On symmetry reduction of some differential equations and classification of low-dimensional nonconjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$

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Symmetry reduction of partial differential equations with non-trivial symmetry groups is an important method for the investigation of these equations. To realize this aim we can use functional bases of invariants of nonconjugate subgroups of their symmetry groups. The details on this theme can be found, for example, in [1, 2, 3]).

The papers [4, 5] are devoted to the symmetry reduction of some differential equations in the spaces $M(1,3) \times R(u)$ and $M(1,4) \times R(u)$, which are invariant with respect to the Poincaré group $P(1,4)$. However, it turned out that the reduced equations, which are obtained with the help of non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$ of given rank are of different types. It means that using only the rank of those nonconjugate subalgebras we cannot explain differences in the properties of the reduced equations. Here, and in what follows $M(1,3)$ and $M(1,4)$ are four- and five- dimensional Minkowsky spaces, correspondingly; $R(u)$ is the real number axis of the dependent variable u.

It should be noted that Grundland, Harnad, and Winternitz [6] have, for the first time, pointed out this fact.

It is known that the nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$ of the same rank may have different structural properties. Therefore, to explain the differences in the properties of the above mentioned reduced equations, we suggest to try to investigate the connections between structural properties of nonconjugate subalgebras of the same rank of the Lie algebra of the group $P(1,4)$ and the properties of the reduced equations corresponding to them.

Using the classification [7] of one- and two- dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$ into classes of isomorphic subalgebras, we have classified non-singular manifolds in the space $M(1,4) \times R(u)$ invariant under these subalgebras. The details can be found in [8].

Until now, we have classified the functional bases of invariants in the space $M(1,3)\times R(u)$ of one- and two- dimensional non-conjugate subalgebras of the Lie algebra of the group $P(1,4)$ using the classification of those subalgebras. In other words, we have established the connection between classification of one- and two- dimensional non-conjugate subalgebras of the Lie algebra of the group $P(1,4)$ and their invariants in the space $M(1,3) \times R(u)$.

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