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# TYPE CURVES FOR SELECTED PROBLEMS OF FLOW TO WELLS IN CONFINED AQUIFERS



BOOK 3 CHAPTER B3

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## <sup>-</sup>echniques of Water-Resources Investigations of the United States Geological Survey

Chapter B3

## YPE CURVES FOR SELECTED PROBLEMS OF -LOW TO WELLS IN CONFINED AQUIFERS

By J. E. Reed

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Book 3 APPLICATIONS OF HYDRAULICS

#### UNITED STATES DEPARTMENT OF THE INTERIOR

#### CECIL D. ANDRUS, Secretary

#### GEOLOGICAL SURVEY

H. William Menard, Director

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## PREFACE

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## SYMBOLS AND DIMENSIONS

Numbers in parentheses indicate the solutions to which the definition applies. If no number appears, the symbol has only one definition in this report]

imbol	Dimension	Description
ï	Dimensionless	$\sqrt{K_z/K_r}$
	L	Aquifer thickness.
	L	Thickness of confining bed (4, 6, 7, 11); specifically the upper confining bed (5).
1.	L	Thickness of lower confining bed.
<b>*</b> **	L	Depth from top of aquifer to top of pumped well screen.
- Child	L	Depth from top of aquifer to top of observation-well screen
1		Change in water level in well
Å.	ī.	Initial head increase in well
Į.	Ĩ.	Change in water level in acuifer
10 	17-1	Hydraulic conductivity of aquifer
29 27	17-1	Hydraulic conductivity of the acuitar in the radial direction
	LT = 1	Hydraulic conductivity of the equiler in the vertical direction.
*. • ;	LT L/T-1	Hydraulic on ductivity of one aquiter in the vertical uncertain.
		Hydraune conductivity of comming bed (4, 6, 7); Specifically the upper comming
3	I (1))	
pi.		Hydraune conductivity of lower comming bed.
ř.		Depth from top of aquifer to bottom of pumped well screen.
Ng		Depth from top of aquifer to bottom of observation-well screen.
	$L^{3}T^{-1}$	Discharge rate.
<i>.</i> <b>(</b> )	$L^{3}T^{-1}$	Discharge rate.
1	L	Radial distance from center of pumping, flowing, or injecting well.
1	L	Radius of well casing or open hole in the interval where the water level changes.
r	L	Effective radius of well screen or open hole for pumping, flowing, or injecting well.
	Dimensionless	Storage coefficient.
*	$L^{-1}$	Specific storage of aquifer.
, x	$L^{+1}$	Specific storage of confining beds.
,	Dimensionless	Storage coefficient of upper confining bed.
"	Dimensionless	Storage coefficient of lower confining bed.
	L	Drawdown in head (change in water level).
	L	Drawdown in upper confining bed.
	L	Drawdown in lower confining bed.
•	L	Constant drawdown in discharging well.
	$L^{2}T^{-1}$	Transmissivity.
rr. Tru. Tun	$L^2T^{-1}$	Components of the transmissivity tensor in any orthogonal x-, y-axis system.
. T	$L^2 T^{-1}$	Transmissivities along two principal axes, $\epsilon$ and $n$ , such that $T_{n} = 0$ .
ε' ηη	T	Time.
	Dimensionless	Variable of integration
	Dimensionless	$r^{2}S/4Tt(2, 6)$ ; variable of integration (3, 7, 9)
	Dimensionless	Variable of integration
	Dimensionless	Dummy variable (2, 5); variable of integration (3)
v	I.	Distances from the numbed well for an arbitrary rectangular coordinate system
y	15	(10)
	Dimensionless	Variable of integration $(1, 2, 4, 5, 6)$
	I	Depth from top of aquifer also specifically the depth to bottom of a piezometer (2)
	L	6): depth below top of upper confining bed (5)
	Dimonsionloss	Dummy variable (10)
	Dimonsionloss	$T_{1}/S_{r^{2}}$
	Dimensionless	Variable of integration
	Dimonsionloss	Angle between x axis and c axis
	Dimensionless	The network of the number will in a goodinate system calinger with principal area of
η	L	transmissivity toncor
	Dimonsionloss	r/r
	Dimensionless	$\Gamma/\Gamma_{\rm H}$ , $T_{\rm h}/\rho_{\rm m}$ ?
	Dimensionless	

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## TYPE CURVES FOR SELECTED PROBLEMS OF FLOW TO WELLS IN CONFINED AQUIFERS

#### By J. E. Reed

#### Abstract

This report presents type curves and related material for 11 conditions of flow to wells in confined aquifers. These solutions, compiled from hydrologic literature, span an interval of time from Theis (1935) to Papadopulos; Bredehoeft, and Cooper (1973). Solutions are presented for constant discharge, constant drawdown, and variable discharge for pumping wells that fully penetrate leaky and nonleaky aquifers. Solutions for wells that partially penetrate leaky and nonleaky aquifers are included. Also, solutions are included for the effect of finite well radius and the sudden injection of a volume of water for nonleaky aquifers. Each problem includes the partial differential equation, boundary and initial conditions, and solutions. Programs in FORTRAN for calculating additional function values are included for most of the solutions.

## Introduction

The purpose of this report is to assemble, under one cover and in a standard format, the more commonly used type-curve solutions for confined ground-water flow toward a well in an infinite aquifer. Some of these solutions are only published in several different journals; some of these journals are not readily obtainable. Other solutions which are included in several references (for example, Ferris and others, 1962; Walton, 1962; Hantush, 1964a; Lohman, 1972) are included here for completeness.

The need for a compendium of type curves for aquifer-test analysis was recognized by Robert W. Stallman, who initiated the work on it. However, ill health and the press of other duties prevented him from personally carrying out his concept, but he never ceased to advocate the need for the compendium. Although it is reduced in scope from his original concept, this report should be recognized to be a result of Stallman's foresight and endeavors in the field of ground-water hydrology.

The type-curve method was devised by C. V. Theis (Wenzel, 1942, p. 88) to determine the two unknown parameters, S and T, in the equations

$$\mathbf{s} = (Q/4\pi T)W(u)$$

and

$$u = r^2 S / (4Tt),$$

where s is the drawdown in water level in response to the pumping rate Q in an aquifer with transmissivity T and storage coefficient S. The distance r from the pumping well, and the elapsed time t since pumping began, combine with S and T to define a dimensionless variable u and corresponding dimensionless response W(u). Briefly, the method consists of plotting a function curve or type curve, such as (l/u, W(u))on logarithmic-scale graph paper, and plotting the time-drawdown (t-s) data on a second sheet having the same scales. This is equivalent to expressing the preceding equations as

$$\log s = \log Q/4\pi T + \log W(u)$$

and

$$\log 1/u = \log t + \log 4T/r^2S$$

If the two sheets are superimposed and matched, keeping coordinate axes parallel, as shown in figure 0.1, the respective coordinate



FIGURE 0.1.—Relation of 1/u, W(u) type curve and t, s data plot. Modified from Stallman (1971, p. 5, fig. 1).

axes will be related by constant factors:  $s/W(u)=C_1$  and  $t/(1/u)=C_2$ . The values of these two constants are

$$C_1 = Q/(4\pi T)$$

and

$$C_2 = r^2 S / (4T)$$

Thus, a common match point for the two curves may be chosen, and the four coordinate points—W(u), 1/u, s, and t—recorded for the common match point. T can be obtained from the equation  $T = QW(u)/(4\pi s)$ , and then S can be solved from the equation  $S = 4Tut/r^2$ , where W(u), 1/u, s, and t are the match-point values.

It is apparent that the type curves, and data, can be plotted in several ways. That is, the function curve, using W(u) as an example, could be plotted as (u,W(u)) with corresponding data plots of (1/t,s) or  $(r^2/t,s)$ ; or could be plotted as (1/u,W(u)) with corresponding data plots of (t,s) or  $(t/r^2,s)$ . The type-curve method is covered more fully by Ferris, Knowles, Brown, and Stallman (1962, p. 94).

The type curves presented in this report are shown on two different plots. One plot has both logarithmic scales with 1.85 inches per logcycle, such as K and E 467522.<sup>1</sup> The other plot is arithmetic-logarithmic scale with the logarithmic scale 2 inches per log-cycle and the arithmetic scale with divisions at multiples of 0.1, 0.5, and 1.0 inches, such as K and E 466213.

Other methods exist for analysis of aquifertest data. Among them are methods based on plots of data on semi-log paper, developed by

<sup>&#</sup>x27;The use of brand names in this report is for identification purposes only and does not imply endorsement by the U.S. Geological Survey.

Jacob (Ferris and others, 1962, p. 98) and by Hantush (1956, p. 703). These methods are useful, but they are beyond the scope of this report.

Aquifer tests deal with only one component of the natural flow system. The isolation of the effects of one stress upon the system is based upon the technique of superposition. This technique requires that the natural flow system can be approximated as a linear system, one in which total flow is the addition of the individual flow components resulting from distinct stresses.

The use of the principle of superposition is implied in most aquifer-test analyses. The term "superposition," as here applied, is derived from the theory of linear differential equations. If the partial-differential equation is linear (in the dependent variable and its derivatives), two or more solutions, each for a given set of boundary and initial conditions, can be summed algebraically to obtain a solution for the combined conditions. For instance, consider a situation (fig. 0.2) where a well has been pumping for some time at a constant rate  $Q_0$ , and the drawdown trend for that pumping rate has been established. Assume that the pumping rate increases by some amount  $\Delta Q$  at some time  $t_1$ . Then the drawdown for that step incrase in rate will be the change in drawdown from that occurring due to the pumpage  $Q_0$ .

Programs, written in FORTRAN, for calculating additional function values are included for most of the solutions. Some of the type-curve solutions would require an unreasonably long tabulation to include all the possible combinations of parameters. An alternative to a tabulation is the computer program that can calculate type-curve values for the parameters desired by the user. The programs could be easily modified to calculate aquifer response to more than one well, such as well fields or image-well systems (Ferris and others, 1962, p. 144). The programs have been tested and are probably reasonably free from error. However, because of the large number of possible parameter combinations, it was possible to test only a sample of possible parameter values. Therefore, errors might occur in future use of these programs.

"An aquifer test is a controlled field experiment made to determine the hydraulic properties of water-bearing and associated rocks" (Stallman, 1971). The areal variability of hydraulic properties in an aquifer limits aquifer tests to integrating these properties within the



FIGURE 0.2.—The application of the principle of superposition to aquifer tests.

cone of depression produced during the test. Aquifer-test solutions are based on idealized representations of the aquifer, its boundaries, and the nature of the stress on the aquifer. The type-curve solutions presented in this report all have certain assumptions in common. The common assumptions are that the aquifer is horizontal and infinite in areal extent, that water is confined by less permeable beds above and below the aquifer, that the formation parameters are uniform in space and constant in time, that flow is laminar, and that water is released from storage instantaneously with a decline in head. Also implicit is the assumption that hydraulic potential or head is the only cause of flow in the system and that thermal, chemical, density, or other forces are not affecting flow. In addition to these common assumptions are special assumptions that characterize each solution summary. An important first step in aquifer-test analysis is deciding which simplified representations most closely match the usually complex field conditions.

Generally the best start in the analysis of aquifer-test data is with the most general set of type curves that apply to the situation,  $k_{ZD}$ , ing in mind limitations of the method and effects that cause departures from the theoretical results. For example, the most general set of type curves for constant discharge presented in this report is for leaky aquifers with storage of water in the confining beds, solution 5. This includes, as a limiting case, the curve for a nonleaky aquifer. The most severe limitation on this set of curves is that they apply only at early times, as specified in solution 5.

Some of the effects that cause departure from the theoretical curves are partial penetration, finite well radius, and variable discharge for the pumped well. The effects of partial penetration must be considered when r/b < 1.5, and because vertical-horizontal anisotropy is probably a common condition, these effects should be considered for r/b < 10. The effect of finite well radius should be considered for early times, as specified in *solution* 8. The effects of variable discharge depend upon the manner of the variation. A change in discharge is more important if the change is monotonic, either continually increasing or decreasing. This fact is shown by the type curves for *solution* 11, where a monotonic change of 10 percent caused a significant departure from the Theis curve. If the discharge variation consists of random "noise" about a constant discharge, a 10percent variation is not significant. The most general set of type curves for tests on flowing wells is *solution* 7, for leaky aquifers, which includes nonleaky aquifers as a limiting case. The only set of curves for slug tests is given in *solution* 9.

A recurring problem in type-curve solution for unknown hydrologic parameters is that of nonuniqueness. That is, function curves for different parameter values sometimes have similar shapes. An example of this is given by Stallman (1971, p. 19 and fig. 6). He indicated that the selection of the conceptual model is very important in interpreting the test results. Equally important is adequate testing of the conceptual model. Corroboration of the conceptual model is indicated by similar results for hydrologic parameters from data collected at varying distances from the pumped well, depths within the aquifer, and at different observation times. However, proof of suitability of the conceptual model ultimately rests on field investigations and not on curve matching.

As an example of similar curve shapes for different situations, consider the case of constant discharge in a nonleaky aquifer with exponentially varying thickness. The thickness, b, is equal to  $b_0 \exp[-2(X-X_0)/a]$ , where  $b_0$ and  $X_0$  are the thickness and X-coordinate, respectively, at the site of the discharging well and a is a parameter. The drawdown for this situation is given by Hantush (1962, p. 1529):

$$s = (Q/4\pi Kb_0) \exp(r/a\cos\Theta) W(u,r/a),$$

where

$$W(u,\beta) = \int_{u}^{\infty} (\exp(-y - \beta^2/4y)/y) \, dy,$$
$$u = r^2 S / 4Kt$$

Q is the discharge, r is the distance from the discharging well,  $\Theta$  is the angle, with apex at the discharging well, between the observation

well and the positive X-axis, K is the hydraulic conductivity of the aquifer, and  $S_s$  is the specific storage coefficient of the aquifer. This solution is similar to the equation describing drawdown in a leaky artesian aquifer (Hantush, 1956, p. 702), which is

$$s = (Q/4\pi T) W(u,r/B),$$

with T = Kb,  $B = \sqrt{Tb'/K'}$ , and b' and K' are the thickness and hydraulic conductivity, respectively, of the leaky confining bed. The other symbols are used as above.

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These two functions have the same shape when plotted on logarithmic paper, and drawdown resulting from one function could be matched to a type curve of the other function. Suppose, as an example, that the "observed data" are described by the function for the aquifer with exponentially changing thickness. Suppose, also, that the hydrologist is unaware of the variation in thickness and that the family of type curves for leaky aquifers without storage in the confining beds, solution 4, has been chosen for analysis of the "observed data." Matching the data plots to the type curves and solving for unknown parameters by the methods suggested in solution 4 gives for the ratio of  $K_a$ , the apparent hydraulic conductivity, to K, the true hydraulic conductivity,  $K_a/$  $K = \exp((r/a) \cos \Theta)$ . The ratio would be close to one only in the vicinity of the discharging well. The diffusivity,  $K/S_s$ , would be determined correctly, but the apparent specific storage coefficient would have the same percentage error as the apparent hydraulic conductivity. Most important of all, the erroneous conclusion would be that the aquifer is leaky, with leakage parameter  $B = \sqrt{Kbb'/K'} = a$ . This somewhat contrived example illustrates a principle in the interpretation of aquifer-test data. Conclusions about the hydrologic constraints on the response of the aquifer to pumping should not be based on the shape of the data curves. Inferences may be made from these curves, but they must be verified by other hydrologic and geologic data. Therefore, proof of the suitability of the conceptual model must come from field investigations.

Many of the old reports of the U.S. Geological Survey contain references to the terms "coefficient of transmissibility" and "field coefficient of permeability." These terms, which were expressed in inconsistent units of gallons and feet, have been replaced by transmissivity and hydraulic conductivity (Lohman and others, 1972, p. 4 and p. 13). Transmissivity and hydraulic conductivity are not solely properties of the porous medium; they are also determined by the kinematic viscosity of the liquid, which is a function of temperature. Field determinations of transmissivity or hydraulic conductivity are made at prevailing field temperatures, and no corrections for temperature are made.

## Summaries of Type-Curve Solutions for Confined Ground-Water Flow Toward a Well in an Infinite Aquifer

Solution 1: Constant discharge from a fully penetrating well in a nonleaky aquifer (Theis equation)

Assumptions:

- 1. Well discharges at a constant rate, Q.
- 2. Well is of infinitesimal diameter and fully penetrates the aquifer.
- 3. Aquifer is not leaky.
- 4. Discharge from the well is derived exclusively from storage in the aquifer.

Differential equation:

$$\partial^2 s / \partial r^2 + (l/r) (\partial s / \partial r) = (S/T) (\partial s / \partial t)$$

Boundary and initial conditions:

$$s(r,0) = 0, r \ge 0$$
 (1)

$$s(x,t) = 0, t \ge 0 \tag{2}$$

$$Q = \begin{cases} 0, t < 0 \\ (3) \end{cases}$$

$$r \frac{\partial s}{\partial t} = -\frac{Q}{2\pi t^{2}}, \ t \ge 0$$
(4)

 $2\pi T$ 

Equation 1 states that initially drawdown is zero everywhere in the aquifer. Equation 2

states that the drawdown approaches zero as the distance from the well approaches infinity. Equation 3 states that the discharge from the well is constant throughout the pumping period. Equation 4 states that near the pumping well the flow toward the well is equal to its discharge.

Solution (Theis, 1935):

$$s = \frac{Q}{4\pi T} \int_{u}^{\infty} \frac{e^{-u}}{y} dy$$
$$u = \frac{r^{2}S}{4Tt},$$

where

 $\int_{u}^{\infty} \frac{e^{-u}}{y} \, dy = W(u) = -0.577216 - \log_{e} u + u$  $-\frac{u^{2}}{2!2} + \frac{u^{3}}{3!3} - \frac{u^{4}}{4!4} + \dots$ 

Comments:

Assumptions made are applicable to artesian aquifers (fig. 1.1). However, the solution may be applied to unconfined aquifers if drawdown is small compared with the saturated thickness of the aquifer and if water in the sediments through which the water table has fallen is discharged instantaneously with the fall of the water table. According to assumption 2, this solution does not consider the effect of the change in storage within the pumping well. Assumption 2 is acceptable if

$$t > 2.5 \times 10^2 r_c^2 / T$$

(Papadopulos and Cooper, 1967, p. 242), where  $r_c$  is the radius of the well casing in the interval over which the water-level declines, and other symbols are as defined previously. Figure 1.2 on plate 1 is a logarithmic graph of  $W(u)=4\pi sT/Q$  plotted on the vertical coordinates versus  $1/u = 4Tt/(r^2S)$  plotted on the horizontal coordinates. The test data should be plotted with s on the vertical coordinates and corresponding values of t or  $t/r^2$  on the horizontal coordinates.

Values of W(u) for u between 0 and 170 may be computed by using subroutine EXPI of the IBM System/360 Scientific Subroutine Package. Table 1.1 gives values of W(u) for selected values of 1/u between  $1 \times 10^{-1}$  and  $9 \times 10^{14}$ , as calculated by this subroutine.



FIGURE 1.1.—Cross section through a discharging well in a nonleaky aquifer.

1/u	$1/u \times 10^{-1}$	1	10	102	103	10'	105	104
1.0	'0.00000	0.21938	1.82292	4.03793	6.33154	8.63322	10.93572	13.23830
1.2	.00003	.29255	1.98932	4.21859	6.51369	8.81553	11.11804	13.42062
1.5	.00017	.39841	2.19641	4.44007	6.73667	9.03866	11.34118	13.64376
2.0	.00115	.55977	2.46790	4.72610	7.02419	9.32632	11.62886	13.93144
2.5	.00378	.70238	2.68126	4.94824	7.24723	9.54945	11.85201	14.15459
3.0	.00857	.82889	2.85704	5.12990	7.42949	9.73177	12.03433	14.33691
3.5	.01566	.94208	3.00650	5.28357	7.58359	9.88592	12.18847	14.49106
4.0	.02491	1.04428	3.13651	5.41675	7.71708	10.01944	12.32201	14.62459
5.0	.04890	1.22265	3.35471	5.63939	7.94018	10.24258	12.54515	14.84773
6.0	.07833	1.37451	3.53372	5.82138	8.12247	10.42490	12.72747	15.03006
7.0	.11131	1.50661	3.68551	5.97529	8.27659	10.57905	12.88162	15.18421
8.0	.14641	1.62342	3.81727	6.10865	8.41011	10.71258	13.01515	15.31774
9.0	.18266	1.72811	3.93367	6.22629	8.52787	10.83036	13.13294	15.43551
1/4	$1/u \times 10^7$	10*	10*	1010	10"	10'2	1013	10''
1.0	15.54087	17.84344	20.14604	09 44862	24.75121	27.05379	29.35638	31.65897
1.2	15,72320	18.02577	20.32835	22.44002	24,93353	27.23611	29.53870	31.84128
1.5	15.94634	18.24892	20.55150	22.0000	25.15668	27.45926	29.76184	32.06442
2.0	16.23401	18.53659	20.83919	23 14177	25.44435	27.74693	30.04953	32.35211
2.5	16.45715	18.75974	21.06233	23 36491	25.66750	27.97008	30.27267	32.57526
3.0	16.63948	18.94206	21.24464	23 54723	25.84982	28.15240	30.45499	32.75757
3.5	16.79362	19.09621	21.39880	23 70139	26.00397	28.30655	30.60915	32.91173
4.0	16.92715	19.22975	21.53233	23 83492	26.13750	28.44008	30.74268	33.04526
5.0	17.15030	19.45288	21.75548	24.05806	26.360 <u>64</u>	28.66322	30.96582	33.26840
6.0	17.33263	19.63521	21.93779	24.24039	26:54297	28.84555	31.14813	33.45071
7.0	17.48677	19.78937	22.09195	24.39453	26.697	28.99969	31.30229	33.60487
8.0	17.62030	19.92290	22.22548	24.52806	26.83064	29.13324	31.43582	33.73840
9.0	17.73808	20.04068	22.34326	24,64584	26.94843	29.25102	31.55360	33.85619

#### TABLE 1.1.—Values of Theis equation W(u) for values of 1/u

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'Value shown as 0.00000 is nonzero but less than 0.000005.

TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

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## Solution 2: Constant discharge from a partially penetrating well in a nonleaky aquifer

#### Assumptions:

- 1. Well discharges at a constant rate, Q.
- 2. Well is of infinitesimal diameter and is screened in only part of the aquifer.
- 3. Aquifer has radial-vertical anisotropy.
- . 4. Aquifer is not leaky.
  - 5. Discharge from the well is derived exclusively from storage in the aquifer.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + a^2 \frac{\partial^2 s}{\partial z^2} = \frac{S}{T} \frac{\partial s}{\partial t}$$
$$a^2 = K_z / K_r$$

This is the differential equation for nonsteady radial and vertical flow in a homogeneous confined aquifer with radial-vertical anisotropy.

Boundary and initial conditions:

1

$$s(r, z, 0) = 0, r \ge 0, 0 \le z \le b$$
 (1)  
 $s(x, z, t) = 0, t \ge 0$  (2)

$$\frac{\partial s(r,0,t)}{\partial z} = 0, \ r \ge 0, \ t \ge 0$$

$$\frac{\partial s(r,0,t)}{\partial z} = 0, \ r \ge 0, \ t \ge 0$$

$$(3)$$

$$\frac{\partial s(r,b,t)}{\partial z} = 0, \ r \ge 0, \ t \ge 0$$

$$\lim_{r \to 0} r \frac{\partial s}{\partial r} = \int_{0}^{0} \frac{Q}{2\pi K_r (l-d)}, \quad \substack{0 < z < d \\ d < z < l \\ l < z < b}$$
(5)

Equation 1 states that initially the drawdown is zero everywhere in the aquifer. Equation 2 states that the drawdown approaches zero as the distance from the pumped well approaches infinity. Equations 3 and 4 state that there is no vertical flow at the upper and lower boundaries of the aquifer. This means that vertical head gradients in the aquifer are caused by the geometric placement of the pumping well screen, and not by leakage. Equation 5 states that near the pumping well the flow is radial, that the flow toward the well is equal to its discharge, that the discharge is distributed uniformly over the well screen, and that no radial flow occurs above and below the screen. Solution:

I. For the drawdown in a piezometer, a solution by Hantush (1961a, p. 85, and 1964a, p. 353) is given by

$$s = \frac{Q}{4\pi T} \left[ W(u) + f\left( u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{z}{b} \right) \right], \tag{6}$$

where

$$W(u) = \int_{u}^{\infty} \frac{e^{-u}}{y} \, dy$$

and

$$f\left(u,\frac{ar}{b},\frac{l}{b},\frac{d}{b},\frac{z}{b}\right) = \frac{2b}{\pi(l-d)} \sum_{n=1}^{\infty} \frac{1}{n} \cdot \sin\frac{n\pi l}{b} - \sin\frac{n\pi d}{b} \cos\frac{n\pi z}{b} W\left(u,\frac{n\pi ar}{b}\right)$$
(7)

$$W(u, x) = \int_{\mathcal{U}}^{\infty} (\exp(-y - x^2/4y)/y) \, dy$$

$$u = \frac{\sqrt{5}}{4Tt}$$
$$a = \sqrt{K_{\rm s}/K_{\rm r}}$$

 $r^2S$ 

An alternate form of this solution for a=1 is given by Hantush (1961a, p. 85):

$$s = \frac{Qb}{8\pi T(l-d)} \Big[ M\left(u, \frac{l+z}{r}\right) + M\left(u, \frac{l-z}{r}\right) \\ + f'\left(u, \frac{b}{r}, \frac{l}{r}, \frac{z}{r}\right) - M\left(u, \frac{d+z}{r}\right) - M\left(u, \frac{d-z}{r}\right) \\ - f'\left(u, \frac{b}{r}, \frac{d}{r}, \frac{z}{r}\right) \Big],$$
(8)

in which

$$f'\left(u,\frac{b}{r},\frac{x}{r},\frac{z}{r}\right) = \sum_{1}^{\infty} \left[M\left(u,\frac{2nb+x+z}{r}\right) - M\left(u,\frac{2nb-x-z}{r}\right) + M\left(u,\frac{2nb+x-z}{r}\right) - M\left(u,\frac{2nb-x+z}{r}\right)\right]$$
(9)

and

$$M(u,\beta) = \int_{u}^{\infty} \frac{e^{-y}}{y} \operatorname{erf}(\beta \sqrt{y}) \, dy$$
$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^{-x}} \, dy \, .$$

II. For the drawdown in an observation well (Hantush, 1961a, p. 90, and 1964a, p. 353),

$$s = \frac{Q}{4\pi T} \left[ W(u) + \bar{f}\left(u, \frac{ar}{b}, \frac{l}{b}, \frac{d}{b}, \frac{l'}{b}, \frac{d'}{b}\right) \right], \quad (10)$$

where W(u) is as defined previously and

$$\overline{f}\left(u,\frac{ar}{b},\frac{l}{b},\frac{d}{b},\frac{l'}{b},\frac{d'}{b}\right) = \frac{2b^2}{\pi^2(l-d)(l'-d')}$$
$$\cdot \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\sin\frac{n\pi l}{b} - \sin\frac{n\pi d}{b}\right)$$
$$\cdot \left(\sin\frac{n\pi l'}{b} - \sin\frac{n\pi d'}{b}\right) W\left(u,\frac{n\pi ar}{b}\right), \qquad (11)$$

where W(u,x) and u are as defined previously.

Comments:

Assumptions apply to conditions shown in figure 2.1. The effects of partial penetration need to be considered for ar/b < 1.5. There must be a type curve for each value of ar/b, d/b, l/b, and either z/b for piezometer, or l'/b and d'/b for observation wells. Because the number of possible type curves is large, only samples of curves for selected values of the parameters are shown in figure 2.2 on plate 1.

For large values of time, that is, for  $t > b^2 S/(2a^2T)$  or  $t > bS/(2K_z)$ , the effects of partial penetration are constant in time, and

$$W\left(u,\frac{n\pi ar}{b}\right)$$

can be approximated by

$$2K_{0}\left(\frac{n\pi ar}{b}\right)$$

(Hantush, 1961a, p. 92).  $K_0(x)$  is the modified Bessel function of the second kind of order zero. Equation 6 then becomes

$$s = \frac{Q}{4\pi T} W(u) + \delta s = \frac{Q}{4\pi T} \left[ W(u) + f_s \right],$$



FIGURE 2.1.—Cross section through a discharging well that is screened in a part of a nonleaky aquifer.

where 
$$\partial s = \frac{Q}{4\pi T} f_s$$

and  $f_s$  is given in equation 7

with 
$$W\left(u, \frac{n\pi ar}{b}\right)$$
 replaced by  $2K_0\left(\frac{n\pi ar}{b}\right)$ .

Figure 2.3 shows plots of  $f_s$  as tabulated by Weeks (1969, p. 202-207). In using these curves, it should be noted that  $f_s$  for a given r, b, and  $z_1$ ,  $l_1$ ,  $d_1$  is equal to  $f_s$  for the same r, b, and  $z_2=b-z_1$ ,  $l_2=b-d_1$ , and  $d_2=b-l_1$ . Figure 2.3 can be used to find  $f_s$  by interpolation and then constructing type curves of  $W(u) + f_s$  in the manner described by Weeks (1964, p. D195). For small values of time

$$t < \frac{(2b-l-z)^2 S}{20T}$$

(Hantush, 1961b, p. 172), equation 8 can be approximated by

$$s = \frac{Qb}{8\pi T(l-d)} \left[ M\left(u, \frac{l+z}{r}\right) - M\left(u, \frac{d+z}{r}\right) + M\left(u, \frac{l-z}{r}\right) - M\left(u, \frac{d-z}{r}\right) \right].$$



FIGURE 2.3.—The drawdown correction factor  $f_i$  versus ar/b, from tables of Weeks (1969).

An extensive table of  $M(u,\beta)$  has been prepared by Hantush (1961c).

Although r/b for a given observation well probably would be known, however, the conductivity ratio  $a^2$  would not be. Thus, it would not be known which ar/b curve should be matched. In other words, not only T and S, but also the conductivity ratio  $a^2$  must be determined. A criterion for determining the match between data curves and type curves is that the values of ar/b for different observation wells should all indicate the same "a". Plotting the drawdown data for several observation wells on a single  $t/r^2$  plot and matching to sets of type curves, a different set for each "a", is a useful approach.

Figure 2.2 was prepared from data calculated by the FORTRAN program listed in table 2.1. This program computes "s" from either equation 6 or 10, depending on the input data. The input data consist of cards containing the parameters coded in specific formats. Readers unfamiliar with FORTRAN format items should consult a FORTRAN language manual. The first card contains: the aquifer thickness (b), coded in columns 1-5, in format F5.1; the depth to bottom of pumped well screen (l), coded in columns 6-10, in format F5.1; the



FIGURE 2.3.—Continued.

depth to top of pumped well screen (d), coded in columns 11–15, in format F5.1; the number of observation wells and (or) piezometers, coded in columns 16–20, in format I5; the smallest value of 1/u for which computation is desired, coded in columns 21–30, in format E10.4; the largest value of 1/u for which computation is desired, coded in columns 31–40, in format E10.4. The ratio of the largest 1/u value to the smallest 1/u value should be less than  $10^{12}$ . Following this card is a group of cards containing one card for each observation well or piezometer. These cards are coded for an observation well as: distance from pumped well multiplied by the square root of the ratio of the vertical to horizontal conductivity  $(r\sqrt{K_s/K_r})$ , in columns 1-5, in format F5.1; depth to bottom of observation well screen (l'), coded in columns 6-10, in format F5.1; depth to top of observation well screen (d'), coded in columns 11-15, in format F5.1. A card would be coded for a piezometer as follows: distance from pumped well multiplied by the square root of the ratio of the vertical to horizontal conductivity  $(r\sqrt{K_s/K_r})$ , in columns 1-5, in format F5.1; and total depth of piezometer (z), in columns 11-15, in format F5.1. The output from this program is tables of computed function values,



FIGURE 2.3.-Continued.

an example of which is shown in figure 2.4. Subroutines DQL12, BESK, and EXPI are from the IBM Scientific Subroutine Package and a discussion of them is in the IBM SSP manual.

## Solution 3: Constant drawdown in a well in a nonleaky aquifer

Assumptions:

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- 1. Water level in well is changed instantaneously by  $s_w$  at t = 0.
- 2. Well is of finite diameter and fully penetrates the aquifer.

- 3. Aquifer is not leaky.
- 4. Discharge from the well is derived exclusively from storage in the aquifer.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} = \frac{S}{T} \frac{\partial s}{\partial t}$$

This is the differential equation describing nonsteady radial flow in a homogeneous isotropic confined aquifer.

Boundary and initial conditions:

$$s(r,0) = 0, r \ge r_w \tag{1}$$



FIGURE 2.3.—Continued.

$$s(r_w,t) = \begin{cases} 0, t < 0 \\ s_w = \text{constant}, t \ge 0 \end{cases}$$
(2)

 $s(\infty,t) = 0, t \ge 0 \tag{3}$ 

Equation 1 states that initially the drawdown is zero everywhere in the aquifer. Equation 2 states that, as the well is approached, drawdown in the aquifer approaches the constant drawdown in the well, implying no entrance loss to the well. Equation 3 states that the drawdown approaches zero as the distance from the well approaches infinity. Solutions:

I. For the well discharge (Jacob and Lohman, 1952, p. 560):

$$Q = 2\pi T s_w G(\alpha),$$

where

$$G(\alpha) = \frac{4\alpha}{\pi} \int_0^\infty x e^{-\alpha x^2} \left\{ \frac{\pi}{2} + \tan^{-1} \left[ \frac{Y_0(x)}{J_0(x)} \right] \right\} dx$$
  
and 
$$\alpha = \frac{Tt}{Sr_w^2}.$$

II. For the drawdown in water level (Hantush, 1964a, p. 343):



FIGURE 2.3.—Continued.

$$s = s_w A(\tau, \rho),$$

where  $A(\tau,\rho) = 1$ 

R: 1

$$-\frac{2}{\pi} \int_{0}^{\infty} \frac{J_{0}(u) Y_{0}(\rho u) - Y_{0}(u) J_{0}(\rho u)}{J_{0}^{2}(u) + Y_{0}^{2}(u)} \exp(-\tau u^{2}) \frac{du}{u},$$
  
and  
$$\tau = \alpha = \frac{Tt}{Sr_{w}^{2}},$$
  
$$\rho = \frac{r}{r_{w}}.$$

Comments:

Boundary condition 2 requires a constant drawdown in the discharging well, a condition

most commonly fulfilled by a flowing well, although figure 3.1 shows the water level to be below land surface.

Figure 3.2 on plate 1 is a plot from Lohman (1972, p. 24) of dimensionless discharge  $(G(\alpha))$  versus dimensionless time  $(\alpha)$ . Additional values in the range  $\alpha$  greater than  $1 \times 10^{12}$  were calculated from  $G(\alpha) \approx 2/\log(2.2458\alpha)$  (Hantush, 1964a, p. 312). Function values for  $G(\alpha)$  are given in table 3.1. The data curve consists of measured well discharge versus time. After the data and type curves are matched, transmissivity can be calculated from  $T = Q/2\pi s_w G(\alpha)$ , and the storage coefficient can be



FIGURE 2.3.—Continued.

calculated from  $S = Tt/\alpha r_w^2$ , where  $(\alpha, G(\alpha))$  and (t,Q) are matching points on the type curve and data curve, respectively.

Similarly, data curves of drawdown versus time may be matched to figure 3.3 on plate 1; this is a plot of dimensionless drawdown  $(A(\tau,\rho)=s/s_w)$  versus dimensionless time  $(\tau/\rho^2$  $= Tt/Sr^2)$ . After the data and type curves are matched, the hydraulic diffusivity of the aquifer can be calculated from the equality  $T/S = (\tau/\rho^2)(r^2/t)$ . Usually  $s_w$  is known, and some of the uncertainty of curve matching can be eliminated by plotting  $s/s_w$  versus t because only horizontal translation is then required. If  $r_w$  is also known, the particular curve to be matched can be determined from the relation  $\rho = r/r_w$ . Generally, however, the effective radius,  $r_w$ , differs from the actual radius and is not known. The effective radius can often be estimated from a knowledge of the construction of the well and the water-bearing material, or it can be determined from step-drawdown tests (Rorabaugh, 1953). Figure 3.3 was plotted from table 3.2. For  $\tau \leq 1 \times 10^3$ , the data are from Hantush (1964a, p. 310). For  $\tau > 1 \times 10^3$ , values of drawdown in a leaky aquifer, as  $r_w/B \rightarrow 0$ , were used. (See solution 7.) Where 0.000 occurs in table 3.2,  $A(\tau, \rho)$  is less than 0.0005.



FIGURE 2.3.—Continued.





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FIGURE 3.1.—Cross section through a well with constant drawdown in a nonleaky aquifer.

### Solution 4: Constant discharge from a fully penetrating well in a leaky aquifer

Assumptions:

- 1. Well discharges at a constant rate, Q.
- 2. Well is of infinitesimal diameter and fully penetrates the aquifer.
- Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').
- 4. Confining bed is overlain, or underlain, by an infinite constant-head plane source.
- 5. Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
- 6. Flow in the aquifer is two-dimensional and radial in the horizontal plane and flow in the confining bed is vertical. This assumption is approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining bed.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} - \frac{sK'}{Tb'} = \frac{S}{T} \frac{\partial s}{\partial t}$$

This is the differential equation describing nonsteady radial flow in a homogeneous isotropic aquifer with leakage proportional to drawdown.

Boundary and initial conditions:

$$\mathbf{s}(\mathbf{x},t) = \mathbf{0}, \ t \ge \mathbf{0} \tag{2}$$

$$Q = \int_{\text{constant}>0, t \ge 0}^{0, t < 0}$$
(3)

$$\lim_{r \to 0} r \quad \frac{\partial s}{\partial r} = -\frac{Q}{2\pi T} \tag{4}$$

Equation 1 states that the initial drawdown is zero. Equation 2 states that drawdown is small at a large distance from the pumping well. Equation 3 states that the discharge from the well is constant and begins at t=0. Equation 4 states that near the pumping well the flow toward the well is equal to its discharge.

Table	3.1.—	Values	of	$G(\alpha)$
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[Modified from	Lohman	(1972,	p.	24)]	
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α	$\alpha \times 10^{-4}$	10-3	10 2	10. 1	1	10	10 <sup>2</sup>	103	10'	10 <sup>5</sup>
1		18.34	6.13	2.249	0.985	0.534	0.346	0.251	0.1964	0.1608
2		13.11	4.47	1.716	.803	.461	.311	.232	.1841	.1524
3		10.79	3.74	1.477	.719	.427	.294	.222	.1777	.1479
4		9.41	3.30	1.333	.667	.405	.283	.215	.1733	.1449
5		8.47	3.00	1.234	.630	.389	.274	.210	.1701	.1426
6		7.77	2.78	1.160	.602	.377	.268	.206	.1675	.1408
7		7.23	2.60	1.103	.580		.263	.203	.1654	.1393
8		6.79	2.46	1.057	.562	.359	.258	.200	.1636	.1380
9		6.43	2.35	1.018	.547	.352	.254	.198	.1621	.1369
α	$\alpha \times 10^{6}$	107	10*	10*	1010	10'1	1012	1013	10"	10"
1	0 1360	0.1177	0 1037	0.0927	0.0838	0.0764	0.0704	0.0651	0.0605	0 0566
2	1299	.1131	1002	0899	.0814	0744	0686	0636	0593	0555
3	1266	.1106	0982	0883	0801	0733	0677	0628	0586	0549
4	1244	1089	.0968	0872	0792	.0726	0671	.0622	0581	
5	1227	1076	0958	0864	0785	0720	0666	0618	0577	
6	1213	1066	0950	0857	0779	0716	0662	0615	0574	
7	1202	1057	0943	0851	0774	0712	0658	0612	0572	
8	1192	1049	0937	0846	0770	0709	0655	0609	0569	
9		1043	0932	0842	0767	0706	0653	0607	0567	
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#### TABLE 3.2.—Values of $A(\tau,\rho)$ [Values of $A(\tau,\rho)$ for $\tau \le 10^{\circ}$ modified from Hantush (1964a, p. 310)]

							ρ			
1	r		5	10	20	50	100	200	500	1000
1	;	× 1	0.002							
2			.022							
3			.049	0.000						
5			.101	.002						
Ž			.142	.006						
1	>	< 10	.188	.016	0.000					
2			.277	.057	.001					
4			.358	.054	.004					
5			.381	.146	.016					
7			.414	.184	.031					
1	×	× 10 <sup>2</sup>	.446	.222	.053	0.000				
1.5			.479	291	.085	.001				
3			.528	.328	.146	.009				
5			.559	.372	.194	.026	0.000			
7		/ 103	.578	.397	.223	.044	.001			
1.5		× 10°	.090	.422	.204	.066	.012			
2			.627	.467	.309	.116	.021	0.000		
3			.644	.490	.338	.147	.039	.001		
5 7			.662	.517	.372	.186	.068	.006		
í	×	< 10⁴	.685	.535	.413	.237	.114	.025		
$\bar{1}.5$			.696	.566	.435	.264	.142	.043	0.000	
2			.704	.577	.450	.283	.161	.058	.001	
3			.715	.592	.469	.308	.188	.081	.005	0.000
7			.734	.620	.506	.355	.242	.134	.025	.001
i	>	×10 <sup>5</sup>	.742	.631	.520	.373	.263	.156	.039	.002
1.5			.750	.642	.532	.392	.285	.180	.058	.007
2 3			.755	.650	.544	.405	.300	.197	.072	.013
5			.771	.672	.574	.443	.345	.247	.122	.044
ž			.776	.680	.584	.456	.360	.264	.141	.059
1	>	< 10 <sup>6</sup>	.782	.688	.594	.470	.376	.282	.160	.076
1.5			.788	.696	.604	.484	.392	.301	.181	.096
3			.797	.709	.622	.506	.405	.331	.216	.132
5			.803	.718	.633	.521	.436	.352	.240	.157
7			.807	.724	.641	.531	.448	.365	.255	.173
15	7	10'	.011 815	.730	.048	.541	.409 479	.378 309	.270	.190
2			.818	.740	.662	.558	.480	.402	.299	.221
3			.822	.746	.669	.568	.492	.415	.314	.238
5			.827	.753	.678	.580	.506	.431	.333	.258
1	×	1.08	.030	.157 762	.004 690	.087	.014	.441 459	.344	285
1.5			.837	.766	.696	.603	.533	.463	.370	.300
2			.839	.770	.701	.609	.540	.470	.379	.310
3 5			.842	.774	.706	.617 696	.549	.481	.391	.323
7			.849	.783	.714	.632	.567	.494	.400	.340
1	>	< 10 <sup>9</sup>	.851	.787	.723	.638	.574	.510	.425	.361
1.5			.854	.791	.728	.645	.582	.519	.435	.372
2			.856	.794	.731	.649	.587	.525	.443	.380
5			.861	.802	.742	.663	.603	.544	.464	.405
7			.863	.804	.746	.668	.609	.550	.472	.413
1	X	1010	.865	.807	.749	.673	.615	.557	.480	.422
1.5			.867 860	.810 813	.753	.678	.621	.564	.488	.431
วี			.871	.816	.760	.687	.631	.576	.502	.447
5			.874	.819	.765	.693	.638	.584	.512	.457
7	~	1011	.875	.821	.768	.696	.643	.589	.518	.464
15	^	10.1	.877	.824	.770	.700	.647	.594	.524	.471
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#### TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

Solution (Hantush and Jacob, 1955, p. 98):

$$s = \frac{Q}{4\pi T} \int_{u}^{\infty} \frac{e^{-z} \frac{e^{-z}}{4Bz}}{z}$$
(5)

where  $u = r^2 S / 4Tt$ 

$$B = \sqrt{\frac{Tb'}{K'}}.$$
 (6)

Comments:

As pointed out by Hantush and Jacob (1954; p. 917), leakage is three-dimensional, but if the difference in hydraulic conductivities of the aquifer and confining bed are sufficiently great, the flow may be assumed to be vertical in the confining bed and radial in the aquifer. This relationship has been quantified by Hantush (1967, p. 587) in the condition b/B < 0.1. In terms of relative conductivities, this would be  $K/K' > 100 \ b/b'$ . Assumption 5, that there is no change in storage of water in the confining bed, was investigated by Neuman and Witherspoon (1969b, p. 821). They concluded that this assumption would not affect the solution if

$$\beta < 0.01$$
, where  $\beta = \frac{r}{4b} \sqrt{\frac{K'S_s'}{KS_s}}$ .

Assumption 4, that there is no drawdown in water level in the source bed lying above the confining bed, was also examined by Neuman and Witherspoon (1969a, p. 810). They indicated that drawdown in the source bed would have negligible effect on drawdown in the pumped aquifer for short times, that is, when

 $\frac{Tt}{r^2S} < 1.6 \frac{\beta^2}{(r/B)^4}$  Also, they indicated (1969a, p. 811) that neglect of drawdown in the source bed is justified if  $T_s > 100T$ , where  $T_s$  represents the transmissivity of the source bed. Figure 4.1, a cross section through the discharging well, shows geometric relationships. Figure 4.2 on plate 1 shows plots of dimensionless drawdown compared to dimensionless time, using the notation of Cooper (1963) from Lohman (1972, pl. 3). Cooper expressed equations 5 and 6 as

$$L(u,v) = \int_{u}^{\infty} \frac{e^{-\frac{y-\frac{y}{y}}{y}}}{y} dy, \qquad (7)$$



FIGURE 4.1.—Cross section through a discharging well in a leaky aquifer.

with

$$v = \frac{r}{2}\sqrt{\frac{K'}{Tb'}}.$$
 (8)

Cooper's type curves and equation 5 express the same function with r/B=2v. Hantush (1961e) has a tabulation of equation 5, parts of which are included in table 4.1.

The observed data may be plotted in two ways (Cooper, 1963, p. C51). The measured drawdown in any one well is plotted versus  $t/r^2$ ; the data are then matched to the solid-line type curves of figure 4.2. The data points are alined with the solid-line type curves either on one of them or between two of them. The parameters are then computed from the coordinates of the match points  $(t/r^2,s)$  and (1/u, L(u,v)), and an interpolated value of v from the equations

 $T = \frac{Q}{4\pi} \frac{L(u,v)}{s} \,,$ 

$$S = 4T \frac{t/r^{2}}{1/u},$$
 (10)  
$$\frac{K'}{b'} = 4T \frac{v^{2}}{r^{2}}.$$

and

Drawdown measured at the same time but in different observation wells at different distances can be plotted versus  $t/r^2$  and matched to the dashed-line type curves of figure 4.2. The data are matched so as to aline with the dashed-line curves, either on one or between two of them. From the match-point coordinates  $(s,t/r^2)$  and (L(u,v),1/u) and an interpolated value of  $v^2/u$ , T and S are computed from equations 9 and 10 and the remaining parameter from

$$K'/b' = S \frac{v^2/u}{t}$$

The region 
$$v_1^2/u \ge 8$$
 and

e condi-

$$L(u,v) \ge 10^{-2}$$
 corresponds to steady-stattions.

