Alexander M. Ostrowski (1893–1986): His life, work, and students*

Walter Gautschi

As a former student of Professor Ostrowski — one of his last — I am delighted to recall here the life and work of one of the great mathematicians of the 20th century. Needless to say that, in view of Ostrowski's immense and vastly diverse mathematical legacy, this can be done only in a most summary fashion. Further literature on Ostrowski can be found in some of the references at the end of this article. We also assemble a complete list of his Ph.D. students and trace the careers of some of them.

1. His life

The mother of Alexander

Alexander Markovich Ostrowski was born in Kiev on September 25, 1893, the son of Mark Ostrowski, a merchant in Kiev, and Vera Rashevskaya. He attended primary school in Kiev and a private school for a year before entering the Kiev School of Commerce. There, his teachers soon became aware of Alexander's extraordinary talents in mathematics and recommended him to Dmitry Aleksandrovich Grave, a professor of mathematics at the University of Kiev. Grave himself had been a student of Chebyshev in St. Petersburg before assuming a position at the University of Kharkov and, in 1902, the chair of mathematics at the University of Kiev. He is considered the founder of the Russian school of algebra, having worked on Galois theory, ideals, and equations of the fifth degree. The seminar on algebra he ran at the University of Kiev was famous at the time.

*Expanded version of a lecture presented at a meeting of the Ostrowski Foundation in Bellinzona, Switzerland, May 24–25, 2002, and published in Italian in [14].

D. A. Grave

After a few personal interviews with Alexander, Grave became convinced of Alexander's exceptional abilities and accepted him — then a boy of 15 years — as a full-fledged member of his seminar. Alexander attended the seminar for three years while, at the same time, completing his studies at the School of Commerce. During this time, with Grave's assistance, he wrote his first mathematical paper, a long memoir on Galois fields, written in Ukrainian, which a few years later (in 1913) appeared in print.

When the time came to enroll at the university, Ostrowski was denied entrance to the University of Kiev on purely bureaucratic grounds: he graduated from the School of Commerce and not from High School! This prompted Grave to write to E. Landau and K. Hensel and to ask for their help. Both responded favorably, inviting Ostrowski to come to Germany. Ostrowski opted for Hensel's offer to study with him at the University of Marburg. Two years into his stay at Marburg, another disruptive event occurred — the outbreak of World War I — which left Ostrowski a civil prisoner. Only thanks to the intervention of Hensel, the restrictions on his movements were eased somewhat, and he was allowed to use the university library. That was all he really needed. During this period of isolation, Ostrowski almost single-handedly developed his now famous theory of valuation on fields.

Alexander, ca. 1915

After the war was over and peace was restored between the Ukraine and Germany, Ostrowski in 1918 moved on to Göttingen, the world center of mathematics at that time. There, he soon stood out among the students by his phenomenal memory and his already vast and broadly based knowledge of the mathematical literature. One student later recalled that the tedious task of literature search, in Göttingen, was extremely simple: all one had to do was to ask the Russian student Alexander Ostrowski and one got the answer — instantly and exhaustively! At one time, he even had to come to the rescue of David Hilbert, when during one of his lectures Hilbert needed, as he put it, a beautiful theorem whose author unfortunately he could not recall. It was Ostrowski who had to whisper to him: "But, Herr Geheimrat, it is one of your own theorems!"

David Hilbert

Not surprisingly, therefore, Felix Klein, always keen in recognizing young talents, became interested in Ostrowski, took him on as one of his assistants, and entrusted him, together with R. Fricke, with editing the first volume of his collected works. In 1920, Ostrowski graduated *summa cum laude* with a thesis written under the guidance of Hilbert and Landau. This, too, caused quite a stir, since it answered, in part, Hilbert's 18th problem. Ostrowski succeeded in proving, among other things, that the Dirichlet zeta series $\zeta(x,s) = 1^{-s}x + 2^{-s}x^2 + 3^{-s}x^3 + \cdots$ does not satisfy an algebraic partial differential equation.

After his graduation, Ostrowski left Göttingen for Hamburg, where as assistant of E. Hecke he worked on his Habilitation Thesis. Dealing with

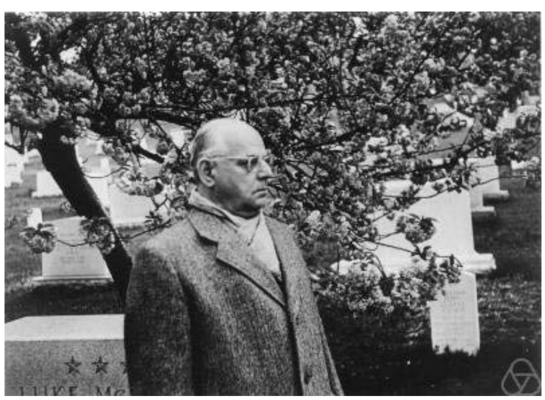
Ostrowski, the skater

modules over polynomial rings, this work was also inspired by Hilbert. The habilitation took place in 1922, at which time he returned to Göttingen to teach on recent developments in complex function theory and to receive habilitation once again in 1923. The academic year 1925–26 saw him as a

