What "Ease-Off" shows about Bevel and Hypoid Gears

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This article is a follow-up to "The Basics of Spiral Bevel Gears," which appeared in our January/February 2001 issue.

Introduction

The configuration of flank corrections on bevel gears is subject to relatively narrow restrictions. As far as the gear set is concerned, the requirement is for the greatest possible contact zone to minimize flank compression. However, sufficient reserves in tooth depth and longitudinal direction for tooth contact displacement should be present. From the machine—and particularly from the tool—point of view, there are restrictions as to the type and magnitude of crowning that can be realized. Crowning is a circular correction. Different kinds of crowning are distinguished by their direction. Length crowning, for example, is a circular (or 2nd order) material removal, starting at a reference point and extending in tooth length or face width.

Design philosophies connected with the particular cutting methods create additional limits by concentrating on given tools and mechanisms, although machines with expanded capabilities can be built.

Commonly Known Crowning Effects

TRADITIONAL FACE MILLING:

- crowning through differing cutter heads, 5-cut-method, through special blade arrangement and cutter head tilt;
- profile crowning through curved blade profile;
- flank twist (bias) through extra generating motions, or root angle tilt.

TRADITIONAL FACE HOBBING:

- length crowning through two-part cutter head;
- length crowning through special blade arrangement and tilted cutter head;
- profile crowning through curved blade profile.

MODERN FACE MILLING AND FACE HOBBING:

- length crowning through cutter tilt;
- flank twist through root angle tilt;
- profile crowning through curved blades;
- free form corrections through universal machine motions.

With the *CAGE*™ for Windows program for bevel gear calculation, a tool has been developed enabling precise calculation and analysis of all important gear cutting methods.

In addition to its application in plant operations for recalculation of flank compression and root tension, this complex calculation system is a suitable instrument for carrying out examinations that were impossible in the past. Hereby, it is possible to significantly reduce the amount of time required by commonly applied methodology for design, manufacture, and subsequent testing. Furthermore, it is possible to experiment with new possibilities where the manufacture of prototypes for this purpose would be extremely difficult.

Crowning and Ease-Off

Continuous gear corrections emanating from the mean point of the tooth flank and radiating in all directions are called "crowning." In a first approach, the corrections may be described in terms of circular- or parabolic-shaped lengthwise "crowning" on the flank and profile crowning on the tooth profile. The direction of the contact lines has a special importance with regard to these two curvatures. Contact lines with different directions, but with the same lengthwise and profile crowning, *deliver tooth flank corrections with completely different effects.*

Through the kinematic formation in bevel gear machines, flank surfaces are always created during

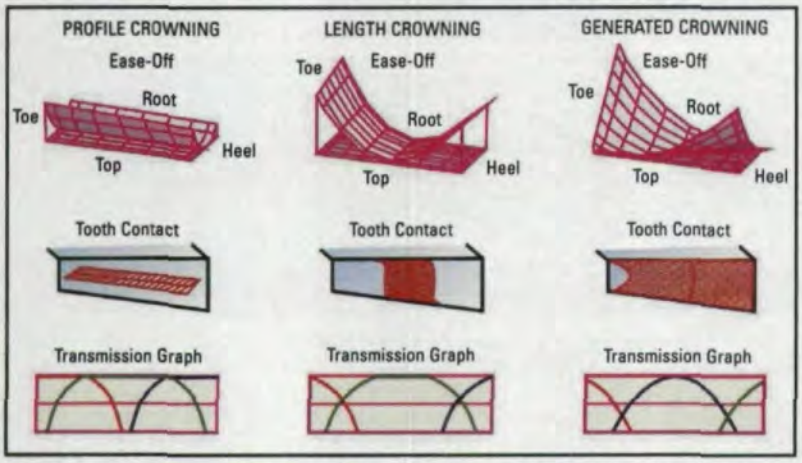

Figure 1—Basic elements for ease-off configuration: profile crowning, length crowning and generated crowning.

the generating process that fulfill a doubly constant differentiability. The usable tooth flank ends in the region where this condition is no longer fulfilled. This is the case, for example, with flank mutilations and undercuts. Protuberance corners in the blade profile are not reflected as corners in the flank, since, through the generating process, the order of the mathematical function of the generated profile lies at least one above that of the tool profile. This rule applies to a certain catalog of blade profile shapes (Ref. 1). In the case of elimination of the generating motion (non-generated ring gears), it is possible to recognize protuberance bends, for example, based only on the contact distances or the shape of the ease-off.

