Inference of Probability Threshold Curves for Implicit Modeling with Uncertainty

Steve Mancell and Clayton Deutsch

Implicit modeling of boundaries is a popular industry technique utilized over a wide array of boundary types. The modeling of these boundaries is an integral early step to resource estimation and the subsequent decision making. Therefore, it is imperative to properly position boundaries and efficiently quantify the associated uncertainty. Uncertainty is inherent and ubiquitous to any boundary in earth sciences. The uncertainty is due to a lack of knowledge derived from data sparsity and unknown randomness in geological phenomena (Caers, 2011). The relationship between indicator estimated probabilities and appropriate thresholds for uncertainty is illustrated by a Probability-Threshold curve (PTC). The experimental curves from known truths are used to understand the uncertainty with varying geological scenarios. The proper uncertainty threshold value derives from extensive study of the curves and scenarios ultimately giving access to boundary uncertainty around a nearest neighbour thresholded base case boundary model.

Introduction

A boundary represents the extent of a domain whereby a transition occurs. These domains and boundaries can be geological in nature, as in folds and faults, or geostatistcal, as in domains for defining populations for common analysis or stationary domains (Wilde & Deutsch, 2011). Uncertainty in the location of boundaries should be quantified when modeling and most techniques are incapable, ineffective, simplistic and/or expensive in doing so. The correct boundary position is important in order to accurately and precisely estimate the location, tonnage, and uncertainty of a resource (Manchuk & Deutsch, 2019).

Indicator formulisms that are mapped out using kriging or radial basis functions (RBF) are another avenue that can be pursued in boundary modeling. The random field is thresholded to a proportion provided by conditioning data in concert with geological knowledge. This can be used to extract a boundary.

A workflow outlining Probability Threshold Curves (PTCs) show the relationship between thresholding values and probability for an indicator estimate. By finding the appropriate threshold, the extraction of an unbiased and fair boundary model is possible. Uncertainty assessment from further studies of the PTC relationship and corresponding acceptable low and high thresholds is an essential aspect of the final boundary model. These thresholds correspond to eroded and dilated boundary models, which give access to a zone of uncertainty. The only direct access to the true PTC is from experimental PTCs construction from synthetic data. By simulating multiple possible truths from the data configuration, sampling the truths, indicator estimating the synthetic samples, and comparing the volumes and characteristics of the estimates to the corresponding truths, an analysis of the relationship and the nature of the uncertainty follows. Through extensive construction of experimental PTCs over a wide array of model types, the study of patterns and characteristics of the PTCs is possible. The behaviour of the PTCs over thousands of models gives an understanding of the function that describes uncertainty. Moreover, a database of various models and their characteristics summarizes the simple shape of the function and its five parameters.

Experimental Workflow for Probability-Threshold Curves

The experimental workflow for producing PTCs begins with the creation of synthetic truths. By simulating 'Truth' realizations of boundaries from input data, sampling the realizations, and interpolating those samples, one can build a set of models. The comparison of the models and their corresponding truths over incremental threshold values allows for the determination of the true PTC. One can create the desired data configurations or source varying shapes from a library in order to build an array of true scenarios. The inspection of the PTC and associated thresholds allow for the assessment of boundary uncertainty for the final model.

The workflow begins with the simulation of 'truths' from a given data set. The use of the `GSLIB` executable `BLOCKSIS`, in conjunction with global proportions and conditioning data, allows for the creation of multiple true models. The local data may have constraining data points added in order to help alleviate the issues arising from short-scale variability in the `BLOCKSIS` algorithm. This variability produces unrealistic noise distal to conditioning data. The T=100 truths represent possible exhaustive realities given conditioning data and global proportions.

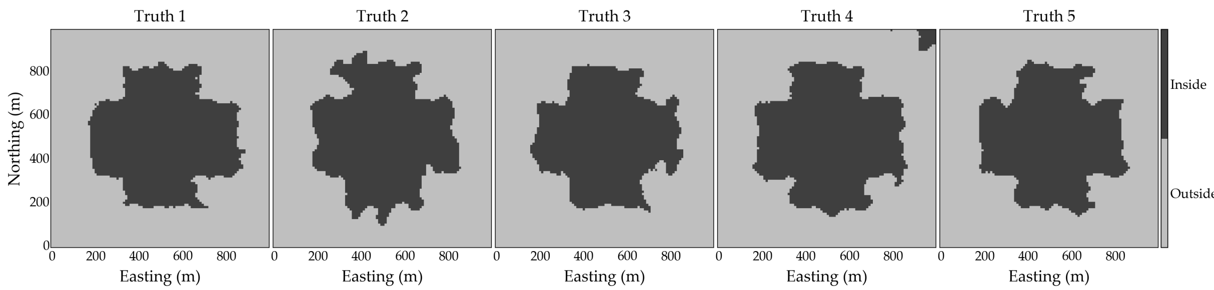


Figure 1: Simulated Truths from input data and global proportions using BLOCKSIS executable.

After the construction of the simulated truths, the GSAMPLE executable is run for gridded drilling at varying drill hole spacings (e.g. d= 10m, 25m, 50m, 100m, 150m, 200m, 250m). The total drilled data comprises 700 sets of data from the original T=100 truths. The omnidirectional variography is carried out uniquely to each set of synthetic drill data, and the indicator kriging estimation using KT3DN CCG software renders $i^*_{T,d}$=700 models.

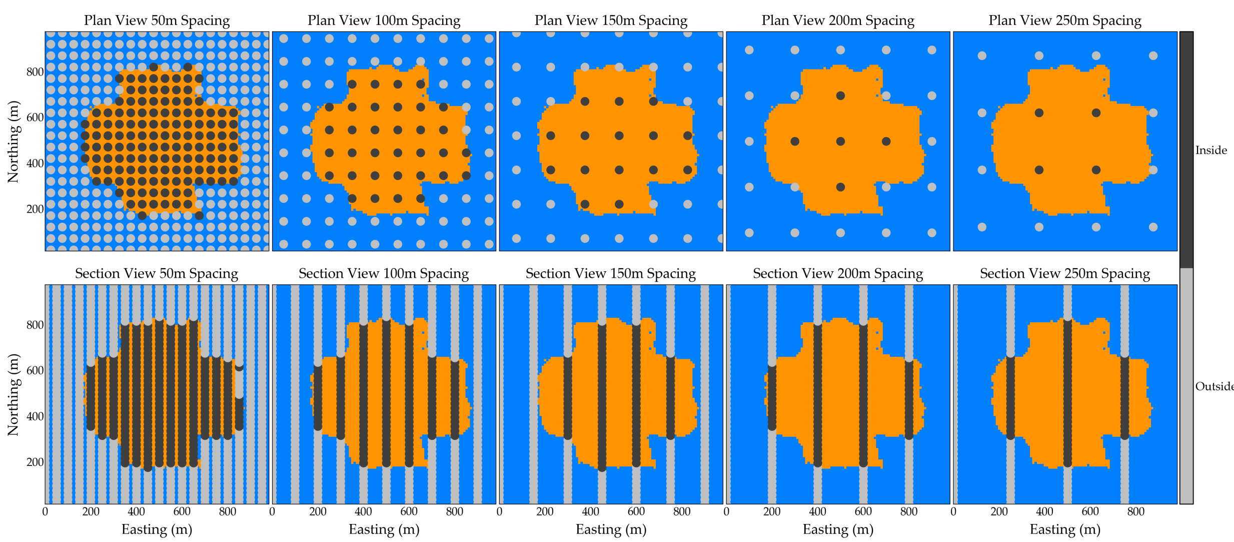


Figure 2: Truth Realization 1 with drill hole spacings 50, 100, 150, 200, 250m overlain in plan (*top*) & section (*bottom*). The 10m and 25m spacings are tight at resolution

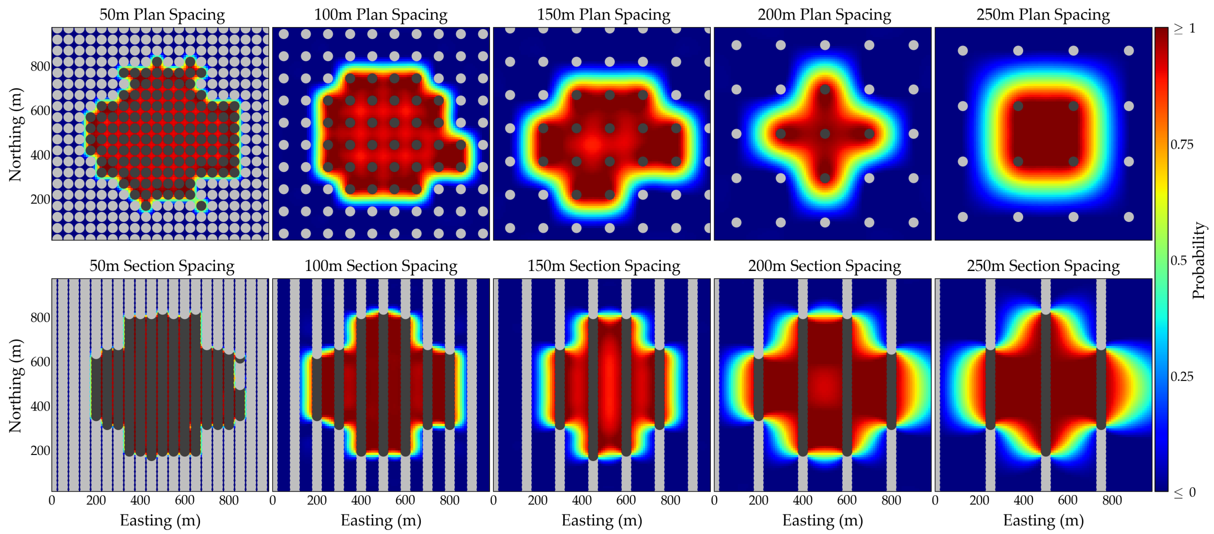


Figure 3: Truth 1 indicator estimates at respective drill hole spacings 50, 100, 150, 200, 250m in plan (*top*) & section (*bottom*). The 10m and 25m spacings are tight at resolution

A thresholding process of the $i^*_{T,d}$=700 indicator estimates at p05 increments from p0 to p100 ensues combining for a total of $i^{*,p}_{T,d}$=14,000 models. Comparison between the $i^{*,p}_{T,d}$=14,000 threshold model volumes and their respective *T*=100 truth realizations follows whereby if the threshold model at a particular drill hole spacing is larger in volume than the corresponding truth, it returns a value of 1; otherwise, a value of 0 is assigned.

