

# Design of Minimum Length Nozzle Using Method of Characteristics

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**Abstract:** The nozzle efficiency is largely affected by the nozzle contour. As a result, an optimum geometrical design of a solid rocket motor nozzle is designed in order to achieve maximum thrust and velocity. Two types of nozzle exit configurations are well-thought-out in the design process, conical and contoured. A conical nozzle is used for its simplicity of production, but a contoured nozzle is preferred because it increases performance while reducing weight, a contour nozzle also adjusts the flow such that the exhaust product is in a less or more axial direction, reducing divergence losses. In this case, we'll look at a contour-shaped nozzle that can optimise thrust as needed. Process of Designing a contour nozzle (minimum length nozzle) utilising method of characteristics (MOC) is carried out MOC is preferred owing to its widespread usage in the industry.

**Keyword:** Solid rocket motor, Nozzle, Thrust, contour nozzle, minimum length nozzle, Method of characteristics.

## 1. INTRODUCTION TO ROCKET PROPULSION

### Introduction

Rocket propulsion systems are classified in a variety of ways, including chemical, nuclear, and solar energy sources, as well as fundamental functions such as booster stages, sustained or upper stages, attitude control, orbit station maintaining, and the sort of vehicle they drive (aircraft, missile, assisted take off, space vehicle, etc.) or according to their size, propellant type, structure, or the number of rocket propulsion units utilised in a specific vehicle, as well as the thrust generation mechanism. The kind of propellant utilised in a rocket can be used to classify the propulsion system. Liquid and solid propellants are the two most common kinds.

### Solid propellant rocket motor

Solid propellants (fuel/oxidizer) have been utilised in military applications such as missiles because they can be stored for a long period without considerable propellant deterioration and because they can be launched reliably most of the time. The grain is a solid propellant (or charge) that includes all of the chemical components required for full combustion. It is designed to burn smoothly and at a predefined

pace on all exposed internal grain surfaces once ignited. As the propellant is burnt and consumed, the inside cavity increases. The heated gases that arise travel through the supersonic nozzle, providing thrust. There are no feed systems or valves, and there are no or very few moving components throughout the system.

## 2. NOZZLE

The exhaust gas is accelerated out of the nozzle to create thrust in a divergent nozzle, also known as a DE LAVAL nozzle. The heat generated in the combustion chamber of a solid rocket engine is generally converted into kinetic energy in the exhaust using a convergent divergent nozzle. Newton's 3rd law of motion describes how a rocket engine employs a nozzle to accelerate heated exhaust. The mass flow rate through the engine, the exit velocity of the flow, and the pressure at the engine's exit determine the amount of thrust generated. The rocket nozzle design determines the value of these three flow variables. The fundamental thermodynamic principles may be used to estimate throat area, nozzle half angle, and expansion ratio for simple nozzles with non-contoured conical outputs.

To decrease divergence loss, enhance particular impulse somewhat, and minimise nozzle length and mass, a more sophisticated contoured (or bell-shaped) nozzle is utilised.

### Minimum Length Supersonic Nozzle

Supersonic nozzles are used to increase a flow to desired supersonic levels in a number of technical applications. The use of supersonic nozzles is possible. There are two categories of nozzles: progressive-expansion nozzles and nozzles with a minimum length. Gradual-expansion nozzles are widely employed in situations where maintaining a high-quality flow at the appropriate exit conditions is critical. For other applications, the huge weight and length penalties associated with gradual expansion nozzles make them impractical, hence minimum-length nozzles with a sharp corner to give the initial expansion are usually utilised. The flow may be separated into simple and non-simple areas for both gradual-expansion and minimum-length nozzles. Mach wave reflections and intersects describe a non-simple area. It is preferable to limit the non-simple zone as much as feasible in order to achieve the criterion of consistent conditions at the nozzle outlet. This can be accomplished by designing the nozzle surface so that no Mach waves (e.g., characteristics) are generated or reflected while the flow is straightened. As a result, the Method of Characteristics is used to build a supersonic nozzle that fits these parameters. The design of both a gradual-expansion and a minimum-length nozzle is illustrated in this paper.

