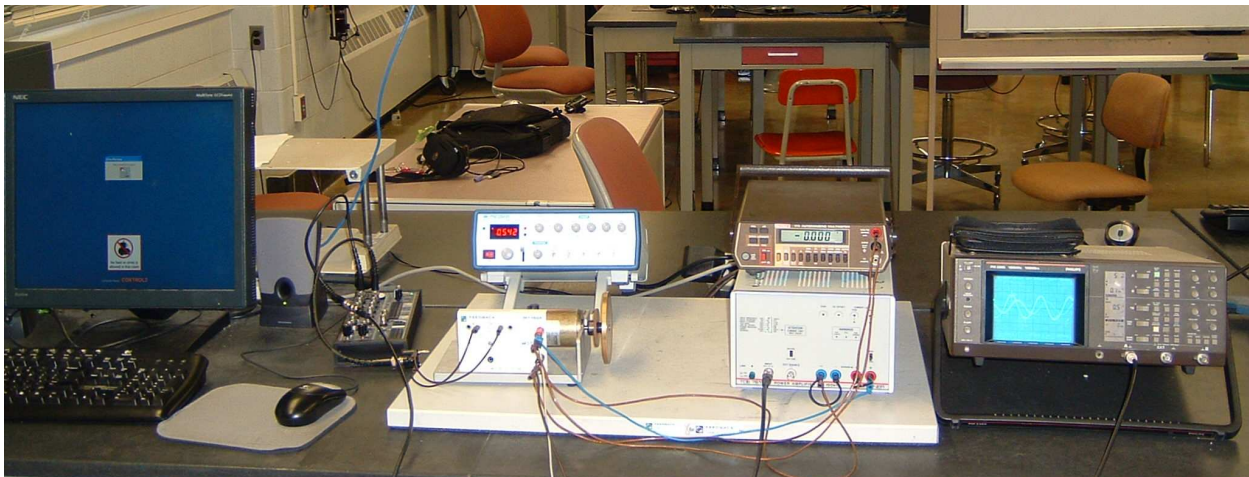


ME 451: Control Systems Laboratory

Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48824-1226

ME451 Laboratory Experiment #4

Sinusoidal Response of a First Order Plant DC Servo Motor



***** Please bring your Experiment #1 Short Form for this lab*****

ME451 Laboratory Manual Pages, Last Revised: March 31, 2008
Send comments to: Dr. Clark Radcliffe, Professor

References:

- C.L. Phillips and R.D. Harbor, *Feedback Control Systems*, Prentice Hall, 4th Ed.
Section 2.7, pp. 38-43: Electromechanical Systems
Section 4.1, pp. 116-120: Time Response of First Order Systems
Section 4.4, pp. 129-132: Frequency Response of Systems
Section 8.1, 8.2, pp. 275-293: Frequency Responses, Bode Diagrams
Appendix B, pp. 635-650: Laplace Transform

1. Objective

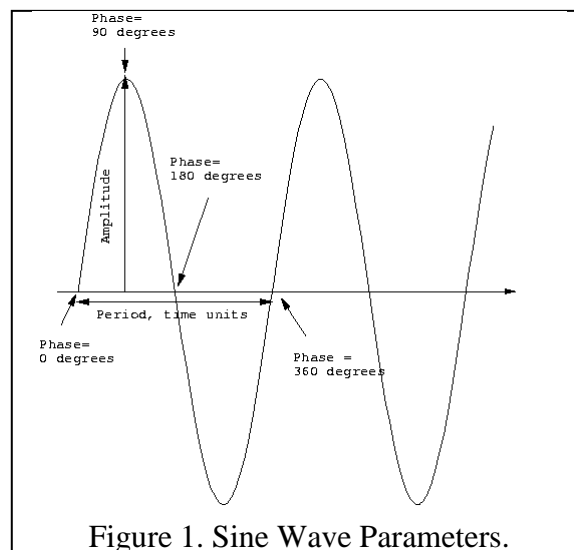
The response of a linear system to a sinusoidal input is useful for predicting its behavior for arbitrary periodic inputs, but more importantly, for compensator design. For first-order systems, the sinusoidal response depends on the DC gain, K , but primarily on the time constant, τ . Both K and τ are functions of system parameters. The objective of this experiment is to investigate the effect of system parameters on system response to a sinusoidal input.

