{w})$, for then the retrospective rating agreement would operate to encourage carelessness in the adjustment of claims, neglect of salvage possibilities and laxity in loss prevention, on the part of the insurer. Nor should $R(t'')$ so greatly exceed $R(t')$ that the insured might save money by withhold-

* Except commissions under certain contingent commission agreements, which we leave out of account since such agreements are not properly applicable to retrospectively rated insurances.

ing reports of loss-events from the insurer and bearing the ensuing losses and expenses himself, for—at least in the case of liability insurances—such withholding of reports may prove to be very costly to both the insured and the insurer. * Apparently, then, the following condition must be satisfied:

$$R(t'') - R(t') \leq \delta(t', t'') + a(t', t'') \quad \text{for every pair, } t', t'', \text{ such that } t' < t'', \tag{II}$$

where $\delta(t' t'')$ is the *minimum* increment in s which would effect an increase in t from t' to t'', and $a(t', t'')$ is some *non-munificent* allowance for an increase in w concomitant with such an increase in t. From (4), (6) and (7), it follows that $\delta(t', t'') = t'' - t'$. Precisely what the allowance $a(t', t'')$ should be, we need not decide. It is sufficient to observe that at most $a(t', t'')$ should certainly not exceed the mathematical expectation of the increment in w concomitant with an increase in t from t' to t''; and that expectation is given as the difference $W(t'') - W(t')$ where $W(t)$ is any member of the class of functions to which the context of (9) refers, provided that neither t' nor t'' belongs to a certain set Z, dependent upon the function $W(t)$, whereof the probability that t will fall in Z is zero. It follows that

$$R(t'') - R(t') \leq t'' - t' + W(t'') - W(t') \tag{10}$$

must be satisfied for every pair, t', t'', such that $t' < t''$, with the possible exception of any pair of which at least one member belongs to Z.

It may be argued that, if (II) were violated only for values t', t'', such that $t' < t'' < \zeta$, where ζ is some number such that the event: $\mathbf{t} \geq \zeta$, is practically certain to occur, then the probability of any ultimate benefit to the insurer arising out of its own carelessness, or of any ultimate saving to the insured as a result of withholding reports, would be so small as to be of no practical importance. Hence, with respect to increments in $R(t)$, practicability demands only that (II) be satisfied for every pair, t', t'', such that $t' < t''$ and $t'' \geq \zeta'$, where ζ' is the least upper bound of the numbers ζ. Likewise, it may be argued that, in respect to monotonicity of $R(t)$, practicability demands only that $R(t') \leq R(t'')$ be satisfied for every pair, t' t'', such that $t' < t''$ and $t'' \geq \zeta'$.

But it is well known that there are sets of insurances under which the probability that no loss whatsoever will be incurred is by no means inconsiderable, yet to which it would not be impracticable to apply a retrospective rating agreement. J may be such a set, for in the interest of generality we have refrained from making any assumption about the probability distribution of s, except that $\mathbf{E}(\mathbf{s})$ exists. If J is such a set, then zero is the only non-negative number ζ of which it may be said that the event: $\mathbf{t} \geq \zeta$, is practically certain to occur. In that case, $\zeta' = 0$, and the conditions (I) and (II) as originally stated must be satisfied if $\mathbf{u} = R(\mathbf{t})$ is to be a practicable retrospective rating formula.

By definition, the underwriting profit to the insurer on the insurances included in J, is $\mathbf{u} - \mathbf{s} - \mathbf{w}$, which is a random variable.† Clearly, $\mathbf{E}(\mathbf{u} - \mathbf{s} - \mathbf{w})$ must exist, that is to say, the Lebesgue-Stieltjes integral

$$\int_0^\infty (u - s - w)\, dF(u - s - w)$$ must exist and have a finite value, for otherwise

* Cf. Perryman [10, p. 7].

† Cramér [5, p. 154, 155, 162-164].

the Law of Large Numbers would not hold—in fact, it could not even be meaningfully stated—with respect to the net underwriting profit on any portfolio of insurances of which those included in J might form a part.* Since $\mathbf{u} = \mathbf{s} + \mathbf{w} + (\mathbf{u} - \mathbf{s} - \mathbf{w})$ and since, by assumption, both $\mathbf{E}(\mathbf{s})$ and $\mathbf{E}(\mathbf{w})$ exist, it follows † that

$\mathbf{E}(\mathbf{u})$ *must exist; and the number* p *defined by the equation*

$$p = \mathbf{E}(\mathbf{u}) - \mathbf{E}(\mathbf{s}) - \mathbf{E}(\mathbf{w}),$$

must satisfy such criteria of reasonableness as may be applicable in practice to the mathematical expectation of underwriting profit under the retrospective rating agreement in accordance with which $\mathbf{u}$ *is to be determined.* (III)

As a necessary consequence of (10) and (III), we have the following proposition: $R(0)$ *must not be less than* $\mathbf{E}(\mathbf{s} - \mathbf{t}) + W(0) + p.$ (11)

Proof: Substituting t for t'' and 0 for t' in (10), we have

$$R(t) - R(0) \leq t + W(t) - W(0),$$

which must hold for all values of $t \geq 0$, with the possible exception of a set Z whereof $\int_Z dF(t) = 0$. Therefore, since $F(t)$ is non-decreasing,

$$\int_0^\infty R(t)dF(t) - R(0)\int_0^\infty dF(t) \leq \int_0^\infty t\,dF(t) + \int_0^\infty W(t)dF(t) - W(0)\int_0^\infty dF(t).$$

But, since $R(\mathrm{t}) = \mathbf{u}$, $\int_0^\infty R(t)dF(t) = \mathbf{E}(\mathbf{u}) = \mathbf{E}(\mathbf{s}) + \mathbf{E}(\mathbf{w}) + p$, by (2) and (III).

By (1), $\int_0^\infty dF(t) = 1$; by (2) and (8), $\int_0^\infty t\,dF(t) = \mathbf{E}(\mathrm{t})$;

* Uspensky [12, Chp. X, esp. Par. 7, p. 191].

† Cramér [5, p. 172-173]. The existence of E(u) can also be shown to be a necessary consequence of (8), (9), (10), (I) and the fact that $R(t')$ and $W(t')$ must be uniquely defined for $t' = 0$.

and by (2) and (9), $\int_0^\infty W(t)dF(t) = \mathbf{E}(w)$.

Hence: $\mathbf{E}(s) + \mathbf{E}(w) + p - R(0) \leq \mathbf{E}(t) + \mathbf{E}(w) - W(0)$, from which (11) follows.

Since, by (8), $\mathbf{E}(s - t) \geq 0$, and since $W(0) + p$ is certainly positive, (11) implies that $R(0)$ must be positive. Therefore, in view of (I), $R(t)$ *must be positive for every possible value of* t.

It may appear that this conclusion is an obvious condition of the practicability of $u = R(t)$ as a retrospective rating formula, independent of any of the conditions (I), (II) and (III). However, this is not the case. If it were practicable to violate (I) or (II), then a practicable formula could be devised under which u would be negative for some values of t. Moreover, if there is a number $\zeta > 0$ such that t is practically certain to be not less than ζ, then under certain conditions relative to the probability distributions of s and t, it would be possible to devise a practicable formula under which u would be negative for some values of t, without violating either (I) or (II) for any pair, t', t'', such that $t' < t''$ and $t'' \geq \zeta$.

In addition to the conditions (I), (II), (III) and their logical consequences, which the function $R(t)$ *must* satisfy in order that it shall be a practicable retrospective rating formula, there is at least one condition not implied by (I), (II) and (III) which it is desirable (though not necessary) that $R(t)$ satisfy, to wit: *that* $R(t)$ *be bounded.*

If $R(t)$ is bounded, then it has a least upper bound G and, since $R(t)$ must be monotone non-decreasing, either (i) there exists a number g such that $R(t) = G$ for every $t \geq g$, and $R(t) < G$ for every $t < g$, or (ii) no such number g exists, but $\lim R(t) = G$ as $t \to \infty$. In accordance with custom, we shall call G the *Maximum Retrospective Premium*; and in case (i) we shall say that G is attained at the point g, while in case (ii) G is never attained.

If $\mathbf{E}(n)$, i.e., the expected number of loss-events, is large—say—more than 500, and if $\mu_2(n)$, i.e., the variance of n, is not much greater than $\mathbf{E}(n)$, the insured in negotiating the retrospective rating agreement may be much less interested in a Maximum Retrospective Premium than he is in the limits λ_1, λ_2, , , , placed upon v_1, v_2 , , , by rules of the form (7) in accordance with which the value of t shall be determined. In fact, if for every k, $\lambda_k = \Lambda$ then—even if $R(t)$ were unbounded—the insured would be exposed to no greater economic risk under the retrospective rating agreement, than that which he would have retained under a fixed-premium excess insurance against loss greater than Λ resulting from any one loss-event, provided—of course—that $R(t)$ satisfies conditions (I), (II) and (III). By proper selection of the value of Λ, the latter risk could in any case be so limited as to be quite bearable by the insured. These facts constitute a sufficient reason why $R(t)$ need not be bounded in order to be practicable as a retrospective rating formula.

Nevertheless, however great $\mathbf{E}(\mathbf{n})$ may be and however much the insured may be interested in prescribing the limits $\lambda_1, \lambda_2, , , ,$ he will certainly prefer an agreement in which a definite limit G is placed upon the possible values of $\mathbf{u}$, to one under which there is no such limit. Moreover, it should be observed that if $R(\mathbf{t})$ is bounded, then the existence of $\mathbf{E}(\mathbf{u})$ and of all of the moments of the distribution of $\mathbf{u}$ about any point whatever, is assured, and every analytical difficulty which might arise if some moment of that distribution were not known to exist, is thus avoided.

Finally, there are two conditions which cannot very well be expressed in categorical terms, but which it is desirable that $R(\mathbf{t})$ satisfy in some degree depending upon the circumstances of its proposed application. The first of these conditions is that, *given any particular value of* $\mathbf{t}$, *the corresponding value of* $\mathbf{u}$ *shall be computable to a certain number of significant figures with relatively small expenditure of time and effort.* For example, a function $R(\mathbf{t})$ such that the computation of its value to six significant figures for a particular value of $\mathbf{t}$, would require ten hours of labor by a skilled computer working with all available tables and computing devices, would certainly not be a very suitable formula for the rating of risks on each of which $\mathbf{E}(\mathbf{u})$ is \$5,000. Yet it might be quite satisfactory as a formula for the rating of a risk on which $\mathbf{E}(\mathbf{u})$ is \$500,000.

The second such condition is that, *in the case of any particular set of insurances, it shall be possible with relatively small expenditure of time and effort to determine the values of whatever parameters appear in the formula* $\mathbf{u} = R(\mathbf{t})$ *so that there is only a small probability that* $|\ \mathbf{E}(\mathbf{u}) - \bar{u}\ |$ *exceeds* η, where $\bar{u}$ is some predetermined value which it is desired that $\mathbf{E}(\mathbf{u})$ shall take, and η is some number which is relatively small in comparison with $\bar{u}$, e.g., $.05\bar{u}$. That $R(\mathbf{t})$ should satisfy this condition arises out of the fact that, in practice, estimates of $\mathbf{E}(\mathbf{s})$, $\mathbf{E}(\mathbf{w})$ and p [which we will represent by the symbols $\overline{\mathbf{E}(\mathbf{s})}$, $\overline{\mathbf{E}(\mathbf{w})}$ and $\bar{p}$] are given, * and it is required to determine the parameters in $R(\mathbf{t})$ so that condition (III) shall be satisfied. Thus, the sum of those three estimates is a predetermined value $\bar{u}$ which it is desired that $\mathbf{E}(\mathbf{u})$ shall take, for if $\mathbf{E}(\mathbf{u})$ does indeed take that value, then that condition will be satisfied provided the algebraic sum of $\bar{p}$ and the errors of the estimates $\overline{\mathbf{E}(\mathbf{s})}$ and $\overline{\mathbf{E}(\mathbf{w})}$ is a number which satisfies the criteria of reasonableness referred to therein. But in the course of any determination of the aforesaid parameters, $\mathbf{E}(\mathbf{u})$ can only be expressed in terms of characteristics of the probability distribution of $\mathbf{t}$. In practice, none of these characteristics is ever exactly known. Estimates of their values must be made from data relative to past experience on a properly selected sample of insurances, and such estimates are— of course—subject to error. Hence it is never possible to say (after the values of those parameters have

* These estimates are usually called the *Expected Losses*, the *Expense Allowance* and the *Profit or Contingency Loading*, respectively.

been determined) precisely what value $\mathbf{E}(\mathbf{u})$ *does indeed* take. At best, one can only say how small is the probability that $\mathbf{E}(\mathbf{u})$ differs from $\bar{u}$ by more than η, which is the reason why the condition under discussion in this paragraph has been phrased as it is in the first sentence hereof.

C. Sectionally Linear Retrospective Rating Formulae in General

In many categories of insurance, the losses which insurers undertake to bear, though they do indeed have about them a certain air of fortuity, are nevertheless subject to control by the insured to an extent which is often surprisingly great, through one or several of the following means: (i) rigorous inspection of premises, material and equipment; (ii) installation of special devices designed to prevent the occurrence of loss-events or to minimize the losses resulting therefrom; (iii) revision of working methods so as to eliminate hazardous operations; (iv) careful selection and training of employees; (v) cooperation with insurers in the adjustment of claims, in efforts toward salvage, and in proceedings to recover damages from negligent third parties; and (vi) in the case of Compensation losses, the re-employment and rehabilitation of injured workmen.

However, with the possible exception of (v), all of these means are costly to the insured. Their employment frequently appears to be in conflict with his immediate interests. Under non-retrospective methods of rating his risks for insurance, any reduction in premium rates which he may obtain as a result of their employment, is generally delayed or spread piecemeal over a period of years, even though their employment brings about an immediate reduction in loss rates.

The fundamental premise upon which the most widely applicable argument in favor of retrospective rating is based, is that the insured will be much more likely to employ those means vigorously and persistently if—in addition to the rather indefinite moral and long-term economic incentives which are always present—he has a very definite and immediate economic incentive to do so. Under a retrospective rating agreement made in accordance with any of the approved plans, such an incentive exists so long as the sum of the modified values of the losses incurred under the insurances which are subject to the agreement, is less than the "allowance for losses in the Maximum Retrospective Premium," i.e., the lower bound (g) of the values of t for which $u = G$ by formula (5).

But if that premise is sound, then *it would be well to have such an incentive persist throughout the entire term of the agreement, regardless of how great the sum of the losses becomes.*

Now the incentive provided by an agreement under which **u** is determined by formula (5) arises out of the fact that, for every increment (Δt) in the sum of the modified losses after that sum has attained the value h and until it attains the value g, the cost to the insured of the very insurances covering the events causing such an increment, is forthwith increased by an amount equal to $C\Delta t$. Obviously, if C is a constant, such an incentive cannot be made to persist regardless of how great the sum of the modified losses becomes, unless **u** is to be unbounded, i.e., unless there is to be no Maximum Retrospective Premium. Yet, as we have previously observed, the insured will certainly prefer an agreement under which **u** is bounded. In fact, except under extraordinary circumstances, he will refuse to be interested in any proposal in which G is greater than $4\mathbf{E}(\mathbf{s})$.

In many cases, given values of $\lambda_1, \lambda_2, , ,$ e.g., λ_k = \$10,000 for every k, it is possible to select a value of g so great that **t**, the sum of the modified losses, is practically certain not to exceed g, and then to select some two of the other parameters, B, C, h, H and G in formula (5), so that—after the remaining three have been determined—C is found to be not too small—say—not less than .30, and neither H nor G is so large that the insured would refuse to be interested in any proposal in which they were set forth as the minimum and maximum values of **u**. In any *such* case, (5) may be regarded as a satisfactory formula, for an agreement drawn in accordance with (5) wherein the parameters have been so selected and determined, will be an agreement under which a definite and immediate economic incentive for the insured to prevent losses is practically certain to persist throughout its term. However, it must be admitted that "practically certain" is a term having no precise meaning.

But in other cases [unless the upper bound of the sequence $\lambda_1, \lambda_2, \ldots$ is so small that the retrospective rating agreement would have little value as a device to encourage the insured's efforts to hold down the value of each $\mathbf{a}_k$, or unless that sequence is constructed in some special way, e.g., monotonically decreasing so that λ_k is very small for values of k which are large relative to $\mathbf{E}(\mathbf{n})$, in consequence of which evaluation of the parameters in (5) would be very difficult] it turns out that, as soon as one selects g so great that **t** is practically certain not to exceed g, then either H or G must be too large to be of any interest to the insured, or else C must be so small that the prospect of a saving of $C\Delta t$ in premium for the insurances in question, could hardly be considered (at least for small values of Δt) as an incentive for the insured to take actions he would not otherwise be inclined to take which might prevent an increase of Δt in losses. In any of *these* cases, or—for that matter—*in any case*, an agreement under which a considerable incentive of the kind under discussion will persist until the sum of the modified losses has attained a certain value, say g_1 or g_2, while some lesser incentive will persist until that sum has

attained any value g_m which one chooses to specify, may be drawn in accordance with the following formula:

$$\mathbf{u} = \begin{cases} H & \text{if } \mathbf{t} \leq h \\ B_1 + C_1\,\mathbf{t} & \text{if } h \leq \mathbf{t} \leq g_1 \\ B_2 + C_2\,\mathbf{t} & \text{if } g_1 \leq \mathbf{t} \leq g_2 \\ \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots \\ B_m + C_m\,\mathbf{t} & \text{if } g_{m-1} \leq \mathbf{t} \leq g_m \\ G & \text{if } \mathbf{t} \geq g_m \end{cases} \tag{12}$$

in which:

$$\left.\begin{array}{llll} B_1 + C_1 h = H & \cdots\cdots & \cdots\cdots & \\ B_2 + C_2 g_1 = B_1 + C_1 g_1 & & B_m + C_m g_{m-1} = B_{m-1} + C_{m-1} g_{m-1} & \\ \cdots\cdots & \cdots\cdots & G = B_m + C_m g_m & \end{array}\right\} \tag{13}$$

It is apparent that, when the relationship between $\mathbf{u}$ and $\mathbf{t}$ is of the form (12), the graph of $\mathbf{u}$ plotted against $\mathbf{t}$ is a segmental arc composed of $m + 2$ straight line segments joined together in succession so that the right end-point of any one of them is the left end-point of the next. The slope of the first and the last is zero; while the slopes of the m intervening segments are $C_1, C_2, \ , \ , C_m$ respectively. For the purpose of drawing an agreement under which the incentives shall be as stated in the preceding paragraph, it is generally sufficient to have $m = 2$ or 3. However, in the interest of generality we shall treat the case in which m may be any finite positive integer. Clearly, (5) is only a special case of (12), i.e., the case in which $m = 1$.

In (12) there are $3m + 3$ parameters (m B's, m C's, m g's, h, H and G) and in (13) there are $m + 1$ independent equations relating them to each other. Hence $2m + 2$ of the parameters must be determined by means other than the system (13). Values for $2m + 1$ of them may be selected arbitrarily—with a view toward the design of a formula under which the insured will have an adequate incentive to prevent losses, provided—however—that the values selected must be such that $\mathbf{u}$ as a function $R(\mathbf{t})$ shall satisfy the conditions (I) and (II) set forth in Section B. But $\mathbf{u}$ must also satisfy condition (III), i.e., $\mathbf{E}(\mathbf{u})$ must equal $\mathbf{E}(\mathbf{s}) + \mathbf{E}(\mathbf{w}) + p$, where p is some number satisfying the criteria referred to in (III). Accordingly, after integrating the function of $\mathbf{t}$ defined by (12) with respect to $F(t)$ over the interval $(0, \infty)$, and collecting terms, we have:

$$H + C_1 \Xi(h) + \sum_{j=1}^{m-1} (C_{j+1} - C_j)\, \Xi(g_j) - C_m \Xi(g_m) = \mathbf{E}(\mathrm{s}) + \mathbf{E}(\mathrm{w}) + p, \qquad (14)$$

in which $\Xi(g_j) = \int_{g_j}^{\infty} (t - g_j)\, dF(t)$ and $\Xi(h)$ is of course defined by the same equation with h in place of g_j.

In case $\lambda_k = \infty$ for every k, so that t = s, the function of two variables, r and P, first studied by Paul Dorweiler [7] and now generally called the *Excess Pure Premium Ratio*, when defined in terms of operations such as summation performed upon a finite number of observed values of losses or loss-ratios, may be regarded as an *estimate* of the value of $\Xi(g)/\mathbf{E}(\mathrm{s})$ for $g = rP$ and $\mathbf{E}(\mathrm{s}) = aP$, where a is a certain constant usually called the "permissible loss-ratio." But when defined as by Stefan Peters [**11**, p. 589] or as by A. L. Bailey [**1**, p. 67] in terms of a frequency function, the Excess Pure Premium Ratio is the *same* as $\Xi(g)/\mathbf{E}(\mathrm{s})$ for $g = rP$ and $\mathbf{E}(\mathrm{s}) = aP$, provided t = s and provided the probability distribution of s is of the continuous type. For $m = 1$, the left side of (14) reduces to an expression equivalent to the product of P by the left side of Perryman's fundamental equation [**10**, p. 7] when the symbols $H'p$ and $G'p$ appearing therein are interpreted to mean $\Xi(h)/P$ and $\Xi(g)/P$, respectively.

Equation (14) and the $m + 1$ equations in (13) form a system of $m + 2$ independent equations; so that after a value for p and values for $2m + 1$ of the parameters in (12) have been selected, the values of the remaining $m + 2$ parameters are uniquely determined by that system, as soon as a particular set J of insurances has been specified and values of $\lambda_1, \lambda_2, , ,$ prescribed, so that $\mathbf{E}(\mathrm{s})$, $\mathbf{E}(\mathrm{w})$ and $F(t)$ are theoretically fixed.

However, the actual computation of the numerical values of the remaining $m + 2$ parameters may be very difficult. In fact, unless a table is at hand from which a reliable estimate of $\Xi(g)$ for any particular value of g may be obtained, the time and effort required to complete a satisfactory approximation to the values uniquely determined as stated above, may be so great as to prohibit the use of a formula of the type (12). Even when such tables are at hand, one generally has to resort to trial and error methods unless every one but one of the $2m + 2$ parameters appearing in (14) is either included in the set of $2m + 1$ parameters to which values have been pre-assigned by selection, or else its value can be determined from the pre-assigned values and the equations in (13).

A reliable estimate of $\Xi(g)$ for any particular value of g may be obtained by the proper use of a table of Excess Pure Premium Ratios *provided* the distribution of total losses underlying or implied by that table for risks of "standard premium size" P equal to $\mathbf{E}(t)/a$, is approximately the same as $F(t)$. Otherwise, estimates of $\Xi(g)$ obtained by use of that table cannot be regarded as reliable.* Now, sets of insurances to which it is practicable and desirable to apply retrospective rating agreements differ widely among themselves in respect to the characteristics of the probability distributions of total modified losses incurred thereunder, even though we confine our attention to a single particular prescription of values for $\lambda_1, \lambda_2, , ,$ e.g., λ_k = \$10,000 for every k, and to sets for each of which $\mathbf{E}(t)$ has the same value under that prescription. Therefore, if one is always to have at hand an appropriate table for use in drawing a retrospective rating agreement under which the ultimate premium u for a set J of insurances is to be determined by a formula of the type (12), regardless of the nature of the set J, then a great many different tables [*either* of Excess Pure Premium Ratios or of values of some other function from which reliable estimates of $\Xi(g)$ may be obtained] will have to be constructed. But the construction of any one such table for any one distribution function $F(t)$ e.g., the construction of any one column in a table of Excess Pure Premium Ratios, requires a very considerable expenditure of time and effort; † and the construction of a number of such tables for a number of different distribution functions all having the same first moment, $\mathbf{E}(t)$, about the point $t = 0$, would be a rather formidable task. When one considers that this would have to be done for each of a series of values of $\mathbf{E}(t)$ ranging from \$3,000 to \$500,000, it is clear that the entire project would be a prodigious undertaking indeed. Consequently, (12) can hardly be said to satisfy the condition expressed in the last paragraph of Section B. All of the statements relative to (12) made in this paragraph or in the preceding one, apply also to (5), since (5) is only a special case of (12).

D. A Class of Non-linear Retrospective Rating Formulae

The facts set forth in the preceding Section, impel us to inquire whether there may not be a class of functions which are fundamentally better adapted to serve as retrospective rating formulae than those belonging to the class of sectionally linear functions defined by (12). The principal thesis of this paper is that there is indeed at least one such class, to wit, the class defined by the equation:

* Cf. Bailey [2, Part VI, Section A].

† For methods of constructing tables of Excess Pure Premium Ratios, see Dorweiler [7 and 8] Valerius [13], Peters [11] and Bailey [2, Part VI, Section B]. Each of the methods outlined or exemplified in these papers, involves a very considerable expenditure of time and effort, not by reason of any defect in the method, but by reason of the character of the function $\Xi(g)$.

$$\mathbf{u} = \mathfrak{R}(\mathbf{t}) = G - Ae^{-\beta \mathbf{t}} \qquad \text{for } \mathbf{t} \geqq 0, \tag{15}$$

in which G, A and β are parameters having positive real values independent of every characteristic or consequence of any particular loss-event, and $\mathbf{t}$ is defined by (6). Thus, all that was said of $R(\mathbf{t})$ in Section A applies to $\mathfrak{R}(\mathbf{t})$; $\mathfrak{R}(\mathbf{t})$ is defined for every non-negative real value of $\mathbf{t}$; and for $\mathbf{t} < 0$, $\mathfrak{R}(\mathbf{t})$ is not defined.

Since A and β are real and positive, $\mathfrak{R}(\mathbf{t})$ is monotone non-decreasing, and so satisfies condition (I) of Section B.

If t', t'' be any pair of possible values of $\mathbf{t}$, such that $t' < t''$, it can easily be shown * that $\mathfrak{R}(t'') - \mathfrak{R}(t') < A\beta(t'' - t')$. Hence, it is always possible by a suitable restriction upon the values of $A\beta$ to ensure that $\mathfrak{R}(\mathbf{t})$ shall satisfy condition (II) of Section B. For example, (II) will certainly be satisfied if $A\beta \leqq 1$.

$\mathfrak{R}(\mathbf{t})$ has a lower bound, $G - A$, which it attains only at the point $\mathbf{t} = 0$, and an upper bound, $G = \lim_{t \to \infty} \mathfrak{R}(t)$, which it never attains. Hence, under (15), $G - A$ may be called the *Minimum Retrospective Premium*; G, the *Maximum Retrospective Premium*; and A, the *amplitude* or "swing."

Since $\mathfrak{R}(\mathbf{t})$ is bounded, the existence of $\mathbf{E}(\mathbf{u})$ under (15) is assured. Hence, $\mathfrak{R}(\mathbf{t})$ will satisfy (III) of Section B as soon as the values of the parameters G, A and β have been determined so that

$$\mathbf{E}(\mathbf{u}) = \int_0^\infty (G - Ae^{-\beta \mathbf{t}})\, dF(t) = \mathbf{E}(\mathbf{s}) + \mathbf{E}(\mathbf{w}) + p, \tag{16}$$

where p is some number satisfying the criteria referred to in (III).

In any case in which numerical values have been assigned to G, A and β, the value of $\mathbf{u} = \mathfrak{R}(\mathbf{t})$ for any particular value of $\mathbf{t}$ may be computed to six significant figures in a very few minutes with the help of a seven-place table of common logarithms and an ordinary calculating machine, by means of the formula

$$\mathbf{u} = G - \text{antilog}_{10} \left\{ \log_{10} A - \beta\, \mathbf{t} \log_{10} e \right\}, \tag{15'}$$

which follows at once from (15). Six significant figures will be sufficient in almost all cases encountered in practice. But of course, greater accuracy may be attained by the use of more elaborate tables, or by expanding $e^{-\beta \mathbf{t}}$ in a power series and computing the sum of a sufficiently large number of terms.

* See (17).

At every point in the domain of t, $\mathfrak{R}(t)$ possesses a positive first differential coefficient, $\mathfrak{R}'(t) = A\beta e^{-\beta t}$. Hence, under an agreement drawn in accordance with (15), the insured would always have a definite and immediate economic incentive to prevent losses, regardless of the height to which the sum of the losses already incurred may have arisen. For at every point in the stochastic process by which the sum of the modified losses attains its ultimate value t, the premium which the insured will be obligated ultimately to pay to the insurer is increasing (or the return to which the insured will be entitled out of the amount he has already paid to the insurer is decreasing) at the rate of $A\beta e^{-\beta t}$ per monetary unit of increase in the sum of the modified losses, where t is the attained value of that sum at that point.

Thus, $A\beta e^{-\beta t}$ may be regarded as an index of such incentive under such an agreement. It has its maximum value $A\beta$ at $t = 0$; it decreases as t increases, approaching 0 asymptotically as $t \to \infty$. Admittedly, in any particular case, for very large values of t the incentive will be so small as to be inconsiderable. But, in every case in which the making of a retrospective rating agreement could be at all justified, it is possible by proper choice of the values of λ_1, λ_2, , , A and β, to construct such an agreement in accordance with (15), under which the incentive will be quite considerable, say $A\beta e^{-\beta t} \geq .30$, for every $t \leq t'$, where t' is some number of which the a priori probability that t will exceed t' is small, say $1 - F(t') < .05$, while some lesser incentive (smaller and continuously diminishing as t increases) will persist for every $t > t'$, without making G so great that the insured will not accept the agreement, or λ_k so small that the agreement would have little value as a device to encourage the insured's efforts to hold down the value of a_k.

Thus, we have shown that under a proper choice of values of G, A and β, $\mathfrak{R}(t)$ satisfies all of the necessary conditions of practicability as a retrospective rating formula, and that $\mathfrak{R}(t)$ also possesses all of the properties which it is desirable that such a formula should possess, except that we have not yet shown that $\mathfrak{R}(t)$ satisfies the condition expressed in the last paragraph of Section B. That it does indeed satisfy that condition in a great many cases, is the theme of the following Section.

E. Evaluation of the Parameters G, A and β.

In (5) there are three parameters, G, A and β, to which numerical values must be assigned; and (16) is the only equation relating their values one to another which must be satisfied. Hence, values for two of them may be selected arbitrarily—or with a view toward the design of a formula which meets the insured's requirements and at the same time offers the insured an adequate incentive to prevent losses, provided—however—that the two values selected

and the third which is consequently determined by (16) must, when taken together, be such that $\mathfrak{R}(t)$ satisfies condition (II) of Section B. [As previously remarked, $\mathfrak{R}(t)$ always satisfies condition (I) since A and β are real and positive by definition.]

If we set $\dfrac{\alpha(t', t'')}{t'' - t'} = \rho(t', t'')$, then, since $\delta(t', t'') = t'' - t'$,

condition (II) with respect to $\mathfrak{R}(t)$ may be expressed thus:

$$\frac{\mathfrak{R}(t'') - \mathfrak{R}(t')}{t'' - t'} \leqq 1 + \rho(t', t'')$$

for every pair, t', t'', such that $t' < t''$.

But, by the Theorem of the Mean and the fact that $\mathfrak{R}'(t) = A\beta e^{-\beta t}$ is a decreasing function,

$$\frac{\mathfrak{R}(t'') - \mathfrak{R}(t')}{t'' - t'} < A\beta e^{-\beta t'} \tag{17}$$

for every such pair. Therefore, if $A\beta e^{-\beta t'} \leqq 1 + \rho(t', t'')$ (18)

for every such pair, (II) will certainly be satisfied. In the case of any particular set J of insurances, all of the facts upon which the allowance $\alpha(t', t'')$ depends (e.g., the past expense experience on similar insurances, the tax rates applicable to $\mathbf{u}$, whether or not the term *losses* is to be defined to include allocated claim expenses, etc.) will be known or can be ascertained; whereupon a value can be assigned to $\alpha(0, \bar{t})$ where $\bar{t}$ is some relatively small possible value of t, e.g., .01 $\mathbf{E}(t)$, from which the value of $\rho(0, \bar{t})$ can be computed. $\rho(t', t'')$ can be considered constant for small values of t' and t''. Hence, if $A\beta \leqq 1 + \rho(0, \bar{t})$, then (18) will be satisfied for small values of t' and t''; and when it is satisfied for small values, it will usually be found that $A\beta e^{-\beta t}$, decreases fast enough as t increases so that (18) is satisfied for all values of t' and t'', $t' < t''$.

In accordance with (11), in order that the values of the parameters G, A and β shall be such that $\mathfrak{R}(t)$ satisfies both (II) and (III), it is necessary that $\mathfrak{R}(0)$ be not less than $\mathbf{E}(s - t) + W(0) + p$, which is positive. Now, $\mathfrak{R}(0) = G - A$. Therefore, the choice of values for two of the parameters must be governed not only by (18), but also by the following relationship:

$$G - A \geqq \mathbf{E}(s - t) + W(0) + p > 0 \tag{19}$$

Of course, any set of values of G, A and β which satisfies (16) and (18), will also satisfy (19). But (19) is useful as a guide in selecting the values for two

of the parameters which are to be substituted in (16) in order to obtain a value for the third.

Let us turn our attention now to the integral which appears in (16).

$$\int_0^\infty (G - Ae^{-\beta t})dF(t) = G\int_0^\infty dF(t) - A\int_0^\infty e^{-\beta t}\,dF(t) \qquad (20)$$

The integral appearing in the second term on the right is the Laplace-Stieltjes transform of $F(t)$, which we shall denote by $\mathfrak{L}\{F(t),\beta\}$.

In general, if x be any random variable, we shall write

$$\int_0^\infty e^{-\beta x}\,dF(x) = \mathfrak{L}\{F(x),\beta\}. \qquad (21)$$

If the probability distribution of x is of the continuous type, the Lebesgue-Stieltjes integral in (21) has the same value as the ordinary Riemann integral, $\int_0^\infty e^{-\beta x} f(x)dx$, in which $f(x) = F'(x)$. We shall denote this latter integral, which is the Laplace transform of the frequency function $f(x)$, by $\mathfrak{L}\{f(x),\beta\}$. These notations emphasize the fact that the Laplace-Stieltjes transform is a function of β, of which the character is determined as soon as the probability distribution of the random variable x is known.

If in (21) we were to replace 0 as the lower limit of integration by $-\infty$ * we would have the bilateral Laplace-Stieltjes transform, $\mathfrak{L}_2\{F(x),\beta\}$, which —in view of (2)— is the same as $\mathbf{E}(e^{-\beta x})$. $\mathfrak{L}_2\{F(x), -i\theta\}$ regarded as a function of θ, is the familiar *characteristic function* of the distribution of x; and $\mathfrak{L}_2\{F(x), -\theta\}$ regarded as function of θ, is often called the *moment-generating function* of the distribution. If x is a random variable (like $\mathbf{n}$, $\mathbf{a}_k$, $\mathbf{v}_k$, $\mathbf{s}$, $\mathbf{t}$) which cannot take a negative value, then of course $\mathfrak{L}_2\{F(x),\beta\} = \mathfrak{L}\{F(x),\beta\}$. Furthermore, if x is any such variable, then $\int_0^\infty dF(x) = 1$, from which it follows (by virtue of certain fundamental theorems on Lebesgue-Stieltjes integrals, * and since $0 < e^{-\beta x} \leq 1$ for $x \geq 0$) that $\mathfrak{L}\{F(x),\beta\}$ exists and $0 < \mathfrak{L}\{F(x),\beta\} \leq 1$ for every real value of $\beta \geq 0$. In

* Cramér [5, p. 63].

particular, $\mathfrak{L}\{F(x),0\} = 1$. It can also be shown * that $\mathfrak{L}\{F(x),\beta\}$ is continuous at every point in the interval $0 \leq \beta < \infty$, and that it decreases monotonically from 1 at $\beta = 0$ and approaches $F(0)$ as $\beta \to \infty$.

In view of (20), (21) and the fact that $\int_0^\infty dF(t) = 1$, equation (16) may be rewritten as follows:

$$G - A\,\mathfrak{L}\{F(t),\beta\} = \mathbf{E}(\mathrm{s}) + \mathbf{E}(\mathrm{w}) + p \tag{22}$$

In the structure of retrospective rating formulae of the non-linear type (15), this single equation (22) is the analogue of the system of $m + 2$ equations consisting of (14) and the $m + 1$ equations (13) in the structure of retrospective rating formulae of the sectionally linear type (12). Or perhaps it would be better to say that (22) is the analogue of (14) and that in the structure of formulae of the type (15) there is no analogue of the $m + 1$ equations (13), for (13) is really a part of the definition of the class of functions from which sectionally linear retrospective rating formulae are chosen, whereas the class of non-linear functions under discussion is defined by the single equation (15) and the assertion that G, A and β are real and positive.

When a particular set J of insurances has been specified and values of $\lambda_1, \lambda_2, \ldots$ prescribed, $\mathbf{E}(\mathrm{s})$, $\mathbf{E}(\mathrm{w})$ and $F(t)$ are theoretically fixed, so that as soon as values of p and two of the parameters G, A and β have been selected, the value of the third is uniquely determined by (22).

If $F(t)$ is (or can be closely approximated by) a function whose Laplace-Stieltjes transform can be expressed in a few simple terms easy to evaluate numerically, then the value of the third parameter can be computed without difficulty. For example, if the probability distribution of t is of the continuous type, and if the frequency function of t is:

$$\left.\begin{aligned} f(t) &= 0 && \text{for } t \leq 0, \\ f(t) &= \frac{c^b}{\Gamma(b)}\, t^{b-1}\, e^{-ct} && \text{for } t > 0, \end{aligned}\right\} \tag{23}$$

then
$$\mathfrak{L}\{F(t),\beta\} = \mathfrak{L}\{f(t),\beta\} = \left(1 + \frac{\beta}{c}\right)^{-b} \tag{24}$$

* By application of the first two theorems in Section 7.3 of Cramér [5].

On substituting this last expression for $\mathfrak{L}\{F(t),\beta\}$ in (22), one sees at once that the value of any one of the three parameters, G, A or β, may be computed without much effort as soon as values for the other two have been selected and the values of b, c and $\mathbf{E}(s) + \mathbf{E}(w) + p$ are known.

Frequency functions of the class defined by (23), which is a sub-class of those of Pearson Type III, will be found to fit distributions of t very satisfactorily in many cases; * and in those cases it is already clear that non-linear functions of the form (15) are better adapted than sectionally linear functions of the form (5) or (12) to serve as retrospective rating formulae.

Of course, it may be argued that if $F(t)$ is such that, for any particular g, the numerical value of $\Xi(g)$ can easily be found, then the values of any $m + 2$ of the parameters in (12) can be computed without difficulty as soon as values for the other $2m + 1$ of them have been selected. But the functions which can be satisfactorily fitted to distributions of t without extraordinary labor, do not have that property. They are of such a character that integrals of the form $\Xi(g) = \int_g^\infty (t - g)dF(t)$ are not easy to evaluate for $g > 0$; whereas some of them [notably those corresponding to the frequency functions defined by (23)] have Laplace-Stieltjes transforms which are very easy to evaluate for any given β, and which in addition are such that, if the value of the transform be given, the corresponding value of β may very quickly be computed.

In any case, whether or not $F(t)$ can be satisfactorily approximated by a function whose Laplace-Stieltjes transform has the virtue discussed above, $\mathfrak{L}\{F(t),\beta\}$ itself possesses two properties by reason of which the non-linear functions defined by (15) are especially well adapted to serve as retrospective rating formulae. The first of these is set forth in the following paragraph.

By a MacLaurin expansion of the factor $e^{-\beta x}$ appearing in (21), we have:

$$\mathfrak{L}\{F(x),\beta\} = \int_0^\infty \left\{1 - \beta x + \frac{(\beta x)^2}{\lfloor 2} - \frac{(\beta x)^3}{\lfloor 3} + \ldots + q_m(\beta,x)\right\} dF(x)$$

in which $q_m(\beta, x)$, the remainder after the term of degree $m - 1$, is given by the equation: $q_m(\beta, x) = (-1)^m e^{-\beta cx} \frac{(\beta x)^m}{\lfloor m}$, where $0 < c < 1$. Now, if x is any random variable (like n, a_k, v_k, s, t) which cannot take a negative

* Cf. A. L. Bailey [1, p. 78 and Table 9].

value, then $\int_0^\infty dF(x) = 1$ and $\int_0^\infty \frac{(\beta x)^k}{\lfloor k} dF(x) = \frac{\beta^k}{\lfloor k} \mathbf{E}(\mathrm{x}^k)$ for every $k \geqq 0$,

in which $\mathbf{E}(\mathrm{x}^k)$ is the kth moment of the distribution of x about the point $\mathrm{x} = 0$. Therefore, if $\mathbf{E}(\mathrm{x}^m)$ exists, then

$$\mathfrak{L}\{F(x),\beta\} = 1 - \beta\mathbf{E}(\mathrm{x}) + \frac{\beta^2}{\lfloor 2} \mathbf{E}(\mathrm{x}^2) - \frac{\beta^3}{\lfloor 3} \mathbf{E}(\mathrm{x}^3) + \ldots + Q_m(\beta),$$

in which $Q_m(\beta) = \int_0^\infty q_m(\beta, x) dF(x)$. So $|Q_m(\beta)| \leq \left| \frac{\beta^m}{\lfloor m} \mathbf{E}(\mathrm{x}^m) \right|$

Thus, if the first $m - 1$ moments of the distribution of t about the point $\mathrm{t} = 0$ are known, then for any particular value of β, an estimate of the transform $\mathfrak{L}\{F(t),\beta\}$ is immediately given by the polynomial

$$1 - \beta\mathbf{E}(\mathrm{t}) + \frac{\beta^2}{\lfloor 2} \mathrm{E}(\mathrm{t}^2) - \ldots + (-1)^{m-1} \frac{\beta^{m-1}}{\lfloor m - 1} \mathbf{E}(\mathrm{t}^{m-1}); \qquad (25)$$

and the absolute difference between that estimate and the true value of the transform is less than $\left| \frac{\beta^m}{\lfloor m} \mathbf{E}(\mathrm{t}^m) \right|$, which is very small for sufficienty small values of β.