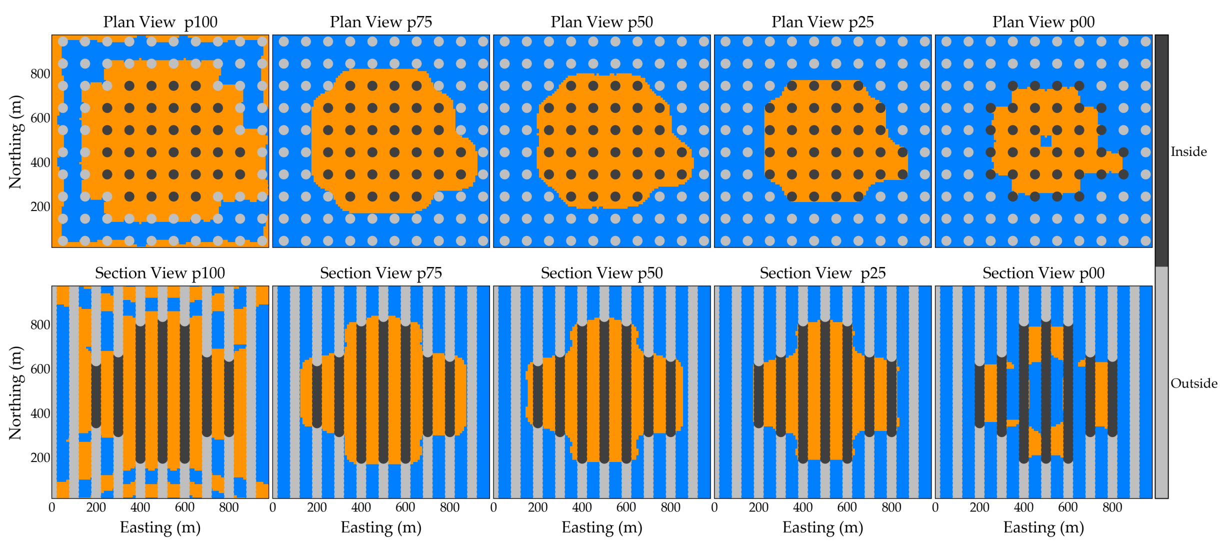


Figure 4: Indicator kriged models with 100m drillhole spacing and thresholds in plan (*top*) & section (*bottom*). The resulting models are compared to the corresponding Truth volumes

The cumulative number of models at specific thresholds that are larger than corresponding truths are summed and divided by the number of T=100 Truths. The final data input to the PTC curve is an extrapolation of the threshold model comparison data for specific drill hole spacings plot against the thresholding values. The value of 1 on the ordinate probability axis signifies that all of the threshold models are larger in volume than the truths at that particular spacing. Conversely, a value of 0 indicates that all the models were smaller than the corresponding truths. The nature of the curve between probabilities of 1 and 0 varies as it transitions and defines the zone of uncertainty. The behaviour of the curves and the study of what parameters affect them lead to the building of a database of varying input model characteristics such as shape, size, drill orientation, and structure.

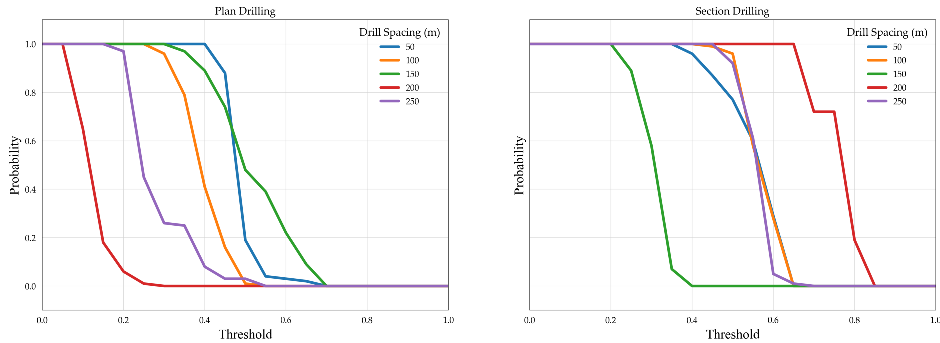


Figure 5: Probability Threshold curves for varying drill hole spacings in plan (*left*) and section view (*right*). A probability of 1 signifies that all threshold models at that spacing are larger than the truths. Conversely, the probability of 0 indicates that all truths are larger than the threshold models

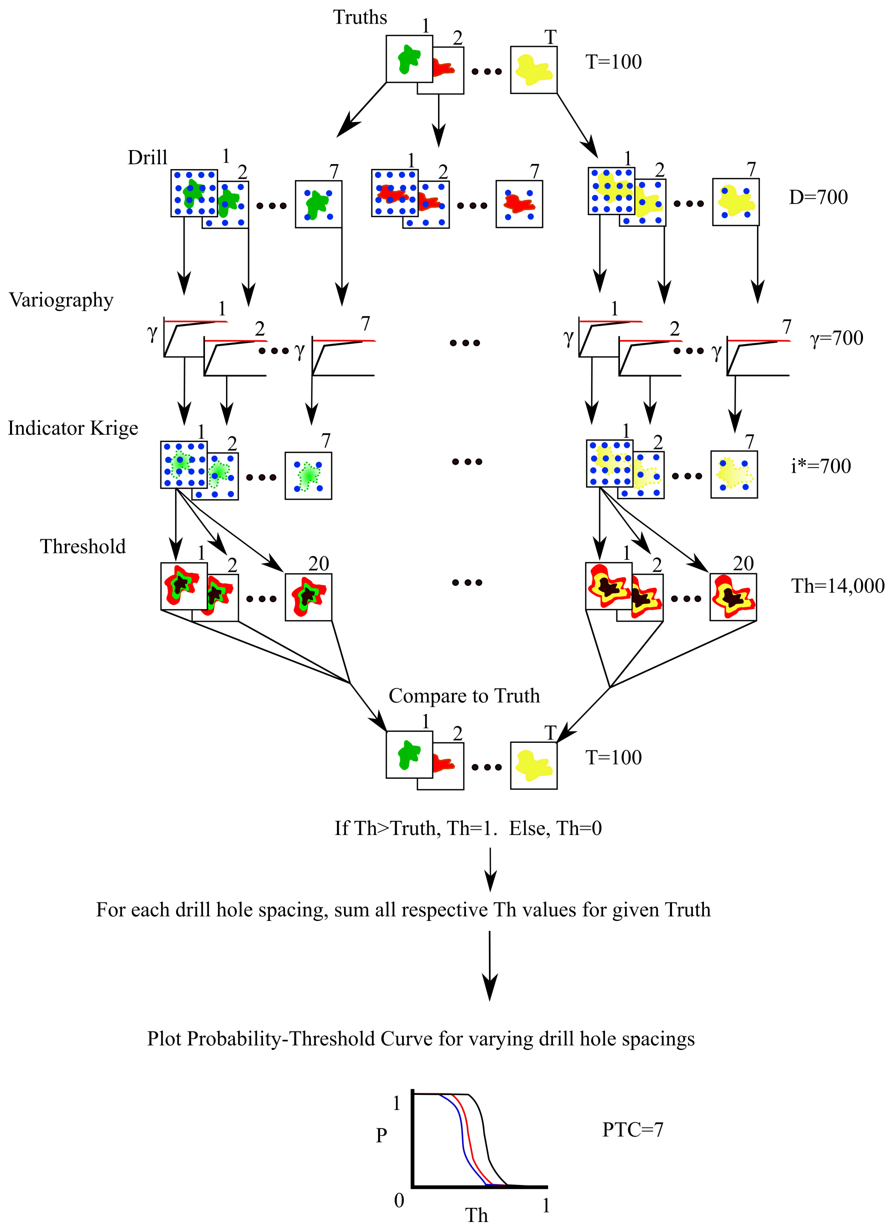


Figure 6: Schematic for the construction of Probability Threshold curves for varying drill hole spacings. Simulated truths at various spacings, variography is carried out on each synthetic data set and fed into `KT3DN` software. The krige models are thresholded from p100 to p0 in p05 increments and compared to their respective truths. If the models are larger than the truth, the model is assigned a value of 1; otherwise, the value is 0. The model values are summed and divided by the number of truths to return a probability of a threshold model being larger than the truth for a specific drill hole spacing. The end product is an experimental PTC for each drill hole spacing showing the probability for a model to be larger than the truth

Multiple Scenarios. In order to quantify and understand uncertainty in the context of complex and variable geological realities, there must be a study of multiple scenarios and the resulting PTCs must be understood. The behaviour of PTCs as they relate to changes in the size of a domain relative to geology, varying geological shapes, drill spacing ratio to geology size, and other geological attributes give insight. A subset of shapes can be seen in Figure 7 that represent synthetic truths for testing. The experimental shapes are run through the PTC workflow over varying drill spacings in plan and section view (Figure 8). Plotting multiple scenario curves together illustrates the variation in model uncertainty. Standardization of the resulting PTCs, centers the curves for closer observation and comparison purposes in Figure 8.