## 3. NOZZLE DESIGN

For designing a profile of a nozzle Different methods have been proposed, Method of characteristics and G.V.R. Rao approximation method. Method of characteristics is the

method that's discussed below for designing a minimum length nozzle.

**General Theory**

The nonlinear differential equation of the velocity potential can be used to explain the physical circumstances of a two-dimensional, steady, isentropic, irrotational flow. The technique of characteristics is a mathematical framework that may be used to discover solutions to the aforementioned velocity potential while meeting certain boundary conditions, resulting in the governing partial differential equations (PDEs) becoming ordinary differential equations (ODEs).

**Method of Characteristics**

In a supersonic flow, characteristics are 'lines' oriented in specified directions along which disturbances (pressure waves) propagate. The Method of Characteristics (MOC) is a numerical approach for addressing two-dimensional compressible flow problems, among other things. Flow parameters such as direction and velocity may be estimated at different sites across a flow field using this method.

The following are the three qualities of characteristics:

Property 1: A curve or line in a two-dimensional supersonic flow along which physical disturbances propagate at the local speed of sound relative to the gas is a characteristic.

Property 2: A characteristic is a curve along which the flow qualities are continuous, even though the initial derivatives are discontinuous, and the derivatives are indefinite.

Property 3: A characteristic is a curve that may be used to convert the controlling partial differential equations into an ordinary differential equation (s).

**Minimum Length Nozzle (MLN) using Method of Characteristics (MOC)**

**Characteristic Lines**

Two-Dimensional Irrotational Flow For steady, two-dimensional, irrotational flow, Equation 3.1, which is the whole velocity potential

$$\left(1 - \frac{\Phi_x^2}{a^2}\right)\Phi_{xx} + \left(1 - \frac{\Phi_y^2}{a^2}\right)\Phi_{yy} - \frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy} = 0 \quad (3.1)$$

Note:  $\Phi$  is the full-velocity potential

equation, is used to determine the characteristic lines. Note that  $\Phi$  is the velocity potential.

Equation 3.2 shows the velocity potential by definition, and then equations 3.3 and 3.4 are derived.

$$\Phi_x = u, \quad \Phi_y = v \quad \text{and} \quad V = u\hat{i} + v\hat{j} \quad (3.2)$$

$$d\Phi_x = \frac{\partial\Phi_x}{\partial x}dx + \frac{\partial\Phi_x}{\partial y}dy = \Phi_{xx}dx + \Phi_{xy}dy \quad (3.3)$$

$$d\Phi_y = \frac{\partial\Phi_y}{\partial x}dx + \frac{\partial\Phi_y}{\partial y}dy = \Phi_{xy}dx + \Phi_{yy}dy \quad (3.4)$$

Equations 3.6, 3.7, and 3.8 are produced by substituting the equation 3.2 into equations 3.1, 3.3, and 3.4.

$$\left(1 - \frac{u^2}{a^2}\right)\Phi_{xx} - \frac{2uv}{a^2}\Phi_{xy} + \left(1 - \frac{v^2}{a^2}\right)\Phi_{yy} = 0 \quad (3.6)$$

$$(dx)\Phi_{xx} + (dy)\Phi_{xy} = du \quad (3.7)$$

$$(dx)\Phi_{xy} + (dy)\Phi_{yy} = dv \quad (3.8)$$

Equations 3.6 over 3.8 are a system of synchronized, linear, algebraic equations in the variables  $\Phi_{xx}$ ,  $\Phi_{yy}$ , and  $\Phi_{xy}$ . By applying Cramer's rule, the explanation for  $\Phi_{xy}$  is to be the following equation 3.9.

$$\Phi_{xy} = \frac{\begin{vmatrix} 1 - \frac{u^2}{a^2} & 0 & 1 - \frac{v^2}{a^2} \\ dx & du & 0 \\ 0 & dv & dy \end{vmatrix}}{\begin{vmatrix} 1 - \frac{u^2}{a^2} & -\frac{2uv}{a^2} & 1 - \frac{v^2}{a^2} \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix}} = \frac{N}{D} \quad (3.9)$$

A point A and its surrounding neighbourhood in an arbitrary flow field are represented in Figure 3.1. The velocity potential's derivative,  $\Phi_{xy}$ , has a specific value at point A. The solution for  $\Phi_{xy}$  at point A for an arbitrary dx and dy option, and dy for an arbitrary direction away from point A specified by the dx and dy option. There are equivalent values of the change in velocity du and dv for the given dx and dy.

