FORMATION OF CARRIER TRIPLE BEAT SIGNAL IN A COMMUNICATION CHANNEL (INTERMODULATION)

Geoffrey Bature Dafe

Abstract

When more than two carrier signals are present in a communication channel, third - order intermodulation interference can be created by the multiplication of the three fundamental carriers, this is called Carrier Triple Beat (CTB). These spurious signals (CTB) are in bands at a level 6dB higher than the third-order intermodulation product created from two signals because there is no second harmonic involved in the production of the interference signal. Intermodulation interference frequency (IF) bandwidth. When the bandwidth is less than one octave, even-order - intermodulation product such as second and fourth order intermodulations interference, are out of band, and can be filtered out. Odd-order intermodulations such as third - and fifth-order products are in-band interference, with sidebands as close to the carrier as the spacing of the desired carriers. After several factors consideration on the interference levels, a summary of the table of results was obtained that shows a variation in the number of carriers for different carrier triple beat signals at different levels below third-order intercepts. This is very useful for RF design engineers in linearising the design of systems that the interference level is suppressed below the desired dynamic range.

Introduction

Most electronic devices suffer from intermodulation interference signals, that only specifically designed filters can be used to attenuate them from being amplified and passing through as noise into the system. The most stubborn being the interference from the third order. This third order intermodulation, which is created by the mixing of the fundamental of one signal and the second harmonic of the order signal, is usually higher in amplitude than the fifth order. Intermodulation product and therefore of primary concern.

When more than two carriers are present in a channel, third-order intermodulation interference can be created by the multiplication of three fundamental carriers; this is called Carrier Triple Beat (CTB). These spurious signals (CTB) are in band at a level 6dBhigher than the third-order intermodulation products created from two signals because there is to second harmonic involved in the production of the interference signal. The level of CTB interference is further enhanced **by** the fact that multiple CTB signals can occur in the frequency band. The number of interference signals that can be superimposed on any particular channel is related to the number of desired carrier present. Statistically, more CTB interference occurs in the center of the band.

Methodology: Determining Intermodulation Interference Levels

Experimentally, this was carried out in the laboratory using a 2GHz spectrum analyzer and three separate carrier signals from different RF amplifiers fed into the analyzer and the output signal viewed and recorded. CTB interference is a spurious signal created from the interaction of three or more signals summed together in a non-linear device. The level of the interference signal is related to the levels of the input signals and the non-linearity of the device. In designing a system, the required operating signal levels and the respective acceptable spurious level determine the acceptable nonlinearity of the equipment. Given the system's non-linear characteristic, the acceptable spurious level and the output level of the carriers, the maximum number of carriers can be determined.

To determine the maximum operating signal, the nonlinear characteristic of the components are defined and the resultant spurious responses are evaluated. The same analysis is performed in reverse to define the acceptable nonlinearity given an operating signal range. A generalized non linear system can be represented by a Taylor Series Expansion of the non-linear transfer characteristic, thus:

$$S_0 = \alpha_0 + \alpha S| + \alpha_2 Si^2 + \alpha_3 Si^3 + \alpha_4 Si^4 + \ldots \quad (1)$$

Where S_0 = output signal, Si = input signal a_n = Coefficient of the device (n=0,1,2,3,4, ...)

For a linear system, $a_n = 0$ for n> 1 If the device is A.C. coupled, $a_0 = 0$

If *S/-Elcos* (coit) + *Elcos* (co$_2$t) + *Elcos* (ro.it)

Where El = Peak amplitude, w, w_2. and w3= respective radian frequencies.

When Si is applied to a non-linear system, intermodulation products are created in all the higher order terms, proportion to the coefficients of the respective terms

Second Order Intermodulation of CTB

The second order intermodulation product of three in-band signals is usually almost an octave away from the desired carriers. In a narrowband system, these signals can easily be filtered and are therefore, not considered in the spurious analysis. The second order intermodulation term is expected as Second order = 2^{nd} order = $a_2si = a_2$ [Elcos (coit) + Elcos ($o>_2t$) + Elcos (cjo_3t)]^2

Expanding the terms.