Contact distances in the gear flank area of the ease-off topography show the interplay between pinion and gear flank during the mesh; they also show the corrections made by the actual generating gear versus a theoretical conjugate generating gear independent of whether this "exact" generating gear even exists for the observed method.

Geometrical Flank Corrections

Simple variations in ease-off are created by different blade profiles, cutter head radii and machine settings. This group of parameters leads to changes in the generating gear geometry. By means of the topography, it is possible to recognize directly the action that led to such a change in flank correction, both qualitatively and quantitatively. Flank direction, profile direction and lengthwise and profile crowning are hereby influenced to a large degree. The left column of Figure 1 shows a gear with profile crowning only. Pure length crowning is shown in the center of Figure 1. It is generated by different curvature of convex and concave flanks or a suitable cutter head tilt. The right sequence in Figure 1 is the result of generated crowning as it results from a third order modified ratio of roll or from a root angle tilt. Since this correction works along the path of contact, from roll position to roll position, it creates a parabolic transmission error but has no influence to the relative contact between two interacting contact lines (one of the pinion and one of the gear).

However, crowning along the contact lines and crowning in the path-of-contact direction are relevant and must be taken into consideration during the design phase. Depending on the length of the contact lines, a crowning is desired that stems from both the expected Hertzian contact and the sought-after displacement behavior. The required corrections in the path-of-contact direction are also derived from the displacement behavior of the total gear set and, in addition, from the minimization of meshing impact. The resulting orientations of the main directions of

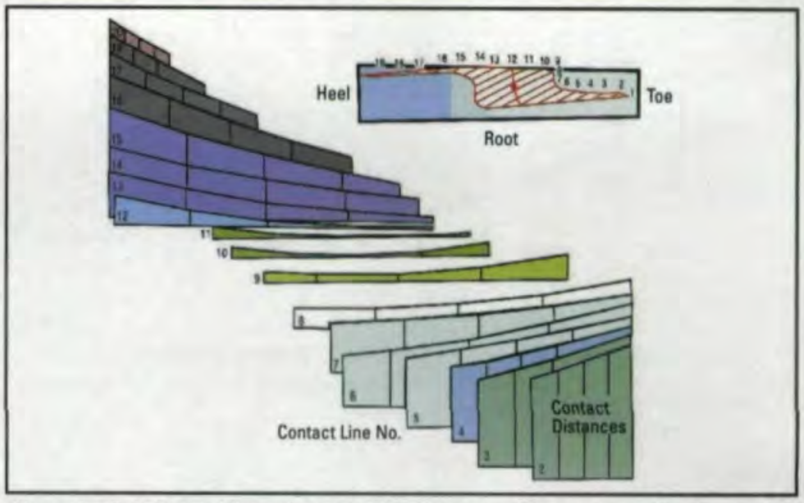

Figure 2—Relative positions of contact lines between pinion and gear flanks—straight bottom line represents the conjugate reference.