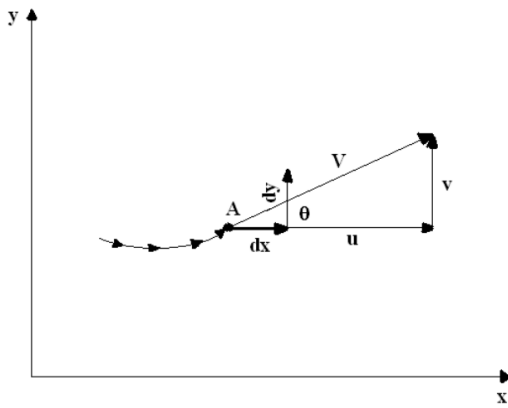


Figure 3.1 streamline geometry

The slope of the characteristic lines is shown below in equation 3.10 and equation 3.11.

$$\left(\frac{dy}{dx}\right)_{char} = \frac{-uv/a^2 \pm \sqrt{(u^2+v^2)/a^2} - 1}{1 - (u^2/a^2)} \quad (3.10)$$

$$\frac{u^2+v^2}{a^2} - 1 = \frac{V^2}{a^2} - 1 = M^2 - 1 \quad (3.11)$$

There are three key points to remember:

- When  $M > 1$ , each point of the flow field has two actual characteristics. Furthermore, equation 3.1 is a hyperbolic partial differential equation in this case.
- If  $M = 1$ , each point of the flow has just one actual feature. A parabolic partial differential equation is Equation 3.1.
- If  $M$  is less than one, the properties are imaginary, and equation 3.1 is an elliptic partial differential equation.

Because each point in a flow has two real characteristics, the method of characteristics (MOC) becomes a suitable strategy for solving supersonic flow. The equation 3.10 for a steady, two-dimensional supersonic flow is investigated. Study the streamline as shown in Figure 3.1. At point A,  $u = V \cos \theta$  and  $v = V \sin \theta$ . Equation 3.10 develops equation 3.12 shown below.

$$\left(\frac{dy}{dx}\right)_{char} = \frac{-V^2 \cos \theta \sin \theta \pm \sqrt{\frac{V^2}{a^2} (\cos^2 \theta + \sin^2 \theta) - 1}}{1 - \frac{V^2}{a^2} \cos^2 \theta} \quad (3.11)$$

Since,  $\mu = \sin^{-1}(1/M)$ , equation 3.12 is obtained as follows.

$$V^2/a^2 = M^2 = 1/\sin^2 \mu \quad (3.12)$$

Therefore, the following equation 3.13 is obtained.

$$\left(\frac{dy}{dx}\right)_{char} = \frac{-\cos \theta \sin \theta \pm \sqrt{\cos^2 \theta + \sin^2 \theta} - 1}{\sin^2 \mu} \frac{1}{\left[1 - \frac{\cos^2 \theta}{\sin^2 \mu}\right]} \quad (3.13)$$

The slope of the characteristic lines is given by equation 3.14, which is derived from trigonometry and algebra.

$$\left(\frac{dy}{dx}\right)_{char} = \tan(\theta \mp \mu) \quad (3.14)$$

Figure 3.2 depicts a graphical representation of equation 3.14. The streamline establishes an angle  $\theta$  with the x axis at point A. There are two characteristics that pass-through point A, one at an angle  $\mu$  above the streamline and the other at an angle  $\mu$  below it.

