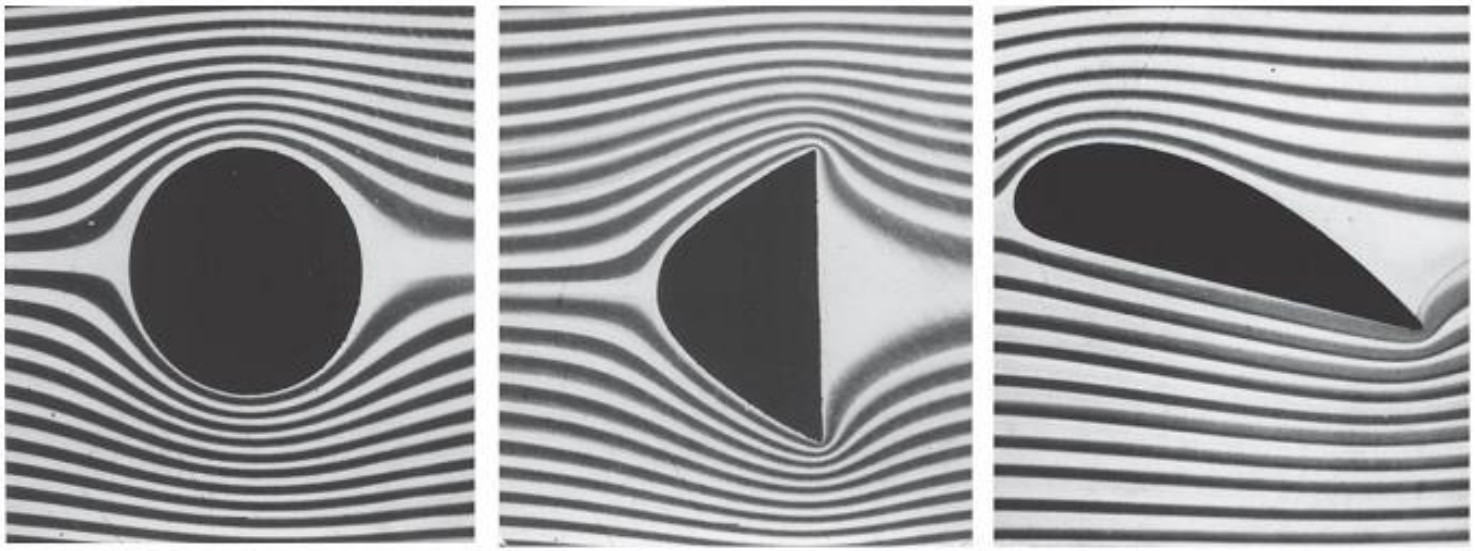
Hidrodinamika

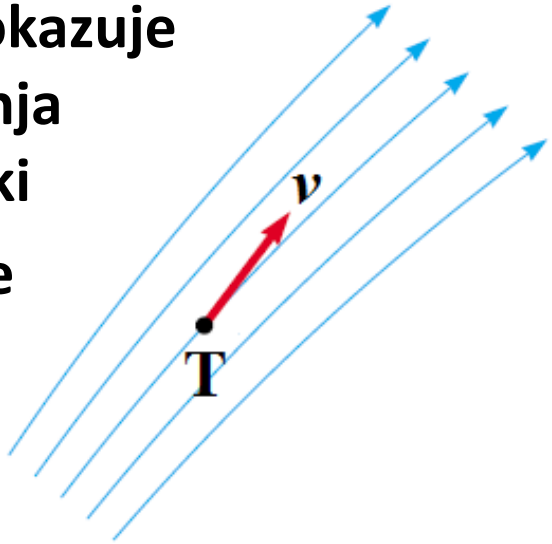
Idealan fluid*

- fluid je nestlačiv
- temperatura je stalna
- tok fluida je stacionaran (brzina fluida stalna u vremenu)
- tok fluida je laminaran – slojevit (nije turbulentan)
- fluid nije viskozan (nema trenja među slojevima fluida)