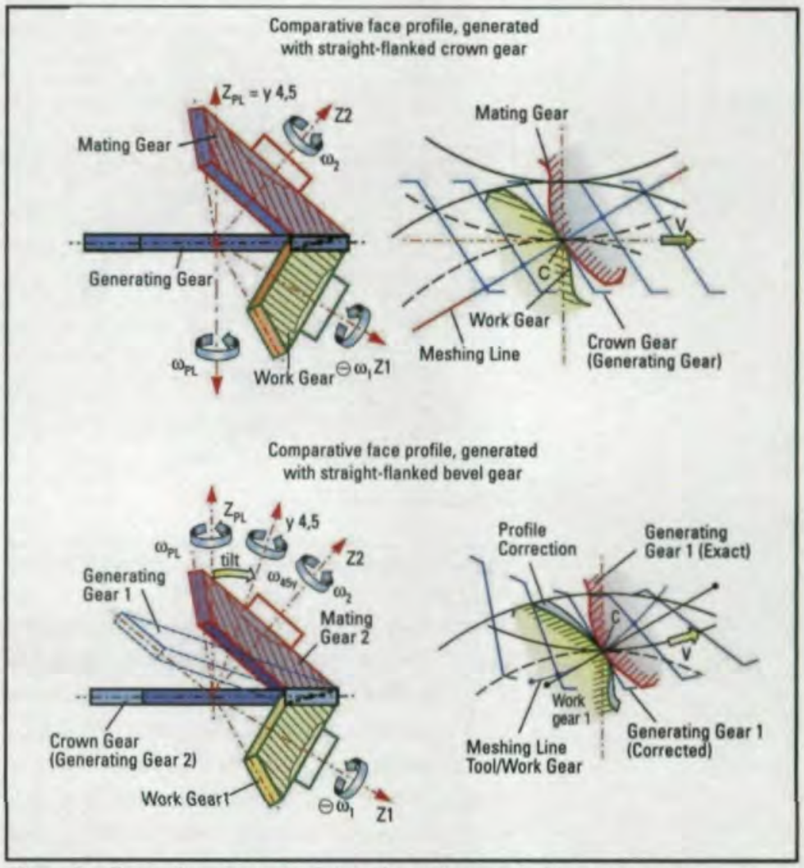

Figure 3—Generation of generated crowning using a conical generating gear.

curvature of the ease-off are those along the contact lines and in the path-of-contact direction, respectively.

The geometrical generating gear corrections dealt with previously do not offer the possibility of deviating from the longitudinal tooth direction or tooth profile direction as the main directions of curvature. Flank corrections thus developed always lead to tooth bearing forms as shown in Figure 1, left and center. The path of contact is developed through the low points of the contact lines caused by the curves they "cut out" of the ease-off topography. If one observes the contact distances over the

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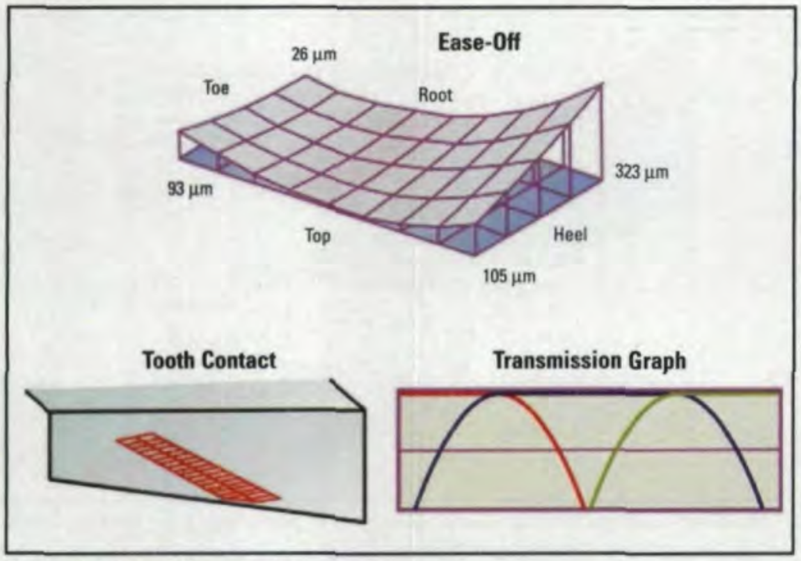

Figure 4—Contact line crowning.