Rockefeller Research Fellow at Oxford, Cambridge, and Edinburgh. Shortly after returning to Göttingen, he received —and accepted— a call to the

Ostrowski, in his 40s and 50s, and at 60

University of Basel. The local newspaper (on the occasion of Ostrowski's 80th birthday) could not help recalling that 200 years earlier, the university lost Euler to St. Petersburg because, according to legend, he found himself at the losing end of a lottery system then in use for choosing candidates (in reality, he was probably considered too young for a professorship at the university). Now, however, the university hit the jackpot by bringing Ostrowski from Russia to Basel!



Ostrowski, Washington, D.C., 1964

Ostrowski remained in Basel for his entire academic career, acquiring the Basel citizenship in 1950. It was here where the bulk of his mathematical work unfolded. Much of it lies in the realm of pure mathematics, but important impulses received from repeated visits to the United States in the late forties and early fifties stirred his interest in more applied problems, particularly numerical methods in conformal mapping and problems, then emerging, relating to the iterative solution of large systems of linear algebraic equations. He went about this work with great enthusiasm, even exuberance, having been heard, in the halls of the *National Bureau of Standards*, to exclaim Gottfried Keller's lines "Trinkt, o Augen, was die Wimper hält, von dem goldnen Überfluss der Welt!"[1]. And indeed, exciting problems of pressing significance began to burst forward at this time and demanded nothing less than farsighted and imaginative uses of advanced mathematical techniques.

Margret Ostrowski, 1970

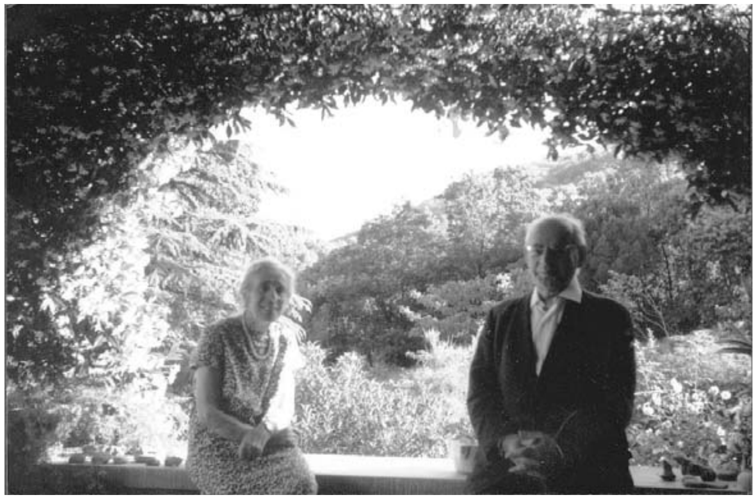
In 1949, Ostrowski married Margret Sachs, a psychoanalyst from the school of Carl Gustav Jung and at one time, as she once revealed to me, a secretary and confidante of Carl Spitteler[2]. Her warm and charming personality greatly helped soften the severe lifestyle of Ostrowski, the scholar, and brought into their lives some measure of joyfulness. This, in fact, is the time the author got to know the Ostrowskis, having become his student and assistant, and, on several occasions, having had the pleasure of being a guest at their house in the old part of the city.

Ostrowski retired from the University in 1958. This did not bring an end to his scientific activities. On the contrary! He continued, perhaps at an even accelerated pace, to produce new and important results until his late eighties. At the age of 90, he was still able to oversee the publication by Birkhäuser of his collected papers, which appeared 1983–85 in six volumes.

75th birthday, Buffalo

After Ostrowski's retirement, he and his wife took up residence in Montagnola, where they earlier had built a beautiful villa — Casa Almarost (ALexander MARgret OSTrowski), as they named it — overlooking the Lake of Lugano. They were always happy to receive visitors at Almarost, and their gracious hospitality was legendary. Mrs. Ostrowski, knowing well the inclinations of mathematicians, always led them down to Ostrowski's library in

[1]As recalled, and kindly related to the author, by Olga Taussky-Todd.
[2]Swiss poet (1845–1924), 1919 Nobel Laureate in Literature.



Margret and Alexander Ostrowski at Almarost

order to leave them alone for a while, so they could catch up on the newest mathematics and mathematical gossip. The walls of the library were filled with books, not all mathematical, but also a good many on science fiction and mystery stories, Ostrowski's favored pastime reading.

Mrs. Ostrowski passed away in 1982, four years before Ostrowski's death in 1986. They are buried in the lovely cemetery of Gentilino, not far from the grave of Hermann Hesse, with whom they were friends.