We will experiment with an armature controlled DC servo motor, which we approximate will behave as a first-order system with voltage as the input and angular velocity as the output. The transfer function of the system will be obtained to identify specific parameters that affect sinusoidal response. We seek primarily the system parameters which affect the gain and time constant. We will vary these parameters to experimentally verify the change in the sinusoidal response.

2. Background

2.1. Sinusoidal Command:

A sine wave: is characterized by 3 parameters: amplitude A_C , period T (seconds) and phase ϕ (radians)



The phase ϕ (radians) of a sine wave is a relative quantity; since the sine function can take any argument and has no absolute starting point. Mathematically, a sine wave $y(t)$ varying with time t is described by:

$$y(t) = A_c \sin(\omega t + \phi) \quad (1)$$

Here A_c is the amplitude and can have any units (feet, volts, psi, etc.) the quantity $y(t)$ represents in its physical form. The phase ϕ has units of angle, either degrees or radians. In the equation as written ω has a unit of angle per unit time, because the quantity ωt must have units of angle. The angular frequency ω (rad/sec) is 2π times the circular frequency f (cycles/sec = Hertz). The circular frequency, f (Hz) is the inverse of the wave period (seconds/cycle).

$$\omega = 2\pi f = 2\pi/T \quad (2)$$

2.2. Sinusoidal response:

Consider a linear process with a sinusoidal input, whose output is observed. Below we examine the output and input to observe potential differences in the signals.

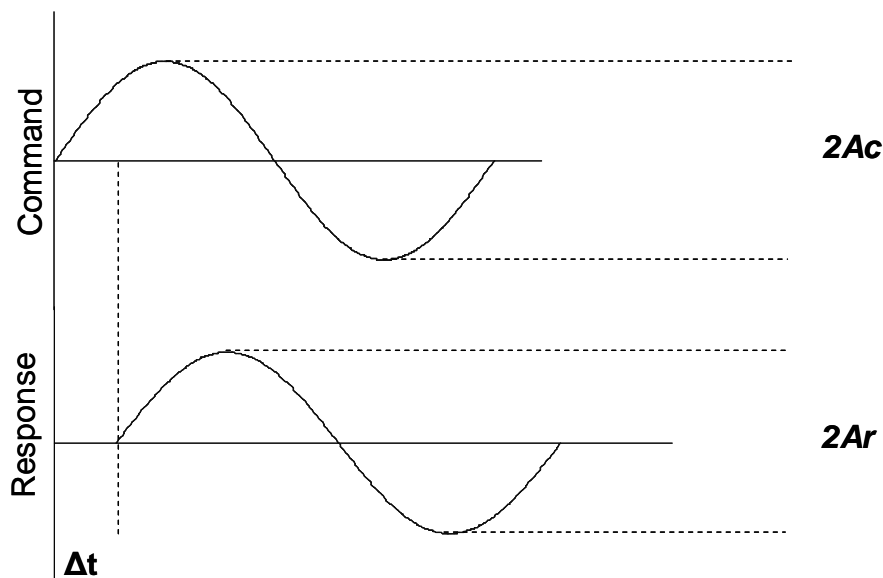


Figure 2. Sinusoidal Command versus Sinusoidal Response.

- The output will be a sinusoid of the same frequency as the input.
- The ratio of the output amplitude to that of the input amplitude (often called process gain) will in general vary with the frequency of the sine wave input.
- The difference in phase between the input and output sine waves will also depend on the frequency.

We call the ratio of the output amplitude to input amplitude for any given time the process **Gain**. This is a function of time, as the gain changes throughout the wave if there is any phase angle (described below with eq. (4)).

$$G(j\omega) = \frac{y_R(t)}{y_C(t)} = \frac{A_R \sin(\omega_R t + \phi_R)}{A_C \sin(\omega_C t + \phi_C)} \quad (3)$$

The **gain amplitude** is defined as $|G(j\omega)| = A_R/A_C$ and the difference between input and output phases is the **phase angle**

$$\angle G(j\omega) = (\phi_R - \phi_C) = \frac{2\pi}{T} \Delta t \text{ (radians)} = \frac{360}{T} \Delta t \text{ (degrees)} \quad (4)$$

