Antennas

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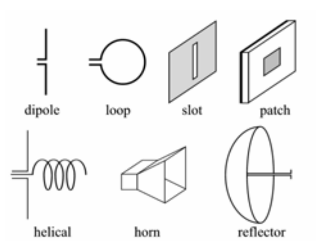
Antennas

Transmitting Antenna: Any structure designed to efficiently radiate electromagnetic radiation in a preferred direction is called a *transmitting antenna*.

Wires passing an alternating current emit, or *radiate*, electromagnetic energy. The shape and size of the current carrying structure determines how much energy is radiated as well as the direction of radiation.



Receiving Antenna: Any structure designed to efficiently receive electromagnetic radiation is called a transmitting antenna

We also know that an electromagnetic field will induce current in a wire. The shape and size of the structure determines how efficiently the field is converted into current, or put another way, determines how well the radiation is captured. The shape and size also determines from which direction the radiation is preferentially captured.

Antennas – Radiation Power

Let us consider a transmitting antenna (transmitter) is located at the origin of a spherical coordinate system.
In the far-field, the radiated waves resemble plane waves propagating in the radiation direction and time-harmonic fields can be related by the chapter 5 equations.

Electric and Magnetic Fields:

$$\mathbf{E}_s = -\eta_o \mathbf{a}_r \times \mathbf{H}_s$$

and

$$\mathbf{H}_s = \frac{1}{\eta_o}\mathbf{a}_r \times \mathbf{E}_s$$

The time-averaged power density vector of the wave is found by the Poynting Theorem

Power Density:

$$\mathbf{P}(r,\theta,\phi) = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_s \times \mathbf{H}_s^*\right]$$

$$\mathbf{P}(r,\theta,\phi) = P(r,\theta,\phi)\mathbf{a_r}$$

The total power radiated by the antenna is found by integrating over a closed spherical surface,

Radiated Power: $$P_{rad} = \oint \mathbf{P}(r,\theta,\phi) \bullet d\mathbf{S} = \int\int P(r,\theta,\phi) r^2 \sin\theta \, d\theta \, d\phi$$

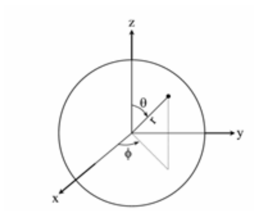
Antennas – Radiation Patterns

Radiation patterns usually indicate either electric field intensity or power intensity. Magnetic field intensity has the same radiation pattern as the electric field intensity, related by η_o

It is customary to divide the field or power component by its maximum value and to plot a normalized function

Normalized radiation intensity:

$$P_n(\theta,\phi) = \frac{P(r,\theta,\phi)}{P_{\max}}$$



Isotropic antenna: The antenna radiates electromagnetic waves equally in all directions.

$$P_n(\theta,\phi)_{iso} = 1$$

Antennas – Radiation Patterns

Radiation Pattern:

A directional antenna radiates and receives preferentially in some direction.

It is customary, then, to take slices of the pattern and generate two-dimensional plots.

The polar plot can also be in terms of decibels.

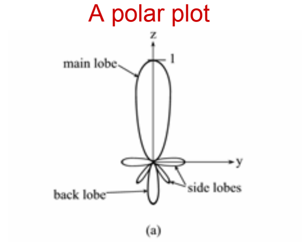
$$E_n(\theta,\phi) = \frac{E(r,\theta,\phi)}{E_{max}}$$

$$E_n(\theta,\phi)(dB) = 20\log\left[E_n(\theta,\phi)\right]$$

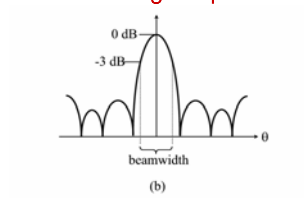
It is interesting to note that a normalized electric field pattern in dB will be identical to the power pattern in dB.

$$P_n(\theta,\phi)(dB) = 10\log\left[P_n(\theta,\phi)\right]$$

A polar plot



A rectangular plot



Antennas – Radiation Patterns

Radiation Pattern:

It is clear in Figure that in some very specific directions there are zeros, or nulls, in the pattern indicating no radiation.

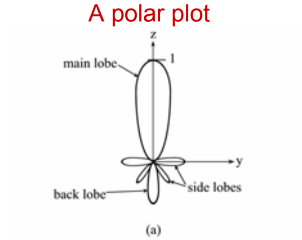
The protuberances between the nulls are referred to as lobes, and the main, or major, lobe is in the direction of maximum radiation.