Now, in the case of any particular set J of insurances, all of the moments, $\mathbf{E}(\mathrm{t}^k)$ for $k = 1, 2, 3, \ldots$, certainly exist, since the range of possible values of t is bounded by the total amount of wealth in the world. But none of them is ever exactly known. However. estimates of their values may be made from data relative to past experience on a properly selected sample of insurances; and although the variances of the sampling distributions of moments of order $k > 3$ are very great, the contribution to the probable error of (25) resulting from the substitution therein of such an estimate of $\mathbf{E}(\mathrm{t}^k)$ in place of $\mathbf{E}(\mathrm{t}^k)$ itself, will be very small for small values of β, provided—of course—that the sample from which such estimate was derived, was very large and properly chosen. Thus, both the absolute difference $|Q_m(\beta)|$ and the probable error introduced into (25) by the substitution of estimates of the moments therein appearing, in place of the moments themselves, will be small in comparison with the true value of the transform $\mathfrak{L}\{F(t),\beta\}$ for sufficiently small values of β.

Fortunately, in many cases, the only values of β which are of any practical interest, are ones which are very small indeed. For, in general, $A\beta$ can not

be greater than 1.25 without violating condition (II) of Section B, at least for small values of t''; and in retrospective rating formulae of the type (15), the amplitude or "swing" A can be quite high, e.g., 3 times the Expected Losses, without in effect shifting almost all of the risk back to the insured. In many cases, values of $A < 2\mathbf{E}(t)$ will be of no interest; and in those cases, only values of $\beta < 1.25/2\mathrm{E}(t)$ will be of any practical importance.

For example, suppose that $\mathbf{E}(t)$ has been estimated to be \$50,000 with a probable error of ϵ_1, that A is to be \$100,000, and that $A\beta$ is to be 1.00. Then $\beta = .00001$. Suppose further that it is known that $\mathbf{E}(t^4) < (\$10)^{19}$, and that estimates, $\overline{\mathbf{E}(t^2)}$ and $\overline{\mathbf{E}(t^3)}$, of $\mathbf{E}(t^2)$ and $\mathbf{E}(t^3)$ are at hand, of which the probable errors are ϵ_2 and ϵ_3, respectively. Then

$1 - \beta(50{,}000) + \frac{\beta^2}{\lfloor 2} \overline{\mathbf{E}(t^2)} - \frac{\beta^3}{\lfloor 3} \overline{\mathbf{E}(t^3)}$ gives an estimate (L) of the value of $\mathfrak{L}\{F(t), \beta\}$ for $\beta = .00001$, of which the error made by neglecting all moments of order greater than 3, is less than .0042, and the probable error ($\bar{\epsilon}$) due to the use of estimates of $\mathbf{E}(t)$, $\mathbf{E}(t^2)$ and $\mathbf{E}(t^3)$ is less than

$.00001\epsilon_1 + \frac{(.00001)^2 \epsilon_2}{\lfloor 2} + \frac{(.00001)^3 \epsilon_3}{\lfloor 3}$. When a value $\bar{u}$ for the right side of equation (22) has been given, L may be substituted for $\mathfrak{L}\{F(t), \beta\}$ and \$100,000 substituted for A in the left side, and the equation solved for G. With G equal to the value thus obtained, $A = \$100{,}000$, and $\beta = .00001$, $\mathbf{E}(u)$ will fall short of $\bar{u}$ by less than \$420 with a probable error less than $\bar{\epsilon}$ times \$100,000.

Thus, it is apparent that—in many cases—the parameters in (15) can be determined in a manner satisfying the condition set forth in the last paragraph of Section B, without any knowledge of the shape of the probability distribution of the sum of the modified losses, that is to say—without any knowledge of the function $F(t)$, provided only that one has reliable estimates of the first three or four moments of that distribution about the point $t = 0$. But in no case can any such statement be made relative to the parameters in (5) or (12). In order to determine them, one must have at hand reliable estimates of the value of the function $\Xi(g)$ for at least some values of g; and in order to obtain such estimates, one must have knowledge of the shape of the aforesaid distribution.

The second of the two properties of the transform $\mathfrak{L}\{F(t), \beta\}$ by reason of which the non-linear functions defined by (15) are especially well adapted to serve as retrospective rating formulae, is expressed in the following

Theorem. *The Laplace-Stieltjes transform of the distribution function of the sum of any finite number (N) of independent random variables, none of which can take a negative value, is equal to the product of the Laplace-Stieltjes transforms of the distribution functions of the N terms in that sum.*

The following proof for the case in which $N = 2$ can easily be extended to the case in which N is any finite number.

Let x and y be any two independent random variables, neither of which can take a negative value; and let $t = x + y$. Then $e^{-\beta x}$ and $e^{-\beta y}$ are independent random variables, and

$$\mathbf{E}(e^{-\beta t}) = \mathbf{E}(e^{-\beta(x+y)}) = \mathbf{E}(e^{-\beta x}e^{-\beta y}) = \mathbf{E}(e^{-\beta x}) \times \mathbf{E}(e^{-\beta y}). \ *$$

But, by (2) and (21), in view of the hypothesis that neither x nor y can take a negative value,

$$\mathbf{E}(e^{-\beta t}) = \mathfrak{L}\{F(t), \beta\}; \quad \mathbf{E}(e^{-\beta x}) = \mathfrak{L}\{F(x), \beta\}; \quad \mathbf{E}(e^{-\beta y}) = \mathfrak{L}\{F(y), \beta\}.$$

Hence: $$\mathfrak{L}\{F(t), \beta\} = \mathfrak{L}\{F(x), \beta\} \times \mathfrak{L}\{F(y), \beta\}.$$

Now, suppose that the set J to which a single retrospective rating agreement is to be applied, consists of N insurances, $j_1, j_2, \ , \ , j_N$, such that the sums, $t_1, t_2, \ , \ , t_N$, of the modified losses incurred under $j_1, j_2, \ , \ , j_N$, respectively, are independent random variables. Then
$t = t_1 + t_2 + \ldots . + t_N$; and by the foregoing theorem,

$$\mathfrak{L}\{F(t), \beta\} = \mathfrak{L}\{F(t_1), \beta\} \times \mathfrak{L}\{F(t_2), \beta\} \times \ldots . \times \mathfrak{L}\{F(t_N), \beta\}. \quad (26)$$

Thus, if for each of the insurances $j_1, j_2, \ , \ , j_N$, we know the Laplace-Stieltjes transform of the distribution function of the sum of the modified losses incurred thereunder, then $\mathfrak{L}\{F(t), \beta\}$ is immediately given by this simple equation (26), the right side of which may then be substituted for $\mathfrak{L}\{F(t), \beta\}$ in (22) in order to determine the value of any one of the parameters G, A and β as soon as values for the other two have been selected.

The analysis of J into components, $j_1, j_2, \ , \ , j_N$, need not follow traditional lines of insurance classification. For example, suppose J consists of group life insurance, non-occupational disability insurance, and Workmen's Compensation insurance covering all employees of some one corporation. Then j_1 may be defined to be the first and the last of these insurances in so far as they apply to events resulting in the death of one or several employees by reason of which an obligation arises under the Workmen's Compensation Law to pay benefits in addition to those afforded by the group life insurance; j_2

* Cramér [5, 14.5 and 15.3.4].

may be defined to be the group life insurance in so far as it applies to deaths not resulting from events to which j_1 applies; j_3 may be defined to be the non-occupational disability insurance; and j_4 may be defined to be the Workmen's Compensation insurance in so far as it applies to events other than those to which j_1 applies. Under such definitions, the random variables $\mathbf{t}_1$, $\mathbf{t}_2$, $\mathbf{t}_3$ and $\mathbf{t}_4$ may – with negligible error – be considered independent.

Now, let us consider any single one (j_i) of the insurances, $j_1, j_2, \ , \ , j_N$, of which the set J is composed. Let $\mathbf{m}$ be the number of loss-events under j_i, and $\mathbf{v}_1, \mathbf{v}_2, \ , \ , \mathbf{v}_m$ be the modified losses consequent thereto. Suppose (i) that $\mathbf{v}_1, \mathbf{v}_2, \ , \ , \mathbf{v}_m$ are independent and (ii) that—prior to the actual occurrence of any loss-event—their distribution functions are identical, so that the distribution function of any one of them may be represented by $F_i(v)$. Let $\mathbf{t}_i = \mathbf{v}_1 + \mathbf{v}_2 + \ldots + \mathbf{v}_m$, and let $\mathbf{t}_i^{(m)}$ represent the value of $\mathbf{t}_i$ on the hypothesis that $\mathbf{m}$ takes the value m.

Then $\mathfrak{L}\{F(t_i), \beta\} = \mathbf{E}(e^{-\beta \mathbf{t}_i}) = \int_0^\infty \underset{\mathbf{m}=m}{\mathbf{E}}(e^{-\beta \mathbf{t}_i})\, dF(m)$, where $\underset{\mathbf{m}=m}{\mathbf{E}}(e^{-\beta \mathbf{t}_i})$ is the *conditional mathematical expectation* (or *conditional mean value*) of $e^{-\beta \mathbf{t}_i}$) on the hypothesis that $\mathbf{m} = m$,* given by the integral $\int_0^\infty e^{-\beta \mathbf{t}_i^{(m)}} dF(t_i^{(m)})$, which is the Laplace-Steiltjes transform of $F(t_i^{(m)})$. But $\mathbf{t}_i^{(m)}$ is the sum of m independent random variables each of which has the distribution function $F_i(v)$.

Hence, by the Theorem stated above, $\underset{\mathbf{m}=m}{\mathbf{E}}(e^{-\beta \mathbf{t}_i}) = \left[\mathfrak{L}\{F_i(v), \beta\}\right]^m$.

Therefore:
$$\mathfrak{L}\{F(t_i), \beta\} = \int_0^\infty \left[\mathfrak{L}\{F_i(v), \beta\}\right]^m dF(m). \tag{27}$$

Now, suppose (iii) that $\mathbf{m}$ has a Poisson distribution, that is to say, suppose $F(m) = \sum_{k=0}^{m} \frac{(\omega_i)^k}{\lfloor k} e^{-\omega_i}$, where $\omega_i = \mathbf{E}(\mathbf{m})$. Then the Lebesgue-Stietljes integral on the right side of (27) becomes equivalent to the infinite series

* Kolmogoroff [9, Fünftes Kapitel, § 4].

$\sum_{k=0}^{\infty} \frac{(\omega_i)^k}{\lfloor k} e^{-\omega_i}\Big[\mathfrak{L}\{F_i(v), \beta\}\Big]^k$, which converges for all real non-negative values of ω_i and β, to e^{θ_i} where $\theta_i = \omega_i\Big[\mathfrak{L}\{F_i(v), \beta\} - 1\Big]$. (28)

Given any particular set of insurances to which it would be practicable and desirable to apply a retrospective rating agreement, it is generally possible to define the members $j_1, j_2, \ , \ , j_N$, and the terms *loss* and *loss-event* in such a way that (a) the conditions (i), (ii) and (iii) are realized, or can be considered as realized for all practical purposes, in the case of each of the insurances $j_1, j_2, \ , \ , j_N$; * and that (b) the sums, $t_1, t_2, \ , \ , t_N$, of the modified losses incurred under them, respectively, are independent random variables, or can for all practical purposes be so considered. Then each of the transforms on the right side of (26) may be expressed in the form (28), so that we have

$$\mathfrak{L}\{F(t), \beta\} = e^{\theta_1} \times e^{\theta_2} \times \ldots . \times e^{\theta_N} = e^{\Phi}$$

$$\text{where} \quad \Phi = \sum_{i=1}^{N} \omega_i\Big[\mathfrak{L}\{F_i(v), \beta\} - 1\Big]. \qquad (29)$$

Thus we have shown that, if for each insurance (j_i) included in the set J, we have a realiable estimate $(\bar{\omega}_i)$ of the mathematical expectation of the number of loss-events, and reliable estimates, $L_i(\beta)$, of the Laplace-Stieltjes transform of the distribution function of the modified losses each resulting from a single loss-event, for the values of β in which we may be interested, then a reliable estimate of $\mathfrak{L}\{F(t), \beta\}$ for each of those values of β may be obtained by a fairly simple computation, indicated by (29). If, for any particular value of β, Φ' be the estimate of Φ obtained by substituting the estimates $\bar{\omega}_i$ and $L_i(\beta)$ for ω_i and $\mathfrak{L}\{F_i(v), \beta\}$ in (29), and if ϵ be the *actual* error of Φ', then the absolute difference, $\big|\ \mathbf{E}(u) - \bar{u}\ \big|$ will be $Ae^{\Phi'} \times \big|\ 1 - e^{-\epsilon}\ \big|$, when the parameters G and A have been determined so as to satisfy equation (22) with $e^{\Phi'}$ substituted in place of $\mathfrak{L}\{F(t), \beta\}$ and $\bar{u}$ in place of $\mathbf{E}(s) + \mathbf{E}(w) + p$. It is clear, therefore, that the parameters in (15) can be determined in a manner eminently satisfying the condition set forth in the last paragraph of Section B in every case in which sufficiently reliable estimates, $\bar{\omega}_i$ and $L_i(\beta)$ for $i = 1, 2, \ , \ , N$ and for the values of β in which we may be interested, can be obtained with relatively small expenditure of time and effort.

* Concerning (iii), see Bailey [1, Part I, Section A]. A very interesting discussion of imperfect realizations of condition (iii) will be found in Carleton [3]. Note that when (iii) is not realized even approximately, the transform of $F(t_i)$ may still be evaluated in terms of the transform of $F_i(v)$ and the distrition of m by means of (27), provided (i) and (ii) are satisfied.

Such estimates can indeed be so obtained from statistics which are regularly compiled by insurers and their rating organizations, in the cases of a great many sets of insurances to which it is practicable and desirable to apply retrospective rating agreements. For example, let us consider Workmen's Compensation and Employers' Liability insurances, with respect to which we may assume that only those rules of the form (7) in which $\lambda_k = \Lambda$ for every k, will be of any interest. Each such insurance with respect to operations falling within several Manual classes may be considered to be several differennt insurances, one (j_i) for each such class. Then $\bar{w}_i$ may be obtained by multiplying the exposures of the ith class for the risk in question, by the observed mean accident frequency per unit of exposure for all exposures of that class within—say—the three latest Policy Years, modified in accordance with an appropriate individual risk experience rating procedure.

For each jurisdiction, all Manual classes could be divided into a small number of groups, e.g., five groups, such that the distribution function of actual losses each resulting from a single accident arising out of operations of any one class, could be shown to be approximately the same as the distribution function of actual losses resulting from a single accident arising out of operations of any other class in the same group. For each such group, a table of values of the transform $\mathfrak{L}\{F(v), \beta\}$ [where $F(v)$ is the distribution function of modified losses each resulting from a single accident, which is characteristic of that group] could be constructed for values of β ranging from 0 to .01 and for each of several values of Λ, e.g., \$5,000, \$10,000, \$15,000, \$20,000 and ∞. Values of $\beta > .01$ can be shown to be of no practical interest. For small values of β, $\mathfrak{L}\{F(v), \beta\}$ could be very satisfactorily approximated by polynomials of the form (25) with v in place of t. The construction of such tables would not be a very laborious task, especially if it were found that for some groups the distribution of v could be satisfactorily approximated by a frequency function of the form (23). From such tables, the estimtes $L_i(\beta)$ could be read directly or determined by interpolation.

Finally, it may be asserted that no rule comparable in simplicity to either (26) or (29) can be stated for the evaluation of $\Xi(g)$ in terms of statistics relative to individual members of a set consisting of several different insurances. Therefore, when any such set is to be the subject of a single retrospective rating agreement, the determination of the parameters in a sectionally linear formula of the type (5) or (12) to a prescribed degree of accuracy, will necessarily involve a greater expenditure of time and effort than the determination of the parameters in a non-linear formula of the type (15), to the same degree of accuracy.

BIBLIOGRAPHY

1. A. L. Bailey, *Sampling Theory in Casualty Insurance, Introduction and Parts I and II*, P.C.A.S. Vol. XXIX, No. 59 (1942) pp. 50-93.

2. ———— *Sampling Theory in Casualty Insurance, Parts III-VII*, P.C.A.S. Vol. XXX, No. 60 (1943) pp. 31-65.

3. John Carleton, *Non-Random Accident Distributions and the Poisson Series*, P.C.A.S. Vol. XXXII, No. 62 (1945) pp. 21-26.

4. T. O. Carlson, *An Actuarial Analysis of Retrospective Rating*, P.C.A.S. Vol. XXVIII, No. 58 (1942) pp. 283-323.

5. Harald Cramér, *Mathematical Methods of Statistics*, Princeton: Princeton University Press, 1946.

6. ———— *Random Variables and Probability Distributions*, London: Cambridge University Press, 1937.

7. Paul Dorweiler, *Observations on Making Rates for Excess Compensation Insurance*, P.C.A.S. Vol. XIII, No. 28 (1927) pp. 154-180.

8. ———— *On Graduating Excess Pure Premium Ratios*, P.C.A.S. Vol. XXVIII, No. 57 (1941) pp. 132-157.

9. A. Kolmogoroff, *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Berlin: 1933; New York: Chelsea Publishing Co., 1946.

10. F. S. Perryman, *Possible Values for Retrospective Rating Plans*, P.C.A.S. Vol. XXXI, No. 61 (1944) pp. 5-34.

11. Stefan Peters, *Discussion of Dorweiler* [8], P.C.A.S. Vol. XXVIII, No. 58 (1942) pp. 588-590.

12. J. V. Uspensky, *Introduction to Mathematical Probability*, New York and London: McGraw-Hill, 1937.

13. N. M. Valerius, *Risk Distributions Underlying Insurance Charges in the Retrospective Rating Plan*, P.C.A.S. Vol. XXIX, No. 59 (1942) pp. 94-121.

SEASONAL FLUCTUATION IN LOSS RATIOS FOR AUTOMOBILE BODILY INJURY COVERAGE

BY

JOHN W. CLARKE

Actuaries, underwriters, executives, claim adjusters, agents, and others connected with the business of casualty, indemnity, and fire insurance have long recognized that the number and amount of claims rise and fall with the seasons of the year. A glance at the figures reported monthly for fire claims by the National Board of Fire Underwriters shows the seasonal swing on top of the general increase in burning rate that has plagued the fire insurance business for the past few years. Companies writing accident and health insurance and using a notice average reserve frequently use a lower figure for average cost per claim during the winter months than during the spring and summer months. This paper attempts to analyze the claim figures for automobile bodily injury liability to determine the quantitative as well as the qualitative effect of the seasonal variation in a particular line of insurance. The paper is divided into two chief parts, the development of adjustment factors, and the application of these factors.

DEVELOPMENT OF ADJUSTMENT FACTORS:

The most striking figures that show the seasonal fluctuation in automobile accidents are the figures for fatalities each month put out by the National Safety Council. Table I below brings the situation to the fore:

TABLE I

Percentage of Fatalities in Automobile Accidents in the United States by Month
National Safety Council

	Per cent of Total				*Per cent of Total*				*Per cent of Total*		
Month	*1939-41*	*1944*	*1947*	*Month*	*1939-41*	*1944*	*1947*	*Month*	*1939-41*	*1944*	*1947*
Jan.	7.6	9.8	7.4	May	7.3	7.3	7.8	Sept.	9.3	8.2	8.9
Feb.	6.2	7.8	6.6	June	7.6	6.9	7.5	Oct.	10.1	9.0	9.3
March	6.8	7.8	7.1	July	8.5	6.9	8.6	Nov.	10.1	9.8	9.4
April	6.8	7.3	7.3	Aug.	9.0	7.8	9.5	Dec.	10.7	11.4	10.6

The years were selected as typical prewar, wartime, and postwar experiences. The prewar pattern of 1939-41 is repeated in 1947 except that the proportions for January and December are low in 1947. The 1944 figures show the effect of

the wartime curtailment of summer (non-essential) driving. The National Safety Council lays great stress on the daylight versus darkness driving in the summer and winter months. Pedestrian deaths, for example, reached a minimum at 23% below the monthly average for the summer months of 1947. In the late fall, short days brought darkness during the peak traffic hours and pedestrian deaths reached a maximum at 42% above the monthly average.

Although mileage increases during the summer months, the death rate related to miles traveled decreases in the summer. The death rate per 100,000,000 vehicle miles in 1947 varied from about 9.2 in January to a minimum of 7.3 in June to a maximum of 10.2 in November and fell slightly to 10.0 in December.

These figures indicate very clearly the existence of the seasonal trend. They are, however, of little use to an insurance company because they do not distinguish insureds exposed and because fatalities are a small proportion of automobile liability claims. Therefore, the figures of one insurance company have been analyzed to determine the trend in insurance claims. These figures will be used in the balance of this paper.

The company whose figures are being analyzed pays its claims under the automobile bodily injury liability coverage through a network of claim adjusting offices spread over the United States and the southern part of Canada. The Canadian business represents about two per cent. of the total. Claims are reported to these offices and the adjusters there make up a file and report to the home office. These reports to the home office must be submitted within 30 days and are usually submitted sooner. At the time of the report the adjuster estimates the amount the company can expect to pay on each individual claim. A series of tests over the years has proved these estimates to be reasonably accurate in the aggregate. The response of estimates to increasing cost per claim has been somewhat slow, especially during 1948. Further discussion of this factor, showing the negligible effect, is given later in the section entitled "further comments on adjustment factors."

When the adjuster's report is received in the home office, the actuarial department prepares a punched card. The charge date to incur the claim on the books is determined by the date punched on this card. All reports received in a given month are punched as of that month. The claims reported plus an estimated figure for claims incurred but not reported equal the incurred claims for the month and are so reported in the monthly financial statements.

The basic raw data were recorded from the actuarial punched cards. The years 1937-41 give the last prewar experience. The years 1946 and 1947 are years in which the exposure increased during the year as additional business was written. During 1948, the written premiums continued to increase but much of the increase was from rate adjustment rather than new policies. These raw data show immediately the seasonal fluctuation. They are not, however, of much value for determining factors to damp this seasonal fluctuation. Therefore, an adjustment was made in the figures to compensate for the different number of days in different months. Ratios were then calculated for the adjusted figures to the average for the appropriate period. These ratios show the seasonal trend even more clearly.

Two facts inherent in the data should be considered for their possible effect on the results. The company writes a substantial volume of Massachusetts

statutory coverage. This business all expires on December 31 of each year but it does not renew automatically on January 1 of the following year. This produces less exposure in the early months of the year. The second factor is the increasing exposure by number of policies in general.

A check was made of the effect of increasing exposure by number of policies for the years 1947 and 1948. The ratio of the number of claims actually reported to the number of policies in force was computed each month. During 1947, the number of policies increased from 708,000 at the end of January to 858,000 at the end of December, and during 1948 from 843,000 to 927,000. The increase thus fell off from 21% in 1947 to 10% in 1948. The effect of the Massachusetts statutory business expiring in December and failing to renew at once is seen in the drop of 15,000 policies in force from December 1947 to January 1948. A special study of the Massachusetts figures shows, taking the number for twelve months of 1948 at 100%, that only 58% were on the books at the end of January and 76% at the end of February. This shows the powerful influence of the slow renewal of that business. Because January and February are high claim months, this slow renewal holds down the January and February claim figures.

The ratio of premium on the policies in force to the number of policies in force shows a steady increase over the twelve months. These figures represent all business in force during the month, not new business paid during the month. This amount of premium increased from $22,185,000 at the end of January, 1947 to $31,320,000 at the end of December, 1947, and during 1948 from $30,837,000 to $37,370,000. Again the percentage increase fell from 41% to 21%.

The real financial effect on a company's operations is wrought by the amounts rather than the number of claims where the amount of a claim exceeds substantially the expense of paying it. The ratios by amounts for the periods 1937-41 and 1946-48 were analyzed to obtain correction factors to reduce the effect of seasonal fluctuation. These ratios were graduated by the Henderson-Whittaker "A" Difference Equation method to smooth the chance fluctuations. The value of a=1 in the formula was chosen to minimize the smoothing effect and maximize the reproduction of the underlying curve. The graduating equations were:

$$u'_x = u'_{x-1} - \tfrac{1}{3}\, u'_{x-2} + \tfrac{1}{3}\, u''_x$$

$$u_x = u_{x+1} - \tfrac{1}{3}\, u_{x+2} + \tfrac{1}{3}\, u'^x$$

where u" is the ungraduated function, u' an intermediate series, and u the final graduated value. This method produced satisfactory values. Several graduations by fitting to forms of curves distorted the data badly. Graphic graduation reproduces substantially the data determined by the difference equation.

Ratios were computed for the periods 1937-41 and 1946-48 for the monthly values in the raw data by amounts. The data adjusted for the number of days in the month and graduated gives the pattern to be expected in the parent universe of the sample with equal divisions. The difference between the adjusted and the graduated adjusted ratios is the correction toward the true picture with equal intervals. Application of these differences to the ratios in the raw data

gives the true pattern with intervals of actual months. The final answer was rounded to the nearest whole per cent. These figures are given in Table II.

TABLE II

Graduated and Corrected Monthly Amount Ratios
Automobile Bodily Injury Liability

	Graduated Adjusted Ratios		*Basic Ratios*		*Final Corrected Ratios*	
Month	*1937-41*	*1946-48*	*1937-41*	*1946-48*	*1937-41*	*1946-48*
Jan.	110.4	112.3	112.4	114.7	113	114
Feb.	103.7	105.8	99.5	100.2	96	98
Mar.	97.0	99.5	101.1	103.4	99	101
Apr.	91.6	94.2	80.9	88.7	91	93
May	89.5	91.4	93.0	89.5	91	93
June	89.6	91.2	89.2	90.9	88	90
July	91.4	92.6	92.1	95.9	93	94
Aug.	94.7	95.0	97.6	96.8	97	97
Sept.	98.9	98.3	100.3	94.2	98	97
Oct.	103.9	102.3	104.4	112.4	106	104
Nov.	110.5	106.1	102.9	94.8	109	105
Dec.	118.9	111.5	126.7	118.3	121	114

The prewar period ratios show a much greater variation than the postwar period ratios. The pattern, however, is very much the same. Although the data for 1937-41 show a variation of 33 points from lowest to highest against 24 points for the 1946-48 data, each has a minimum in June and a maximum in December in the final corrected ratios. The pattern and size of the figures indicate that adjustments made on the basis of the 1946-48 figures would be conservative. These 1946-48 figures will be used in discussing the effect of this seasonal fluctuation on statement figures.

APPLICATION OF ADJUSTMENT FACTORS:

Each insurance company licensed to operate in one or more states must file with the state authorities an annual statement. One item in this statement is the liability for unearned premiums. This liability is computed in general as the pro-rata portion of the premiums from the date of the statement, December 31, to the date of expiry of the policy then in force. The reserve for unearned

premiums computed on a straight pro-rata basis may be overstated or understated, depending on the date of the statement and the distribution of business. A policy written in June, for example, has had some over-average and some under-average loss months on December 31, with an approximately equal share of each yet to come. A policy written in September has, however, on December 31 already passed through three of its worst months and has all its good months yet to come. Clearly the liability for its unearned premium is overstated.

On the assumptions that a company writes the same amount of business in each month and calculates its unearned premium reserve by assuming that all policies written in a month are written in the middle of the month, a set of figures has been calculated to adjust the unearned premium reserve at the end of each month. The first set of figures is gross, assuming that the loss dollar and the expense dollar can be released together. The second set of figures assumes a normal 53% loss ratio and releases only the loss dollar. The 53% is based on the Bureau permissible loss ratio at 55.4% less 2.4% for allocated loss expense included. These figures are listed in Table III.

TABLE III

Ratios Percent to Apply to Unearned Premium Reserve at End of Month Indicated to Adjust for Seasonal Fluctuation Automobile Bodily Injury Liability

Month	*Gross Ratios*	*Corrected to Normal 53% Loss Basis*
Dec.	−0.8333%	−0.4417%
Jan.	−3.1667	−1.6783
Feb.	−2.8333	−1.5017
Mar.	−3.0000	−1.5900
Apr.	−1.8333	−0.9717
May	−0.6667	−0.3533
June	1.0000	0.5300
July	2.0000	1.0600
Aug.	2.5000	1.3250
Sept.	3.0000	1.5900
Oct.	2.3333	1.2367
Nov.	1.5000	0.7950

As a test of the difference between the theoretical and an actual distribution of business, the adjustment was calculated as of December 31, 1948 for the company whose figures are being analyzed. This company calculates its unearned premium on the assumption that the business written in a month averages to the middle of the month. It writes a large amount of Massachusetts business, all expiring December 31, but is otherwise normal. The adjustment amounted to a reduction of about $111,000 in the unearned premium reserve, or 0.676 per cent. on the gross basis. This was a reduction of about $58,800 or 0.36 per cent. on the normal 53% loss ratio basis.

This reduction in the unearned premium reserve anticipates a profit yet to be realized. It is fallacious only if the experience does not develop as expected and as cancelations on a pro-rata or short term rate basis may change the picture. As to the development of the experience, the comparison of the 1946-48 figures used with the 1937-41 figures makes the adjustment look conservative. The cancelation problem is fluid. As of January 1, a cancelation of any policy that runs up to but not beyond May 31 produces a profit and a cancelation of a policy running beyond May 31 produces a loss.

Although this type of adjustment is proper on theoretical grounds and can be applied easily as part of the procedure of calculating the reserve for unearned premium, the small relative size of the change produced vitiates its value for this purpose. Application of such an adjustment in the annual statement would require a change in the rules for calculating unearned premium. The amount of the adjustment in statement reserves does not warrant any consideration of a change in rules.

A more important use for the adjustment is to reduce fluctuations in monthly loss ratios because of normal seasonal variation so that any real non-seasonal trend can manifest itself. All insurance men are familiar with the trend in the automobile loss ratio—bad in the winter, good in the summer. All make mental adjustments in looking at the bare figures. The percentages here derived can be used as a quantitative correction.

The simplest and most direct application is to increase or decrease the earned premium for the month by the per cent. that the corrected ratio differs from 100% or by .53 times such per cent. The revised loss ratio of claims to premium computed on this basis takes account automatically of increases in business by the writing of more policies. It debits or credits a portion of any rate adjustments effective in the earned premium. Another method is to adjust the losses themselves. The best way to do this seems to be to compute a normal loss at 53% of the earned premium, obtain the correction as a per cent. of the normal loss, and compute the adjusted amount of claim as actual plus or minus the correction. These adjustments are not intended for use in financial statements but as underwriting, rate-making, and production guides. The adjustments must themselves be watched to see that they sum to a small figure for the whole year.

Each of these adjustments was applied to the monthly figures of the company being analyzed and the results are given in Table IV. The net adjustment on claims was a net increase of $12,000 on $20,000,000 or about 0.06 per cent. The premium adjustment on the basis of the .53 modification amounted also to about $12,000 on over $37,000,000 or about 0.03 per cent.

TABLE IV

Monthy Loss Ratio Adjusted for Seasonal Fluctuation
Automobile Bodily Injury Liability

Month	*1948 Loss Ratio*	*Loss Ratio Claims Adjusted*	*Loss Ratio Premiums Adjusted*
Jan.	66.2	58.7	61.6
Feb.	50.9	52.0	51.4
Mar.	57.3	56.8	57.0
Apr.	52.9	56.6	54.9
May	48.6	52.3	50.4
June	52.2	57.5	55.2
July	49.3	52.5	51.0
Aug.	46.8	48.4	47.5
Sept.	54.9	56.5	55.8
Oct.	51.7	49.6	50.6
Nov.	51.3	48.6	49.9
Dec.	60.7	53.3	56.5
Total	53.4		

The loss ratios in Table IV include an adjustment for claims incurred but not reported and are after all adjustments for changes and corrections. The effect of rate increases and other improvements in the automobile bodily injury liability line is clear from all three columns. It is startlingly clear from the second and third columns. The spread of the loss ratio is reduced from 19.4 points to 14.1 points in the premium adjustment and only 10.3 points in the claim adjustment.

The value of figures of this kind to an underwriter should be great. Unlike the life insurance business where the sins and graces of the father are usually visited upon the son, the casualty business reacts quickly to underwriting judgment and adequacy of the rate structure. With these figures, the underwriter can go a long way toward eliminating one variable from his study of how good his judgment has been.

More important than the question of underwriting profit and loss itself is the question of capacity. Many casualty and fire companies in the past few years have been plagued by the reduction in surplus from financing new business. Increased rates for a time further aggravate this problem before they relieve it. A basic question in establishing an underwriting program for the next year becomes how much new business can be financed out of surplus and how much will that surplus be. The bare loss ratio may well lead to an excess of optimism or pessimism according to the time of the year at which it is examined. True, a study of the cumulative loss ratio will reduce the tendency to excess, but this cumulative ratio is often studied along with the ratio for the last few months as a guide for projection. Specifically on the figures in Table IV, the months of October and November are really better than they look, and December is only normal instead of bad. This type of figure is particularly important when the figures of the last few years are distorted and require a great deal of mental adjustment before they can be used for comparisons.

FURTHER COMMENTS ON ADJUSTMENT FACTORS:

During the course of the study of seasonal fluctuations of claims, many other analyses of the figures were made beside those reported here. The figures by number showed the same pattern as the figures by amount but to a lesser degree. As would, therefore, be expected, the figures for average amount of claim showed a similar pattern overlaid in the post-war experience on the general increase. The only startling item was a large average amount in January 1948 that offset a small number. This appears to be a normal accidental fluctuation in data of this size.

The statement was made earlier in this paper that the exposure was increasing during the period because of increasing numbers of policies and increasing average size. The ratios of claims to policies in force by number again reveal the general seasonal pattern of lower in the summer and higher in the winter. The ratio by number is only part of the picture, and figures for average size claim were also studied. These, too, show the seasonal pattern. Hence the product of number and amount shows a definite pattern. Figures for average annual premium on the policies in force by month show the trend of the increase. The major part of this increase is, of course, rate changes. Some part may be due to an increase in the number of cars per policy.

The figures used to compute the adjustment ratios were based on adjusters' original estimates. Mention was made earlier that these estimates had fallen behind the rising claim cost in the post-war experience. These estimates are changed by the adjusters as development of the claim picture warrants. The changes, however, are recorded in the month received rather than related back to the month in which the claim is first reported. Thus the effect of the changes on the record because of the way it is compiled is not a true seasonal effect. The average claim after changes was about 10% greater in 1946 and 1947 and about 15% greater in 1948. Calculations similar to those made for the reported figures were made for the figures after correction for changes reported in the month. The results were similar. The figures for the graduated ratios of the claims adjusted for the number of days in the month are set out in Table V.

TABLE V

Comparison of Reported and Changed Estimates
Automobile Bodily Injury Liability

Month	*Graduated Monthly Original Report*	*Ratios 1946-48 After Changes*
Jan.	112.3	111.2
Feb.	105.8	105.2
Mar.	99.5	99.3
Apr.	94.2	94.0
May	91.4	90.8
June	91.2	90.1
July	92.6	91.7
Aug.	95.0	95.2
Sept.	98.3	99.6
Oct.	102.3	103.7
Nov.	106.1	107.2
Dec.	111.5	111.5

Both sets of figures produce the same answer within the limits of accuracy of the method and the data. Hence either set could be used. The choice was made in favor of original reports for several reasons. The original report picture is clear and shows the true seasonal effect. The actuary watches the changes and makes corrections for his estimates of incurred claims as the changes indicate. These changes are a measure of the adjusters' inability to evaluate claims accurately in the initial stages of development, particularly during a period of changing economic conditions. The prewar experience shows adjusters do very well in a reasonably stable claim situation. The correction adjustments are calculated to apply to the total figure for incurred claims including the incurred but not reported. Hence the use of adjustments based on the corrected estimates would produce a double correction when the actuary corrects his estimates for the same effect. The question is academic because the figures are so close, but the reasoning may be of some interest.

CONCLUSION:

The effect of the adjustment for seasonal fluctuation in claims applied to the unearned premium reserve is too small to be of any practical importance. The effect of the adjustment in bringing out the true picture of the change in loss ratio by month may have considerable value for underwriting and pro-

duction planning. Any company using such figures would probably prefer to calculate such an adjustment on the basis of its own experience. The National Safety Council makes a big point of the relation between the hours of dusk and the hours of heavy vehicle and pedestrian traffic. The company whose figures have been analyzed does not have monthly state records in satisfactory form and amount for a geographic analysis. Perhaps this might be undertaken by companies that write business principally in the north or south of the country.

The author wishes to express his gratitude to Mr. H. T. Barber who suggested the original study and to Mr. Barber and Mr. T. F. Tarbell for helpful criticism in preparing this paper.

FURTHER REMARRIAGE EXPERIENCE

BY

ROBERT J. MYERS

Until recent years the only actuaries concerned with remarriage rates were those who dealt with workmen's compensation insurance where monthly survivor benefits are frequently paid to widows subject to their not remarrying. Of course, others such as demographers and sociologists have been interested in remarriage data. Also, to some extent, this subject has been of concern to actuaries and tax experts involved in inheritance matters.

The classic work on remarriage experience in American actuarial literature is "An American Remarriage Table" by William F. Roeber and Ralph M. Marshall, *Proceedings*, Vol XIX, page 279. This paper analyzed insurance company data in regard to workmen's compensation for the period 1921-29[1] which had been collected especially to study remarriage experience. From this analysis there was prepared the American Remarriage Table, with mortality rates from the United States White Females Life Table for 1910[2]. The American Remarriage Table is on a select and ultimate basis (5 year select period).

Since the passage of the 1939 Amendments to the Social Security Act and the 1946 Amendments to the Railroad Retirement Act, actuaries engaged in social insurance work have been concerned with the element of remarriage as these two systems now provide monthly payments to widows subject to their not remarrying. One of the studies that was undertaken to "modernize" the American Remarriage Table involved revising the mortality basis. A number of these revised tables were prepared for internal use in the Social Security Board (now the Social Security Administration). Later when the Railroad Retirement Board became interested in this subject, its staff also prepared certain modifications. One of these is published along with certain commutation columns in "A Revised American Remarriage Table" by Abraham M. Niessen, *Record of the American Institute of Actuaries*, Vol. XXXVIII, page 5. This revised table is based on remarriage probabilities which are 50% higher than those in the American Remarriage Table, with this increase being graded down beyond age 54, until for ages 59 and over the original rates were used. At the same time the mortality basis for the revised table was derived from U.S. white female mortality in 1945, increased to allow for a presumed excess mortality of widows as compared with the general female population.

[1] The relatively insignificant amount of data for durations at widowhood beyond the 6th year was not used, and the sizable experience for the 6th year was considered to be "ultimate."

[2] Actually this table is based on the 1910 Census as to the population exposed to risk, combined with the deaths in 1909-11.

This paper will present the results of several studies on remarriage experience made by the author over the course of the past few years. Included are analyses of actual remarriage experience under the Employees' Compensation system, which is in effect the workmen's compensation program for Federal Government employees, and under the Old-Age and Survivors Insurance system. In addition, a set of two modified remarriage tables will be presented, along with commutation columns, using as bases the remarriage rates of the American Remarriage Table and mortality rates for the U.S. white female population for 1939-41.

REMARRIAGE EXPERIENCE UNDER EMPLOYEES' COMPENSATION SYSTEM

The Employees' Compensation program, which was formerly administered by the Employees' Compensation Commission—an independent agency—but is now a Bureau within the Federal Security Agency, provides in effect workmen's compensation benefits for Federal Government employees. In the *14th Annual Report of the Commission* (covering fiscal year 1930) and in the *30th Annual Report of the Commission* (covering fiscal year 1946), there are set forth various statistical data which can serve as a basis for a study of their remarriage experience.