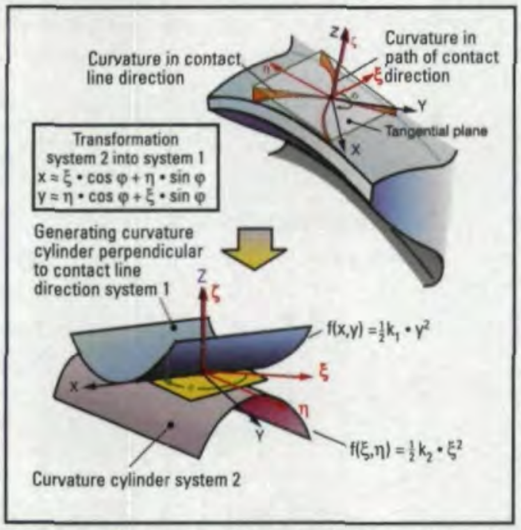

Figure 5—Curvature relationships in the tooth bearing center area.

contact lines, as seen from the ease-off topography (Fig. 2), then the point of contact can be identified by the smallest contact distance.

The smallest contact distance yields the deviation in the angle of rotation. The angled lines of contact, in combination with a symmetrical lengthwise and profile crowning, lead to the typical shapes of the crowning along the contact lines shown in Figure 2.

Accordingly, entrance and exit of tooth contact takes place at the boundaries of the gear flanks, while—only in the mean flank zone—a minimum of the circular crowning function takes place within the flank itself (contact lines 9 to 12, Fig. 2). In order to eliminate the natural twist in case of circular length and profile crowning, which is recognizable in the contact distances in Figure 2, it is necessary that a twist with opposite tendency be applied to the ease-off topography.

Kinematic Flank Corrections

Manipulation of the ease-off, in the sense of twist, belongs to the group of higher-order correction procedures. These procedures lead to changes in the kinematics of the generating gear. Additionally, one may add varied generating ratios (modified-roll, non-linear supplement motions and conical generating gears by applying cutter tilt). The way a conical generating gear functions is shown in Figure 3. The development of gear and pinion flanks by generating with a generating gear with a 90° pitch angle (crown gear) is shown in the top section of the figure. In the cross section (in the middle of the gear face width) in the right section of Figure 3, the profile of the gears is formed by generating with a straight-profiled crown gear. Because the diameters increase from inside to outside when the transmission ratios are the same, the profile curvature decreases continuously toward the heel. The difference shown in the lower right hand portion of Figure 3 occurs because the straight profiled generating flank is not moved horizontally during generation (crown gear), but a rotation of the straight profile takes place through the conical shape of the generating gear body.

The generating roll angle changes over a series of contact lines. The right-hand column in Figure 1 shows the ease-off topography resulting from this procedure. We are speaking here of a purely generated crowning; the contact lines of the tooth bearing marking compound are visible over their entire length, just as with those of the uncorrected conjugate gear. The tooth bearing covers the entire active flank region. The conical generating gear is obtained in the gear machine by tilting the cutter head perpendicular to the axis of the generating gear. An identical flank correction can be obtained through a modified, non-constant generating ratio of roll. The effects of a contact line crowning are shown in Figure 4 (Ref. 2). The effect is opposite to the generating crowning in Figure 1. No transmission error is generated along the path of contact. This effect, which results in a high bias contact, cannot be used independently but is an additional ease-off "design element."

Construction of Ease-Off Topography

For higher developed bevel and hypoid gear sets that are intended for specific uses, it is recommended that an optimal ease-off be "constructed" based on loading-induced deformation and deflection behavior. Correction topography developed in this manner can be described in all points by two curvature cylinders with certain diameters and certain orientations (Refs. 3 & 4), as shown at the bottom of Figure 5. As seen from the preceding sections, through the kinematic generation of bevel gear flanks—if one disregards bends in blade edges—

there always result correction functions that are doubly-constant differentiable, i.e. smooth and bend-free. Construction and prescription of the ease-off topography must reasonably be performed within the framework of these conditions. The ease-off must also be viewed in conjunction with the basic geometrical parameters (especially spiral angle and lengthwise tooth curvature.) To be sure, those are not recognizable on the correction surface, but, for example, they still exhibit strong influence on displacement behavior (Ref. 4).