2^{nd} order = a_2 [Elcos (cod) ^{2} + [Elcos (to_2t)^2 + [Elcos ($o>_3t$)]^2 + 2[Elcos (cojt)]^2 . Elcos (co_2t)^2 + 2[Elcos (cod) . Elcos (co_3t) + 2[Elcos ($u>_3t$). Elcos (co_2t)] .... (2)

All of the terms are the sum or difference of two carriers closely spaced (narrow band). A trigonometric expansion would put all of the interference signals almost an octave away. For systems with a bandwidth less than an octave, these products can be filtered and therefore their effects on the systems performance are negated. For the purpose of this analysis the second order effects of three carriers beating with each other will be considered negligible.

CTB Intermodulation Interference Signals

CTB intermodulation products are spurious signals due to the cube of the input signal multiplied by the a_3 coefficients of the Taylor serious expansion. A trigonometric expansion of this term confirms that the interference signals are in band therefore cannot be filtered out.

An Analysis of Third Order Intermodulation of CTB Signals

If *Si* consists of three signals of *Equation Amplitudes:*

Si = Elcos (o»|t) + Elcos (co_2t) + Elcos (co_3t)

Third-order the CTB = 3^{rd} order = $a_3S/^3$ [Elcos (co,t) + Elcos (co_2t) + Elcos (co_3t)]^3 .... (3)

Expanding the terms (reader can decide to break it down the way he understands also),

3^{rd} order = a_3 [Elcos (cofi) + Elcos (co_2t) + Elcos (io_3t)]. {[Elcos (cod)]^2 + [Elcos (w_2t)]^2 + [Elcos (co_3t)]^2 + 2[Eicos (cod). Elcos (co_2t)] + 2[Elcos (cofi)] . Elcos (co_3t) + 2[Elcos ($u>_3t$) . Elcos (co_2t)]

....(4)

Multiplying out and combing terms (reader can decide to break it down yhe way he understands also, for detailed derivation),

3^{rd} order = a_3EI^3 {[cos (cofi)^3 + [cos (co_2t)]^3 + [cos (co_3t)]^3

+3/2[2cos(a>it) + 2cos (co_2t) + 2cos (co_3t)

+ l/2cos(2coi - $®_2$)t) + +l/2cos(2u)| + co_2)t)

+l/2cos(2a)| - co_3)t) + +l/2cos(2o)| + $cr>_3$)t)

+ l/2cos($2co_2$ - coi)t) + +l/2cos($2co_2$ + (0])t)

+ l/2cos($2co_3$ - (0|)t) + +l/2cos($2to_3$ + coi)t)

+l/2cos($2co_2$ - co_3)t) + +l/2cos($2co_2$+co_3)t)

+l/2cos($2co_3$ - $©_2$)t) + +l/2cos($2oo_3$ + (o_2)t)

+(6/4)cos(co, -oo_2+ co_3)t) + +(6/4)COS(OJI- co_2 _ co_3)t)

$+(6/4)\cos(\omega_1 + \omega_{2} + \omega_3)t)$ + $+(6/4)\cos(\omega_1 + \omega_2 - \omega_3)t)\}$............(5)

Assuming that all their frequencies ($\omega_1, \omega_2, \omega_3$.....) are located in a new narrow band (much less than an octave) and considering only in the band terms, other than the fundamental

3^{rd} order (in-Band) = $a_3EI^3 \bullet \{1/2\cos((2\omega_1 - \omega_2)t) + 1/2\cos(2\omega_1 - \omega_3)t)$
$+ 1/2\cos(2\omega_2 - \omega_1)t) + 1/2\cos(2\omega_3 - \omega_1)t) + 1/2\cos(2\omega_2 - \omega_3)t)$
$+ 1/2\cos(2\omega_3 - \omega_2)t) + (6/4)\cos(\omega_1 - \omega_{2} + \omega_3)t) + (6/4)\cos(\omega_1 - \omega_{2} . \omega_3)t)$
$+ (6/4)\cos(\omega_1 + \omega_2 - \omega_3)t)\}$..(6)

The resulting frequencies are 2 Signals 3^{rd} order $(2\omega_2 - \omega_1)$ and CTB. The relative amplitudes are $(1/2)\ a_3EI^3$ and $(6/4)\ a_3EI^3$, respectively.