The more extensive data in the 30th Annual Report had been compiled such that the tabulated exposure for the first year of widowhood was not a full year, and thus preliminary studies which incorrectly assumed a full year of exposure appeared to show that the actual experience was much lower than the expected for that year. In brief, the difficulty was that the exact exposure in each case was rounded to the nearest half year and then tabulated so that there resulted an exposure at duration zero (representing exposure for the first ¼ year), an exposure at duration ½, an exposure at duration 1, etc. When the first two groups were combined to yield the exposure for less than 1 year of widowhood, an incorrect result was produced, since actually the exposure included only the first ¾ of a year of widowhood; correspondingly, what was tabulated as second year of widowhood was really from duration ¾ to duration 1¾, etc. While this made a serious understatement for the first year of widowhood, the exposure being for only ¾ year instead of a full year, the error was not as great for the succeeding years because there was a full year of exposure in each, although durations since widowhood were ¼ of a year misplaced. From the actual tabulations[3], which were subdivided into the exposure to the nearest half year, it was possible to approximate the exposure on a correct full year basis for each duration, and this has been done for the succeeding analysis.

Before proceeding with the analysis of the data described above, which covers the period from 1916 (when the system began) to the middle of 1945, there will first be analyzed the earlier experience, namely, that from 1916 to the end of 1929, as published in the 14th Annual Report. These data were given in very detailed form, so that it was possible to build up the exposures on a full-year basis. The experience has been traced through on a select basis

[3] Made available through the courtesy of Mr. Edward F. Brayer, Chief Statistician, Bureau of Employees' Compensation, Federal Security Agency.

for the first five years of widowhood for various age groups at widowhood, and is thereafter on an ultimate basis by attained age with all durations combined. However, for the latter it was necessary to redistribute the data and arrange them in the age groups shown; this was done by osculatory interpolation.

Table 1 compares the actual remarriage experience for the period 1916-29 with that expected according to the American Remarriage Table, showing both actual remarriages and the ratio of actual to expected. By coincidence, the ratio of actual remarriages to those expected was 100% for all ages and durations combined. However, although the American Remarriage Table gave such close correspondence in the aggregate, there were significant differences by age and duration. The ratio was considerably below 100% for durations 0 and 1, but on the other hand was almost 150% for the relatively limited experience in the ultimate years. Considered by age, the ratio of actual to expected tended to be considerably higher for the younger ages. Thus, for the select period, the ratio was about 100% for ages at widowhood up to 30, about 90% for ages 31 to 45, and appreciably below 50% thereafter; correspondingly, in the ultimate durations the ratio was close to 200% at the youngest ages and somewhat lower at the middle ages (there was practically no ultimate data beyond age 50).

Table 1

U.S. EMPLOYEES' COMPENSATION SYSTEM
COMPARISON OF ACTUAL REMARRIAGE EXPERIENCE WITH THAT EXPECTED ACCORDING TO THE AMERICAN REMARRIAGE TABLE, 9/7/16 TO 12/31/29

Age at entry	*Duration since husband's death*						*Age attained*	*Duration 5 or more*
	0	*1*	*2*	*3*	*4*	*0-4*		
	Actual Remarriages[a]							
Under 21	3	11	7	1	5	27	21-25	2.3
21-25	6	12	16	9	15	58	26-30	14.9
26-30	5	17	15	15	6	58	31-35	23.7
31-35	4	11	11	11	4	41	36-40	15.8
36-40	4	9	5	5	1	24	41-45	7.9
41-45	2	4	5	1	2	14	46-50	8.4
46-50	–	2	1	–	1	4	51-55	2.0
51-55	1	–	–	–	–	1	56-60	.9
56-60	–	1	–	–	–	1	61-65	.1
61 & over	–	–	–	–	–	–	66 & over	–
Total	25	67	60	42	34	228	Total	76.0

Ratio of Actual to Expected Remarriages

Under 21	81%	128%	137%	27%	250%	117%	21-25	153%
21-25	63	57	113	81	263	94	26-30	194
26-30	56	84	111	134	103	97	31-35	199
31-35	61	73	103	120	83	88	36-40	136
36-40	103	103	83	94	86	90	41-45	93
41-45	87	77	135	31	118	87	46-50	179
46-50	—	69	50	—	100	45	51-55	77
51-55	143	—	—	—	—	20	56-60	56
56-60	—	125	—	—	—	38	61-65	10
61 & over	—	—	—	—	—	—	66 & over	—
Total	66	79	104	88	137	90	Total	146

[a]Actual remarriages for duration 5 or more are shown with one decimal since data had to be redistributed (by osculatory interpolation) so as to be in age groups shown.

Note: Total actual remarriages for all ages and durations combined: 304 with ratio of actual to expected of 99.8%.

Next, considering the data for the period up to 1945, there again had to be a redistribution of the ultimate data, as in the previous table. In addition there was the redistribution of all the data as described previously, so as to correct for the method of duration classification. Table 2 summarizes data on the remarriage experience for the 29-year period, while a comparison of the scope and extent of these data with those underlying the American Remarriage Table are as follows:

Item	*American Remarriage Table*	*Employees' Compensation*	
		1916-29	*1916-45*
Widows in Experience	10,699	1,915	4,922
Remarriages	1,187	304	940
Deaths	363	127	602
Exposure (life-years)	37,040	11,500	47,300

For all ages and durations combined, the ratio of actual to expected remarriages was about 106%, or somewhat higher than for the period up to 1929; in part, this was due to relatively more of the experience being in the ultimate years. The ratio for the select period was 99%, as against 90% for the earlier experience, while the ratio for the ultimate period was 121%, as contrasted with 146%. In general, the experience for each year of the select period closely approximated 100%, with deviations therefrom appearing to be random. Considering the select period by age, it may be seen that the ratio

was significantly higher for those widowed up to age 25, being about 125%; on the other hand, for those widowed beyond age 45 the ratio was quite low, being roughly 50%. Next, considering the ultimate data, there is the same tendency for a very high ratio (about 150%) for attained ages through 40, and ratios of about 100% thereafter, with again a very low ratio at the highest ages.

Table 2

U. S. EMPLOYEES' COMPENSATION SYSTEM
COMPARISON OF ACTUAL REMARRIAGE EXPERIENCE WITH THAT EXPECTED ACCORDING TO THE AMERICAN REMARRIAGE TABLE, 9/7/16 TO 6/30/45

Age at entry	*Duration since husband's death* 0	1	2	3	4	0-4	*Age attained*	*Duration* 5 or more
				Actual Remarriages[a]				
Under 21	11.0	24.0	17.5	4.5	10.5	67.5	21-25	12.5
21-25	27.5	47.0	44.5	33.0	25.0	177.0	26-30	33.6
26-30	16.0	39.5	43.0	33.0	19.0	150.5	31-35	63.3
31-35	15.0	31.5	23.5	17.5	14.0	101.5	36-40	77.2
36-40	14.5	23.5	14.0	12.0	9.0	73.0	41-45	43.5
41-45	5.0	8.0	9.0	7.0	6.5	35.5	46-50	38.0
46-50	1.5	4.5	4.5	2.5	1.0	14.0	51-55	19.5
51-55	2.0	—	1.5	1.5	1.5	5.5	56-60	11.8
56-60	.5	1.5	—	—	1.0	3.0	61-65	7.0
61 & over	.5	.5	—	—	—	1.0	66 & over	5.1
Total	93.5	180.0	157.5	111.0	86.5	628.5	Total	311.5
				Ratio of Actual to Expected Remarriages				
Under 21	121%	123%	151%	54%	233%	127%	21-25	291%
21-25	117	92	135	128	195	121	26-30	152
26-30	65	72	117	111	121	93	31-35	136
31-35	91	87	94	82	120	92	36-40	147
36-40	138	102	89	88	118	103	41-45	98
41-45	82	58	94	82	135	83	46-50	114
46-50	45	61	85	53	37	60	51-55	91
51-55	111	—	52	56	33	42	56-60	85
56-60	56	75	—	—	143	48	61-65	80
61 & over	62	29	—	—	—	19	66 & over	55
Total	96	84	111	95	138	99	Total	121

[a]Actual remarriages are shown with one decimal since data had to be redistributed to allow for method of classifying by duration (see text), while data for duration 5 or more had to be further redistributed as described in footnote a of Table 1.

Note: Total actual remarriages for all ages and durations combined: 940 with ratio of actual to expected of 105.7%.

Table 2a summarizes the remarriage experience for the period 1930-45, being in effect obtained by subtraction of Table 1 from Table 2. As would be expected, the ratio of actual to expected remarriages in the select period is slightly above 100%, while that for the ultimate period is about 115%. Thus, it might be said that the most recent remarriage experience is about 10% above the American Remarriage Table regardless of duration, although by age at widowhood the actual is materially above the expected at the youngest ages.

Table 2a

U. S. EMPLOYEES' COMPENSATION SYSTEM
COMPARISON OF ACTUAL REMARRIAGE EXPERIENCE WITH THAT EXPECTED ACCORDING TO THE AMERICAN REMARRIAGE TABLE, 1/1/30 TO 6/30/45 (BY SUBTRACTION)

Age at entry	*Duration since husband's death* 0	1	2	3	4	0-4	*Age attained*	*Duration 5 or more*
				Actual Remarriages[a]				
Under 21	8.0	13.0	10.5	3.5	5.5	40.5	21-25	10.2
21-25	21.5	35.0	28.5	24.0	10.0	119.0	26-30	18.7
26-30	11.0	22.5	28.0	18.0	13.0	92.5	31-35	39.6
31-35	11.0	20.5	12.5	6.5	10.0	60.5	36-40	61.4
36-40	10.5	14.5	9.0	7.0	8.0	49.0	41-45	35.6
41-45	3.0	4.0	4.0	6.0	4.5	21.5	46-50	29.6
46-50	1.5	2.5	3.5	2.5	—	10.0	51-55	17.5
51-55	1.0	—	1.5	1.5	.5	4.5	56-60	10.9
56-60	.5	.5	—	—	1.0	2.0	61-65	6.9
61 & over	.5	.5	—	—	—	1.0	66 & over	5.1
Total	68.5	113.0	97.5	69.0	52.5	400.5	Total	235.5
				Ratio of Actual to Expected Remarriages				
Under 21	148%	119%	162%	76%	220%	135%	21-25	364%
21-25	152	117	152	164	141	141	26-30	130
26-30	71	65	121	97	131	91	31-35	114
31-35	112	96	88	53	145	94	36-40	150
36-40	159	101	92	83	167	111	41-45	99
41-45	79	47	68	113	145	81	46-50	103
46-50	71	56	106	86	—	69	51-55	93
51-55	91	—	83	88	50	56	56-60	89
56-60	100	42	—	—	250	54	61-65	90
61 & over	125	62	—	—	—	37	66 & over	61
Total	115	88	115	99	139	105	Total	115

[a]Actual remarriages are shown with one decimal since data had to be redistributed to allow for method of classifying by duration (see text), while data for duration 5 or more had to be further redistributed as described in footnote a of Table 1.

Note: Total actual remarriages for all ages and durations combined: 636 with ratio of actual to expected of 108.8%.

It did not seem worth while conducting a mortality investigation for the experience in the period through 1929 because of the small amount of data and, more important, because the data were grouped broadly at the older ages (i.e., age 61 and over) where most of the deaths occurred so that any significant analysis was virtually impossible. Since the tabulations were obtained in detail in regard to the period through 1945, it was possible to obtain adequate data at the advanced ages for this period.

Table 3 compares the actual aggregate (i.e., without regard to duration of widowhood) mortality experience for the period 1916-45 with the expected according to two U. S. population life tables. Probably the most appropriate table covering this period is that for U. S. White Females for 1930-39, since this fell within the period although toward the latter end (which seems desirable since the benefit roll built up steadily over the period, and thus the latter years should be more heavily weighted). In addition, comparison has been made with the U. S. White Females Table for 1939-41 which is the most recent, complete official life table.

Table 3

U. S. EMPLOYEES' COMPENSATION SYSTEM
COMPARISON OF ACTUAL MORTALITY EXPERIENCE WITH THAT EXPECTED ACCORDING TO TWO U. S. POPULATION LIFE TABLES, 9/17/16 TO 6/30/45

Attained Age	*Actual Deaths*[a]	*Ratio of Actual to Expected Deaths According to U. S. White Females Table for*	
		1930-39	*1939-41*
Under 31	24.2	198%	288%
31-35	11.9	80	108
36-40	27.1	116	153
41-45	34.9	105	126
46-50	43.1	86	101
51-55	59.9	88	101
56-60	82.5	100	110
61-65	85.8	82	92
66-70	92.7	89	98
71-75	70.7	91	97
76-80	43.7	85	89
81 and over	25.5	110	113
Total	602.0	94	105

[a] Shown with one decimal since data had to be redistributed (by osculatory interpolation) so as to be in attained age groups shown.

In the aggregate, the ratio of actual to expected deaths was slightly below 100% according to the 1930-39 table and slightly above using the 1939-41 table. This experience thus indicates that the mortality of the widow beneficiaries is at least as favorable as that of the general female population. There was no particularly significant trend in the ratio by age, except that for the very youngest ages it was very high, being almost 200% according to the 1930-39 table.

Table 4 analyzes the mortality experience on a select and ultimate basis by duration of widowhood, using the 1930-39 table as the basis for the expected deaths. The ratio for the select period was 104%, as against 89% for the ultimate period, with no particular trend by attained age. In large part this higher ratio for the select period was due to the high mortality experience for those widowed at age 30 and below. When considered by duration in the select period for all ages at widowhood combined, there was some indication of higher mortality in the first year of widowhood, where the ratio was 130%, as against about 100% for each of the other four durations; again a large part of this is due to the experience of those widowed at age 30 and under, the ratios for those widowed beyond age 30 being 109, 87, 85, 99 and 97%, respectively, for the five durations.

Table 4

U. S. EMPLOYEES' COMPENSATION SYSTEM
COMPARISON OF ACTUAL MORTALITY EXPERIENCE WITH THAT EXPECTED BY U. S. WHITE FEMALES 1930-39 LIFE TABLE, BY AGE AT ENTRY AND DURATION OF WIDOWHOOD, 9/17/16 TO 6/30/45

Age or Duration	*Actual Deaths*[a]	*Ratio of Actual to Expected Deaths*
First 5 Years of Widowhood, by Age at Widowhood		
Under 31	25.5	193%
31-40	26.0	115
41-50	31.5	89
51-55	23.0	103
56-60	16.5	80
61 & over	37.0	94
Total	159.5	104
First 5 Years of Widowhood, by Duration of Widowhood		
0	42.5	130%
1	28.5	89
2	32.0	106
3	28.5	97
4	28.0	96
Total	159.5	104

After First 5 Years of Widowhood, by Attained Age		
Under 41	19.9	105%
41-45	19.7	103
46-50	24.8	78
51-55	42.5	92
56-60	65.3	106
61-65	54.3	70
66-70	76.1	85
71-75	70.7	91
76-80	43.7	85
81 & over	25.5	110
Total	442.5	89

[a]Shown with one decimal since data had to be redistributed (by osculatory interpolation) so as to be in attained age groups shown.

In summary, this study has shown that the remarriage experience of the Employees' Compensation program for the roughly 29 years of experience up through 1945 closely paralleled that expected according to the American Remarriage Table, although at the very youngest ages the tabular rates were exceeded, and at the very oldest ages they were not nearly attained. In regard to mortality experience, the beneficiaries have had close to normal population mortality, with an indication of considerable excess mortality for the very youngest ages at widowhood, and also to some extent in the first year of widowhood.

REMARRIAGE EXPERIENCE UNDER OLD-AGE AND SURVIVORS INSURANCE SYSTEM

Under the Old-Age and Survivors Insurance system, monthly benefits are payable to two categories of widows of covered workers, namely, (1) regardless of age so long as an eligible child under 18 is present (termed "widow's current" benefits) and (2) after age 65 (termed "widow's" benefits). In both instances benefit payments to the widow cease upon her remarriage. The relative size of the data in the Old-Age and Survivors Insurance experience as against those on which the American Remarriage Table was based may be judged by the number of remarriages. Under the Old-Age and Survivors Insurance system for 1940-46, the period for which an extensive theoretical investigation will be made subsequently in this section (although rough consideration will also be given to the period 1940-48), the actual number of remarriages was 36,628 as compared with only 1,187 in the experience on which the American Remarriage Table was based.

To date, because of the manpower shortages during the war and postwar period, it has been impossible to conduct an exact actuarial investigation as to the remarriage experience under the program. However, certain rough studies have been made, and these give some very good and interesting indications of the experience. It is to be hoped that in the future, it may be possible to make further studies but on a more precise basis.

One of the major difficulties with the actual data available up to now is that they are based on date of administrative action rather than date of demographic event (i.e., date of widowhood, date of remarriage, or date of death).

Table 5 shows, in the first column, the proportion of the possible cases where claims for widow's current benefits are actually filed. Because of the maximum benefit provision[4] and because of the widow engaging in covered employment[5], there are many instances where widows do not file a claim. In 1940 this proportion was 10%, and since then there has been a steady increase to 20% for 1947 awards. The second column shows the proportion of the widows who have actually filed claim for widow's current benefits and who are receiving such benefits. A very significant proportion does not receive benefits, principally because of covered employment. This proportion has increased from 7% at the end of 1940 to 27% at the end of 1948. Combining these two factors, it would appear that over 40% of the widows who would otherwise be eligible are not receiving benefits because of covered employment or because of the maximum provisions.

Table 5

OLD-AGE AND SURVIVORS INSURANCE SYSTEM
DATA ON WIDOW'S CURRENT BENEFICIARIES

Calendar Year	*Awards in Year as % of Awards for Deaths of Married Men with Children*	*Beneficiaries in Current Payment Status as % of Benefits in Force at end of Year*
1940	90.1	92.7
1941	88.0	88.1
1942	86.4	80.2
1943	84.0	73.7
1944	83.2	73.3
1945	84.6	76.3
1946	82.2	74.4
1947	80.1	73.6
1948	*	72.8

*Not available.

From the data on widow's current benefits in force (including those which are not in current payment status) and from the data on terminations of widow's current benefits classified by reason for termination, it is possible to determine rough termination probabilities by cause. It should be emphasized that these are only approximate figures because the data are classified by time of administrative action, rather than by time of demographic event.

Table 6 presents the calculated termination probabilities for individual calendar years. Based on the experience of 1940, about 10% of the widow's current beneficiaries would terminate in a year. This proportion gradually increased and levelled out to about 14% during the war years 1942-45, but in the three postwar years, 1946-48, rose to a new level of 18%.

[4] Such that the maximum amount is payable in respect to the children so that nothing additional would be payable to the widow if she were to file.

[5] In which case the benefit is not paid while earnings are $15 or more per month.

The reason for this trend is apparent when the probabilities by cause of termination are considered. About 8% of the widows terminate each year because the last child attained age 18; this figure has shown a slight upward trend over the 9 years considered. In 1940 only about 2% of the widows were terminated because of remarriage; this figure increased steadily until it was at a level of about 5% for 1942-45 and then jumped sharply to 9% in 1946 and has decreased somewhat since then (reflecting the general trend in marriages throughout the country). Normally the probability of termination because of remarriage would be expected to increase for a few years and then level off, since the remarriage rate is generally rather low in the first year of widowhood, but increases to a fairly sizeable peak in the second and third years and decreases thereafter.

Table 6

OLD-AGE AND SURVIVORS INSURANCE SYSTEM
TERMINATION PROBABILITIES[a] FOR WIDOW'S CURRENT BENEFICIARIES, BY CAUSE OF TERMINATION

Year	*Last Child Attained 18*	*Marriage of Widow*	*Death of Widow*	*Marriage or Death of Last Child*	*Total*[b]
1940	75	19	4	2	103
1941	82	38	5	4	129
1942	80	49	5	5	139
1943	82	51	5	4	142
1944	82	49	4	3	139
1945	80	53	5	3	141
1946	79	92	5	4	179
1947	83	82	4	4	178
1948	88	71	4	5	173

The other two major causes of termination, namely, death of the widow and marriage or death of the last child, each account for only about 2% of the terminations each year. Incidentally, the probability of death of the widow (namely, 4-5 per thousand) is relatively low. Subsidiary studies which have been made indicate that this is about the "expected" probability on the basis of over-all female population mortality. Once again there is evidence that widows do not have higher mortality than the general female population.

The actual experience may be compared with that which might be expected in regard to the two major causes of termination (namely, last child attained age 18 and remarriage of widow). There have been developed "expected" termination rates for widow's current benefits on the basis of stationary conditions (namely, the same number of widows being created each year, with a

[a]Number of terminations in year divided by average number in force at beginning and end of year. Expressed as rate per thousand.

[b]Includes a relatively small number of terminations for other causes, except in 1947 and 1948 there were a sizable number of terminations due to entitlement to larger veteran's pension and railroad retirement benefits (with combined probability of about 4 per thousand).

fixed age distribution of both the widows and the orphan children). The termination rates for the last child attaining 18 were determined from the Richmond Family Composition Study[6], while the termination rates for remarriage of the widow were based on the theoretical rates of the American Remarriage Table. The figures were all derived on the basis of year of operation of the program. For comparability it was necessary to modify the actual termination probabilities, as shown in Table 6, so as to convert them to "rates." (The probabilities in Table 6 are dependent upon other causes of termination, whereas rates in Table 7 completely eliminate any such subsidiary effects.)

Table 7 compares the actual and "expected" termination rates for widow's current beneficiaries. In regard to terminations because the last child attained age 18, the actual and expected rates have been quite close, although in recent years the actual has fallen somewhat below the expected. This may be explained by the relatively large number of beneficiaries arising from war deaths; in these cases the children were younger on the average than in the theoretical stationary population[7] used to obtain the "expected" rates, and so there were relatively fewer attainments of age 18.

Table 7

OLD-AGE AND SURVIVORS INSURANCE SYSTEM
ACTUAL AGGREGATE TERMINATION RATES[a] FOR WIDOW'S CURRENT BENEFICIARIES COMPARED WITH "EXPECTED" RATES[b] FOR LAST CHILD ATTAINING AGE 18 AND FOR MARRIAGE OF WIDOW

Year of Operation[c]	*Last Child Attained 18*			*Marriage of Widow*		
	Actual	*Expected*	*Ratio*	*Actual*	*Expected*	*Ratio*
1	72	69	104%	18	18	100%
2	79	72	110	37	27	137
3	77	75	103	48	32	150
4	79	79	100	49	33	148
5	78	82	95	48	33	145
6	77	84	92	52	32	163
7	76	86	88	88	31	284
8	79	88	90	79	30	263
9	85	90	94	69	29	238
10	*	92	*	*	28	*
15	*	104	*	*	26	*
18	*	115	*	*	26	*

*Not available

[a] Termination probabilities from Table 6 adjusted so as to be on a "rate" basis (ignoring effect of all other causes of decrement). Expressed as a rate per thousand.

[b] Based on theoretical distribution of youngest children in Richmond Family Composition Study data and on American Remarriage Table, respectively, both applied to a constant number of new cases created each year.

[c] First year of operation was 1940.

[6] A country-wide sample survey which was processed in Richmond, Va. For details see *Social Security Bulletin*, April 1939, page 9.

[7] This was determined from normal mortality circumstances where there are relatively few deaths of young fathers.

On the other hand, the ratios of the actual to "expected" termination rates because of remarriage of the widow have been substantially above 100%. Thus, for the 3rd to the 6th years of operation (1942-45), the ratio was close to 150%, while in the postwar years it was well over 200%. This would seem to indicate that the remarriage rates of the American Remarriage Table are considerably lower than actual experience under the Old-Age and Survivors Insurance program. Of course, any thorough analysis should take into account the actual exposure to remarriage by age and duration since widowhood. In the crude study of Table 7, the expected remarriage rate was based on applying the tabular rates to the distribution of the theoretical widow's current population arising under the operation of the system. It would appear that these ratios of actual to expected tend to be inflated, in part at least because of the effect of the war in creating relatively more young widows for whom remarriages can be expected to be high. Accordingly, based on this limited experience and crude analysis, it might well be reasonable that the remarriage experience of widow's current beneficiaries is perhaps 50% above that of the American Remarriage Table.

A better study of the remarriage experience under the Old-Age and Survivors Insurance system can be made on a theoretical projection basis which, as will be noted, has certain possible weaknesses, but which is the best that can be done under the circumstances. There is available a tabulation of widow's current benefit data by attained age for the years 1940-46, showing, for each of these calendar years, the number of awards and terminations. The terminations are subdivided into remarriages, deaths, and other causes. The total "in force" is shown for December 31 of each year. Since the experience of each calendar year is not subdivided according to year of award, a select analysis of the experience cannot be made by exact methods but an approximate procedure can be developed.

The method used involved constructing an approximate select "in force" for each year, and then applying American Remarriage Table rates to obtain the "expected" remarriages.

The select "in force" was obtained by using continuance probabilities applied to the awards of each year by age and then adjusting the results so that the total calculated "in force" equalled the actual "in force" at the end of each calendar year. The continuance probabilities presented the probability of a widow's current beneficiary remaining "in force" according to age at widowhood and duration of widowhood, and thus took account of the probabilities of termination because of death, remarriage, and attainment of age 18 of the youngest child. The latter of these three factors is by far the most important cause of termination of widow's current benefits and was based on the age data as to children in the Richmond Family Composition Study referred to previously. The mortality factor in these continuance probabilities was based on the U. S. White Females Life Table for 1929-31, using somewhat antiquated mortality as an allowance for the believed excess higher mortality of widows (a subject which will be discussed in more detail later). The remarriage basis was taken to be 100% of the American Remarriage Table rates. Although at first glance it might appear that the use of these remarriage rates in building

up the exposed to risk might lead to serious error, for very rough and approximate purposes this is not the case because the remarriage element is only a small element involved in continuance factors[8].

The method of adjustment described in the previous paragraph may be made clear by an actual description of the processes involved. The awards of 1940 were shown by age at widowhood, and the number of these who continued in force on December 31, 1940 was also shown. Using these continuance probabilities, the "in force" on December 31, 1940 were projected to December 31, 1941; similarly, the new entrants (or awards of 1941) were projected for an average of ½ year to the same date. These projected figures were then adjusted age by age so that their total equalled the actual beneficiaries "in force". Then these adjusted figures were projected for one full year by using the proper continuance probabilities, as was also done for the 1942 awards, projected for ½ year. The grand total of the projected 1940-42 awards as of December 31, 1942, was then compared with the actual "in force" on that date, and the projected figures adjusted accordingly. In this fashion there was built up an adjusted "in force" as of the end of each calendar year subdivided first by calendar year of widowhood (which, of course, can be used with proper adjustment, to be described subsequently, as a duration demarkation) and then by age at widowhood. Of course, too, in the actual exposure, there would be considered for each year the awards of that year.

The actual adjustment factors used against the projected "in force" at the end of the year to bring the projected into conformity with the actual "in force" were as follows:

Calendar Year	Factor Applied to Projected "In Force"
1940	None required
1941	.9747
1942	.9723
1943	.9724
1944	.9752
1945	.9712
1946	.9411

The remarriage rates used against the exposed to risk to determine the "expected" remarriages were derived from the probabilities of remarriage appearing in the American Remarriage Table, which were first transformed to

[8] This might be made clearer by considering a specific example along related lines. Suppose that we are investigating the mortality experience of a group of persons age 50 where the death rate is approximately 10 per 1000. The continuance probability for this group would then be .99, while the mortality or discontinuance rate would be .01 according to the expected basis. However, if actual mortality for this group were double the expected, then the continuance probability would actually be .98 or a difference of only 1%, whereas there was a difference of 100% in mortality rates.

rates by eliminating the mortality element present and then adjusted to conform to the form of the Old-Age and Survivors data. One adjustment was necessary to take account of the fact that observation in the Old-Age and Survivors Insurance data did not start at date of widowhood, but rather at the later date of administrative action, herein assumed to be three months after widowhood. A further adjustment was also required since the American Remarriage Table is tabulated according to age last birthday, whereas the Old-Age and Survivors Insurance data are shown by age in year of award, or on the average, nearest birthday at date of award.

In accordance with these reasons for adjustment, the following actual procedure was adopted to allow for the data being based on date of administrative action. It was assumed that the experience in the calendar year of award was an average exposure for ½ year and that the exposure began three months after widowhood rather than at exact date of widowhood. Similarly, the exposure in the first calendar year after the year of widowhood was assumed to extend from duration nine months to one year and nine months, while in the second calendar year after the year of widowhood the exposure extended from duration one year and nine months to two years and nine months, and so forth for subsequent calendar years. This required adjustment of the tabular remarriage probabilities shown in the American Remarriage Table so as to put them on a rate basis[9] and on a corresponding duration basis. Expressed in notational form, the required rates are as follows (where $x-\frac{1}{4}$ is exact age at widowhood and thus x is exact age at time of filing claim):

Calendar year of award	$\frac{1}{2}\,{}^{(rq)}[x-\frac{1}{4}] + \frac{1}{4}$
Next calendar year	${}^{(rq)}[x-\frac{1}{4}] + \frac{3}{4}$
Following calendar year.	${}^{(rq)}[x-\frac{1}{4}] + 1\frac{3}{4}$, etc.

For each age at widowhood the probabilities or remarriage shown in the American Remarriage Table bear a fixed relationship to the probability for duration 0, namely as follows:

Duration	Ratio to Probability for Duration 0
0	1.00
1	2.45
2	1.93
3	1.89
4	1.15
5 (Ultimate)	.81

[9] By the formula, $(rq)_x = \frac{r^r_x}{1-\frac{1}{2}\,q^r_x}$ where (rq) is the remarriage rate, r^r is the tabular probability of remarriage and q^r is the tabular probability of dying unremarried.

Interpolation on these ratios was performed so as to get ratios for the remarriage rates required[10], with the following results:

Duration	Ratio to Rate for Duration 0-1
¼- ¾	.56
¾-1¾	2.15
1¾-2¾	2.12
2¾-3¾	1.91
3¾-4¾	1.31
4¾-5¾	.89
5¾-6¾ and over	.81

Thus knowing $(rq)_{[x-¼]}$, all the required rates were derived from the above ratios.

These rates were applied to the adjusted exposures described previously. Accordingly, expected remarriages were obtained for each calendar year for each age at widowhood and each duration. These were then collected by attained age for each calendar year since that was the only breakdown available for the actual remarriages.

Ratios of actual to expected remarriages, by attained age groups, are shown in Table 8 for single calendar years, and for the prewar years 1940-41, the war years 1942-45, and all years 1940-46 combined.

The most outstanding feature of this table is the great increase in the ratios of actual to expected remarriages for 1946. This is undoubtedly a true reflection of the large number of marriages taking place in that year. However, the fact that these ratios are aggregate should be considered in interpreting them. Because of this, the ratios for any calendar year are affected by the proportion of the experience occuring at the various durations since widowhood. For example, in the assumed distribution developed herein, 100% of the 1940 exposure is included between ¼ and ¾ of a year after widowhood, whereas in 1946 only 13% of the exposure occurred between these durations.

The trend, by age, of these ratios of actual to "expected" remarriages is fairly consistent for all calendar years. The ratios start off somewhere over 100%, increase slightly until some age between 25 and 45 and then generally decrease to below 100% in the highest age group. For all calendar years combined, the ratio starts around 160% in the youngest age group, increases to approximately 170% in the 30-34 year age group, and then tapers off to 83% in the 55-64 year age group.

[10] A minor, and probably insignificant, theoretical error occurs here since the original ratios applied to remarriage probabilities rather than to remarriage rates.

TABLE 8

OLD-AGE AND SURVIVORS INSURANCE SYSTEM RATIOS OF ACTUAL TO EXPECTED[a] REMARRIAGES FOR WIDOW'S CURRENT BENEFICIARIES

	Attained Age								
Year	Under 25	25-29	30-34	35-39	40-44	45-49	50-54	55-64	All Ages to 65
1940	102%	106%	76%	86%	78%	42%	50%	133%	87%
1941	119	139	134	116	82	65	45	36	114
1942	141	141	149	136	114	111	69	53	132
1943	128	132	153	146	138	121	91	60	135
1944	108	129	140	148	151	143	107	96	133
1945	130	131	150	155	159	140	100	78	140
1946	237	234	230	232	217	196	153	110	226
1940-46	161	167	170	167	159	143	107	83	161
1940-41	116	133	125	111	82	61	45	51	109
1942-45	125	132	148	148	145	132	95	76	136

[a] Expected remarriages calculated on basis of American Remarriage Table.

In general, the ratios of actual to "expected" in any age group increase each calendar year. Undoubtedly, the major part of this trend is due to an actual increase in the rate of remarriage. To some extent, however, it may result from the fact, as suggested above, that later durations are brought into the experience as the calendar year advances. This result would then appear if the experience remarriage rates at later durations after widowhood are consistently higher than those in the American Remarriage Table. Tending to offset this is the fact that the greater part of the exposure in any calendar year is at the early durations.

MODIFIED AMERICAN REMARRIAGE TABLES

In preparing revised remarriage tables the considerations described previously led to the conclusion that actual remarriage rates now might be considerably above those in the original American Remarriage Table. As a remarriage basis it seemed desirable to have two alternatives, namely, (1) the basic rates in the original American Remarriage Table and (2) 150% of such rates[11].

Likewise, of course, there is need for bringing in more modern mortality. In the opinion of the author and as indicated in the previous analysis, there is not any clear evidence to indicate that the mortality of widows is higher than that of the total female population. The experience of life insurance companies with settlement options has not indicated high mortality for widows. The results from census data may be biased since the deaths in the numerator may be cor-

[11] It might have been desirable to have graded this percentage down at the very older ages as did Niessen in his paper referred to previously, but it was felt that this refinement was unnecessary considering the rough general nature of the adjustment.

rectly reported as to marital status, but the population in the denominator may well be understated since the widows, especially those at the younger ages, may report themselves as other than widowed. Accordingly, the mortality basis adopted has been that according to the U. S. White Females Life Table for 1939-41, which is the latest complete, published official table[12]. Since 1939-41 mortality has continued to improve, so to that extent it might be said that there is some allowance for higher mortality for widows.

The first step was to derive rates of remarriage from the American Remarriage Table which shows probabilities only[13]. The formula used was the usual one (see footnote 9). The probability of surviving unremarried from a year may be represented by $p'_x = 1 - \frac{q_x + (rq)_x - q_x \cdot (rq)_x}{1 - \frac{1}{2} q_x \cdot (rq)_x}$ where q and (rq) are rates of death and remarriage, respectively.

After obtaining all values of p'_{x+t}, l'_{x+t} was derived by working backward from l_{79}, taken from the U. S. White Females Table for 1939-41.

Two sets of tables of l_x, D_x and N_x (at 3% interest) have been prepared. One set uses remarriage rates equal to those described above, and the other set uses 150% of these rates. A table of probabilities or remarriage has also been worked out on the 100% basis.

Table 9 shows the number of widows surviving unremarried according to the 100% remarriage rates, while Table 10 gives similar data for the 150% rates.

Tables 11 to 14 give the D's and N's for these two tables using an interest rate of 3%. Finally Table 15 gives the remarriage probabilities on the 100% basis; it should be emphasized that these are not rates since they represent a combination of remarriage rates with the U. S. White Female 1939-41 mortality.

The author gratefully acknowledges the great help he has received from Walter E. Wilcox, an Associate of the Society of Actuaries, who assisted him in the development of the "projections" study of Old-Age and Survivors Insurance experience and who supervised the computations involved in the modified American Remarriage Table.

[12] "United States Life Tables and Actuarial Tables 1939-1941" by Thomas N. E. Greville, United States Government Printing Office, Washington, D. C. 1946.

[13] In this connection it might be pointed out that there was perhaps a minor theoretical error present in the development of the original American Remarriage Table. In the special study made remarriage *probabilities* were developed, and these were used as probabilities combined with mortality *rates*. The remarriage *probabilities* originally derived should have been converted into *rates* so as to eliminate the mortality element in the investigation. Then these remarriage *rates* should have been combined with the mortality *rates* from the official life table to get the proper remarriage probabilities. However, the theoretical adjustment involved would be relatively small because the actual mortality in the investigation probably did not differ significantly (considering the small size of the sample) from the theoretical or tabular *rates*.

Table 9

100% AMERICAN REMARRIAGE TABLES, 1939-41a/

Number Living Unmarried

Age at Entry [x]	Years Elapsed Since Husband's Death: 0 $l^r_{[x]}$	1 $l^r_{[x]+1}$	2 $l^r_{[x]+2}$	3 $l^r_{[x]+3}$	4 $l^r_{[x]+4}$	5 or more l^r_{x+5}	Age Attained x + 5
18	391,581	364,143	302,228	261,560	227,207	208,896	23
19	354,817	331,374	278,381	243,124	213,069	196,903	24
20	323,262	303,168	257,599	226,937	200,565	186,237	25
21	295,966	278,667	239,378	212,637	189,445	176,733	26
22	272,256	257,328	223,340	199,983	179,557	168,252	27
23	251,940	238,960	209,367	188,841	170,750	160,669	28
24	234,089	222,787	196,986	178,901	162,875	153,865	29
25	218,363	208,506	185,971	170,035	155,793	147,737	30
26	204,643	196,007	176,238	162,127	149,457	142,223	31
27	192,573	184,970	167,675	155,064	143,754	137,242	32
28	181,872	175,168	159,836	148,719	138,599	132,736	33
29	172,460	166,520	152,960	143,039	133,963	128,654	34
30	163,959	158,691	146,705	137,865	129,734	124,940	35
31	156,435	151,751	141,107	133,208	125,908	121,565	36
32	149,795	145,595	136,108	129,015	122,429	118,483	37
33	143,758	139,997	131,543	125,183	119,251	115,660	38
34	138,326	134,954	127,406	121,685	116,336	113,069	39
35	133,504	130,457	123,689	118,535	113,694	110,695	40
36	129,074	126,329	120,272	115,625	111,238	108,490	41
37	125,102	122,607	117,160	112,960	108,977	106,454	42
38	121,522	119,248	114,343	110,527	106,888	104,555	43
39	118,265	116,182	111,750	108,272	104,938	102,772	44
40	115,301	113,384	109,369	106,186	103,128	101,098	45
41	112,582	110,817	107,162	104,235	101,416	99,513	46
42	110,078	108,442	105,103	102,407	99,804	98,011	47
43	107,824	106,285	103,210	100,709	98,278	96,567	48
44	105,700	104,240	101,404	99,078	96,801	95,167	49

Table 9 -- Continued

Age at Entry [x]	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or more	
	$l^r_{[x]}$	$l^r_{[x]+1}$	$l^r_{[x]+2}$	$l^r_{[x]+3}$	$l^r_{[x]+4}$	l^r_{x+5}	
45	103,712	102,323	99,700	97,519	95,378	93,805	50
46	101,897	100,562	98,101	96,036	94,004	92,476	51
47	100,184	98,888	96,570	94,603	92,652	91,156	52
48	98,558	97,295	95,101	93,206	91,321	89,841	53
49	96,951	95,708	93,621	91,802	89,986	88,524	54
50	95,486	94,247	92,229	90,450	88,667	87,192	55
51	94,047	92,807	90,842	89,087	87,323	85,838	56
52	92,658	91,403	89,474	87,729	85,959	84,440	57
53	91,246	89,971	88,065	86,323	84,550	83,000	58
54	89,852	88,553	86,653	84,899	83,108	81,505	59
55	88,482	87,134	85,224	83,434	81,603	79,949	60
56	87,062	85,670	83,736	81,915	80,045	78,327	61
57	85,602	84,160	82,205	80,336	78,414	76,627	62
58	84,135	82,628	80,628	78,707	76,720	74,850	63
59	82,673	81,056	78,990	76,996	74,939	72,985	64
60	81,067	79,407	77,277	75,204	73,066	71,020	65
61	79,373	77,642	75,453	73,308	71,091	68,949	66
62	77,640	75,817	73,551	71,327	69,018	66,774	67
63	75,826	73,908	71,565	69,248	66,841	64,480	68
64	73,952	71,932	69,491	67,069	64,551	62,068	69
65	71,953	69,827	67,285	64,758	62,130	59,529	70
66	69,820	67,590	64,954	62,318	59,583	56,861	71
67	67,570	65,229	62,500	59,763	56,913	54,071	72
68	65,242	62,783	59,943	57,084	54,120	51,167	73
69	62,757	60,178	57,239	54,272	51,204	48,141	74
70	60,107	57,413	54,387	51,328	48,175	45,024	75
71	57,382	54,566	51,445	48,290	45,055	41,834	76
72	54,473	51,554	48,363	45,141	41,855	38,590	77
73	51,481	48,460	45,209	41,930	39,609	35,323	78

Functions beyond age at entry 73 are ultimate, depending upon mortality rates only

a/ Based on remarriage rates from American Remarriage Table combined with U.S. White Females 1939-41 mortality rates.

