A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
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SIMPLE MATEMATICAL MODEL FOR PREDICTION OF RECRYSTALLIZATION IN COPPER

PROSTY MODEL MATEMATYCZNEGO PRZEWIDYWANIA REKRYSTALIZACJI W MIEDZI

Experiments were performed on cold deformed copper annealed at varied temperatures in order to test the effect of deformation on recrystallization kinetics. A simple mathematical model based on Avrami equation was presented for prediction of the recrystallization progress with respect to effect of deformation and annealing temperature. Appropriate constants for mathematical equations were determined by statistical fitting of experimental data. Presented equations for calculation of annealing time that is necessary for receiving 5 % and 95 % recrystallized volume can be used in technological practice.

Keywords: copper recrystallization, recrystallization kinetics, modeling, Avrami equation

Przeprowadzono badania wpływu wielkości odkształcenia i temperatury wyżarzania na kinetykę rekrystalizacji miedzi. W oparciu o proste zależności matematyczne oparte na równaniu Avramiego pokazano prosty sposób przewidywania postępów rekrystalizacji w zależności od stopnia wstępnego odkształcenia i warunków wyżarzania. Pokazano sposób wykorzystania liniowej ekstrapolacji wyników eksperymentalnych do określenia odpowiednich stałych w równaniach i sposób wyznaczenia równań opisujących kinetykę rekrystalizacji. Podano równania, które mogą być wykorzystane w warunkach praktyki przemysłowej do określenia warunków wyżarzania miedzi w celu uzyskania 5% lub 95% objętości zrekrystalizowanej.

1. Introduction

Industrial processing of metallic materials is often based on cold deformation procedures used in order to receive desired shape of a product. Cold deformation leads to effective strain hardening and reduction of the material plasticity. Thus an intermediate annealing procedure is necessary to continue further shaping of the product. In result of recovery and recrystallization processes, restored plastic properties and effective softening of the product allows further processing of the material. However, annealing of the product is an energy and time consuming procedure. From economical point of view, it is not a favorable part of any working process.

Determining of heat treatment conditions is a crucial point of each material processing procedure both for reduction of time/energy costs and receiving desired material properties. As the most of industrial processes are based on computer-controlled processing parameters, the mathematical models are undoubtedly welcomed for prediction of recrystallization progress in annealed materials. Simple relations are more convenient for practical use than already published complicated mathematical models for *static* or *post-dynamic* recrystallization processes [1-6].

Recrystallization kinetics for single-phase materials mostly depends on the material strain hardening as well as deformation conditions, initial structure and material composition, annealing parameters such as heating rate and annealing temperature. The most effective components of driving force for recrystallization process result from the stored energy related to dislocation density and arrangement of dislocation substructure resulted from foregoing recovery process.

Nucleation time and *primary recrystallization* time are typical time-periods for *static discontinuous recrystallization* that are commonly observed for cold deformed and annealed metals. *Primary recrystallization* is controlled by the nucleation density and diffusion-controlled velocity of recrystallization front migration alike [7-11]. The recrystallized volume vs. annealing time relation can be described by the Avrami equation [9]:

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$$X_v = 1 - \exp(-kt^n) \tag{1}$$

where:

k – constant depending on annealing temperature and dislocation density resulted from the initial structure and pre-deformation conditions
n – constant depending on the material composition

In spite of detailed discussion on recrystallization process in the literature, simple mathematical relations of the strain and temperature effects on recrystallization time at used annealing procedures are very helpful in technological practice. A simple approach to mathematical prediction of the recrystallization progress in high purity copper deformed with varied strains and annealed at given temperatures was considered below. The mathematical model based on an Avrami equation was used and experimentally verified for the copper deformed by cold drawing.

2. Experimental

Annealing experiments were performed on high purity electrolytic copper that is processed by CONTIROD® hot rolling and multi-step cold drawing for production of fine wires less than 0.1 mm in diameter. High purity of copper is necessary to avoid the wire rupture during final sequences of multi-step cold drawing procedure. Tested material composition was shown in Table 1.

TABLE 1
Chemical analysis results for copper used in experiments

Elements, ppm												
Cu, wt.%	Ag	Pb	Fe	Ni	S	Zn	Bi	Sb	As	Sn	Te	Se
>99.99	10	2	1	2	3	2	<1	<1	<1	<1	<1	<1

Preliminary recrystallized copper wire of 8.0 mm in diameter was cold deformed by multi-step laboratory drawing at the rate of ~0.2 m/s. As the material temperature was slightly increased during drawing, the sample was air cooled before each drawing pass. Annealing of cold deformed material within temperature range of 443 K - 533 K was followed by structural observations performed on samples sectioned along the wire axis. Optical microscopy observations and statistical points-method was used for determination of recrystallized volume at partly recrystallized samples. Details of statistical analysis and experimental results were summarized in PhD-thesis by Kwapisinski [13]. Selected results for samples with 5 % – 95% recrystallized volume were used for further mathematical processing described bellow.

3. Mathematical approach to copper recrystallization kinetics

Specific recrystallized volume for primary recrystallization time can be described by classical Avrami equation (1). The recrystallized volume (X_v) is usually displayed vs. logarithm of annealing time. In many instances of kinetics studies, presented s-shaped recrystallization characteristics are shifted left in the graph if annealing temperature is raised or higher deformation of the material is used. The slope of S-shaped curves remains practically constant and depends on the material chemical composition.

A simple statistic procedure described bellow can be used for mathematical description of experimental results. Transformation of equation (1) to linear relationship is necessary for statistical determination of k and n constants:

$$\ln\left(\ln\frac{1}{1-X_v}\right) = \ln k + n \cdot \ln t \tag{2}$$

Then, a last-square method is used for fitting the set of data points according to linear relation:

$$y = n_i x + b_i \tag{3}$$

where:

$$y = \ln\left(\ln\tfrac{1}{1-X_v}\right)$$
$$x = \ln(t)\,.$$

Parameters n_i and b_i were statistically calculated for each set of data points related to given true strain (ε_i) and annealing temperature (T_i). Results are shown in Tab. 2.

TABLE 2

Parameters n and k for the equation (2) calculated from linear approximation of experimental data displayed in Fig. 1

True strain	Annealing temperature, K	Slope, n_i	k_i
0.46	533	1.1111	0.0142
0.46	503	1.0833	0.0034
1.02	503	1.0784	0.0227
1.02	473	1.0711	0.0085
1.54	503	1.0619	0.0702
1.54	473	1.0834	0.0092
2.03	463	1.0821	0.0238
2.03	443	1.0805	0.0024
2.54	463	1.1076	0.0485
2.54	443	1.0755	0.0028
2.77	463	1.0311	0.0661
2.77	443	1.0080	0.0057

Average = $\overline{n}$ 1.0728

If assume a negligible effect of strain and annealing temperature on the slope of *S*-shaped recrystallization characteristics, the value of *n*-constant can be averaged for all experimental data sets:

$$\bar{n} = \frac{\sum_{i}^{r} n_i}{r} \tag{4}$$

where

n_i – slope parameter for given data set at given ε_i and T_i

r – number of experimens performed at given ε_i and T_i

Fitting of experimental data cannot obviously be succsessful if previously calculated b_i – values are used for equasion (3). Thus, after averaging n_i – values also adequate correction of b_i – values has to be done. Let each set of data points is extrapolated by linear relationschip with using calculated $\overline{n}$ – value:

$$y = \overline{n}x + b_k, \tag{5}$$

where

y, x – experimental data point coordinates received for ginen ε_i and T_i values

b_k – new parameter that must be recalculated to maintain the best linear fitting of experimental data

Simple procedure is recommended for calculation of a new b_k – value. Lets calculate average values of $\overline{x}$ and $\overline{y}$ for each set of *j*-data-points received at given ε_i and T_i

$$\bar{x} = \frac{\sum_{j}^{m} (\ln t)_j}{m}$$

$$\bar{y} = \frac{\sum_{j}^{m} \left\{\ln\left[\ln\left(\frac{1}{1-X_v}\right)\right]\right\}_j}{m},$$

where:

m – number of data points for one set of data points related to given strain and annealing temperature

If averaged $\overline{n}$ – value is used for fitting data poins by linear relationschip, a new b_k – value can be calculated as follows:

$$b_k = \frac{\overline{y}}{m} - \bar{n} \cdot \frac{\overline{x}}{m} \tag{6}$$

It is trivial statement that the value of b_k – constant can be calculated directly from the formula:

$$b_k = \frac{\sum_{j}^{m} \left\{\ln\left[\ln\left(\frac{1}{1-X_v}\right)\right]\right\}_j}{m} - \bar{n}\frac{\sum_{j}^{m} (\ln t)_j}{m}.$$

Thus the final linear relationschip for given set of data points is writen as follows:

$$y = \bar{n}x + b_k. \tag{7}$$

According to received b_k value, a new corrected k_k – parameter was calculated for every (ε_i /T_i) data set (Tab. 3). Experimental data and related fitting lines based on b_k and $\overline{n}$ constants are displayed in Fig. 1.

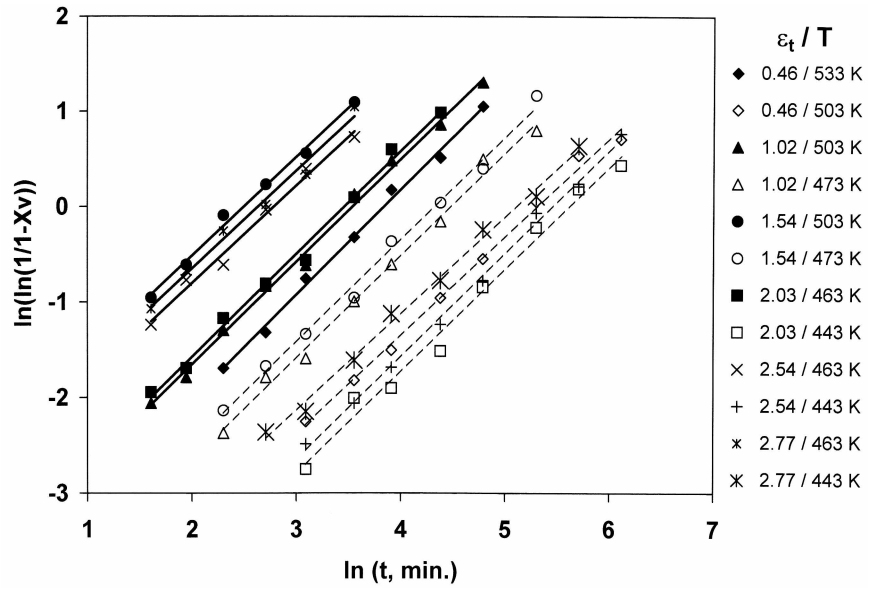

Fig. 1. Recrystallized volume fraction vs. annealing time for high purity copper deformed by cold drawing. Linear fitting of data sets was performed according to relation (7) True strain and annealing temperature is marked in legend

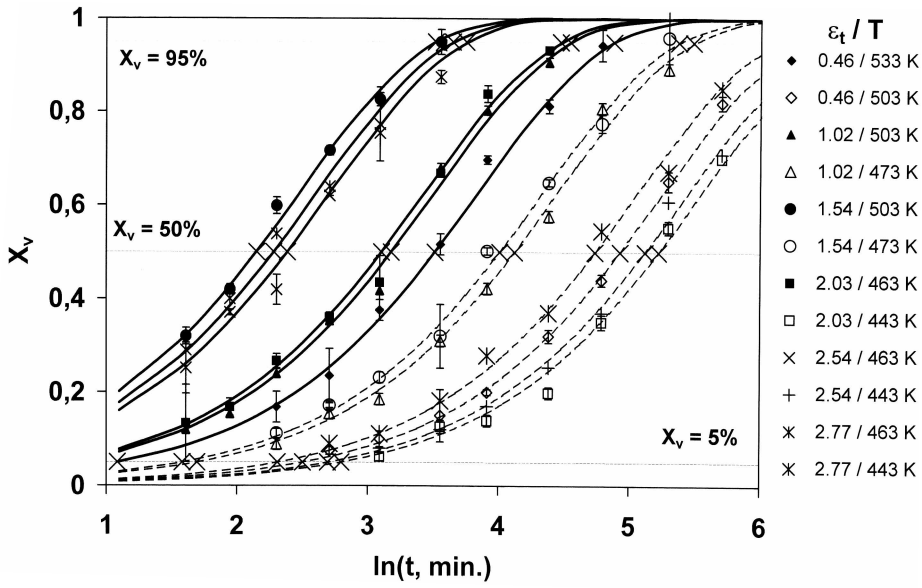

Fig. 2. Avrami plot of recrystallization fraction vs. annealing time. Annealing time for recrystallized volume of 5% and 50% and 95% was marked in the figure. True strain values and annealing temperatures are shown in legend

Avrami graph, i.e. X_v vs. ln$t$ relation, is displayed in Fig. 2. The parameters $\overline{n}$ and k_k were used for equation (1) in order to fit every data set (ε_i, T_i):

$$n = \bar{n} \tag{8}$$

$$k_k = \exp b_k. \tag{9}$$

In line with the received mathematical relations, annealing time for 5% and 50% and 95% recystallized volume ($X_v = 0.05$, $X_v = 0.5$, $X_v = 0.95$) was calculated for every set of ε_i/T_i data. The results are marked in Fig. 2 and listed in Tab. 3.

TABLE 3

Corrected k_k parameters calculated according to equation (9). Recrystallization time for $X_v = 0.05$, $X_v = 0.5$, $X_v = 0.95$ was calculated according to equation (1) (see: Fig. 2)

True strain	Annealing temperature, K	k_k corrected for constant $\overline{n}$	$t_{0.05}$ min	$t_{0.5}$ min	$t_{0.95}$ min
0.46	533	0.0160	2.96	33.54	131.25
0.46	503	0.0035	12.21	138.30	541.20
1.02	503	0.0229	2.12	24.01	93.97
1.02	473	0.0083	5.44	61.63	241.18
1.54	503	0.0690	0.76	8.59	33.61
1.54	473	0.0094	4.86	55.07	215.49
2.03	463	0.0248	1.97	22.29	87.24
2.03	443	0.0026	16.41	185.79	727.02
2.54	463	0.0537	0.96	10.85	42.46
2.54	443	0.0029	14.79	167.49	655.42
2.77	463	0.0600	0.86	9.78	38.29
2.77	443	0.0043	10.08	114.16	446.71

It is commonly accepted that recrystallization time for cold deformed single-phase materials mainly depends on the material composition, annealing temperature and stored energy resulted form preliminary deformation conditions. Migration of recrystallization fronts during primary recrystallization is diffusion controlled process dependent on driving force P. The driving force for pure metals depends on dislocation density gradient nearby migrating grain boundary. If suppose that temperature-related diffusion process is the main controlling factor for the grain boundary migration, the velocity of migrating recrystallization front, v_f, can be described by the following equation:

$$v_f = P \cdot \exp\left(-\frac{Q}{RT}\right), \tag{10}$$

where:

P – driving force dependent on dislocation density and their arrangement
Q – activation energy
R – gas constant
T – annealing temperature.

Activation energy is mainly depended on the chemical composition of the material and does not practically depend on the material deformation and annealing temperature. If suppose that the Q – value is strain independent and P – value is unchanged during recrystallization process, adequate half-recrystallization time (for X_v = 50%) can be calculated from the formula:

$$t_{50\%} = A \cdot \exp\left(\frac{Q}{RT}\right), \tag{11}$$

where:

A – constant parameter depending on the material composition and preliminary deformation conditions

Fitting of experimental data requires the formula (11) to be transformed into linear relation:

$$\ln(t_{50\%}) = \ln A + \frac{Q}{R} \cdot T^{-1}. \tag{12}$$

According to the relation (12), calculated $t_{0.5}$ values for copper are displayed in Fig. 3. As mentioned above, samples were annealed at temperatures convenient for experimental determination of recrystallized volume (X_v) vs. annealing time. Therefore, experiments were limited to the annealing time range of 5–450 min.

It was assumed that the slope of linear relation, $\frac{Q_i}{R}$, was varied in Fig. 3 as a result of some experimental scatter rather than any variation of Q-value or other effects. Therefore, the calculation of activation energy was performed after averaging the line slope parameters calculated from linear fitting of experimental data shown in Fig. 3 and listed in Tab. 4. Received average activation energy of $\overline{Q}$ = 152 kJ/mol · K was found to be close to the Q-values reported in the literature i.e. 109 kJ/mol · K - 146 kJ/mol · K kJ/mol[8,10,11,14].

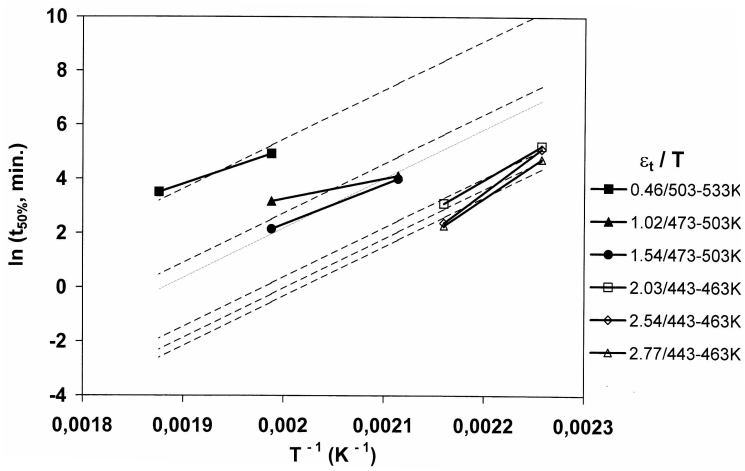

Fig. 3. Recrystallization half-time vs. reverse annealing temperature. Linear approximation was performed according to the procedure described in the text. Strain value and annealing temperatures are marked in the figure legend

Taking into account a constant $\overline{Q}$ value, appropriate correction of A-values for each $(\varepsilon_i - \Delta T_i,)$-set of data was necessary. Thus, a new corrected A_{cor}-values were calculated in the same manner as described above for b_k and adequate equations (4) - (7):

$$A_{cor} = \frac{\overline{y}}{l} - \bar{n}_{\varepsilon} \cdot \frac{\overline{x}}{l}, \tag{13}$$

where:

$\overline{n}_{\varepsilon}$ – averaged slope for linear approximation of data points shown in Fig. 3

l – number od annealing temperatures for samples deformed to given strain (here: $l = 2$)

$$x = T^{-1}$$
$$y = \ln(t_{50\%}).$$

The effect of strain on A_{cor} value is shown in Fig. 4 and corresponding data are listed in Tab. 4. Received data can be approximated with correlation coefficient of $R = 0.9765$ if an arbitrary chosen exponential function is used:

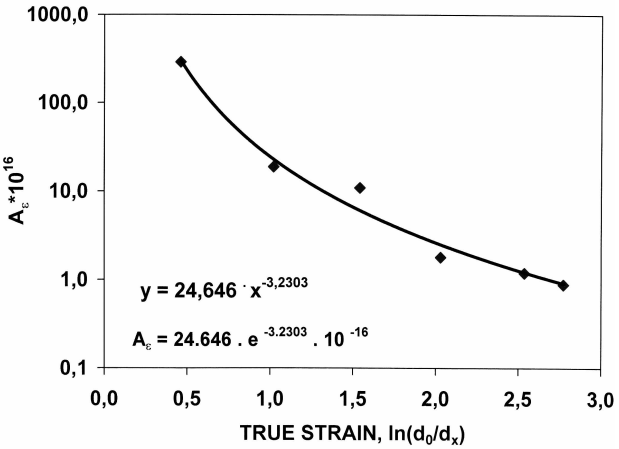

Fig. 4. Effect of deformation on A_{cor} – value. Experimental data were approximated by means an exponential equation displayed in the figure

$$A_{cor} = 24.646 \cdot \varepsilon_t^{-3.2303} \cdot 10^{-16}. \tag{14}$$

Including strain-depending A_{cor} parameter and averaged $\overline{Q}$-value into formula (11), following relation of half-recrystallization time vs. deformation and annealing temperature is received:

$$t_{50\%} = 24.646 \cdot \varepsilon_t^{-3.2303} \cdot \exp\left(\frac{\bar{Q}}{RT}\right) \cdot 10^{-16} \text{ [min]}. \tag{15}$$

TABLE 4

Line equation parameters calculated for equation (12) (see Fig. 3). Line slope parameters, i.e. Q/R values, were calculated for each set of experimental data (ε_t, ΔT). A-values were calculated from equation (13) assuming constant activation energy of Q=152.3 kJ/mol · K

True strain	Temperature range, K	Experimental line slope, Q/R	A_{cor} calculated for Q=152.3 kJ/mol · K
0.46	503-533	12.7	$290.0 \cdot 10^6$
1.02	473-503	7.5	$19.0 \cdot 10^6$
1.54	473-503	14.7	$11.0 \cdot 10^6$
2.03	443-463	21.7	$1.8 \cdot 10^6$
2.54	443-463	28.1	$1.2 \cdot 10^6$
2.77	443-463	25.2	$0.9 \cdot 10^6$

Average Q/R, $\overline{n}_\varepsilon = 18.313$

Exponential relation is useful in practice for calculation of annealing temperature T or/and true strain ε_t that is necessary for receiving 50% recrystallization volume during annealing at given temperature. Simple transformation of formula (15) results in following relations:

$$T = \frac{\bar{Q}}{R \ln\left(\frac{t_{50\%}}{24.646 \cdot \varepsilon_t^{-3.2303} \cdot 10^{-16}}\right)} \quad [\mathrm{K}] \qquad (16)$$

$$\varepsilon_t = \left(\frac{24.646 \cdot \exp\left(\frac{\bar{Q}}{RT}\right) \cdot 10^{-16}}{t_{50\%}}\right) \qquad (17)$$

From practical point of view, the most desired equations are related to recrystallization start and finish time. Using similar mathematical procedure as described above, the time value for the recrystallization start and finish, i.e. $X_v = 5\%$ and $X_v = 95\%$ respectively, was found to fit the following relation:

$$t_{5\%} = 2.1725 \cdot \varepsilon_t^{-3.2553} \cdot \exp\left(\frac{\bar{Q}}{RT}\right) \cdot 10^{-16} \quad [\mathrm{min}] \quad (18)$$

$$t_{95\%} = 96.069 \cdot \varepsilon_t^{-3.2559} \cdot \exp\left(\frac{\bar{Q}}{RT}\right) \cdot 10^{-16} \quad [\mathrm{min}] \quad (19)$$

Both strain value and annealing temperature that are necessary for the recrystallization start and finish within selected period of time can be easily calculated from relations (15), (18) and (19). For example, an annealing temperature that is necessary for receiving 5% and 95% recrystallized volume during annealing of cold deformed copper was calculated for annealing time of 0.1 min. and 60 min. (Fig. 5). For samples deformed close to critical strain ($\varepsilon_t \sim 0.03$) and annealed for 60 min., recrystallization starts at the temperature of ~630 K and $X_v = 95\%$ is expected at 730 K. If the strain is increased up to $\varepsilon_t = 10$, recrystallization start and finish is expected at 380 K and 410 K respectively. Increasing the annealing temperature up to 490 K, reduction of recrystallization finish time down to 0.1 min is expected for heavy deformed material.

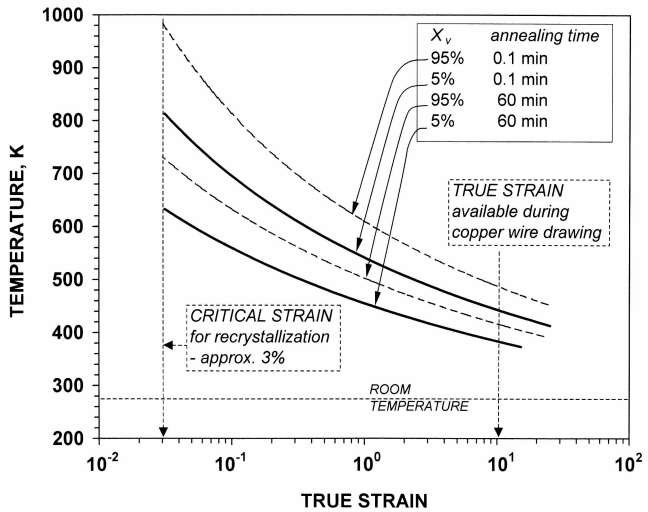

Fig. 5. Effect of strain on start and finish recrystallization temperature for copper annealed 60 min and 0,1 min calculated according to the relation (18) and (19)

It is worth stressing that the estimation of the recrystallization time at very high annealing temperatures is useful for prediction of the material softening during some industrial multi-step hot deformation processes performed at high strain rates.

High temperature multi-step deformation is commonly used in order to attain an efficient material processing. Evaluation of the structural processes during time-intervals between hot rolling passes or hot forging procedures is important for controlling the structure and final properties of the material. In many cases an intense working of a product can be performed with large cross-section reduction due to at least partial recrystallization of the material even if the processing is performed at relatively low temperature range.

4. Structural aspect of copper heavy drawing

Presented mathematical procedure for prediction of recrystallization progress is useful for determining of technological parameters for some hot working processes. The method is mostly assigned for prediction of structural changes during annealing of cold deformed material or hot multi-step deformation with adequately high deformation rates. For example, both the rolling temperature and time interval between rolling passes can be calculated for each deformation step if complete or partial recrystallization is required during multi-step hot rolling. Due to controlled recrystallization, the material fracture becomes retarded and intense material processing can be continued. Moreover, determination of technological parameters for recrystallization controlled rolling and other thermomechanical processing is important for mathematical modeling of grain size refinement and demanded properties of a product.

It is generally believed that recrystallization process does not operate in cold deformed materials if the metal melting temperature is high enough. For example, recrystallization of cold deformed copper is observed usually within the temperature range of 470 K – 650 K. However, recrystallization temperature is reduced with strain as it was shown formerly. As a result, some recrystallization effects can be expected at heavy deformed copper if the material temperature slightly increases due to heat release during deformation process.

Structural observations performed for copper wire deformed at room temperature with strain of $\varepsilon_t = 2.77$ confirmed conclusion mentioned above. Typical recrystallization effects are shown in Fig. 6. Similar recrystallization fronts were also observed for other samples deformed to larger strains [13].

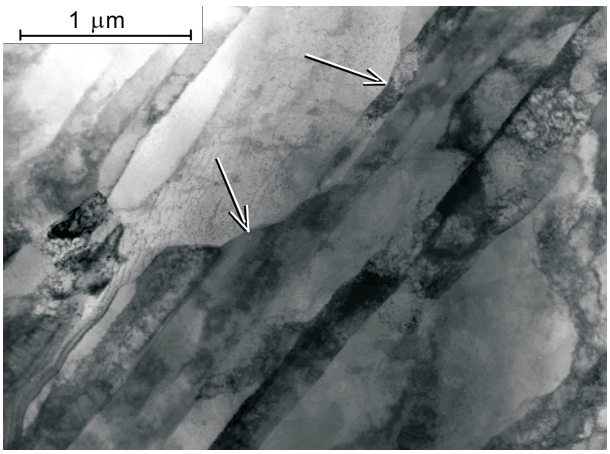

Fig. 6. TEM structure of cold drown copper wire deformed with $\varepsilon_t = 2.77$ at room temperature. Recrystallization front is marked in the figure

5. Discussion

Structural detection of recrystallization progress for cold deformed and annealed copper is usually performed at temperature between 470 K and 530 K, as the range of recrystallization time is convenient for laboratory recrystallization tests. Determining of recrystallization start and finish time beyond mentioned temperature range is difficult in usual experimental practice. For example, recrystallization of a material can simply be finished during heating up the sample and estimation of start and finish time becomes impossible at very high annealing temperature. On the other hand, many industrial working processes such as multi-step hot rolling are performed at high temperature range. Prediction of recrystallization progress within short period of time between rolling passes is important for determining the best parameters of recrystallization controlled rolling procedures [14,15].

Cold rolling or drawing industrial technologies usually include an intermediate annealing procedure in order to reduce strain hardening, restore plastic properties, and enable further product forming. Thus, a proper determining of recrystallization conditions becomes a crucial point both for controlled working processes at high temperatures and minimizing energy consumption at intermediate annealing at cold processing technologies alike.

In order to meet technological requirements, mathematical formulas (18) and (19) were found to control the recrystallization process in industrial practice. Simple transformation of described mathematical relation between recrystallized volume, strain, annealing temperature and time allows calculation of optional parameter vs. others if necessary (see equations (15), (16), (17)). For

example, calculated recrystallization time at high enough annealing temperature is reduced to a few seconds. It means that hot processing of copper, performed with high deformation rates, usually results in recrystallized structure of the product.

On the other hand, recrystallization of heavy deformed copper can also develop if prolonged annealing is performed at temperature considerably bellow 470 K. The most effective reduction of recrystallization temperature is usually observed for high purity of the metal and high deformation values. Some structural recrystallization effects for heavy cold drawn copper wire have been already reported by Vingrove and Clairns [16,17]. However, the authors concluded that recrystallization process is related to intense recovery process rather than the nucleation and growth of new grains. Increased misorientation angle for some subboundaries above 2-5 degree was assumed to be an evidence for the recrystallization development. Thus, the recrystallization development was ascribed to the recovery process rather than diffusion-controlled migration of high-angle grain boundaries.

At technological practice, the heat release due to deformation process as well as friction heating effect result in increased temperature of the material. During industrial multi-step wire drawing, the material temperature periodically increases while the wire passes thorough consecutive die even if the wire is intensively cooled by an oil lubrication system. It is observed that drawing of high purity copper can result in strain hardening stabilization at large strains that allow achieving a large wire diameter reduction in technological drawing practice [13]. If very high deformation rate and drawing speed of 10-30 m/s are taken to consideration, approaching suitable temperature for partial recrystallization of copper seems to be readily available. For example, mathematical relations (18) and (19) indicate possibility of partial recrystallization for copper wire deformed with ε_t = 10 after 0.1 min at the temperature range of 440 K – 480 K (see: Fig. 5). If strain hardening is consistently reduced, multi-step drawing of the copper can be continued to large diameter reduction without any intermediate annealing procedures.

To support conclusion mentioned above, transmission electron microscopy structural observations were performed for copper samples deformed with high strains. Typical structure of the copper wire cold deformed with strain of ε_t = 2.77 was shown in Fig. 6. The sample diameter was reduced from 8 mm to 2 mm by multi-step laboratory drawing at room temperature. It should be stressed that the wire was air-cooled before successive passes and material temperature practically never exceeded ~350 K. In spite of low deformation temperature, some structural changes due to partial recrystallization of the material were often observed (Fig. 6).

According to relations (18) and (19), the nucleation of recrystallization process is not practically expected for annealing temperatures bellow 373 K. Although, the recrystallization of heavy deformed copper was reported by G. Mima *et all*. [19] for samples annealed bellow 350K. Presented above equations are based on experimental data received at experimentally convenient annealing temperature range that does not include the temperature value mentioned above. It is worth to stress that extrapolation method described formerly can lead to some misfit between experimental results and simplified mathematical predictions of recrystallization process beyond used conditions range. For example, intensified shear bands development at high strains accelerates recrystallization due to effective nucleation of recrystallized grains at shear bands or micro-deformation bands [20-23]. As a result, the recrystallization process may become faster than that expected from the extrapolation method presented above. On the other side, some other processes such as impurities dragging by migrating boundary can effectively reduce a driving force at low dislocation density at materials slightly deformed just above the critical strain. Finally, some deviation of experimental data from extrapolated relation can also be expected for slightly deformed material.

Moreover, constant values for equations (15) – (19) are related to specific copper initial grain size of 20 μm – 60 μm and deformation by drawing method. It should be stressed that the driving force for recrystallization process can become substantially changed if deformation conditions and the material purity and initial structure of the material are changed. As a result, enlarged misfit between experimental and recrystallization parameter values calculated from equations (15) – (19) is expected. Thus, an experimental correction of adequate constants at the equations is recommended for any other materials.

6. Conclusions

1. Avrami equation was used for prediction of recrystallization progress in cold deformed and annealed copper. Simple mathematical relation between recrystallization conditions was determined according to presented mathematical procedure and statistical fitting of experimental data.
2. The effect of cold deformation value on copper recrystallization parameters (T, t, X_v) was incorporated to Avrami equation by fitting an arbitrary chosen exponential relation of A-parameter vs. true strain value (ε_t). Adequate relations for calculation of start and

finish recrystallization time corresponding to X_v = 5% and X_v = 95% respectively, were presented.

3. Activation energy of $\overline{Q}$ = 152 kJ/mol · K for recrystallization of cold deformed copper was calculated. The $\overline{Q}$ – value found to be close to the data reported in the literature.

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REFERENCES

[1] V. Erukhimovitch, J. Baram, Modelling recrystallization kinetics, Mater. Sci. Eng. A **214**, 78-83 (1996).

[2] R. Sandstrom, R. Lagneborg, A model for static recrystallization after hot deformation, Acta Met. **23**, 481-491 (1975).

[3] L. M. Clarebrough, M. E. Hargreaves, The influence of grain size on the stored energy and mechanical properties of copper, M. H. Loretto, Acta Met. **6**, 725 (1958).

[4] A. M. Elwazri, E. Essadiqi, S. Yue, Kinetics of metadynamic recrystallization in microalloyed hypereutectoid steels, ISIJ Intern. **44**, 744-752 (2004).

[5] E. Woldt, New kinetic model for primary recrystallization of pure metals, Metall. Mater. Trans. A **32**, 2465-2473 (2001).

[6] D. Muljono, M. Ferry, D. P. Dunne, Continuous annealing - modelling the kinetics of recrystallization, Proc. 4^{-th} Intern. Conf. REX'99, JIMIS-10, edit. T. Sakai, H. G. Suzuki, 671-676 (1999).

[7] H. Mecking, F. Hirsh, Recrystallization of Metallic Materials, edit. F. Hassner, Stuttgart, 1970.

[8] R. E. Reed - Hill, Physical Metallugy principles, Edit. D. Van Nostrand Company – USA, 1973.

[9] R. V. Muraleedharan, Variation of slope in Avrami plots, Scripta Mater. **40**, 1367-1370 (1999).

[10] K. Brown, M. Hatherly, Recrystallization kinetics of rolled copper single crystals, Jour. Inst. Metals **98**, 317-320 (1970).

[11] S. Murphy, C. J. Ball, The recrystallization of tough-pitch copper, Journ. Inst. Met. **100**, 225 (1972).

[12] H. D. Mengelberg, M. Meixner, K. Luke, Zur Kinetik Der Rekristallisation von Kupfer, Acta Met. **13**, 835-844 (1965).

[13] P. Kwapisinski, PhD. Thesis – in polish, AGH-University of Science and Technology, Faculty of Non-Ferrous Metals, 2003.

[14] T. Siwecki, Modelling of microstructure evolution during recrystallization controlled rolling, ISIJ Intern. **32**, 368-376(1992).

[15] T. Siwecki, G. Engberg, Recrystallization controlled rolling of steels, in Proc. Conf., Thermo-Mechanical Processing in Theory, Modelling & Practice, Stockholm 1996, Edit, ASM Intern. 121-144 (1997).

[16] J. H. Clairns, J. Clough, M. A. P. Dewey, J. Nutting, The structure and mechanical properties of heavily deformed copper, Journ. Inst. Met. **99**, 93 (1971).

[17] A. L. Vingrove, Some aspects of relating structure to properties of heavy deformed copper, Journ. Inst. Met. **100**, 313 (1972).

[18] M. Szczerba, L. Błaż, Static and Dynamic Recrystallization in CuAl Single Crystals, in Proc. 7^{-th} RISO Intern. Symp., "Annealing Processes, Recovery, Recrystallization and Grain Growth", 561-566 (1986).

[19] G. Mima, Y. Ogino, J. Sato, Annealing spectrum of heavily drawn copper wire, Jour. Inst. Metals **96**, 49-58 (1968).

[20] L. Blaz, M. Szczerba, The role of deformation bands in the development of recrystallization in Cu15at.%Al single crystals, Archiwum Hutnictwa **30**, 557-570 (1985).

[21] A. Korbel, Structural and mechanical aspects of homogeneous and non-homogeneous deformation in solids, in "Localization and Fracture Phenomena in inelastic solids", Edit. P. Perzyna, Intern. Center Mech. Sc., SpringerWienNewYork – Courses and lectures No 386, 21-91 (1998).

[22] H. Paul, J. H. Driver, Z. Jasieński, Shear banding and recrystallization nucleation in a Cu-2%Al alloy single crystal, Acta Mater. **50**, 815-830 (2002).

[23] J. A. Hines, K. S. Vecchio, Recrystallization kinetics within adiabatic shear bands, Acta Mater. **45**, 635-649 (1997).

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