## Lecture 7 Sept. 10, 2020

$>$ Symmetry in Nature and in Molecules
> Symmetry Operations
$>$ Symmetry Elements
> Point Groups and Assignments

## Symmetry



Intuitively, we know symmetry when we see it.
But how do we put in quantitative terms that allows us to compare, assign, classify?

## Symmetry: mirror planes



## Symmetry


$\mathrm{C}_{3}$ Rotation Axis

## Chirality: Non-superimposable mirror images (aka optical isomers or enantiomers)



Specifically, a chiral compound can contain no improper axis of rotation $\left(S_{n}\right)$, which includes planes of symmetry and inversion center. Asymmetric molecules are always chiral.

## Symmetry in Nature and in Molecules

The symmetry of a molecule is determined by the existence of symmetry operations performed with respect to symmetry elements. A symmetry element is a line, a plane or a point within or through an object, about which a rotation or reflection leaves the object in an orientation indistinguishable from the original. A plane of symmetry is designated by the symbol $\sigma$ (or sometimes $s$ ), and the reflection operation is the coincidence of atoms on one side of the plane with corresponding atoms on the other side, as though reflected in a mirror. A center or point of symmetry is labeled $i$, and the inversion operation demonstrates coincidence of each atom with an identical one on a line passing through and an equal distance from the inversion point. Finally, a rotational axis is designated $C_{n}$, where the degrees of rotation that restore the object is $360 / \mathrm{n}\left(\mathrm{C}_{2}=1800\right.$ rotation, $\mathrm{C}_{3}=1200$ rotation, $\mathrm{C}_{4}=90$ rotation, $\mathrm{C}_{5}=720$ rotation). $\mathrm{C}_{1}$ is called the identity operation E because it returns the original orientation. An object having no symmetry elements other than E is called asymmetric. Such an object is necessarily chiral. Since a plane or point of symmetry involves a reflection operation, the presence of such an element makes an object achiral. One or more rotational axes of symmetry may exist in both chiral, dissymmetric, and achiral objects.

## Symmetry Operations and Symmetry Elements

## Definitions:

$>$ A symmetry operation is an operation on a body such that, after the operation has been carried out, the result is indistinguishable from the original body (every point of the body is coincident with an equivalent point or the same point of the body in its original orientation).
$>$ A symmetry element is a geometrical entity such as a line, a plane, or a point, with respect to which one or more symmetry operations may be carried out

| Symmetry Operation | Symmetry Element | Notation |
| :--- | :--- | :---: |
| Identity | - | E |
| Reflection in a plane | Plane of symmetry | $\sigma$ |
| Proper rotation | Rotation axis (line) | $\mathrm{C}_{\mathrm{n}}$ |
| Rotation followed by reflection in  <br> the plane perpendicular to the Improper rotation axis <br> rotation axis (line) | $\mathrm{S}_{\mathrm{n}}$ |  |
| Inversion | Center of inversion |  |



## Let's look for these in molecules

## what is a point group?

A collection of symmetry elements for a specific symmetry, intersecting at a specific point for molecules, and displayed in a character table.

## The $C_{1}$ point group:

Molecules that have no symmetry elements at all except the trivial one where they are rotated through $360^{\circ}$ and remain unchanged, belong to the $\boldsymbol{C}_{1}$ point group. In other words, they have an axis of $360^{\circ} / 360^{\circ}=1$-fold, so have a $\boldsymbol{C}_{1}$ axis. Examples are:


Two symmetry approaches or notations

Schoenflies notation
(for molecules-interior lines, points, planes)

Herrman-Mauguin (crystallographic point groups
-translations, crystal classes )

## The $C_{n}$ point groups:

These have a $C_{n}$ axis as their only symmetry element. They generally resemble propellers which have the front and back different. Important examples are (hydrogens omitted for clarity):


Notes
(i) symmetry operations more fundamental, but elements often easier to spot.
(ii) some symmetry elements give rise to more than one operation - especially rotation - as above.

## ROTATIONS - AXES OF SYMMETRY

Some examples for different types of molecule: e.g.




Line in molecular plane, bisecting HOH angle is a rotation axis, giving indistinguishable configuration on rotation by $180^{\circ}$.

By VSEPR - trigonal, planar, all bonds equal, all angles $120^{\circ}$. Take as axis a line perpendicular to molecular plane, passing through $B$ atom.

axis perpendicular
to plane
N.B. all rotations CLOCKWISE when viewed along -z direction.


## Symbol for axes of symmetry

where rotation about axis gives indistinguishable configuration every $(360 / n)^{0}$ (i.e. an n-fold axis)

Thus $\mathrm{H}_{2} \mathrm{O}$ has a $\mathrm{C}_{2}$ (two-fold) axis, $\mathrm{BF}_{3}$ a $\mathrm{C}_{3}$ (three-fold) axis. One axis can give rise to >1 rotation, e.g. for $\mathrm{BF}_{3}$, what if we rotate by $240^{\circ}$ ?