Table 10

150% AMERICAN REMARRIAGE TABLES, 1939-41[a/]

Number Living Unmarried

Age at Entry [x]	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or more	
	$l^r_{[x]}$	$l^r_{[x]+1}$	$l^r_{[x]+2}$	$l^r_{[x]+3}$	$l^r_{[x]+4}$	l^r_{x+5}	
18	835,358	748,038	557,722	445,531	358,100	315,117	23
19	716,519	645,992	491,535	398,512	324,379	288,250	24
20	620,222	562,839	436,369	358,813	296,591	265,069	25
21	541,202	494,166	390,055	325,025	272,150	245,006	26
22	475,978	437,191	350,946	296,202	251,076	227,598	27
23	422,509	390,242	318,110	271,615	232,826	212,435	28
24	377,612	350,598	290,011	250,329	216,938	199,156	29
25	339,538	316,857	265,786	231,898	203,008	187,468	30
26	307,604	288,419	245,081	215,894	190,814	177,180	31
27	280,508	264,171	227,174	201,985	180,096	168,075	32
28	257,242	243,269	211,586	189,755	170,614	159,992	33
29	237,430	225,414	198,130	179,094	162,263	152,823	34
30	220,066	209,701	186,183	169,577	154,788	146,414	35
31	205,084	196,101	175,714	161,179	148,151	140,692	36
32	192,217	184,353	166,567	153,768	142,208	135,540	37
33	180,773	173,898	158,385	147,110	136,868	130,906	38
34	170,751	164,732	151,132	141,166	132,075	126,721	39
35	161,993	156,663	144,708	135,894	127,784	122,949	40
36	154,126	149,434	138,917	131,097	123,863	119,508	41
37	147,262	143,078	133,769	126,801	120,321	116,372	42
38	141,096	137,374	129,137	122,910	117,089	113,496	43
39	135,652	132,303	124,975	119,380	114,124	110,849	44
40	130,787	127,765	121,231	116,195	111,438	108,414	45
41	126,390	123,665	117,801	113,236	108,923	106,143	46
42	122,414	119,950	114,681	110,541	106,620	104,046	47
43	118,898	116,622	111,844	108,077	104,480	102,069	48
44	115,676	113,561	109,216	105,766	102,443	100,184	49

Table 10 — Continued

Age at Entry (x)	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or more	
	$l^F_{[x]}$	$l^F_{[x]+1}$	$l^F_{[x]+2}$	$l^F_{[x]+3}$	$l^F_{[x]+4}$	l^F_{x+5}	
45	112,668	110,700	108,751	103,576	100,510	98,383	50
46	110,008	108,162	104,516	101,567	98,692	96,667	51
47	107,543	105,777	102,405	99,638	96,941	94,991	52
48	105,249	103,577	100,440	97,821	95,251	93,357	53
49	103,017	101,400	98,474	96,004	93,582	91,756	54
50	101,060	99,477	96,686	94,313	91,978	90,166	55
51	99,153	97,600	94,935	92,640	90,375	88,578	56
52	97,353	95,813	93,241	90,998	88,757	86,952	57
53	95,570	94,027	91,523	89,324	87,119	85,307	58
54	93,846	92,294	89,847	87,665	85,479	83,620	59
55	92,181	90,595	88,171	85,981	83,771	81,884	60
56	90,485	88,868	86,450	84,254	82,032	80,094	61
57	88,769	87,115	84,705	82,488	80,231	78,237	62
58	87,063	85,358	82,930	80,677	78,379	76,308	63
59	85,374	83,602	81,117	78,807	76,447	74,303	64
60	83,602	81,750	79,229	76,848	74,423	72,201	65
61	81,708	79,798	77,236	74,798	72,309	69,997	66
62	79,805	77,806	75,183	72,682	70,110	67,700	67
63	77,827	75,737	73,048	70,463	67,809	65,289	68
64	75,806	73,617	70,840	68,158	65,401	62,766	69
65	73,640	71,350	68,482	65,712	62,861	60,120	70
66	71,329	68,947	66,011	63,147	60,199	57,351	71
67	68,918	66,430	63,420	60,472	57,426	54,466	72
68	66,450	63,849	60,746	57,686	54,542	51,475	73
69	63,819	61,110	57,926	54,774	51,537	48,376	74
70	60,998	58,188	54,950	51,729	48,430	45,194	75
71	58,140	55,215	51,900	48,600	45,239	41,946	76
72	55,077	52,068	48,713	45,372	41,980	38,655	77
73	51,954	48,856	45,468	42,090	38,682	35,351	78

Functions beyond age at entry 73 are ultimate, depending upon mortality rates only.

a/ Based on remarriage rates (150%) from American Remarriage Table combined with U.S. White Females 1939-41 mortality rates.

Table 11

100% AMERICAN REMARRIAGE TABLES, 1939-41a/

$D^{r}_{(x)+n}$ Columns - Interest at 3 Percent

Age at Entry (x)	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or more	
	$D^{r}_{(x)}$	$D^{r}_{(x)+1}$	$D^{r}_{(x)+2}$	$D^{r}_{(x)+3}$	$D^{r}_{(x)+4}$	D^{r}_{x+5}	
18	230,013	207,666	167,336	140,601	118,578	105,846	23
19	202,347	183,474	149,643	126,885	107,960	96,863	24
20	178,982	162,968	134,439	114,987	98,665	88,948	25
21	159,096	145,434	121,291	104,603	90,480	81,950	26
22	142,089	130,386	109,869	95,513	83,260	75,745	27
23	127,656	117,553	99,995	87,565	76,870	70,225	28
24	115,156	106,404	91,341	80,539	71,189	65,292	29
25	104,291	96,683	83,722	74,318	66,110	60,866	30
26	94,892	88,240	77,030	68,798	61,574	56,887	31
27	86,694	80,846	71,110	63,884	57,500	53,296	32
28	79,492	74,332	65,850	59,486	53,823	50,045	33
29	73,183	68,604	61,182	55,547	50,508	47,093	34
30	67,549	63,474	56,971	51,979	47,488	44,402	35
31	62,572	58,931	53,201	48,760	44,746	41,944	36
32	58,171	54,893	49,822	45,850	42,242	39,690	37
33	54,201	51,245	46,748	43,192	39,947	37,616	38
34	50,634	47,960	43,959	40,762	37,835	35,702	39
35	47,445	45,012	41,434	38,551	35,899	33,934	40
36	44,635	42,318	39,116	36,509	34,101	32,290	41
37	41,907	39,875	36,994	34,629	32,435	30,761	42
38	39,522	37,653	35,053	32,896	30,886	29,332	43
39	37,343	35,616	33,260	31,286	29,440	27,992	44
40	35,346	33,746	31,603	29,790	28,089	26,734	45
41	33,508	32,022	30,064	28,391	26,818	25,549	46
42	31,808	30,423	28,627	27,080	25,623	24,430	47
43	30,249	28,949	27,293	25,856	24,497	23,369	48
44	28,790	27,565	26,034	24,696	23,426	22,369	49

Table 11 (Continued)

100% AMERICAN REMARRIAGE TABLES, 1939-41a/

$D^r_{[x]+n}$ Columns - Interest at 3 Percent

Age at Entry [x]	Years Elapsed Since Husband's Death: 0	1	2	3	4	5 or more	Age Attained x+5
	$D^r_{[x]}$	$D^r_{[x]+1}$	$D^r_{[x]+2}$	$D^r_{[x]+3}$	$D^r_{[x]+4}$	D^r_{x+5}	
45	27,425	26,270	24,851	23,600	22,409	21,398	50
46	26,161	25,066	23,740	22,564	21,443	20,480	51
47	24,972	23,931	22,689	21,580	20,519	19,600	52
48	23,851	22,859	21,693	20,642	19,635	18,754	53
49	22,779	21,832	20,734	19,739	18,785	17,941	54
50	21,781	20,872	19,830	18,881	17,970	17,157	55
51	20,828	19,955	18,963	18,055	17,182	16,398	56
52	19,923	19,080	18,134	17,262	16,421	15,661	57
53	19,048	18,234	17,328	16,491	15,682	14,946	58
54	18,210	17,424	16,554	15,746	14,965	14,249	59
55	17,410	16,646	15,807	15,024	14,266	13,570	60
56	16,632	15,889	15,078	14,321	13,586	12,907	61
57	15,877	15,155	14,371	13,636	12,922	12,260	62
58	15,150	14,445	13,685	12,970	12,274	11,626	63
59	14,446	13,758	13,017	12,319	11,640	11,007	64
60	13,760	13,085	12,364	11,681	11,019	10,398	65
61	13,080	12,422	11,720	11,055	10,409	9,801.0	66
62	12,422	11,777	11,092	10,443	9,810.8	9,215.4	67
63	11,778	11,146	10,478	9,843.5	9,224.7	8,639.6	68
64	11,152	10,532	9,878.1	9,256.1	8,649.1	8,074.2	69
65	10,535	9,925.8	9,285.9	8,676.9	8,082.2	7,518.3	70
66	9,924.8	9,328.0	8,703.1	8,106.7	7,525.2	6,972.2	71
67	9,325.3	8,740.0	8,130.4	7,547.9	6,978.6	6,437.0	72
68	8,741.7	8,167.2	7,570.6	6,999.6	6,442.8	5,913.4	73
69	8,163.8	7,600.3	7,018.6	6,460.9	5,918.2	5,402.1	74
70	7,591.3	7,039.9	6,474.6	5,932.5	5,405.9	4,905.1	75
71	7,036.1	6,495.9	5,946.0	5,418.8	4,908.5	4,424.9	76
72	6,484.8	5,958.6	5,427.0	4,917.9	4,427.1	3,962.8	77
73	5,950.2	5,437.9	4,925.3	4,436.0	3,964.8	3,521.7	78

Functions Beyond Age At Entry 73 Are Ultimate Depending Upon Mortality Rates Only

a/ Based on 100% remarriage rates from American Remarriage Table combined with U.S. White Females 1939-41 mortality rates.

Table 12

150% AMERICAN REMARRIAGE TABLES, 1939-41[a]/

$D^{r}_{[x]+n}$ Columns - Interest at 3 Percent

Age at Entry [x]	Years Elapsed Since Husband's Death: 0	1	2	3	4	5 or more	Age Attained x+5
	$D^{r}_{[x]}$	$D^{r}_{[x]+1}$	$D^{r}_{[x]+2}$	$D^{r}_{[x]+3}$	$D^{r}_{[x]+4}$	D^{r}_{x+5}	
18	490,685	426,596	308,797	239,495	186,890	159,667	23
19	408,621	357,670	264,224	207,981	164,644	141,800	24
20	343,402	302,554	227,738	181,808	145,903	126,599	25
21	290,923	257,902	197,638	159,891	129,980	113,608	26
22	248,410	221,521	172,642	141,468	116,423	102,462	27
23	214,082	191,973	151,931	125,947	104,816	92,850	28
24	185,760	167,448	134,477	112,695	94,819	84,511	29
25	162,165	146,925	119,654	101,357	86,146	77,234	30
26	142,634	129,843	107,119	91,614	78,613	70,870	31
27	126,282	115,463	96,400	83,215	72,036	65,270	32
28	112,435	103,230	87,171	75,900	66,256	60,321	33
29	100,752	92,868	79,249	69,549	61,177	55,940	34
30	90,664	83,878	72,302	63,935	56,659	52,033	35
31	82,031	76,153	66,249	58,999	52,650	48,543	36
32	74,645	69,506	60,971	54,647	49,066	45,404	37
33	68,156	63,654	56,287	50,758	45,848	42,574	38
34	62,503	58,543	52,145	47,288	42,954	40,013	39
35	57,570	54,054	48,475	44,196	40,348	37,691	40
36	53,178	50,058	45,179	41,394	37,971	35,569	41
37	49,330	46,533	42,238	38,872	35,811	33,627	42
38	45,888	43,376	39,588	36,581	33,834	31,841	43
39	42,833	40,558	37,196	34,496	32,017	30,192	44
40	40,094	38,026	35,031	32,598	30,353	28,669	45
41	37,617	35,734	33,048	30,842	28,803	27,251	46
42	35,373	33,651	31,236	29,231	27,373	25,934	47
43	33,356	31,765	29,576	27,747	26,043	24,701	48
44	31,507	30,030	28,040	26,363	24,791	23,538	49

Table 12 (Continued)

150% AMERICAN REMARRIAGE TABLES, 1939-41a/

$D^r_{[x]+n}$ Columns - Interest at 3 Percent

Age at Entry [x]	Years Elapsed Since Husband's Death: 0	1	2	3	4	5 or more	Age Attained x+5
	$D^r_{[x]}$	$D^r_{[x]+1}$	$D^r_{[x]+2}$	$D^r_{[x]+3}$	$D^r_{[x]+4}$	D^r_{x+5}	
45	29,794	28,421	26,609	25,065	23,615	22,442	50
46	28,243	26,960	25,293	23,861	22,512	21,408	51
47	26,806	25,598	24,060	22,728	21,469	20,424	52
48	25,470	24,335	22,911	21,664	20,480	19,488	53
49	24,204	23,130	21,808	20,642	19,535	18,596	54
50	23,052	22,030	20,789	19,688	18,641	17,742	55
51	21,959	20,985	19,818	18,775	17,783	16,922	56
52	20,932	20,001	18,897	17,905	16,956	16,127	57
53	19,950	19,056	18,009	17,064	16,158	15,361	58
54	19,020	18,160	17,164	16,259	15,392	14,619	59
55	18,138	17,307	16,353	15,483	14,645	13,898	60
56	17,286	16,483	15,567	14,730	13,924	13,199	61
57	16,464	15,687	14,809	14,001	13,221	12,517	62
58	15,677	14,923	14,076	13,295	12,540	11,853	63
59	14,926	14,190	13,367	12,608	11,875	11,205	64
60	14,190	13,472	12,676	11,937	11,223	10,571	65
61	13,465	12,767	11,997	11,280	10,587	9,950.0	66
62	12,768	12,086	11,338	10,642	9,966.1	9,343.2	67
63	12,089	11,422	10,695	10,016	9,358.3	8,748.0	68
64	11,432	10,778	10,070	9,406.4	8,763.0	8,165.0	69
65	10,782	10,142	9,451.1	8,804.7	8,177.3	7,593.0	70
66	10,139	9,515.3	8,844.7	8,214.5	7,603.0	7,032.3	71
67	9,511.3	8,900.9	8,250.1	7,637.4	7,041.5	6,484.0	72
68	8,903.6	8,305.9	7,672.0	7,073.4	6,493.1	5,949.5	73
69	8,302.0	7,718.0	7,102.8	6,520.7	5,956.6	5,428.5	74
70	7,703.9	7,135.0	6,541.6	5,978.8	5,434.5	4,923.7	75
71	7,129.1	6,573.2	5,998.6	5,453.6	4,928.6	4,436.7	76
72	6,556.8	6,018.0	5,466.3	4,943.1	4,440.3	3,969.5	77
73	6,004.8	5,482.3	4,953.5	4,451.9	3,972.3	3,524.5	78

Functions Beyond Age At Entry 73 Are Ultimate Depending Upon Mortality Rates Only

a/ Based on 150% remarriage rates from American Remarriage Table combined with U.S. White Females 1939-41 mortality rates.

Table 13

100% AMERICAN REMARRIAGE TABLES, 1939-41[a/]

$N^{r}_{[x]+n}$ Columns - Interest at 3 Percent

Age at Entry [x]	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or More	
	$N^{r}_{[x]}$	$N^{r}_{[x]+1}$	$N^{r}_{[x]+2}$	$N^{r}_{[x]+3}$	$N^{r}_{[x]+4}$	N^{r}_{x+5}	
18	2,544,998	2,314,985	2,107,319	1,939,983	1,799,382	1,680,804	23
19	2,345,267	2,142,920	1,959,445	1,809,803	1,682,918	1,574,958	24
20	2,168,136	1,989,154	1,826,186	1,691,747	1,576,760	1,478,095	25
21	2,010,051	1,850,955	1,705,521	1,584,230	1,479,627	1,389,147	26
22	1,868,314	1,726,225	1,595,839	1,485,970	1,390,457	1,307,197	27
23	1,741,091	1,613,435	1,495,882	1,395,887	1,308,322	1,231,452	28
24	1,625,856	1,510,700	1,404,296	1,312,965	1,232,416	1,161,227	29
25	1,521,069	1,416,768	1,320,085	1,236,363	1,162,045	1,095,935	30
26	1,425,603	1,330,711	1,242,471	1,165,441	1,096,643	1,035,069	31
27	1,338,216	1,251,522	1,170,676	1,099,566	1,035,682	978,182	32
28	1,257,869	1,178,377	1,104,045	1,038,195	978,709	924,886	33
29	1,183,865	1,110,682	1,042,078	980,896	925,349	874,841	34
30	1,115,209	1,047,660	984,186	927,215	875,236	827,748	35
31	1,051,556	988,984	930,053	876,852	828,092	783,346	36
32	992,380	934,209	879,316	829,494	783,644	741,402	37
33	937,045	882,844	831,599	784,851	741,659	701,712	38
34	885,246	834,612	786,652	742,693	701,931	664,096	39
35	836,735	789,290	744,278	702,844	664,293	628,394	40
36	791,039	746,504	704,186	665,070	628,561	594,460	41
37	748,010	706,103	666,228	629,234	594,605	562,170	42
38	707,419	667,897	630,244	595,191	562,295	531,409	43
39	669,022	631,679	596,063	562,803	531,517	502,077	44
40	632,659	597,313	563,567	531,964	502,174	474,085	45
41	598,154	564,646	532,624	502,560	474,169	447,351	46
42	565,363	533,555	503,132	474,505	447,425	421,802	47
43	534,216	503,967	475,018	447,725	421,869	397,372	48
44	504,514	475,724	448,159	422,125	397,429	374,003	49

Table 13 (Continued)

100% AMERICAN REMARRIAGE TABLES, 1939-41[a/]

$N^{r}_{[x]+n}$ Columns - Interest at 3 Percent

Age at Entry [x]	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or More	
	$N^{r}_{[x]}$	$N^{r}_{[x]+1}$	$N^{r}_{[x]+2}$	$N^{r}_{[x]+3}$	$N^{r}_{[x]+4}$	N^{r}_{x+5}	
45	476,199	448,774	422,504	397,653	374,053	351,644	50
46	449,220	423,059	397,993	374,253	351,689	330,246	51
47	423,457	398,485	374,554	351,865	330,285	309,766	52
48	398,846	374,995	352,136	330,443	309,801	290,166	53
49	375,281	352,502	330,670	309,936	290,197	271,412	54
50	352,805	331,024	310,152	290,322	271,441	253,471	55
51	331,297	310,469	290,514	271,551	253,496	236,314	56
52	310,736	290,813	271,733	253,599	236,337	219,916	57
53	291,038	271,990	253,756	236,428	219,937	204,255	58
54	272,208	253,938	236,574	220,020	204,274	189,309	59
55	254,213	236,803	220,157	204,350	189,326	175,060	60
56	236,996	220,364	204,475	189,397	175,076	161,490	61
57	220,544	204,667	189,512	175,141	161,505	148,583	62
58	204,847	189,697	175,252	161,567	148,597	136,323	63
59	189,877	175,431	161,673	148,656	136,337	124,697	64
60	175,599	161,839	148,754	136,390	124,709	113,690	65
61	161,978	148,898	136,476	124,756	113,701	103,292.0	66
62	149,035	136,613	124,836	113,744	103,301.8	93,491.0	67
63	136,745	124,967	113,821	103,343.8	93,500.3	84,275.6	68
64	125,103	113,951	103,419.3	93,541.2	84,285.1	75,636.0	69
65	114,067	103,532.6	93,606.8	84,320.9	75,644.0	67,561.8	70
66	103,631.3	93,706.5	84,378.5	75,675.4	67,568.7	60,043.5	71
67	93,793.5	84,468.2	75,728.2	67,597.8	60,049.9	53,071.3	72
68	84,556.2	75,814.5	67,647.3	60,076.7	53,077.1	46,634.3	73
69	75,882.7	67,718.9	60,118.6	53,100.0	46,639.1	40,720.9	74
70	67,763.0	60,171.7	53,131.8	46,657.2	40,724.7	35,318.8	75
71	60,219.0	53,182.9	46,687.0	40,741.0	35,322.2	30,413.7	76
72	53,204.2	46,719.4	40,760.8	35,333.8	30,415.9	25,988.8	77
73	46,739.2	40,789.0	35,351.1	30,425.8	25,990.8	22,026.0	78

Functions beyond age at entry 73 are ultimate, depending upon mortality rates only.

a/ Based on remarriage rates (100%) from American Remarriage Table combined with U.S. White Females 1939-41 mortality rates.

Table 14

150% AMERICAN REMARRIAGE TABLES, 1939-41[a/]

$N^{r}_{[x]+n}$ Columns - Interest at 3 Percent

Age at Entry. [x]	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or More	
	$N^{r}_{[x]}$	$N^{r}_{[x]+1}$	$N^{r}_{[x]+2}$	$N^{r}_{[x]+3}$	$N^{r}_{[x]+4}$	N^{r}_{x+5}	
18	3,691,699	3,200,914	2,774,318	2,465,521	2,226,026	2,039,136	23
19	3,282,609	2,873,988	2,516,318	2,252,094	2,044,113	1,879,469	24
20	2,939,074	2,595,672	2,293,118	2,065,380	1,883,572	1,737,669	25
21	2,647,404	2,356,481	2,098,579	1,900,941	1,741,050	1,611,070	26
22	2,397,926	2,149,516	1,927,995	1,755,353	1,613,885	1,497,462	27
23	2,183,749	1,969,667	1,777,694	1,625,763	1,499,816	1,395,000	28
24	1,997,349	1,811,589	1,644,141	1,509,664	1,396,969	1,302,150	29
25	1,833,886	1,671,721	1,524,796	1,405,142	1,303,785	1,217,639	30
26	1,690,228	1,547,594	1,417,751	1,310,632	1,219,018	1,140,405	31
27	1,562,931	1,436,649	1,321,186	1,224,786	1,141,571	1,069,535	32
28	1,449,257	1,336,822	1,233,592	1,146,421	1,070,521	1,004,265	33
29	1,347,539	1,246,787	1,153,919	1,074,670	1,005,121	943,944	34
30	1,255,442	1,164,778	1,080,900	1,008,598	944,663	888,004	35
31	1,172,053	1,090,022	1,013,869	947,620	888,621	835,971	36
32	1,096,263	1,021,618	952,112	891,141	836,494	787,428	37
33	1,026,727	958,571	894,917	838,630	787,872	742,824	38
34	962,883	900,380	841,837	789,692	742,404	699,450	39
35	904,080	846,510	792,456	743,981	699,785	659,437	40
36	849,526	796,348	746,290	701,111	659,717	621,746	41
37	798,961	749,631	703,098	660,860	621,988	586,177	42
38	751,817	705,929	662,553	622,965	586,384	552,550	43
39	707,809	664,976	624,418	587,222	552,726	520,709	44
40	666,619	626,525	588,499	553,468	520,870	490,517	45
41	627,892	590,275	554,541	521,483	490,651	461,848	46
42	591,461	556,088	522,437	491,201	461,970	434,597	47
43	557,150	523,794	492,029	462,453	434,706	408,663	48
44	524,693	493,186	463,156	435,116	408,753	383,962	49

Table 14 (Continued)

150% AMERICAN REMARRIAGE TABLES, 1939-41[a/]

$D^{r}_{[x]+n}$ Columns - Interest at 3 Percent

Age at Entry [x]	Years Elapsed Since Husband's Death						Age attained x+5
	0	1	2	3	4	5 or more	
	$D^{r}_{[x]}$	$D^{r}_{[x]+1}$	$D^{r}_{[x]+2}$	$D^{r}_{[x]+3}$	$D^{r}_{[x]+4}$	D^{r}_{x+5}	
45	493,928	464,134	435,713	409,104	384,039	360,424	50
46	464,851	436,608	409,648	384,355	360,494	337,982	51
47	437,235	410,429	384,831	360,771	338,043	316,874	52
48	411,010	385,540	361,205	338,294	316,630	296,150	53
49	385,981	361,777	338,647	316,839	296,197	276,662	54
50	362,266	339,214	317,184	296,396	276,707	258,066	55
51	339,644	317,685	296,700	276,882	258,107	240,324	56
52	318,093	297,161	277,160	258,263	240,358	223,402	57
53	297,512	277,562	258,506	240,497	223,433	207,275	58
54	277,909	258,889	240,729	223,565	207,306	191,914	59
55	259,221	241,083	223,776	207,423	191,940	177,295	60
56	241,387	224,101	207,618	192,051	177,321	163,397	61
57	224,380	207,916	192,223	177,420	163,419	150,198	62
58	208,192	192,515	177,592	163,516	150,221	137,681	63
59	192,794	177,868	163,678	150,311	137,703	125,828	64
60	178,121	163,931	150,459	137,783	125,845	114,623	65
61	164,148	150,683	137,916	125,919	114,639	104,052.2	66
62	150,902	138,134	126,046	114,710	104,068.3	94,102.2	67
63	138,339	126,250	114,828	104,133	94,117.3	84,769.0	68
64	126,460	115,028	104,250	94,180.4	84,774.0	76,011.0	69
65	115,203	104,421	94,279.1	84,828.0	76,023.3	67,846.0	70
66	104,569	94,430.5	84,915.2	76,070.5	67,856.0	60,253.0	71
67	94,561.9	85,050.6	76,149.7	67,899.6	60,282.2	53,220.7	72
68	85,184.7	76,281.1	67,975.2	60,303.2	53,229.8	46,736.7	73
69	76,387.3	68,085.3	60,367.3	53,264.5	46,743.8	40,787.2	74
70	68,152.5	60,448.6	53,313.6	46,772.0	40,793.2	35,358.7	75
71	60,518.1	53,389.0	46,815.8	40,817.2	35,363.6	30,435.0	76
72	53,422.8	46,866.0	40,848.0	35,381.7	30,438.6	25,998.3	77
73	46,893.6	40,888.8	35,406.5	30,453.0	26,001.1	22,028.8	78

Functions beyond age at entry 73 are ultimate, depending upon mortality rates only.

a/ Based on remarriage rates (150%) from American Remarriage Table combined with U.S. White Females 1939-41 mortality rates.

Table 15

100% AMERICAN REMARRIAGE TABLES, 1939-41[a]/

$r^{r}_{[x]}$ Probability of Remarriage

Age at Entry [x]	Years Elapsed Since Husband's Death: 0	1	2	3	4	5 or more	Age Attained x+5
	$r^{r}_{[x]}$	$r^{r}_{[x]+1}$	$r^{r}_{[x]+2}$	$r^{r}_{[x]+3}$	$r^{r}_{[x]+4}$	r^{r}_{x+5}	
18	.0689	.1688	.1332	.1299	.0790	.0558	23
19	.0648	.1586	.1252	.1221	.0742	.0525	24
20	.0608	.1489	.1175	.1146	.0697	.0493	25
21	.0570	.1395	.1101	.1074	.0653	.0462	26
22	.0533	.1305	.1029	.1004	.0611	.0432	27
23	.0499	.1222	.0963	.0940	.0571	.0404	28
24	.0466	.1141	.0900	.0877	.0533	.0378	29
25	.0434	.1063	.0838	.0818	.0496	.0352	30
26	.0404	.0990	.0781	.0761	.0462	.0328	31
27	.0376	.0921	.0726	.0708	.0431	.0305	32
28	.0349	.0855	.0674	.0658	.0400	.0283	33
29	.0324	.0793	.0626	.0611	.0372	.0263	34
30	.0300	.0733	.0579	.0565	.0344	.0243	35
31	.0277	.0678	.0535	.0522	.0318	.0226	36
32	.0257	.0627	.0495	.0483	.0294	.0208	37
33	.0237	.0578	.0456	.0445	.0271	.0192	38
34	.0218	.0532	.0420	.0409	.0249	.0176	39
35	.0201	.0490	.0386	.0376	.0230	.0163	40
36	.0184	.0449	.0354	.0345	.0211	.0149	41
37	.0169	.0412	.0324	.0316	.0193	.0137	42
38	.0155	.0377	.0297	.0290	.0177	.0126	43
39	.0142	.0345	.0272	.0266	.0162	.0115	44
40	.0130	.0315	.0249	.0243	.0149	.0105	45
41	.0118	.0288	.0228	.0222	.0136	.0096	46
42	.0107	.0263	.0208	.0202	.0124	.0087	47
43	.0098	.0241	.0191	.0185	.0114	.0080	48
44	.0090	.0220	.0174	.0169	.0104	.0073	49

Table 15 (Continued)

100% AMERICAN REMARRIAGE TABLES, 1939-41a/

$r^{r}_{[x]}$ Probability of Remarriage

Age at Entry [x]	Years Elapsed Since Husband's Death						Age Attained x+5
	0	1	2	3	4	5 or more	
	$r^{r}_{[x]}$	$r^{r}_{[x]+1}$	$r^{r}_{[x]+2}$	$r^{r}_{[x]+3}$	$r^{r}_{[x]+4}$	r^{r}_{x+5}	
45	.0082	.0200	.0159	.0154	.0095	.0066	50
46	.0075	.0184	.0146	.0141	.0087	.0061	51
47	.0069	.0169	.0134	.0131	.0080	.0056	52
48	.0063	.0155	.0124	.0121	.0074	.0051	53
49	.0058	.0142	.0113	.0110	.0067	.0047	54
50	.0054	.0132	.0105	.0102	.0063	.0043	55
51	.0050	.0123	.0098	.0095	.0058	.0041	56
52	.0047	.0115	.0092	.0089	.0055	.0038	57
53	.0044	.0108	.0086	.0083	.0051	.0036	58
54	.0041	.0102	.0081	.0078	.0049	.0034	59
55	.0040	.0097	.0077	.0075	.0046	.0032	60
56	.0038	.0093	.0073	.0071	.0044	.0031	61
57	.0036	.0088	.0070	.0068	.0042	.0029	62
58	.0035	.0085	.0067	.0066	.0041	.0028	63
59	.0034	.0084	.0066	.0064	.0040	.0028	64
60	.0034	.0082	.0065	.0063	.0039	.0028	65
61	.0032	.0079	.0063	.0061	.0037	.0027	66
62	.0032	.0078	.0061	.0060	.0036	.0027	67
63	.0032	.0076	.0060	.0059	.0036	.0026	68
64	.0032	.0076	.0060	.0059	.0036	.0026	69
65	.0032	.0076	.0059	.0058	.0035	.0025	70
66	.0031	.0074	.0058	.0056	.0034	.0024	71
67	.0030	.0071	.0055	.0055	.0033	.0023	72
68	.0028	.0070	.0055	.0054	.0032	.0023	73
69	.0027	.0067	.0053	.0052	.0031	.0022	74
70	.0025	.0062	.0049	.0048	.0029	.0020	75
71	.0024	.0058	.0047	.0046	.0027	.0019	76
72	.0021	.0052	.0042	.0041	.0024	.0017	77
73	.0019	.0046	.0038	.0037	.0022	.0015	78

a/ Based on 100% remarriage rates from American Remarriage Table combined with U.S. White Females 1939-41 mortality rates.

VALUATION OF DEATH BENEFITS UNDER U. S. LONGSHOREMEN'S AND HARBOR WORKERS' COMPENSATION ACT AS AMENDED JUNE 24, 1948

BY

SYLVIA POTOFSKY

The amendment to the U. S. Longshoremen's & Harbor Workers' Compensation Act effective June 24, 1948 removed the previous maximum limitation of $7500 for total payments in any one death claim. Accordingly, in contrast to the situation before the 1948 amendment when most death claims were valued at the $7500 maximum, individual computations based on age and relationship of dependents will now have to be made for the purpose of determining loss reserves or for filing of loss data with rate-making agencies.

The purpose of this paper is to present a practical method for valuing the loss reserves on the basis of the revised benefit provisions.

The following is an outline of the death benefits provided under the Longshoremen's & Harbor Workers' Compensation Act.

Dependent	*Death Benefits*
Widow (or widower)	35% of wages payable during widowhood with two years' compensation in one sum on remarriage.
Parents or grandparents	25% of wages payable during dependency.
Brothers, sisters, grandchildren	15% of wages payable until age 18.
Children	Where there is no widow, 35% of wages for one child and 15% for each additional child, shared equally, payable until age 18.
	Where there is a widow, 15% of wages for each child, increased when widow dies or remarries to 35% for one child and 15% for each additional child, shared equally, payable until age 18.

Maximum for all dependents combined may not exceed 66⅔% of wages.

For computation of lump sum awards payable to dependents, the Longshoremen's & Harbor Workers' Compensation Act specifies the use of the American Experience Table of Mortality, the remarriage tables of the Dutch Royal Insurance Institution and interest at 4% per annum.

However, for the purpose of estimating loss reserves under the U. S. Longshoremen's & Harbor Workers' Compensation Act other than for use in awards which fix the actual commuted amounts payable to dependents, it would appear practicable to employ the basic annuity values which are already available in connection with the New York Workmen's Compensation Law and some of the tabular values already published in New York Workmen's Compensation Special Bulletins No. 207 and No. 222, if we accept as the basis for valuation the Survivorship Annuitants Table of Mortality instead of the American Experience Table and 3% instead of 4% interest per annum. It should be recognized that the 3% interest rate and the Survivorship Annuitants Table will result in higher values.

If we are willing to accept the basic premise of 3% interest per annum and the use of the Survivorship Annuitants Table of Mortality as the basis for valuation of death benefits under the Longshoremen's & Harbor Workers' Compensation Act, the following is a method of arriving at the necessary formulae and tabular values for the various classes of dependents:

WIDOW (OR WIDOWER)

Death benefits to widow (or widower) are 35% of wages payable until death or remarriage plus two years' compensation in one sum on remarriage. The formula for present value of benefits to widow (or widower) per $100 annual wages is $35\,\bar{a}_{x'} + 70\,\bar{E}_{x'}$ where $\bar{a}_{x'}$ indicates an annuity of 1 per annum payable continuously to widow age x until death or remarriage and $\bar{E}_{x'}$ indicates the present value of 1 payable at moment of remarriage.

Using values of $\bar{a}_{x'}$ and $\bar{E}_{x'}$ based on the Survivorship Annuitants Table of Mortality and the Dutch Royal remarriage table and 3% interest, Table 1 of the Appendix has been constructed showing the present value per $100 annual wages of benefits to widow (or widower) under the Longshoremen's & Harbor Workers' Compensation Act.

PARENTS OR GRANDPARENTS

Death benefits to dependent parents or grandparents are 25% of wages payable during dependency (which for actuarial computation purposes is assumed to be until death). These benefits are the same as benefits to dependent parents or grandparents under the N. Y. Workmen's Compensation Law prior to 7/1/48.

Therefore, the present value of benefits to parents or grandparents under the U. S. Longshoremen's & Harbor Workers' Act may be found directly from Table X published on page 20 of N. Y. Special Bulletin No. 207.

BROTHERS, SISTERS, GRANDCHILDREN

Death benefits to dependent brothers, sisters or grandchildren are 15% of wages payable until age 18. These benefits are the same as benefits to these dependents under the N. Y. Workmen's Compensation Law prior to 7/1/48.

Therefore, the present value of benefits to brothers, sisters, or grandchildren under the U. S. Longshoremen's & Harbor Workers' Act may be

found directly from Table IX published on page 19 of N. Y. Special Bulletin No. 207.

CHILDREN

For valuation of children's benefits, it was necessary to derive a set of formulae to express the present value of benefits in terms of joint-life annuities. The method used is explained fully in the article by Mr. Joseph H. Woodward, "Valuation of the Benefit to Widow & Children, Provided by the N. Y. Workmen's Compensation Law as Amended in 1922" published in T. A. S. A. Vol. XXIV page 414.

As explained in this article it is assumed that the force of mortality with regard to children's lives is constant for all ages 0 to 18. Therefore, the present value of an annuity involving joint-lives of children depends only on the total number of children (independent of their ages) and the number of years until oldest child reaches age 18. The expression ${}_r\bar{a}_{\overline{18-y_n}|}$ represents the present value of an annuity during joint lives of r children payable until oldest child, age y_n reaches age 18. Similarly, the expression, ${}_r\bar{a}_{x:\overline{18-y_n}|}$ represents the present value of an annuity involving joint-lives of r children and the life and probability of remarriage of a widow age x payable until oldest child, age y_n reaches age 18.

Where there is only one child, the benefit under the Longshoremen's & Harbor Workers' Compensation Act is 15% of wages while the widow is living and unmarried and 35% after widow dies or remarries, payable until child reaches age 18. Therefore, the present value of benefits per $100 annual wages is $35\,{}_1\bar{a}_{\overline{18-y_1}|} - 20\,{}_1\bar{a}_{x:\overline{18-y_1}|}$ where x is age of widow and y_1 is age of child.

However, where there is more than one child it is necessary to consider that the total benefits to the widow and all children may not exceed the 66⅔% maximum provided under the Longshoremen's & Harbor Workers' Compensation Act. Where widow's age is x and there are n children age $y_1, y_2, y_3, \cdots y_n$ in ascending order of age, the benefit in respect of child age y_n in addition to benefits payable in respect of the widow and $(n-1)$ youngest children is:

while widow is alive and unmarried

15% while none of $(n-1)$ youngest children is alive and under age 18,

15% while exactly one of $(n-1)$ youngest children is alive and under age 18,

1⅔% while exactly two of $(n-1)$ youngest children are alive and under age 18,

0% while three or more of $(n-1)$ youngest children are alive and under age 18.

after death or remarriage of widow

35% while none of $(n-1)$ youngest children is alive and under age 18,

15% while exactly one of $(n-1)$ youngest children is alive and under age 18,

15% while exactly two of $(n-1)$ youngest children are alive and under age 18,

1⅔% while exactly three of *(n−1)* youngest children are alive and under age 18,

0% while four or more of *(n−1)* youngest children are alive and under age 18.

The benefits described above may be expressed in terms of annuities in the form $\bar{a}_{yn:\overline{18\text{-}y_n|}} \dfrac{[r]}{y_1\, y_2\, y_3 \cdots y_{n-1}}$ which is defined as present value of payments until oldest child y_n reaches age 18 while exactly r out of the remaining *(n−1)* children are living, and also in the form $\bar{a}_{x'yn:\overline{18\text{-}y_n|}} \dfrac{[r]}{y_1\, y_2\, y_3 \cdots y_{n-1}}$ which is the present value of payments until oldest child y_n reaches age 18 while exactly r out of the remaining *(n−1)* children are living and the widow age x is living and unmarried.

Now the present value of the children's benefits may be expressed as follows:

$$15\,\bar{a}_{x'yn:\overline{18\text{-}y_n|}} \frac{[0]}{y_1\, y_2\, y_3 \cdots y_{n-1}} + 15\,\bar{a}_{x'yn:\overline{18\text{-}y_n|}} \frac{[1]}{y_1\, y_2\, y_3 \cdots y_{n-1}}$$

$$+ 1\tfrac{2}{3}\,\bar{a}_{x'yn:\overline{18\text{-}y_n|}} \frac{[2]}{y_1\, y_2\, y_3 \cdots y_{n-1}}$$

$$+ 35\left(\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[0]}{y_1\, y_2\, y_3 \cdots y_{n-1}} - \bar{a}_{x'yn:\overline{18\text{-}y_n|}} \frac{(0)}{y_1\, y_2\, y_3 \cdots y_{n-1}}\right)$$

$$+ 15\left(\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[1]}{y_1\, y_2\, y_3 \cdots y_{n-1}} - \bar{a}_{x'yn:\overline{18\text{-}y_n|}} \frac{[1]}{y_1\, y_2\, y_3 \cdots y_{n-1}}\right)$$

$$+ 15\left(\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[2]}{y_1\, y_2\, y_3 \cdots y_{n-1}} - \bar{a}_{x'yn:\overline{18\text{-}y_n|}} \frac{[2]}{y_1\, y_2\, y_3 \cdots y_{n-1}}\right)$$

$$+ 1\tfrac{2}{3}\left(\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[3]}{y_1\, y_2\, y_3 \cdots y_{n-1}} - \bar{a}_{x'yn:\overline{18\text{-}y_n|}} \frac{[3]}{y_1\, y_2\, y_3 \cdots y_{n-1}}\right)$$

$$= 35\,\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[0]}{y_1\, y_2\, y_3 \cdots y_{n-1}} + 15\,\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[1]}{y_1\, y_2\, y_3 \cdots y_{n-1}}$$

$$+ 15\,\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[2]}{y_1\, y_2\, y_3 \cdots y_{n-1}} + 1\tfrac{2}{3}\,\bar{a}_{yn:\overline{18\text{-}y_n|}} \frac{[3]}{y_1\, y_2\, y_3 \cdots y_{n-1}}$$

$$- \ 20\ \bar{a}_{x'yn\,:\,\overline{18\text{-}y_n|}}\ \frac{[0]}{y_1\ y_2\ y_3\ \cdots\ y_{n-1}} \ - \ 13\tfrac{1}{3}\ \bar{a}_{x'yn\,:\,\overline{18\text{-}y_n|}}\ \frac{[2]}{y_1\ y_2\ y_3\ \cdots\ y_{n-1}}$$

$$- \ 1\tfrac{2}{3}\ \bar{a}_{x'yn\,:\,\overline{18\text{-}y_n|}}\ \frac{[3]}{y_1\ y_2\ y_3\ \cdots\ y_{n-1}} \qquad (1)$$

As indicated above, it has been assumed that the force of mortality is constant for all children's ages from 0 to 18. The probability that a child (regardless of age) will live for one year is s (where the force of mortality $\mu = .00522 = -\log_e s$).