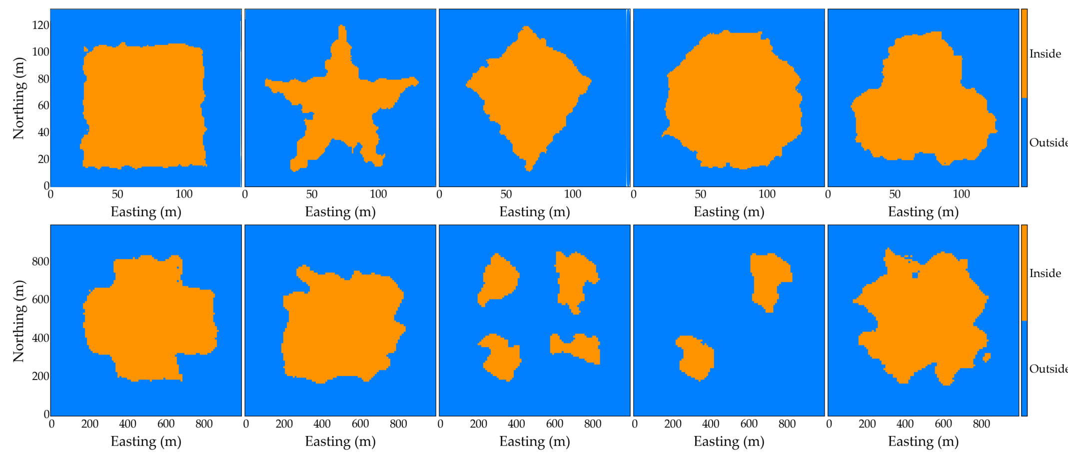


Figure 7: Truth 1 values for 10 example scenarios testing varying geological shapes

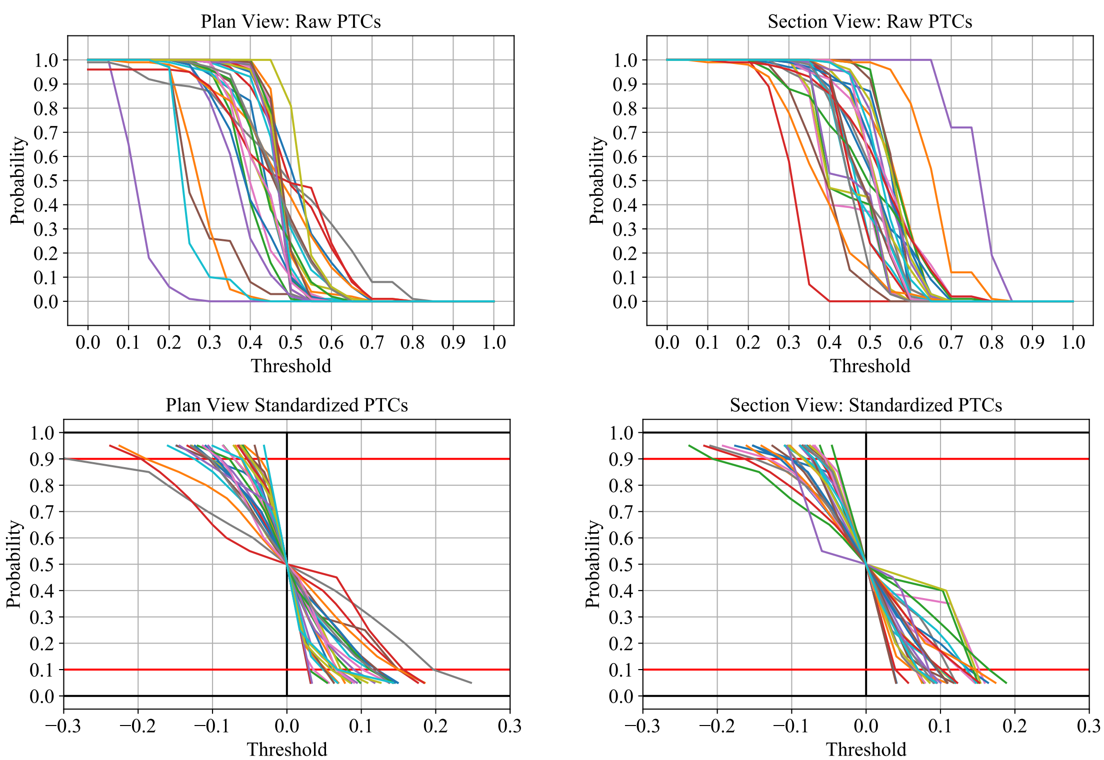


Figure 8: Raw PTC curves for a subset of 5 different geological scenarios at varying drill spacings in plan and section view (*top row*). Standardized PTCs of same scenarios and drill spacings (*bottom row*)

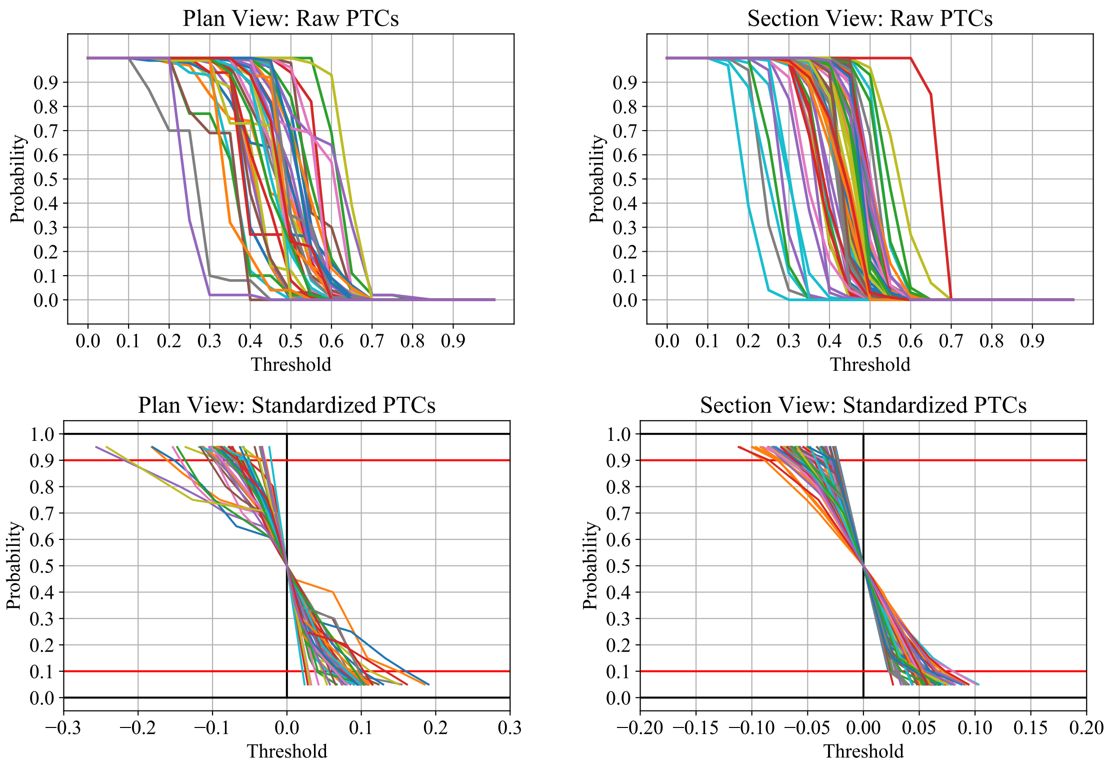


Figure 9: Raw PTC curves for a subset of 15 different geological scenarios at varying drill spacings in plan and section view (*top row*). Standardized PTCs of same scenarios and drill spacings (*bottom row*)

The tails of the PTCs are trimmed as the behaviour at the extremities is erratic and represents highly unlikely large or small models. Moreover, the threshold values for final uncertainty in the model will likely be within a finer range centered around the unbiased NN model threshold. Figure 4 illustrates how the high and low threshold values ($p100$ & $p00$) correspond to unrealistic models and justifies trimming the curves for comparison purposes. The standardized PTCs show how over many scenarios with varying geology and model characteristics, the uncertainty bandwidth at the p95, p05 follow similar linearity with values ranging predominantly between 0.05 and 0.15.