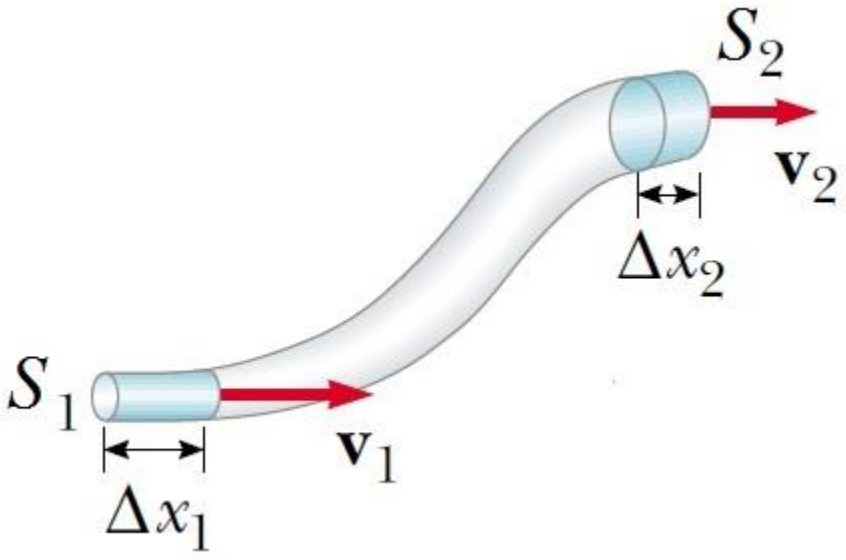


Strujnice

- nagib tangente pokazuje smjer brzine gibanja fluida u nekoj točki
- brzina je veća gdje su strujnice gušće



Jednadžba kontinuiteta*



$$\Delta x_1 = v_1 \Delta t \qquad \Delta x_2 = v_2 \Delta t$$

$$m_1 = \rho S_1 \Delta x_1 = \rho S_1 v_1 \Delta t$$

$$m_2 = \rho S_2 v_2 \Delta t$$

$$m_1 = m_2$$

zakon očuvanja mase

$$S_1 v_1 = S_2 v_2$$

$$q_m = \rho S v = \text{konst}$$

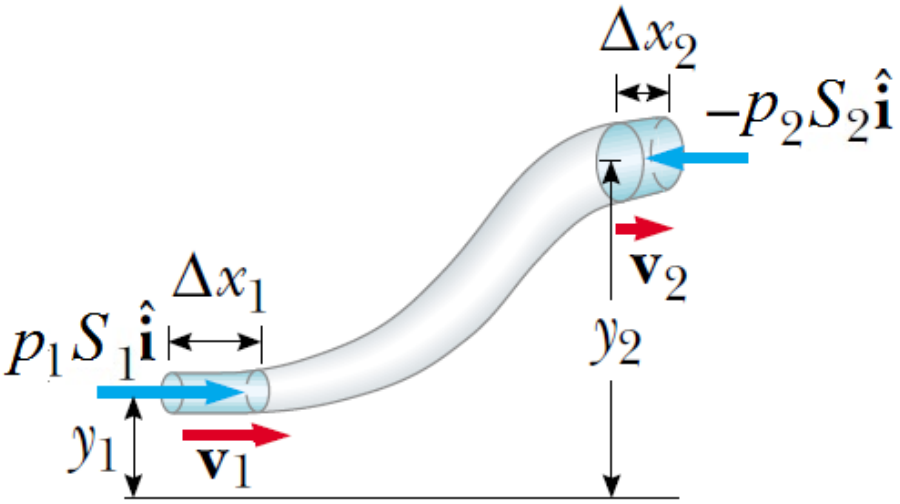
maseni tok

$$q_v = S v = \text{konst}$$

volumni tok

$$q_v = \int_S \vec{v} \cdot d\vec{S}$$

Bernoullijeva jednadžba*



$$W_1 = \vec{F}_1 \cdot \Delta\vec{x}_1 = p_1 S_1 \Delta x_1 = p_1 \Delta V_1$$

$$W_2 = \vec{F}_2 \cdot \Delta\vec{x}_2 = -p_2 S_2 \Delta x_2 = -p_2 \Delta V_2$$

$$\Delta V_1 = \Delta V_2 = \Delta V$$

$$\Delta W = W_1 + W_2 = (p_1 - p_2)\Delta V$$

$$\Delta W = \Delta E_k + \Delta E_p$$

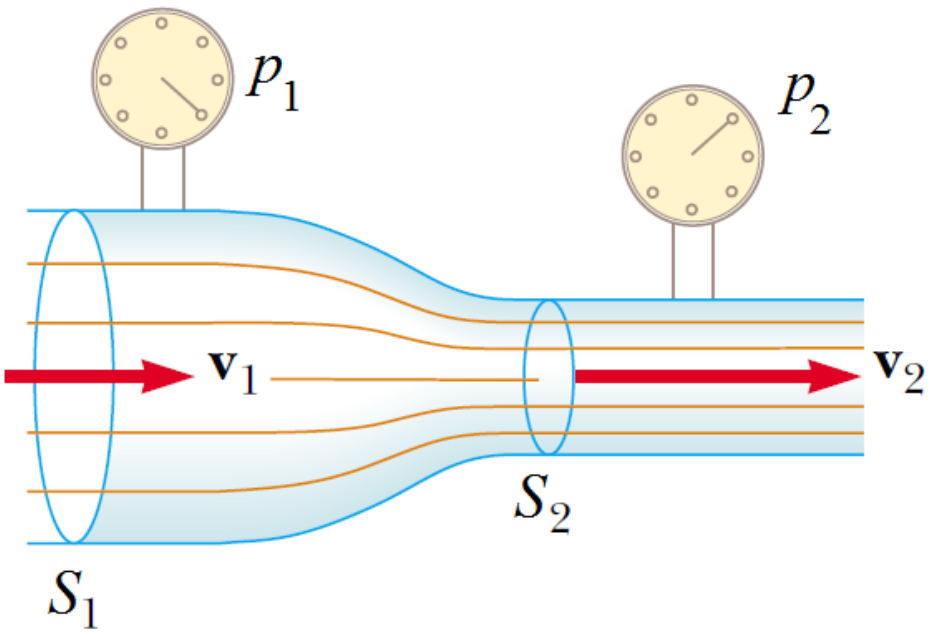
$$\Delta E_k = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 \qquad \Delta E_p = \Delta m g y_2 - \Delta m g y_1$$