Therefore the probability that y_n will live one year, that exactly r out of the remaining $(n-1)$ children will live one year and that the remaining $(n-1-r)$ children will die within one year is

$$_{n-1}C_r\ (s)\ (s)^r\ (1\text{-}s)^{n-1-r} \ = \ _{n-1}C_r\ (s^{r+1})\ (1\text{-}s)^{n-1-r}$$

$$= \ _{n-1}C_r\Big[s^{r+1} \ - \ (_{n-1-r}C_1)\ s^{r+2} \ + \ (_{n-1-r}C_2)\ s^{r+3} \ - \ (_{n-1-r}C_3)\ s^{r+4} \ + \ \text{etc.}\Big]$$

The joint-life annuity $\ _r\bar{a}_{\overline{\ t\ }} = \int_{t=o}^{t=18-y} (vs^r)^t\,dt$

Therefore

$$\bar{a}_{yn\,:\,\overline{18\text{-}y_n|}}\ \frac{[r]}{y_1\ y_2\ y_3\ \cdots\ y_{n-1}} \ = \ _{n-1}C_r\Big[(_{n-1-r}C_0)\ _{r+1}\bar{a}_{\overline{18\text{-}y_n|}} \ -$$

$$(_{n-1-r}C_1)_{r+2}\ \bar{a}_{\overline{18\text{-}y_n|}} \ + \ (_{n-1-r}C_2)\ _{r+3}\bar{a}_{\overline{18\text{-}y_n|}} \ - \ \text{etc.}\Big]\ .$$

$$\bar{a}_{x'yn\,:\,\overline{18\text{-}y_n|}}\ \frac{[r]}{y_1\ y_2\ y_3\ \cdots\ y_{n-1}} \ = \ _{n-1}C_r\Big[(_{n-1-r}C_0)\ _{r+1}\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ -$$

$$(_{n-1-r}C_1)\ _{r+2}\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ + \ (_{n-1-r}C_2)\ _{r+3}\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ - \ \text{etc.}\Big]$$

In terms of joint-life annuities formula (1) becomes:

$$\begin{aligned}
&35\ (_{n-1}C_0)\ (_{n-1}C_0 \ \times \ _1\bar{a}_{\overline{18\text{-}y_n|}} \ - \ _{n-1}C_1 \ \times \ _2\bar{a}_{\overline{18\text{-}y_n|}} \ + \ \text{etc.}) \\
&+ \ 15\ (_{n-1}C_1)\ (_{n-2}C_0 \ \times \ _2\bar{a}_{\overline{18\text{-}y_n|}} \ - \ _{n-2}C_1 \ \times \ _3\bar{a}_{\overline{18\text{-}y_n|}} \ + \ \text{etc.}) \\
&+ \ 15\ (_{n-1}C_2)\ (_{n-3}C_0 \ \times \ _3\bar{a}_{\overline{18\text{-}y_n|}} \ - \ _{n-3}C_1 \ \times \ _4\bar{a}_{\overline{18\text{-}y_n|}} \ + \ \text{etc.}) \\
&+ \ 1\tfrac{2}{3}\ (_{n-1}C_3)\ (_{n-4}C_0 \ \times \ _4\bar{a}_{\overline{18\text{-}y_n|}} \ - \ _{n-4}C_1 \ \times \ _5\bar{a}_{\overline{18\text{-}y_n|}} \ + \ \text{etc.}) \qquad (2) \\
&- \ 20\ (_{n-1}C_0)\ (_{n-1}C_0 \ \times \ _1\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ - \ _{n-1}C_1 \ \times \ _2\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ + \ \text{etc.}) \\
&- \ 13\tfrac{1}{3}\ (_{n-1}C_2)\ (_{n-3}C_0 \ \times \ _3\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ - \ _{n-3}C_1 \ \times \ _4\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ + \ \text{etc.}) \\
&- \ 1\tfrac{2}{3}\ (_{n-1}C_3)\ (_{n-4}C_0 \ \times \ _4\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ - \ _{n-4}C_1 \ \times \ _5\bar{a}_{x'\,:\,\overline{18\text{-}y_n|}} \ + \ \text{etc.})
\end{aligned}$$

Substituting n= 1, 2, 3, 4, 5 respectively in above formula (2), we obtain the present value of benefits to youngest, second youngest, third youngest, fourth youngest and fifth youngest child as follows:

Youngest Child

$$35\ {}_1\bar{a}_{\overline{18\text{-}y_1}|} \ -\ 20\ {}_1\bar{a}_{x'\,:\,\overline{18\text{-}y_1}|} \qquad \ldots\ldots(3)$$

Second Youngest Child

$$35\ ({}_1C_0)\ ({}_1\bar{a}_{\overline{18\text{-}y_2}|} \ -\ {}_2\bar{a}_{\overline{18\text{-}y_2}|}) \ +\ 15\ ({}_1C_1)\ ({}_2\bar{a}_{\overline{18\text{-}y_2}|})$$
$$-\ 20\ ({}_1C_0)\ ({}_1\bar{a}_{x'\,:\,\overline{18\text{-}y_2}|} \ -\ {}_2\bar{a}_{x'\,:\overline{18\text{-}y_2}|})$$
$$=\ 35\ {}_1\bar{a}_{\overline{18\text{-}y_2}|} \ -\ 20\ {}_2\bar{a}_{\overline{18\text{-}y_2}|} \ -\ 20\,{}_1\bar{a}_{x'\,:\,\overline{18\text{-}y_2}|} \ +\ 20\ {}_2\bar{a}_{x'\,:\,\overline{18\text{-}y_2}|} \qquad \ldots\ldots(4)$$

Third Youngest Child

$$35\ ({}_2C_0)\ ({}_1\bar{a}_{\overline{18\text{-}y_3}|} \ -\ 2\ {}_2\bar{a}_{\overline{18\text{-}y_3}|} \ +\ {}_3\bar{a}_{\overline{18\text{-}y_3}|})$$
$$+\ 15\ ({}_2C_1)\ ({}_2\bar{a}_{\overline{18\text{-}y_3}|} \ -\ {}_3\bar{a}_{\overline{18\text{-}y_3}|}) \ +\ 15\ ({}_2C_2)\ ({}_3\bar{a}_{\overline{18\text{-}y_3}|})$$
$$-\ 20\ ({}_2C_0)\ ({}_1\bar{a}_{x'\,:\,\overline{18\text{-}y_3}|} \ -\ 2\ {}_2\bar{a}_{x'\,:\,\overline{18\text{-}y_3}|} \ +\ {}_3\bar{a}_{x'\,:\,\overline{18\text{-}y_3}|})$$
$$-\ 13\tfrac{1}{3}\ ({}_2C_2)\ ({}_3\bar{a}_{x'\,:\,\overline{18\text{-}y_3}|})$$
$$=\ 35\ {}_1\bar{a}_{\overline{18\text{-}y_3}|} \ -\ 40\ {}_2\bar{a}_{\overline{18\ y_3}|} \ +\ 20\ {}_3\bar{a}_{\overline{18\text{-}y_3}|} \ -\ 20\ {}_1\bar{a}_{x'\,:\,\overline{18\text{-}y_3}|}$$
$$+\ 40\ {}_2\bar{a}_{x'\,:\,\overline{18\text{-}y_3}|} \ -\ 33\tfrac{1}{3}\ {}_3\bar{a}_{x'\,:\,\overline{18\text{-}y_3}|} \qquad \ldots\ldots(5)$$

Fourth Youngest Child

$$35\ ({}_3C_0)\ ({}_1\bar{a}_{\overline{18\text{-}y_4}|} \ -\ 3\ {}_2\bar{a}_{\overline{18\text{-}y_4}|} \ +\ 3\ {}_3\bar{a}_{\overline{18\text{-}y_4}|} \ -\ {}_4\bar{a}_{\overline{18\text{-}y_4}|})$$
$$+\ 15\ ({}_3C_1)\ ({}_2\bar{a}_{\overline{18\text{-}y_4}|} \ -\ 2\ {}_3\bar{a}_{\overline{18\text{-}y_4}|} \ +\ {}_4\bar{a}_{\overline{18\text{-}y_4}|})$$
$$+\ 15\ ({}_3C_2)\ ({}_3\bar{a}_{\overline{18\text{-}y_4}|} \ -\ {}_4\bar{a}_{\overline{18\text{-}y_4}|}) \ +\ 1\tfrac{2}{3}\ ({}_3C_3)\ ({}_4\bar{a}_{\overline{18\text{-}y_4}|})$$
$$-\ 20\ ({}_3C_0)\ ({}_1\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \ -\ 3\ {}_2\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \ +\ 3\ {}_3\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \ -\ {}_4\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|})$$
$$-\ 13\tfrac{1}{3}\ ({}_3C_2)\ ({}_3\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \ -\ {}_4\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|}) \ -\ 1\tfrac{2}{3}\ ({}_3C_3)\ ({}_4\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|})$$
$$=\ 35\ {}_1\bar{a}_{\overline{18\text{-}y_4}|} \ -\ 60\ {}_2\bar{a}_{\overline{18\text{-}y_4}|} \ +\ 60\ {}_3\bar{a}_{\overline{18\text{-}y_4}|} \ -\ 33\tfrac{1}{3}\ {}_4\bar{a}_{\overline{18\text{-}y_4}|}$$
$$-\ 20\ {}_1\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \ +\ 60\ {}_2\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \ -\ 100\ {}_3\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \ +\ 58\tfrac{1}{3}\ {}_4\bar{a}_{x'\,:\,\overline{18\text{-}y_4}|} \qquad \ldots(6)$$

Fifth Youngest Child

$$35\,({}_4C_0)\,({}_1\bar{a}_{\overline{18\text{-}y5}|} - 4\,{}_2\bar{a}_{\overline{18\text{-}y5}|} + 6\,{}_3\bar{a}_{\overline{18\text{-}y5}|} - 4\,{}_4\bar{a}_{\overline{18\text{-}y5}|} + {}_5\bar{a}_{\overline{18\text{-}y5}|})$$

$$+ 15\,({}_4C_1)\,({}_2\bar{a}_{\overline{18\text{-}y5}|} - 3\,{}_3\bar{a}_{\overline{18\text{-}y5}|} + 3\,{}_4\bar{a}_{\overline{18\text{-}y5}|} - {}_5\bar{a}_{\overline{18\text{-}y5}|})$$

$$+ 15\,({}_4C_2)\,({}_3\bar{a}_{\overline{18\text{-}y5}|} - 2\,{}_4\bar{a}_{\overline{18\text{-}y5}|} + {}_5\bar{a}_{\overline{18\text{-}y5}|})$$

$$+ 1\tfrac{2}{3}\,({}_4C_3)\,({}_4\bar{a}_{\overline{18\text{-}y5}|} - {}_5\bar{a}_{\overline{18\text{-}y5}|})$$

$$- 20\,({}_4C_0)\,({}_1\bar{a}_{x':\overline{18\text{-}y5}|} - 4\,{}_2\bar{a}_{x':\overline{18\text{-}y5}|} + 6\,{}_3\bar{a}_{x':\overline{18\text{-}y5}|} - 4\,{}_4\bar{a}_{x':\overline{18\text{-}y5}|} + {}_5\bar{a}_{x':\overline{18\text{-}y5}|})$$

$$- 13\tfrac{1}{3}\,({}_4C_2)\,({}_3\bar{a}_{x':\overline{18\text{-}y5}|} - 2\,{}_4\bar{a}_{x':\overline{18\text{-}y5}|} + {}_5\bar{a}_{x':\overline{18\text{-}y5}|})$$

$$- 1\tfrac{1}{3}\,({}_4C_3)\,({}_4\bar{a}_{x':\overline{18\text{-}y5}|} - {}_5\bar{a}_{x':\overline{18\text{-}y5}|})$$

$$= 35\,{}_1\bar{a}_{\overline{18\text{-}y5}|} - 80\,{}_2\bar{a}_{\overline{18\text{-}y5}|} + 120\,{}_3\bar{a}_{\overline{18\text{-}y5}|} - 133\tfrac{1}{3}\,{}_4\bar{a}_{\overline{18\text{-}y5}|}$$

$$+ 58\tfrac{1}{3}\,{}_5\bar{a}_{\overline{18\text{-}y5}|} - 20\,{}_1\bar{a}_{x':\overline{18\text{-}y5}|} + 80\,{}_2\bar{a}_{x':\overline{18\text{-}y5}|}$$

$$- 200\,{}_3\bar{a}_{x':\overline{18\text{-}y5}|} + 233\tfrac{1}{3}\,{}_4\bar{a}_{x':\overline{18\text{-}y5}|} - 93\tfrac{1}{3}\,{}_5\bar{a}_{x':\overline{18\text{-}y5}|} \qquad \ldots(7)$$

Tables 2 and 3 of the Appendix have been constructed showing the present value per $100 annual wages of benefits under the Longshoremen's & Harbor Workers' Compensation Act to youngest child and second youngest child respectively where there is a widow, based on formula (3) and (4) above.

Table 4 of the Appendix shows the present value per $100 annual wages of benefits under the Longshoremen's & Harbor Workers' Compensation Act to youngest, second, third, fourth and fifth youngest child where there is no widow. The formulae for these children's benefits are obtained by omitting terms involving widow x from the general children's formulae (3), (4), (5), (6) and (7), as follows:

Youngest Child (No widow)

$$35\,{}_1\bar{a}_{\overline{18\text{-}y1}|}$$

Second Youngest Child (No widow)

$$35\,{}_1\bar{a}_{\overline{18\text{-}y2}|} - 20\,{}_2\bar{a}_{\overline{18\text{-}y2}|}$$

Third Youngest Child (No widow)

$$35\,{}_1\bar{a}_{\overline{18\text{-}y3}|} - 40\,{}_2\bar{a}_{\overline{18\text{-}y3}|} + 20\,{}_3\bar{a}_{\overline{18\text{-}y3}|}$$

Fourth Youngest Child (No widow)

$$35\ {}_1\bar{a}_{\overline{18-y4}|} - 60\ {}_2\bar{a}_{\overline{18-y4}|} + 60\ {}_3\bar{a}_{\overline{18-y4}|} - 33\tfrac{1}{3}\ {}_4\bar{a}_{\overline{18-y4}|}$$

Fifth Youngest Child (No widow)

$$35\ {}_1\bar{a}_{\overline{18-y5}|} - 80\ {}_2\bar{a}_{\overline{18-y5}|} + 120\ {}_3\bar{a}_{\overline{18-y5}|} - 133\tfrac{1}{3}\ {}_4\bar{a}_{\overline{18-y5}|} + 58\tfrac{1}{3}\ {}_5\bar{a}_{\overline{18-y5}|}$$

Regarding the calculation of present values of benefits to the third, fourth and fifth youngest children where there is a widow it would not appear necessary to construct separate tables as has been done for the first and second youngest children. Instead, it would appear expedient to calculate individual values wherever these are necessary by employing, in various combinations, appropriate values which are available in Table 4 of the Appendix, and in Tables IV, V, VI, VII, VIII and IX of N. Y. Special Bulletin No. 222, as follows:

Third Youngest Child

Formula (5) for present value of benefits to third youngest child is

$$35\ {}_1\bar{a}_{\overline{18-y3}|} - 40\ {}_2\bar{a}_{\overline{18-y3}|} + 20\ {}_3\bar{a}_{\overline{18-y3}|} - 20\ {}_1\bar{a}_{x':\overline{18-y3}|} + 40\ {}_2\bar{a}_{x':\overline{18-y3}|} - 33\tfrac{1}{3}\ {}_3\bar{a}_{x':\overline{18-y3}|}$$

Instead of separately calculating each term of this formula, we can break up the formula into the following 6 component parts for each of which tabular values have already been calculated and are available in Table 4 of the Appendix and Tables IV, V, VI and IX of N. Y. Special Bulletin No. 222.

Component Part of Formula (5)	*Tabular Values*
(1) $35\ {}_1\bar{a}_{\overline{18-y3}|} - 40\ {}_2\bar{a}_{\overline{18-y3}|} + 20\ {}_3\bar{a}_{\overline{18-y3}|}$	Table 4 (column 3)
(2) $-\ 5\tfrac{1}{3}\ (30\ {}_1\bar{a}_{\overline{18-y3}|})$	Table IX (column 1) Multiplied by–5⅓
(3) $+\ 3\tfrac{1}{3}\ (30\ {}_1\bar{a}_{\overline{18-y3}|} - 23\tfrac{1}{3}\ {}_3\bar{a}_{\overline{18-y3}|})$	Table IX (column 2) Multiplied by 3⅓
(4) $+\ 10\tfrac{2}{3}\ (30\ {}_1\bar{a}_{\overline{18-y3}|} - 10\ {}_1\bar{a}_{x':\overline{18-y3}|}$	Table IV Multiplied by 10⅔
(5) $-\ 5\tfrac{1}{3}\ (30\ {}_1\bar{a}_{\overline{18-y3}|} - 10\ {}_1\bar{a}_{x':\overline{18-y3}|} - 3\tfrac{1}{3}\ {}_2\bar{a}_{x':\overline{18-y3}|})$	Table V Multiplied by–5⅓
(6) $-\ 3\tfrac{1}{3}\ (30\ {}_1\bar{a}_{\overline{18-y3}|} - 23\tfrac{1}{3}\ {}_3\bar{a}_{\overline{18-y3}|} - 10\ {}_1\bar{a}_{x':\overline{18-y3}|} - 6\tfrac{2}{3}\ {}_2\bar{a}_{x':\overline{18-y3}|} + 10\ {}_3\bar{a}_{x':\overline{18-y3}|})$	Table VI Multiplied by–3⅓

For example, where widow's age x *is 35* and child's age y_3 *is 9*, the present value of child's benefits is calculated as follows:

(1) Table 4 (column 3)		116.03
(2) Table IX (column 1)	multiplied by $-5\frac{1}{3} = -5\frac{1}{3}$ (231.85)	$= -1236.53$
(3) Table IX (column 2)	" " $3\frac{1}{3} = 3\frac{1}{3}$ (59.30)	$= +\ 197.66$
(4) Table IV	" " $10\frac{2}{3} = 10\frac{2}{3}$ (167.62)	$= +1787.95$
(5) Table V	" " $-5\frac{1}{3} = -5\frac{1}{3}$ (146.65)	$= -\ 782.13$
(6) Table VI	" " $-3\frac{1}{3} = -3\frac{1}{3}$ (14.76)	$= -\ 49.20$
TOTAL		33.78

The value obtained in this case by substituting the actual joint-life annuity values in formula (5) for third youngest child is 33.76.

Fourth Youngest Child

Formula (6) for the present value of benefits to the fourth youngest child is

$$35\ {}_1\bar{a}_{\overline{18\text{-}y_4|}} - 60\ {}_2\bar{a}_{\overline{18\text{-}y_4|}} + 60\ {}_3\bar{a}_{\overline{18\text{-}y_4|}} - 33\tfrac{1}{3}\ {}_4\bar{a}_{\overline{18\text{-}y_4|}}$$
$$- 20\ {}_1\bar{a}_{x':\overline{18\text{-}y_4|}} + 60\ {}_2\bar{a}_{x':\overline{18\text{-}y_4|}} - 100\ {}_3\bar{a}_{x':\overline{18\text{-}y_4|}} + 58\tfrac{1}{3}\ {}_4\bar{a}_{x':\overline{18\text{-}y_4|}}$$

This may be broken up into the following 8 component parts:

Component Part of Formula (6)	*Tabular Values*
(1) $35\ {}_1\bar{a}_{\overline{18\text{-}y_4|}} - 60\ {}_2\bar{a}_{\overline{18\text{-}y_4|}} + 60\ {}_3\bar{a}_{\overline{18\text{-}y_4|}} - 33\frac{1}{3}\ {}_4\bar{a}_{\overline{18\text{-}y_4|}}$	Table 4 (column 4)
(2) $-\ \frac{1}{3}\ (30\ {}_1\bar{a}_{\overline{18\text{-}y_4|}})$	Table IX (column 1) Multiplied by $-\frac{1}{3}$
(3) $-\ 7\frac{1}{2}\ (30\ {}_1\bar{a}_{\overline{18\text{-}y_4|}} - 23\frac{1}{3}\ {}_3\bar{a}_{\overline{18\text{-}y_4|}})$	Table IX (column 2) Multiplied by $-7\frac{1}{2}$
(4) $+\ 5\frac{5}{6}\ (30\ {}_1\bar{a}_{\overline{18\text{-}y_4|}} - 70\ {}_3\bar{a}_{\overline{18\text{-}y_4|}} + 40\ {}_4\bar{a}_{\overline{18\text{-}y_4|}})$	Table IX (column 3) Multiplied by $5\frac{5}{6}$
(5) $+\ 15\frac{5}{6}\ (30\ {}_1\bar{a}_{\overline{18\text{-}y_4|}} - 10\ {}_1\bar{a}_{x':\overline{18\text{-}y_4|}})$	Table IV Multiplied by $15\frac{5}{6}$
(6) $-\ 15\frac{1}{2}\ (30\ {}_1\bar{a}_{\overline{18\text{-}y_4|}} - 10\ {}_1\bar{a}_{x':\overline{18\text{-}y_4|}} - 3\frac{1}{3}\ {}_2\bar{a}_{x':\overline{18\text{-}y_4|}})$	Table V Multiplied by $-15\frac{1}{2}$

(7) $+ 7\frac{1}{2}\,(30\ {}_1\bar{a}_{\overline{18-y_4}|} - 23\frac{1}{3}\ {}_3\bar{a}_{\overline{18-y_4}|} - 10\ {}_1\bar{a}_{x':\overline{18-y_4}|} - 6\frac{2}{3}\ {}_2\bar{a}_{x':\overline{18-y_4}|} + 10\ {}_3\bar{a}_{x':\overline{18-y_4}|})$ — Table VI Multiplied by $7\frac{1}{2}$

(8) $- 5\frac{5}{6}\,(30\ {}_1\bar{a}_{\overline{18-y_4}|} - 70\ {}_3\bar{a}_{\overline{18-y_4}|} + 40\ {}_4\bar{a}_{\overline{18-y_4}|} - 10\ {}_1\bar{a}_{x':\overline{18-y_4}|} - 10\ {}_2\bar{a}_{x':\overline{18-y_4}|} + 30\ {}_3\bar{a}_{x':\overline{18-y_4}|} - 10\ {}_4\bar{a}_{x':\overline{18-y_4}|})$ — Table VII Multipied by $-5\frac{5}{6}$

For example, where widow's age x *is 40* and child's age y_4 *is 12*, the present value of child's benefits is calculated as follows:

(1) Table 4 (column 4)				12.20
(2) Table IX (column 1)	multiplied by	$-\frac{1}{3}$ =	$-\frac{1}{3}$ (162.48) =	−54.16
(3) Table IX (column 2)	" "	$-7\frac{1}{2}$ =	$-7\frac{1}{2}$ (39.84) =	−298.80
(4) Table IX (column 3)	" "	$5\frac{5}{6}$ =	$5\frac{5}{6}$ (1.70) =	+ 9.92
(5) Table IV	" "	$15\frac{5}{6}$ =	$15\frac{5}{6}$ (112.89) =	+1787.42
(6) Table V	" "	$-15\frac{1}{2}$ =	$-15\frac{1}{2}$ (96.60) =	−1497.30
(7) Table VI	" "	$7\frac{1}{2}$ =	$7\frac{1}{2}$ (5.84) =	+ 43.80
(8) Table VII	" "	$-5\frac{5}{6}$ =	$-5\frac{5}{6}$ (.26) =	−1.52
TOTAL				1.56

The value obtained in this case by substituting the actual joint-life annuity values in formula (6) for the fourth youngest child is 1.53.

Fifth Youngest Child

Formula (7) for the present value of benefits to the fifth youngest child is

$$35\ {}_1\bar{a}_{\overline{18-y_5}|} - 80\ {}_2\bar{a}_{\overline{18-y_5}|} + 120\ {}_3\bar{a}_{\overline{18-y_5}|} - 133\frac{1}{3}\ {}_4\bar{a}_{\overline{18-y_5}|} + 58\frac{1}{3}\ {}_5\bar{a}_{\overline{18-y_5}|} - 20\ {}_1\bar{a}_{x':\overline{18-y_5}|} + 80\ {}_2\bar{a}_{x':\overline{18-y_5}|} - 200\ {}_3\bar{a}_{x':\overline{18-y_5}|} + 233\frac{1}{3}\ {}_4\bar{a}_{x':\overline{18-y_5}|} - 93\frac{1}{3}\ {}_5\bar{a}_{x':\overline{18-y_5}|}$$

This may be broken up into the following 10 component parts:

Component Part of Formula (7)	*Tabular Values*
(1) $35\ {}_1\bar{a}_{\overline{18-y_5}|} - 80\ {}_2\bar{a}_{\overline{18-y_5}|} + 120\ {}_3\bar{a}_{\overline{18-y_5}|} - 133\frac{1}{3}\ {}_4\bar{a}_{\overline{18-y_5}|} + 58\frac{1}{3}\ {}_5\bar{a}_{\overline{18-y_5}|}$	Table 4 (column 5)

(2) $- 59\frac{1}{3}\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|})$ — Table IX (column 1) Multiplied by $-59\frac{1}{3}$

(3) $+ 118\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 23\frac{1}{3}\ {}_3\bar{a}\ \overline{18\text{-}y5|})$ — Table IX (column 2) Multiplied by 118

(4) $- 88\frac{2}{3}\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 70\ {}_3\bar{a}\ \overline{18\text{-}y5|} + 40\ {}_4\bar{a}\ \overline{18\text{-}y5|})$ — Table IX (column 3) Multiplied by $-88\frac{2}{3}$

(5) $+ 28\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 140\ {}_3\bar{a}\ \overline{18\text{-}y5|} + 160\ {}_4\bar{a}\ \overline{18\text{-}y5|} - 50\ {}_5\bar{a}\ \overline{18\text{-}y5|})$ — Table IX (column 4) Multiplied by 28

(6) $+ 1\frac{1}{3}\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 10\ {}_1\bar{a}_{x'} : \overline{18\text{-}y5|})$ — Table IV Multiplied by $1\frac{1}{3}$

(7) $+ 58\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 10\ {}_1\bar{a}_{x'} : \overline{18\text{-}y5|} - 3\frac{1}{3}\ {}_2\bar{a}_{x'} : \overline{18\text{-}y5|})$ — Table V Multiplied by 58

(8) $- 118\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 23\frac{1}{3}\ {}_3\bar{a}\ \overline{18\text{-}y5|} - 10\ {}_1\bar{a}_{x'} : \overline{18\text{-}y5|} - 6\frac{2}{3}\ {}_2\bar{a}_{x'} : \overline{18\text{-}y5|} + 10\ {}_3\bar{a}_{x'} : \overline{18\text{-}y5|})$ — Table VI Multiplied by -118

(9) $+ 88\frac{2}{3}\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 70\ {}_3\bar{a}\ \overline{18\text{-}y5|} + 40\ {}_4\bar{a}\ \overline{18\text{-}y5|} - 10\ {}_1\bar{a}_{x'} : \overline{18\text{-}y5|} - 10\ {}_2\bar{a}_{x'} : \overline{18\text{-}y5|} + 30\ {}_3\bar{a}_{x'} : \overline{18\text{-}y5|} - 10\ {}_4\bar{a}_{x'} : \overline{18\text{-}y5|})$ — Table VII Multiplied by $88\frac{2}{3}$

(10) $- 28\ (30\ {}_1\bar{a}\ \overline{18\text{-}y5|} - 140\ {}_3\bar{a}\ \overline{18\text{-}y5|} + 160\ {}_4\bar{a}\ \overline{18\text{-}y5|} - 50\ {}_5\bar{a}\ \overline{18\text{-}y5|} - 10\ {}_1\bar{a}_{x'} : \overline{18\text{-}y5|} - 13\frac{1}{3}\ {}_2\bar{a}_{x'} : \overline{18\text{-}y5|} + 60\ {}_3\bar{a}_{x'} : \overline{18\text{-}y5|} - 40\ {}_4\bar{a}_{x'} : \overline{18\text{-}y5|} + 3\frac{1}{3}\ {}_5\bar{a}_{x'} : \overline{18\text{-}y5|})$ — Table VIII Multipied by -28

Calculation of individual values for the fifth youngest child may be made by substituting tabular values for each of the above 10 component parts of formula (7).

TABLES FOR VALUATION OF DEATH BENEFITS UNDER U.S. LONGSHOREMEN'S & HARBOR WORKERS COMPENSATION ACT EFFECTIVE 6/24/48*

*Note: These tables are based on
Survivorship Annuitants' Table of Mortality,
Dutch Royal Remarriage Tables,
and 3% interest.

TABLE 1*

WIDOW OR WIDOWER (AT 35¢)

Present Value of Compensation per $100 Annual Wages

Payable Until Death or Remarriage

$$35\,\bar{a}_{x'} + 70\,\bar{E}_{x'}$$

Age	Present Value	Age	Present Value	Age	Present Value
15	316.41	45	504.41	75	231.51
16	323.75	46	589.40	76	219.07
17	331.87	47	583.24	77	206.94
18	340.78	48	576.06	78	195.15
19	350.52	49	567.91	79	183.72
20	361.12	50	558.96	80	172.67
21	372.55	51	549.20	81	162.00
22	384.81	52	538.76	82	151.74
23	397.87	53	527.72	83	141.89
24	411.64	54	516.08	84	132.46
25	426.03	55	504.03	85	123.46
26	440.92	56	491.49	86	114.89
27	456.15	57	478.60	87	106.75
28	471.57	58	465.43	88	99.05
29	486.97	59	451.94	89	91.79
30	502.15	60	438.22	90	84.95
31	516.86	61	424.35	91	78.52
32	530.92	62	410.32	92	72.52
33	544.12	63	396.23	93	66.91
34	556.26	64	382.03	94	61.72
35	567.16	65	367.84	95	56.87
36	576.72	66	353.75	96	52.43
37	584.82	67	339.58	97	48.34
38	591.37	68	325.48	98	44.56
39	596.33	69	311.50	99	41.09
40	599.76	70	297.66	100	37.94
41	601.58	71	283.99	101	35.07
42	601.89	72	270.51	102	32.48
43	600.77	73	257.25	103	30.14
44	598.26	74	244.25		

*Note: Tables 1, 2, 3 and 4 are based on Survivorship Annuitants Table of Mortality, Dutch Royal Remarriage Tables, and 3% interest.

TABLE 2

YOUNGEST CHILD

Present Value Per $100 Annual Wages, Payable Until Age 18

$35, \bar{a}_{\overline{18-y}|} - 20, \bar{a}_{x:\overline{18-y}|}$

Age of Widow (x)	Age of Child (y)																		Age of Widow (x)
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
15	346.72	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	15
16	344.84	327.64	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	16
17	342.73	325.66	308.12	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	17
18	340.39	323.47	306.07	288.20	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	18
19	337.80	321.03	303.80	286.10	267.95	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	19
20	334.94	318.35	301.29	283.77	265.82	247.43	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	20
21	331.81	315.40	298.54	281.23	263.47	245.29	226.71	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	21
22	328.39	312.20	295.55	278.45	260.92	242.97	224.62	205.90	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	22
23	324.69	308.72	292.30	275.44	258.15	240.44	222.34	203.87	185.08	.....	.....	.....	.....	.....	.....	.....	.....	.....	23
24	320.71	304.98	288.81	272.20	255.17	237.73	219.89	201.69	183.17	164.37	.....	.....	.....	.....	.....	.....	.....	.....	24
25	316.47	301.00	285.09	268.75	251.99	234.83	217.28	199.37	181.13	162.62	143.88	.....	.....	.....	.....	.....	.....	.....	25
26	311.98	296.78	281.15	265.10	248.63	231.76	214.52	196.91	178.97	160.76	142.32	123.72	.....	.....	.....	.....	.....	.....	26
27	307.28	292.36	277.02	261.27	245.11	228.55	211.62	194.33	176.71	158.81	140.69	122.39	104.02	.....	.....	.....	.....	.....	27
28	302.39	287.76	272.72	257.28	241.44	225.20	208.60	191.64	174.35	156.78	138.98	121.00	102.93	84.87	.....	.....	.....	.....	28
29	297.36	283.03	268.30	253.18	237.66	221.76	205.49	188.87	171.92	154.69	137.22	119.57	101.81	84.04	66.36	.....	.....	.....	29
30	292.24	278.21	263.80	248.99	233.81	218.24	202.31	186.04	169.44	152.56	135.43	118.11	100.67	83.19	65.78	48.58	.....	.....	30
31	287.09	273.36	259.26	244.78	229.92	214.70	199.12	183.19	166.94	150.41	133.62	116.64	99.52	82.34	65.20	48.23	31.58	.....	31
32	281.95	268.53	254.74	240.58	226.05	211.16	195.92	180.34	164.44	148.26	131.81	115.16	98.36	81.48	64.62	47.88	31.41	15.38	32
33	276.88	263.75	250.27	236.42	222.22	207.66	192.76	177.52	161.97	146.13	130.02	113.70	97.21	80.63	64.04	47.53	31.25	15.34	33
34	271.95	259.10	245.91	232.37	218.48	204.25	189.67	174.77	159.55	144.04	128.27	112.27	96.09	79.80	63.47	47.19	31.08	15.30	34
35	267.20	254.62	241.71	228.46	214.88	200.95	186.69	172.11	157.21	142.03	126.57	110.89	95.01	79.00	62.92	46.86	30.93	15.25	35
36	262.67	250.35	237.71	224.73	211.43	197.79	183.84	169.56	154.97	140.09	124.95	109.56	93.97	78.22	62.39	46.54	30.78	15.21	36
37	258.41	246.32	233.92	221.21	208.17	194.81	181.13	167.14	152.84	138.26	123.40	108.29	92.97	77.49	61.88	46.24	30.63	15.18	37
38	254.47	242.59	230.41	217.93	205.13	192.03	178.61	164.89	150.86	136.54	121.95	107.11	92.05	76.80	61.41	45.95	30.50	15.14	38
39	250.86	239.17	227.19	214.91	202.34	189.46	176.28	162.80	149.02	134.95	120.61	106.01	91.18	76.16	60.97	45.69	30.37	15.11	39

TABLE 2

YOUNGEST CHILD

(Concluded)

Age of Widow (x)	Age of Child (y)																		Age of Widow (x)
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
40	247.58	236.06	224.25	212.16	199.78	187.11	174.14	160.88	147.33	133.49	119.37	105.00	90.38	75.56	60.56	45.44	30.25	15.08	40
41	244.69	233.29	221.64	209.71	197.50	185.00	172.22	159.15	145.80	132.16	118.25	104.08	89.66	75.02	60.19	45.22	30.15	15.05	41
42	242.16	230.88	219.34	207.54	195.47	183.13	170.51	157.62	144.44	130.98	117.25	103.25	89.01	74.54	59.86	45.02	30.05	15.02	42
43	240.00	228.80	217.35	205.66	193.71	181.49	169.01	156.26	143.23	129.92	116.35	102.51	88.42	74.10	59.56	44.84	29.97	15.00	43
44	238.21	227.06	215.68	204.07	192.20	180.09	167.72	155.08	142.18	129.01	115.57	101.87	87.91	73.71	59.29	44.67	29.89	14.98	44
45	236.81	225.68	214.34	202.77	190.97	178.93	166.64	154.10	141.29	128.23	114.90	101.31	87.47	73.38	59.06	44.54	29.82	14.96	45
46	235.74	224.61	213.28	201.74	189.98	177.98	165.75	153.28	140.55	127.57	114.33	100.84	87.09	73.09	58.86	44.41	29.76	14.95	46
47	235.04	223.86	212.54	200.99	189.24	177.27	165.07	152.64	139.96	127.04	113.87	100.45	86.78	72.86	58.70	44.31	29.71	14.93	47
48	234.69	223.47	212.08	200.51	188.74	176.76	164.57	152.16	139.52	126.63	113.51	100.14	86.52	72.66	58.56	44.22	29.67	14.92	48
49	234.69	223.39	211.93	200.30	188.48	176.48	164.27	151.85	139.22	126.35	113.25	99.91	86.33	72.51	58.45	44.16	29.64	14.91	49
50	235.00	223.59	212.03	200.32	188.44	176.38	164.13	151.69	139.03	126.17	113.07	99.75	86.19	72.40	58.37	44.10	29.61	14.91	50
51	.....	224.12	212.44	200.61	188.64	176.50	164.19	151.69	139.00	126.10	113.00	99.67	86.11	72.33	58.32	44.07	29.59	14.90	51
52	.....	.....	213.11	201.15	189.06	176.81	164.41	151.84	139.09	126.15	113.01	99.66	86.09	72.30	58.29	44.05	29.58	14.90	52
53	.....	.....	.....	201.93	189.69	177.32	164.80	152.14	139.30	126.29	113.10	99.71	86.11	72.31	58.28	44.04	29.58	14.89	53
54	.....	.....	.....	.....	190.57	178.04	165.39	152.60	139.66	126.56	113.29	99.84	86.20	72.36	58.31	44.05	29.58	14.90	54
55	.....	.....	.....	.....	.....	178.93	166.12	153.18	140.12	126.91	113.55	100.02	86.32	72.43	58.35	44.07	29.59	14.90	55
56	.....	.....	.....	.....	.....	.....	167.04	153.94	140.73	127.38	113.91	100.28	86.50	72.55	58.42	44.11	29.60	14.90	56
57	.....	.....	.....	.....	.....	.....	.....	154.84	141.46	127.96	114.35	100.61	86.73	72.71	58.52	44.16	29.63	14.91	57
58	.....	.....	.....	.....	.....	.....	.....	.....	142.31	128.64	114.87	101.00	87.01	72.89	58.63	44.22	29.65	14.91	58
59	.....	.....	.....	.....	.....	.....	.....	.....	.....	129.44	115.50	101.48	87.35	73.13	58.78	44.30	29.68	14.92	59
60	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	116.24	102.04	87.76	73.40	58.95	44.40	29.73	14.93	60
61	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	102.66	88.22	73.72	59.15	44.51	29.78	14.94	61
62	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	88.74	74.09	59.39	44.64	29.83	14.96	62
63	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	74.49	59.65	44.78	29.89	14.97	63
64	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	59.95	44.96	29.98	14.99	64
65	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	45.15	30.06	15.02	65
66	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	30.14	15.03	66

TABLE 3

SECOND YOUNGEST CHILD

Present Value Per $100 Annual Wages, Payable Until Age 18

$$35,\overline{a}_{\overline{18-y_2}|} - 20_{\,2}\overline{a}_{\overline{18-y_2}|} - 20,\overline{a}_{x':\overline{18-y_2}|} + 20_{\,2}\overline{a}_{x':\overline{18-y_2}|}$$

Age of Widow (x)	Age of Child (y)																		Age of Widow (x)
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
15	208.27	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	15
16	208.17	199.22	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	16
17	208.06	199.12	189.92	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	17
18	207.94	199.01	189.82	180.36	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	18
19	207.81	198.89	189.72	180.27	170.55	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	19
20	207.66	198.76	189.60	180.17	170.46	160.47	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	20
21	207.50	198.62	189.47	180.05	170.36	160.38	150.11	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	21
22	207.32	198.46	189.33	179.93	170.25	160.29	150.03	139.47	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	22
23	207.13	198.29	189.18	179.80	170.14	160.19	149.95	139.40	128.54	.....	.....	.....	.....	.....	.....	.....	.....	.....	23
24	206.93	198.10	189.02	179.65	170.01	160.08	149.86	139.33	128.48	117.31	.....	.....	.....	.....	.....	.....	.....	.....	24
25	206.71	197.91	188.84	179.50	169.88	159.97	149.76	139.25	128.42	117.26	105.77	.....	.....	.....	.....	.....	.....	.....	25
26	206.48	197.70	188.66	179.34	169.74	159.85	149.66	139.16	128.35	117.21	105.73	93.90	.....	.....	.....	.....	.....	.....	26
27	206.24	197.48	188.46	179.17	169.59	159.72	149.55	139.08	128.28	117.15	105.69	93.87	81.69	.....	.....	.....	.....	.....	27
28	205.99	197.26	188.26	178.99	169.44	159.59	149.44	138.98	128.21	117.10	105.64	93.84	81.67	69.12	.....	.....	.....	.....	28
29	205.73	197.03	188.06	178.81	169.28	159.46	149.33	138.89	128.13	117.04	105.60	93.81	81.65	69.10	56.16	.....	.....	.....	29
30	205.47	196.79	187.85	178.63	169.12	159.32	149.21	138.79	128.05	116.98	105.55	93.77	81.62	69.10	56.15	42.80	.....	.....	30
31	205.20	196.56	187.64	178.44	168.96	159.18	149.10	138.70	127.97	116.91	105.51	93.74	81.60	69.07	56.15	42.79	29.00	.....	31
32	204.94	196.32	187.43	178.26	168.80	159.04	148.98	138.60	127.90	116.85	105.46	93.71	81.58	69.06	56.15	42.79	29.00	14.75	32
33	204.68	196.09	187.22	178.07	168.64	158.90	148.86	138.50	127.82	116.79	105.41	93.67	81.58	69.05	56.13	42.79	29.00	14.75	33
34	204.43	195.86	187.02	177.89	168.48	158.77	148.75	138.41	127.74	116.73	105.37	93.64	81.53	69.03	56.12	42.78	29.00	14.75	34
35	204.19	195.64	186.82	177.72	168.33	158.64	148.64	138.32	127.67	116.68	105.33	93.61	81.51	69.02	56.11	42.78	29.00	14.75	35
36	203.96	195.44	186.64	177.56	168.19	158.52	148.54	138.24	127.60	116.62	105.28	93.58	81.49	69.01	56.11	42.78	29.00	14.75	36
37	203.75	195.24	186.46	177.41	168.05	158.40	148.44	138.15	127.53	116.57	105.25	93.55	81.47	68.99	56.10	42.77	29.00	14.75	37
38	203.55	195.06	186.30	177.26	167.93	158.30	148.35	138.08	127.47	116.52	105.21	93.52	81.45	68.98	56.09	42.77	29.00	14.75	38
39	203.37	194.90	186.16	177.13	167.81	158.20	148.26	138.01	127.42	116.47	105.17	93.50	81.44	68.97	56.09	42.77	28.99	14.75	39