The top portion of Figure 5 shows the flank correction in contact line and path-of-contact direction relative to the uncorrected flank point (mean point). Herefrom result the curvatures and directions of curvature for the curvature cylinder system in the bottom part of Figure 5. If only linear, generating-cradle-dependent kinematics that act symmetrically toward the mean point are used in addition to the usual geometrical generating gear corrections, then the observation of the differential curvatures of a single flank point—namely the mean point—is sufficient to describe the entire correction. For the realization of the correction surface, traditionally all that are available are the three basic crowning elements shown in Figure 4, for which the directions of curvature have already been determined. Based on that, it is now possible to further simplify the curvature system shown in Figure 5; a third curvature cylinder can now be added, whereby—in this case—the positions (orientation of the lengthwise axes) of the cylinders are fixed. In order to realize the prescribed flank corrections, therefore, only three free curvatures remain to be defined. This is relatively easy for the computer, but very restrictive for the ease-off design.

Variation of the Crowning

In the following sections, some of the examination results of practical gear examples will be discussed. In the three gear sets shown in Figure 6, changes were made only in the length crowning (Ref. 5). All other identifying magnitudes selected were identical to those in the table of Figure 6. In the center of the figure is the sequence of the load-free marking-compound tooth bearing. The length crowning increases from left to right. The starting point was the average length crowning for which the tooth bearing can be seen in the center of Figure 6. The individual reaction of the tooth bearings to changes in length crowning is entirely characteristic. The flat curve of the path of contact (left in Figure 6) twists and, with increasing lengthwise crowning, constantly becomes steeper, while the tooth bearing surface is simultaneously reduced. The entrance and exit of the path of contact in all

Pairing	Crowning	ΔR_{b-RI}	ΔR_{b-RA}	α_n	β_m	Z_1	Z_2	M_m	AV	D_{02}
1	large	14 mm	0	20°	36.6°	11	36	12 mm	0	630 mm
2	medium	5 mm	0	20°	36.6°	11	36	12 mm	0	630 mm
3	small	2.4 mm	0	20°	36.6°	11	36	12 mm	0	630 mm

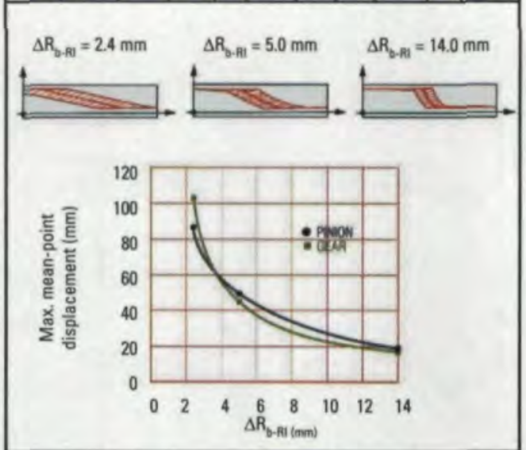

Figure 6—Examination of the influence of crowning alteration only.

cases runs precisely on the edges of the active contact area.

The objective of an increase in crowning is usually the reduction of sensitivity toward axis position displacements. By limiting the correction change to length crowning only, however, the opposite is more often achieved. When used in a gear assembly or for individual axle drives, a tooth bearing displacement resulting from deformation in the direction of heel and tooth top (gear, as opposed to pinion) through the transmitted load is to be expected. The length crowning correction, however, considers only the offset components in the longitudinal direction, i.e., toward the heel. The sensitivity of the tooth bearing in tooth-depth direction, on the other hand, has increased. This fact is exemplified in the lengthening of the entrance and exit zones. If one observes the axis-displacement spectrum (V-H Test) and measures the mean point movement in the direction of the face width, the result is shown in the graph in Figure 6. The gear set with greater lengthwise crowning appears to be highly insensitive.