$$(p_1 - p_2)\Delta V = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 + \Delta m g y_2 - \Delta m g y_1$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{konst.}$$

Venturijeva cijev*



$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

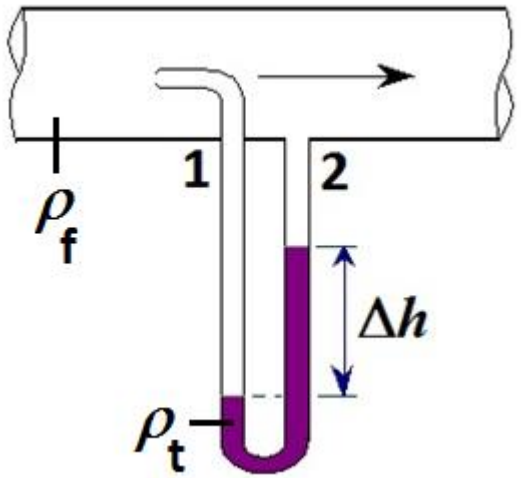
$$S_1 v_1 = S_2 v_2$$

$$v_1 = \frac{S_2}{S_1} v_2$$

$$p_1 + \frac{1}{2}\rho \frac{S_2^2}{S_1^2} v_2^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 = S_1 \sqrt{\frac{2(p_1 - p_2)}{\rho(S_1^2 - S_2^2)}}$$

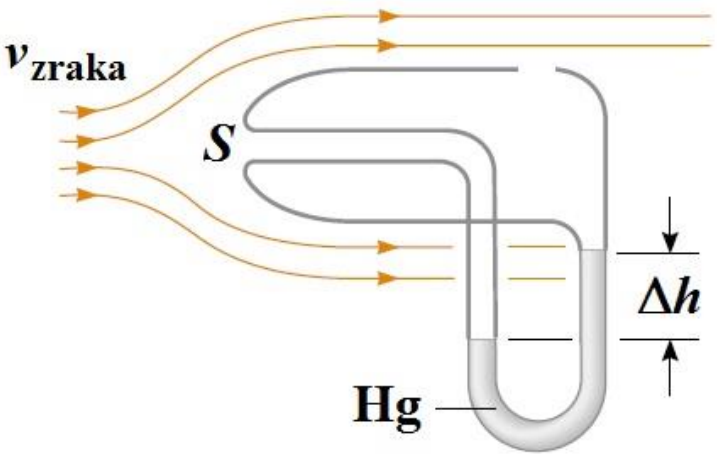
Pitot-Prandtlova cijev*



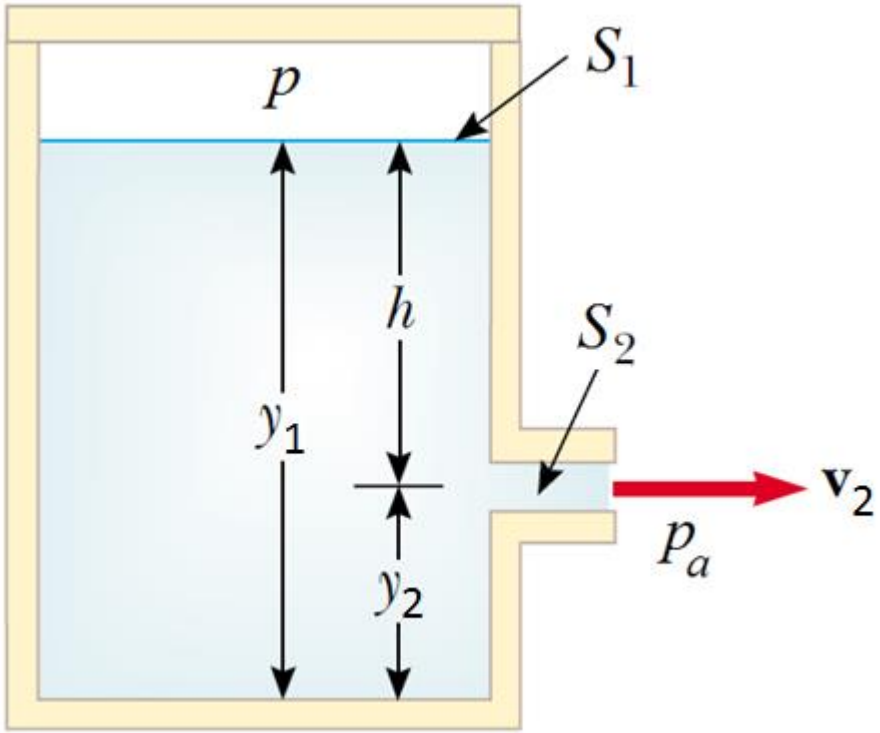
- cijev 1: $p_1 = p + \frac{1}{2}\rho_f v^2$
- cijev 2: $p_2 = p$