PTC Function Fitting. Access to the truth and replicates is not possible in reality. There is one true boundary that is unknown and must be estimated. Therefore, following the experimental PTC workflow is impossible in reality. However, by building an experimental PTC database of different boundary model scenarios with differing characteristics, one gain insight into boundary uncertainty. The experimental PTCs for the varying drill hole spacings are fit with a function that is parameterized by five variables: an upper limit, base case probability, lower limit, upper exponent, and lower exponent. The curve is fit with an upper and lower function with demarcation posited to be the p50, where half of the models are larger than their corresponding truths. Equation 1 shows the function parameters and a non-linear PTC is in Figure 10.

$$t = \begin{cases} \left(\frac{P-0.5}{0.5}\right)^{\omega_h} (b-h)+h, & for\ P > 0.5 \\ \left(\frac{P}{0.5}\right)^{\omega_l} (l-b)+b, & for\ P < 0.5 \end{cases} \tag{1}$$

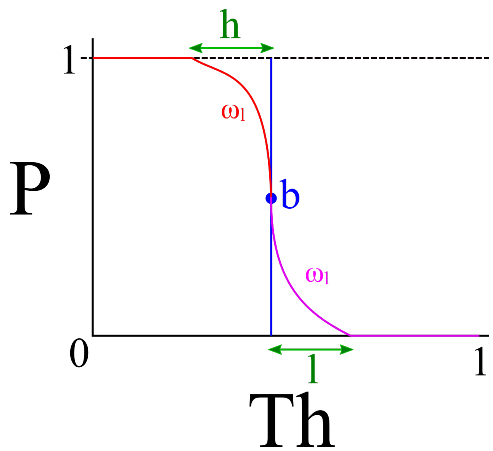


Figure 10: Parameterization of PTC function form. The upper (h) and lower (l) limits, and base case point, b, control the extents and centering, respectively. The w values control the curvature of the function

Where the threshold curve value (t), for probability greater than 0.5, is defined by the probability (P), an upper curvature parameter (ω_h), the base case probability (b), and the upper limit (h). For probabilities below 0.5, the threshold value is a function of the probability value (P), lower curvature parameter (ω_l), lower limit (l), and base case probability parameter (b).

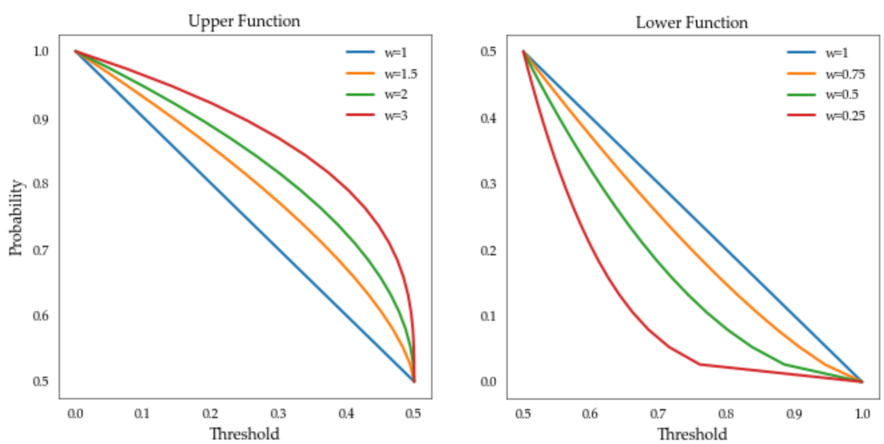


Figure 11: Curvature parameterization of PTC function form. The w_h values control the curvature of the upper function, while the w_l controls the curvature of the lower function

By fitting the functions to the PTCs from synthetic data, the construction of a database of parameterized function values can help identify specific model characteristics and their potential effects on the curves. With changing input parameters and simulating new truths, the process of creating PTCs and fit functions to build a substantial dataset of function parameters gives insight to boundary uncertainty and the relationship to PTCs.

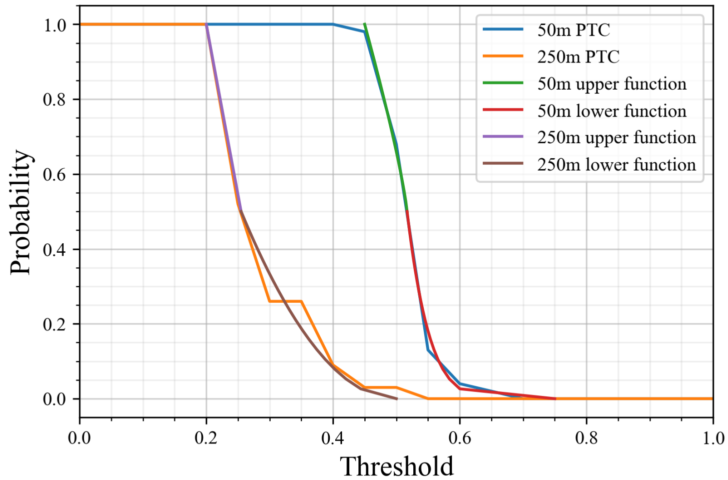


Figure 12: Fitted functions for 50m & 250m drill hole spacings. A total of 5 parameters would be produced for each curve: base case p50 (b), upper limit (h), lower limit (l), upper curve exponent (w_h), and lower curve exponent (w_l).

The function fitting for further analysis assumes the function is linear. The b-value for the PTC is the $p50$ model where half of the models are larger than the truths and half smaller. The final model's base case will come from the NN model threshold. The upper and lower limits given by l & h define the slope of the linear function. A linear model for PTC is in Figure 13.

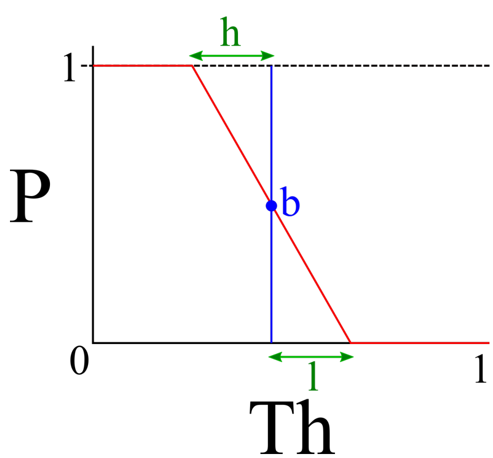


Figure 13: Parameterization of PTC function form. The upper (h) and lower (l) limits, and base case point, b, control the extents and centering, respectively

Increasing h and l decreases the slope of the function resulting in increasing uncertainty as the threshold models transition below the corresponding true model volumes.

Delta Values

The linear function for PTCs is parameterized by the upper and lower limits at $p95$ and $p05$ for h and l, respectively. The delta values, the difference between b and h & l, for the different scenarios gives insight into the uncertainty threshold for dilated and eroded boundary extraction. The upper and lower limits are categorized together as the delta values. Figures 14 & 15 show the delta h & l values over n=311 geological scenarios.

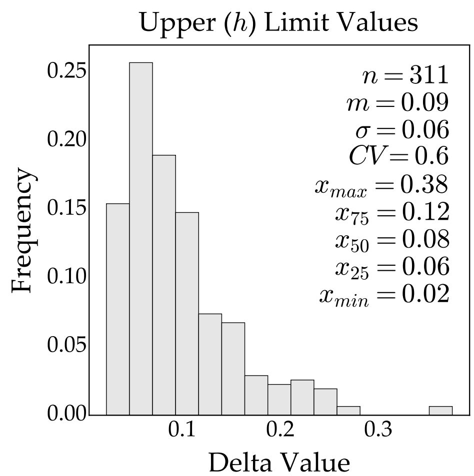


Figure 14: Upper (h) limit delta values for multiple scenarios

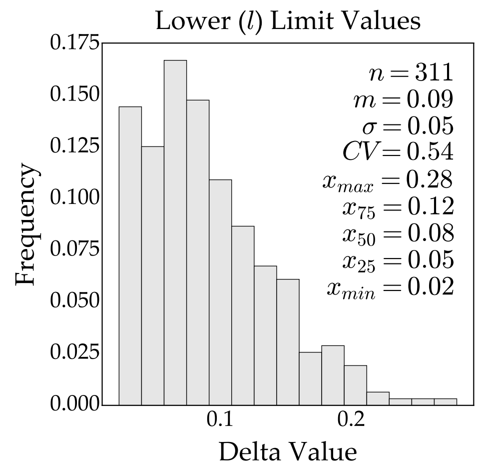


Figure 15: Lower (l) limit delta values for multiple scenarios

The means for the delta values are the same at 0.09. The standard deviations are near at 0.06 and 0.05 for upper and lower delta values, respectively. The higher delta values signify higher uncertainty in the scenarios.

For delta value comparisons between plan view and section view drilling, the results are in Figure 16 & 17.

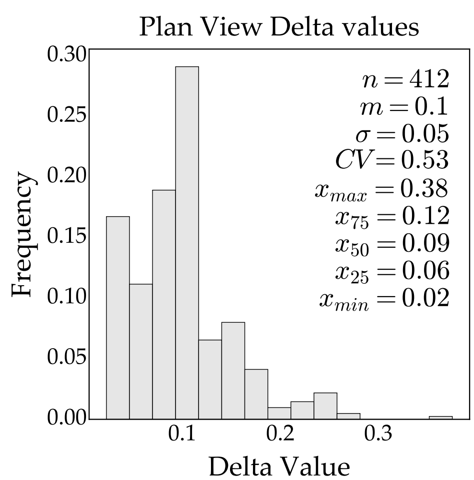


Figure 16: Delta values for all plan view drilling scenarios

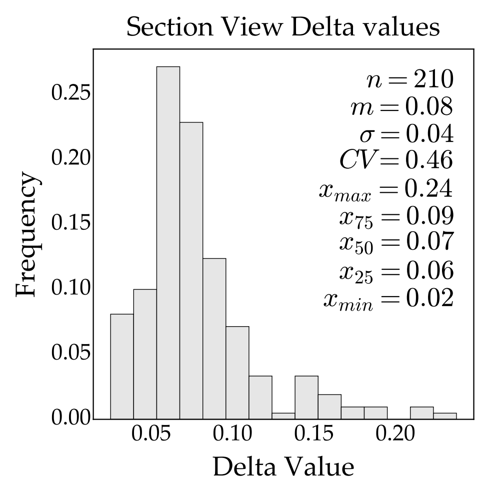


Figure 17: Delta values for all section view drilling scenarios

The section view scenarios have lower mean values and standard deviations. Referring to Figure 4, the data informing the section estimates is tightly spaced downhole. In contrast, plan view estimates have sparse data leading to higher delta values and uncertainty.