There are also side lobes and back lobes. These other lobes divert power away from the main beam and are desired as small as possible.

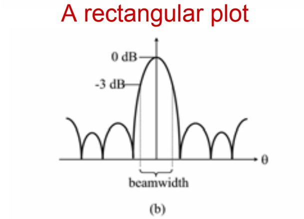
Beam Width:

One measure of a beam's directional nature is the *beamwidth*, also called the half-power beamwidth or 3-dB beamwidth.

A polar plot



A rectangular plot



Antennas – Pattern Solid Angle

Antenna Pattern Solid Angle:

A *differential solid angle*, $d\Omega$, in sr, is defined as

$$d\Omega = \sin\theta d\theta d\phi.$$

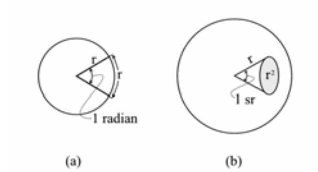
For a sphere, the solid angle is found by integrating

$$\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta d\theta d\phi = 4\pi(sr).$$

An antenna's pattern solid angle,

$$\boxed{\Omega_p = \iint P_n(\theta,\phi)\,d\Omega}$$

All of the radiation emitted by the antenna is concentrated in a cone of solid angle Ω_p over which the radiation is constant and equal to the antenna's maximum radiation value.



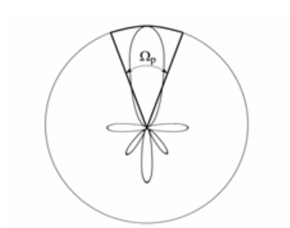
A radian is defined with the aid of Figure a). It is the angle subtended by an arc along the perimeter of the circle with length equal to the radius.
A steradian may be defined using Figure (b). Here, one steradian (sr) is subtended by an area r2 at the surface of a sphere of radius r.

Antennas – Directivity

Directivity:

The *directive gain*,, of an antenna is the ratio of the normalized power in a particular direction to the average normalized power, or

$$D(\theta,\phi) = \frac{P_n(\theta,\phi)}{P_n(\theta,\phi)_{avg}}$$



Where the normalized power's average value taken over the entire spherical solid angle is

$$P_n(\theta,\phi)_{avg} = \frac{\iint P_n(\theta,\phi)\,d\Omega}{\iint d\Omega} = \frac{\Omega_p}{4\pi}$$

The *directivity, Dmax*, is the maximum directive gain,

$$D_{\max} = D(\theta,\phi)_{\max} = \frac{P_n(\theta,\phi)_{\max}}{P_n(\theta,\phi)_{avg}}$$

$$\boxed{D_{\max} = \frac{4\pi}{\Omega_p}}$$ Using $P_n(\theta,\phi)_{\max} = 1$

Example

8.1: In free space, suppose a wave propagating radially away from an antenna at the origin has

$$\mathbf{H}_s = \frac{I_s}{r}\sin\theta\,\mathbf{a}_\phi$$

where the driving current phasor $I_s = I_o e^{j\alpha}$

Find (1) $\boldsymbol{E}_s$

$$\mathbf{E}_s = -\eta_o\mathbf{a}_r \times \mathbf{H}_s = -\eta_o\mathbf{a}_r \times \frac{I_s}{r}\sin\theta\,\mathbf{a}_\phi = -\eta_o\frac{I_s}{r}\sin\theta\left(\mathbf{a}_r \times \mathbf{a}_\phi\right) = \frac{\eta_o I_s}{r}\sin\theta\,\mathbf{a}_\theta$$

Find (2) $\mathbf{P}(r,\theta,\phi)$

$$\mathbf{P}(r,\theta,\phi) = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_s \times \mathbf{H}_s^*\right] = \frac{1}{2}\mathrm{Re}\left[\left(\frac{\eta_o I_s}{r}\sin\theta\,\mathbf{a}_\theta\right)\times\left(\frac{I_s}{r}\sin\theta\,\mathbf{a}_\phi\right)^*\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\left(\frac{\eta_o I_o e^{j\alpha}}{r}\sin\theta\,\mathbf{a}_\theta\right)\times\left(\frac{I_o e^{j\alpha}}{r}\sin\theta\,\mathbf{a}_\phi\right)^*\right] = \frac{1}{2}\mathrm{Re}\left[\left(\frac{\eta_o I_o e^{j\alpha}}{r}\sin\theta\,\mathbf{a}_\theta\right)\times\left(\frac{I_o e^{-j\alpha}}{r}\sin\theta\,\mathbf{a}_\phi\right)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\eta_o\frac{I_o^2}{r^2}\sin^2\theta\left(\mathbf{a}_\theta \times \mathbf{a}_\phi\right)\right] = \boxed{\frac{1}{2}\eta_o\frac{I_o^2}{r^2}\sin^2\theta}\,\mathbf{a}_r$$