2.3. Frequency response of first-order systems to sinusoidal commands:

The standard form of transfer function of a first-order system is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \quad (5)$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of the output and input variables, respectively, K is the DC gain, and τ is the time constant. For a sinusoidal input

$$u(t) = A \sin(\omega t), \quad U(s) = \frac{A \omega}{s^2 + \omega^2} \quad (6)$$

The response of the system, in Laplace domain, can be written as

$$Y(s) = \frac{KA\omega}{(s^2 + \omega^2)(\tau s + 1)} \quad (7)$$

Assuming poles of $G(s)$ are in the left-half plane, the steady-state response of the system (after transients have decayed) can be written as

$$y(t) = A |G(j\omega)| \sin(\omega t + \phi), \quad \phi = \angle G(j\omega) \quad (8)$$

It is clear from (8) that a sinusoidal input produces a sinusoidal output. The amplitude of the output is scaled by a factor of $|G(j\omega)|$ and the phase lags behind the input by $\angle G(j\omega)$. For the standard first-order system in (5), given the values of K and τ , the “gain” $|G(j\omega)|$ and the “phase” $\angle G(j\omega)$ can be expressed as a function of ω ,

$$|G(j\omega)| = \frac{K}{\sqrt{1 + \omega^2 \tau^2}} \quad \angle G(j\omega) = -\arctan(\omega\tau) \quad (9)$$

For a process we may plot $|G(j\omega)|$ and $\angle G(j\omega)$ as functions of frequency together in what is called a Frequency Response Diagram. Amplitude ratio $|G(j\omega)|$ may be thought of as the frequency dependent process gain. It may be expressed in dimensional or dimensionless form.

The latter is preferred. In practice the interesting range of $|G(j\omega)|$ may cover several orders of magnitude. For this reason it is often expressed on a logarithmic scale in decibels (dB). This can only be used when the $|G(j\omega)|$ has been made dimensionless, and the relevant definition is:

$$Gain(dB) = 20\text{Log}_{10}(|G(j\omega)|) \quad (10)$$

On a logarithmic scale the open loop gains $|G(j\omega)|$ and $\angle G(j\omega)$ can be plotted to generate what are known as gain and phase plots, or Bode diagrams. The frequency response diagrams for a standard first-order system are shown in Fig.3. Notice on these diagrams that for a first order system, there are no peaks in gain, the gain roll off (slope as ω goes to ∞) is always 20 dB/dec, and the phase lag does not drop below -90° .

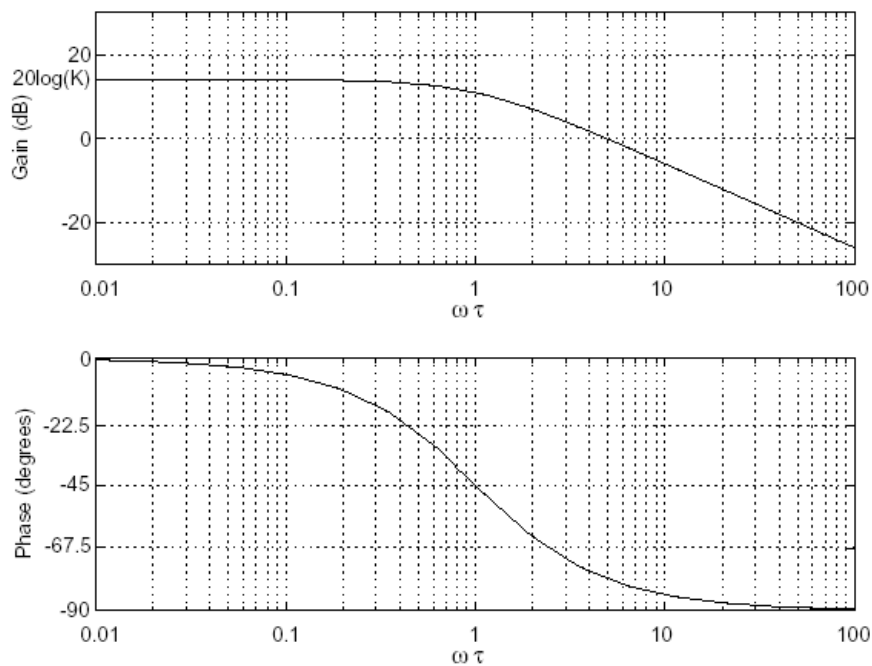


Figure 3. Frequency response of a first-order system to a sinusoidal command.