Figure 18 shows the delta values over all geologies, drill hole spacings, in both plan and section view. The delta values describe the uncertainty relationship for the indicator estimates to the corresponding thresholds in a linear relationship.

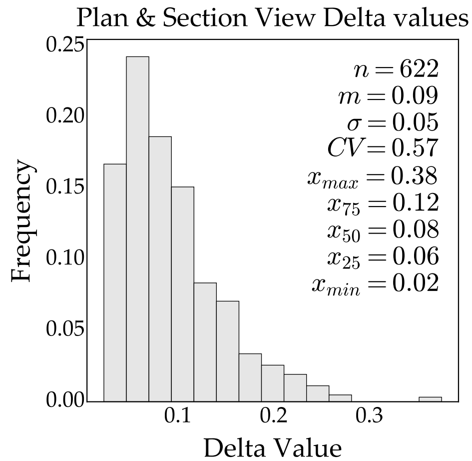


Figure 18: Delta values for all geological scenarios

The mean for delta values trimmed at $p95/p05$ is 0.09 with a standard deviation of 0.05. The distribution is skewed to lower delta values with only 25% of values over 0.12.

Delta & Drill Spacing

Understanding the controls on the delta values is crucial for determining appropriate uncertainty thresholds. Distinct patterns emerge when assessing the experimental standardized PTCs. An obvious predictor for increased uncertainty is the amount of informing data. The increasing drill hole spacings result in less conditioning data for the indicator estimation. The higher delta values associated with increasing drill hole spacings lead to more uncertainty in the boundary model. The higher uncertainty is reflected in the PTCs by a shallower slope. Figure 19 shows a scenario with Truths 1-5 drilled with increasing data spacing and the corresponding $p50$ models. The PTCs for the example are in Figure 20.

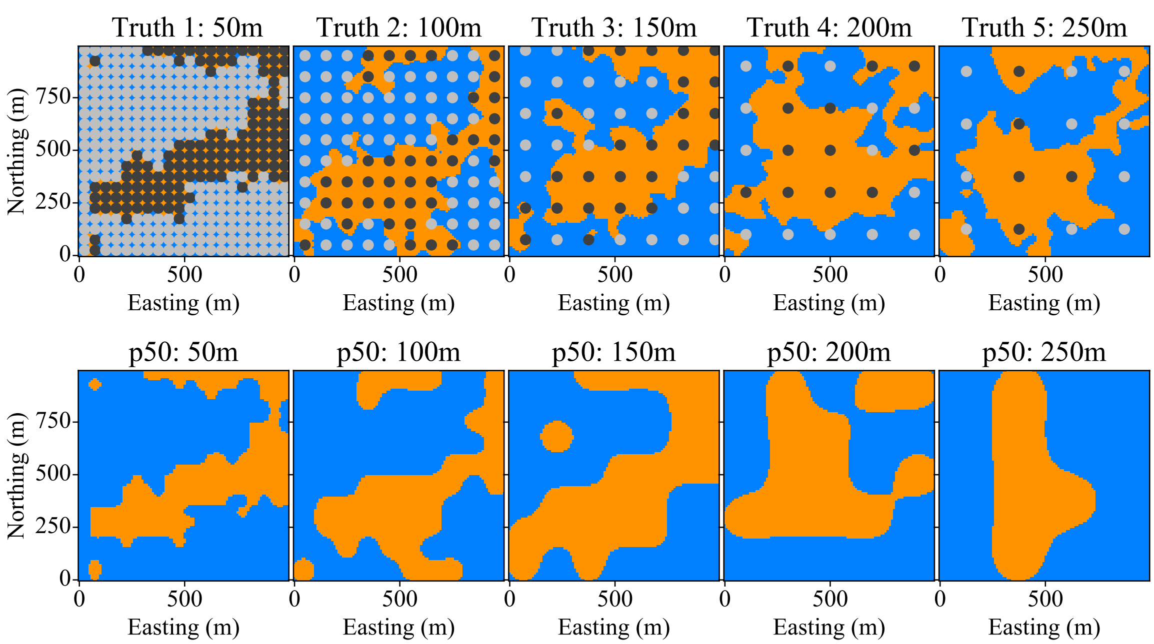


Figure 19: Truths 1-5 with increasing data spacing (*top row*). The 10m and 25m drill hole spacings are tight at resolution and are not included. Corresponding $p50$ indicator threshold models (*bottom row*)

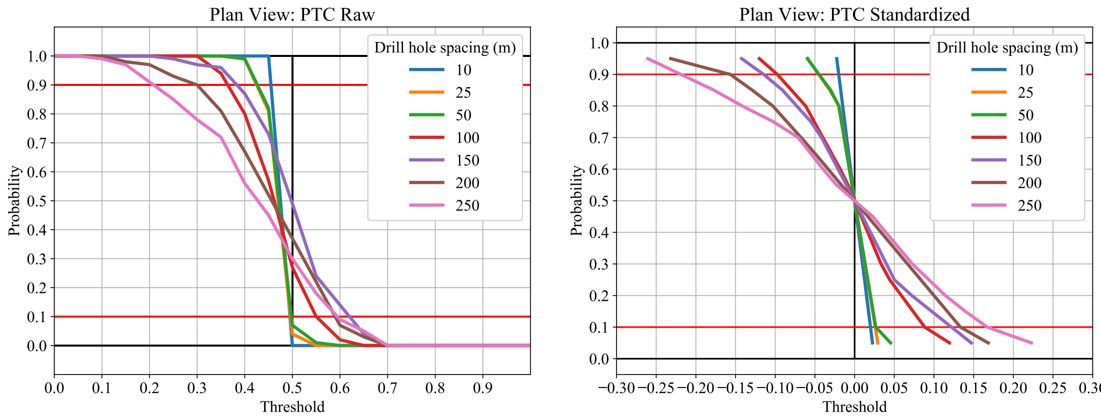


Figure 20: Raw PTC curves of geological scenario with seven drill spacings in plan view (*left*). Standardized PTCs of same scenario and drill spacings trimmed at p95-p05 (*right*)

The example illustrates how increasing sample spacing leads to larger delta values, shallower curves, and, therefore, increased uncertainty in the boundary model. The model for approximation is also appropriate as the PTC curvature is reasonably linear. Section view drilling contains significantly more samples compared to plan view. The synthetic drill holes provide ample information for the estimation algorithm. The results of section drilling are observed in Figure 21 with corresponding PTCs in Figure 22.

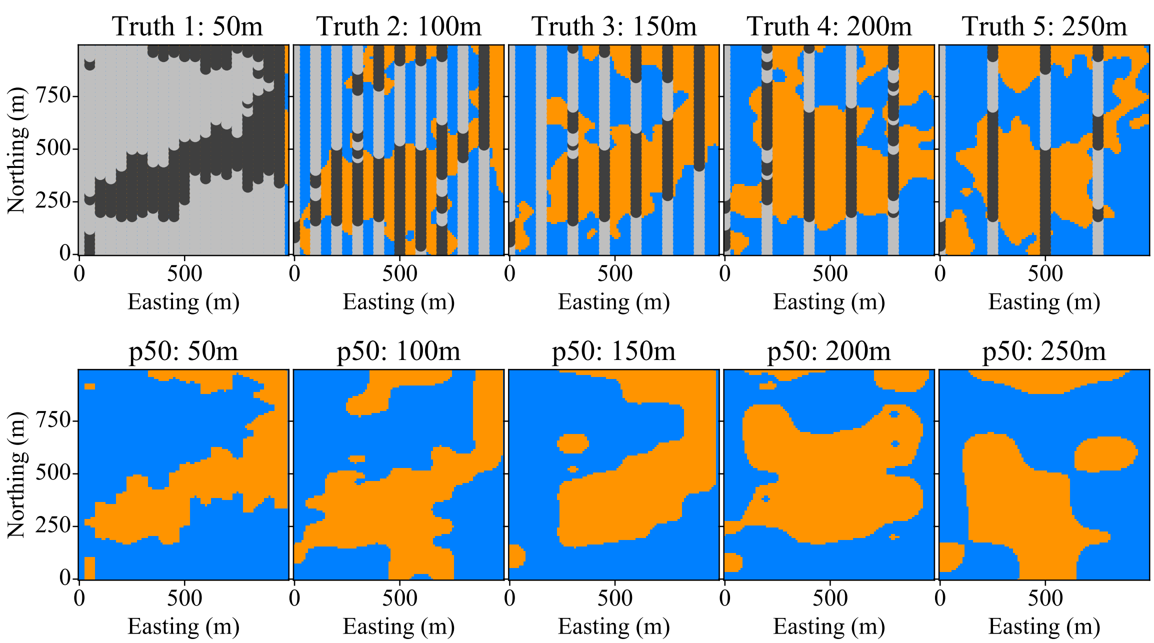


Figure 21: Truths 1-5 with increasing data spacing (*top row*). The 10m and 25m drill hole spacings are tight at resolution and are not included. Corresponding p50 indicator threshold models (*bottom row*)