$$p_1 - p_2 = \rho_t g \Delta h$$

$$v^2 = \sqrt{\frac{2g\Delta h\,\rho_t}{\rho_f}}$$



Torricellijev zakon istjecanja*



$$S_1 v_1 = S_2 v_2$$

$$S_2 \ll S_1 \qquad v_1 \ll v_2 \qquad v_1 \approx 0$$

$$p + \rho g y_1 = p_a + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

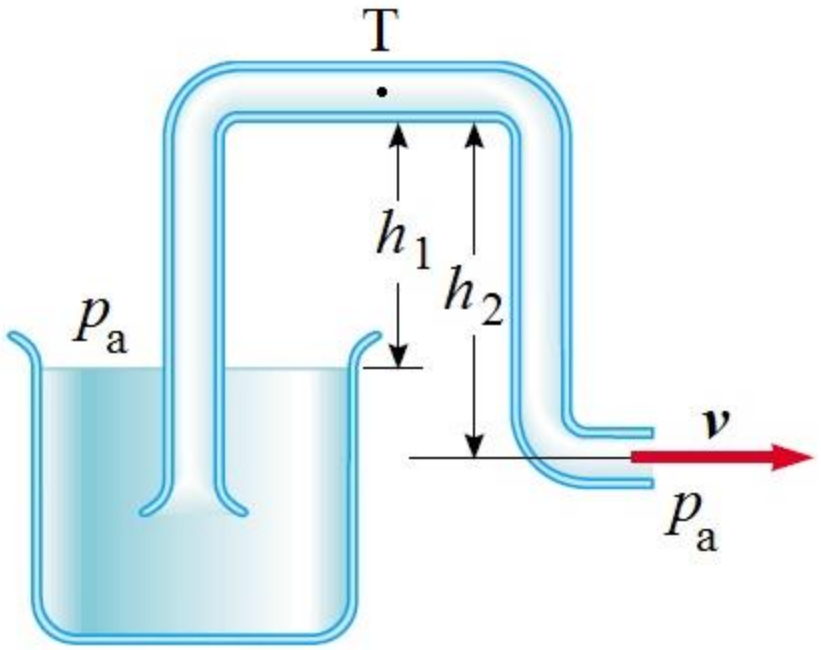
$$y_1 - y_2 = h$$

- **ako je posuda otvorena**

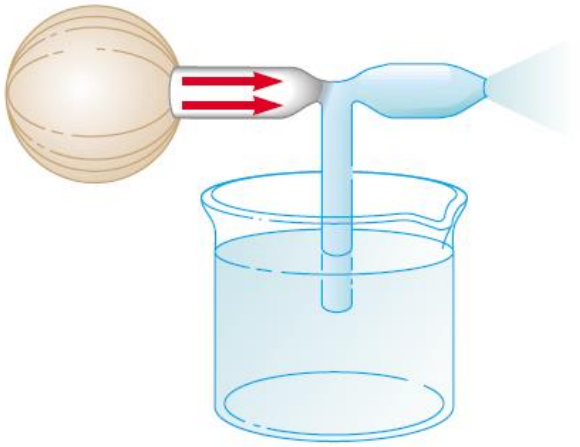
$$p = p_a$$

$$v_2 = \sqrt{2gh}$$

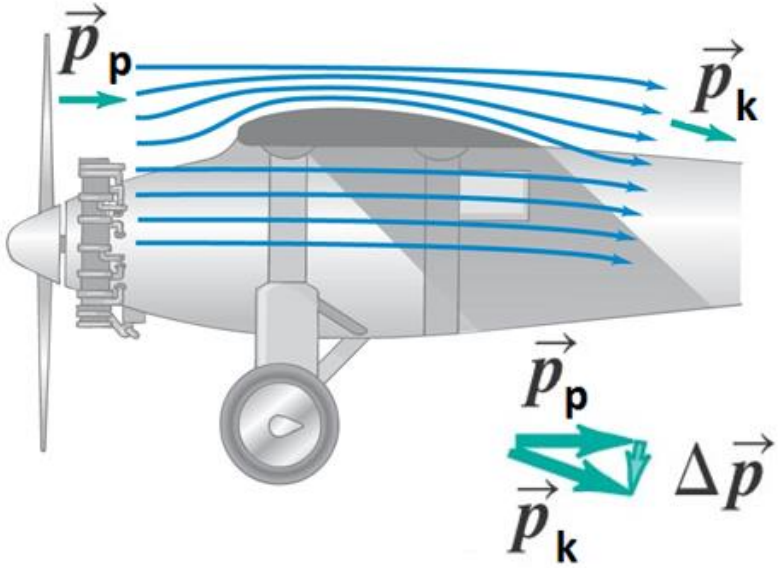
Sisaljka



Rasprsivač



Avionsko krilo

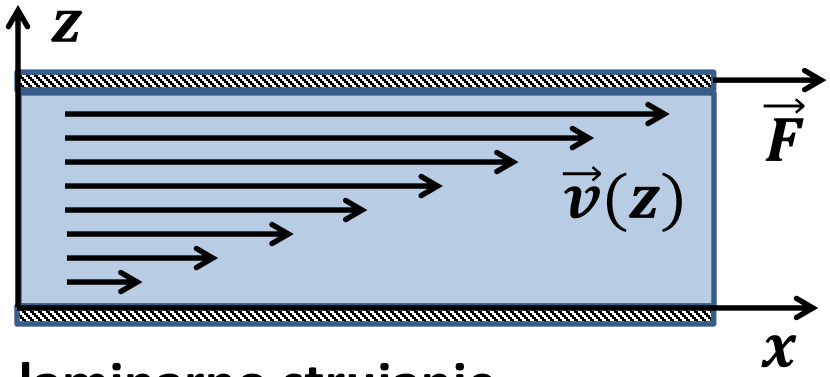


$$p_L = p_a - \rho g h_1$$

$$p_D = p_a - \rho g h_2$$

$$\Delta p = p_L - p_D = \rho g (h_2 - h_1)$$