2.4. DC servo motor system

Recall the armature controlled DC servo motor in laboratory experiment #1 and shown here again in Fig.4. The system variables are

- e_a armature voltage
- e_b back EMF
- i_a armature current
- T torque produced by motor
- θ angular position of motor shaft
- Ω angular velocity of motor shaft

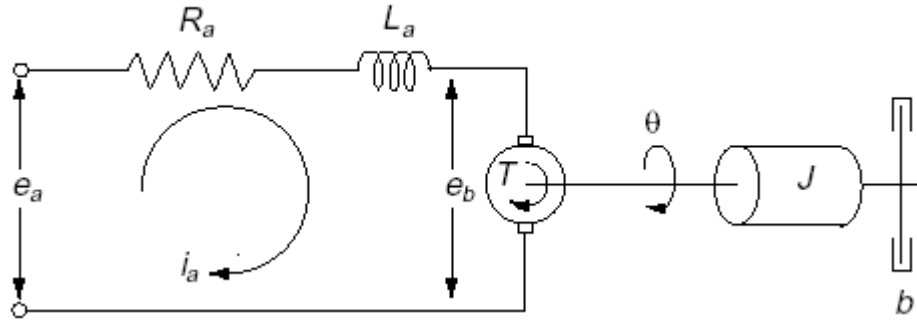


Figure 4: Conceptual Model of the DC Servo-motor

The parameters of the system, shown in Fig. 4, include

- R_a armature resistance
- L_a armature inductance
- J moment of inertia of motor shaft
- b coefficient of viscous friction

The system parameters, not shown in Fig.4, include

- K_T torque constant
- K_b back EMF constant

The torque constant K_T models the relationship between the electric current i_a and motor torque T while the back EMF constant K_b models the relationship between the motor speed Ω and the electrical back emf produced by the DC motor,

$$T = K_T i_a \quad (11a)$$

$$e_b = K_b \Omega \quad (11b)$$

The transfer function of the servo motor, with armature drive e_a as input and motor speed Ω as output, can be written (Phillips and Harbor, Chapter 2) as

$$G(s) = \frac{\Omega(s)}{e_a(s)} = \frac{K_T}{JL_a s^2 + (bL_a + JR_a)s + (bR_a + K_T K_b)} \quad (12)$$

Typically, the inductance of a motor is relatively small and can be neglected at low frequency yielding the approximation,

$$G(s) = \frac{\Omega(s)}{e_a(s)} = \frac{K_T}{JR_a s + (bR_a + K_T K_b)} \quad (13)$$

Rearranging the terms in (13) results in the standard form (5),

$$G(s) = \frac{\Omega(s)}{e_a(s)} = \frac{K_T / (bR_a + K_T K_b)}{[JR_a s / (bR_a + K_T K_b)]s + 1} \quad (14)$$

Where the motor's DC gain,

$$K_m = \frac{K_T}{bR_a + K_T K_b} \quad (15a)$$

and the motor's time constant,

$$\tau_m = \frac{JR_a}{bR_a + K_T K_b} \quad (15b)$$

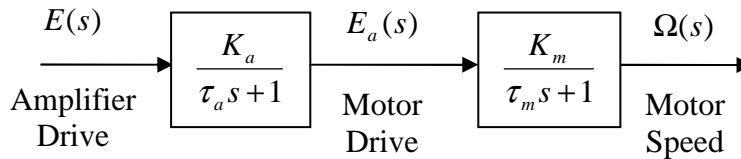


Figure 5: Block Diagram of the Amplifier and Motor

A block diagram of the motor transfer function is shown in Fig.5. Very often, an amplifier is used to generate the armature voltage to the motor. The amplifier, which can be modeled as its own first-order system with gain K_a and time constant τ_a , is also shown in Fig.5. The amplifier time constant, τ_a , is small enough to assume it has no effect on low frequency signals, leaving the amplifier first-order system nothing but the gain K_a . Together, the motor and the amplifier can be modeled as a single first-order system with DC gain