Mach lines are the distinguishing lines. The angle  $\theta + \mu$  characteristic is referred to as a C+ characteristic, and it is a left-running characteristic. A C- characteristic is a right-running characteristic that is given by the angle  $\theta - \mu$ . Because the flow parameters change from point to point in the flow, the characteristics are curved.

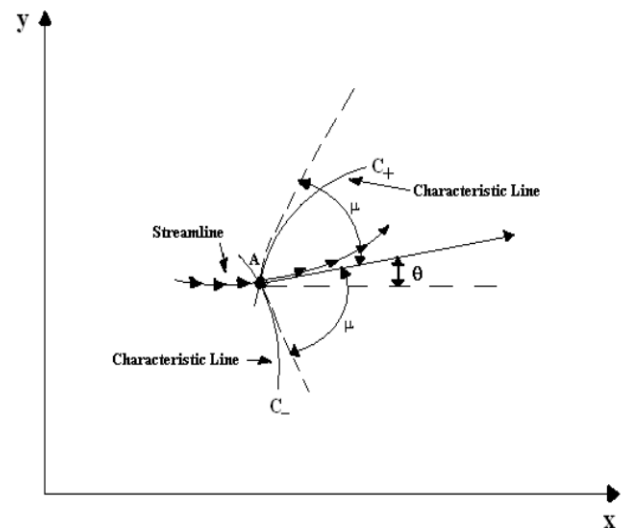


Figure 3.2 left and right characteristic lines

### Compatibility Equations

The compatibility equation is the following equation 3.15

$$\frac{dv}{du} = \frac{-\left[1 - (u^2/a^2)\right] dy}{\left[1 - (v^2/a^2)\right] dx} \quad (3.15)$$

To maintain the flow field derivatives small, N is zero only when D is zero, according to equation 3.9. When D = 0, only directions along the characteristic lines are examined, and when N = 0, only directions along the characteristic lines are examined. As a result, equation 3.15 is only valid along characteristic lines. As a result, equation 3.16 is as follows:

$$\frac{dy}{dx} \equiv \left( \frac{dy}{dx} \right)_{char} \quad (3.16)$$

The compatibility equation therefore is shown in equation 3.18 below

$$d\theta = \mp \frac{dV}{V} \quad (3.18)$$

The equation's negative version is applied to the C- characteristic, whereas the positive form is applied to the C+ characteristic. For Prandtl-Meyer flow, equation 3.18 is equal to equation 3.19 given below.

$$d\theta = \sqrt{M^2 - 1} \frac{dV}{V} \quad (3.19)$$

Equation 3.19 is substituted by algebraic compatibility equations 3.20 and 3.21 since the equations are equivalent

$$(3.20)$$

$$\theta + v(M) = \text{cnst} = K_- \text{ (along the } C_- \text{ characteristic)} \quad (3.21)$$

$$\theta - v(M) = \text{cnst} = K_+ \text{ (along the } C_+ \text{ characteristic)}$$

### Compatibility Equations Point by Point Along the Characteristics

#### Internal Flow

If the flow field conditions are known at two points in the flow, the conditions at a third point can be found. The third point is located by the intersection of the C- characteristic through the first point and the C+ characteristic through the second point, as shown in Figure 3.3 below.

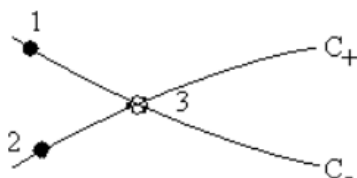


Figure 3.3-unit process for MOC

The  $\theta$  and  $v$  for the third point are found in terms of the known values of  $K_+$  and  $K_-$  as shown in equation 3.22 and equation 3.23 below

$$\theta = \frac{1}{2} \left[ (K_-)_1 + (K_+)_2 \right] \quad (3.22)$$

$$v = \frac{1}{2} \left[ (K_-)_1 - (K_+)_2 \right] \quad (3.23)$$

As a result of the known values at the first and second points, the flow conditions at the third point may now be calculated. Equation 3.24 is used to get the Mach number.

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1} \quad (3.24)$$

After calculating the Mach number, the pressure, temperature, and density may be determined using isentropic flow relations, as illustrated in the equations below.