Viskoznost*



laminarno strujanje

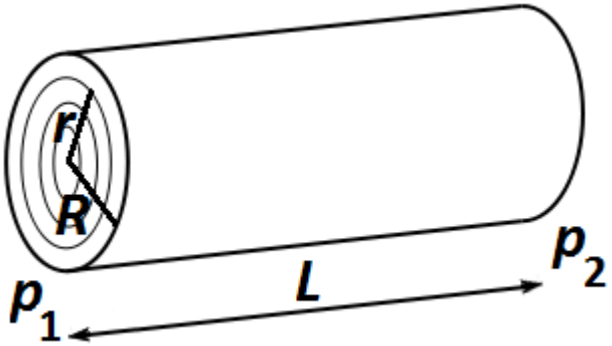
$$F_\eta = \eta S \frac{dv}{dz}$$

koeficijent dinamičke viskoznosti

(njutnovski fluidi)

$$S = 2r\pi L$$

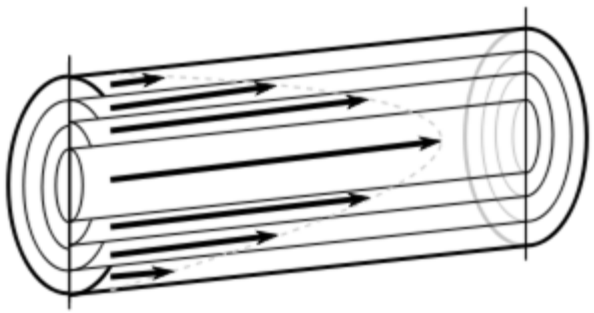
$$F = \Delta p r^2 \pi \qquad F_\eta = -F$$



$$dv = -\frac{\Delta p}{2\eta L} r dr \qquad v(R) = 0$$

$$v(r) = \frac{\Delta p}{4\eta L}(R^2 - r^2) \qquad v(r) = v_0\left(1 - \frac{r^2}{R^2}\right)$$

$$dV = dSdl = 2r\pi\, dr\, dl = 2r\pi\, dr\, v(r)dt$$



$$dq_v = \frac{dV}{dt} = 2r\pi\, dr\, v(r) = \frac{\pi\Delta p}{2\eta L}\left(R^2 - r^2\right) r dr$$

$$q_v = \int dq_v = \frac{\pi\Delta p}{2\eta L}\int_0^R \left(R^2 - r^2\right) r dr$$