Ostrowski's merits are not restricted to research alone; they are eminent also on the didactic level, and he exerted a major influence on mathematical publishing. With regard to teaching, his three volumes on the differential and integral calculus [22], which began to appear in the mid-1940s, and in particular the extensive collection of exercises, later published separately with solutions [23], are splendid models of mathematical exposition, which still today serve to educate generations of mathematicians and scientists. His book on the solution of nonlinear equations and systems of equations, published in the United States in 1960 and going through several edi-

Ostrowski at the age of 90

tions [24], [25], continues to be one of the standard works in the field. And last but not least, he had well over a dozen doctoral students, some having attained international stature of their own, and all remaining grateful to him for having opened to them the beauty of mathematics and imparted on them his high standards of intellectual integrity. On the publishing front, Ostrowski was a long-time consultant to the Birkhäuser-Verlag and was instrumental in establishing and supervising their well-known Green Series of textbooks. To a good extent, he can be credited for Birkhäuser having attained the leading position it now occupies in mathematical publishing.

Cemetery of Gentilino

Ostrowski's achievements did not remain unrecognized. He was awarded three honorary doctorates, one from the Federal Institute of Technology (ETH Zurich) in 1958, one from the University of Besançon in 1967, and another in 1968 from the University of Waterloo.

In the early 1980s Professor and Mrs. Ostrowski established an International Prize to be awarded every two years after their deaths [13]. It is to recognize the best achievements made in the preceding five years in Pure Mathematics and the theoretical foundations of Numerical Analysis. So far, eleven prizes have been awarded, the first in 1989 to Louis de Branges for his proof of the Bieberbach conjecture, the fourth in 1995 to Andrew Wiles for his proof of Fermat's last theorem. Characteristically of Ostrowski's view of mathematics as an international and universal science, he expressly stipulated that the award should be made "entirely without regard to politics, race, religion, place of domicile, nationality, or age." This high esteem of scientific merits, regardless of political, personal, or other shortcomings of those attaining them, came across already in 1949, when he had the courage of inviting Bieberbach — then disgraced by his Nazi past and ostracized by the European intelligentsia — to spend a semester as guest of the University of Basel and conduct a seminar on geometric constructions. Undoubtedly, it was Ostrowski who successfully persuaded Birkhäuser to publish the seminar in book form [3].

2. His work

Let us now take a quick look at Ostrowski's mathematical work. A first appreciation of the vast scope of this work can be gained from the headings in the six volumes of his collected papers [27]:

Vol. 1 Determinants, Linear Algebra, Algebraic Equations;

Vol. 2 Multivariate Algebra, Formal Algebra;

Vol. 3 Number Theory, Geometry, Topology, Convergence;

Vol. 4 Real Function Theory, Differential Equations, Differential Transformations;

Vol. 5 Complex Function Theory;

Vol. 6 Conformal Mapping, Numerical Analysis, Miscellany.

Much of this work is at the highest levels of mathematics and can be indicated here only by key words and phrases. The same applies to work that, although more accessible, is difficult to adequately summarize in a few words. From the remaining papers, a few results are selected in chronological order and briefly sketched in “excerpts”, hoping in this way to provide a glimpse into Ostrowski's world of mathematics. We go through this work volume by volume and add dates to indicate the period of his life in which the respective papers have been written.

2.1. Volume 1

Key words: Sign rules of Descartes, Budan–Fourier, and Runge (1928–65); critique and correction of Gauss's first and fourth proof of the Fundamental Theorem of Algebra (1933); long memoir on Graeffe's method (1940); linear iterative methods for symmetric matrices (1954); general theory of vector and matrix norms (1955); convergence of the Rayleigh quotient iteration for computing real eigenvalues of a matrix (1958–59); Perron–Frobenius theory of nonnegative matrices (1963–64)

Excerpt 1.1. Matrices with *dominant diagonal* (1937),

$$A = [a_{ij}], \quad d_i := |a_{ii}| - \sum_{j \neq i} |a_{ij}| > 0, \quad \text{all } i.$$

Hadamard in 1899 proved that for such matrices $\det A \neq 0$. Ostrowski sharpens this to $|\det A| \geq \prod_i d_i$.

Excerpt 1.2. *M-matrices* (1937),

$$A = [a_{ij}], \quad a_{ii} > 0, \quad a_{ij} \le 0 \ \ (i \ne j),$$

$$a_{11} > 0, \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \quad \dots, \quad \det A > 0.$$

Theorem. *If A is an M-matrix, then $A^{-1} \ge \mathbf{0}$.*

The theory of M-matrices and the related theory of H-matrices, stemming from Ostrowski's 1937 paper, have proved to be powerful tools in the analysis of iterative methods for solving large systems of linear equations. In addition, this theory forms the basis for the general theory of eigenvalue inclusion regions for matrices, as in the case of the well-known Gershgorin Theorem. See also Excerpt 2.2.