$$K = \frac{K_a K_T}{bR_a + K_T K_b} \quad (16)$$

and the time constant

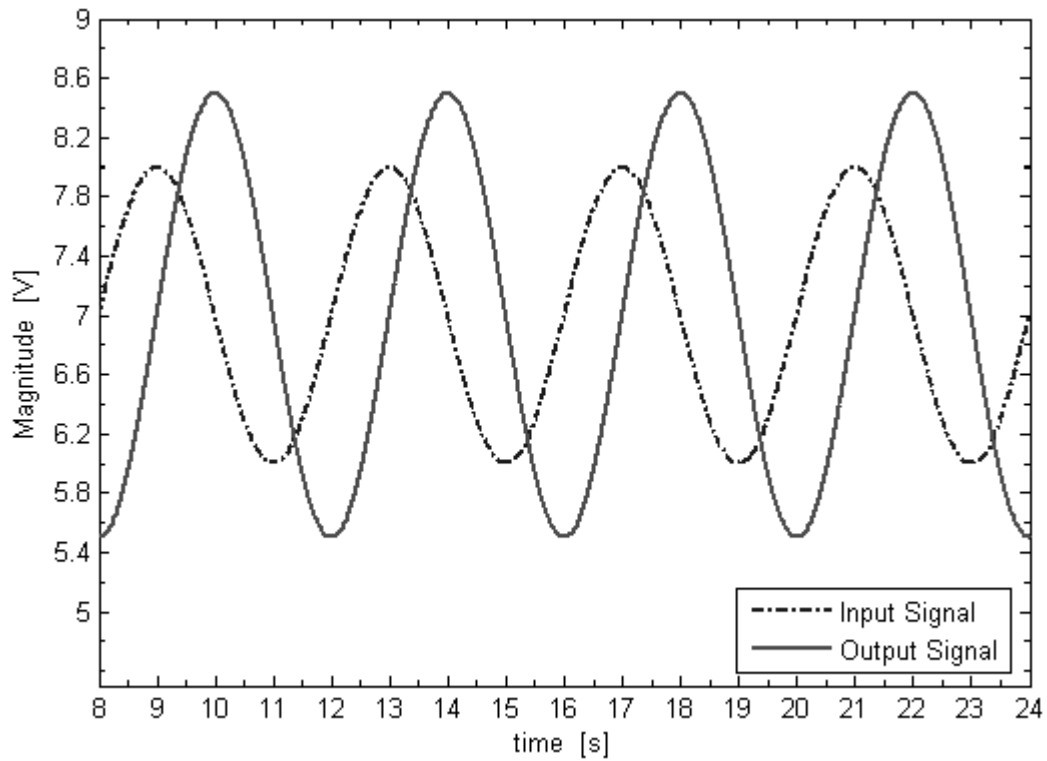
$$\tau = \frac{JR_a}{bR_a + K_T K_b} \quad (17)$$

Comparing Eqs.(5) and (16), it becomes clear that the amplifier plus motor system's DC gain comprised of the motor and amplifier (K) is the product of the motor DC gain (K_m) and the gain of the amplifier (K_a). A comparison of Eqs.(9) and (17) indicates that the time constant of the system comprised of the motor and amplifier (τ), and the motor time constant (τ_m), are the same.

One of the primary objectives of this experiment is to study the effects on the sinusoidal response of the motor when varying the DC gain and the time constant. We will vary K by varying the amplifier gain K_a . The time constant τ will be varied by changing the inertia of the motor shaft J . Physically; this will be done by mounting an inertia disk on the motor shaft.

Pre-Lab Sample Questions

Use the plot below to answer questions 1-3:



1) What is the phase angle at this frequency, in degrees?

