The mechanism of Plana's Calendar

Andrea Bacciotti¹

¹Politecnico di Torino, Italy, andrea.bacciotti@polito.it

III. THE EASTER PROBLEM

Abstract – In this note we illustrate the main features and the mechanism of the so-called Plana's calendar, a perpetual calendar covering a period of 4000 years. The amount of information provided by Plana's calendar is not limited to the simple correspondence *day* of the week \longleftrightarrow date, but include movable Christian feasts.

I. INTRODUCTION

To menage complex social activities such as job, religion, commerce, political institutions, the mankind needs to develop tools for measuring and organizing time. Roughly speaking, such tools are clocks (for period of time less than one day) and calendars (for periods longer than a day). Since the most ancient civilized countries, calendar modeling has been related to the periodic motion of celestial bodies. Of course, from the Earth's point of view, the most important celestial bodies are the Sun and the Moon. Unfortunately, the periods of their motions are difficult to compare, and cannot be reduced each other easily. Other merely cultural traditions (for instance, the 7 days week) further complicate thinks. In summary, to design a calendar is not an easy task. This may not be a problem as far as we operate over short periods, small areas, and the amount of the required information is minimal. But the amount of information and the need of precision becomes more and more crucial, because of the evolution of the human society and the growth of complexity.

II. ANCIENT CALENDAR SYSTEMS

The most important ancient cultures developed a calendar system¹, but ancient calendars were rather imprecise. For instance, Roman calendar was based on the assumption *1 year* = 365 days, with an error of about 1/4 day each year. Around 40BC, the accumulated delay (w.r.t. the natural sequence of seasons) was about 3 months.

The Roman general Giulio Cesare promoted a reform. The leap year was introduced and the calendar was rephased. The error was strongly reduced, but not definitively removed, so that at the beginning of sixteenthcentury, the calendar anticipated the natural sequence of seasons of about 10 days.

Note that at that time, the importance of the calendar from the religious point of view was greatly increased.

According to the hierarchical order of Christian feasts, Easter comes the first. To strengthen the unity and the central power of the Roman Church against heresies, and to reinforce consensus, especially in the initial centuries of the Christian era, celebration of Easter at a common date was an important issue.

The First Council of Nicaea (325AD) decided the following rule to compute Easter date:

> Easter day is the first SUNDAY after the first FULL MOON after the SPRING EQUINOX

(later, the spring equinox was conventionally fixed on March 21). To apply this rule is difficult because it involves three natural periodic phenomena (Earth's orbit and rotation, Moon's orbit) and one cultural requirement (the seven days week) whose periods are difficult to compare. Finding a method or a formula for computing the date of Eastern was considerer a challenge by many scientists. The famous physicist and mathematician C.F. Gauss published three papers about this problem (1800, 1802, 1807, see [2]). Also G. Peano wrote some papers on this subjects (see for instance [3]).

The error accumulated at the beginning of sixteenth century made questionable to fix the right date. Pope Gregorio XIII decided to promote a new reform, to further reduce the error. But, in order to re-design and re-phase correctly the calendar, it was necessary to measure the anticipation with the best accuracy. To this purpose, in 1575 the dominican Egnazio Danti, astronomer and mathematician, member of the committee formed by the Pope, convinced Grand Duke Cosimo I Medici to instal some instruments for astronomic measures on the facade of Santa Maria Novella in Firenze: an armillary sphere and twelve sundials (Figures **??**. In particular, on the equinox day the shadow of the armillary sphere projects exactly a cross on the facade.

The reform elaborated by the committee was applied in 1582: as well known, the days from October 5 to October 14 were suppressed, and corrections were applied to the rule of leap years.

IV. THE CHALLENGE OF THE PERPETUAL CALENDAR

A *perpetual calendar* is a calendar which provides information about time organization for a (potentially) arbi-

¹Readers interested to computations related to astronomical observations are referred, for instance, to [1]



Fig. 1. Firenze, Santa Maria Novella: armillary sphere (on the right of the facade)



Fig. 2. Firenze, Santa Maria Novella: 12 sundials (on the left of the facade)

trarily large period of time. Today, in the computer era, perpetual calendars are available on line. But, at the beginning of nineteenth century, the realization of a perpetual calendar was a true challenge.

The degree of difficulty depends on the desired information. Actually, if we limit ourself to the simple correspondence *date* \longleftrightarrow *day of the week*, it is not so difficult to devise a perpetual calendar. For instance, according to the old Roman calendar system, there are only 7 possible calendars for every year ($365 \equiv 1 \mod(7)$). Each calendar is identified by the day corresponding to January 1. This implies that a same calendar can be re-used after 7 years.

Instead, according to the Julian calendar system, there are 14 possible calendars for every year. The same calendar can be re-used every 28 years $(28 = \text{L.C.M.}\{4, 7\})$.