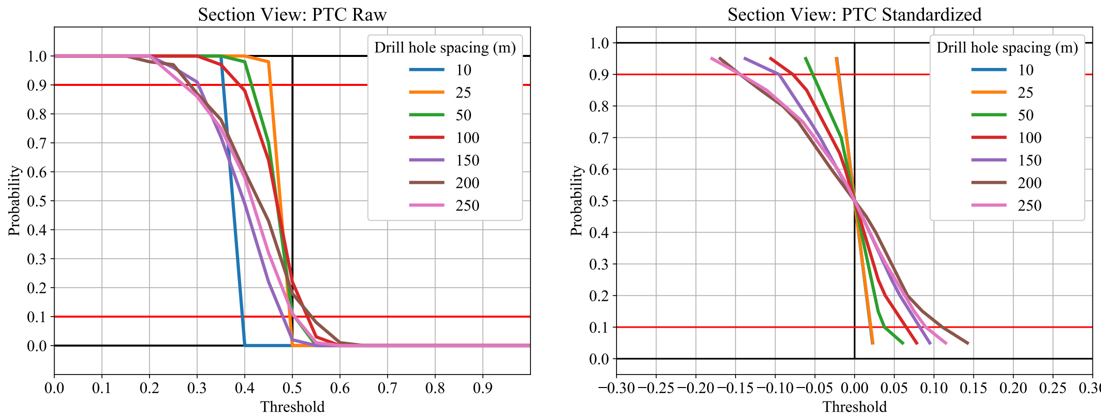


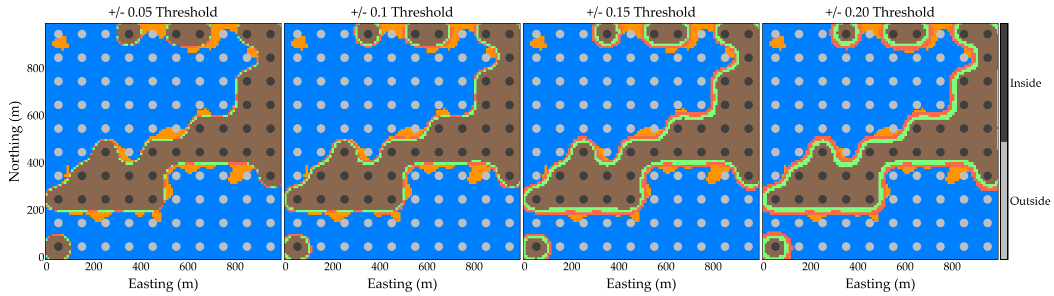
Figure 22: Raw PTC curves of geological scenario with seven drill spacings in section view (*left*). Standardized PTCs of same scenario and drill spacings trimmed at $p95$-$p05$ (*right*)

The delta values in section view are considerably less than in plan view for wider spacings. The lower uncertainty is attributable to an increase in conditioning data downhole. The tighter drill hole spacings have similar delta values; however, with increasing drill spacing, the delta values diverge.

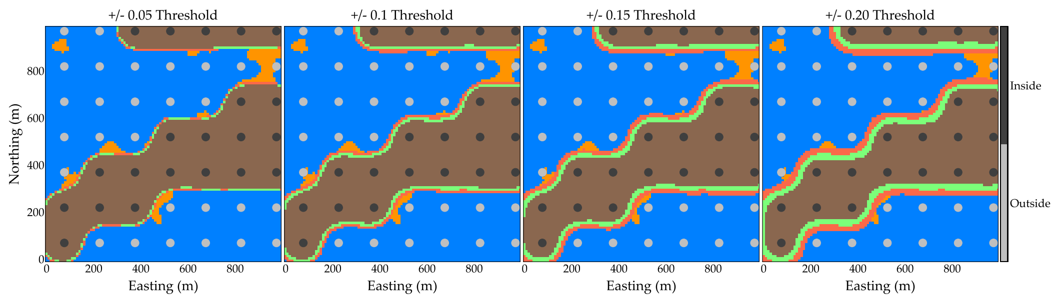
Uncertainty Thresholds

The delta values threshold the indicator estimates above and below the NN thresholded model. Eroded and dilated cases, in conjunction with the NN threshold model, form the final boundary model with uncertainty. The delta values over all scenarios are generally between 0.05-0.20 in section and plan view. In order to understand the effect of increasing delta values, the indicator threshold workflow including uncertainty assessment is visually inspected.

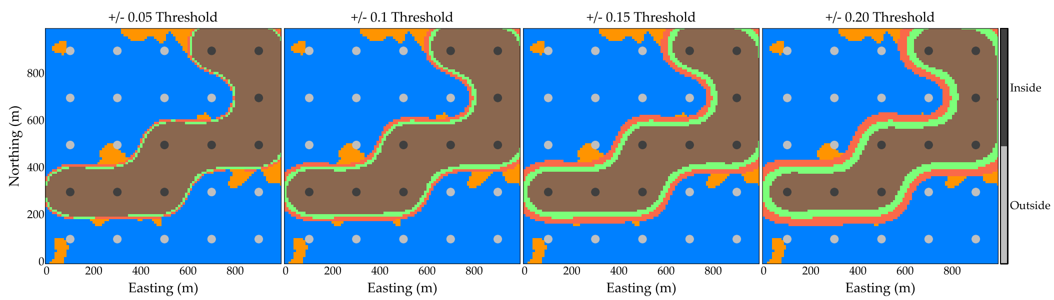
Figure 23 shows varying drill spacings and delta values used to threshold for boundary uncertainty. The 10, 25, & 50m drill spacing models are tight at resolution with indiscernible uncertainty bandwidths and thus not shown. Uncertainty bandwidths grow with increasing delta values and drill spacing. A delta value of 0.05 corresponds to a constrictive bandwidth with unrealistic uncertainty given the spacings. The remaining threshold models appear realistic given the conditioning data. The gridded nature of the drilling results in a rather uniform bandwidth; however, noticable increases in uncertainty are evident at estimation locations between diagonal inside data. The 250m spacing models exemplify this as the eroded cases are disjointed, the base cases thinly center between the data, and the dilated cases expand to give reasonable uncertainty. The methodology's adherence to the spatial configuration of the conditioning data results in reasonable bandwidths for uncertainty.



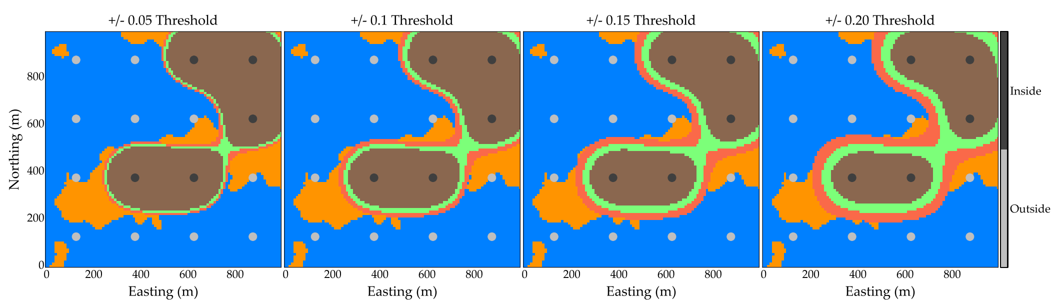
(a) Threshold models for 100m gridded drill spacing models showing uncertainty bandwidths



(b) Threshold models for 150m gridded drill spacing models showing uncertainty bandwidths



(c) Threshold models for 200m gridded drill spacing models showing uncertainty bandwidths



(d) Threshold models for 250m gridded drill spacing models showing uncertainty bandwidths

Figure 23: Boundary models with uncertainty bandwidths from threshold values of +/- 0.05, 0.1, 0.15,& 0.2. Eroded models in *brown*, NN thresholded base case models in *green*, and dilated models in *red*. The plan view models overlie the corresponding truth (*orange*) and underly the conditioning data informing the thresholded indicator estimates.

The volumes of the eroded and dilated boundaries as a percentage of the underlying truth volume are in Tables 1 & 2. The percentages for 10m spacing hold constant in two instances showing the minimal uncertainty change between higher thresholds at tight spacings.

Table 1: Eroded boundary threshold volumes as percentage of true volume for different plan view drill spacings

Spacing (m)	Threshold Volumes (%)			
	0.05	**0.10**	**0.15**	**0.20**
10	-1.2	-5.0	-5.2	-5.2
25	-0.1	-1.0	-1.7	-2.6
50	-4.3	-6.8	-8.1	-9.8
100	-8.2	-11.4	-17.1	-22.1
150	19.0	12.9	7.7	0.9
200	3.9	-2.3	-9.2	-17.0
250	-12.1	-19.2	-25.7	-32.8

Table 2: Dilated boundary threshold volumes as percentage of true volume for different plan view drill spacings

Spacing (m)	Threshold Volumes (%)			
	0.05	**0.10**	**0.15**	**0.20**
10	4.4	4.6	4.6	4.8
25	3.1	3.8	4.2	5.7
50	4.4	5.6	7.4	9.7
100	1.0	4.6	10.4	14.3
150	29.8	35.9	41.4	47.9
200	16.1	22.2	28.7	34.8
250	3.3	10.3	17.4	24.7

The 150m spacing eroded model is larger than the truth, a result of having the highest percentage of inside conditioning data at 42.9%. Morever, the configuration of the conditioning data is proximal to the true boundary leading to the estimate expanding beyond (Figure 24).