Answer: Phase angle = -90°

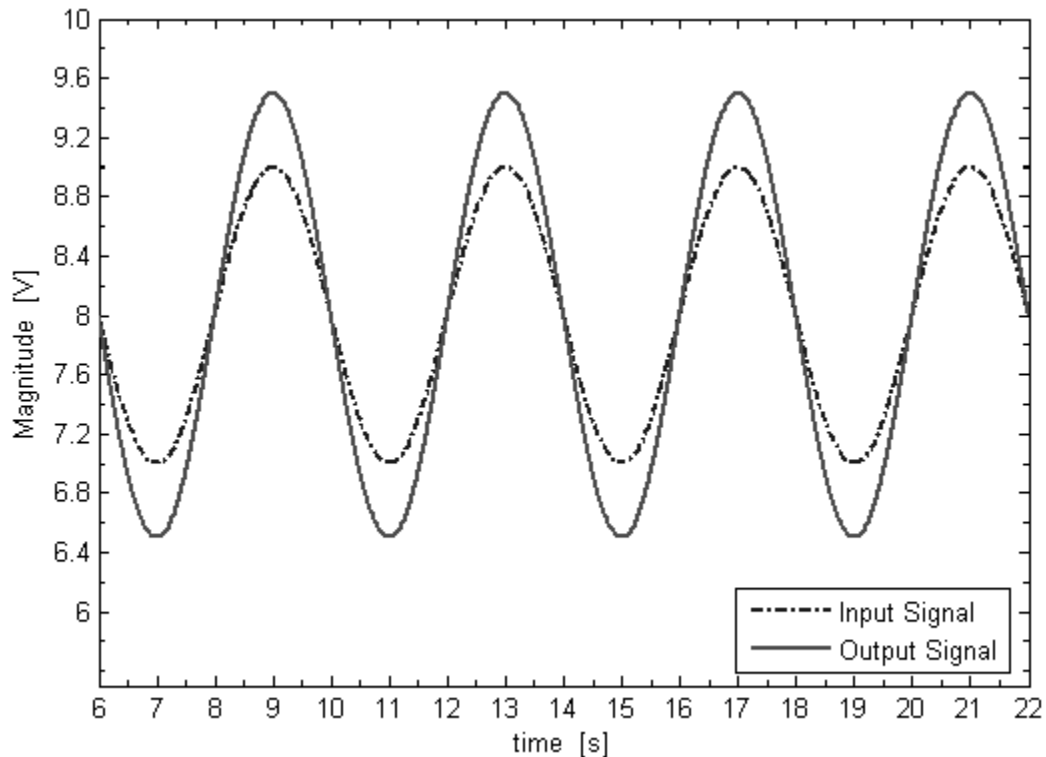
2) What is the non-dimensional gain at this frequency?

Answer: Gain = 1.5

3) What is the gain at this frequency in decibels?

Answer: Gain = 3.5 dB

Use the plot below, and Figure 3, to answer questions 4-7



- 4) Assume that the output above is the output of an ideal first order system to the given input. If the input frequency was increased by a factor of 10, would the non-dimensional gain go down, go up, or stay approximately the same?

Answer: Go down

- 5) Assume that the output above is the output of an ideal first order system to the given input. If the input frequency was decreased by a factor of 10, would the non-dimensional gain go down, go up, or stay approximately the same?

Answer: Stay approximately the same

- 6) Assume that the output above is the output of an ideal first order system to the given input. If the input frequency was increased by a factor of 10, would the phase angle become more negative, become more positive, or stay approximately the same?

Answer: Become more negative

- 7) Assume that the output above is the output of an ideal first order system to the given input. If the input frequency was decreased by a factor of 10, would the phase angle become more negative, become more positive, or stay approximately the same?

Answer: Stay approximately the same

3. Description of Experimental Setup

3.1. Hardware and software

The hardware set up is similar to Experiment 1, except for the fact that we are using a signal generator similar to the op-amp experiment (Experiment 3) to generate sinusoidal voltage signals. See Figures 6 & 7 for the exact equipment setup.

Components:

1. DC motor-tachometer unit MT150F

The motor speed will be measured from the tachometer voltage with the scaling factor of 333 rpm/V.

2. Oscilloscope

The oscilloscope will be used to measure and display voltage signals as functions of time.

3. Signal Generator

This unit will be used to generate a sinusoidal signal of a given frequency and amplitude.

4. Matlab script “tsfreq.m”

The script generates frequency response plots using data obtained from experiments.