Excerpt 1.3. *Continuity of the roots* of an algebraic equation (1939).

It is well known that the roots of an algebraic equation depend continuously on the coefficients of the equation. Ostrowski gives us a quantitative formulation of this fact.

Theorem. *Let x_ν, y_ν be the zeros of*

$$p(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n, \quad a_0 a_n \ne 0,$$

resp.

$$q(z) = b_0 z^n + b_1 z^{n-1} + \cdots + b_n, \quad b_0 b_n \ne 0.$$

If

$$b_\nu - a_\nu = \varepsilon_\nu a_\nu, \quad |\varepsilon_\nu| \le \varepsilon, \quad 16 n \varepsilon^{1/n} \le 1,$$

then

$$\left| \frac{x_\nu - y_\nu}{x_\nu} \right| \le 15 n \varepsilon^{1/n}.$$

Excerpt 1.4. Convergence of the *successive overrelaxation method* (1954).

The iterative solution of large (nonsingular) systems of linear algebraic equations

$$Ax = b, \quad A \in \mathbb{R}^{n \times n}, \ b \in \mathbb{R}^n,$$

was an object of intense study in the 1950s culminating in the "successive overrelaxation method" (SOR)

$$Dx^{k+1} = \omega(b - Lx^{k+1} - Ux^k) - (\omega - 1)Dx^k, \quad k = 0, 1, 2, \dots,$$

where ω is a real parameter and D, L, U are, respectively, the diagonal, lower triangular, and upper triangular part of A. The method is said to converge if $\lim_{k\to\infty} x^k = A^{-1}b$ for arbitrary b and arbitrary $x^0 \in \mathbb{R}^n$.

Ostrowski–Reich Theorem. *If A is symmetric with positive diagonal elements, and $0 < \omega < 2$, then SOR converges if and only if A is positive definite.*

Reich proved the theorem for $\omega = 1$ in 1949. Ostrowski proved it for general ω in $(0, 2)$, even when $\omega = \omega_k$ depends on k but remains in any compact subinterval of $(0, 2)$.

Excerpt 1.5. A little mathematical jewel (1979).

Theorem. *Let p and q be polynomials of degrees m and n, respectively. Define*

$$M_f = \max_{|z|=1} |f(z)|.$$

Then

$$\gamma M_p M_q \le M_{pq} \le M_p M_q, \quad \gamma = \sin^m \frac{\pi}{8m} \sin^n \frac{\pi}{8n}.$$

The interest here lies in the lower bound, the upper one being trivial. It is true that this lower bound may be quite small, especially if m and/or n are large. But jewels need not be useful as long as they shine!

2.2. Volume 2

Key words: Algebra of finite fields (1913); theory of valuation on a field (1913–17); necessary and sufficient conditions for the existence of a finite basis for a system of polynomials in several variables (1918–20); various questions of irreducibility (1922, 1975–77); theory of invariants of binary forms (1924); arithmetic theory of fields (1934); structure of polynomial rings (1936); convergence of block iterative methods (1961); Kronecker's elimination theory for polynomial rings (1977).

The fact, proved by Ostrowski in 1917, that the fields of real and complex numbers are the only fields, up to isomorphisms, which are complete (Ostrowski used the older term "perfect" for "complete") with respect to an Archimedean valuation is known today as "Ostrowski's Theorem" in valuation theory (P. Roquette [31]).

Excerpt 2.1. Evaluation of polynomials (1954). If

$$p(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n,$$

then, by *Horner's rule*, $p(x) = p_n$, where

$$p_0 = a_0, \quad p_\nu = x p_{\nu-1} + a_\nu, \quad \nu = 1, 2, \ldots, n.$$

Complexity: n additions, n multiplications.

Theorem. *Horner's rule is optimal for addition and optimal for multiplication when $n \le 4$.*

It has later been shown by V. Ja. Pan [28] that Horner's scheme indeed is *not* optimal with respect to multiplication when $n > 4$.

Because of this paper, the year 1954 is generally considered "the year of birth of algebraic complexity theory" (P. Bürgisser and M. Clausen [5]).

Excerpt 2.2. Metric properties of *block matrices* (1961),

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix}, \quad A_{\nu\mu} \in \mathbb{R}^{\nu\times\mu}.$$

Question. *Is Hadamard's theorem still valid if* $|\cdot|$ *is replaced by* $\|\cdot\|$?