TABLE 3

SECOND YOUNGEST CHILD

(Concluded)

Age of Widow (x)	Age of Child (y)																		Age of Widow (x)
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
40	203.21	194.75	186.02	177.01	167.71	158.11	148.19	137.94	127.36	116.44	105.14	93.48	81.42	68.96	56.08	42.77	28.99	14.75	40
41	203.07	194.62	185.91	176.91	167.62	158.02	148.12	137.89	127.32	116.40	105.11	93.46	81.41	68.95	56.08	42.76	28.99	14.75	41
42	202.94	194.51	185.80	176.81	167.54	157.95	148.06	137.83	127.27	116.36	105.09	93.44	81.39	68.94	56.07	42.76	28.99	14.74	42
43	202.84	194.41	185.72	176.74	167.46	157.89	148.00	137.79	127.24	116.33	105.07	93.42	81.38	68.94	56.07	42.76	28.99	14.74	43
44	202.76	194.34	185.64	176.67	167.41	157.84	147.96	137.75	127.20	116.31	105.05	93.41	81.37	68.93	56.06	42.76	28.99	14.74	44
45	202.70	194.28	185.59	176.62	167.36	157.80	147.92	137.72	127.18	116.29	105.03	93.39	81.36	68.93	56.06	42.76	28.99	14.74	45
46	202.66	194.24	185.55	176.58	167.32	157.76	147.89	137.69	127.16	116.27	105.03	93.38	81.36	68.92	56.06	42.75	28.99	14.74	46
47	202.63	194.21	185.52	176.55	167.30	157.74	147.87	137.67	127.14	116.25	105.00	93.37	81.35	68.92	56.06	42.75	28.99	14.74	47
48	202.63	194.20	185.51	176.54	167.28	157.72	147.85	137.66	127.13	116.24	105.00	93.37	81.35	68.91	56.05	42.75	28.99	14.74	48
49	202.65	194.21	185.51	176.54	167.24	157.72	147.84	137.65	127.12	116.24	104.99	93.36	81.34	68.91	56.05	42.75	28.99	14.74	49
50	202.68	194.23	185.53	176.55	167.28	157.72	147.84	137.65	127.11	116.23	104.99	93.36	81.34	68.91	56.05	42.75	28.99	14.74	50
51	.....	194.28	185.56	176.57	167.29	157.73	147.85	137.65	127.12	116.23	104.98	93.36	81.34	68.91	56.05	42.75	28.99	14.74	51
52	.....	.....	185.60	176.60	167.32	157.74	147.86	137.66	127.12	116.23	104.99	93.36	81.34	68.91	56.05	42.75	28.99	14.74	52
53	.....	.....	.....	176.65	167.36	157.77	147.88	137.67	127.13	116.24	104.99	93.36	81.34	68.91	56.05	42.75	28.99	14.74	53
54	.....	.....	.....	.....	167.40	157.81	147.91	137.69	127.14	116.25	105.00	93.36	81.32	68.91	56.05	42.75	28.99	14.74	54
55	.....	.....	.....	.....	.....	157.85	147.94	137.71	127.16	116.26	105.00	93.37	81.34	68.91	56.05	42.75	28.99	14.74	55
56	.....	.....	.....	.....	.....	.....	147.98	137.74	127.18	116.28	105.01	93.38	81.35	68.91	56.05	42.75	28.99	14.74	56
57	.....	.....	.....	.....	.....	.....	.....	137.78	127.21	116.29	105.03	93.38	81.35	68.92	56.05	42.75	28.99	14.74	57
58	.....	.....	.....	.....	.....	.....	.....	.....	127.24	116.32	105.04	93.39	81.36	68.92	56.06	42.75	28.99	14.74	58
59	.....	.....	.....	.....	.....	.....	.....	.....	.....	116.34	105.06	93.41	81.37	68.92	56.06	42.75	28.99	14.74	59
60	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	105.08	93.42	81.37	68.93	56.06	42.75	28.99	14.74	60
61	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	93.43	81.38	68.93	56.06	42.76	28.99	14.74	61
62	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	81.39	68.94	56.07	42.76	28.99	14.74	62
63	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	68.95	56.07	42.76	28.99	14.74	63
64	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	56.07	42.76	28.99	14.74	64
65	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	42.76	28.99	14.75	65
66	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	28.99	14.75	66

TABLE 4

CHILDREN (NO WIDOW)

(Taken in Ascending Order of Age)

PRESENT VALUE PER $100 ANNUAL WAGES PAYABLE UNTIL AGE 18

Age of Child	First Child (1)	Second Child (2)	Third Child (3)	Fourth Child (4)	Fifth Child (5)	Age of Child
0	468.29	—	—	—	—	0
1	449.24	202.52	—	—	—	1
2	429.52	193.14	184.55	—	—	2
3	409.11	183.48	175.73	35.03	—	3
4	387.96	173.54	166.60	32.38	3.50	4
5	366.08	163.31	157.16	29.75	3.03	5
6	343.41	152.78	147.40	27.13	2.59	6
7	319.95	141.94	137.29	24.54	2.18	7
8	295.65	130.79	126.84	21.98	1.81	8
9	270.49	119.31	116.03	19.45	1.47	9
10	244.45	107.50	104.84	16.98	1.16	10
11	217.48	95.35	93.26	14.55	.89	11
12	189.56	82.86	81.27	12.20	.65	12
13	160.65	70.00	68.87	9.91	.45	13
14	130.71	56.77	56.03	7.71	.29	14
15	99.72	43.17	42.74	5.61	.16	15
16	67.63	29.18	28.98	3.61	.07	16
17	34.40	14.79	14.74	1.74	.02	17

INTERNATIONAL ACTUARIAL NOTATION

BY

F. S. PERRYMAN

The Council of the Society announces that the revised International Actuarial Notation, particulars of which are given below, will be adopted in the *Proceedings* of the Society as from the present Number. The revised Notation will also be used by the Examination Committee for all examinations subsequent to those in May, 1950. The revised Notation has been adopted by the Society of Actuaries and also by the Institute of Actuaries in England, the Faculty of Actuaries in Scotland and by actuarial societies in various other foreign countries.

Before giving an account of the circumstances leading up to the present revision and a complete account of the Notation it is desirable to state here the principal changes which are now being introducd:

"(1) Interest. In place of the symbol $j_{(m)}$=the nominal rate of interest convertible m times in a year when the effective rate is i, the symbol $i^{(m)}$ has been substituted; the former definitions of $i^{(m)}$ and $\bar{i}$ as the effective rate of interest when interest is convertible m times a year and momently respectively have been abolished.

"(2) Annuities-due. In place of the symbol a for an annuity under which the first payment is to be made at once the symbol ä has been adopted. This adoption of the trema ¨ as a symbol of acceleration has permitted the introduction of s̈ for the accumulated amount of an annuity-due at the end of the term, for which no symbol previously existed.

"(3) Mortality. The use of Q for the probability of dying within a longer term than 1 year has been omitted.

"(4) Compound symbols. The use of D in a compound symbol to indicate decreasing benefits has been introduced.

"(5) Commutation columns. The old definition of $N_x = D_{x+1} + D_{x+2} + D_{x+3}$ etc. has been abolished in favour of $N_x = D_x + D_{x+1} + D_{x+2}$ etc.

This renders the use of $\mathbb{N}_x$ and $\mathbf{N}_x$ for the latter series unnecessary."

The complete statement of the revised Notation and its history follows:

The existing international actuarial notation was founded on the 'Key to the Notation' given in the *Institute of Actuaries Text-Book*, Part II, *Life Contingencies*, by George King, and is embodied in an explanatory statement adopted unanimously by the Second International Actuarial Congress held in London in May 1898 and printed on pp. 618–40 of the *Transactions* of that Congress. At the Third International Congress held in Paris in June 1900 a further statement by Dr Sprague was submitted which rearranged the symbols in different orders and grouped them on different principles but did not introduce any changes in the symbols themselves beyond two slight additions. This statement is printed on pp. 622–51 of the *Transactions* of that Congress. At the Eleventh International Congress held in Paris in June 1937 an International Committee was appointed to consider the question of notation and given the duty of submitting definite proposals to the next Congress. This Committee met in Brussels in July 1938 and again in July 1939, and certain changes would have been proposed for adoption at Lucerne in 1940. Owing to the war this could not be done, but in order not to lose the benefit of these changes for an indefinite period the Institute of Actuaries and the Faculty of Actuaries have decided to adopt at a date to be mutually arranged those recommendations which had received unanimous or substantial support at the meetings of the Committee.

As many students and members of the profession have not ready access to the above-mentioned *Transactions* it has been thought desirable to prepare and issue the following statement of the International Actuarial Notation after embodying in it the changes arising from the recommendations mentioned above.

The general principles on which the system is based are as follows:

To each fundamental symbolic letter are attached signs and letters each having its own signification.

The lower space to the left is reserved for signs indicating the conditions relative to the duration of the operations and to their position with regard to time.

The lower space to the right is reserved for signs indicating the conditions relative to ages and the order of succession of the events.

The upper space to the right is reserved for signs indicating the periodicity of the events.

The upper space to the left is free, and in it can be placed signs corresponding to other notions.

In what follows these two conventions are used:

A letter enclosed in brackets, thus (x), denotes 'a person aged x'. A letter or number enclosed in a right angle, thus $\overline{n}|$ or $\overline{15}|$, denotes a term-certain of years.

FUNDAMENTAL SYMBOLIC LETTERS

Interest

i = the effective rate of interest, namely, the total interest earned on 1 in a year on the assumption that the actual interest (if receivable otherwise than yearly) is invested forthwith as it becomes due on the same terms as the original principal.

$v = (1+i)^{-1}$ = the present value of 1 due a year hence.
$d = 1-v$ = the discount on 1 due a year hence.
$\delta = \log_e(1+i) = -\log_e(1-d)$ = the force of interest or the force of discount.

Mortality Tables

l = number living.
d = number dying.
p = probability of living.
q = probability of dying.
μ = force of mortality.
m = central death rate.
a = present value of an annuity.
s = amount of an annuity.
e = expectation of life.
A = present value of an assurance.
E = present value of an endowment.
P, π = premium per annum. P generally refers to net premiums, π to special premiums.
V = policy value.
W = paid-up policy.

The methods of using the foregoing principal letters and their precise meaning when added to by suffixes, etc., follow.

Interest

$i^{(m)} = m\{(1+i)^{1/m}-1\}$ = the nominal rate of interest, convertible m times a year.
$a_{\overline{n}|} = v+v^2+\ldots+v^n$ = the value of an annuity-certain of 1 per annum for n years, the payments being made at the end of each year.
$\ddot{a}_{\overline{n}|} = 1+v+v^2+\ldots+v^{n-1}$ = the value of a similar annuity, the payments being made at the beginning of each year.
$s_{\overline{n}|} = 1+(1+i)+(1+i)^2+\ldots+(1+i)^{n-1}$ = the amount of an annuity-certain of 1 per annum for n years, the payments being made at the end of each year.
$\ddot{s}_{\overline{n}|} = (1+i)+(1+i)^2+\ldots+(1+i)^n$ = the amount of a similar annuity, the payments being made at the beginning of each year.

The diaeresis or trema above the letters a and s is used as a symbol of acceleration of payments.

Mortality Tables

The ages of the lives involved are denoted by letters placed as suffixes in the lower space to the right. Thus:

l_x = the number of persons who attain age x according to the mortality table.
$d_x = l_x - l_{x+1}$ = the number of persons who die between ages x and $x+1$ according to the mortality table.
p_x = the probability that (x) will live 1 year.
q_x = the probability that (x) will die within 1 year.

$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx}$ = the force of mortality at age x.

m_x = the central death-rate for the year of age x to $x+1 = d_x \Big/ \int_0^1 l_{x+t}\,dt$.

e_x = the curtate 'expectation of life' (or average after-lifetime) of (x).

In the following it is always to be understood (unless otherwise expressed) that the annual payment of an annuity is 1, that the sum assured in any case is 1, and that the symbols indicate the present values:

a_x = an annuity, first payment at the end of a year, to continue during the life of (x).

$\ddot{a}_x = 1 + a_x$ = an 'annuity-due' to continue during the life of (x), the first payment to be made at once.

A_x = an assurance payable at the end of the year of death of (x).

Note. $e_x = a_x$ at rate of interest $i = 0$.

A letter or number at the lower left corner of the principal symbol denotes the number of years involved in the probability or benefit in question. Thus:

${}_np_x$ = the probability that (x) will live n years.

${}_nq_x$ = the probability that (x) will die within n years.

Note. When $n = 1$ it is customary to omit it, as shown on page 2, provided no ambiguity is introduced.

${}_nE_x = v^n {}_np_x$ = the value of an endowment on (x) payable at the end of n years if (x) be then alive.

If the letter or number comes before a perpendicular bar it shows that a period of deferment is meant. Thus:

${}_{n|}q_x$ = the probability that (x) will die in a year, deferred n years; that is, that he will die in the $(n+1)$th year.

${}_{n|}a_x$ = an annuity on (x) deferred n years; that is, that the first payment is to be made at the end of $(n+1)$ years.

${}_{n|t}a_x$ = an intercepted or deferred temporary annuity on (x) deferred n years and, after that, to run for t years.

A letter or number in brackets at the upper right corner of the principal symbol shows the number of intervals into which the year is to be divided. Thus:

$a_x^{(m)}$ = an annuity on (x) payable by m instalments of $1/m$ each throughout the year, the first payment being one of $1/m$ at the end of the first $1/m$th of a year.

$\ddot{a}_x^{(m)}$ = a similar annuity but the first payment of $1/m$ is to be made at once, so that

$\ddot{a}_x^{(m)} = 1/m + a_x^{(m)}$.

$A_x^{(m)}$ = an assurance payable at the end of that fraction $1/m$ of a year in which (x) dies.

If $m \to \infty$ then instead of writing (∞) a bar is placed over the principal symbol. Thus:

$\bar{a}$ = a continuous or momently annuity.

$\overline{A}$ = an assurance payable at the moment of death.

A small circle placed over the principal symbol shows that the benefit is to be complete. Thus:

$\mathring{a}$ = a complete annuity.

$\mathring{e}$ = the complete expectation of life.

Note. Some consider that $\bar{e}$ would be as appropriate as $\mathring{e}$. As $e_x = a_x$ at rate of interest $i = 0$, so also the complete expectation of life $= \bar{a}_x$ at rate of interest $i = 0$.

When more than one life is involved the following rules are observed:

If there are two or more letters or numbers in a suffix without any distinguishing mark, joint lives are intended. Thus:

$l_{xy} = l_x \times l_y, \quad d_{xy} = l_{xy} - l_{x+1:y+1}.$

Note. When, for the sake of distinctness, it is desired to separate the letters or numbers in a suffix, a colon is placed between them. A colon is used instead of a point or comma to avoid confusion with decimals when numbers are involved.

a_{xyz} = an annuity, first payment at the end of a year, to continue during the joint lives of (x), (y) and (z).

A_{xyz} = an assurance payable at the end of the year of the failure of the joint lives (x), (y) and (z).

In place of a life a term-certain may be involved. Thus:

$a_{x:\overline{n}|}$ = an annuity to continue during the joint duration of the life of (x) and a term of n years certain; that is, a temporary annuity for n years on the life of (x).

$A_{x:\overline{n}|}$ = an assurance payable at the end of the year of death of (x) if he die within n years, or at the end of n years if (x) be then alive; that is, an endowment assurance for n years.

If a perpendicular bar separates the letters in the suffix, then the status after the bar is to follow the status before the bar. Thus:

$a_{y|x}$ = a reversionary annuity, that is, an annuity on the life of (x) after the death of (y).

$A_{z|xy}$ = an assurance payable on the failure of the joint lives (x) and (y) provided both these lives survive (z).

If a horizontal bar appears above the suffix then survivors of the lives, and not joint lives, are intended. The number of survivors can be denoted by a letter or number over the right end of the bar. If that letter, say r, is not distinguished by any mark, then the meaning is *at least* r survivors; but if it is enclosed in square brackets, $[r]$, then the meaning is *exactly* r survivors. If no letter or

number appears over the bar, then unity is supposed and the meaning is *at least one* survivor. Thus:

$a_{\overline{xyz}}$ = an annuity payable so long as at least one of the three lives (x), (y) and (z) is alive.

$a^{\overline{2}}_{\overline{xyz}}$ = an annuity payable so long as at least two of the three lives (x), (y) and (z) are alive.

$p^{[2]}_{\overline{xyz}}$ = probability that exactly two of the three lives (x), (y) and (z) will survive a year.

${}_nq_{\overline{xy}}$ = probability that the survivor of the two lives (x) and (y) will die within n years $= {}_nq_x \times {}_nq_y$.

${}_nA_{\overline{xy}}$ = an assurance payable at the end of the year of death of the survivor of the lives (x) and (y) provided the death occurs within n years.

When numerals are placed above or below the letters of the suffix, they designate the order in which the lives are to fail. The numeral placed *over* the suffix points out the life whose failure will finally determine the event; and the numerals placed *under* the suffix indicate the order in which the other lives involved are to fail. Thus:

A^1_{xy} = an assurance payable at the end of the year of death of (x) if he dies first of the two lives (x) and (y).

A^2_{xyz} = an assurance payable at the end of the year of death of (x) if he dies second of the three lives (x), (y) and (z).

$A^2_{\underset{1}{xyz}}$ = an assurance payable at the end of the year of death of (x) if he dies second of the three lives, (y) having died first.

$A_{\overline{xy}:\underset{3}{z}}$ = an assurance payable at the end of the year of death of the survivor of (x) and (y) if he dies before (z).

$A^1_{x:\overline{n}|}$ = an assurance payable at the end of the year of death of (x) if he dies within a term of n years.

$\left.\begin{matrix} a^{\overline{\ }}_{\underset{1}{yz}|x} \\ \text{or} \\ a^{\overline{2}}_{yz|x} \end{matrix}\right\}$ = an annuity to (x) after the failure of the survivor of (y) and (z), provided (z) fails before (y).

Note. Sometimes to make quite clear that a joint-life status is involved a symbol ⌐¬ is placed above the lives included. Thus $A^{\overline{1}}_{xy:\overline{n}|}$ = a joint life temporary assurance on (x) and (y).

In the case of reversionary annuities, distinction has sometimes to be made between those where the times of year at which payments are to take place are determined at the outset and those where the times depend on the failure of the preceding status. Thus:

$a_{y|x}$ = annuity to (x), first payment at the end of the year of the death of (y) or, on the average, about 6 months after his death.

$\hat{a}_{y|x}$ = annuity to (x), first payment 1 year after the death of (y).

$\hat{\hat{a}}_{y|x}$ = complete annuity to (x), first payment 1 year after the death of (y).

ANNUAL PREMIUMS

The symbol P with the appropriate suffix or suffixes is used in simple cases, where no misunderstanding can occur, to denote the annual premium for a benefit. Thus:

P_x = the annual premium for an assurance payable at the end of the year of death of (x).

$P_{x:\overline{n}|}$ = the annual premium for an endowment assurance on (x) payable after n years or at the end of the year of death of (x) if he die within n years.

P^{1}_{xy} = the annual premium for a contingent assurance payable at the end of the year of death of (x) if he die before (y).

In all these cases it is optional to use the symbol P in conjunction with the principal symbol denoting the benefit. Thus instead of $P_{x:\overline{n}|}$ we may write $P(A_{x:\overline{n}|})$. In the more complicated cases it is necessary to use the two symbols in this way. Suffixes, etc., showing the conditions of the benefit are to be attached to the principal letter, and those showing the conditions of payment of the premium are to be attached to the subsidiary symbol P. Thus:

${}_nP(\overline{A}_x)$ = the annual premium payable for n years only for an assurance payable at the moment of the death of (x).

$P_{xy}(A_x)$ = the annual premium payable during the joint lives of (x) and (y) for an assurance payable at the end of the year of death of (x).

${}_nP({}_{n|}a_x)$ = the annual premium payable for n years only for an annuity on (x) deferred n years.

${}_tP^{(m)}(A_{x:\overline{n}|})$ = the annual premium payable for t years only, by m instalments throughout the year, for an endowment assurance for n years on (x) (see below as to $P^{(m)}$).

Notes. (1) As a general rule the symbol P could be used without the principal symbol in the case of assurances where the sum assured is payable at the end of the year of death, but if it is payable at other times, or if the benefit is an annuity, then the principal symbol should be used.

(2) $P^{(m)}_x$. A point which was not brought out when the international system was adopted is that there are two kinds of premiums payable m times a year, viz. those which cease on payment of the instalment immediately preceding death and those which continue to be payable to the end of the year of death. To distinguish the latter the m is sometimes enclosed in square brackets, thus $P^{[m]}$.

POLICY VALUES AND PAID-UP POLICIES

${}_tV_x$ = the value of an ordinary whole-life assurance on (x) which has been t years in force, the premium then just due being unpaid.

${}_tW_x$ = the paid-up policy the present value of which is ${}_tV_x$.

The symbols V and W may, in simple cases, be used alone, but in the more complicated cases it is necessary to insert the full symbol for the benefit thus:

${}_tV^{(m)}(\overline{A}_{x:\overline{n}|})$ (corresponding to $P^{(m)}(\overline{A}_{x:\overline{n}|})$), ${}_tV({}_{n|}a_x)$.

Note. As a general rule V or W can be used as the main symbol if the sum assured is payable at the end of the year of death and the premium is payable periodically throughout the duration of the assurance. If the premium is payable for a limited number of years, say n, the policy value after t years could be written ${}_tV[{}_nP(A)]$, or, if desired ${}^n_tV(A)$.

In investigations where modified premiums and policy values are in question such modification may be denoted by adding accents to the symbols. Thus, when a premium other than the net premium (a valuation premium) is used in a valuation it may be denoted by P′ and the corresponding policy value by V′. Similarly, the office (or commercial) premium may be denoted by P″ and the corresponding paid-up policy by W″.

COMPOUND SYMBOLS

(Ia) = an annuity
(IA) = an assurance
} commencing at 1 and increasing 1 per annum.

If the whole benefit is to be temporary the symbol of limitation is placed outside the brackets. Thus:

$(Ia)_{x:\overline{n}|}$ = a temporary increasing annuity.

$(IA)^1_{x:\overline{n}|}$ = a temporary increasing assurance.

If only the increase is to be temporary but the benefit is to continue thereafter, then the symbol of limitation is placed immediately after the symbol I. Thus:

$(I_{\overline{n}|}a)_x$ = a whole-life annuity
$(I_{\overline{n}|}A)_x$ = a whole-life assurance
} increasing for n years and thereafter stationary.

If the benefit is a decreasing one, the corresponding symbol is D. From the nature of the case this decrease must have a limit, as otherwise negative values might be implied. Thus:

$(D_{\overline{n}|}A)^1_{x:\overline{n}|}$ = a temporary assurance commencing at n and decreasing by 1 in each successive year.

If the benefit is a varying one the corresponding symbol is v. Thus:

(va) = a varying annuity.

COMMUTATION COLUMNS

Single lives

$$D_x = v^x l_x,$$
$$N_x = D_x + D_{x+1} + D_{x+2} + \text{ etc.},$$
$$S_x = N_x + N_{x+1} + N_{x+2} + \text{ etc.},$$
$$C_x = v^{x+1} d_x,$$
$$M_x = C_x + C_{x+1} + C_{x+2} + \text{ etc.},$$
$$R_x = M_x + M_{x+1} + M_{x+2} + \text{ etc.}$$

When it is desired to construct the assurance columns so as to give directly assurances payable at the moment of death the symbols are distinguished by a bar placed over them. Thus:

$\overline{C}_x = v^{x+\frac{1}{2}} d_x$ which is an approximation to $\int_0^1 v^{x+t} \mu_{x+t} l_{x+t} dt$.

$\overline{M}_x = \overline{C}_x + \overline{C}_{x+1} + \overline{C}_{x+2} +$ etc.

$\overline{R}_x = \overline{M}_x + \overline{M}_{x+1} + \overline{M}_{x+2} +$ etc.

Joint lives

$D_{xy} = v^{\frac{1}{2}(x+y)} l_{xy}$,

$N_{xy} = D_{xy} + D_{x+1:y+1} + D_{x+2:y+2} +$ etc.,

$C_{xy} = v^{\frac{1}{2}(x+y)+1} d_{xy}$,

$M_{xy} = C_{xy} + C_{x+1:y+1} + C_{x+2:y+2} +$ etc.,

$C^1_{xy} = v^{\frac{1}{2}(x+y)+1} d_x l_{y+\frac{1}{2}}$,

$M^1_{xy} = C^1_{xy} + C^1_{x+1:y+1} + C^1_{x+2:y+2} +$ etc.

SELECTION

If the suffix to a symbol which denotes the age is enclosed in a square bracket it indicates the age at which the life was selected. To this may be added, outside the bracket, the number of years which have elapsed since selection, so that the total suffix denotes the present age. Thus:

$l_{[x]+t}$ = the number in the select life table who were selected at age x and have attained age $x+t$.

$d_{[x]+t} = l_{[x]+t} - l_{[x]+t+1}$.

$a_{[x]}$ = value of an annuity on a life now aged x and now select.

$a_{[x-n]+n}$ = value of an annuity on a life now aged x and select n years ago at age $x-n$.

$N_{[x]} = D_{[x]} + D_{[x]+1} + D_{[x]+2} + \ldots$.

$\ddot{a}_{[x]} = N_{[x]} \div D_{[x]} = 1 + a_{[x]}$,

and similarly for other functions.

When Dr Sprague presented his statement, he mentioned that an objection had been raised that the notation in some cases offers the choice of two symbols for the same benefit. For instance, a temporary annuity may be denoted either by ${}_n a_x$ or by $a_{x:\overline{n}|}$. This is, he says, a necessary consequence of the principles underlying the system, and neither of the alternative forms could have been suppressed without injury to the symmetry of the system.

REVIEWS OF PUBLICATIONS

CLARENCE A. KULP, BOOK REVIEW EDITOR

Dictionary of Insurance Terms. Edited by Ralph H. Blanchard. Chamber of Commerce of the United States, Washington, 1949. Pp. 74.

The business of insurance is a technical business employing many technical terms which are peculiar to it. It uses also, but with specific meanings and connotations, many terms in general use. The same term occasionally is used in different senses in different forms of insurance. Under these circumstances the usefulness of a dictionary of insurance terms to students and those concerned with the insurance business is self-evident.

This *Dictionary of Insurance Terms* is the most comprehensive American dictionary which has appeared in the insurance field. It contains 765 entries, with definitions of 569 terms and 224 cross-references. The terms selected for inclusion have been carefully chosen. One would find it difficult to support a contention that any is superfluous. On the contrary, the exclusions may be questioned. The definitions given are concisely and clearly stated and, with unusually few exceptions, accurate. By any criteria which may be applied, the *Dictionary* is a distinct contribution to the literature of insurance.

In a first edition, regardless of the care which may have been exercised, there are almost certain to be grounds for criticism both as to the content and scope of the treatment. The errors are for the most part minor. The term, obligor, is cross-referenced to the term, surety, and the definition of corporate suretyship uses obligor synonymously with surety. In legal literature, the party designated as obligor usually is the principal who is primarily obligated to the obligee. An industrial accident is defined as "an accident arising out of one's employment," whereas many accidents occurring in the course of, but not necessarily arising out of, one's occupation are classified as industrial accidents. The term, apportionment, is defined synonymously with contribution, although it frequently is used to refer to the allocation of a blanket policy over the individual items covered by it.

In several definitions clarity of precision is sacrificed for brevity. The terms, expense ratio and loss ratio, are defined without indication as to whether the expenses and losses are on an incurred or paid basis or whether the premiums to which they are related are written or earned premiums. It would seem desirable to indicate in the definitions the different bases on which such ratios are stated. Overinsurance and underinsurance are defined respectively as insurance exceeding or less in amount than "the possible loss to which it applies," whereas a standard lower than the full value of the property or the maximum possible loss may be applied. The definition of proximate cause is so brief as to make an understanding of the term elusive. Twisting usually connotes a change of insurer induced by misrepresentation or incomplete comparison, whereas any change of insurer falls within the perimeter of the definition which is presented. The meaning of the term, underwriter, is limited to insurers or those involved in the selection of risks, although in life insurance it has become generally accepted as including the insurance agent.

There are a number of important omissions. Among terms defined, but used with meanings not included in the definitions, are appraisal in the sense of valuation, audit in the sense of a survey of insurance needs and coverage, and a schedule policy as a form providing specific coverage for a number of items of property. Only 4 abbreviations are defined, one of which includes reference to an organization of insurers. If the insurance organizations frequently referred to by their initials are to be identified, and it would be useful if they were, there are many more which should be included. Even if this group of abbreviations should be excluded, designations as C.L.U. and C.P.C.U. should appear, and frequently used terms such as the F.C. and S. clause and F.P.A. in marine insurance should be identified. It would be desirable also to include the abbreviations of common shipping terms such as C.I.F., C and F, F.A.S. and F.O.B. Among important terms omitted from the entries are accrued liabilities, allied lines, annuity-due, collision clause (marine insurance), contributory negligence, divided coverage (burglary insurance), exemption statutes (life insurance), guest laws (automobile insurance), indemnitor's agreements (suretyship), insurance buyers, item, jettison, loss reserves, materiality, mortality tables—select, ultimate and aggregate, the nationwide definition (inland marine insurance), natural premium (life insurance), negligence, policy fee, probability, seaworthiness, second injury funds, sue and labor clause, suicide clauses and the waiver of inventory clause.

These criticisms fade into relative insignificance when the great bulk of excellent, concise definitions is considered. The *Dictionary* should prove a useful, practical tool and handy reference for anyone seeking enlightenment as to many perplexing insurance terms.

*C. M. KAHLER

Length of Life. Louis I. Dublin, Alfred J. Lotka and Mortimer Spiegelman. Revised Edition. Ronald Press, New York, 1949. Pp. xxv, 379.

This valuable addition to the lengthening shelf of treatises on the life table appears as the plans for the 1950 Census—with its ultimate surprises—approach their final stage. The Metropolitan Life Insurance Company has to its credit the study of vital statistics over several decades. Much of the material in the new volume has first appeared in their *Monthly Statistical Bulletin* or in papers presented to various learned societies. The senior authors are widely known for the first edition of this book and many other books in which they pioneered in and popularized the analysis of morbidity, mortality, population and related data, for example, *The Economic Value of a Man.*

The book is full of an amazing number of little statistical analyses. Interesting items dealing with the longevity of the Presidents of the United States, the histories of the old mortality tables, the apparent influence of geographical residence, of urban or of rural environment, of sex, marital condition, family and personal health history and of occupation on mortality are competently presented. Causes of death, birth rates, marriage, medical and sanitary science,

*Guest reviewer

forecasts and their tools, cohorts and generation life tables—these again are developed logically. The brief treatment of the differences between insurance company experience and that developed in respect to the whole population should be useful. Too often this important distinction is ignored.

The diagrams are particularly good in omitting hard-to-read logarithmic or semi-logarithmic scales, and showing graphs in full scale instead of deceiving the eye by setting curves improperly near the base lines. They also avoid the use of human figures, whose three-dimensional bulk unfits them for two-dimensional signs.

The tables are well-arranged and carefully captioned. There is a list of sources, and two indexes, one of names of authors and one of subjects.

Subjects of special interest include the following:

Presumptive extent of paternal orphanhood in the United States

Selection in life insurance

International comparisons

Generation life tables

Inheritance of longevity

Error, bias and fraud as limitations on the accuracy of government life tables

The comprehensive list of mortality tables near the back of the book sets out for reference many life expectancy values and mortality rates in form convenient for comparison. Unfortunately as with any reference work of this scope, misquotation and misunderstanding seem unavoidable, since the non-figure-wise select their missiles to suit their purposes: to prove progress or stagnation, or to allot praise or blame. The expectation of life values especially carries erroneous implications in most amateur citations. We shall expect longer future life-times at age zero for those born in 1950 than will be shown in the new life table that we expect to see developed by Dr. Greville. In the warfare that we can see forming on the pros and cons of the "welfare state" and of the improvability of the nation's health, this treatise will be used by both sides and various guerrilla groups, sometimes correctly, but usually rather recklessly. To a very large extent these values are not "facts" but "appearances". The tools provided by these indices can be a helpful guide, but sharp tools in unskilled hands may be carelessly used. In spite of this danger, the book is one of great value.

W. R. WILLIAMSON

Motor Insurance. A. G. M. Batten and W. A. Dinsdale. 2nd Edition. Stone & Cox, Ltd., London, 1947. Pp. viii, 325.

Third Party Insurance. A. G. M. Batten and W. A. Dinsdale. 2nd Edition. Stone & Cox, Ltd., London, 1947. Pp. x, 265.

These two companion volumes were prepared to "be of use to all connected with the business, but particularly to students in their preparation for the examinations of the Chartered Insurance Institute." Fundamentally they are

text books and as such are not designed to convince but merely to inform. For this reason and also because they are British and the reviewer's entire insurance experience has been American, they are rather immune to critical analysis, but they should make interesting reading for any American insurance man possessed of wide-angle vision.

The language barrier is surprisingly obdurate. In insurance there have been always technical terms unintelligible to the layman. This jargon has grown up independently on the two sides of the Atlantic. Nor are the difficulties all one-sided. This reviewer was recently visited by a British underwriter with international responsibilities who explained that he had no trouble understanding policies written in French, German, Greek, Italian and Arabic, but that our American policies and manuals really had him baffled. Such expressions as "exposure" and "retrospective" were considerably worse than Greek to him. On our own part we have learned, of course, to understand readily such expressions as "lift insurance" and "chemists' indemnities" but we had more trouble with "stop cock risks," "teagles," "drovers' proposals," "electricity undertakings," "charabanc," "windscreen," and "knock for knock agreements." It is interesting to know that in Great Britain the application for insurance is the "proposal," an automobile dealer is a "motor trader," and a deductible form is an "excess."

Possibly the most striking aspect of British insurance in the eyes of an American is the absence of rate regulation. To American champions of free enterprise and sometime critics of British socialism this would seem inconsistent and a reversal of the expected roles. What dreams of pre-S.E.U.A. bliss the following words must conjure up to the legally inhibited liability underwriter of today!

> Underwriters are free to use any methods [of rating] which appear to them to be the most suitable for the many and varied types of risk offered to them.

Again these sugared words!

> Where the limit of indemnity is unusually large . . . insurers may insist upon basing or partly basing their premium on the limit of indemnity.

Here truly is a happy hunting ground where the actuary also has been, if not banished, at least put in his place where he may weep and gnash his teeth without interfering with the business.

Heaven is made even more perfect by the existence of the *Road Traffic Acts*, which, among other things, make insurance compulsory on all drivers of motor vehicles. Here also the insurer has the advantage, for there is no assigned risk plan and in the words of the authors "insurers are not philanthropists and cannot be expected to bear losses far in excess of the premiums received, year after year, without raising their voices in complaint." In the light of the *Road Traffic Acts*, however,

> Insurers feel themselves to be morally bound to afford insurance facilities in all possible cases. Otherwise a vehicle owner may be prevented from using his vehicle on the public highway and few insurers would care, without very good reason, to incur the odium of putting a man out of business unnecessarily. Declinature is justified, therefore, only if the

continuance of a policy and the consequent appearance of a driver upon the road is contrary to the interests and safety of the public generally.

It is plain to see that motor-vehicle drivers, or at least bad drivers, do not constitute a strong voters' bloc in the British Isles. The victims, however, are another matter, and there is a degree of social maturity, startling to us, in the concept of the Motor Insurer's Bureau. This was formed by the voluntary action of the insurers and "is designed to provide compensation for third parties who would ordinarily have a valid claim for damages arising out of motor vehicle road accidents, but where they are deprived of compensation by the absence of insurance, or by ineffective insurance."

Here we have a striking contrast, perhaps reflecting a basic difference in our standards and *mores,* between the American "voluntary" assigned risk plan and the British "voluntary" Motor Insurer's Bureau. It may be that the difference arises from differing social ideologies or it may be merely the result of a preponderance of drivers in this country and of pedestrians in Great Britain.

In case the reader is curious, the "knock for knock agreement" is an economical and realistic device for a well-insured tight little island. It is simply an agreement between companies that when their insureds' automobiles smash each other up the companies will waive possible subrogation on the physical damage and each company will pay for its own "knocks."

But enough. A recital of the interesting oddities, curious (to us) customs, and strange wisdoms of British insurance could keep afoot indefinitely without limping. Basically, however, there are great controlling similarities. Our common foundation of common law, our related cultures, and our importation for the past century of British insurance capital and personnel, have operated to maintain in our insurance patterns a sisterhood though not a twinship.

DUDLEY M. PRUITT

PUBLICATIONS RECEIVED:

1. *How Strong is Your Casualty Company?* Roger Kenney. Published by author, Dedham, Mass., 1949. Pp. ix, 246.
2. *Inland Marine and Transportation Insurance.* William H. Rodda. Prentice-Hall, New York City, 1949. Pp. xvi, 539.
3. *Law's State Chart. Companies Licensed December 31, 1949. Fire and Marine Insurance Companies.* Harrison Law. Published by same, Nutley, N. J. Pamphlet. Pp. 16.
4. *Man and The Motor Car.* Revised Edition. Accident Prevention Department, Association of Casualty and Surety Companies, New York City, 1949. Pp. xv, 318.

OBITUARY

ROLAND BENJAMIN

1884-1949

Roland Benjamin, a Charter Member of the Casualty Actuarial Society, died at his home in Baltimore, Maryland, on July 2, 1949 after a short illness, at the age of sixty-five.

Born in Baltimore, June 29, 1884, he attended Baltimore City College and was associated with the Merchants National Bank before entering the employ of the Fidelity and Deposit Company of Maryland as a clerk in its accounting department July 1, 1906. His advancement was rapid and he was appointed assistant treasurer in January, 1913 and six months later was named comptroller of the Company. He became treasurer on January 21, 1919, serving both the Fidelity and Deposit Company of Maryland and its affiliate, the American Bonding Company of Baltimore, in that capacity until his retirement June 30, 1949. In addition, he was a director of the Company, having been elected to that office January 14, 1948.

Mr. Benjamin always maintained an active interest in the affairs of the Society up to the time of his election as treasurer, when increased responsibilities prevented his attendance at its meetings, but he continued to follow the reports of the Society's proceedings with keen interest.

OBITUARY

JOHN FROBERG

1893-1949

Mr. John Froberg, an associate member of the Casualty Actuarial Society, died on October 11, 1949 following a long illness.

Mr. Froberg served his entire business life with the California Inspection Rating Bureau. He started with the Bureau when experience rating for workmen's compensation was first adopted in California and for many years supervised the Bureau's experience rating department. Later he was made a joint manager of the Bureau and at the time of his death was the Bureau's chief executive with the title of Chairman.

Mr. Froberg's position was one which required tact as well as technical knowledge. His continuous advancement to the top position in the Bureau is adequate testimony as to his ability in both respects. He will be missed by the many friends he made over the years.

OBITUARY

FRED S. GARRISON

1879-1949

Fred S. Garrison, a Fellow of the Casualty Actuarial Society since 1915, died at his home in West Hartford on November 14, 1949.

Mr. Garrison was born in Chatham, New Jersey on March 19, 1879, and attended the local schools of that city. He was a veteran of 52 years in the insurance business, commencing his career with the United States Mutual Accident Association in 1894. He later served with the United States Casualty Company, and the New Amsterdam Casualty Company where he became Assistant Secretary. In 1915 Mr. Garrison joined The Travelers Indemnity Company to organize the Burglary and Plate Glass Departments of that company. He became Assistant Secretary of The Travelers in 1916 and Secretary in 1930.

Over the many years until his retirement in 1947, Mr. Garrison gave unstintingly of his efforts in making substantial contributions to the advancement and development of the Burglary insurance field. He was widely known and respected by Burglary underwriters and producers throughout the United States and Canada, and at the time of his retirement was unofficially considered as the dean of Burglary insurance underwriters.