Magnitude: $P(r,\theta,\phi) = \frac{1}{2}\eta_o\frac{I_o^2}{r^2}\sin^2\theta$

Find (3) P_{rad}

$$P_{rad} = \oint \mathbf{P}(r,\theta,\phi)\bullet d\mathbf{S} = \int\int P(r,\theta,\phi)\,r^2\sin\theta\,d\theta\,d\phi$$

$$P_{rad} = \int\int\left(\frac{1}{2}\eta_o\frac{I_o^2}{r^2}\sin^2\theta\right)r^2\sin\theta\,d\theta\,d\phi$$

$$P_{rad} = \left(\frac{1}{2}\eta_o\frac{I_o^2}{r^2}\right)\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\,d\theta\,d\phi$$

$$P_{rad} = \left(\frac{1}{2}\eta_o\frac{I_o^2}{r^2}\right)\left(\int_0^{\pi}\sin^3\theta\,d\theta\right)\left(\int_0^{2\pi}d\phi\right)$$

$$P_{rad} == \left(\frac{1}{2}\eta_o\frac{I_o^2}{r^2}\right)\left(\frac{4}{3}\right)(2\pi) = \frac{4}{3}\pi\eta_o I_o^2$$

We make use of the formula

$$\int\sin^3\theta\,d\theta = -\cos\theta + \frac{\cos^3\theta}{3}$$

$$\int_0^{\pi}\sin^3\theta\,d\theta = \left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^{\pi}$$

$$= \left[\left(-\cos\pi + \frac{\cos^3\pi}{3}\right) - \left(-\cos 0 + \frac{\cos^3 0}{3}\right)\right]$$

$$= \left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] = 2 - \frac{2}{3} = \frac{4}{3}$$

Find (4) $P_n(r,\theta,\phi)$ Normalized Power Pattern

$$P(r,\theta,\phi) = \frac{1}{2}\eta_o\frac{I_o^2}{r^2}\sin^2\theta$$

when $\theta = \frac{\pi}{2}$

$$P_{\max} = \frac{1}{2}\eta_o\frac{I_o^2}{r^2}$$

$$P_n(\theta,\phi) = \frac{P(r,\theta,\phi)}{P_{\max}}$$

$$P_n(\theta,\phi) = \sin^2\theta$$

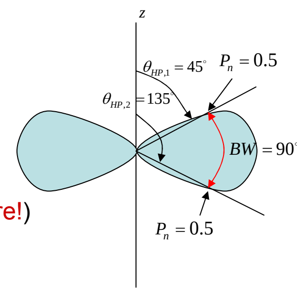
Find (5) Beam Width

$$P_n(\theta,\phi)=\sin^2\theta \Rightarrow \frac{1}{2}=\sin^2\theta_{HP} \qquad \sin\theta_{HP}=\pm\frac{1}{\sqrt{2}}$$

$$\sin\theta_{HP}=+\frac{1}{\sqrt{2}}$$

$\theta_{HP,1}=45^\circ$ and $\theta_{HP,2}=135^\circ$

$$Beamwidth(BW)=135^\circ-45^\circ=90^\circ$$



(6) Pattern Solid Angle Ω_p (Integrate over the entire sphere!)

$$\boxed{\Omega_p=\int\int P_n(\theta,\phi)\,d\Omega}$$

$$\Omega_P=\iint\sin^2\theta\sin\theta d\theta d\phi=\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\,d\theta\,d\phi=\left(\int_0^{\pi}\sin^3\theta\,d\theta\right)\left(\int_0^{2\pi}d\phi\right)=\left(\frac{4}{3}\right)(2\pi)=\frac{8\pi}{3}$$

(7) directivity D_{max}

$$D_{\max}=\frac{4\pi}{\Omega_P}=\frac{4\pi}{\frac{8\pi}{3}}=\frac{2}{3}=1.5$$

(8) Half-power Pattern Solid Angle $\Omega_{p,HP}$ (Integrate over the beamwidth!)