Answer: Yes, if

$$\begin{bmatrix} \|A_{11}\|^* & -\|A_{12}\| & \dots & -\|A_{1n}\| \\ -\|A_{21}\| & \|A_{22}\|^* & \dots & -\|A_{2n}\| \\ \vdots & \vdots & & \vdots \\ -\|A_{n1}\| & -\|A_{n2}\| & \dots & \|A_{nn}\|^* \end{bmatrix}$$

is an M-matrix, where

$$\|B\|^* = \min_{\|x\|=1} \|Bx\|, \quad \|B\| = \max_{\|x\|=1} \|Bx\|.$$

2.3. Volume 3

Key words: Existence of a "regular" basis for polynomials with coefficients in a finite arithmetic field that take on integer values for integer arguments (1919); arithmetic theory of algebraic numbers (1919); Diophantine equations and approximations (1921–27, 1964–82); existence criterion for a common zero of two real functions continuous inside and on the boundary of a disk (1933); topology of oriented line elements (1935); evolutes and evolvents of a plane curve (1955) and an oval in particular (1957); differential geometry of plane parallel curves (1955); Ermakov's convergence and divergence criteria for $\int^\infty f(x)\,dx$ (1955); necessary and sufficient conditions for two line elements to be connectable by a curve with monotone curvature (1956); behavior of fixed-point iterates in the case of divergence (1956); summation of slowly convergent positive or alternating series (1972).

Excerpt 3.1. Infinite products (1930),

$$x_0 = x, \quad x_{\nu+1} = \varphi(x_\nu), \quad \nu = 0, 1, 2, \dots,$$
$$\prod_{\nu=0}^{\infty} (1 + x_\nu) = \Phi(x).$$

Example. Euler's product $\varphi(x) = x^2$, $\Phi(x) = (1 - x)^{-1}$.

Problem. Determine *all* products which converge in a neighborhood of $x = 0$, and for which φ is rational and Φ algebraic.

Solution: completely enumerated.

Excerpt 3.2. "Normal" power series (1930),

$$\sum_{\nu=-\infty}^{\infty} a_\nu z^\nu \text{ with } a_\nu \geq 0,\ a_\nu^2 \geq a_{\nu-1} a_{\nu+1},$$

and all coefficients between two positive ones are also positive.

Theorem. *The product of two normal power series, if it exists, is also normal.*

2.4. Volume 4

Key words: Dirichlet series and algebraic differential equations, thesis Göttingen (1919); strengthening, or simplifying, proofs of many known results from real analysis (1919–38); various classes of contact transformations in the sense of S. Lie (1941–42); invertible transformations of line elements (1942); conditions of integrability for partial differential equations (1943); indefinite integrals of "elementary" functions, Liouville Theory (1946); convex functions in the sense of Schur with applications to spectral properties of Hermitian matrices (1952); theory of characteristics for first-order partial differential equations (1956); points of attraction and repulsion for fixed-point iteration in Euclidean space (1957); univalence of nonlinear transformations in Euclidean space (1958); a decomposition of an ordinary second-order matrix differential operator (1961); theory of Fourier transforms (1966); study of the remainder term in the Euler–Maclaurin formula (1969-70); asymptotic expansion of integrals containing a large parameter (1975).

A technique introduced in the 1946 paper on Liouville's Theory is now known in the literature as the "Hermite–Ostrowski method" (J. H. Davenport, Y. Siret, and E. Tournier [7]). This work has attained renewed relevance because of its use in formal integration techniques of computer algebra.

Excerpt 4.1. The (frequently cited) *Ostrowski–Grüss inequality* (1970),

$$\left| \int_0^1 f(x)g(x)\,dx - \int_0^1 f(x)\,dx \int_0^1 g(x)dx \right| \le \frac{1}{8} \operatorname*{osc}_{[0,1]} f \max_{[0,1]} |g'|.$$

Excerpt 4.2. Generalized *Cauchy–Frullani integral* (1976),

$$\int_0^\infty \frac{f(at) - f(bt)}{t}\,dt = [M(f) - m(f)] \ln \frac{a}{b}, \quad a > 0,\ b > 0,$$

where

$$M(f) = \lim_{x\to\infty} \frac{1}{x} \int_1^x f(t)\,dt, \quad m(f) = \lim_{x\downarrow 0}\, x \int_x^1 \frac{f(t)}{t^2}\,dt.$$

In the original version of the formula, there were point evaluations, $f(\infty)$ and $f(0)$, in place of the mean values $M(f)$ and $m(f)$.

2.5. Volume 5

Key words: Gap theorems for power series and related phenomena of "overconvergence" (1921–30); investigations related to Picard's theorem (1925–33); quasi-analytic functions, the theory of Carleman (1929); analytic continuation of power series and Dirichlet series (1933, 1955).

Excerpt 5.1. Alternative characterization of *normal families of meromorphic functions* (1925).

Theorem. *A family $\mathcal{F}$ of meromorphic functions is normal* (*i.e., precompact*) *if and only if it is equicontinuous with respect to the spherical metric.*

Excerpt 5.2. *Carleman's theorem* on quasianalytic functions, as reformulated by Ostrowski (1929).