His wit, his wisdom and his courtesy will be sorely missed by his many friends throughout the Casualty insurance field.

OBITUARY

ROLLAND VAUGHN MOTHERSILL

1886-1949

Rolland Vaughn Mothersill, an associate of this Society since November 21, 1919, died on July 25, 1949. He was born in Holly, Michigan on April 24, 1886. He graduated from the University of Michigan in 1911.

After being employed with a life company in Spokane, Washington he was with the California Insurance Department for about five years. He was afterward with the Standard Accident Insurance Company and for a time was with General Motors as an Insurance buyer.

On June 21, 1921 he became the first Actuary and Secretary of the Minnesota Compensation Board. He remained with the Board until 1926 when he became one of the founders of the Anchor Casualty Company. He served successively as Vice-President, Executive Vice-President, President and, finally, as Chairman of the Board of that company.

Mr. Mothersill during his life time had a varied insurance career in the west. Although he was not often present at meetings of the Society he was well known in the west and middle-west and was active in club work there.

OBITUARY

ALBERT HENRY MOWBRAY

1881-1949

Albert Henry Mowbray, Professor of Insurance at the University of California and consulting actuary, died on January 7, 1949, following a heart attack which he had suffered on the same day.

Mr. Mowbray was a Charter Member and past President of the Casualty Actuarial Society, a Fellow of the Acturial Society of America, the Insurance Institute of America, the American Statistical Association and the American Association for the Advancement of Science, and he was a member of the American Mathematical Society, the American Economic Association and the American Association of University Teachers of Insurance.

He was born on March 30, 1881 in San Francisco and educated at the University of California at Berkeley where he received the degree, Bachelor of Arts, in 1904. He was a member of the Phi Beta Kappa and the Sigma Xi Societies and the Phi Kappa Sigma fraternity.

The first position taken by Mr. Mowbray after his graduation from college was as an actuarial clerk with the New York Life Insurance Company, from 1905 to 1907. Following this, he held the position of actuary for the Insurance Departments of the States of North Carolina and California before becoming an instructor at the University of California for the school year 1910-1911. The year 1910 also marked the beginning of Mr. Mowbray's period of activity in actuarial consulting work, a field in which he was to carry on so extensively in his more mature years. In 1913, he resumed employment in the insurance business as an actuary, serving in this capacity during the next ten years successively with the Liberty Mutual Insurance Company in Boston, the Industrial Commission of New York and the National Council on Compensation Insurance. In 1923 he was made Associate Professor at the University of California and in 1928 a full Professor. From 1923, also, to the date of his death, Mr. Mowbray gave part of his time to consulting work. He was prominent in this field, acting as consultant for the California State Insurance Department, California State Insurance Fund, State Insurance Fund of Utah, and many other official bodies as well as private concerns.

Mr. Mowbray was called upon to serve the Federal Government as an actuarial consultant for President Roosevelt's Committee on Economic Security in 1935 and he was a member of the Advisory Council on Social Security, during the years 1937 to 1949.

Mr. Mowbray contributed many distinguished and valuable papers at meetings of the Casualty Actuarial Society, over twenty appearing in the Proceedings during the years since 1914, the year of the Society's establishment. He also contributed many book reviews and discussions, totaling over

fifty during the many years of his active participation in the work of the Society. Mr. Mowbray has also made valuable contributions to the Actuarial Society of America and other technical societies.

As an author, however, Mr. Mowbray was best known for his text-book, "Insurance, Its Theory and Practice in the U. S.," the first edition of which was published in 1930. Later editions appeared in 1937 and in 1946. A pioneer in the casualty actuarial field, he came to be regarded as one of the leading compensation actuaries of the country more than a quarter of a century ago. His grasp of technical insurance problems, however, covered the whole field of insurance as evidenced by the scope of his excellent text-book.

Although Mr. Mowbray was not so active in the affairs of the Society after taking up residence in California in 1923, he never lost his interest in its aims and purposes. He was always a strong believer in the importance of education and sound training as the foundation for an insurance career and his connection with the University of California gave him the opportunity to impart to thousands of young men and women a knowledge of the fundamentals of insurance.

It is hard to say whether his participation in the development of actuarial techniques, during the earlier part of his career, or his work as an insurance educator in his later years represents the greater contribution to the business. Suffice it to say that they are both notable and involve the qualities of thoroughness and intellectual integrity which were so inherently characteristic of all his undertakings. Mr. Mowbray's death is a loss to the insurance business, to the Society, and to all those who had the privilege of knowing him.

OBITUARY

LLOYD ARTHUR HEBER WARREN

1879-1949

Lloyd A. H. Warren, a Fellow of the Society, died suddenly at his home in Winnipeg on October 7, 1949.

Lloyd Warren was born on November 18, 1879 in Balderson, Ontario. He entered Queen's University in 1899 and received the degree of Master of Arts in 1904. During the next two years he studied at Clark University and spent a short time on the staff of the Sun Life Insurance Company in Montreal as an Actuarial Clerk. In 1907, he was appointed to the staff of the University of Manitoba as Assistant-Professor of Mathematics and continued to serve that University for the next forty-two years.

In 1911 he continued his studies at the University of Chicago and received the degree of Doctor of Philosophy in Applied Mathematics in 1913. His interest in Astronomy was recognized by the Royal Astronomical Society which elected him a Fellow in 1914. His chief interest, however, was in Actuarial Science and he continued to further his knowledge in that field and to teach courses in Actuarial Science at the University of Manitoba. He became a Fellow of the American Institute of Actuaries in 1934 and also a Fellow of the Casualty Actuarial Society. He was a co-author of a text in Algebra which was adopted by the American Institute of Actuaries as part of the required reading for students who wished to write the examinations of the Institute for the degree of Associate. His achievements were recognized by the University of Manitoba by successive appointments as Associate-professor and Professor of Mathematics and by the appointment as the first Professor of Actuarial Science in the newly formed School of Commerce in 1935. He spent the remainder of his life building up the department of Actuarial Science until it became widely known in Canada and the United States. He retired on August 31, 1949.

Professor Warren will always be remembered by his many students who hold positions in many parts of the United States and Canada for the interest which he took in their welfare, and his excellence as a teacher will always be remembered with gratitude. His persistence in his own studies inspired them to follow his example and his own high ideals gave them a goal which they could strive to attain. The Actuarial profession has benefited greatly by his splendid contribution which will be remembered with gratitude by Actuaries on this continent.

OBITUARY

LEE J. WOLFE

1882-1949

Lee J. Wolfe, a Charter Member of the Society, died suddenly in New York City on April 28, 1949.

Mr. Wolfe was born on April 3, 1882 in the City of Baltimore, Maryland. He was educated at the College of the City of New York. After a period of apprenticeship in the Actuarial Department of the old Provident Savings Life, during which he studied actuarial science, he entered the consulting actuarial field in which he was engaged for the remainder of his life.

Lee J. Wolfe first joined with his brother, the late S. Herbert Wolfe, also a Charter Member of the Society, in the formation of the partnership of S. H. and Lee J. Wolfe in 1905. In 1942, the partnership name was changed to Wolfe, Corcoran and Linder of which firm Mr. Wolfe was the senior partner at the time of his death.

During the early years of his consulting business, and particularly at the time of the Armstrong Investigation, Mr. Wolfe undertook many assignments in behalf of state insurance commissioners, and continued in close association with the National Association of Insurance Commissioners by attending their meetings regularly until quite recently. His consulting activities were wide and varied, consisting of not only actuarial work, but also auditing and insurance accounting, and were not confined to the casualty field but extended to life insurance, fraternal insurance, fire insurance, pension work, and practically all kinds of actuarial activities. His knowledge of tax legislation and tax problems was particularly wide, and he was co-author of a book entitled "Inheritance Tax Calculations."

Besides membership in the Casualty Actuarial Society, Mr. Wolfe was also a Fellow of the American Statistical Society, the Royal Statistical Society, and the American Institute of Accountants. He was also a Certified Public Accountant in the State of New York and in several other states.

Mr. Wolfe's consulting functions required extensive travel and he had literally hundreds of intimate friends throughout the whole United States as well as in his home city. Within his wide acquaintance, he was held in the highest regard for his extensive knowledge of so many aspects of the insurance business, his high ability, and his personality and genuine friendliness. His many friends will greatly miss him.

ABSTRACT FROM THE MINUTES OF THE MEETING

MAY 23 and 24, 1949

The semi-annual meeting of the Casualty Actuarial Society was held at the Red Lion Inn, Stockbridge, Massachusetts, on Monday and Tuesday, May 23 and 24, 1949.

President Cahill called the meeting to order at 2:30 P.M. on May 23rd, the roll was called, showing the following forty-six Fellows and twelve Associates present:

FELLOWS

ALLEN, E. S.	FONDILLER	PERRYMAN
BAILEY	FULLER	PRUITT
BARBER	GARDINER	RODERMUND
BARTER	GODDARD	ROSS
BATHO, E. R.	GRAHAM, C. M.	ROWELL
BERKELEY	HAUGH	SCHLOSS
BURLING	JOHNSON	SINNOTT
CAHILL	KOLE	SMICK
CARLSON	KORMES	SMITH, S. E.
CARLETON	LINDER	UHTHOFF
COATES, C. S.	MASTERSON	VALERIUS
COGSWELL	MATTHEWS	VAN TUYL
CROUSE	MAYCRINK	WIEDER
ELLIOTT	MCCONNELL	WILLIAMS
FARLEY	MILLER	WILLIAMSON
	OBERHAUS	

ASSOCIATES

BATHO, B.	MAYER	SCAMMON
DOWLING	MACKEEN	STOKE
FURNIVALL	PENNOCK	VERGANO
LUFKIN	PERRY	WHITE

By invitation, a number of officials of Casualty Insurance Companies and other organizations were present.

The minutes of the meeting held November 19, 1948 were approved as printed in the *Proceedings*. Samuel M. Ross was elected Librarian of the Society.

No formal papers were presented at this meeting of the Society.

An informal dinner had been held on the evening of May 22, 1949, also on the evening of May 23, 1949.

Informal discussion of the following topics was participated in by the members of the Society and by representatives of insurance organizations.

President Cahill turned the meeting over to Vice-President Barber for a discussion on "The function of Casualty Insurance Companies under State Disability Benefit Laws, with particular reference to New York." Recess was then declared until the following day.

On May 24th, the meeting was called to order at 10 A.M. by President Cahill, who later turned the meeting over to Vice-President Goddard for a discussion on "Retrospective rating for combinations of lines (Plan D) and what is ahead."

Upon motion, the meeting adjourned at noon.

ABSTRACT FROM THE MINUTES OF THE MEETING

NOVEMBER 18, 1949

The annual meeting of the Casualty Actuarial Society was held at the Hotel Biltmore, New York, on Friday, November 18, 1949. An informal dinner, celebrating the 35th Anniversary of the Society, had been held on Thursday evening, November 17th at the Hotel Biltmore; the dinner group was addressed by the Hon. W. Ellery Allyn, Insurance Commissioner of Connecticut and vice-president of the National Association of Insurance Commissioners, Mr. G. F. Michelbacher, former president of the Society, Hon. Robert E. Dineen, Superintendent of Insurance of the State of New York, and by Mr. L. W. Scammon, Actuary of the Massachusetts Workmen's Compensation Rating and Inspection Bureau.

President Cahill called the annual meeting to order at 10:20 A.M., the roll was called, showing the following 57 Fellows and 21 Associates present:

FELLOWS

AINLEY	EPPINK	MILLS
ALLEN, E. S.	FALLOW	MOORE, G. D.
BAILEY	FONDILLER	OBERHAUS
BARBER	GARDINER	PERRYMAN
BARTER	GINSBURGH	PRUITT
BATHO, E. R.	GODDARD	RESONY
BERKELEY	GRAHAM, C. M.	RODERMUND
BLANCHARD	GRAHAM, W. J.	ROSS
BROWN, F. S.	HAUGH	SALZMANN
BURLING	JOHNSON	SCHLOSS
CAHILL	KOLE	SKILLINGS
CARLSON	KORMES	SMICK
CLARKE	KULP	TARBELL
COGSWELL	LACROIX	UHTHOFF
COMSTOCK	LINDER	VALERIUS
CONSTABLE	MASTERSON	VAN TUYL
CROUSE	MAYCRINK	WIEDER
DAVIES	MCCONNELL	WILLIAMS
ELLIOTT	MILLER, J. H.	WOLFRUM

ASSOCIATES

BLACK, N. C.	HOPE	PENNOCK
DOWLING	MACKEEN	POTOFSKY
GEORGE	LESLIE, JR.	SCAMMON
GROSSMAN	LIVINGSTON	SCHWARTZ
HARWAYNE	LUFKIN	SULLIVAN
HAZAM	MONTGOMERY, J. C.	VERGANO
HEWITT	MUNTERICH	WARREN, C. S

By invitation, a number of officials of casualty insurance companies and insurance organizations were present.

Mr. Cahill read his Presidential address.

The minutes of the meeting held May 23 and 24, 1949 were approved as printed in the *Proceedings*.

The Secretary-Treasurer (Richard Fondiller) read the report of the Council and upon motion it was adopted by the Society. John W. Clarke, Harold F. LaCroix, Jr., John A. Resony and Richard J. Wolfrum had passed the examinations and had been admitted as Fellows: a diploma was presented to each by the President. Frank Harwayne, William J. Hazam, Francis J. Hope and William Leslie, Jr., had passed the examinations and had been admitted as Associates.

The President announced the deaths, during the last year, of five Fellows, Roland Benjamin, Fred S. Garrison, Albert H. Mowbray, Lloyd A. H. Warren, Lee J. Wolfe, and two Associates, John Froberg and Rolland V. Mothersill. Obituary notices appear in this number of the *Proceedings*.

The Auditing Committee (Howard G. Crane, Chairman) reported that the books of the Secretary-Treasurer had been audited and his accounts verified.

The report of the Secretary-Treasurer was read and accepted. The report on Finances follows:

CASUALTY ACTUARIAL SOCIETY
ANNUAL REPORT ON FINANCES

Cash Receipts and Disbursements from October 1, 1948 to Sept. 30, 1949

Income			Disbursements	
On deposit in Marine Midland on October 1, 1948		$ 703.58	Printing & Stationery	$3,604.15
			Postage, Tel., Exp. Etc.	220.15
			Secretarial Work	472.25
Members Dues	$4,088.00		Examination Expense	820.02
Sale of Proceedings	1,155.04		Refund of Exam. Fees	10.00
Examination Fees	743.00		Luncheons & Dinners	1,163.39
Luncheons & Dinners	1,059.75		Library Fund	10.98
Michelbacher Fund	994.35		Storage of Proceedings	48.00
Reprints—Loss			Fire Ins.-Proc. & Library	25.49
Reserve Report	201.15		Purchase of Bond	1,000.00
Maturity of Bonds	3,750.00		Reprints—	
Gain on Maturity	1,250.00		Loss Reserve Report	93.84
Interest on Bonds	12.50		Miscellaneous	46.44
Miscellaneous	13.07			
Foreign Exchange	−2.53	13,264.33	Total	$ 7,514.71
			On deposit September 30, 1949 in Marine Midland Trust Co.	6,453.20
Total		$13,967.91		$13,967.91

Assets		Liabilities	
Cash in Bank	$6,453.20	Michelbacher Fund	$4,002.96
Bond Owned	1,000.00	Fondiller Prize	100.00
		Total Liabilities	$4,102.96
		Surplus	3,350.24
Total Assets	$7,453.20	Total Liabilities & Surplus	$7,453.20

The examination committee (Ernest T. Berkeley, General Chairman) submitted a report of which the following is a summary:

1949 EXAMINATIONS—SUCCESSFUL CANDIDATES

The following is a list of those who passed the examinations held by the Society on May 10 and 11, 1949:

ASSOCIATE EXAMINATIONS

PART I:	J. S. Acheson	Stephen Kuryliw	*H. F. LaCroix, Jr. (Sec. 2)
	J. H. Boyajian	D. B. MacMillan	*F. G. Letwin (Sec. 2)
	R. W. Butcher	Paul Overberg	*Paul S. Liscord, Jr. (Sec. 2)
	G. R. Dinney	W. J. Perkins	J. A. W. Trist
	D. Eckersley	Earl F. Petz, Jr.	Andrew Vogt
	C. H. Graves	Allie V. Resony	Ernest R. Vogt
	Carl A. Haase	A. E. Selwood	Harry Wach
	W. F. Hancock	William D. Smith	A. E. Whiton
	John Harack	P. J. Spellman	Abraham Wind
	W. V. Hauke	James W. Thomas	
PART II:	R. S. Brindise	Carl A. Haase	*William J. Hazem (Sec. 3)
	R. W. Butcher	James B. Haley, Jr.	Paul Overberg
	G. F. S. Clarke	W. V. Hauke	R. B. Pennycock
	Douglas Critchley	B. H. Hazelhurst	I. Rosenberg
	G. R. Dinney	F. E. Hobson	W. G. Saunders
	D. L. Fisher	W. J. Hudson	William D. Smith
	R. F. Flanders	Hugh P. Keay	Roy D. Tofte
	Howard Fosler	N. R. B. King	J. A. W. Trist
	R. E. Galloway	Leon Leckie	Andrew Vogt
	C. H. Graves	Israel Lepkin	D. G. Welland
		C. A. Levitsky	

*Other section of this Part credited to Veterans by the Council under 1948 Rule 4(e).

PART III:	J. H. Biyajian	Frank Harwayne	Paul Overberg
	R. S. Brindise	W. V. Hauke	I. Rosenberg
	R. W. Butcher	W. J. Hazam	W. J. Saunders
	G. F. S. Clarke	F. E. Hobson	J. W. Schlenz
	R. D. Drisko	F. J. Hope	William D. Smith
	D. L. Fisher	W. J. Hudson	J. A. W. Trist
	John D. Gabel	N. R. B. King	Andrew Vogt
	R. E. Galloway	Leon Leckie	D. G. Welland
	Carl A. Haase	Israel Lepkin	Abraham Wind
	John Harack	William Leslie, Jr.	V. S. Zarowski
		Paul S. Liscord, Jr.	
PART IV:	J. B. Franklin, Jr.	W. J. Hazam	Thomas E. Murrin
	Frank Harwayne	Henry Menzel	Allie V. Resony

FELLOWSHIP EXAMINATIONS

PART I:	Robert E. Bruce	C. C. Hewitt, Jr.	G. R. Livingston
	John W. Clarke	Francis J. Hope	John A. Resony
	Frank Harwayne	H. F. LaCroix, Jr.	
PART II:	John W. Clarke	H. F. LaCroix, Jr.	John A. Resony
	C. C. Hewitt, Jr.	G. R. Livingston	Elia Vergano
	Francis J. Hope	R. E. MacKeen	
PART III:	H. F. LaCroix, Jr.	John A. Resony	R. J. Wolfrum
PART IV:	H. F. LaCroix, Jr.	G. C. Munterich	R. J. Wolfrum
		John A. Resony	

The Secretary-Treasurer announced that the Council had elected the following officers:

Editor (re-elected)	Emma C. Maycrink
Librarian (re-elected)	Samuel M. Ross
Chairman–Examination Committee	Roger A. Johnson

In accordance with Constitutional requirements, notice of the following proposed amendments was given. These amendments were, on motion, adopted to read as follows:

Constitution–Article IV–First Sentence.

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, and Editor, a Librarian and a General Chairman of the Examination Committee.

Constitution–Article V–Third Sentence.

The Editor, the Librarian and the General Chairman of the Examination Committee shall be elected annually at the Council meeting preceding the annual meeting of the Society.

By-Laws—Article III—new Sixth Paragraph.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

The annual elections were then held and the following officers and members of the Council were elected.

President	Harmon T. Barber
Vice President	Thomas O. Carlson
Vice President	Norton E. Masterson
Secretary-Treasurer	Richard Fondiller
Editor	Emma C. Maycrink
Librarian	Samuel M. Ross
Chairman—Examination Committee	Roger A. Johnson

Members of the Council:

John W. Carleton	1950
Ernest T. Berkeley	1952
William J. Constable	1952
Dudley M. Pruitt	1952

On behalf of the Committee on Papers, (Arthur N. Matthews, Chairman) Mr. Nels M. Valerius announced that the best paper presented during the last five years was "Possible Values for Retrospective Rating Plans". Mr. Valerius thereupon presented the Third Richard Fondiller prize of One Hundred Dollars to Francis S. Perryman.

The papers appearing in this Volume were presented.

Recess was taken for lunch at the Hotel until 2:15 P.M.

Informal discussion of the following topic was participated in by the members of the Society and by representatives of insurance companies and organizations:

"Uniform accounting of expenses and its effect on ratemaking procedures."

Upon motion, the meeting adjourned at 4:30 P.M.

INDEX TO VOLUME XXXVI

CASUALTY ACTUARIAL SOCIETY

ORGANIZED 1914

1950 YEAR BOOK

(Addendum to Volume XXXVI of the *Proceedings*)

Corrected to January 16, 1950 **No. 29**

FOREWORD

The Casualty Actuarial Society was organized November 7, 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow. The present title was adopted on May 14, 1921. The object of the Society is the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

Prior to 1914, little technical study was given to the actuarial and underwriting problems of most of the branches of casualty insurance. The organization of the Society was brought about through the suggestion of Dr. I. M. Rubinow, who became the first president. The problems surrounding workmen's compensation were at that time the most urgent, and consequently many of the members played a leading part in the development of the scientific basis upon which workmen's compensation insurance now rests.

The members of the Society have also presented original papers to the *Proceedings* upon the scientific formulation of standards for the computation of both rates and reserves in accident and health insurance, liability, burglary, and the various automobile coverages. The presidential addresses constitute a valuable record of the current problems facing the casualty insurance business. Other papers in the *Proceedings* deal with acquisition costs, pension funds, legal decisions, investments, claims, reinsurance, accounting, statutory requirements, loss reserves, statistics, and the examination of casualty companies. The Committee on Remarriage Table submitted a report including tables, printed in *Proceedings* No. 40. The Special Committee on Bases of Exposure submitted a report which is printed in *Proceedings* No. 43. "The Recommendations for Study" appear in *Proceedings* No. 64 and are in effect for the 1950 examinations and thereafter. The Report of the Committee on Mortality for Disabled Lives together with commutation tables and life annuities has been printed in *Proceedings* No. 62. The Committee on Compensation and Liability Loss and Loss Expense Reserves submitted a report which appears in Volume XXXV.

The lower grade of membership in the Society is that of Associate. Examinations have been held every year since organization; they are held on the second Tuesday and following Wednesday during the month of May, in various cities in the United States and Canada. The membership of the Society consists of actuaries, statisticians, and executives who are connected with the principal casualty companies and organizations in the United States and Canada. The Society has a total membership of 270, consisting of 151 Fellows and 119 Associates. The annual meeting of the Society is held in New York in November.

The Society issues a publication entitled the *Proceedings* which contains original papers presented at the meetings. The *Proceedings* also contain discussions of papers, reviews of books and current notes. This Year Book is published annually and "Recommendations for Study" is a pamphlet which outlines the course of study to be followed in connection with the examinations for admission. These two booklets may be obtained free upon application to the Secretary-Treasurer, 90 John Street, New York 7, N. Y.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 18, 1949

THE COUNCIL

**Officers:* HARMON T. BARBER ..... *President*
THOMAS O. CARLSON ..... *Vice-President*
NORTON E. MASTERSON ..... *Vice-President*
RICHARD FONDILLER ..... *Secretary-Treasurer*
EMMA C. MAYCRINK ..... *Editor*
SAMUEL M. ROSS ..... *Librarian*
ROGER A. JOHNSON ..... *Chairman-Examination Committee*

†*Ex-Presidents:* CHARLES J. HAUGH 1951
JAMES M. CAHILL 1953

†*Ex-Vice-Presidents:* HARRY V. WILLIAMS 1951
RUSSELL P. GODDARD 1953

†*Elected:* JOHN W. CARLETON 1950
GEORGE B. ELLIOT 1950
JARVIS FARLEY 1950
CHARLES M. GRAHAM 1951
JOSEPH LINDER 1951
SEYMOUR E. SMITH 1951
ERNEST BERKELEY 1952
WILLIAM J. CONSTABLE 1952
DUDLEY M. PRUITT 1952

* Terms expire at the annual meeting in November 1950.
† Terms expire at the annual meeting in November of the year given.

COMMITTEES

COMMITTEE ON ADMISSIONS

THOMAS F. TARBELL (CHAIRMAN)
WILLIAM J. CONSTABLE
GUSTAV F. MICHELBACHER
FRANCIS S. PERRYMAN
HIRAM O. VAN TUYL

AUDITING COMMITTEE

HOWARD G. CRANE (CHAIRMAN)
CHARLES M. GRAHAM
HAROLD W. SCHLOSS

EDITORIAL COMMITTEE

EMMA C. MAYCRINK (CHAIRMAN)

ASSISTANT EDITORS

CLARENCE A. KULP
JACK J. SMICK

EDUCATIONAL COMMITTEE

ERNEST T. BERKELEY (CHAIRMAN)
ARTHUR L. BAILEY
ROGER A. JOHNSON
CLARENCE A. KULP
JACK J. SMICK

EXAMINATION COMMITTEE

ROGER A. JOHNSON (GENERAL CHAIRMAN)

FELLOWSHIP

SAMUEL M. ROSS (CHAIRMAN)
EDWARD S. ALLEN
CHARLES W. CROUSE

ASSOCIATESHIP

JOHN W. WIEDER (CHAIRMAN)
STEFAN PETERS
DUNBAR R. UHTHOFF

COMMITTEE ON PAPERS

NELS M. VALERIUS (CHAIRMAN)
JOHN L. BARTER
RUSSELL P. GODDARD
EMMA C. MAYCRINK (*ex-officio*)

COMMITTEE ON PROGRAM

HARMON T. BARBER, CHAIRMAN (*ex-officio*)
THOMAS O. CARLSON
WILLIAM J. CONSTABLE
NORTON E. MASTERSON
RICHARD FONDILLER (*ex-officio*)

COMMITTEE ON PUBLICATIONS

HARMON T. BARBER, CHAIRMAN (*ex-officio*)
RICHARD FONDILLER
EMMA C. MAYCRINK
SAMUEL M. ROSS

SPECIAL COMMITTEES

COMMITTEE ON SOCIAL INSURANCE

HAROLD J. GINSBURGH (CHAIRMAN)
JARVIS FARLEY
A. L. KIRKPATRICK
CLARENCE A. KULP
ARTHUR N. MATTHEWS
WILLIAM R. WILLIAMSON

COMMITTEE ON STATUTORY DISABILITY INSURANCE

FRANCIS S. PERRYMAN (CHAIRMAN)
JOHN W. CARLETON
CHARLES J. HAUGH
NORTON E. MASTERSON
HARRY V. WILLIAMS

COMMITTEE ON COMPENSATION AND LIABILITY
LOSS AND LOSS EXPENSE RESERVES

JOSEPH LINDER (CHAIRMAN)
HARMON T. BARBER
JOHN W. CARLETON
NORTON E. MASTERSON
JOHN A. MILLS
E. SHAW SKILLINGS
DUDLEY M. PRUITT

COMMITTEE ON DEVELOPMENT

HARRY V. WILLIAMS (CHAIRMAN)
JOHN W. CARLETON
CLARENCE A. KULP
SYDNEY D. PINNEY
SEYMOUR E. SMITH

COMMITTEE ON PROSPECTUS

JOHN A. MILLS (CHAIRMAN)
JOHN W. CLARKE
GEORGE B. ELLIOTT

MEMBERSHIP OF THE SOCIETY, NOVEMBER 18, 1949

FELLOWS

Those marked (†) were Charter Members at date of organization, November 7, 1914.

Those marked (*) have been admitted as Fellows upon examination by the Society.

Admitted	
*Nov. 21, 1930	AINLEY, JOHN W., Supervising Underwriter, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	ALLEN, EDWARD S., Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York 17, N. Y.
*Nov. 13, 1931	AULT, GILBERT E., Actuary, Church Pension Fund and Church Life Insurance Corporation, 20 Exchange Place, New York 5, N. Y.
Nov. 19, 1948	BAILEY, ARTHUR L., Actuary, New York Insurance Department, 61 Broadway, New York 6, N. Y.
May 23, 1924	BAILEY, WILLIAM B., (Retired), 52 West Hill Drive, West Hartford, Conn.
*Nov. 20, 1924	BARBER, HARMON T., Associate Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	BARKER, LORING M., Actuary, Firemen's Fund Insurance Group, 401 California Street, San Francisco 20, Calif.
*Nov. 20, 1942	BART, ROBERT D., Office Manager, West Bend Aluminum Co., 92 Island Avenue, West Bend, Wis.
*Nov. 18, 1932	BARTER, JOHN L., Vice-President, Hartford Accident & Indemnity Co., 690 Asylum Avenue, Hartford 15, Conn.
*Nov. 13, 1931	BATHO, ELGIN R., Associate Actuary, Berkshire Life Insurance Co., 7 North Street, Pittsfield, Mass.
*Nov. 22, 1934	BERKELEY, ERNEST T., Actuary, Employers Liability Assurance Corporation, Ltd. and American Employers Insurance Company, 110 Milk Street, Boston 7, Mass.
†	BLACK, S. BRUCE, President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Apr. 20, 1917	BLANCHARD, RALPH H., Professor of Insurance, School of Business, Columbia University, New York 27, N. Y.
†	BREIBY, WILLIAM, Vice-President, Pacific Mutual Life Insurance Company, 523 West 6th St., Los Angeles 14, Cal.
*Nov. 18, 1927	BROWN, F. STUART, Assistant to Vice-President, Bankers Indemnity Insurance Company, 15 Washington Street, Newark 2, N. J.
Oct. 22, 1915	BROWN, HERBERT D., (Retired), Glenora-on-Lake Seneca, Dundee, New York.
†	BUCK, GEORGE B., Consulting Actuary, 150 Nassau Street, New York 7, N. Y.

FELLOWS

Admitted	
Apr. 20, 1917	BURHOP, WILLIAM H., Executive Vice-President, Employers Mutual Liability Insurance Company, 407 Grant Street, Wausau, Wis.
*Nov. 23, 1928	BURLING, WILLIAM H., Assistant Secretary, Group Department, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
*Nov. 19, 1929	CAHILL, JAMES M., Secretary, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 18, 1932	CAMERON, FREELAND R., Catalonia Avenue, Coral Gables, Florida.
†	CAMMACK, EDMUND E., Vice-President and Actuary, Aetna Life Insurance Company, Hartford 15, Conn.
*Nov. 17, 1938	CARLETON, JOHN W., Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
*Nov. 21, 1930	CARLSON, THOMAS O., Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
Nov. 18, 1949	CLARKE, JOHN W., Assistant Actuary, Life Department, The Travelers Insurance Company, 700 Main St., Hartford 15, Conn.
*Nov. 13, 1936	CLEARY, ARTHUR E., 162 East 42nd Street, New York 17, N. Y.
*Nov. 15, 1918	COATES, BARRETT N., Coates and Herfurth, Consulting Actuaries, 620 Market Street, San Francisco 4, Calif.
*Nov. 17, 1922	COATES, CLARENCE S., Third Vice-President, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
Oct. 27, 1916	COGSWELL, EDMUND S., First Deputy Commissioner of Insurance Department of Banking and Insurance, Division of Insurance, 100 Nashua Street, Boston 14, Mass.
Feb. 19, 1915	COLLINS, HENRY, (Retired), Timberlane, Route 4, Easton, Md.
*Nov. 23, 1928	COMSTOCK, W. PHILLIPS, Statistician, Preferred Accident Insurance Company, 80 Maiden Lane, New York 7, N. Y.
*Nov. 22, 1934	CONSTABLE, WILLIAM J., President and Treasurer, Excess Insurance Company of America, 99 John Street, New York 7, N. Y.
*Nov. 22, 1934	COOK, EDWIN A., Assistant General Manager and Secretary, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
†	COPELAND, JOHN A., Consulting Actuary, 1520-21 Candler Building, Atlanta, Ga.
*Nov. 18, 1925	CORCORAN, WILLIAM M., Partner, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 19, 1926	CRANE, HOWARD G., Vice-President and Treasurer, General Reinsurance Corporation, and North Star Reinsurance Corporation, 90 John Street, New York 7, N. Y.
*Nov. 22, 1946	CROUSE, CHARLES W., Actuary, Manufacturers Casualty Insurance Company, 1617 Pennsylvania Boulevard, Philadelphia 3, Pa.

FELLOWS

Admitted	
*Nov. 18, 1932	DAVIES, E. ALFRED, (Retired) Falls Village, Conn.
*Nov. 18, 1927	DAVIS, EVELYN M., Woodward, Ryan, Sharp & Davis, Consulting Actuaries, 41 Park Row, New York 7, N. Y.
†	DEKAY, ECKFORD C., President, DeKay & Company, 84 William Street, New York 7, N. Y.
*Nov. 17, 1920	DORWEILER, PAUL, Actuary, Aetna Casualty & Surety Company, Hartford 15, Conn.
*Nov. 24, 1933	EDWARDS, JOHN, Actuary, Ontario Insurance Department, Parliament Buildings, Toronto 2, Ontario, Canada.
*Nov. 15, 1940	ELLIOTT, GEORGE B., General Manager, Pennsylvania Compensation Rating and Inspection Bureau, 938 Public Ledger Building, Independence Square, Philadelphia 6, Pa.
*Nov. 17, 1922	ELSTON, JAMES S., Assistant Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 15, 1935	EPPINK, WALTER T., Actuary, Merchants Mutual Casualty Co., Merchants Mutual Building, Buffalo 5, New York.
†	FACKLER, EDWARD B., Consulting Actuary, Fackler & Company, 8 West 40th Street, New York 18, N. Y.
†	FALLOW, EVERETT S., Actuary, Accident Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 15, 1940	FARLEY, JARVIS, Actuary and Assistant Treasurer, Massachusetts Indemnity Co., 632 Beacon Street, Boston 15, Mass.
†	FARRER, HENRY, Insurance Company of North America, 99 John Street, New York 7, N. Y.
*Nov. 15, 1935	FITZHUGH, GILBERT W., Third Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Feb. 19, 1915	FONDILLER, RICHARD, Consulting Actuary, Woodward and Fondiller, 90 John Street, New York 7, N. Y.
†	FRANKLIN, CHARLES H., (Retired), 6225 Princeton Way, Hawthorne Hills, Seattle, Washington.
*Nov. 18, 1927	FREDRICKSON, CARL H., Actuary, Canadian Underwriters Association, 55 York Street, Toronto, Canada.
*Nov. 22, 1934	FULLER, GARDNER V., Third Vice-President and Risk Experience Manager, Special Risk Department, Lumbermens Mutual Casualty Co., and American Motorist Insurance Co., 4750 Sheridan Road, Chicago 40, Ill.
*Nov. 19, 1948	GARDINER, JAMES B., Manager, Group Contract Bureau, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 20, 1924	GINSBURGH, HAROLD J., Vice-President, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston 16, Mass.
*Nov. 21, 1930	GLENN, J. BRYAN, 5214 First Street, N.W., Washington 11, D.C.
*Nov. 13, 1931	GODDARD, RUSSELL P., Assistant to President, Pennsylvania Manufacturers' Association Casualty Insurance Co., Finance Building, Philadelphia, Pa.

FELLOWS

Admitted	
†	GOODWIN, EDWARD S., 962 Main Street, East Hartford 8, Conn.
*Nov. 19, 1926	GRAHAM, CHARLES M., Chief Self-Insurance Examiner, New York State Workmen's Compensation Board, 80 Center Street, New York 13, N. Y.
†	GRAHAM, WILLIAM J., Consulting Actuary and Insurance Advisor, 1070 Park Avenue, New York 18, N. Y.
†	GREENE, WINFIELD W., Executive Vice-President, General Reinsurance Corporation and North Star Reinsurance Corporation, 90 John Street, New York 7, N. Y.
†	HAMMOND, H. PIERSON, (Retired), 22 Vanderbilt Road, West Hartford, Conn.
Oct. 27, 1916	HARDY, EDWARD R., (Retired), 235 East 22nd Street, New York 10, N. Y.
Oct. 22, 1915	HATCH, LEONARD W., (Retired), 425 Pelham Manor Road, Pelham Manor, New York.
*Nov. 19, 1926	HAUGH, CHARLES J., Secretary, Compensation and Liability Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
Oct. 22, 1915	HODGKINS, LEMUEL G., (Retired), 5 Whitman Road, Worcester 5, Mass.
Oct. 22, 1915	HOLLAND, CHARLES H., Suite 2001, 165 Broadway, New York 6, N. Y.
*Nov. 22, 1934	HOOKER, RUSSELL O., Actuary and Director of Examinations, State of Connecticut Insurance Department, Hartford 2, Conn.
Nov. 18, 1932	HUEBNER, SOLOMON STEPHEN, Professor of Insurance, University of Pennsylvania, Philadelphia 4, Pa.
*Nov. 14, 1947	HUGHEY, M. STANLEY, Procedures Co-ordinator, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
†	HUNTER, ARTHUR, (Retired), 124 Lloyd Road, Montclair, N. J.
Feb. 25, 1916	JACKSON, CHARLES W., (Retired), 74 Quimby Avenue, White Plains, N. Y.
*Nov. 19, 1929	JACKSON, HENRY HOLLISTER, Vice-President & Actuary, National Life Insurance Co., 131 State Street, Montpelier, Vt.
*Nov. 14, 1941	JOHNSON, ROGER A., Actuary, Utica Mutual Insurance Co., 185 Genesee Street, Utica, N. Y.
*Nov. 16, 1939	JONES, HAROLD M., Group Research Division, John Hancock Mutual Life Insurance Company, 197 Clarendon Street, Boston 17, Mass.
*Nov. 19, 1926	KELTON, WILLIAM H., Associate Actuary, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 21, 1919	KIRKPATRICK, A. LOOMIS, Manager Insurance Department, Chamber of Commerce of the U. S. A., 1615 H Street, N.W., Washington 6, D. C.
*Nov. 14, 1941	KOLE, MORRIS B., Associate Actuary, State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.

FELLOWS

Admitted	
*Nov. 24, 1933	KORMES, MARK, Consulting Actuary, 285 Madison Avenue, New York 17, N. Y.
Nov. 23, 1928	KULP, CLARENCE A., Professor of Insurance, University of Pennsylvania, Logan Hall, 36th Street and Woodland Avenue, Philadelphia 4, Pa.
*Nov. 18, 1949	LA CROIX, HAROLD F., JR., Accident Actuarial Department, Travelers Insurance Co., 700 Main St., Hartford 15, Conn.
Nov. 13, 1931	LA MONT, STEWART M., (Retired), Hotel Claremont, Berkeley, Cal.
*Nov. 24, 1933	LANGE, JOHN R., Commissioner of Insurance, State of Wisconsin, State Capitol, Madison 2, Wis.
†	LEAL, JAMES R., Vice-President and Secretary, Interstate Life and Accident Co., Interstate Building, 540 McCallie Avenue, Chattanooga 3, Tenn.
†	LESLIE, WILLIAM, General Manager, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 20, 1924	LINDER, JOSEPH, Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 13, 1936	LYONS, DANIEL J., Second Vice-President, The Guardian Life Insurance Co. of America, 50 Union Square, New York 3, N. Y.
†	MAGOUN, WILLIAM N., (Retired), 33 Fearing Road, Hingham, Mass.
*Nov. 23, 1928	MARSHALL, RALPH M., Assistant Actuary, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 18, 1927	MASTERSON, NORTON E., Vice-President and Actuary, Hardware Mutual Casualty Co. and Hardware Dealers Mutual Fire Insurance Co., 200 Strongs Avenue, Stevens Point, Wis.
*Nov. 19, 1926	MATTHEWS, ARTHUR N., Assistant Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
May 19, 1915	MAYCRINK, EMMA C., Secretary-Treasurer, Association of New York State Mutual Casualty Companies, 60 East 42nd Street, New York 17, N. Y.
*Nov. 15, 1935	MCCONNELL, MATTHEW H., General Accident Fire and Life Assurance Company, Fourth and Walnut Sts., Philadelphia 5, Pa.
*Oct. 31, 1917	MCMANUS, ROBERT J., Statistician, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
†	MICHELBACHER, G. F., President, Great American Indemnity Co., 1 Liberty Street, New York 5, N. Y.
*Nov. 17, 1938	MILLER, JOHN HAYNES, Vice-President and Actuary, Monarch Life Insurance Company, 365 State St., Springfield 1, Mass.
†	MILLIGAN, SAMUEL, Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 18, 1937	MILLS, JOHN A., Vice-President and Actuary, Lumbermens Mutual Casualty Co. and American Motorists Insurance Co., Mutual Insurance Bldg., 4750 Sheridan Road, Chicago 40, Ill.