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\frac{p_o}{p} = \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho_o}{\rho} = \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{1/(\gamma-1)}$$

Assume that the characteristics are straight-line segments connecting the grid points, with average slopes. C- characteristic across the first point is drawn as a straight line with an average slope angle. As indicated in the equation below, the C+ characteristic across the second point is drawn as a straight line with an average slope angle.

$$\left[ \frac{1}{2}(\theta_1 + \theta_3) - \frac{1}{2}(\mu_1 + \mu_3) \right]$$

$$\left[ \frac{1}{2}(\theta_2 + \theta_3) - \frac{1}{2}(\mu_2 + \mu_3) \right]$$

#### Supersonic Nozzle Design

In order to increase the speed of an internal steady flow via a duct from subsonic to supersonic, the duct must be convergent-divergent in form, as shown in Figure 3.4.

Assume that the sonic line is perfectly straight. In the throat, the flow increases to sonic speed. The duct splits downstream of the sonic line. The expansion section in Figure 3.5 is shortened to a point in minimal length nozzles, and the expansion occurs by a centred Prandtl-Meyer wave emerging from a sharp-centre throat with an angle  $\theta_{wmax}$ ,

ML, as shown in Figure 3.5. The length of the supersonic nozzle, L, is the smallest value that can be achieved while maintaining shock-free, isentropic flow.

Assume that the exit Mach numbers of the nozzles in Figures 3.4 and 3.5 are the same. The expansion contour of the minimum-length nozzle depicted in Figure 3.5 exhibits a sharp corner at point a. Only two wave systems meet the fluid: right-running waves from point a and left-running waves from point d. In equation 3.25, the wall's expansion angle downstream of the neck is illustrated.

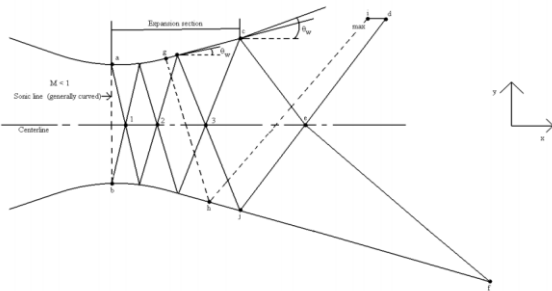


Figure 3.4 schematic of supersonic nozzle design by MOC.

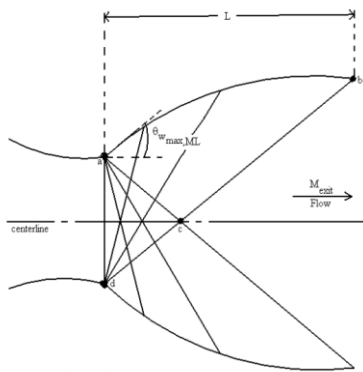


Figure 3.5 schematic of minimum length nozzle

$$\theta_{w_{max,ML}} = \frac{v_M}{2} \quad (3.25)$$

It is point e when the centreline Mach number matches the design exit Mach number. The expansion section comes to an end at point c, which fixes its length as well as the value of wmax. The number of nodes is calculated using the formula below.

$$No. \ of \ Nodes = [2 + (n+1)] [n/2]$$

where n is the number of C<sub>-</sub> characteristics

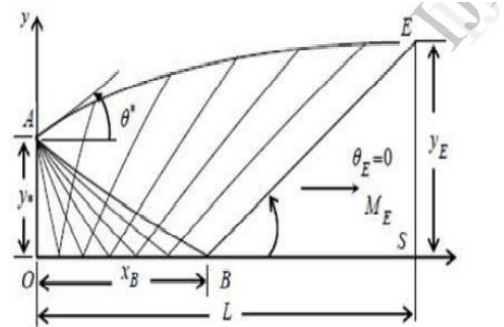


Figure 3.6 Schematic of characteristic lines for minimum length nozzle

#### 4. RESULTS AND DISCUSSION

By compressing the expansion section, the design of a minimal length supersonic nozzle is capable of constructing a minimal length nozzle.