Given a sequence $m = \{m_\nu\}_{\nu=1}^\infty$ of positive numbers m_ν, an infinitely-differentiable function f on $I = [0,\infty)$ is said to belong to the class $C(m)$ if

$$|f^{(\nu)}(x)| \le m_\nu \text{ on } I, \quad \nu = 0, 1, 2, \ldots.$$

The class $C(m)$ is called quasianalytic if $f \in C(m)$ and $f^{(\nu)}(0) = 0$, $\nu = 0, 1, 2, \ldots$, implies $f(x) \equiv 0$ on I.

Ostrowski reformulates, and gives a simplified proof of, one of the main results of Carleman's theory of quasianalytic functions by introducing the function $T(r) = \sup_\nu r^\nu / m_\nu$ (sometimes named after him).

Theorem. *The class $C(m)$ is quasianalytic if and only if*

$$\int_1^\infty \log T(r) \frac{\mathrm{d}r}{r^2} = \infty.$$

Ostrowski's work related to Picard's theorem, though predating R. Nevanlinna's own theory of meromorphic functions, points in the same direction.

2.6. Volume 6

Key words: Constructive proof of the Riemann Mapping Theorem (1929); boundary behavior of conformal maps (1935–36); Newton's method for a single equation and a system of two equations: convergence, error estimates, robustness with respect to rounding (1937–38); convergence of relaxation methods for linear $n \times n$ systems, optimal relaxation parameters for $n = 2$ (1953); iterative solution of a nonlinear integral equation for the boundary function of a conformal map, application to the conformal map of an ellipse onto the disk (1955); "absolute convergence" of iterative methods for solving linear systems (1956); convergence of Steffensen's iteration (1956); approximate solution of homogeneous systems of linear equations (1957); a device of Gauss for speeding up iterative methods (1958); convergence analysis of Muller's method for solving nonlinear equations (1964); convergence of the fixed-point iteration in a metric space in the presence of "rounding errors" (1967); convergence of the method of steepest descent (1967); a descent algorithm for roots of algebraic equations (1969); Newton's method in Banach spaces (1971); a posteriori error bounds in iterative processes (1972–73); probability theory (1946–1980); book reviews, public addresses, obituaries (G. H. Hardy, Wilhelm Süss, Werner Gautschi) (1932–75).

Excerpt 6.1. Matrices close to a triangular matrix (1954),

$$A = [a_{ij}], \quad |a_{ij}| \le m \ (i > j), \quad |a_{ij}| \le M \ (i < j), \quad 0 < m < M.$$

The limit case $m = 0$ corresponds to a triangular matrix with its eigenvalues being the elements on the diagonal. If m is small, one expects the eigenvalues to remain near the diagonal elements. This is expressed by Ostrowski in the following way.

Theorem. *All eigenvalues of A are contained in the union of disks $\bigcup_i D_i$, $D_i = \{z \in \mathbb{C} : |z - a_{ii}| \le \delta(m, M)\}$, where*

$$\delta(m, M) = \frac{M m^{\frac{1}{n}} - m M^{\frac{1}{n}}}{M^{\frac{1}{n}} - m^{\frac{1}{n}}}.$$

The constant $\delta(m, M)$ is best possible.

Excerpt 6.2. The *Moivre–Laplace formula* (1980). If

$$M(n) = \sum_{|\nu - np| \le \eta\sqrt{2npq}} \binom{n}{\nu} p^\nu q^{n-\nu}, \quad 0 < p < 1,\ p + q = 1,\ n > 0,$$

then

$$M(n) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-t^2} dt + \rho(\eta, n),$$

where

$$\rho(\eta, n) = \frac{r_n}{\sqrt{2\pi npq}} e^{-\eta^2} + O(1/n), \quad n \to \infty,$$

and, with $R(x) = x - \lfloor x \rfloor$,

$$r_n = 1 - R(nq + \eta\sqrt{2npq}) - R(np + \eta\sqrt{2npq}).$$

The numbers r_n are everywhere dense in $[-1, 1]$. Prior to Ostrowski's work, the formula has been stated (incorrectly) with 1 in place of r_n.

3. His students

Professor Ostrowski has been the primary advisor ("Referent") for the doctoral students listed below. All dissertations, except one, were written at the Faculty of Mathematics and Natural Sciences of the University of Basel. (The exception is the thesis by Willy Richter.)