The CTB signals are three times higher than the third order intermodulation products when more than two carriers are present.

Two signal, third-order intermodulation distortion.

The resultant frequencies are $2\omega_2 - \omega_2$ and $2\omega_2 - \omega_1$. the relative amplitudes are $3/4\ a_3EI^3$ for both.

CTB Levels Compared Third-order Intermodulation

A tabulation of the third order intermodulation is listed in table 1.

Table 1 third order intermodulation distortion

	Two signals	CTB
Relative amplitude	$3/4\ a_3EI^3$	$(6/4)\ a_3EI^3$

The two-tone, third-order intermodulation product interference is determined by noting the relative single carrier power with respect to the third order intercept point (usually 10dB above the 1 dB compression point) as shown in fig. 1, with formula that

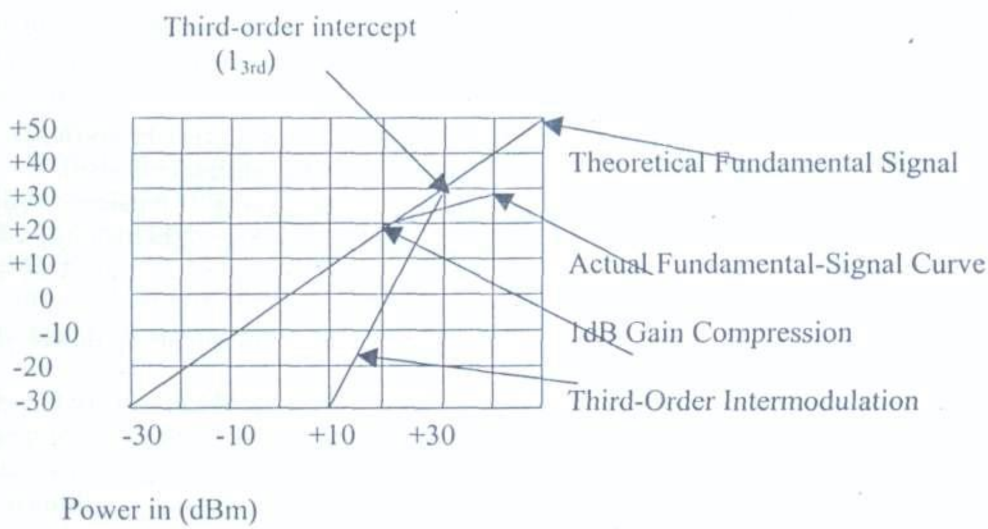

Fig. 1 Third – Order Intermodulation Levels for a typical 10dB gain amplifier

$$IP_3 = -2(I_{3rd} - A) \text{ and } CTB = IP_3 + 6dB$$

Where IP_3 = relative third order intermodulation level (dBc) ; CTB = relative carrier triple beat intetermodulation level (dBc) ; A = signal 1 amplitude = signal 2 amplitude in (dBm); I_{3rd} = third order intercept point (dBm); (dBc) = relative level of the intermodulation with respect to the single carrier amplitude (A).

An example for two output signals of equal amplitude A, as shown in Fig. 2, is

A = +3 dBm I_{3rd} = +20dBm IP_3 = -34 dBc (absolute level = -31 dBm)

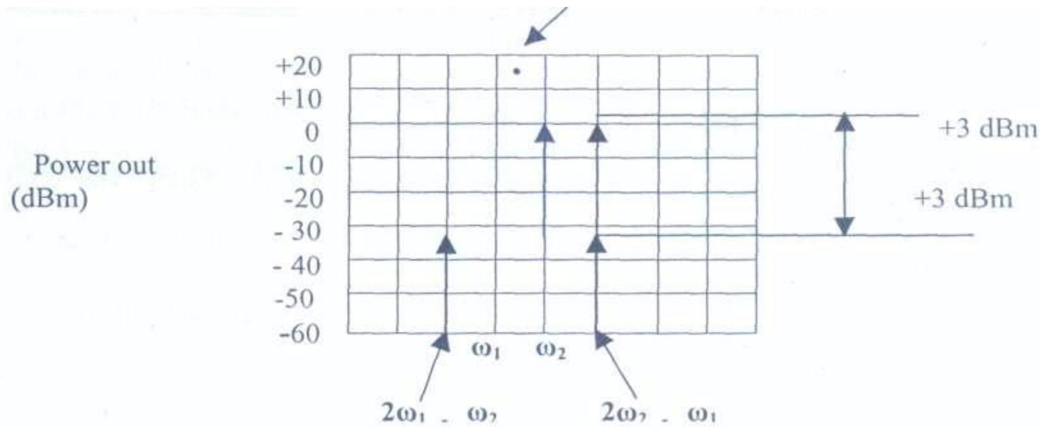

Fig 2 Two-tone, third-order intermodulation levels

Three carrier of equal amplitude A will have a CTB interference of CTB = IP3 – 6dB = -34 dBc + 6 dB = -28 dBc (-25dBm), as shown in fig. 3.

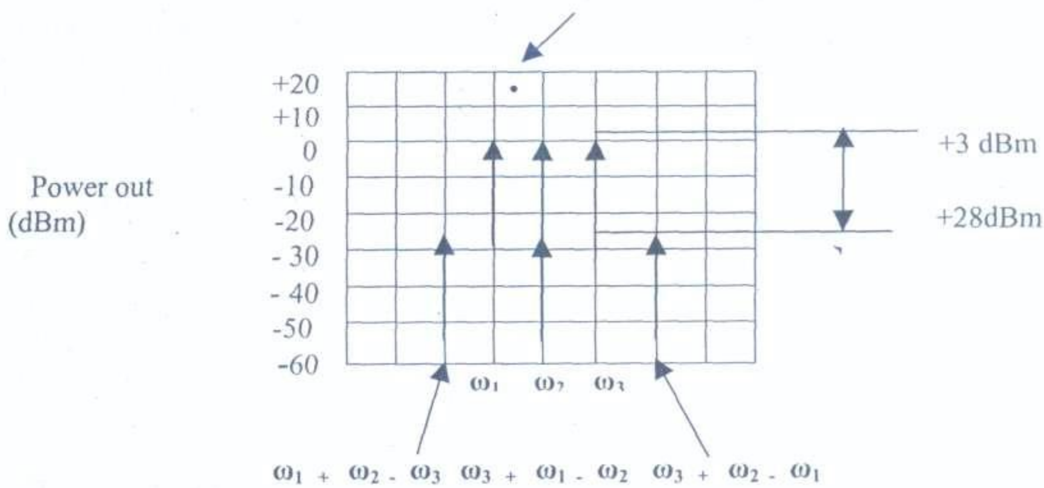

Fig. 3. Carrier Triple beat intermodulation diagram.

The three interference signals each down 28 dBc are products of ω_1, ω_2, ω_3 The lowest interference signal is at frequency $\omega_1 + \omega_2 - \omega_3$. In the center, the interference signal is at $\omega_3 + \omega_1 - \omega_2$ and the highest frequency is at $\omega_3 + \omega_2 - \omega_1$.

Calculating CTB Intermodulation Levels for N Equal Amplitude Signals.

Unlike third order intermodulation interference, CTB signals can overlap each other and add noncoherently. This considerably increases the overall spurious in any givan channel. The total spurious interferences is related to the number of carriers and the position of the carrier, that is, carriers at the ends of the bandwidth have less interfernce products than channels in the center of the band. The number of interference carriers (beats) in any channel is given by

$$\text{beats} = \frac{N^2}{4} + \frac{(N - M)(M-1)}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Where beats = number of interference carriers in the measured channel, N = number of Channels, M = number of the measured channel, $1 \leq M \leq N$

The maximum number of interference carriers occurs in the center of the band (M = N/2). For N >>1, the maximum number of beats (beats_{max}) is given by

$$\text{beats}_{max} = \left(\frac{N^2}{8}\right)$$

The worst-case level of CTB can be arrived at by calculating the levels of each CTB (which is the third-order intermodulation level + 6dB), adding noncoherently the number of beat signals that will fall into the respective band (the worst case being in the center of the band). The CTB interference can therefore, be determined using

$$CTB = 2(I_{3rd} - \text{carrier}) + 6 + 10\log(\text{beat}_{max}) \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Where I_{3rd} = third order intercept point(dBm); Carrier = single carrier output signal level (dBm) Beat_{max} = number of interference products in any channel,
In terms of the total number of carriers N

$$CTB = 2(I_{3rd} - \text{carrier}) + 6 + 10\log = \left[\frac{(N^2)}{8}\right] \qquad (\text{dBc})$$

For example, I_{3rd} = +15; carrier = -20dBm; N=; then CTB will be –45.5 dBc.

Calculating the number of carrier in a given channel

Inversely, the total number of carrier that can be multiplied into a single channel., knowing the required CTB interference level, can be calculated assuming that all of the interference signals are noncoherent and the bandwidth is wide enough for all of the carriers to exist acceptable adjacent channel interference.

The total number of carrier N is given by

$$N = \sqrt{8\left[10^{\frac{(CTB-6+2(I_{3rd}-\text{carrier})}{10}}\right]} \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Where CTB = Maximum acceptance CTB interference level (dBc); I_{3rd} = third-order intercept point (dBm); carrier = single carrier output signal level (dBm); N=number of modulated carrier. As an example, consider, consider CTB = ≤ -57.6 dBc; I_{3rd} = +20 dBm; carrier =-15dBm; the relative third-order intermodulation level is

$$IP_3\ (\text{dBc}) = -2(I_{3rd} - A)$$

Where A = signal 1 amplitude = signal 2 amplitude (dBm); for N output signals, A is taken as A =+3 dBm; I_{3rd} = +20 dBm, and IP_3 = -34 dBc (absolute level = -31 dBm). First check to see that the two-tone, third-order intermodulation interference is below the required specification where A =+3 dBm $IP_3 = -2\,(I_{3rd} - \text{carrier}) = -2[20-(-15)]$ = 70dBc. The obviously meet the desired criteria. Then, the total number of carriers considering CTB interference is

$$N = \sqrt{8\left[10^{\frac{(CTB-6+2(I_{3rd}-\text{carrier})}{10}}\right]}$$, giving **N = 5.909 = 5 carriers**

Results and Discussions

It should be noted that this analysis is valid for CW carriers, which should be considered a worst case signal. Modulated carrier exhibit spectrum spreading which in effect, will lower the intermodulation interference. Table 2, is computed for estimating the number of carriers that a given bandwidth could sustain (neglecting the bandwidth of the carrier, interchannel interference and available system bandwidth). Across the top right hand of Table 11 is the level of each carrier (assuming all the carriers are the same level) below the third-order intercept point. To the left of Table 11, is the acceptable interference level (CTB).

Table 2: Number of carriers in a single channel

Level below third-order intercept	20dB	25dB	30dB	35dB	40dB
CTB			Number carriers		
-20	14	44	141	448	1420
-25	7	25	79	252	797
-30	4	14	44	141	448
-35	2	7	25	79	252
-40	1	4	14	44	141
-45	0	2	7	25	79
-50	0	1	4	14	44
-55	0	0	2	7	25
-60	0	0	1	4	14
-65	0	0	0	2	7
-70	0	0	0	1	4
-75	0	0	0	0	2
-80	0	0	0	0	1

Conclusion

Determining the capacity of a channel more involved than allocating enough bandwidth. CTB interference is an important factor to consider when a channel is loaded with many carriers. It has been shown that the interference level increase as the square of the increase in the number of carriers. The worst-case interference is in the center of the band where there are more combinations of frequencies in a given channels. At the ends of the band, the interference levels go down, but unless the power level of the carriers in the center are lower than the carrier powers at the band edges, it is prudent to assume the worst interference for the system design.

Although the problem is critical as the number of carriers increase, it should be noted that even with only three in band carriers, the interference level is more than 6dB above that calculated for two-signal, third-order intermodulation. It must be noted that this is a worst-case analysis. Most modulated carrier exhibit a band spreading that lower the average spectral density which will some what lower the respective interference.

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