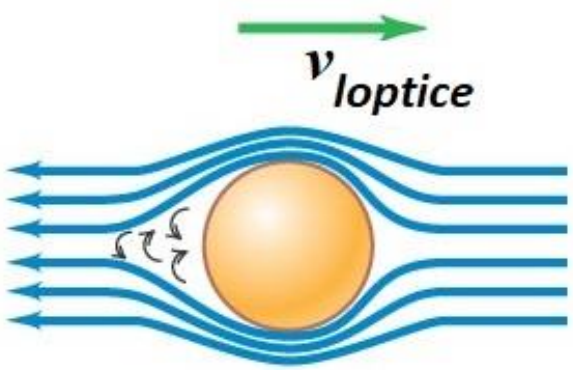
Poiseuilleov zakon protjecanja

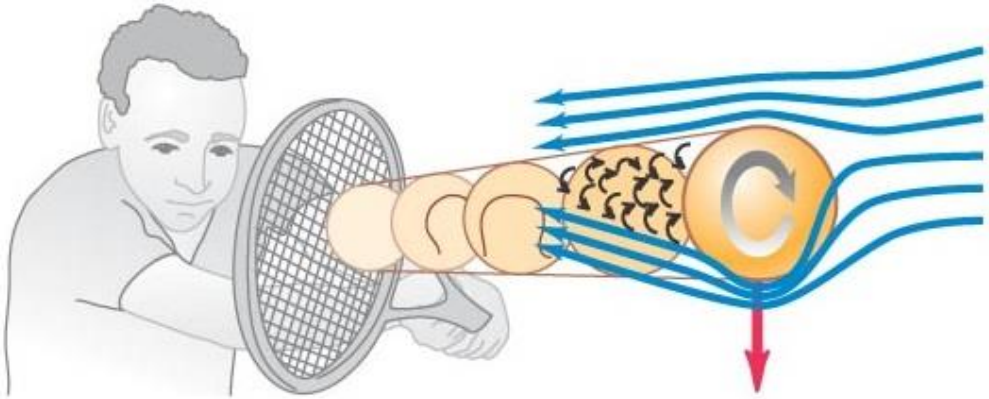
$$q_v = \frac{\pi\Delta p}{8\eta L} R^4$$