1932 Stefan Emanuel Warschawski (1904–1989)
"Über das Randverhalten der Ableitung der Abbildungsfunktion bei konformer Abbildung"

1933 Alwin von Rohr (1903–2001)
"Über die Hilbert–Story'schen invariantenerzeugenden Prozesse"

1934 Leo Leib Krüger (1903–?)
"Über eine Klasse von kontinuierlichen Untergruppen der allgemeinen linearen homogenen projektiven Gruppe des $(2N-1)$-dimensionalen Raumes"

1936 Theodor Samuel Motzkin (1908–1970)
"Beiträge zur Theorie der linearen Ungleichungen"

1938 Caleb Gattegno (1911–1988)
"Le cas essentiellement géodésique dans les équations de Hamilton–Jacobi intégrables par séparation des variables"

1938 Fritz Blumer (1904–1988)
"Untersuchungen zur Theorie der halbregelmässigen Kettenbruchentwicklungen, I & II"

1944 Eduard Batschelet (1914–1979)
"Untersuchungen über die absoluten Beträge der Wurzeln algebraischer, insbesondere kubischer Gleichungen"

1945 Gerhard Stohler (1915–1999)
"Über eine Klasse von einparametrigen Differential-Transformationsgruppen"

1948 Rolf Conzelmann (1916–)
"Beiträge zur Theorie der singulären Integrale bei Funktionen von mehreren Variablen, I & II"

1949 Karl-Felix Moppert (1920–1984)
"Über Relationen zwischen m- und p-Funktionen"

1951 Hermann Georg Wundt (1921–?)
"Eine neue Methode der Periodogramm-Analyse und ihre Anwendung auf die Reihe der Sonnenflecken-Relativzahlen"

1952 Willy Richter (1915–1998)
"Estimation de l'erreur commise dans la méthode de M. W. E. Milne pour l'intégration d'un système de n équations différentielles du premier ordre" (Thèse, Faculté des Sciences, Université de Neuchâtel)

1953 Rudolf Thüring (1924–)
"Studien über den Holditchschen Satz"

1954 Werner Gautschi (1927–1959)
"On norms of matrices and some relations between norms and eigenvalues"

1954 Walter Gautschi (1927–)
"Analyse graphischer Integrationsmethoden"

1959 Hans Richard Gutmann (1907–2001)
"Anwendung Tauberscher Sätze und Lambertscher Reihen in der zahlentheoretischen Asymptotik"

Many of these students have had successful careers either in academia or in secondary school education. Like Ostrowski himself, some of the earlier students came to Basel from abroad: Warschawski from Königsberg; Krüger from Riga; Motzkin from Berlin; and Gattegno from Alexandria, Egypt. All the other students, except Wundt, a native of Aalen, Württemberg, were born and grew up in, or near, Basel.

We have no information about the careers of *von Rohr*, *Krüger*, and *Wundt*.

Warschawski became a Ph.D. student of Ostrowski while the latter was still in Göttingen, and moved with him to Basel, where he completed his thesis in 1932. He returned to Göttingen to start his teaching career but was forced to escape from Nazi persecution. He was able, eventually, to reach the United States, where he developed into a highly respected researcher in the area of conformal mapping. He also distinguished himself as a successful academic administrator by building up to prominence two departments of mathematics, one at the University of Minnesota, the other at the University of California at San Diego. For a biography, see [21].

Motzkin, the son of Leo Motzkin, a prominent member of the Zionist movement who participated at the First Zionist Congress (1897) in Basel and in his youth started on a doctoral dissertation under Kronecker, after completion of his thesis moved to the Hebrew University in Jerusalem, where during World War II he worked as a cryptographer for the British government. In 1948 he emigrated to the United States, where in 1950 he became a member of the Institute of Numerical Analysis at the University of California at Los Angeles and a professor ten years later. Motzkin's work as a mathematician is widely recognized to be brilliant and ingenious. Extremely versatile, he contributed significantly to fields such as linear programming, combinatorics, approximation theory, algebraic geometry, number theory, complex function theory, and numerical analysis. Motzkin numbers and Motzkin paths are mathematical objects still studied extensively in today's literature. See [1] for an obituary.

Gattegno turned his attention to the psychology and didactics of teaching in general, and of teaching mathematics, reading and writing, and foreign languages, in particular. He promoted his innovative and unorthodox approaches in more than 50 books and other publications, conducted seminars throughout the world, founded numerous organizations, and produced relevant teaching material. He earned a second doctorate in psychology in 1952 from the University of Lille. In 1965, Gattegno moved to New York, where he established an educational laboratory and continued his pedagogical activities. For more on Gattegno's life and work, see [29].

Batschelet was a teacher at the Humanistischen Gymnasium Basel from 1939 to 1960 and a Privatdozent at the University of Basel from 1952 to 1957. In 1958 he was awarded the title of extraordinary professor and two years later moved to Washington, D.C. to assume a professorship at the Catholic University. He returned to Switzerland in 1971 where he became professor of mathematics at the University of Zurich. His field of research was statistics and biomathematics; he taught and wrote successful textbooks in this area. See [18] for an obituary.

Moppert, after five years of teaching at schools in Basel, emigrated to Australia, where he assumed a lectureship at the University of Tasmania and in 1958 became a senior lecturer in mathematics at the University of Melbourne. In 1967 he joined the Department of Mathematics at Monash University, where he remained until his death. His mathematical work addressed Riemann surfaces — his thesis topic — and miscellaneous other topics including operators in Hilbert space, Diophantine analysis, Brownian motion, and Euclidean and non-Euclidean geometry. He had a knack for scientific instruments, of which a sundial mounted on one of the walls of the Union Building at Monash, "often a better indicator of the correct time than most other clocks on campus" [6], remains a lasting witness.

Werner Gautschi, a twin brother of the author, emigrated in 1953 to the United States, where during postdoctoral years at Princeton University and the University of California at Berkeley he worked himself into the areas of mathematical statistics and probability theory. He started his academic career in 1956 at Ohio State University, moved to Indiana University at Bloomington in 1957 and two years later back to Ohio State University. Soon after he arrived there, a massive heart attack put an abrupt end to his life and to a very promising career. See [4] and [26] for obituaries.

Walter Gautschi, after two years of postdoctoral work in Rome and at Harvard University, took on positions as a research mathematician at (what was then called) the National Bureau of Standards in Washington, D.C. and at Oak Ridge National Laboratory, Oak Ridge, Tennessee. In 1963 he accepted a professorship in mathematics and computer sciences at Purdue University, where he remained until his retirement in 2000. He worked in the areas of special functions, constructive approximation theory, and numerical analysis, as documented in [15].

Among the students who chose a teaching career at schools in Basel are *Blumer*, Humanistisches Gymnasium (HG), 1932–1973; *Stohler*, Mädchengymnasium (MG) (later Holbein-Gymnasium), 1946–1980; *Conzelmann*, HG (later Mathematisch-Naturwissenschaftliches Gymnasium (MNG)), 1949–1982; *Thüring*, Realgymnasium (RG), 1956–1986; *Gutmann*, RG, 1935–1970 (rector thereof from 1962–1970). Both, Blumer and Conzelmann held also academic positions at the University of Basel, the former a lectorship from 1960 to 1974, the latter a Lehrauftrag in 1956/57, a lectorship from 1958 to 1974, and an extraordinary professorship from 1975 until his retirement in 1984. *Richter*, injured in a military accident and battling tuberculosis, absolved his university studies by correspondence in the military sanatorium of Novaggio and the sanatorium in Leysin during World War II and wrote most of his thesis on the sick-bed. He became a teacher in Neuchâtel, for a few years at the École de Commerce and then at the Gymnase Cantonal until his retirement in 1978.

Ostrowski is listed as secondary advisor (“Korreferent”) to the following students:

1931	Heinrich Johann Ruch (1895–1960) “Über eine Klasse besonders einfacher Modulargleichungen zweiten Grades von der Form $y^2 = R(x)$” (Referent: Otto Spiess)
1942	Ernst Fischer (1914–2000) “Das Zinsfussproblem der Lebensversicherungsrechnung als Interpolationsaufgabe” (Referent: Ernst Zwinggi)
1947	Heinz Hermann Müller (1913–1996) “Scharfe Fassung des Begriffes faisceau in einer gruppentheoretischen Arbeit Camille Jordans” (Referent: Andreas Speiser)
1955	Mario Gottfried Howald (1925–2001) “Die akzessorische Irrationalität der Gleichung fünften Grades” (Referent: Andreas Speiser)

Nothing is known to us about the curricula vitae of these students except for *Howald*, who was teaching at the MNG from 1951 to 1990 (in between for four years at the Gymnasium Bäumlihof). For two years (1962–63) he was working at the Natural Science section of the Goetheanum in Dornach. Besides his teaching activity at the Gymnasien, Howald regularly organized courses in Carona (near Lugano) for amateur astronomers. He is the author of two informative articles [16], [17] on Maupertuis’s Lapland expedition to measure the length of a meridional degree that led to the affirmation of the flatness of the earth near the poles. He also edited, and wrote commentaries to, Daniel Bernoulli’s work on positional astronomy [2] and from 1997 to his death was a member of the Curatorium of the Otto Spiess foundation which supports the Bernoulli edition.

4. Epilogue

To conclude, let me make a few general remarks about Ostrowski’s work. Apart from the kaleidoscopic variety of themes treated by him, a characteristic quality of his work is a strong desire to go to the bottom of things, to unravel the essential features of a problem and the basic concepts needed to deal with it in a satisfactory manner. This is coupled with a relentless drive to be exhaustive. Notable are also his frequent attempts to establish results, even entirely classical ones, under the weakest assumptions possible, and his delight in finding proofs that are short and succinct. A good part of Ostrowski's work has a definite constructive bent, and all of it exhibits a masterly skill in the use of advanced mathematical techniques, particularly analytic techniques of estimation. His work bears the stamp of scholarly thoroughness, coming from a careful study of the literature, not only the contemporary literature, but also, and perhaps more importantly, the original sources.

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