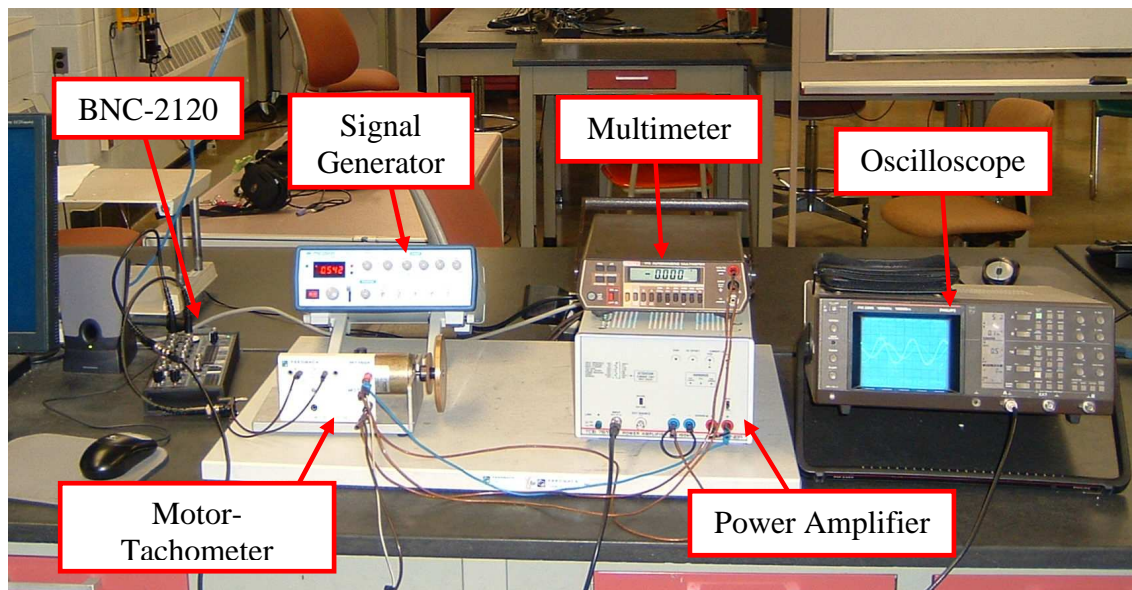


Figure 6 – Equipment Setup

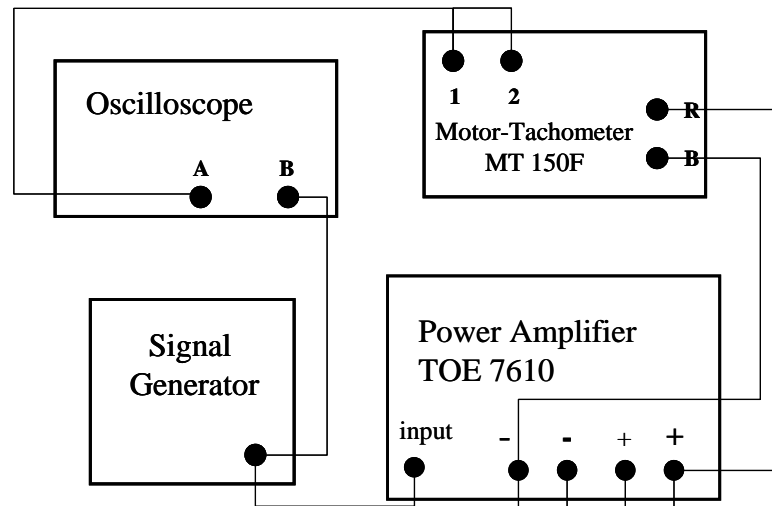


Figure 7-Wiring diagram for Equipment

4. Experimental Procedures

Part A: Steady-state response to sinusoidal input

Procedure: In this experiment you will analyze the response of the DC servo motor to a sinusoidal input. Be sure that the inertia disk is removed for the first part of the lab.

1. **Turn ON the oscilloscope and the amplifier.**
2. **Turn ON the signal generator** - You should notice both input (command) and output (response) sinusoidal signals on the oscilloscope. If you are not able to bring the signals, check if “Mode Func” on signal generator is set to Sine, “Freq” to 0.5 Hz (approx), “Amplitude” so that the peak to peak voltage at the output of the amplifier is 10 Volts max (check using the multimeter), and “Offset” to a voltage level where the motor rotates only in one direction but alternating between higher and lower speeds. This is achieved by keeping the marker on this knob at about 45 degrees (1st quadrant). Pulling the “Offset” knob allows an offset and pushing it in returns the offset to zero.
3. (There is a handout for the oscilloscope on the website. See this for more in-depth instructions on commands and functions to use.) Adjust the oscilloscope such that signals in both channels are completely visible on the screen. Using cursors on the screen, determine the ratio of the response signal and the command signal amplitude, A_r/A_c . Also determine the phase lag Φ between the two signals. See equation (4) in the handout for calculating the phase lag on the oscilloscope. Use the cursors to measure the Δt between the two signals. The period (time to complete one cycle) can then be found by $T=1/f$ where f is the frequency input by the signal generator.
4. Record these results in Table 1. Note: the response signal may not have an exact zero mean on the oscilloscope screen since the mean voltage of the tachometer fluctuates. Therefore, the signal amplitudes should be calculated by dividing the distance between the maximum and the minimum values by 2.