The entire length of the nozzle is reduced when the expansion portion is contracted. The length of the supersonic nozzle is kept to a minimum in the preceding design because the expansion section is kept to a minimum.

In actuality, the expansion part has been compressed to point at the throat's end. using the Prandtl-Meyer equation the characteristic lines are solved in MATLAB all the points are plotted against nozzle axis in which different characteristic lines are seen

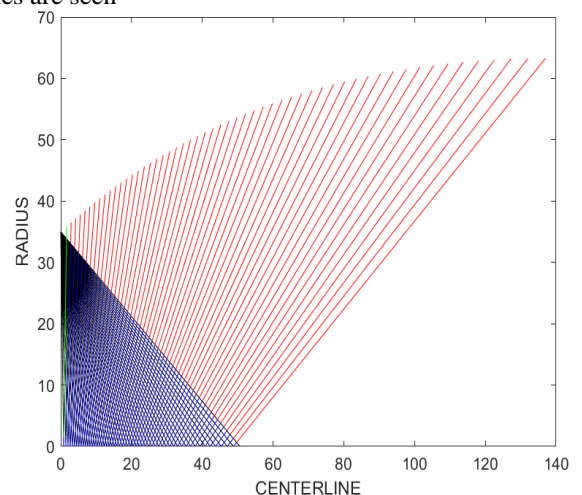


Figure. 4.1 plot of the contour curve with the characteristic lines in MATLAB

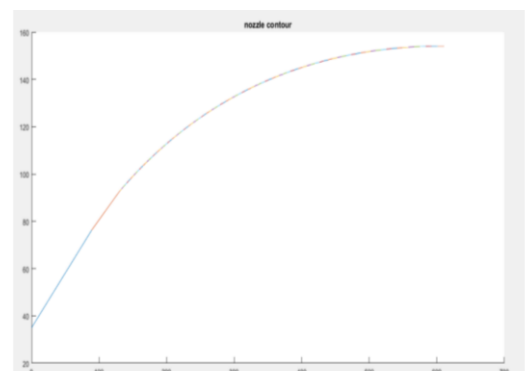


Figure 4.2 plot of wall points

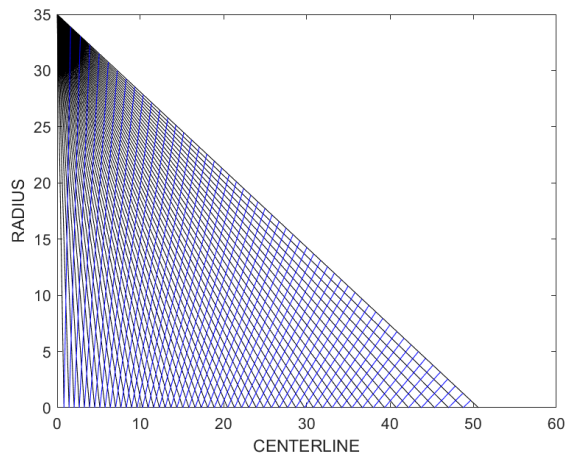


Figure 4.3 points reflected on the wall

These points are later used for plotting a 2D domain FOR CFD analysis You must also choose an optimal number of features, which should be large enough to form a curved (bell-shaped) contour towards the end of the process. Make sure your lines are straight. MOC's overall efficacy may be measured.

## 5. CONCLUSION

There are a variety of uses for supersonic nozzles. Normally, they are exposed to a complicated flow pattern. To achieve the high precision and huge calculations required by current high-speed applications, a computer is required. As a result, a computerised approximation method could be a preferable way to address such an issue. The most acceptable approach to utilise with the supersonic nozzle design is the characteristic method.

By doing several iterations, we can improve our results. Slight modifications in the input parameters algorithm We were able to effectively use the method of characteristics to produce a nozzle shape for a nozzle with a minimum length.

The nozzle points acquired may be imported into any CAD programme for further refining and production. To achieve the greatest outcomes in line with the input specification, it is strongly suggested to combine this strategy with an iterative design approach.

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