$$\boxed{\Omega_{p,HP}=\int\int P_n(\theta,\phi)\,d\Omega}$$

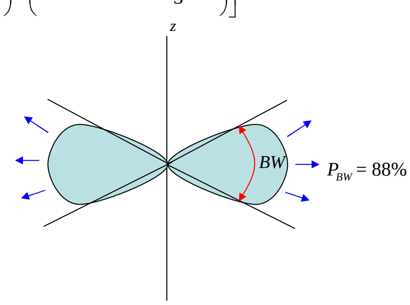
$$\Omega_{P,HP}=\iint\sin^2\theta\sin\theta d\theta d\phi=\int_0^{2\pi}\int_{45^\circ}^{135^\circ}\sin^3\theta\,d\theta\,d\phi=\left(\int_{45^\circ}^{135^\circ}\sin^3\theta\,d\theta\right)\left(\int_0^{2\pi}d\phi\right)=\left(\frac{5}{3\sqrt{2}}\right)(2\pi)=\frac{5\pi\sqrt{2}}{3}$$

$$\int_{45^\circ}^{135^\circ}\sin^3\theta\,d\theta=\left[-\cos\theta+\frac{\cos^3\theta}{3}\right]_{45^\circ}^{135^\circ}=\left[\left(-\cos(135^\circ)+\frac{\cos^3(135^\circ)}{3}\right)-\left(-\cos(45^\circ)+\frac{\cos^3(45^\circ)}{3}\right)\right]$$

$$=\left[\left(\frac{1}{\sqrt{2}}-\frac{1}{6\sqrt{2}}\right)-\left(-\frac{1}{\sqrt{2}}+\frac{1}{6\sqrt{2}}\right)\right]=\frac{2}{\sqrt{2}}-\frac{2}{6\sqrt{2}}=\frac{10}{6\sqrt{2}}=\frac{5}{3\sqrt{2}}$$

Power radiated through the beam width

$$P_{BW}=\frac{\Omega_{P,HP}}{\Omega_P}=\frac{\frac{5\pi\sqrt{2}}{3}}{\frac{8\pi}{3}}=\frac{5\sqrt{2}}{8}\cong 0.88\text{ (or) }88\%$$

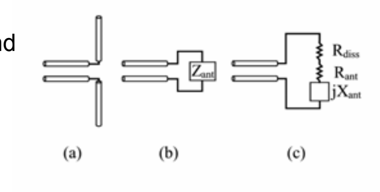


Antennas – Efficiency

Efficiency

Power is fed to an antenna through a T-Line and the antenna appears as a complex impedance

$$Z_{ant} = R_{ant} + jX_{ant}.$$



where the antenna resistance consists of radiation resistance and and a dissipative resistance.

$$R_{ant} = R_{rad} + R_{dis}$$

For the antenna is driven by phasor current $I_o = I_s e^{j\alpha}$

The power radiated by the antenna is

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

The power dissipated by ohmic losses is

$$P_{diss} = \frac{1}{2} I_o^2 R_{diss}$$

An antenna efficiency e can be defined as the ratio of the radiated power to the total power fed to the antenna.

$$e = \frac{P_{rad}}{P_{rad} + P_{diss}} = \frac{R_{rad}}{R_{rad} + R_{diss}}$$

Antennas – Gain

Gain

The power gain, G, of an antenna is very much like its directive gain, but also takes into account efficiency

$$G(\theta,\phi) = eD(\theta,\phi)$$

The maximum power gain

$$G_{\max} = eD_{\max}$$

The maximum power gain is often expressed in dB.

$$G_{\max}(dB) = 10\log_{10}(G_{\max})$$

Example

D8.3: Suppose an antenna has D = 4, R_{rad} = 40 Ω and R_{diss} = 10 Ω. Find antenna efficiency and maximum power gain. (Ans: e = 0.80, G_{max} = 3.2).

Antenna efficiency

$$e = \frac{R_{rad}}{R_{rad} + R_{diss}} = \frac{40}{10+40} = 0.8 \text{ (or) } 80\%$$

Maximum power gain

$$G_{\max} = eD_{\max} = (4)(0.8) = 3.2$$

Maximum power gain in dB

$$G_{\max}(dB) = 10\log_{10}(G_{\max}) = 10\log_{10}(3.2) = 5.05 \text{ dB}$$