Finally, according to the Gregorian calendar system, we still have 14 possible calendars, but the choice of the right calendar to be valid for a given year is more complicated, because of the irregularity of the sequence of the leap years. If we want to include also the information about lunar phase, before to re-use the same calendar we need to wait $53200 = L.C.M.\{400, 7, 19\}$ years (19 years = Meton's cycle).



Fig. 3. Giovanni A.A. Plana

V. GIOVANNI ANTONIO AMEDEO PLANA

Giovanni A.A. Plana (1781-1864), was probably the most eminent astronomer and mathematician active at Torino (Italy), during the first half of nineteenth century.

Born in Voghera (Novara), he attended the secondary school and the university in France (Grenoble, Paris) where he was a student of Lagrange. He was holder of the chair of Astronomy at the University of Torino from 1811 to 1815, during the Napoleonic occupation. After the Napoleon's defeat and the re-establishment of the *ancient régime*, the chair was suppressed and he moved to the chair of Mathematical Analysis. He was also president of the Accademia delle Scienze di Torino.

His principal interest always remained astronomy. His most famous work was the *Theorie du mouvement de la lune* (1832); he constructed also tables of the Moon motion, successfully employed for ship navigation.

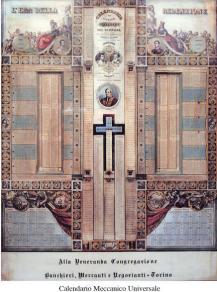
VI. PLANA'S CALENDAR

A perpetual calendar covering a period of 4000 years was constructed in Torino around 1830: although the original preparative documents have not been found, the project is very reasonably attributed to Giovanni Plana. The calendar was presented to a religious institution, the *Congregazione dei Mercanti, Negozianti e Banchieri* and since then, it has been conserved in the Chapel of the Congregazione, a beautiful baroque church in the center of the town. The most remarkable features of this calendar are the following.

• The calendar is a machine, made by wood, paper, strips of cloth.

• The information concern the usual correspondence *day of the week* \longleftrightarrow *date*, Moon phases, tides, Christian feasts.

• It appears as a big picture, equipped by a gilt frame (dimensions: width 99cm, height 125cm, thickness 15cm).



Realizzata nel 1831 da Antonio Amedeo PLANA (1783 - 1864)

Fig. 4. Front vision

It is hung to the wall by a pair of lateral pivots (Figure 4).

• It works as the memory of a modern computer; all the data are pre-computed and stored in the calendar; acting on the mechanism by means of a crank handle, the desired information is displayed through some small windows open on the front.

Although other more famous and ancient mechanisms which may provide similar information exist (Prague, Strasbourg), the Plana's calender is an exemplar unique in the world, because of these very special features.

VII. THE MECHANISM

A perpetual calendar covering so many centuries as Plana's calendar, must face further difficulties: for instance the "discontinuity" due to the 1582 reform.

A scheme of the mechanism is shown in Figure 5. It is constituted by nine cylinders (denoted by C1,...,C9) and three strips os cloth (denoted by S1, S2, S3). The data are printed on paper sheets and pasted on the cylinders, in such a way to form circular or helicoidal tracks.

The angular velocity of cylinders and the scrolling of the strips are coordinated by gears (of suitable ratios) and

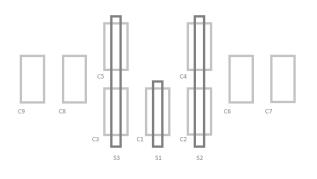


Fig. 5. Scheme of the mechanism

chains. The overall mechanism is activated by a crank handle, which allows us to point on the desired year: the first two digits (century) are read on S1, the second two digits (year) on C1 Once the year has been selected in this way, all the information are displayed and can be read through the small windows:

• C2, C3, C4, C5 data for Easter computation

• C6, C7, C8, C9 correspondence $date \leftrightarrow day of the week$

It is worth noticing that the cylinders and all the gears (including a worm gear) are made by wood.

Plana's calendar was restored some years ago by Giovanni Lanza, an engineer member of the *Congregazione* (see [4]).

REFERENCES

- [1] J. Meeus, "Astronomical algorihtms", 2nd edition, Willmann-Bell, Inc., Richmond 1998.
- [2] C.F. Gauss, "Gesammelte Werke", Göttingen, Dieterich; vol. 6 (1874), pp. 73-79 and pp. 82-86; vol. 11 (1927), pp. 206-210.
- [3] G. Peano, "Sulla riforma del calendario", Atti della Reale Accademia delle Scienze di Torino, vol. 62 (1927), pp. 566-568

[4] G. Lanza, "Calendario Meccanico Universale", *Con*gregazione dei Banchieri, Negozianti e Mercanti