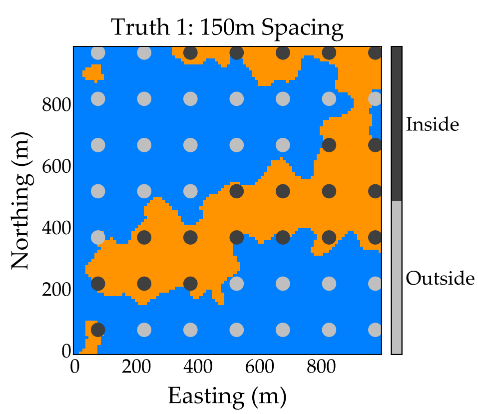
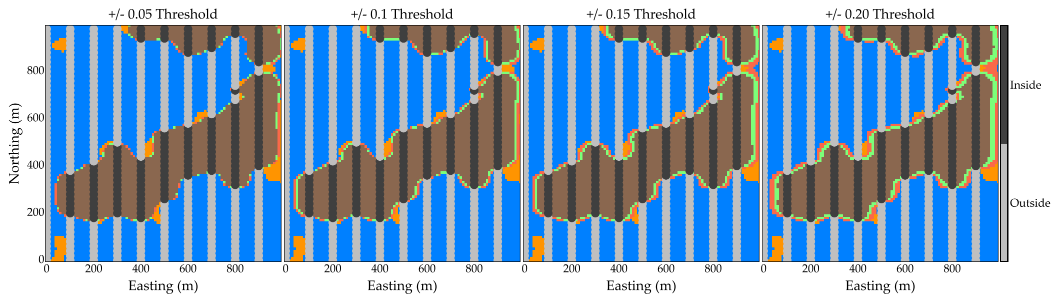
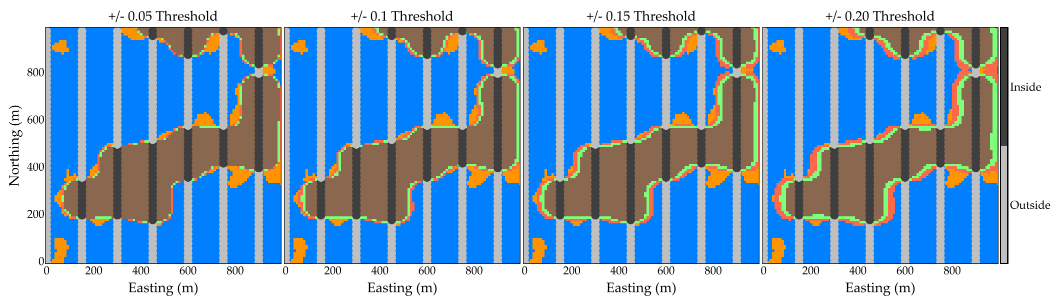


Figure 24: Truth realization 1 with 150m drill spacing

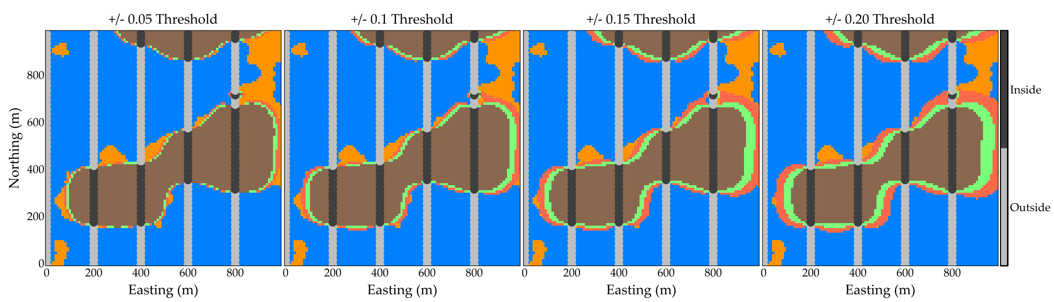
For section view spacing, replicating downhole drilling, the models are informed by more data. Figure 25 shows the same truth, but with section drilling instead of plan view sampling.



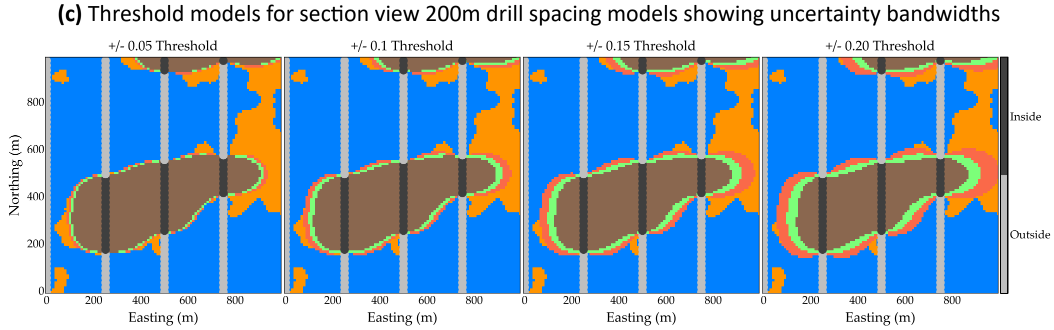
(a) Threshold models for section view 100m drill spacing models showing uncertainty bandwidths



(b) Threshold models for section view 150m drill spacing models showing uncertainty bandwidths



(c) Threshold models for section view 200m drill spacing models showing uncertainty bandwidths



(d) Threshold models for section view 250m drill spacing models showing uncertainty bandwidths

Figure 25: Boundary models with uncertainty bandwidths from threshold values of +/- 0.05, 0.1, 0.15,& 0.2. Eroded models in *brown*, NN thresholded base case models in *green*, and dilated models in *red*. The section view models overlie the corresponding truth (*orange*) and underly the conditioning drillholes informing the thresholded indicator estimates.

The section view drilling presents two disparate sample densities in orthogonal directions. The data is collected densely downhole with sparse data existing along strike of the collars. The asymmetry in the data highlights the efficacy of the uncertainty thresholding as the bandwidths vary smoothly and honor local data configurations. The most pronounced example of this is the +/-0.20 threshold for 250m seen in Figure

25d. Between drill holes the uncertainty bandwidth expands when distal to informing data and contracts proximal to the drill holes. Where the boundaries near the model edges, and away from conditioning data, expansion into the zone of higher uncertainty occurs. The data conforming and flucuating uncertainty is in contrast to SDF modeling where the additive C-parameter for uncertainty results in unrealistic constant bandwidths (Mancell & Deutsch, 2019).

Table 3: Eroded boundary threshold volumes as percentage of true volume for different section view spacings

Spacing (m)	**Threshold Volumes (%)**			
	0.05	**0.10**	**0.15**	**0.20**
10	-6.5	-6.5	-6.5	-6.5
25	-2.0	-3.5	-6.3	-8.9
50	-0.4	-4.4	-6.3	-9.2
100	-0.4	-4.8	-8.0	-11.0
150	-11.4	-16.5	-20.1	-23.8
200	-13.1	-18.8	-23.6	-28.0
250	-30.3	-34.9	-39.0	-42.6

Table 4: Dilated boundary threshold volumes as percentage of true volume for different section view spacings

Spacing (m)	**Threshold Volumes (%)**			
	0.05	**0.10**	**0.15**	**0.20**
10	4.6	4.6	4.6	4.6
25	2.9	4.7	6.5	7.5
50	4.0	8.4	10.1	11.3
100	6.5	9.9	12.7	15.6
150	-2.5	0.4	4.1	8.9
200	-2.2	3.7	9.3	15.9
250	-21.1	-16.1	-9.7	-2.5

The section view volume difference of the models to the truth as a percentage is shown in Tables 3 & 4. The 10m spacing holds constant bandwidths across all thresholds, a function of closely spaced samples resulting in static uncertainty. The 250m dilated boundary volumes are all below the true volume fo the model. The wider spacing misses the North-South structure to the East of the model, leaving a significant amount of geology unsampled. Across the entirety of the models, the correct thresholding for uncertainty coincides with a value between 0.1-0.2. A conservative uncertainty bandwidth coincides with a +/-0.1 thresholding value, while a liberal bandwidth, capturing more uncertainty, derives from a threshold value of +/-0.2.

Volume uncertainty

Comparing volume uncertainty among many geological scenarios, at varying thresholding values, over thousands of truths, at multiple spacings in plan and section view is difficult. An avenue to aid in differentiating uncertainty bandwidths is to standardize the global volumes. For each indicator thresholded model with uncertainty, the model volume is standardized by subtracting and dividing by the corresponding true volume. A value of zero implies the model volume matches the truth volume exactly. A negative value indicates the model is smaller while a positive value means the model is larger in comparison to the underlying true volume. The plan view n=6000 standardized volumes over 1500 geological scenarios with four sample spacings are in Figure 26.