FELLOWS

Admitted	
*Nov. 18, 1921	MONTGOMERY, VICTOR, President, Pacific Employers Insurance Co., 1033 So. Hope Street, Los Angeles 15, Calif.
†	MOORE, GEORGE D., Actuary, 13 Emerson Street, E. Orange, N. J.
*Nov. 17, 1920	MUELLER, LOUIS H., 161 28th Ave., San Francisco 21, Calif.
†	MULLANEY, FRANK R., Vice-President and Secretary, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston 16, Mass.
May 28, 1920	MURPHY, RAY D., Vice-President and Actuary, The Equitable Life Assurance Society of the U. S. A., 393 Seventh Avenue, New York 1, N. Y.
*Nov. 15, 1935	OBERHAUS, THOMAS M., Consulting Actuary, Woodward and Fondiller, 90 John Street, New York 7, N. Y.
†	OLIFIERS, EDWARD, Consulting Actuary, Caixa Postal 8, Pertopolis, Rio, Brazil.
†	ORR, ROBERT K., (Retired), 226 S. Logan Street, Lansing 15, Mich.
*Nov. 21, 1919	OUTWATER, OLIVE E., Actuary, Benefit Association of Railway Employees, 901 Montrose Avenue, Chicago 13, Ill.
*Nov. 21, 1930	PERRYMAN, FRANCIS S., Vice-President and Actuary, Eagle Indemnity Co., Globe Indemnity Co. and Royal Indemnity Co., 150 William Street, New York 8, N. Y.
*Nov. 14, 1941	PETERS, STEFAN, Associate Professor of Insurance, School of Business Administration, 114 South Hall, University of California, Berkeley 4, Calif.
Nov. 19, 1926	PHILLIPS, JESSE S., Chairman of Board, Great American Indemnity Co., 1 Liberty Street, New York 5, N. Y.
*Nov. 24, 1933	PICKETT, SAMUEL C., Rating Supervisor, Insurance Department, State of Connecticut, Hartford 2, Conn.
*Nov. 17, 1922	PINNEY, SYDNEY D., 290 Wolcott Hill Road, Wethersfield 9, Conn.
*Nov. 13, 1931	PRUITT, DUDLEY M., Actuary, General Accident Fire & Life Assurance Corp., Fourth & Walnut Sts., Philadelphia 5, Pa.
*Nov. 18, 1949	RESONY, JOHN A., Casualty Rate Analyst, Connecticut Insurance Department, State Office Building, Hartford, Conn.
May 23, 1919	RICHARDSON, FREDERICK, Coombe, Bradford Abbas, Sherborne, Dorset, England.
*Nov. 19, 1926	RICHTER, OTTO C., Chief Statistician, American Telephone & Telegraph Co., 195 Broadway, New York 7, N. Y.
May 24, 1921	RIEGEL, ROBERT, Professor of Statistics and Insurance, University of Buffalo, Buffalo 14, New York.
*Nov. 14, 1947	RODERMUND, MATTHEW, Assistant Secretary, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
*Nov. 16, 1923	ROEBER, WILLIAM F., General Manager, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 14, 1947	ROSENBERG, NORMAN, Supervising Rate Analyst, California Insurance Department, 621 South Hope St., Los Angeles, Cal.

FELLOWS

Admitted	
*Nov. 17, 1943	ROSS, SAMUEL M., Assistant Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 14, 1947	ROWELL, JOHN H., Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
*Nov. 14, 1947	SALZMANN, RUTH E., Assistant Actuary, Hardware Mutual Casualty Company, Hardware Dealers Mutual Fire Insurance Co., 200 Strongs Ave., Stevens Point, Wis.
*Nov. 20, 1942	SATTERTHWAITE, FRANKLIN E., Quality Control Engineer, Product Service Division, General Electric Company, 1285 Boston Ave., Bridgeport 2, Conn.
*Nov. 19, 1948	SCHLOSS, HAROLD W., Superintendent, Actuarial Department, Royal-Liverpool Group, 150 William St., New York 8, N. Y.
*Nov. 18, 1937	SHAPIRO, GEORGE I., 934 E. 9th Street, Brooklyn 30, N. Y.
*Nov. 13, 1931	SILVERMAN, DAVID, Partner, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 24, 1933	SINNOTT, ROBERT V., Assistant Secretary, Hartford Accident and Indemnity Company, 690 Asylum Ave., Hartford 15, Conn.
*Nov. 19, 1929	SKELDING, ALBERT Z., Actuary, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 19, 1929	SKILLINGS, E. SHAW, Actuary, Allstate Insurance Co., 20 North Wacker Drive, Chicago 6, Ill.
*Nov. 18, 1932	SMICK, JACK J., Consulting Actuary, 65 Cortlandt Street, New Rochelle, N. Y.
*Nov. 15, 1940	SMITH, SEYMOUR E., Assistant Secretary, Casualty Department, The Travelers Insurance Co., Hartford 15, Conn.
*Nov. 24, 1933	ST. JOHN, JOHN B., Consulting Actuary, Box 57, Penllyn, Pa.
Nov. 18, 1927	STONE, EDWARD C., Chairman of the Board, American Employers' Insurance Company, 33 Broad St., Boston 9, Mass.
*Nov. 17, 1920	TARBELL, THOMAS F., Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
†	THOMPSON, JOHN S., President, The Mutual Benefit Life Insurance Co., 300 Broadway, Newark 4, N. J.
†	TRAIN, JOHN L., President, Utica Mutual Insurance Co., 185 Genesee Street, Utica 2, New York
Nov. 17, 1922	TRAVERSI, ANTONIO T., 9 Balfour Street, Wollstonecraft, Sydney, Australia.
*Nov. 19, 1948	TURNER, PAUL A., Assistant Chief Actuary, Joseph Froggatt & Co., Inc., 74 Trinity Place, New York 6, N. Y.
*Nov. 14, 1947	UHTHOFF, D. R., Assistant Actuary, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 23, 1928	VALERIUS, NELS M., Assistant Actuary, Aetna Casualty and Surety Co., Hartford 15, Conn.

FELLOWS

Admitted	
*Nov. 21, 1919	VAN TUYL, HIRAM O., Superintendent, Internal Audit Department, London Guarantee & Accident Co., 55 Fifth Avenue, New York 3, N. Y.
*Nov. 17, 1920	WAITE, ALAN W., Secretary, The Aetna Casualty and Surety Co., 151 Farmington Ave., Hartford 15, Conn.
*Nov. 15, 1935	WAITE, HARRY V., (Retired), 938 Ridge Road, Wethersfield 9, Conn.
*Nov. 14, 1947	WIEDER, JOHN W., JR., Aetna Casualty and Surety Company, Hartford 15, Conn.
*Nov. 15, 1935	WILLIAMS, HARRY V., Assistant Secretary, Hartford Accident and Indemnity Co., 690 Asylum Ave., Hartford 15, Conn.
Nov. 14, 1941	WILLIAMSON, W., RULON, Senior Actuarial Consultant, The Wyatt Company, 3400 Fairhill Drive, Washington 20, D. C.
*Nov. 13, 1931	WITTICK, HERBERT E., Secretary, Pilot Insurance Co., 199 Bay Street, Toronto 1, Canada.
*Nov. 18, 1949	WOLFRUM, RICHARD J., Assistant Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
May 24, 1921	WOOD, ARTHUR B., President, Sun Life Assurance Company of Canada, Montreal, Canada.

ASSOCIATES

Those marked (*) have been admitted as Associates upon examination by the Society.

Admitted	
May 23, 1924	ACKER, MILTON, Manager, General Liability Division, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 15, 1918	ACKERMAN, SAUL B., Professor of Insurance, New York University, Washington Square, New York 6, N. Y.
*Nov. 16, 1939	AIN, SAMUEL N, Consulting Actuary, 120 Broadway, New York 5, N. Y.
Apr. 5, 1928	ALLEN, AUSTIN F., President, Texas Employers' Insurance Association, 530 Interurban Building, P. O. Box 2759, Dallas 1, Texas.
Nov. 15, 1918	ANKERS, R. E., Vice-President and Treasurer, Continental Life Insurance Co., Inc., Investment Building, 15 and K Sts., N.W., Washington 5, D. C.
*Nov. 21, 1930	ARCHIBALD, A. EDWARD, Vice-President and Actuary, Volunteer State Life Insurance Company, Chattanooga 1, Tenn.
*Nov. 24, 1933	BARRON, JAMES C., Asst. Treasurer, General Reinsurance Corporation, 90 John Street, New York 7, N. Y.
*Nov. 23, 1928	BATEMAN, ARTHUR E., c/o Arthur Q. Melendy, Southboro, Mass.
*Nov. 15, 1940	BATHO, BRUCE, Associate Actuary, Life Insurance Company of Georgia, 573 W. Peachtree St., N.E., Atlanta 1, Georgia.
*Nov. 18, 1925	BITTEL, W. HAROLD, Chief Actuary, Department of Banking and Insurance, Trenton 7, New Jersey.
Nov. 17, 1920	BLACK, NELLAS C., Manager, Statistical Department, Maryland Casualty Co., Baltimore 3, Md.
*Nov. 15, 1940	BLACKHALL, JOHN M., California-Western States Life Insurance Company, 10th & J Sts., Sacramento, Calif.
*Nov. 22, 1934	BOMSE, EDWARD L., Supt. New York Met. Special Risks, Royal Indemnity Co., 150 William Street, New York 8, N. Y.
*Nov. 23, 1928	BOWER, P. S., Assistant General Manager and Treasurer, The Great-West Life Assurance Company, Winnipeg, Manitoba, Canada.
*Nov. 15, 1918	BRUNNQUELL, HELMUTH G., (Retired), 1013 East Circle Drive, Milwaukee 2, Wis.
*Oct. 22, 1915	BUFFLER, LOUIS, Director, Underwriting Department, State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.
*Nov. 20, 1924	BUGBEE, J. M., Manager, Automobile Department, Maryland Casualty Co., Box 1228, Baltimore 3, Md.
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.
Nov. 17, 1922	CAVANAUGH, L. D., President, Federal Life Insurance Co., 168 N. Michigan Avenue, Chicago 1, Ill.

ASSOCIATES

Admitted	
*Nov. 18, 1927	CHEN, S. T., Actuary, China United Assurance Society, 104 Bubbling Well Road, Shanghai, China.
*Nov. 24, 1933	CRAWFORD, W. H., Secretary, Fireman's Insurance Co. of Newark, N. J. & Affiliated Fire & Casualty Co.'s Pacific Dept., 220 Bush Street, San Francisco 6, Cal.
*Nov. 18, 1932	CRIMMINS, JOSEPH B., Assistant Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 18, 1925	DAVIS, MALVIN E., Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 24, 1933	DAVIS, REGINALD S., Assistant Commissioner, Division of Real Estate, State of California, 1020 N Street, Sacramento, Calif.
*Nov. 14, 1941	DOWLING, WILLIAM F., Asst. Treasurer, Lumber Mutual Casualty Co., 260 Fourth Avenue, New York 10, N. Y.
June 5, 1925	EGER, FRANK A., Secretary-Comptroller, Indemnity Insurance Co. of North America, 1600 Arch Street, Philadelphia 1, Pa.
*Nov. 16, 1923	FITZ, L. LEROY, Group Department, John Hancock Mutual Life Insurance Company, Boston 17, Mass.
*Nov. 16, 1923	FLEMING, FRANK A., General Manager, Mutual Casualty Insurance Rating Bureau, 60 East 42nd Street, New York 17, N. Y.
*Nov. 13, 1936	FRUECHTEMEYER, FRED J., Assistant to Comptroller, The Andrew Jergens Company, 2535 Spring Grove Ave., Cincinnati 14, Ohio.
*Nov. 19, 1929	FURNIVALL, MAURICE L., Assistant Actuary, Accident Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	GEORGE, HAROLD J., Assistant Actuary, National Life Insurance Co., 131 State St., Montpelier, Vt .
*Nov. 18, 1932	GETMAN, RICHARD A., The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 17, 1922	GIBSON, JOSEPH P., JR., Manager, Casualty Department, American Foreign Insurance Association, 80 Maiden Lane, New York 7, N. Y.
*Nov. 16, 1923	GILDEA, JAMES F., The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	GINGERY, STANLEY W., Assistant Actuary, The Prudential Insurance Co., Newark, N. J.
*Nov. 18, 1927	GREEN, WALTER C., Consulting Actuary, Continental Bank Building, Salt Lake City, Utah.
*Nov. 15, 1940	GROSSMAN, ELI A., Mathematician, United States Life Insurance Co., 84 William St., New York 7, N. Y.
*Nov. 15, 1935	GUERTIN, ALFRED N., Actuary, American Life Convention, 230 N. Michigan Ave., Chicago 1, Ill.
*Nov. 16, 1939	HAGEN, OLAF E., Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.

ASSOCIATES

Admitted	
*Nov. 18, 1921	HAGGARD, ROBERT E., Supervisor, Permanent Disability Rating Bureau, Industrial Accident Commission, 965 Mission Street, San Francisco 3, Calif.
*Nov. 17, 1922	HALL, HARTWELL L., Associate Actuary, Connecticut Insurance Department, 165 Capitol Ave., Hartford 2, Conn.
*Nov. 13, 1936	HAM, HUGH P., Assistant General Manager, The British American Assurance Company, 22 Wellington St. East, Toronto 1, Canada.
Mar. 24, 1932	HARRIS, SCOTT, Executive Vice-President, Joseph Froggatt & Co., Inc., 74 Trinity Place, New York 6, N. Y.
*Mar. 25, 1924	HART, WARD VAN B., Associate Actuary, Connecticut General Life Insurance Co., 55 Elm St., Hartford 15, Conn.
*Nov. 18, 1949	HARWAYNE, FRANK, National Bureau of Casualty Underwriters, 60 John St., New York 7, N. Y.
Nov. 21, 1919	HAYDON, GEORGE F., General Manager, Wisconsin Compensation Rating & Inspection Bureau, 715 N. Van Buren St., Milwaukee 2, Wis.
*Nov. 18, 1949	HAZAM, WILLIAM J., American Mutual Liability Insurance Co., 142 Berkeley St., Boston 16, Mass.
*Nov. 19, 1948	HEWITT, CHARLES C., JR., New Jersey Manufacturers Casualty Insurance Co., 363 W. State St., Trenton, N. J.
Nov. 17, 1927	HIPP, GRADY H., Executive Vice-President, Liberty Life Insurance Co., Greenville, S. C.
*Nov. 16, 1945	HOLZINGER, ERNEST, Actuary, Pension Planning Company, 30 Broad St., New York 4, N. Y.
*Nov. 18, 1949	HOPE, FRANCIS J., Rating and Research Department, Hartford Accident and Indemnity Co., 690 Asylum Avenue, Hartford 15, Conn.
Nov. 19, 1929	JACOBS, CARL N., President, Hardware Mutual Casualty Co., 200 Strongs Ave., Stevens Point, Wis.
*Nov. 18, 1921	JENSEN, EDWARD S., Assistant Secretary, Group Department, Occidental Life Insurance Co. of California, 1151 So. Broadway, Los Angeles 55, Calif.
Nov. 21, 1930	JONES, H. LLOYD, Executive Vice-President and Deputy General Attorney, Phoenix-London Group, 55 Fifth Avenue, New York 3, N. Y.
*Nov. 21, 1919	JONES, LORING D., (Retired), 64 Raymond Ave., Rockville Centre, Long Island, N. Y.
*Nov. 15, 1940	KELLY, ROBERT G., Mathematician, Allied Physics Laboratory, Johns Hopkins University, 8621 Georgia Ave., Silver Spring, Md.
*Nov. 17, 1922	KIRK, CARL L., Deputy Manager, Zurich General Accident & Liability Insurance Co., 135 South LaSalle Street, Chicago 3, Ill.
*Nov. 15, 1935	KITZROW, E. W., Vice-President, Underwriting and Auditing, Hardware Mutual Casualty Co., 200 Strongs Ave., Stevens Point, Wis.

ASSOCIATES

Admitted	
*Nov. 18, 1949	LESLIE, WILLIAM, JR., Superintendent, Special Risks Department, Eagle-Globe-Royal Indemnity Companies, 150 William St., New York 8, N. Y.
*Nov. 19, 1948	LIVINGSTON, GILBERT, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 14, 1947	LUFKIN, ROBERT W., Statistician, Liberty Mutual Insurance Co., 175 Berkeley Street, Boston 17, Mass.
*Nov. 13, 1931	MACKEEN, HAROLD E., Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
Mar. 24, 1932	MAGRATH, JOSEPH J., Administrative Assistant, Chubb & Son, 90 John Street, New York 7, N. Y.
*Nov. 18, 1925	MALMUTH, JACOB, Examiner, New York State Insurance Department, 61 Broadway, New York 6, N. Y.
Mar. 24, 1927	MARSH, CHARLES V. R., (Retired), 617 E. Surf Road, Ocean City, N. J.
*Nov. 13, 1936	MAYER, WILLIAM H., JR., Group Contact Referee, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 17, 1922	MCIVER, R. A., Actuary, Washington National Insurance Co., 610 Church Street, Evanston, Ill.
*Nov. 13, 1931	MILLER, HENRY C., Comptroller-Actuary, California State Compensation Insurance Fund, 450 McAllister Street, San Francisco 2, Calif.
*Nov. 19, 1926	MILNE, JOHN L., Actuary, Philadelphia Life Insurance Company, 111 North Broad Street, Philadelphia 7, Pa.
Nov. 17, 1922	MONTGOMERY, JOHN C., Secretary and Treasurer, Bankers Indemnity Insurance Co., 15 Washington Street, Newark 2, N. J.
May 25, 1923	MOORE, JOSEPH P., Mutual Life and Citizens Assurance Co., Ltd., P.O. Box 1770, Place D'arms, Montreal, Canada.
*Nov. 14, 1947	MUNTERICH, GEORGE C., Lumber Mutual Casualty Insurance Company of New York, 260 Fourth Ave., New York 10, N. Y.
*Nov. 18, 1937	MYERS, ROBERT J., Chief Actuary, Social Security Administration, Washington 25, D. C.
*Nov. 15, 1935	NELSON, S. TYLER, Casualty Actuary, Department of Insurance, State Capitol Building, Springfield, Ill.
*Oct. 27, 1916	NEWELL, WILLIAM, (Retired), 1225 Park Avenue, New York 28, N. Y.
*Nov. 18, 1925	NICHOLSON, EARL, Actuary, Joseph Froggatt & Co., Inc., 74 Trinity Place, New York 6, N. Y.
May 23, 1919	OTTO, WALTER E., President, Michigan Mutual Liability Co., Associated General Fire Co., 163 Madison Avenue, Detroit 26, Mich.
*Nov. 19, 1926	OVERHOLSER, DONALD M., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.
Nov. 20, 1924	PENNOCK, RICHARD M., Actuary, Pennsylvania Manufacturers' Association Casualty Insurance Co., Finance Building, Philadelphia 2, Pa.

ASSOCIATES

Admitted	
*Nov. 14, 1947	PERRY, ROBERT C., Vice-President and Actuary, State Farm Life Insurance Company, Bloomington, Ill.
Nov. 19, 1929	PHILLIPS, JOHN H., Vice-President and Actuary, Employers' Mutual Liability Insurance Co., 407 Grant St., Wausau, Wis.
*Nov. 17, 1920	PIKE, MORRIS, Associate Actuary, John Hancock Mutual Life Insurance Co., Boston 17, Mass.
*Nov. 23, 1928	PIPER, K. B., Vice-President, Provident Life and Accident Insurance Co., 721 Broad St., Chattanooga 2, Tenn.
*Nov. 17, 1922	POORMAN, WILLIAM F., President, Central Life Assurance Society (Mutual), Fifth and Grand Avenues, Des Moines 6, Iowa.
*Nov. 13, 1936	POTOFSKY, SYLVIA, Senior Actuary, The State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.
*Nov. 15, 1918	RAYWID, JOSEPH, President, Joseph Raywid & Co., Inc., 92 William Street, New York 7, N. Y.
Nov. 19, 1932	RICHARDSON, HARRY F., Secretary-Treasurer, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 18, 1932	ROBERTS, JAMES A., Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 18, 1927	SARASON, HARRY M., Statistician, Occidental Life Insurance Company of California, Box 2101, Terminal Annex, Los Angeles 54, Calif.
Nov. 16, 1923	SAWYER, ARTHUR, Royal Indemnity Co., 150 William Street, New York 8, N. Y.
*Nov. 14, 1947	SCAMMON, LAWRENCE W., Actuary, Massachusetts Automobile Rating and Accident Prevention Bureau, Massachusetts Workmen's Compensation Rating and Inspection Bureau, 89 Broad Street, Boston 10, Mass.
*Nov. 14, 1947	SCHWARTZ, MAX J., Examiner, New York State Insurance Department, Albany 1, N. Y.
*Nov. 20, 1930	SEVILLA, EXEQUIEL S., Manager and Actuary, National Life Insurance Co. of the Philippines, Regina Building, P.O. Box 2056, Manila, Philippines.
*Nov. 20, 1924	SHEPPARD, NORRIS E., Professor of Mathematics, University of Toronto, Toronto 5, Canada.
Nov. 15, 1918	SIBLEY, JOHN L., (Retired), 225 Amesburg Road, Haverhill, Mass. c/o Eielson.
*Nov. 18, 1921	SMITH, ARTHUR G., Assistant General Manager, Compensation Insurance Rating Board, Pershing Square Bldg., 125 Park Avenue, New York 17, N. Y.
*Nov. 19, 1926	SOMERVILLE, WILLIAM F., Secretary, St. Paul Mercury Indemnity Co., St. Paul 2, Minn.
*Nov. 18, 1925	SOMMER, ARMAND, Supt. of Agencies, Continental Casualty Co., 910 So. Michigan Avenue, Chicago 5, Ill.
*Nov. 15, 1918	SPENCER, HAROLD S., Statistician, Aetna Casualty and Surety Co., 151 Farmington Ave., Hartford 15, Conn.

ASSOCIATES

Admitted	
Nov. 20, 1924	STELLWAGEN, H. P., Executive Vice-President, Indemnity Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.
*Nov. 16, 1923	STOKE, KENDRICK, Actuary, Michigan Mutual Liability Company, 163 Madison Avenue, Detroit 26, Mich.
*Nov. 21, 1930	SULLIVAN, WALTER F., Assistant Actuary, State Compensation Insurance Fund, 450 McAllister Street, San Francisco 1, Calif.
*Nov. 21, 1919	TRENCH, FREDERICK H., Manager, Underwriting Department, Utica Mutual Insurance Co., 185 Genesee Street, Utica 1, N. Y.
*Nov. 20, 1924	UHL, M. ELIZABETH, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 14, 1947	VERGANO, ELIA, Assistant Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York 17, N. Y.
May 23, 1919	WARREN, CHARLES S., Secretary, Massachusetts Automobile Rating and Accident Prevention Bureau, 89 Broad Street, Boston 10, Mass.
*Nov. 18, 1932	WEINSTEIN, MAX S., Actuary, New York State Employees' Retirement System, 256 Washington Ave., Albany 1, N. Y.
*Nov. 18, 1925	WELLMAN, ALEXANDER C., Vice-President, Protective Life Insurance Co., Birmingham, Ala.
*Nov. 21, 1930	WELLS, WALTER I., Assistant Actuary, State Mutual Life Assurance Co., 340 Main St., Worcester 8, Mass.
Mar. 21, 1929	WHEELER, CHARLES A., Chief Examiner of Casualty Companies, New York State Insurance Department, 61 Broadway, New York 6, N. Y.
*Nov. 18, 1927	WHITBREAD, F. G., Vice-President, Reliance Life Insurance Company, Room 412, Farmers Bank Building, Pittsburgh 22, Pa.
*Nov. 19, 1948	WHITE, AUBREY, Assistant Actuary, Paul Revere Life Insurance Co., 18 Chestnut St., Worcester, Mass.
*Nov. 16, 1939	WITTLAKE, J. CLARKE, Assistant to President, Business Men's Assurance Company, B.M.A. Building, Kansas City 10, Mo.
*Oct. 22, 1915	WOOD, DONALD M., Partner, Childs & Wood, 175 W. Jackson Blvd., Chicago 4, Ill.
*Nov. 18, 1937	WOOD, DONALD M., JR., Childs & Wood, 175 West Jackson Blvd., Chicago 4, Ill.
*Nov. 18, 1927	WOOD, MILTON J., Associate Actuary, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Oct. 22, 1915	WOODMAN, CHARLES E., (Retired), The Brunswick, Waterville, N. Y.
*Nov. 22, 1934	WOODWARD, BARBARA H., Associate Lawyer, Hughes, Hubbard & Ewing, 1 Wall Street, New York 5, N. Y.
*Nov. 18, 1925	WOOLERY, JAMES MYRON, Vice-President and Actuary, Occidental Life Insurance Company, Raleigh, North Carolina.

ASSOCIATES

SCHEDULE OF MEMBERSHIP, NOVEMBER 18, 1949

	Fellows	Associates	Total
Membership, November 19, 1948	153	124	277
Additions:			
By election	—	—	—
By reinstatement		—	
By examination	4	4	8
	157	128	285
Deductions:			
By death	5	2	7
By withdrawal	1	4	5
By transfer from Associate to Fellow	—	3	3
Membership, November 18, 1949	151	119	270

OFFICERS OF THE SOCIETY

Since Date of Organization

Elected	*President*	*Vice-Presidents*	
1914-1915	*Isaac M. Rubinow	*Albert H. Mowbray	*Benedict D. Flynn
1916-1917	*James D. Craig	*Joseph H. Woodward	*Harwood E. Ryan
1918	*Joseph H. Woodward	*Benedict D. Flynn	George D. Moore
1919	*Benedict D. Flynn	George D. Moore	William Leslie
1920	*Albert H. Mowbray	William Leslie	*Leon S. Senior
1921	*Albert H. Mowbray	*Leon S. Senior	*Howard E. Ryan
1922	*Harwood E. Ryan	Gustav F. Michelbacher	Edmund E. Cammack
1923	William Leslie	Gustav F. Michelbacher	Edmund E. Cammack
1924-1925	Gustav F. Michelbacher	*Sanford B. Perkins	Ralph H. Blanchard
1926-1927	*Sanford B. Perkins	George D. Moore	Thomas F. Tarbell
1928-1929	George D. Moore	Sydney D. Pinney	Paul Dorweiler
1930-1931	Thomas F. Tarbell	*Roy A. Wheeler	Winfield W. Greene
1932-1933	Paul Dorweiler	William F. Roeber	*Leon S. Senior
1934-1935	Winfield W. Greene	Ralph H. Blanchard	Charles J. Haugh
1936-1937	*Leon S. Senior	Sydney D. Pinney	Francis S. Perryman
1938-1939	Francis S. Perryman	Harmon T. Barber	William J. Constable
1940	Sydney D. Pinney	Harold J. Ginsburgh	James M. Cahill
1941	Ralph H. Blanchard	Harold J. Ginsburgh	James M. Cahill
1942	Ralph H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	Harold J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945-1946	Charles J. Haugh	James M. Cahill	Harry V. Williams
1947-1948	James M. Cahill	Harmon T. Barber	Russell P. Goddard
1949	Harmon T. Barber	Thomas O. Carlson	Norton E. Masterson

Secretary-Treasurer

1914-1917....*C. E. Scattergood
1918-1949.....R. Fondiller

Editor†

1914.....W. W. Greene
1915-1917.....R. Fondiller
1918.....W. W. Greene
1919-1921....G. F. Michelbacher
1922-1923.....O. E. Outwater
1924-1932.....R. J. McManus
1933-1943.....*C. W. Hobbs
1944-1949.....E. C. Maycrink

Librarian†

1914.....W. W. Greene
1915.....R. Fondiller
1916-1921.....L. I. Dublin
1922-1924.....E. R. Hardy
1925-1937.....W. Breiby
1937-1947.....T. O. Carlson
1948-1949.....S. M. Ross

Chairman – Examination Comm.

1949.....Roger A. Johnson

*Deceased.

†The offices of Editor and Librarian were not separated until 1916.

FELLOWS WHO HAVE DIED

The (†) denotes charter members at date of organization, November 7, 1914.

Admitted		Died
†	Roland Benjamin	July 2, 1949
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
†	Raymond V. Carpenter	Mar. 11, 1947
Feb. 19, 1915	Gorden Case	Feb. 4, 1920
†	Charles T. Conway	July 23, 1921
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
†	Elmer H. Dearth	Mar. 26, 1947
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	David Parks Fackler	Oct. 30, 1924
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Harry Furze	Dec. 26, 1945
Feb. 19, 1915	Fred S. Garrison	Nov. 14, 1949
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
Oct. 22, 1915	George Graham	Apr. 15, 1937
Oct. 22, 1915	Thompson B. Graham	July 24, 1946
May 25, 1923	William A. Granville	Feb. 4, 1943
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Chares E. Hodges	Jan. 22, 1937
†	Frederick L. Hoffman	Feb. 23, 1946
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
†	Charles Hughes	Aug. 27, 1948
Nov. 19, 1929	Robert S. Hull	Nov. 30, 1947
†	Burritt A. Hunt	Sept. 3, 1943
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928

FELLOWS WHO HAVE DIED—*Continued*

Admitted		*Died*
Nov. 19, 1926	Gregory Cook Kelly	Sept. 11, 1948
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Feb. 17, 1915	John M. Laird	June 20, 1942
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
Nov. 16, 1923	D. Ralph McClurg	Apr. 27, 1947
May 23, 1919	Alfred McDougald	July 28, 1944
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	James F. Mitchell	Feb. 9, 1941
†	Henry Moir	June 8, 1937
Nov. 19, 1926	William L. Mooney	Oct. 21, 1948
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Albert H. Mowbray	Jan. 7, 1949
†	Lewis A. Nicholas	Apr. 21, 1940
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
†	Edward B. Phelps	July 24, 1915
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Emil Scheitlin	May 2, 1946
†	Leon S. Senior	Feb. 3, 1940
Apri. 20, 1917	Charles Gordon Smith	June 22, 1938
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Menville Strong	Mar. 30, 1942
Oct. 22, 1915	William R. Strong	Jan. 10, 1946
†	Robert J. Sullivan	July 19, 1934
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
Nov. 18, 1925	Lloyd A. H. Warren	Sept. 30, 1949
May 23, 1919	Archibald A. Welch	May 8, 1945
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	Lee J. Wolfe	Apr. 28, 1949
†	S. Herbert Wolfe	Dec. 31, 1927
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

ASSOCIATES WHO HAVE DIED

Admitted		Died
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
May 25, 1923	Harilaus E. Economidy	Apr. 13, 1948
Nov. 20, 1924	John Froberg	Oct. 11, 1949
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 19, 1929	Harold R. Gordon	July 8, 1948
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 21, 1919	Rolland V. Mothersill	July 25, 1949
Nov. 19, 1929	Fritz Muller	Apr. 27, 1945
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1945
Nov. 18, 1925	James H. Washburn	Aug. 19, 1946
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930

CONSTITUTION

(AS AMENDED NOVEMBER 18, 1949)

ARTICLE I.—*Name.*

This organization shall be called the CASUALTY ACTUARIAL SOCIETY.

ARTICLE II.—*Object.*

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to casualty or social insurance.

ARTICLE III.—*Membership.*

The membership of the Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to office or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any person may, upon nomination to the Council by two Fellows of the Society and approval by the Council of such nomination with not more than one negative vote, become enrolled as an Associate of the Society, provided that he shall pass such examination as the Council may prescribe. Such examination may be waived in the case of a candidate who for a period of not less than two years has been in responsible charge of the Statistical or Actuarial Department of a casualty insurance organization or has had such other practical experience in casualty or social insurance as, in the opinion of the Council renders him qualified for Associateship.

Any person who shall have qualified for Associateship may become a Fellow on passing such final examination as the Council may prescribe. Otherwise, no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Council with not more than three negative votes, followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

ARTICLE IV.—*Officers and Council.*

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, an Editor, a Librarian, and a General Chairman of the Examination Committee. The Council shall be composed of the active officers, nine other Fellows and, during the four years following

the expiration of their terms of office, the ex-Presidents and ex-Vice-Presidents. The Council shall fill vacancies occasioned by death or resignation of any officer or other member of the Council, such appointees to serve until the next annual meeting of the Society.

ARTICLE *V.—Election of Officers and Council.*

The President, Vice-Presidents, and the Secretary-Treasurer shall be elected by a majority ballot at the annual meeting for the term of one year and three members of the Council shall, in a similar manner, be annually elected to serve for three years. The President and Vice-Presidents shall not be eligible for the same office for more than two consecutive years nor shall any retiring member of the Council be eligible for re-election at the same meeting.

The Editor, the Librarian and the General Chairman of the Examination Committee shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by majority ballot of the Society at the annual meeting.

The terms of the officers shall begin at the close of the meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of *Proceedings.*

ARTICLE VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices or may be specified in the by-laws. The duties of the Council shall be to pass upon candidates for membership, to decide upon papers offered for reading at the meetings, to supervise the examination of candidates and prescribe fees therefor, to call meetings, and in general, through the appointment of committees and otherwise, to manage the affairs of the Society.

ARTICLE VII.—*Meetings.*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of ten Fellows. At least two weeks notice of all meetings shall be given by the Secretary.

ARTICLE VIII.—*Quorum.*

Seven members of the Council shall constitute a quorum. Twenty Fellows of the Society shall constitute a quorum.

ARTICLE IX.—*Expulsion or Suspension of Members.*

Except for non-payment of dues no member of the Society shall be expelled or suspended save upon action by the Council with not more than three negative votes followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

ARTICLE X.—*Amendments.*

This constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have been sent to each Fellow by the Secretary.

BY-LAWS

(AS AMENDED NOVEMBER 18, 1949)

ARTICLE I.—*Order of Business.*

At a meeting of the Society the following order of business shall be observed unless the Society votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Report by the Council on business transacted by it since the last meeting of the Society.
5. New Membership.
6. Reports of officers and committees.
7. Election of officers and Council (at annual meetings only).
8. Unfinished business.
9. New business.
10. Reading of papers.
11. Discussion of papers.

ARTICLE II.—*Council Meetings.*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

ARTICLE III.—*Duties of Officers.*

The President, or, in his absence, one of the Vice-Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, send out calls for the said meetings, and, with the approval of the President and Council, carry on the correspondence of the Society. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee of the Council.

The Editor shall, under the general supervison of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings, original papers or reviews written by members, discussions on said papers and other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts and other literary or scientific material collected by the Society.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

ARTICLE IV.—*Dues.*

The Council shall fix the annual dues for Fellows and for Associates. The payment of dues will be waived in the case of Fellows or Associates who have attained the age of seventy years or who, having been members for a period of at least twenty years, shall have attained the age of sixty-five years. Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by vote of the Council, and upon payment of arrears of dues.

ARTICLE V.—*Designation by Initials.*

Fellows of the Society are authorized to append to their names the initials F.C.A.S.; and Associates are authorized to append to their names the initials A.C.A.S.

ARTICLE VI.—*Amendments.*

These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.

SYLLABUS OF EXAMINATIONS

(Effective 1948 and Thereafter)

ASSOCIATESHIP

Part	*Section*	*Subject*
I	1	Descriptive and Analytical Statistics.
	2	Compound Interest and Annuities Certain.
II	3	Differential and Integral Calculus.
	4	Calculus of Finite Differences.
III	5	Probabilities.
	6	Life Contingencies, Life Annuities and Life Assurances.
IV	7	Policy Forms and Underwriting Practice in Casualty Insurance.
	8	Casualty Insurance Rate Making Methods.

FELLOWSHIP

I	9	Insurance Economics.
	10	Insurance Law and Regulation.
II	11	Individual Risk Rating.
	12	Social Insurance.
III	13	Determination of Premium, Loss and Expense Reserves.
	14	Advanced Problems in Casualty Insurance Statistics.
IV	15	Advanced Problems in Casualty Insurance Accounting.
	16	Advanced Problems in the Underwriting and Administration of Casualty Insurance.

RULES REGARDING EXAMINATIONS FOR ADMISSION TO THE CASUALTY ACTUARIAL SOCIETY

1. Dates of Examination.

Examinations will be held on the second Tuesday and following Wednesday during the month of May in each year in such cities as will be convenient for three or more candidates.

2. Filing of Application.

Application for admission to examination should be made on the Society's blank form, which may be obtained from the Secretary-Treasurer. No applications will be considered unless received before the fifteenth day of February preceding the dates of examination. Applications should definitely state for what parts the candidate will appear.

3. Fees.

The examination fee is $3.00 for each part or portion thereof taken, subject to a minimum of $5.00 for each year in which the candidate presents himself; thus for one part, $5.00, for two parts, $6.00, etc. Examination fees are payable to the order of the Society and must be received by the Secretary-Treasurer before the fifteenth day of February preceding the dates of examination.

4. Associateship and Fellowship Examinations.

(a) The examination for Associateship consists of four parts and that for Fellowship consists of four parts. A candidate may take any one or more of the four parts of the Associateship Examination. A candidate may present himself for part of the Fellowship Examination either (a) if he has previously passed the Associateship Examination and all preceding parts of the Fellowship Examination, or (b) if he concurrently presents himself for and submits papers for all unpassed parts of the Associateship Examination and all preceding unpassed parts of the Fellowship Examination. Subject to the foregoing requirements, the candidate will be given credit for any part or parts of either examination which he may pass.

(b) A candidate who has passed the Associateship Examination Parts I-IV prior to 1941, but who has not been enrolled as an Associate because of lack of the experience qualifications required by the examination rules effective prior to 1941, will be enrolled as an Associate upon passing the current Associateship Examination Part IV.

(c) An Associate who has passed no part of the Fellowship Examination under the Syllabus effective prior to 1941 is required, in order to qualify for admission as a Fellow, to pass the current Associateship Examination Part IV and Fellowship Examination Parts I-IV.

(d) A candidate who has passed one or more parts of the Associateship or Fellowship Examinations under the Syllabus effective prior to 1948 will receive credit for the corresponding

parts of the new Syllabus in accordance with the following table:

Parts Passed Under Old Syllabus (Effective Prior to 1948)	*Parts Credited Under New Syllabus (Effective in 1948)*
Associateship, Part I	Associateship, Part I - Section 2
" " II	" " II
" " III	" " I - Section 1
" " IV	" " III
" " V	" " IV
Fellowship, Part I	Fellowship, Part I
" " II	" Parts III & IV - Section 15
" " III	" Parts II & IV - Section 16

Partial examinations will be given to those students requiring same in accordance with the foregoing credits.

5. Alternative to Passing of Fellowship Parts III and IV.

As an alternative to the passing of Parts III and IV of the Fellowship Examination, a candidate may elect to present an original thesis on an approved subject relating to casualty or social insurance. Such thesis must show evidence of ability for original research and the solution of advanced problems in casualty insurance comparable with that required to pass Parts III and IV of the Fellowship Examination, and shall not consist solely of data of an historical nature. Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Committee on Papers of the subject of the thesis and also of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature. All theses must be in the hands of the Secretary-Treasurer before the second Tuesday in May of the year in which they are to be considered. No examination fee will be required in connection with the presentation of a thesis. All theses submitted are, if accepted, to be the property of the Society and may, with the approval of the Council, be printed in the *Proceedings*.

6. Waiver of Examinations for Associate.

The examinations for Associate will be waived under Article III of the Constitution in part or in whole only in case of those candidates who meet the following qualifications and requirements:

1. PARTIAL WAIVER

In case of a candidate who, for a period of at least two years preceding date of application, has been in responsible charge of the actuarial or statistical department of a casualty insurance organization and who has passed examinations of other recognized Actuarial Societies at least equivalent to Parts I, II and III of the

Associateship examinations of this Society, the passing of such parts of the Associateship examinations of this Society will be waived upon approval of the Examination Committee.

An organization whose operations or functions are limited to Accident and Health insurance, or Life Accident and Health insurance, shall not qualify as a casualty insurance organization.

2. FULL WAIVER

(a) The candidate shall be at least thirty-five years of age.

(b) The candidate shall have at least ten years' experience in the casualty actuarial or statistical work or in a phase of casualty insurance which requires a working knowledge of actuarial or statistical procedure or in the teaching of casualty insurance principles in colleges or universities. Experience limited exclusively to the field of accident and health insurance shall not be admissible.

(c) For the two years preceding date of application, the candidate shall have been in responsible charge of the actuarial or statistical department of a casualty insurance organization or shall have occupied an executive position in connection with the phase of casualty work in which he is engaged, or, if engaged in teaching, shall have attained the status of a professor.

(d) The candidate shall have submitted a thesis approved by the Committee on Papers. Such thesis must show evidence of original research and knowledge of casualty insurance and shall not consist of data of an historical nature.

Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Committee on Papers of the subject of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature.

LIBRARY

The Society's library contains all of the references listed in the Recommendations for Study, including the books noted as being out of print with the exception of certain periodicals and publications subject to periodical revision. It also contains numerous other works on casualty actuarial matters. Registered students may have access to the library by receiving from the Society's Secretary-Treasurer the necessary credentials. Books may be withdrawn from the library for a period of two weeks upon payment of a small service fee and necessary postage.

The library is in the immediate charge of Miss Mabel B. Swerig, Librarian of the Insurance Society of New York, 107 William Street, New York 7, N. Y.