5. Repeat step 3 for frequencies shown in Table 1. You do not have use the same exact frequencies, however try to be within 0.5 Hz of that area. **IMPORTANT- record the exact input value that you read on the signal generator beside the preferred values.** You have to input those values into the Matlab program. Record all data in Table 1. To get a more accurate reading, you may need to rescale the horizontal axis of the graph such that 2 to 3 cycles are shown on the graph.
6. Run the Matlab script "tsfreq.m" and enter the values from Table 1 to obtain the frequency response plots. You may need to rescale the axis. Consult your Matlab manual to rescale plot axes.

Questions:

- A.1. From the frequency response plots obtained, determine the DC gain and time constant of the system.
- A.2. What is the approximate roll off (dB/decade) in the gain of the system? How does this match with the roll off in first-order systems?
- A.3. From the phase plot determine the frequency where the system response lags the commanded signal by 45 degrees? Comment on the relation of this frequency with the time constant of the system?
- A.4. Predict the gain of the system when the commanded signal has very high frequency? What would be the phase lag for high frequency?

Part B: Effect of frequency on the amplifier gain K_a and motor gain K_m

Find the amplifier gain and motor gain at 5 Hz and 15 Hz. Use the multi-meter to check the voltage of the amplifier output and motor output to find K_m . Make sure the multi-meter is in AC mode for these measurements. Connect channel B on the oscilloscope to the input of the motor and measure the peak to peak voltage of the amplifier input and output to find K_a . When finished, put oscilloscope channel B back on the motor output.

Questions:

- B.1. What is the effect of changing frequencies on the amplifier gain?
- B.2. What is the effect of changing the frequency on the motor gain?

Part C: Effect of varying rotor inertia J

Procedure: In this section, you will analyze the effect of varying the rotor inertia J on the sinusoidal response of the DC motor. From Eq.(17), it is clear that changing J will change the time constant of the system comprised of the motor and the amplifier.

1. Turn off the power of the amplifier.
2. Securely mount the inertia disk on the motor shaft such that it rotates with the shaft without slippage.
3. Turn on the power of amplifier.
4. Repeat step 3 in part A using frequency values 0.5 and 1 Hz. Record the results in Table 2.

Questions:

- C.1. Is it possible to obtain a Bode diagram with the disk, just like the one we got without the disk? Why/why not?
- C.2. Compare the results in Tables 1 and 2 and comment on the effect of changing the time constant on the gain A_r/A_c , and the phase lag ϕ .

Table 1. Experimental data for DC motor frequency response plots

Run #	Frequency (Hz)	A_r/A_c	ϕ (degrees)	Actual frequency (Hz)
1	0.5			
2	2			
3	4			
4	7			
5	10			
6	12			
7	15			
8	19			
9	22			
10	25			

Table 2. Experimental observations on changing time constant

Run #	Frequency (Hz)	A_r/A_c	ϕ (degrees)	Actual frequency (Hz)
1	0.5			
2	1			

Laboratory Report

Name:

Section:

Date:

A.1. From the frequency response plots obtained, determine the DC gain and time constant of the system.

A.2. What is the approximate roll off (dB/decade) in the gain of the system? How does this match with the roll off in first-order systems?

A.3. From the phase plot determine the frequency where the system response lags the commanded signal by 45 degrees? Comment on the relation of this frequency with the time constant of the system?

A.4. Predict the gain of the system when the commanded signal has very high frequency? What would be the phase lag for high frequency?

Name:

Section:

Date:

B.1. What is the effect of changing frequencies on the amplifier gain?

B.2. What is the effect of changing the frequency on the motor gain?

C.1 Is it possible to obtain a Bode diagram with the disk, just like the one we got without the disk? Why / Why not?

C.2. Compare the results in Tables 1 and 2 and comment on the effect of changing the time constant on the gain A_r/A_c , and the phase lag ϕ .

Name:

Section:

Date:

From your Experiment 1 handout, find the time constants of the motor system (with and without the disc). Do they compare to the time constants found this week? Why or why not?

Summarize the lessons you have learned from this laboratory experience, in few sentences.