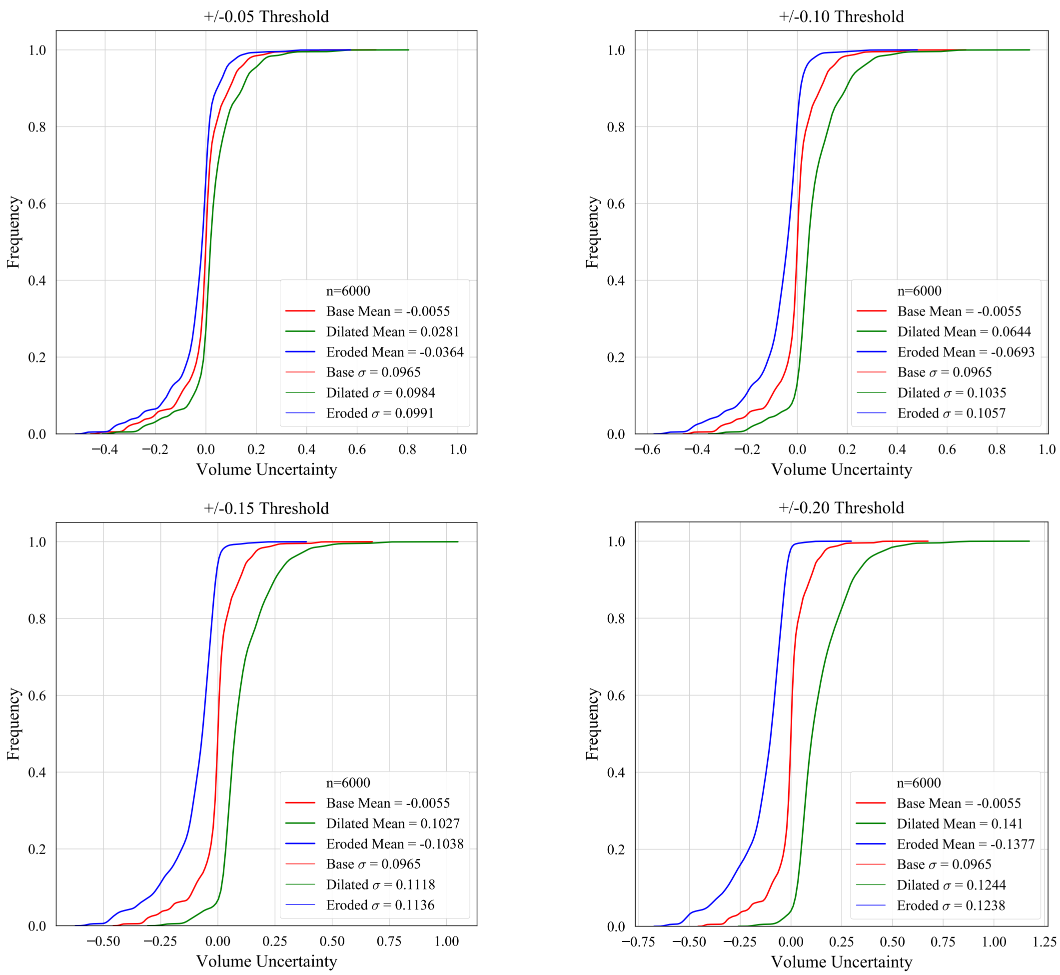


Figure 26: Volume Uncertainty from standardized boundary models for varying thresholds in plan view

The base case volume uncertainties are all equal with same mean and standard deviation. Recall the NN indicator threshold model forms the base boundary model and uncertainty is thresholded above and below to extract dilated and eroded boundaries. Therefore, the base case volume uncertainty is static across the different thresholds. The base case mean is near zero indicating the NN thresholding is overall closely predicting the underlying true volume. The dilated boundary volume means increase as the threshold value increases shifting the curves the right. The opposite occurs with the eroded boundary volumes; as the threshold value increases, the curve shifts left and volumes become smaller. The standard deviations are similar between dilated and eroded distributions of a particular threshold; however, as threshold values increase, the respective standard deviations are elevated.

Figure 27 shows the standardized volume uncertainty for the same underlying truths with section view drilling.

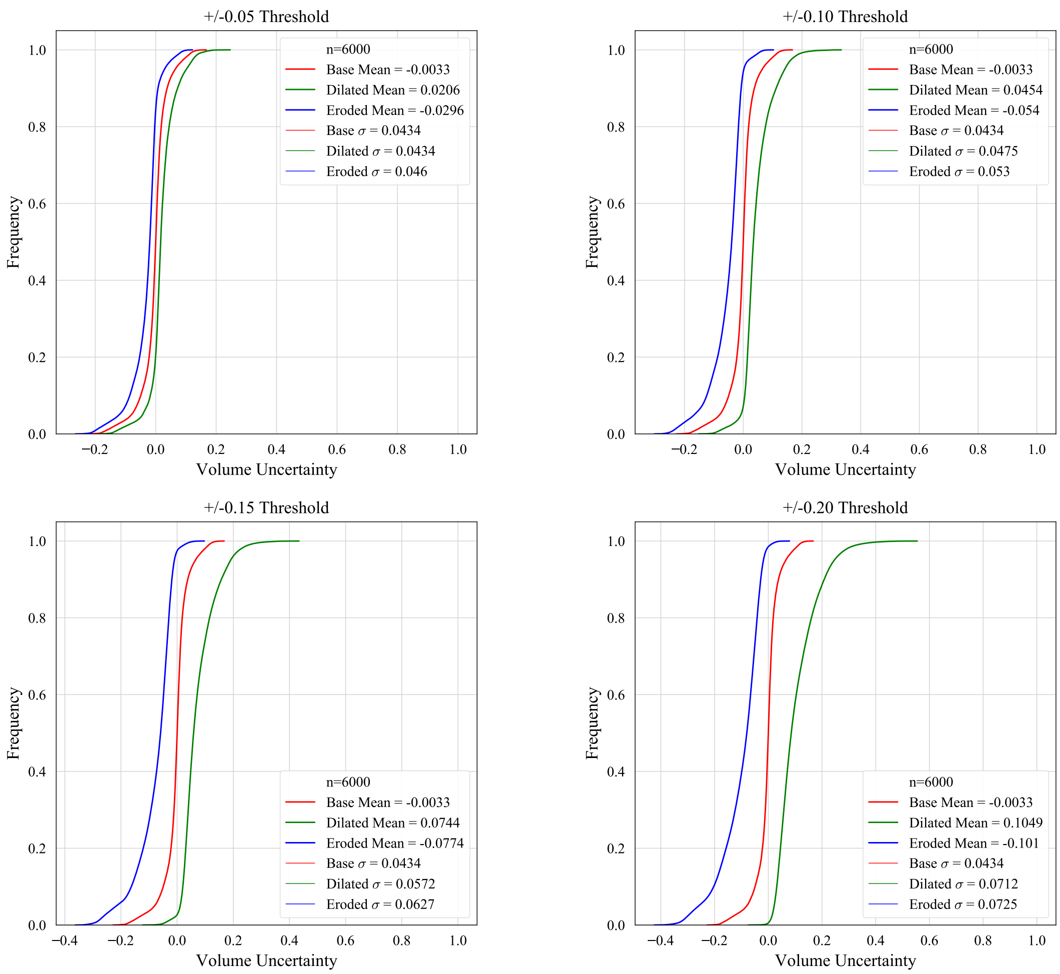


Figure 27: Volume Uncertainty from standardized boundary models for varying thresholds in section view

The section view distributions show the same trends and patterns as plan view, however, with smaller mean values and standard deviations overall. The increase in sampling intuitively leads to smaller uncertainty. For eroded distributions, the upper tails transition more abruptly than the lower tails as models are exceedingly smaller than the truths for the latter. The inverse is true for the dilated distributions – the shift by the threshold value results in the lower tail transitioning faster comparatively to the upper.

The inference of thresholding values for parameterizing probability-threshold curves and assessing uncertainty flows from extensive modeling of synthetic datasets. Verification of the workflow efficacy is accomplished by comparing results to the underlying true model. In reality, access to the truth is unattainable. No two deposits are the same as geology forms from complex and dynamic systems. Uncertainty is ubiquitous and derives from our lack of knowledge. Downstream decision making is greatly aided by an understanding of the uncertainty inherent to boundaries. A thresholding value of +/-0.15 above and below the NN indicator threshold model is recommended. The bandwidth between resulting dilated and eroded boundaries gives realistic uncertainty that honours the spatial configuration of the conditioning data. For a modeler wanting a tighter more conservative bandwidth, a threshold value of +/-0.1 can be chosen. Wider bandwidths can be managed by using a higher threshold of +/-0.2 to increase the probability of the boundary falling within the uncertainty zone.

Conclusion

Uncertainty around boundaries exist anywhere data does not. The quantification and understanding of uncertainty in boundary models is important, as further geostatistical analysis relies heavily on boundary placement. Furthermore, downstream decision making is greatly aided by the ability to ascertain uncertainty and risk. Boundary modeling using indicator thresholding to NN models gives geologically realistic models and the process is straightforward and fast. The relationship between the probability field and thresholding is described by a Probability-Threshold Curve. Extensive study of PTCs shows that uncertainty in the indicator threshold workflow is accessed by further thresholding the estimate by +/-0.15 of the NN calibrated base case. The bandwidth fluctuates and follows the spatial configuration of the conditioning data. In zones with less informing data, the bandwidths expand to give larger uncertainty; in areas of dense sampling, the bandwidths contract.

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