The total displacement characteristic of a gear offers a complete picture of the displacement capability. From left to right in three sequences, Figure 7 shows the diametrically projected characteristic surfaces for three gears with different crowning. The three surfaces placed one above the other, belong together and include, in ordinate direction, the particular displacement of the respective axis. The supporting grid plan illustrates the gear flank, whereby the coordinate origin lies at the heel root. On the average, through a vertical comparison, it is possible to determine a progressive increasing surface inclination in the longitudinal direction. The slope in profile direction tends to decrease with increased length crowning (from ΔR_b = 5 mm to

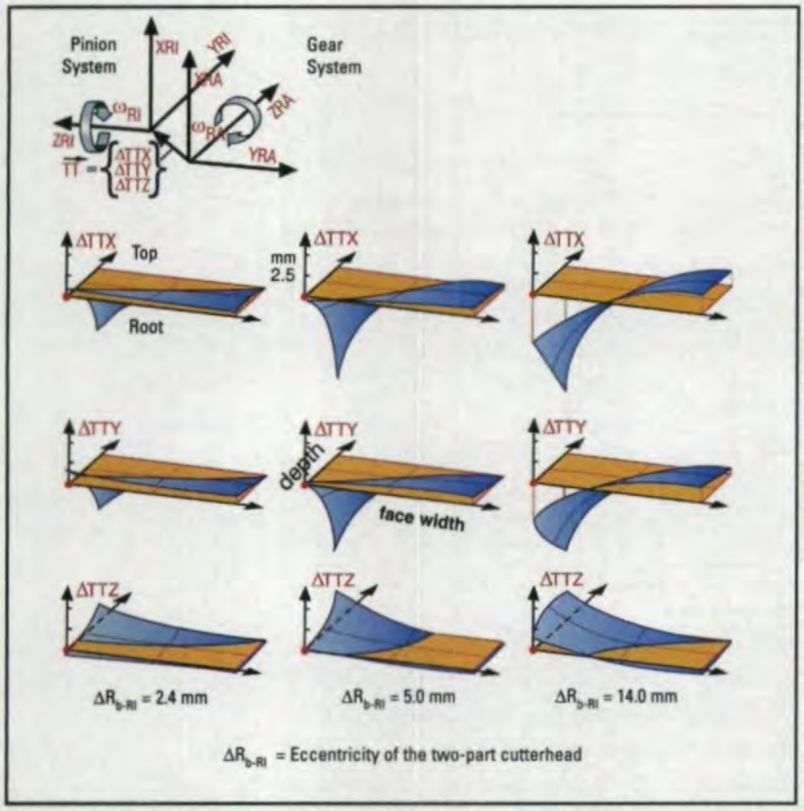

Figure 7—Displacement characteristic surfaces for bevel gears with different crowning.

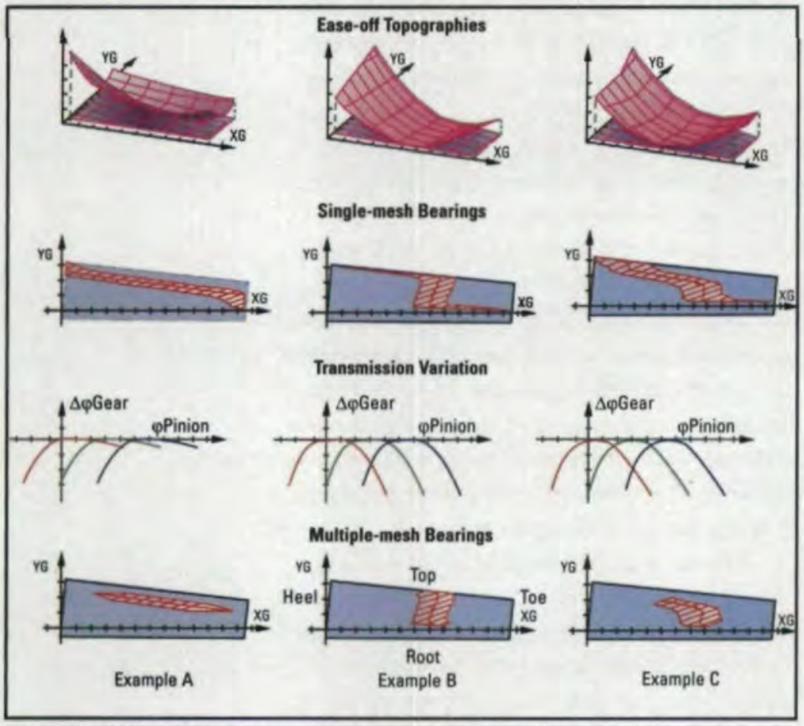

Figure 8—Different contact geometries through variation of length, profile and generated crowning.

