Linear Bounded Automata

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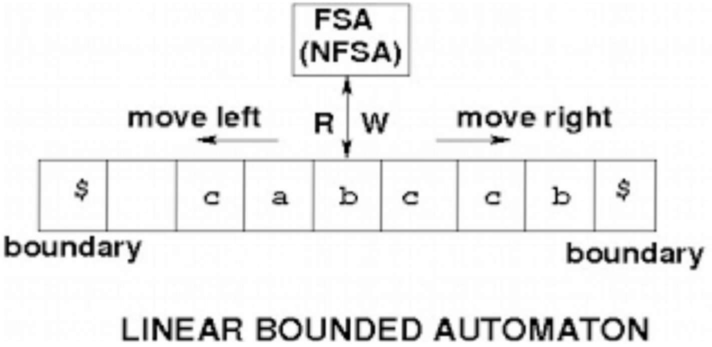
Overview

- Definition
- Results about LBAs

Definition

- A Turing machine that uses only the tape space occupied by the input is called a linear-bounded automaton (LBA).



- A linear bounded automaton is a non-deterministic Turing machine M = (Q, Σ, Γ, δ, s, t, r) such that:

  * There are two special tape symbols < and >(the left end marker and right end marker).

  * The TM begins in the configuration (s, < x >, 0).

  * The TM cannot replace < or > with anything else, nor move the tape head left of < or right of >.

LBA

- An equivalent definition of an LBA is that it uses only k times the amount of space occupied by the input string, where k is a constant fixed for the particular machine.

- Possible to simulate k tape cells with a single tape cell, by increasing the size of the tape alphabet

- Examples: $\{a^n \mid n \text{ is a perfect square}\}$

- Used as a model for actual computers rather than models for the computational process.

Number of configurations

- Suppose that a given LBA M has
  - q states,
  - m characters in the tape alphabet ,
  - the input length is n
- Then M can be in at most

  $\alpha(n) = q * n * m^n$ configurations

  - i.e. With m symbols and a tape which is n cells long, we can have only m^n different tapes.

  - The tape head can be on any of the n cells and we can be executing any of the q states

Results about LBA

- Theorem 1: The halting problem is solvable for LBA.
  - Idea for proof
    - The number of possible configurations for an lba
    - LBA on input w must stop in at most $\alpha(|w|)$ steps

- Corollary: The membership problems for sets accepted by linear bounded automata are solvable

Results about LBA

- Lemma: For any non-deterministic linear bounded automaton there is another which can compute the number of configurations reachable from an input.

  - idea for proof:

    - Enumerate all possible configuration
    - Check whether the nlba can get to them for a given input 'w'

- Theorem 2: The class of sets accepted by non-deterministic LBA is closed under complement.

  - idea for proof:

    1. Find out exactly how many configurations are reachable
    2. examine all of them and if any halting configurations are encountered, reject
    3. Otherwise accept

Results about LBA

Lemma: For every Turing machine there is a linear bounded automaton which accepts the set of strings which are valid halting computations for the Turing machine.

Theorem 3.The emptiness problem is unsolvable for linear bounded

Proof.

- If a Turing machine accepts no inputs then it does not have any valid halting computations.
- Thus the linear bounded automaton which accepts the Turing machine's valid halting computations accepts nothing.
- This means that if we could solve the emptiness problem for linear bounded automata then we could solve it for Turing machines.

Thank You