$$q_v = S\bar{v} = R^2\pi\bar{v}$$

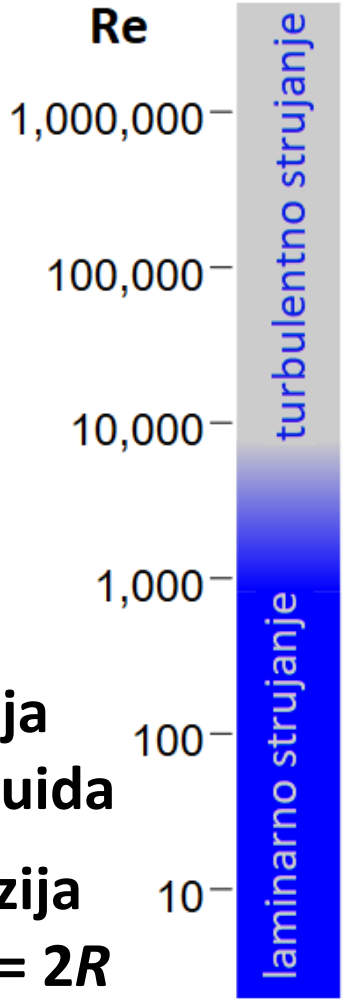
$$F_\eta = 8\pi\eta L\bar{v}$$

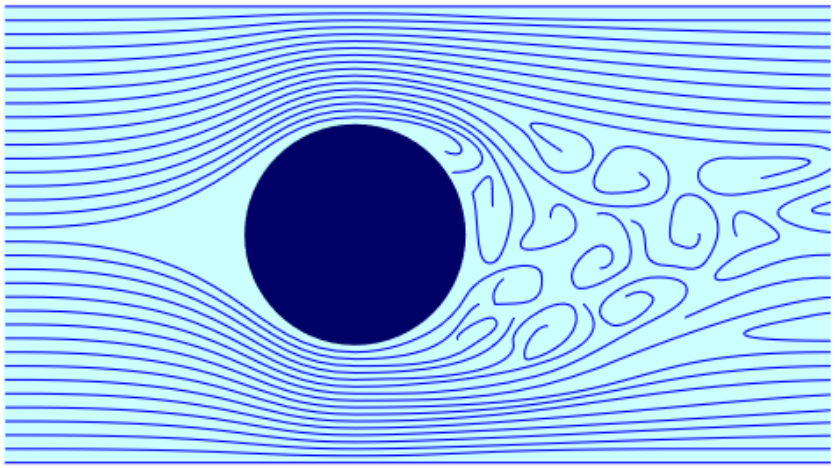
Magnusov efekt

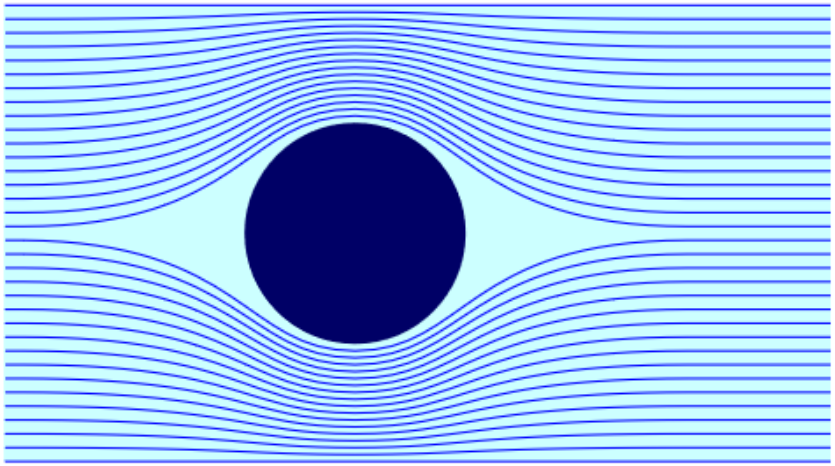




Turbulentno strujanje







- **Reynoldsov broj**

$$Re = \frac{\rho v l}{\eta}$$

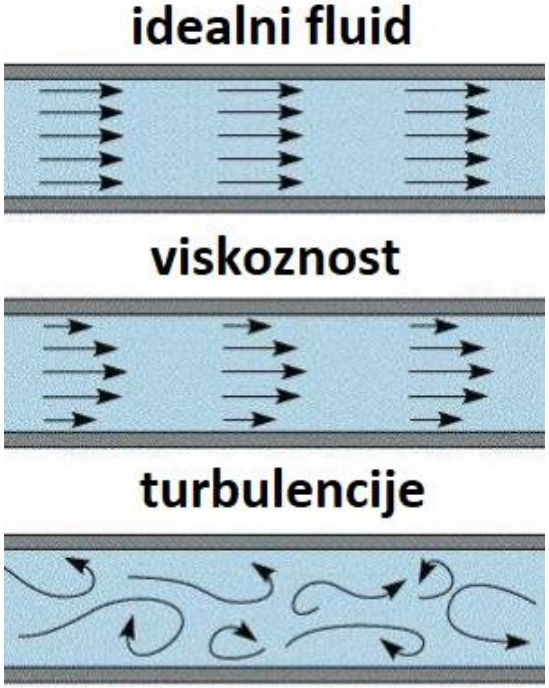
gustoća fluida

srednja brzina fluida

dimenzija tijela (*l* = 2*R* za kuglu)

koef. dinam. viskoznosti

Otpor sredstva



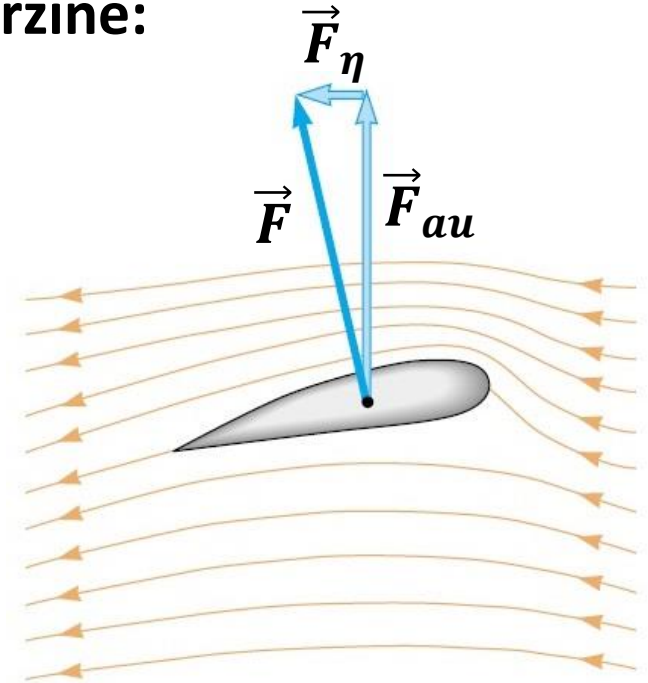
nema otpora sredstva

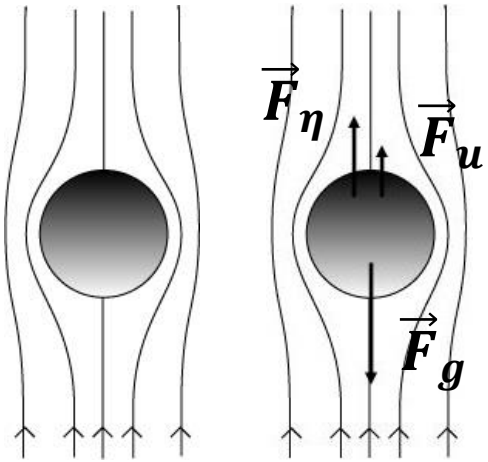
javlja se otpora sredstva

za veće brzine:

koeficijent otpora

$$F_\eta = \frac{1}{2} C_d \rho S v^2$$





Stokesov zakon (za kuglu male brzine)

$$F_\eta = 6\pi\eta R v$$

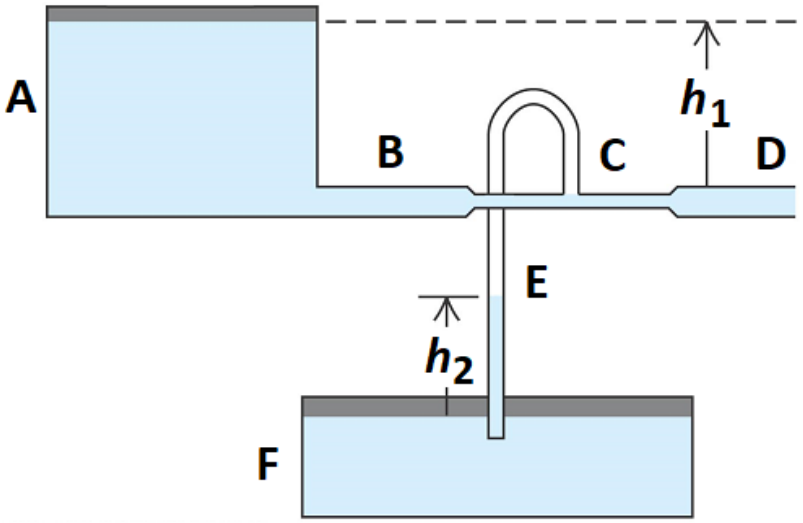
$$\vec{F} = \vec{F}_{au} + \vec{F}_\eta$$

aerodinamički uzgon

$$F_\eta = F_g - F_u = \frac{4}{3} R^3 \pi g (\rho - \rho_f)$$

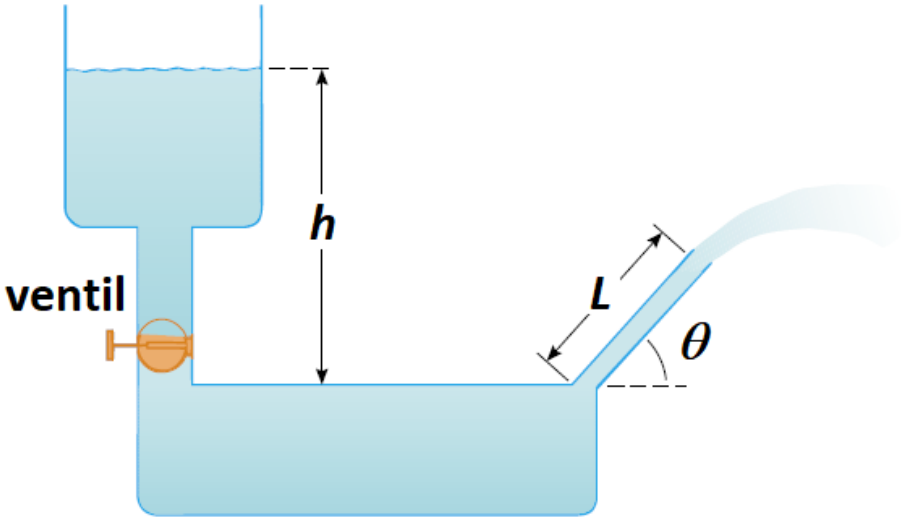
Voda laminarno protječe kroz sustav prikazan na slici. Trenje između slojeva vode je zanemarivo. Veliki spremnici A i F su otvoreni. Površina poprečnog presjeka cijevi C dva je puta manja od površine poprečnih presjeka B i D. Koliki je omjer h_2/h_1?

(3)



Spremnik vode s ventilom spojen je na drugi veliki spremnik kao što je prikazano na slici. Visina površine vode u spremniku s ventilom je 10 m. Cijev duljine 2 m izlazi iz drugog spremnika pod kutom 30° prema horizontali i ima puno manji poprečni presjek od spremnika s ventilom. Ako je ventil otvoren, kolika je maksimalna visina mlaza vode u odnosu na završetak cijevi?

(2.25 m)



Velika arterija koja prenosi krv od srca do pluća ima promjer 5 mm i duljinu 15 cm. Tlak je na početku arterije (kod srca) 380 Pa veći nego na kraju (kod pluća). Viskoznost krvi je 3 mPa·s. Kolika je srednja brzina krvi u toj arteriji?

(0.66 m/s)

Za kap kiše može se uzeti da otpor zraka ima kvadratnu ovisnost o brzini, a koeficijent otpora iznosi 0.5. Kolika je terminalna brzina za kap polumjera 0.2 cm? Gustoća zraka je 1.29 kg/m^3.

(9 m/s)