ΔR_b = 14 mm, Figure 7). This confirms the above mentioned theoretical consideration of more profile sensitivity after increase of length crowning.

The objective of a bevel gear design that is adapted to compensate for gear assembly and shaft/bearing system distortion is obtaining agreement among the offsetting values of the loading spectrum with vertical values in a figure sequence at the desired contact position. However, any increase in crowning also increases the transmission error, particularly under light loads.

Variation of Length Crowning, Profile Crowning and Generated Crowning

This final section shows some calculation examples (contact analysis and FE strength calculation) for the most important tooth bearing types discussed in this paper. The calculations are based on a generated face milled gear set (hypoid offset = 35 mm, ratio = 7/36, shaft angle = 90°; spiral angles = 49°/29°, ring gear outer diameter = 300 mm). In order to obtain certain tooth bearing forms, the gear machine settings were calculated for a high bias (vertical, narrow) tooth bearing (Figure 8, left) and for a tooth bearing with a vertical path of contact (bias neutral, Figure 8, center.) The original machine settings calculated for an aircraft application delivered a tooth bearing as shown in the right-hand sequence of Figure 8. Example B in Figure 8 presents the classical tooth bearing design. The active region of the path of contact runs along the flank center from the top edge vertically to the root across the profile. The tooth is loaded in the region of its greatest strength. When axis deflection occurs during operation, sufficient room remains for the tooth bearing to shift in the longitudinal direction along the tooth. If a shifting of the tooth bearing in the direction of the heel (larger diameter) is expected, it is easy to make a pre-correction in the tooth bearing position by moving it toward the toe (smaller diameter). A disadvantage of this design is that compression peaks appear on the top edges of pinion and gear flanks, even under minimum loading.

A tooth bearing selection according to Example A in Figure 8 has only been used to date when the ratio of tooth depth to tooth width has been very small, for example d/w = 0.1. In this case, in order to utilize the largest possible flank region when length and profile crowning are equal, it follows that a generated crowning should be included. The advantage of that type of gear may be seen in the fact that the contact lines do not expand all the way to the top edges of both gear and pinion teeth (even under full loading); therefore, lower maximum compressions may be expected. The transmission variation curve of Example A demonstrates—in broad regions—much smaller values than that of Example B or C. It follows that the use of generated crowning not only doesn't necessarily yield an increase in rotary deviation, but can actually decrease rotary deviation due to changes in the path of contact curve.

Today, there is a tendency to select a compromise between variants A and B in order to avoid edge contact as much as possible, while still retaining displacement reserves in the longitudinal direction along the tooth. The gear set C in Figure 8 shows such a compromise. The cutter geometry setting differs from gear B primarily through a small protuberance in the pinion blade (about 1° protuberance angle.) The ease-off topography in sequence C of Figure 8 differs somewhat (at the top) from the one in sequence B. This is due to the protuberance in the cutting blades (not shown). Visible in Figure 8 is just the effect, not the protuberance itself.

In order to judge the actual relationships of gearing A and gearing B under load conditions, finite element calculations for two loading levels (gear torque: T_1=500 Nm, T_2=2,000 Nm) were performed. Figure 9 shows the tooth bearings of the gear flanks under load. The surface stress distribution is shown over the active sections of the contact lines.

Example A shows a parabolic compression curve over almost all contact lines, even under heavy loading. At all load levels, Example B demonstrates the expected surface stress peaks during entrance on the gear top edge and during exit on the pinion top edge. The different maximum values for compression between gearing A and gearing B (Figure 9, bottom) permit the conclusion that gearing A can withstand significantly higher torque than gearing B, as seen from the point of view of flank loading alone. It is also interesting to note that, at 2,000 Nm, gearing A has no roll position with single mesh, but demonstrates at all times a contact ratio between 2 and 3. Gearing B, on the other hand, still has a well-defined single meshing area even at loading level 2,000 Nm.