In general $\mathrm{C}_{\mathrm{n}}$ axis (minimum angle of rotation $\left.(360 / n)^{0}\right)$ gives operations $C_{n}{ }^{m}$, where both $m$ and n are integers.

When $m=n$ we have a special case, which introduces a new type of symmetry operation.....

## IDENTITY OPERATION

For $\mathrm{H}_{2} \mathrm{O}, \mathrm{C}_{2}{ }^{2}$ and for $\mathrm{BF}_{3} \mathrm{C}_{3}{ }^{3}$ both bring the molecule to an IDENTICAL arrangement to initial one.

Rotation by $360^{\circ}$ is exactly equivalent to rotation by $0^{\circ}$,
i.e. the operation of doing NOTHING to the molecule.

## xenon tetrafluoride, $\mathrm{XeF}_{4}$



## cyclopentadienide ion, $\mathrm{C}_{5} \mathrm{H}_{5}{ }^{-}$



## benzene, $\mathrm{C}_{6} \mathrm{H}_{6}$



Examples also known of $\mathrm{C}_{7}$ and $\mathrm{C}_{8}$ axes.

If $\mathbf{a} \mathrm{C}_{2 \mathrm{n}}$ axis (i.e. even order) present, then $\mathrm{C}_{\mathrm{n}}$ must also be present:


Therefore there must be a $\mathrm{C}_{2}$ axis coincident with $\mathrm{C}_{4}$, and the operations generated by $\mathrm{C}_{4}$ can be written:

$$
\mathrm{C}_{4}{ }^{1}, \mathrm{C}_{4}{ }^{2}\left(\mathrm{C}_{2}{ }^{1}\right), \mathrm{C}_{4}{ }^{3}, \mathrm{C}_{4}{ }^{4}(\mathrm{E})
$$

Similarly, a $\mathrm{C}_{6}$ axis is accompanied by $\mathrm{C}_{3}$ and $\mathrm{C}_{2}$, and the operations generated by $\mathrm{C}_{6}$ are:

$$
\mathrm{C}_{6}{ }^{1}, \mathrm{C}_{6}{ }^{2}\left(\mathrm{C}_{3}{ }^{1}\right), \mathrm{C}_{6}{ }^{3}\left(\mathrm{C}_{2}{ }^{1}\right), \mathrm{C}_{6}{ }^{4}\left(\mathrm{C}_{3}{ }^{2}\right), \mathrm{C}_{6}{ }^{5}, \mathrm{C}_{6}{ }^{6}(\mathrm{E})
$$

Molecules can possess several distinct axes, e.g. $\mathrm{BF}_{3}$ :


Three $\mathrm{C}_{2}$ axes, one along each B-F bond, perpendicular to $\mathrm{C}_{3}$

Mirror Planes within $\mathrm{BF}_{3}$. Of two types,

Center of Inversion in Molecules: $x, y, z->-x,-y,-z$


## (Proper) Rotation in Chemistry



## Reflection in a Plane in Chemistry



Vertical and Dihedral ( $\sigma_{v}$ that bisects $\mathrm{C}_{2}$ axes) Mirror Planes

Benzene, $\mathrm{C}_{6} \mathrm{H}_{6}: 3 \sigma_{d}$ mirror planes and $3 \sigma_{v}$ mirror planes

side view, one of each type shown


In a molecule that has both $\sigma_{v}$ and $\sigma_{d}$ mirror planes, the $\sigma_{v}$ planes bisect as many atoms as possible and the $\sigma_{d}$ planes bisect as many bonds as possible.

## Symmetry Point Group Assignments

An object may be classified with respect to its symmetry elements or lack thereof. This is done by assigning a symmetry point group, reflecting the combination of symmetry elements present in the structure. For example, bromochlorofluoromethane has no symmetry element other than C1 and is assigned to that point group. All C1 group objects are chiral. Other low symmetry point groups are Cs (only a single plane of symmetry) and Ci (only a point of symmetry). Objects in either of these point groups are achiral. Some objects are highly symmetric and incorporate many symmetry elements. Methane is an example of a high symmetry molecule, having 4 C3 axes, 3 C2 axes and $6 \sigma$ (planes); it belongs to the tetrahedral point group Td. When combinations of rotational axes and planes are present, their relationship is designated by a v (vertical), h (horizontal) or d (diagonal). Thus, a plane containing the principle rotation axis is $\sigma v$, a plane perpendicular to the principle rotation axis is $\sigma \mathrm{h}$, and a plane parallel to the principle rotation axis but bisecting the angle between two C 2 axes is $\sigma \mathrm{d}$. By this notation, the six planes of the methane tetrahedron are all od.

## A Simple Approach to Point Group Assignments



The process used to assign a molecule to a point group is straightforward with a few exceptions. Use this schematic to guide you.


Figure 7-9
Shriver \& Atkins Inorganic Chemistry, Fourth Edition
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## The $D_{n h}$ point groups:



