

# Estimates of mutual inductance

Biot-Savarts law:

$$B = \frac{\mu I}{2\pi r}$$

$B$  is the magnetic flux density at a distance  $r$  from a (long) conductor carrying a current  $I$ .

- $B$  increases with increasing  $I$
- $B$  decreases with increasing  $r$ .

## Example:

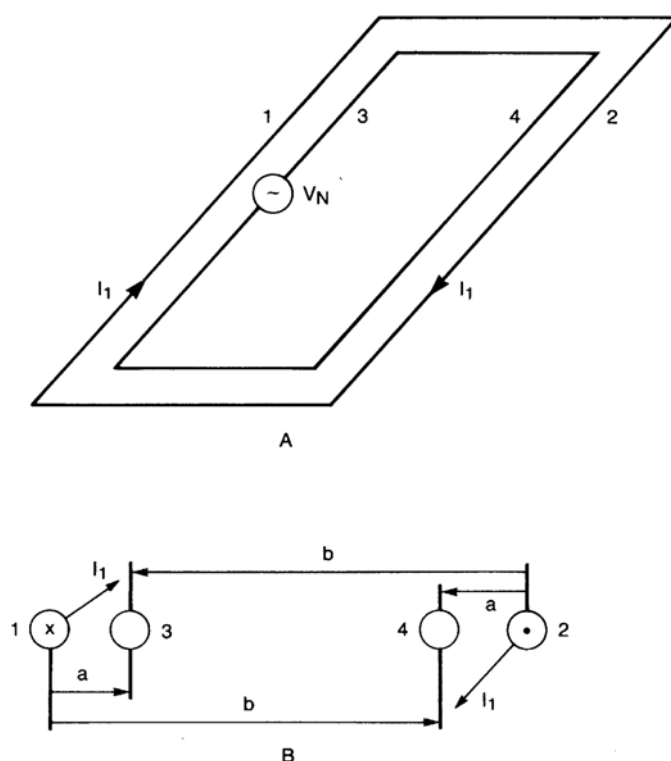


Figure 2-10. (A) Nested coplanar loops; (B) cross-sectional view of A.

Assume the wires along the long “edges” are significantly longer than the wires on the “short edges”.

$I_1$ : Current in outer loop (1 and 2).

$V_N$ : Induced voltage in the inner loop (3 and 4).

a: distance between outer and inner loop.

b: distance between outer loop and inner loop on the opposite side. The flux in the loop consisting of 3 and 4 due to the current in 1:

$$\theta_{12} = \int_a^b \frac{\mu I_1}{2\pi r} dr = \frac{\mu I_1}{2\pi} \ln\left(\frac{b}{a}\right)$$

Conductor 2 generates a similar flux in 3 and 4 in the same direction. Thus the flux is doubled.

Total flux in 3 and 4 due to 1 and 2:

$$\theta_{12} = \left[ \frac{\mu}{\pi} \ln\left(\frac{b}{a}\right) \right] I_1$$

We use the following expression introduced earlier:

$$M_{12} = \frac{\theta_{12}}{I_1}$$

replaces and achieves:

$$M = 4 \times 10^{-7} \ln\left(\frac{b}{a}\right)$$

To find the voltage we include  $M$  in the expression and get:

$$V_N = j\omega M I_1 = j\omega I_1 \cdot 4 \times 10^{-7} \ln\left(\frac{b}{a}\right)$$

Example:

$$f = 10\text{MHz}$$

$$I_1 = 100\mu\text{A}$$

$$a = 10\mu\text{m}$$

$$b = 3000\mu\text{m}$$

$$V_N = 14\text{mV}$$

# The effect of shielding a conductor

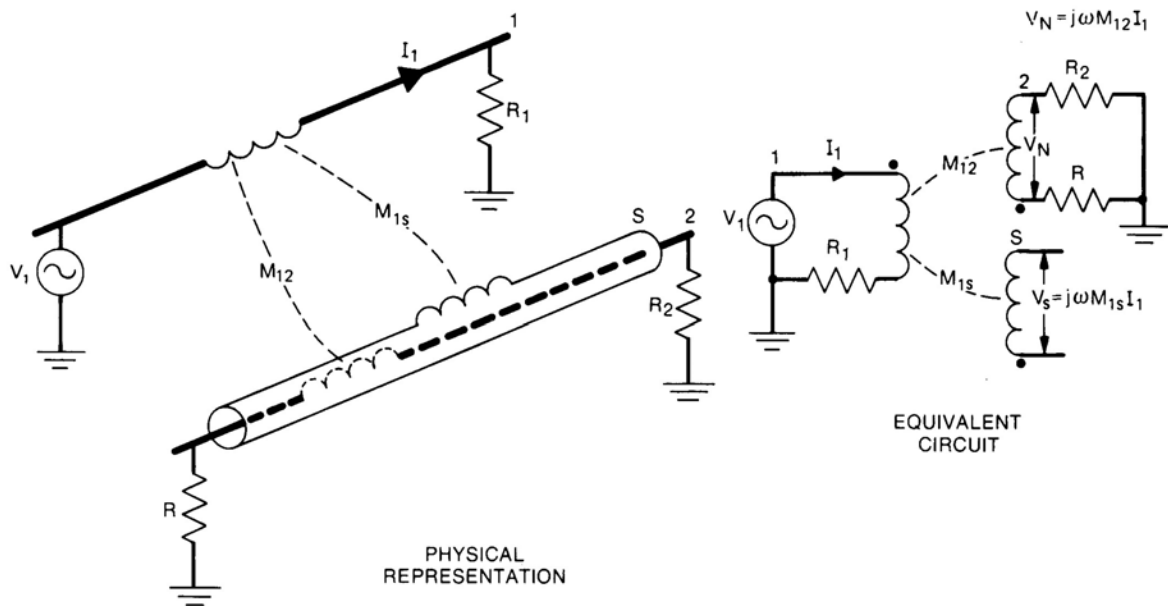


Figure 2-11. Magnetic coupling when a shield is placed around the receptor conductor.

Source (1) generates a voltage in the object (2)

a) First we assume:

The shield is not grounded and non-magnetic

=> The shield has no influence on 2

=> The induced shield voltage is:

$$V_S = j\omega M_{1S} I_1$$

where  $M_{1S}$  is the mutual inductance between the shield and 1.

b)

*Grounding at one end of the shield will have no influence!! (Assuming non-magnetic shielding)*

c) Assume:

The shield is grounded at both ends:

⇒ The voltage induced in the shield will result in a current through the shield.

⇒ This current will induce noise in 2.

What about magnetic coupling between shield and centre conductor? This has to be investigated before we carry on.

# Magnetic coupling between shield and centre conductor

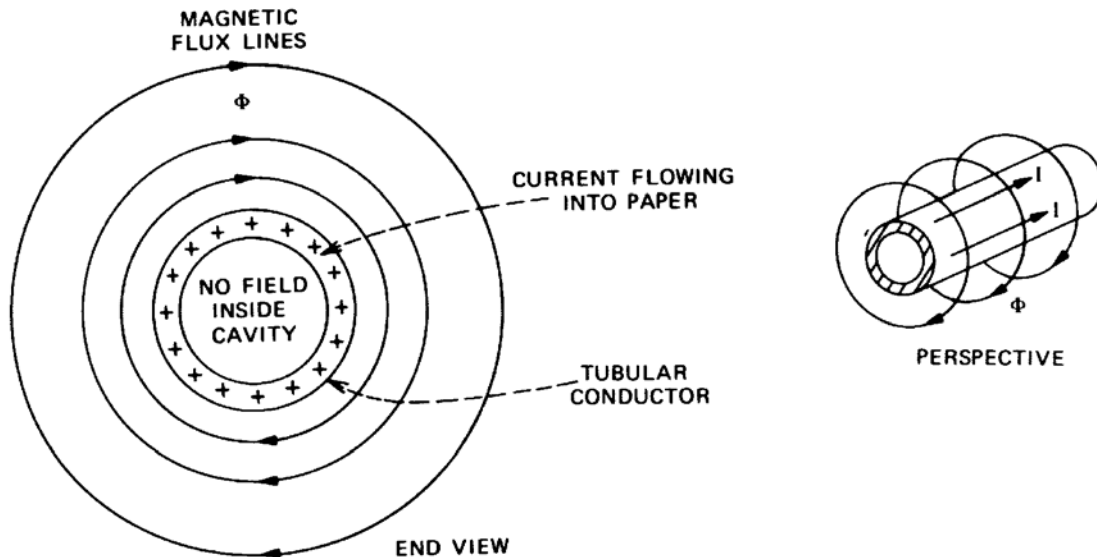


Figure 2-12. Magnetic field produced by current in a tubular conductor.

a) Assume a conductor shaped as a tube.  
(Same thickness everywhere. Uniform current distribution.)

- ⇒ No magnetic field lines inside the pipe
- ⇒ Some magnetic field outside the pipe



b) Conductor inside the shield (Say a coax).

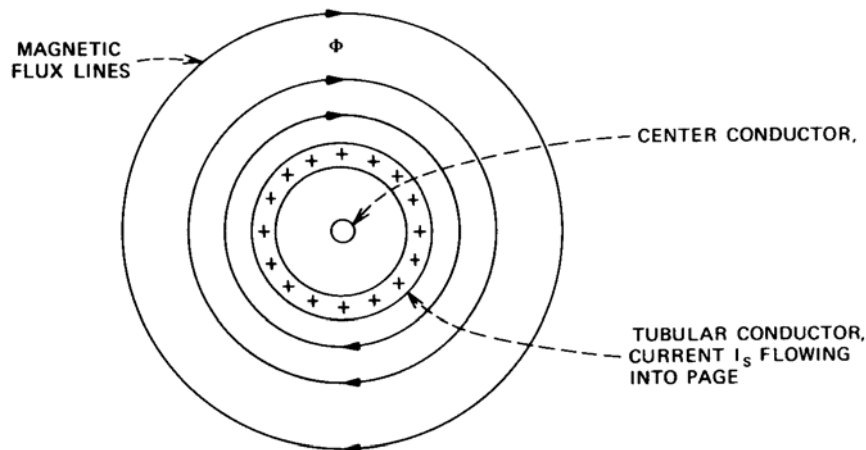


Figure 2-13. Coaxial cable with shield current flowing.

$$L_S = \frac{\phi_S}{I_S}$$

$L_S$ : Shield inductance ("the pipe")

$I_s$ : Current in shield ("the pipe")

$\phi_s$ : Magnetic field generated from screen that enclose both shield and inner conductor.  $\phi_c$  is the magnetic field observed by the centre conductor.

$$M = \frac{\phi_c}{I_s}$$

$M$ : Mutual inductance.

Since  $\phi_s = \phi_c$  we get the important relation:

$$M = L_S$$

*Mutual inductance between shield and inner conductor is equal to the shield inductance!*

Conditions:

- No magnetic field lines in shield
- Uniform distribution of current in shield

Does NOT require:

- That the centre wire is in the geometric centre of the shield (does not have to be a coax).

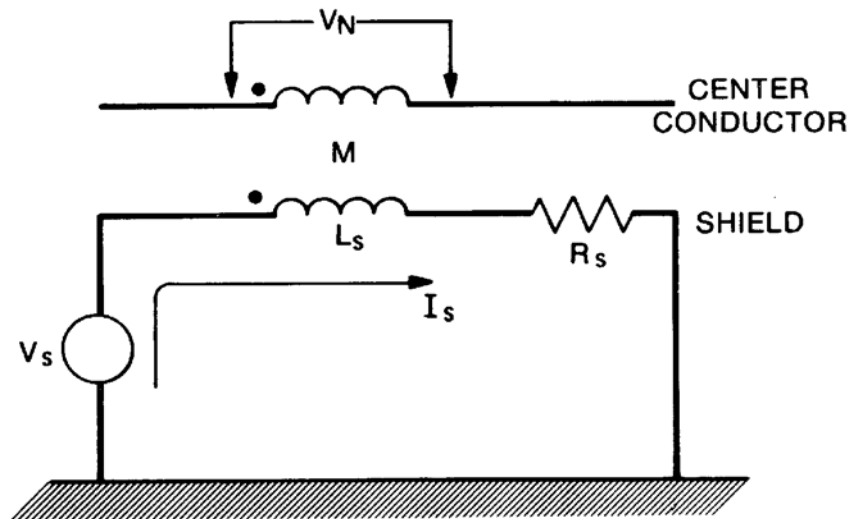


Figure 2-14. Equivalent circuit of shielded conductor.

$V_s$ : Voltage in shield induced from an external source (not drawn)

$I_s$ : Current in shield as a result of  $V_s$  over  $L_s$  and  $R_s$ .

$$V_N = j\omega M I_s$$

$V_N$ : Voltage in centre conductor due to  $I_s$

We find the current  $I_s$ :

$$V_s = I_s (R + j\omega L)$$

and change the order to end up with:

$$I_s = \frac{V_s}{L_s} \left( \frac{1}{j\omega + R_s / L_s} \right)$$

Combined with the previous expression we achieve:

$$V_N = \left( \frac{j\omega M V_S}{L_S} \right) \left( \frac{1}{j\omega + R_S / L_S} \right)$$

Since  $L_S=M$  we may reduce the expression to:

$$V_N = \left( \frac{j\omega}{j\omega + R_S / L_S} \right) V_S$$

The plot for this expression looks like:

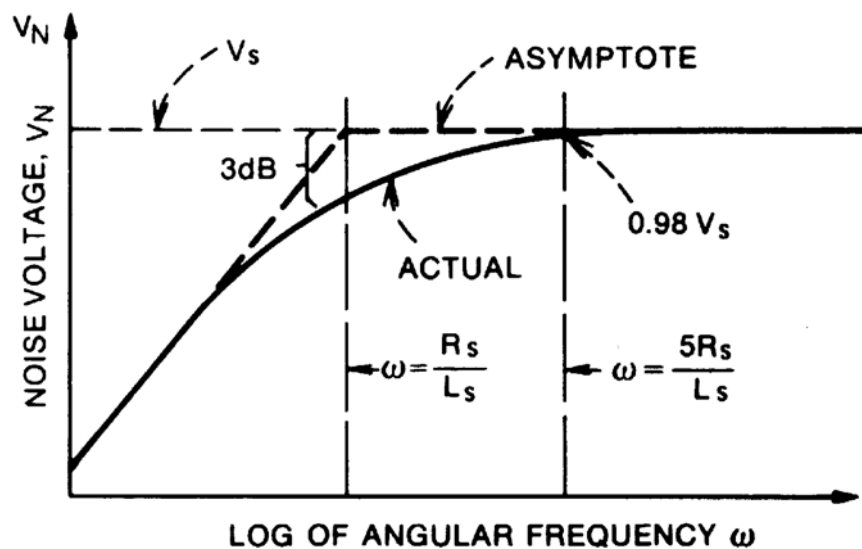


Figure 2-15. Noise voltage in center conductor of coaxial cable due to shield current.

Low frequencies:  $V_N = (j\omega L_S / R_S) V_S$

High frequencies:  $V_N = V_S$

Cut-off frequency:

$$\omega_c = \frac{R_S}{L_S} \qquad f_c = \frac{R_S}{2\pi L_S}$$

Due we wish  $R_s/L_s$  to be small or large?

Small  $R_s/L_s$  means

lower cut-off frequency, faster ascent during cut-off.

Achieved through

- Small resistance in shield
- Large inductance in shield

**Table 2-1 Measured Values of Shield Cutoff Frequency ( $f_c$ )**

Cable	Impedance ( $\Omega$ )	Cutoff Frequency (kHz)	Five Times Cutoff Frequency (kHz)	Remarks
Coaxial cable				
RG-6A	75	0.6	3.0	Double shielded
RG-213	50	0.7	3.5	
RG-214	50	0.7	3.5	Double shielded
RG-62A	93	1.5	7.5	
RG-59C	75	1.6	8.0	
RG-58C	50	2.0	10.0	
Shielded twisted pair				
754E	125	0.8	4.0	Double shielded
24 Ga.	—	2.2	11.0	
22 Ga. <sup>a</sup>	—	7.0	35.0	Aluminum-foil shield
Shielded single				
24 Ga.	—	4.0	20.0	

<sup>a</sup>One pair out of an 11-pair cable (Belden 8775).

frequency than any other. This is due to the increased resistance of its thin aluminum-foil shield.

From the table we see that the cut-off frequency is in the area: 2-20 kHz.

$$Z = \sqrt{\frac{L}{C}}$$

# When shield is connected at both ends

(We jump back to where we were before we looked at the coax.)

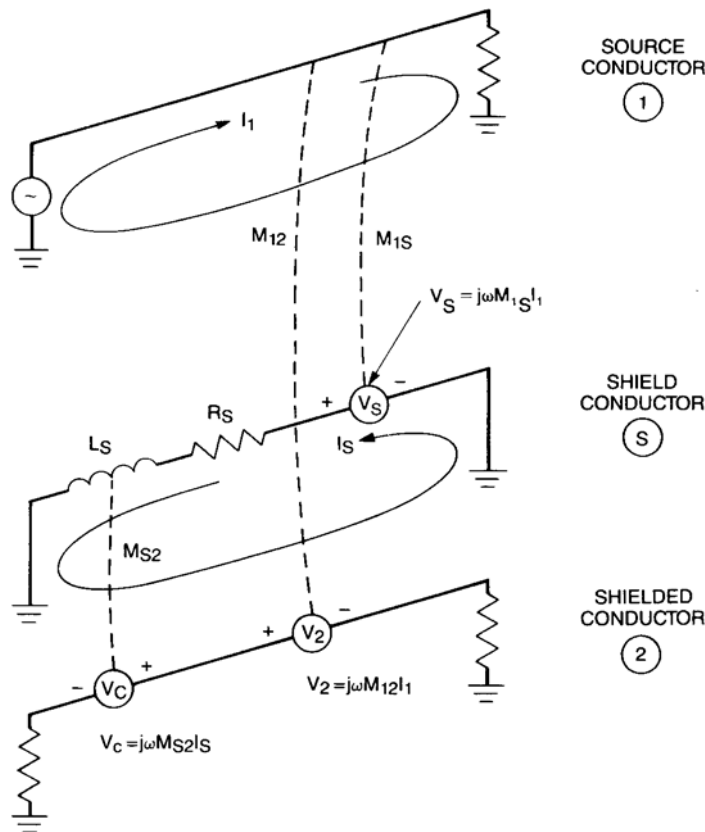


Figure 2-16. Magnetic coupling to a shielded cable with the shield grounded at both ends.

$V_N$  is a result of direct radiation from source and from source via shield.

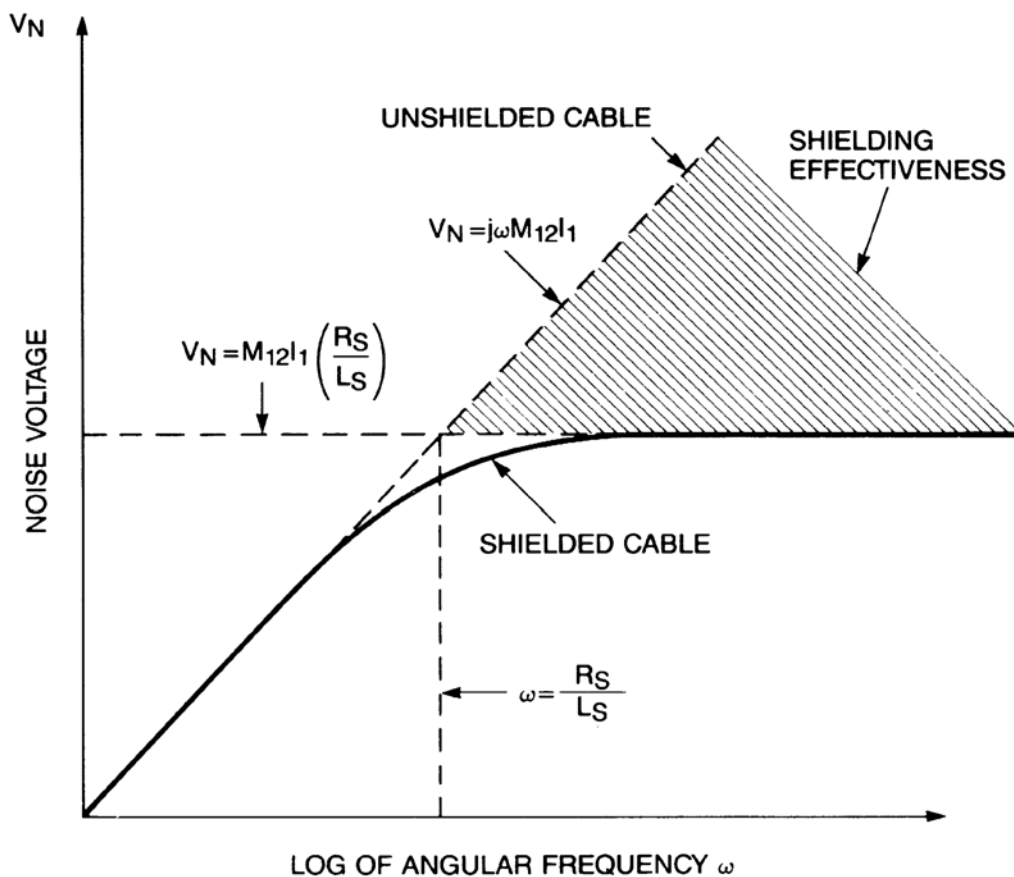
$$V_N = V_2 - V_C$$

Now we can put up the expression for a double grounded shield.

- The shield contributes with the term in parenthesis.
- Without the screen we will only have the term in front of the parenthesis.

Low frequency:  $V_N = j\omega M_{12} I_1$

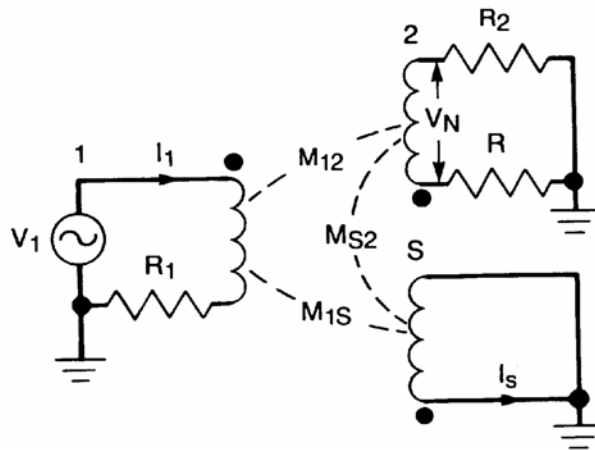
High frequency:  $V_N = M_{12} I_1 (R_S / L_S)$



**Figure 2-17.** Magnetic field coupled noise voltage for an unshielded and shielded cable (shield grounded at both ends) versus frequency.



The drawing shows a transformer model of the system:



**Figure 2-18.** Transformer analogy of magnetic field coupling to a shielded cable when shield is grounded at both ends ( $M_{S2}$  is much larger than  $M_{12}$  or  $M_{1S}$ ).

The shield works as a short-circuit winding that will short connect the voltage in 2.

Shielding to avoid emission:  
First some basics:

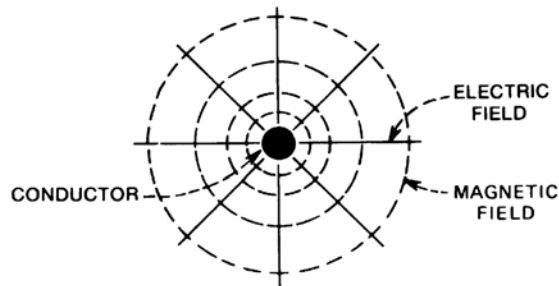


Figure 2-19. Fields around a current-carrying conductor.

A conductor will set up a radial electric field and a circular magnetic field.

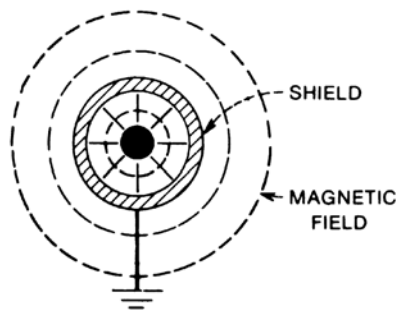


Figure 2-20. Fields around shielded conductor; shield grounded at one point.

A screen grounded at one end will terminate the electrical field.

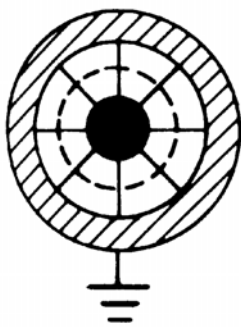
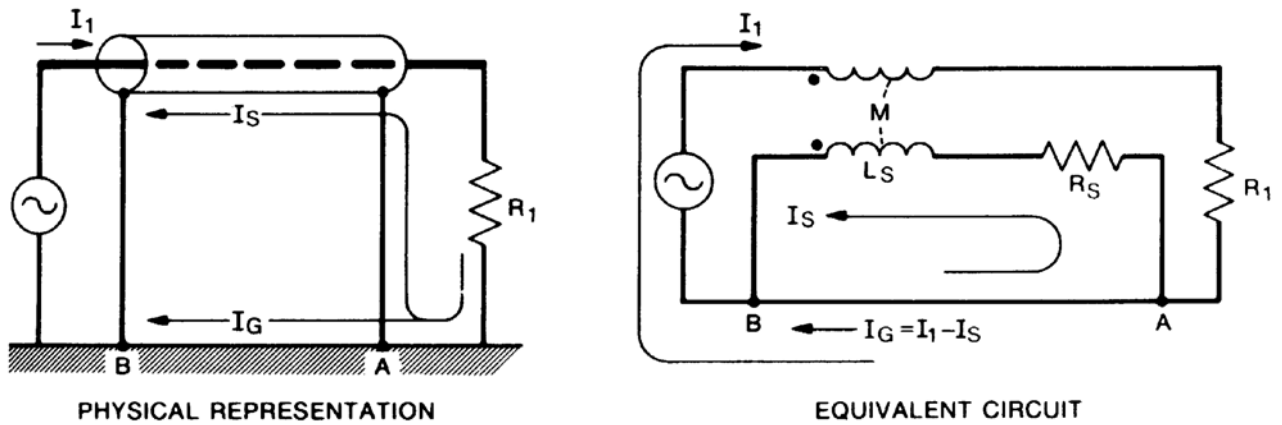


Figure 2-21. Fields around shielded conductor; shield grounded and carrying a current equal to the conductor current but in the opposite direction.

Equal and opposite directed current in screen and centre conductor eliminates the external magnetic field.



**Figure 2-22.** *Division of current between shield and ground plane.*

Hence to reduce the radiation the return current should go in the shield in stead of the ground.

We put up the expression:

$$0 = I_S (j\omega L_S + R_S) - I_1 (j\omega M)$$

Last term is the voltage in the screen as a function of the current in the centre conductor. First term is the same voltage as a function of the current it generates in the shield. Now we assume no resistance in the ground-plane.

We have that  $M=L_S$ .

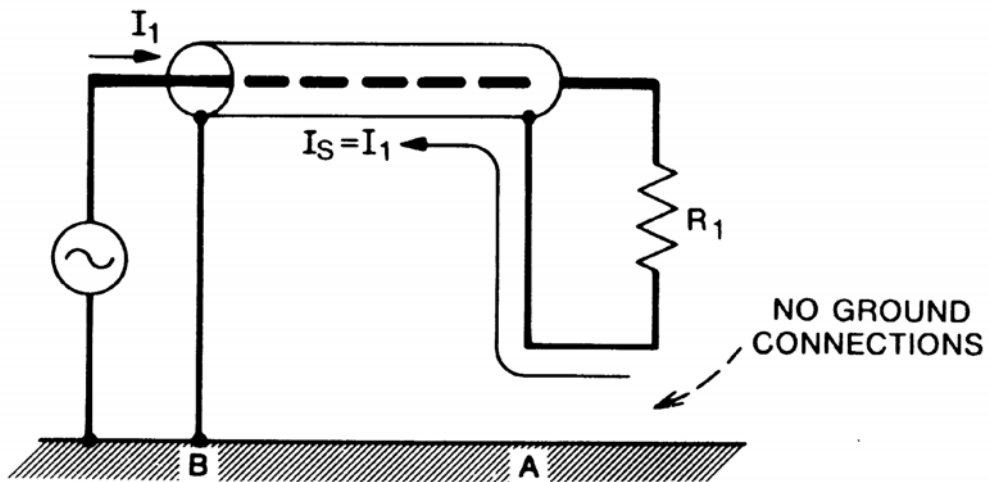
This results in the following expression for  $I_s$ :

$$I_s = I_1 \left( \frac{j\omega}{j\omega + R_s/L_s} \right) = I_1 \left( \frac{j\omega}{j\omega + \omega_c} \right)$$

Low frequencies:  $I_s = I_1 j\omega L_s / R_s$

High frequencies:  $I_s = I_1$

*At higher frequencies we achieve what we want without any effort. The return current passes through the shield instead of through the ground (although the ground may have almost  $0\Omega$  in resistance). The return current is equal to the current in the centre conductor and the magnetic field external to the shield disappears.*



**Figure 2-23.** Without ground at far end, all return current flows through shield.

If the electronic circuitry on the right side is not grounded neither should the shield be grounded. If so all return current pass through the shield and the external magnetic field will be eliminated also at lower frequencies.

# Shielding towards external fields.

To protect towards external magnetic fields the best alternative is to reduce the size of the loops.

*NB! In particular the return current through ground has to be considered. It may find other paths than what the designer expects and result in larger loop areas.*

About shields:

If a shield results in a smaller loop we achieve a better protection. However in this case the increased protection is due to the smaller loop and not due to a magnetic shielding of the centre wire.

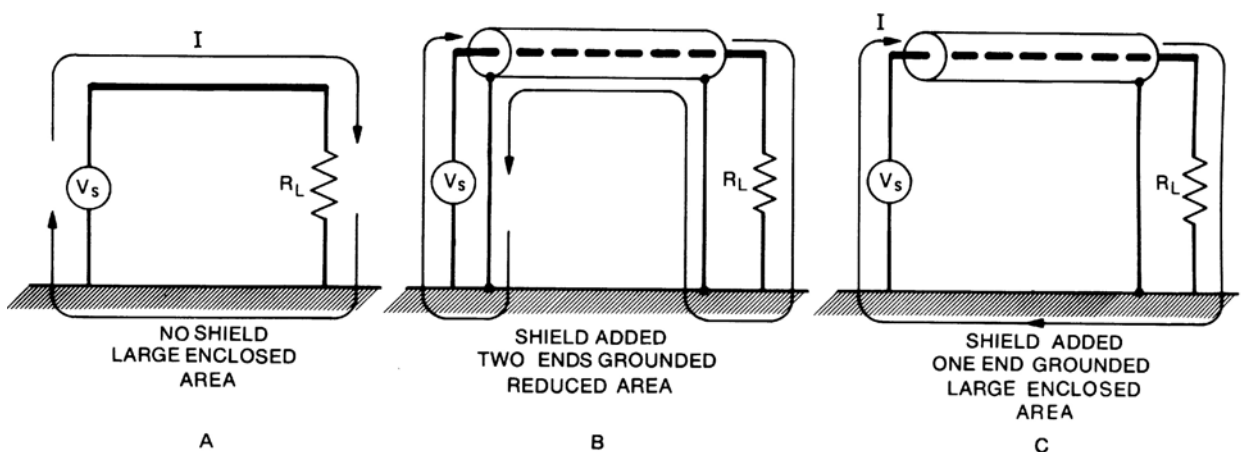


Figure 2-24. Effect of shield on receptor loop area.

The solution in Fig 2-24 B is the best of the three alternatives but have the following limitations:

- Low efficiency at lower frequencies
- Noise generated in the screen will result a voltage drop in the shield and hence be a noise source.
- If the ground potential is different at the two connection points we will have a noise current in the circuit.

*Hence it is magnetic good but electronically bad.*

The noise voltage induced is the shield current times the screen impedance.

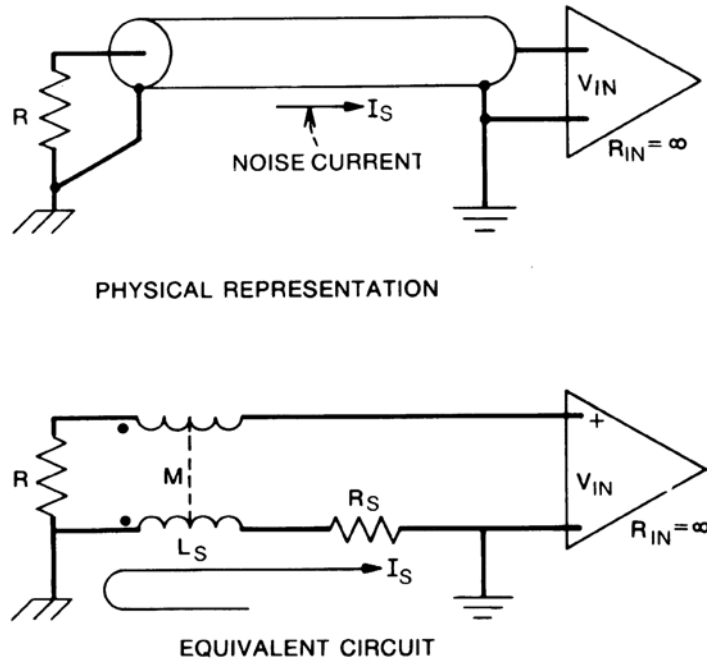


Figure 2-25. Effect of noise current flowing in the shield of a coaxial cable.

We would like to put up an equation for the voltage at the amplifier input that is caused by the noise current through the shield. I.e. we would like to compare the difference on the amplifier input with the difference over the source resistance  $R$ . The current in the shield and centre conductor is equal (but opposite) and since  $M=L_S$  the voltage drop over the coils will be cancelled and we will end up with:

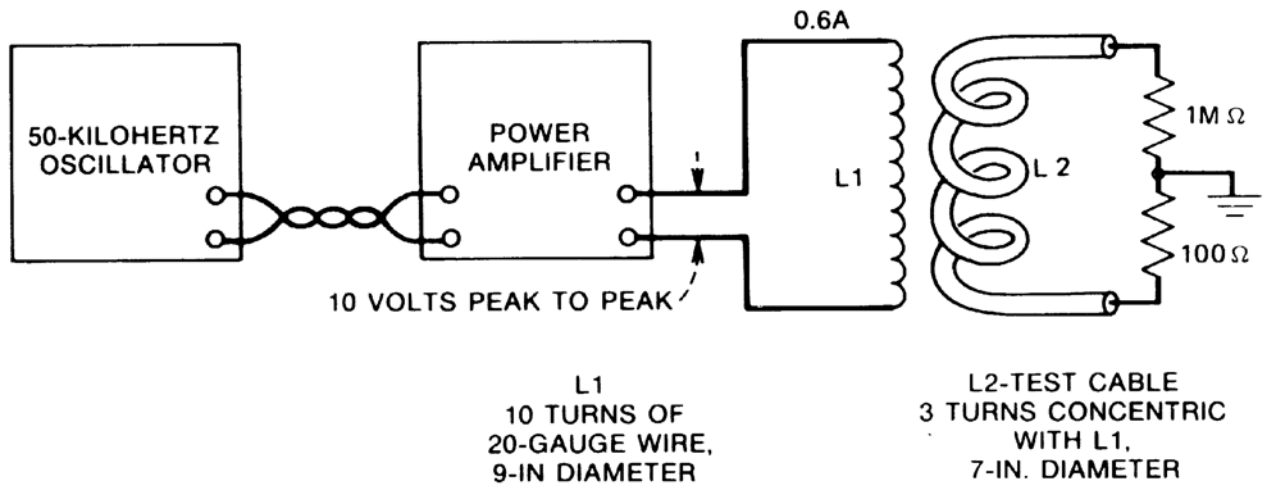
$$V_{IN} = -j\omega MI_S + j\omega L_S I_S + R_S I_S$$

$$V_{IN} = R_S I_S$$



## Example: Lab. test

Test set up:



**Figure 2-27.** Test setup of inductive coupling experiment.

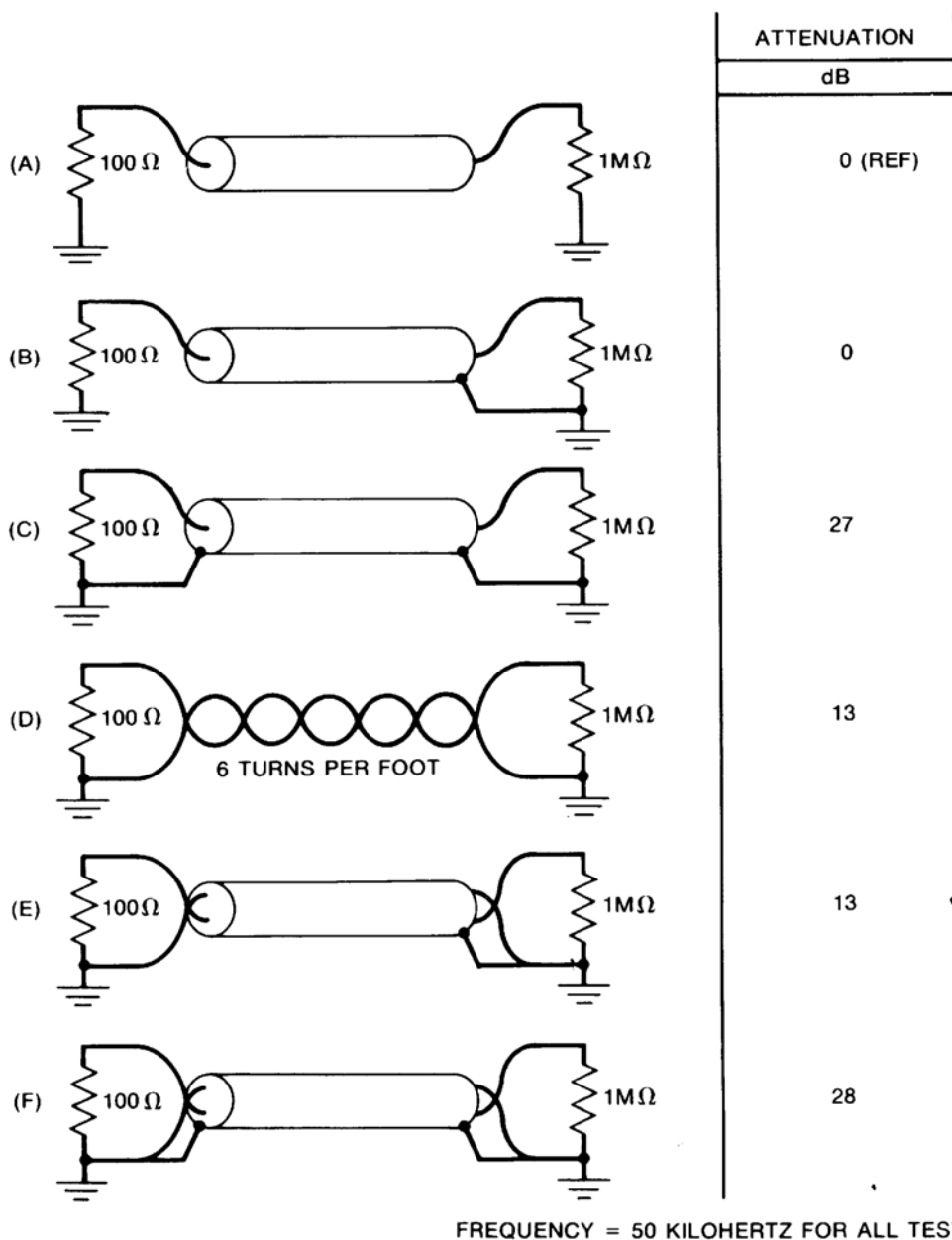
L2 is the target cable to be tested.

The test frequency is 50 kHz which is far above the test cable corner frequencies.

The cable setup A-F (first figures) is grounded through a resistance at both ends.

The cable setups G-K (second figures) have a grounded load at only one end.

Test setup A is used as the reference for the others.



**Figure 2-28.** Results of inductive coupling experiment; all circuits grounded at both ends.

A is the reference.

B: reduces electric fields but since no attenuation is measured the magnetic fields has to be dominating.

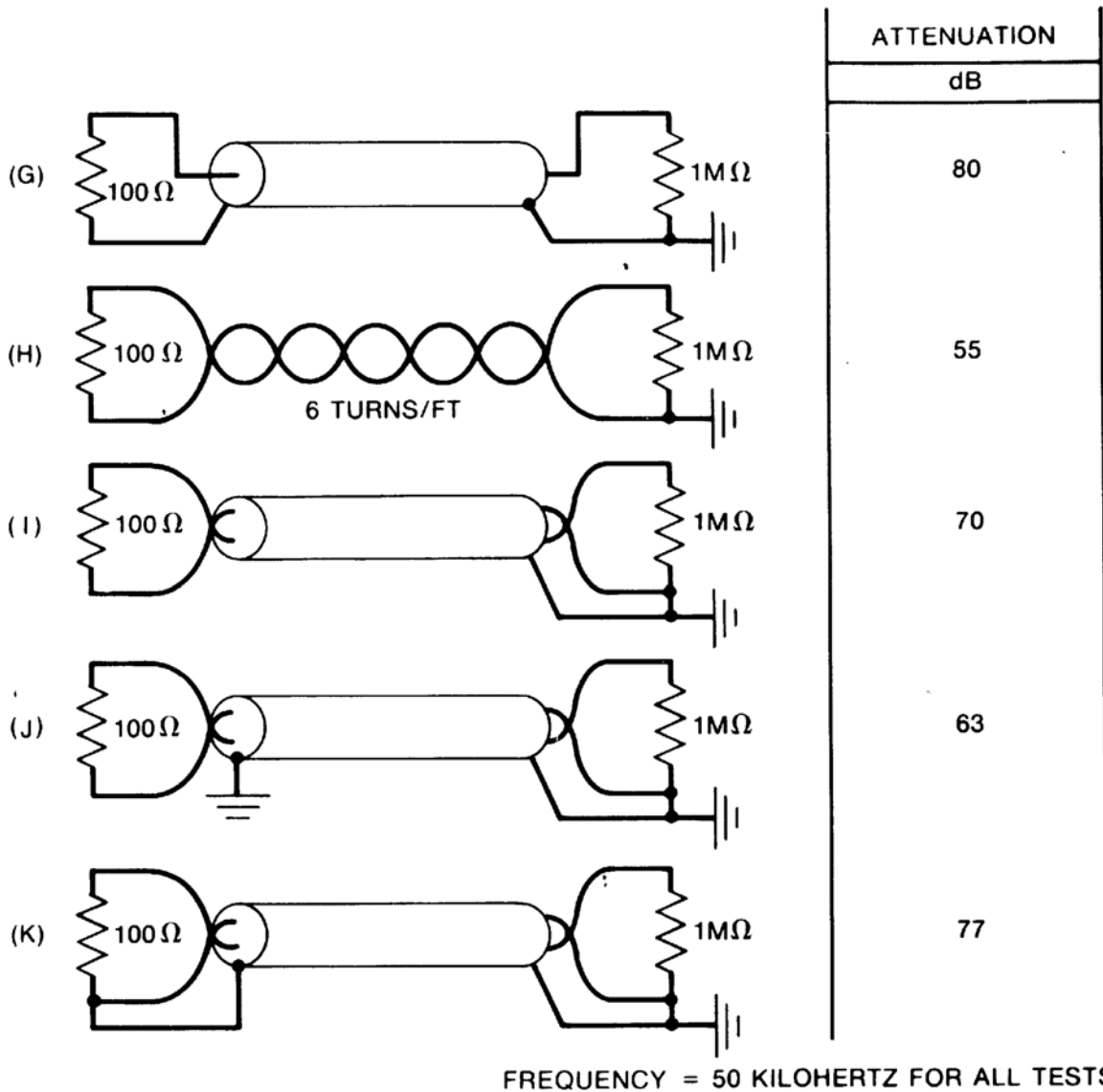
C: Grounded termination in both ends reduces both electric and magnetic field effects.

D: Twisted pair (TP) reduces the effect from the magnetic field but not as good as a coax. (However newer TP has significant improved attenuation).

E: E is D with a one point grounded shield. The grounding of the shield will reduce the electric field effect. No further attenuation shows that the magnetic field is dominating.

F: Grounding the shield at both ends does also reduce the effect from the magnetic field and we achieve a result similar to the two terminal grounded coax in C.

Preferably one end grounding should be used. If the cable has to be grounded at both ends either C or F should be chosen.



**Figure 2-29.** Results of inductive coupling experiment; all circuits grounded at one end only.

G: Grounding at only one end reduces the ground loop and hence less noise is picked up compared to A-F.

H: Old TPs have a significantly worse protection to magnetic fields than coax and no protection to electric fields.

I: I is H with a one end grounded shield. This results in a reduction of the electric field influence. In this case we see that the electric field is so significant that a reduction actually takes place.

More attenuation in G than in I have to be because G has a smaller circuit loop than I in this case. This is in general not the case.

Higher twist density gives better attenuation. One twist for each 5cm corresponds to a Cat 3 cable. Cat. 5 cables have one twist for each cm.

In general we would have preferred I before G at lower frequencies since ground as shield and ground as signal return is separated.

J: At a first though grounding the cable at both ends would reduce the influence from the magnetic field. However by doing so we also establish a ground loop that contributes with noise. In this case we can see that the ground loop noise is larger than the reduction due to the double grounding.

K: By connecting the shield to the right terminal on the twisted pair we establishes a kind of double grounding without adding the large ground loop. However K is often not recommended because noise pick-up on the screen is routed back on one of the pair wires and may influence on the signal. Hence alternative I with a higher twist density is often preferred.

# Selective screening.

An example of selective screening: Shielded loop antenna.

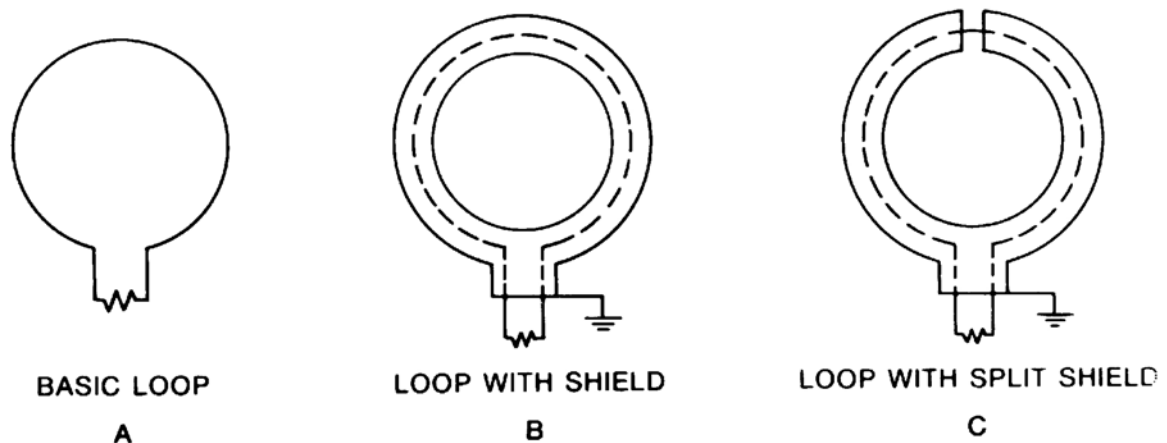
- Shields the signal conductor towards electric fields while magnetic fields penetrate to the antennae. Possible purposes:
  - Radio direction finder
  - Reduce the pick-up of noise in receivers (typically electrical noise is more dominating than magnetic noise).

A) First we look at the basic loop antenna:

The voltage generated by the magnetic field in the antenna is:

$$V_m = 2\pi fBA \cos \theta$$

(B: magnetic field, A: area,  $\theta$ : angle between field and the vector normal to the antenna surface)



**Figure 2-30.** Split shield on loop antenna selectively reduces electric field while passing magnetic field.

The antenna is also a vertical antenna for the electrical field.

$$V_e = \frac{2\pi AE}{\lambda} \cos \theta'$$

$E$ : Electrical field

$2\pi A/\lambda$ : Efficient height of circular antenna.

$\theta'$ : angle between electrical field and the antenna surface.



## B) Full covering shield

... offers protection for electric field  
... however shield current may be present and eliminate both the electric and magnetic field...

## C) Shield with gap

... protects against the electrical field  
... magnetic field generates currents in the central conductor.

## Summary:

A radio signal contains both electrical and magnetic fields. Since typically most of the local noise is electrical fields we may use an antenna that is only sensitive to the magnetic field.

# Coax contra shielded twisted pair.

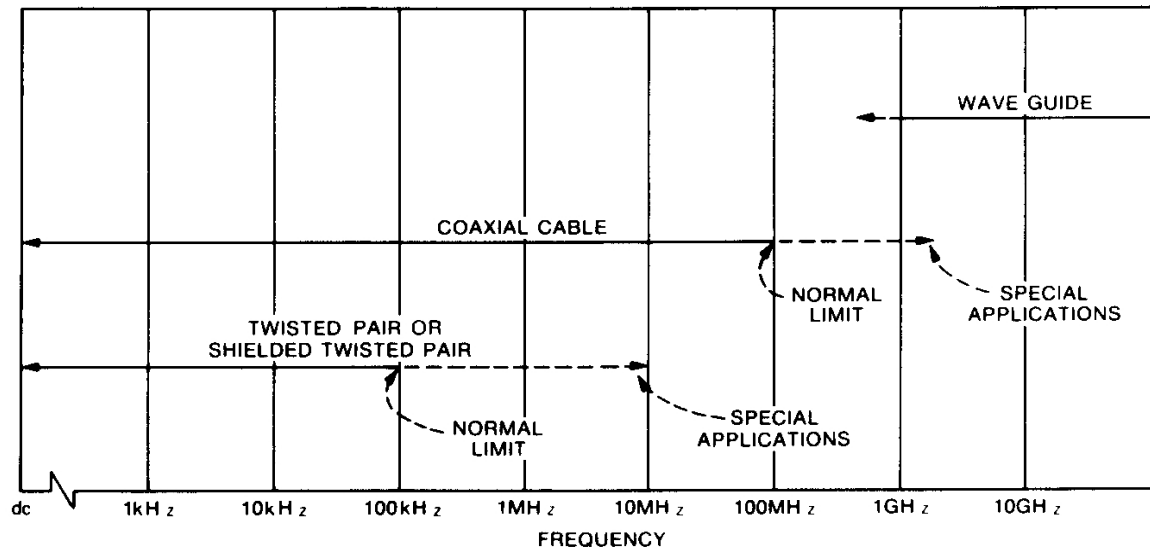


Figure 2-31. Useful frequency range for various transmission lines.

(Figures pr 1988).

Twisted pair: Good at lower frequencies. Good protection to magnetic fields.

Coax: More uniform characteristic impedance. Suited from DC to several hundred MHz.

Wave guides are better at the highest frequencies.

Modern twisted pair cables have better high frequency characteristics and can be used up to several GHz.

Example:

Coax (RG58U): 95pF/m

Cat 5 TP: 56pF/m 100MHz

## **Coax:**

Coax grounded at one point:

- Offers protection to electrical fields.
- Noise currents will generate a noise voltage (noise current x cable resistance) that will be serial to the signal source.

**Triax** (double screen) may solve the problem:

- Noise current in outer screen,
- Signal return current in inner screen.
- However triax cables are expensive and unhandy!

At higher frequencies (>1MHz) a coax will work as a triax due to the skin effect.

## **TP:**

FTP (STP, S/FTP):

- Similar behaviour as the triax at higher frequencies.
- Less expensive and easier to handle.
- Signal current in the twisted pair and noise current in the shield.
- Noise current is induced equal in both of the twisted pair conductors and hence the difference remains equal.

UTP:

- Good protection to magnetic fields
- Bad protection to electrical fields if the termination is not properly balanced.

The strength of STP/FTP cables are at lower frequencies where the magnetic field is the main problem.

Denser twisting increases the frequency range where these cables have an attractive performance!

## **Hose shield (contra foil).**

- Hose shielding is more common than foil.
- Better flexibility and mechanical strength for moving cables. (“Drop cables”)
- Hoses cover 60-98% of the area.
- Somewhat poorer protection to E-fields.
- 5-30dB poorer protection of M-field.
- The hose shielding is less effective (to both E and M fields) at very high frequencies (UHF).

## **Aluminium foil as shield**

Covers almost 100%.

Better protection for E-fields than hose.

Not as mechanically strong as hose.

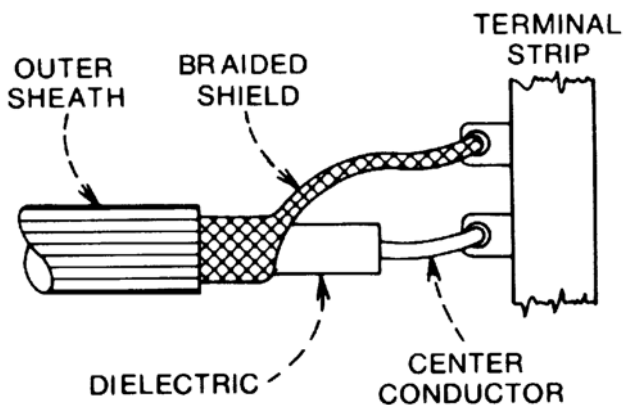
Difficult to terminate properly.

## **Combing hose and foil...**

gives the best from both.

## Cable contacts and "Pigtails"

The discussion so far has assumed that the current has been evenly distributed all over the cable. If the cable shield is not properly terminated the current will not be homogenous distributed.



**Figure 2-33.** *Pigtail shield connection concentrates current on one side of shield.*

A so-called "pigtail" termination will result that the current density is larger at one side of the shield.

To achieve maximum protection the shield should be terminated all 360 degrees at all contacts.

Recommended coax contacts are BNC, UHF or N-type contacts.

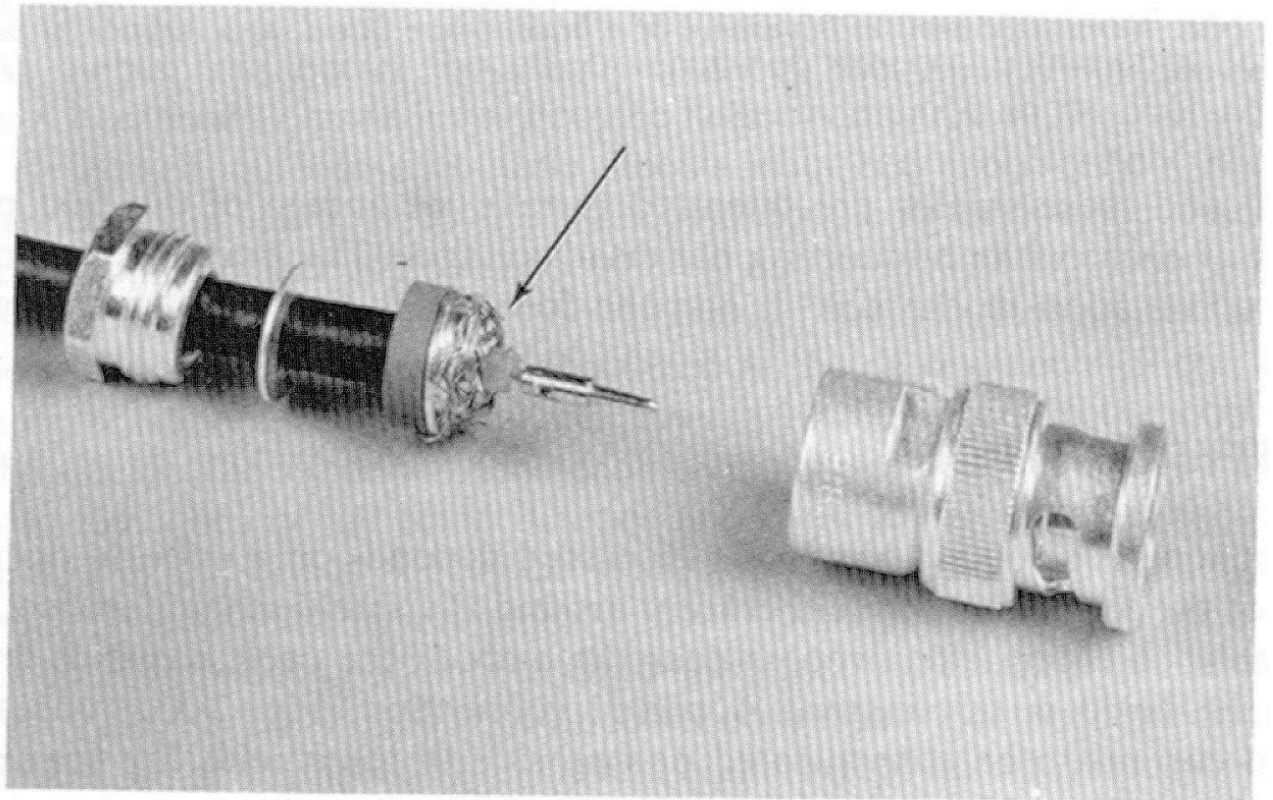
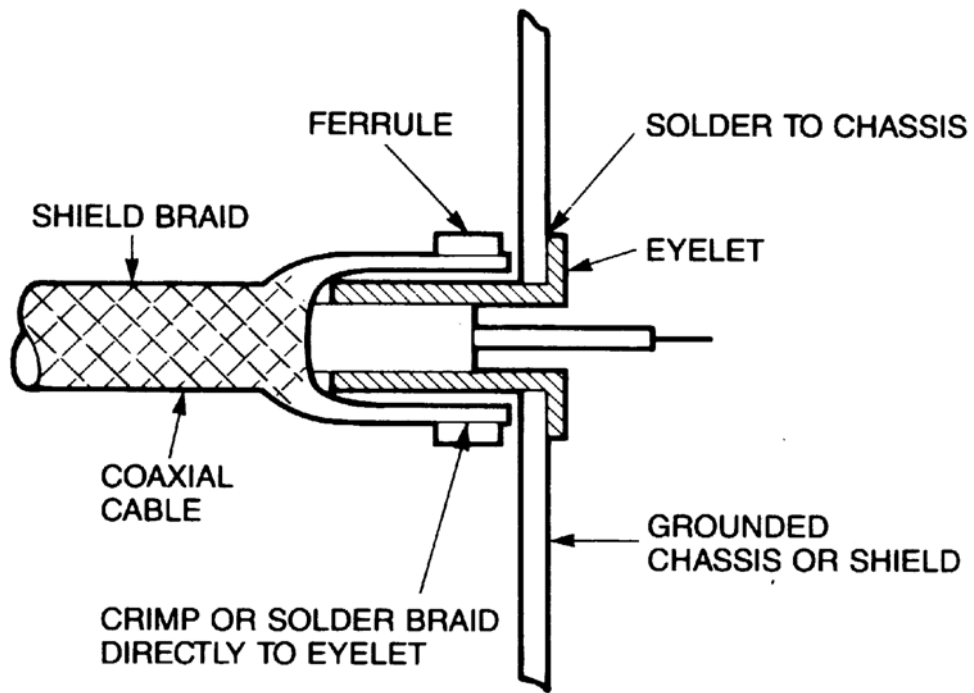
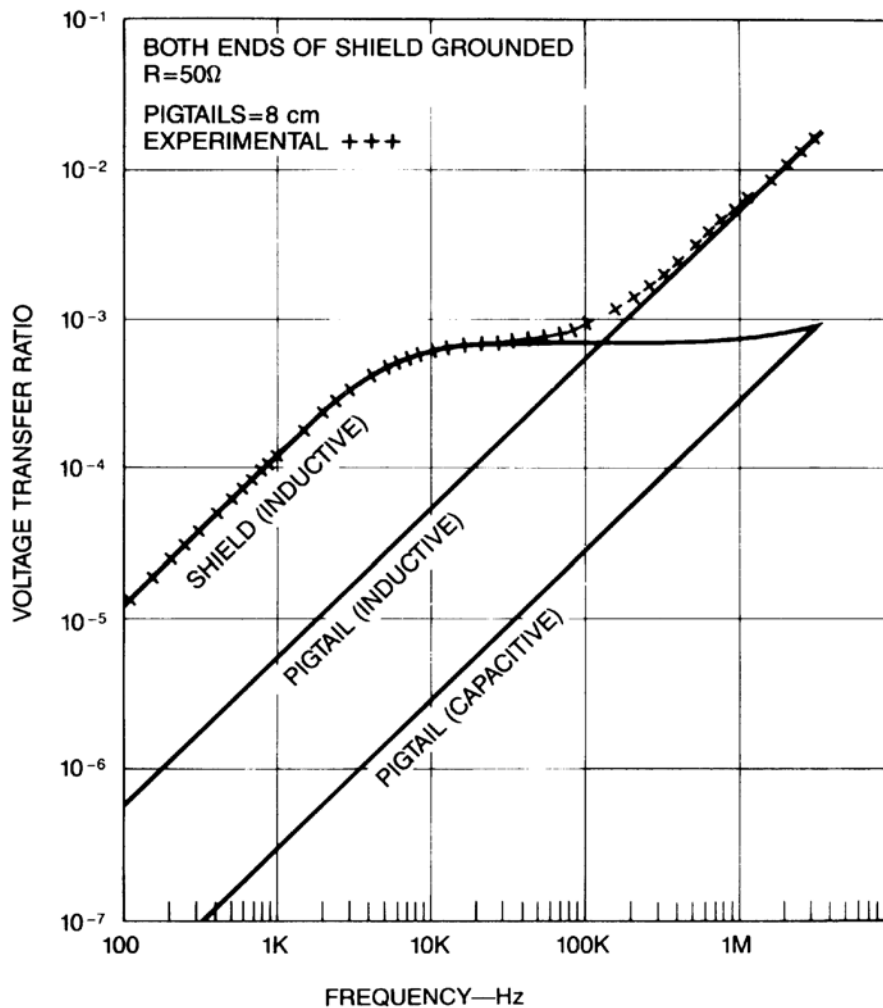


Figure 2-34. *Disassembled BNC connector showing a 360° contact to shield.*





**Figure 2-35.** *One method of terminating a cable with 360° contact to the shield.*

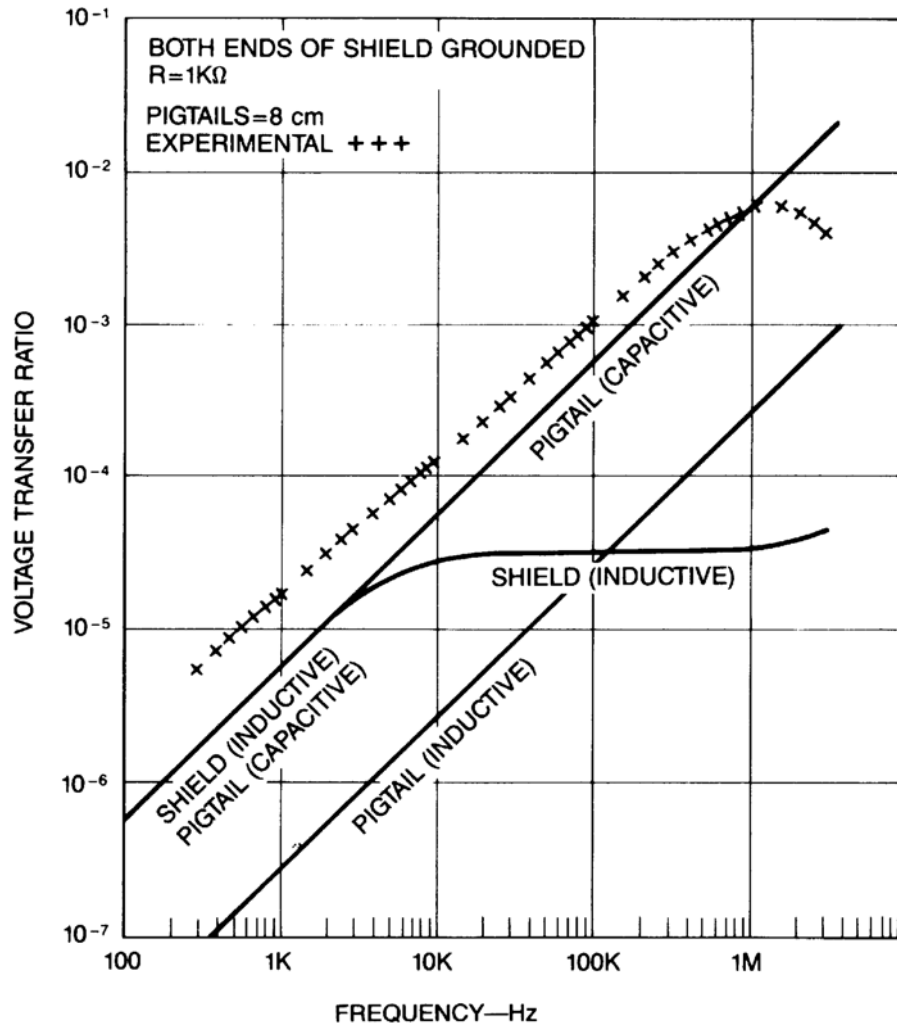


**Figure 2-36.** Coupling to a 3.7-m shielded cable with an 8-cm pigtail termination. Circuit termination equals  $50\Omega$  (from Paul, 1980, © IEEE).

*Example: 3.7m long shielded cable with ca. 8cm pigtail and  $50\Omega$  termination impedance. Three sources for noise:*

- Inductive coupling to shield
- Inductive coupling to pigtail
- Capacitive coupling to pigtail

*In the figure the pigtail inductance is dominating above 100 kHz.*



**Figure 2-37.** Coupling to a 3.7-m shielded cable with an 8-cm pigtail termination. Circuit termination equals  $1000\ \Omega$  (from Paul, 1980, © IEEE).

Example: Like the previous but with  $1000\ \Omega$  termination impedance.

- Inductive coupling in contact (pigtail) decreases
- Capacitive coupling in contact (pigtail) decrease and is dominating above 10 kHz.
- Inductive coupling in shield decreases.
- Total coupling decreases.

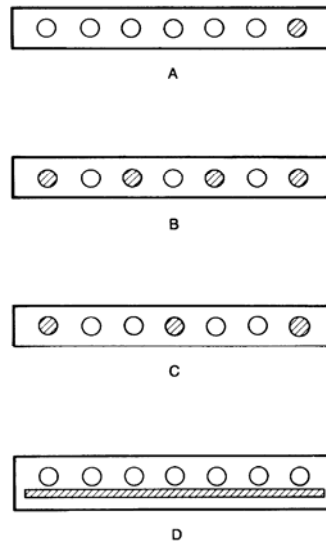
# **Flat cable**

## **(“Licorice cable”/“Lakris kabel”)**

Advantages:

- Less expensive to make contacts when the number of wires are large.
- Conductors are mechanically tied to each other and have a fixed internal order.

An important challenge when using flat cable is the choice of the positions for signals and ground.



○ SIGNAL                      ⊗ GROUND

round; (C) **Figure 2-38.** Ribbon cable configurations: (A) single ground; (B) alternate g.  
ground/signal/signal/ground; (D) signal over ground plane.

- A) One ground wire and the remaining for signals
- Large loops between signals and ground.
  - Common impedance at the ground wire.
  - Crosstalk --- inductive and capacitive.
- ⇒ One of the centre wires should be chosen for ground.
- B) Every second wire is ground
- + Smaller loops
  - + No common impedance
  - + Crosstalk is reduced
  - Less wires for signals
- C) Every third wire is ground
- + Smaller loops
  - +/- Common ground impedance for two signal wires
  - +/- Some crosstalk

D) Ground-plane beneath the wires.

The wires have less distance to the ground-plane than to the each other.

⇒ Smaller loops than for alternative B)

Return current through ground will and close to the conductor it belongs to (discussed in relation to the grounding of M-field shields)

⇒ Smaller loops

But how is the ground-plane contacted?

If the ground-plane is not contacted in its entire width, the ground current will be forced away from the wire it belongs to and efficient loop area will increase.

Shielded flat cable is available but requires full 360 degrees contacts to achieve full effect.

According to Palmgren (1981) the outer conductors have about 7dB less shielding effect than the more central wires.

Flat cables are available in straight version and in a version where two and two wires are twisted together.

## "Long cables"

In the expressions we have studied so far it seems that the capacitive and inductive coupling will increase with frequency infinity. This is not correct. The expressions are only valid when the cables are relatively short compared to the wave length. In this case we can simplify and say that all cable current is in phase. At  $\lambda/4$  the noise coupling starts to decrease. At  $\lambda/2$  it reaches a minimum before it starts to increase again.

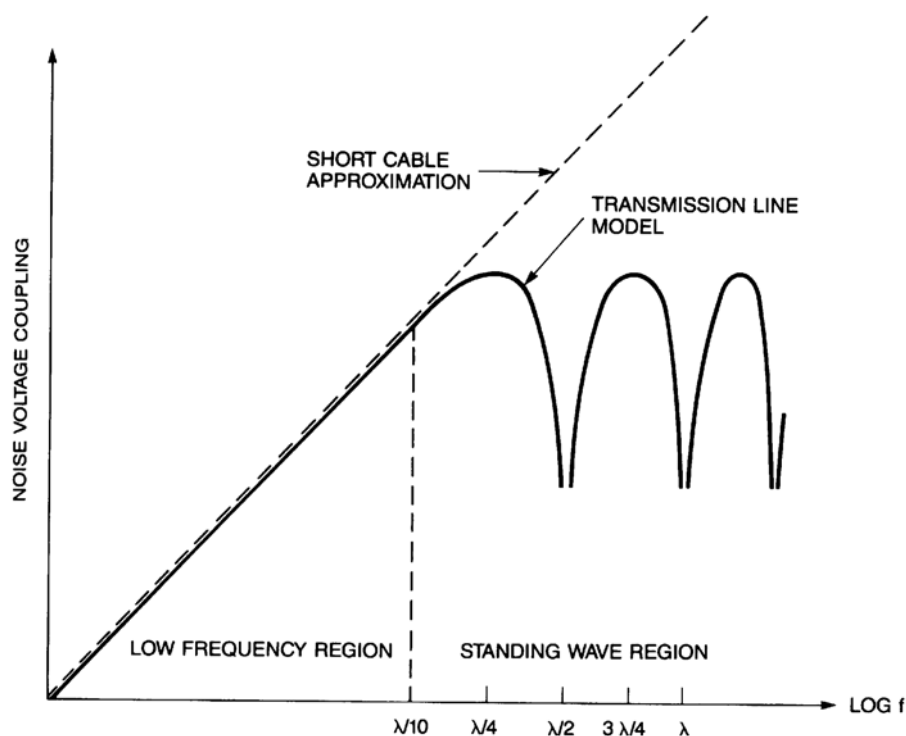


Figure 2-39. Electric field coupling between cables using the short cable approximations and the transmission line model.



Hence we observe:

The coupling factor will have a maximum depending on the cable length and wave length of the noise signal. Maximum is reached when the cable length is about  $\lambda/4$ . Above this frequency the coupling factor will be variable.