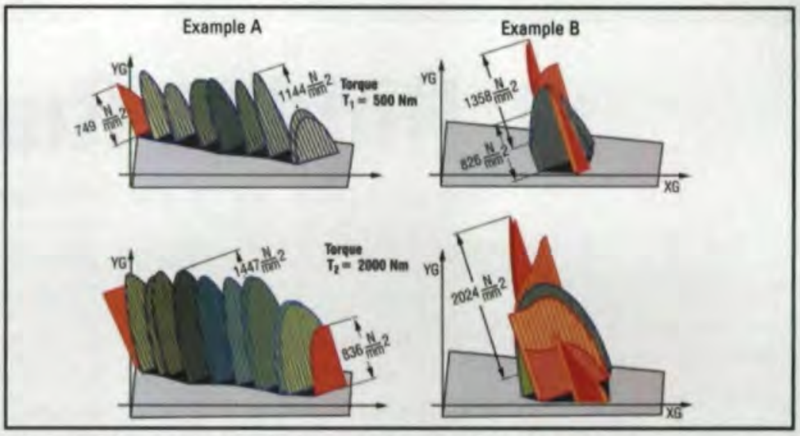

Figure 9—Compression distribution over the active contact line region.

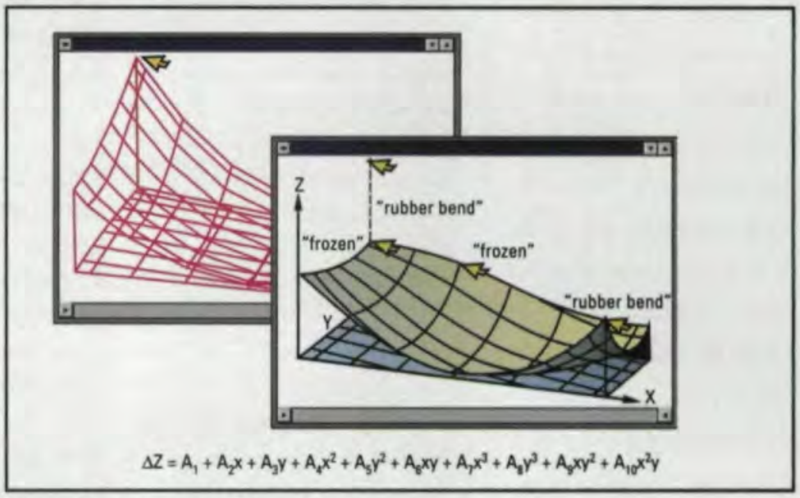

Figure 10—Ease-off by Universal Motions.

Conclusion

Theoretical examinations with analytical calculation tools for flank generation and tooth contact analysis are possible with Windows-based personal computer programs that are easy to operate, are fast and have high flexibility for various parameter studies. The new tools offer the possibility to enter new areas of tooth bearing and ease-off design. Today, the classical tooth bearing configuration is often neglected, and horizontal tooth bearings with large profile and generated crowning are used. However, it appears that a variation of the vertical tooth bearing, the path of contact, which runs slightly inclined over the entire flank width (high bias), presents an improved tooth bearing form that is suited for broad application.

At this time, a new concept of kinematic flank correction is introduced to the bevel and hypoid gear manufacturers. The so-called Universal Motion Control (UMC) adds higher order flank modifications to the second order, mostly geometrically based corrections of this paper. In a completing environment, the new universal motions allow not only the realizing of all combinations of the already discussed ease-offs on coast and drive side independently, but also many shapes that were impossible in the past (Figure 10) (Ref. 6).

Also, to judge flank contact and displacement characteristics, the basic flank form "face hob versus face mill" is of vital importance. The same ease-off and flank contact lead to completely different displacement and roll behavior using one or the other system.

Both a comparison of face hobbing and face milling geometry and the additional enhancement of those systems using the Universal Motion Control could be proposed as "follow-up papers" if the reader's interest is indicated. ○

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