TABLE 4.1.—Selected values of W(u,r/B)

(9)

[From Hantush (1961e)]

u	0.001	0.003	0.01	0.03	0.1	0.3	1	3
$1 \times 10^{-6}$	13.0031	11.8153	9.4425	7.2471	4.8541	2.7449	0.8420	0.0695
2	12.4240	11.6716						
3	12.0581	11.5098	9.4425					
5	11.5795	11.2248	9.4413					
7	11.2570	10.9951	9.4361		•	· · · ·	· · · ·	: · · · · · · · · · · · · · · · · · · ·
$1 \times 10^{-5}$	10.9109	10.7228	9.4176					
2	10.2301	10.1332	9.2961	7.2471				
3	9.8288	9.7635	9.1499	7.2470				` ;
5	9.3213	9.2818	8.8827	7.2450			• • • •	
7	8.9863	8.9580	8.6625	7.2371				
1 × 10-4	8.6308	8.6109	8.3983	7.2122				· · ·
2	7.9390	7.9290	7.8192	7.0685				
3	7.5340	7.5274	7.4534	6.9068	4.8541			and a seten
5	7.0237	7.0197	6.9750	6.6219	4.8530		10 1	
7	6.6876	6.6848	6.6527	6.3923	4,8478			
$1 \times 10^{-3}$	6.3313	6.3293	6.306 <del>9</del>	6.1202	4.8292			
2	5.6393	5.6383	5.6271	5.5314	4.7079	2.7449		
3	5.2348	5.2342	5.2267	5.1627	4.5622	2.7448		
5	4.7260	4.7256	4.7212	4.6829	4.2960	2.7428	,	
7	4.3916	4.3913	4.3882	4.3609	4.0771	2.7350		
$1 \times 10^{-2}$	4.0379	4.0377	4.0356	4.0167	3.8150	2.7104	·	
2	3.3547	3.3546	3.3536	3.3444	3.2442	2.5688		
3	2.9591	2.9590	2.9584	2.9523	2.8873	2.4110	.8420	
5	2.4679	2.4679	2.4675	2.4642	2.4271	2.1371		
7	2.1508	2.1508	2.1506	2.1483	2.1232	1.9206	.8360	
$1 \times 10^{-1}$	1.8229	1.8229	1.8227	1.8213	1.8050	1.6704	.8190	
2	1.2226	1.2226	1.2226	1.2220	1.2155	1.1602	7148	.0695
3	.9057	.9057	.9056	.9053	.9018	.8713	.6010	.0694
5	.5598	.5598	.5598	.5596	.5581	.5453	.4210	.0681
7	.3738	.3738	.3738	.3737	.3729	.3663	2996	.0639
$1 \times 10^{\circ}$	.2194	.2194	.2194	.2193	.2190	.2161	.1855	.0534
2	.0489	.0489	.0489	.0489	.0488	.0485	.0444	.0210
3	.0130	.0130	.0130	.0130	.0130	.0130	.0122	.0071
5	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.0008
7	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001

The drawdown in the steady-state region is given by the equation (Jacob, 1946, eq. 15)

$$s = \frac{Q}{2\pi T} K_0(x),$$

where  $K_0(x)$  is the zero-order modified Bessel function of the second kind and

$$x = r \sqrt{\frac{K'}{Tb'}} .$$

Data for steady-state conditions can be analyzed using figure 4.3 on plate 1. The drawdowns are plotted versus r and matched to figure 4.3. After choosing a convenient match point with coordinates (s,r) and  $(K_0(x),x)$  the parameters are computed from the equations

$$T = \frac{Q}{2\pi s} K_0(x)$$
 and  $\frac{K'}{b'} = \frac{xT}{r^2}$ 

Values of  $K_0(x)$  from Hantush (1956) are given in table 4.2.

A FORTRAN program for generating typecurve function values of equation 7 is listed in table 4.3. Using the notation L(u,v) of Cooper (1963), the function is evaluated as follows. For  $u \ge 1$ ,

$$L(u,v) = \int_{u}^{\infty} (1/y) \exp(-y - v^2/y) \, dy = \int_{u}^{\infty} f(y) \, dy.$$

This integral is transformed into the form

$$\int_0^\infty e^{-x} \left[ \exp\left(-u - \frac{v^2}{x+u}\right) \frac{1}{x+u} \right] dx$$

evaluated by a Gaussian-Laguerre quadrature formula. For  $v^2 < u < 1$ ,

TABLE 4.2.—Selected values of  $K_{a}(x)$ 

[From Hantush	(1956, p	. 704)]
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N	$x = NX10^{-1}$	$x = NX 10^{-1}$	$\frac{x=N}{0.4210}$	
1	4.7212	2.4271		
1.5	4.3159	2.0300	.2138	
2	4.0285	1.7527	.1139	
3		1.3725	.0347	
4		1.1145	.0112	
5		.9244	.0037	
3		.7775		
		.6605		
3		.5653		
		.4867		

$$L(u,v) = \int_{1}^{\infty} f(y) \, dy + \int_{u}^{1} f(y) \, dy.$$

The first integral is evaluated by a Gaussian-Laguerre quadrature formula, as previously described. The second integral is evaluated using a series expansion, as

$$\int_{u}^{1} f(y)dy = s(1)-s(u),$$

where

$$s = \log u \left[ \sum_{n=0}^{\infty} \frac{(v^2)^n}{(n!)^2} \right] + \sum_{m=1}^{\infty} \left[ \frac{(-1)^m}{m} \left[ u^m - \left( \frac{v^2}{u} \right)^m \right] \left[ \sum_{n=0}^{\infty} \frac{(v^2)^n}{(m+n)!n!} \right] \right].$$

For u < 1 and  $u \leq v^2$ ,

$$L(u,v) = 2K_0(2v) - \int_{\frac{U^2}{U}}^{\infty} f(y) \, dy$$

(Cooper, 1963, p. C50),

where  $K_0$  is the zero-order modified Bessel function of the second kind. The integral in the above expression is evaluated by the Gaussian-Laguerre procedure, as described previously.

Input data for this program consist of three cards with the numeric data coded by specific FORTRAN formats. Readers unfamiliar with FORTRAN format items should consult a FORTRAN language manual. The first card contains: the smallest value of 1/u for which computation is desired, coded in columns 1-10in format E10.5; the largest value of 1/u for which computation is desired, coded in columns 11-20 in format E10.5. The table will include a range of 1/u values spanning these two coded values if the span is less than or equal to 12 log cycles. The next two cards contain 12 values of r/B, all coded in format E10.5, in columns 1-10, 11-20, 21-30, 31-40, 41-50, 51-60,61-70, and 71-80 of the first card and columns 1-10, 11-20, 21-30, and 31-40 of the second card. Zero (or blank) coding is permissible in this field, but computation will terminate with the first zero (or blank) value encountered. An example of the output from this program is shown in figure 4.4.

W(U+K/8)

1	R/B								
1/U	0.10E-05	0.30E-05	0.10E-04	0.30E-04	0.10E-03	0.30E-03	0.10E-02	0.30E-02	0.106-01
0.100E 01	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194	0.2194
0.150E 01	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984	0.3984
0.200E 01	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598	0.5598
0.300E 01	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289	0.8289
0.500E 01	1.5556	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1.2226	1+5554
0.700E 01	1.5066	1.5066	1.5066	1.5066	1.5066	1.5066	1.5066	1.5066	1.5065
0.100E 02	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8229	1.8227
0.150E 02	2.1964	2.1964	2.1964	2.1964	2.1964	2.1964	2.1964	2.1964	2.1961
U.200E 02	2.4679	2.4679	2.4679	2.4679	2.4679	2.4679	2.4679	2.4679	2.4675
0.300E 02	2.8570	2.8570	2.8570	2.8570	2.8570	2.8570	2.8570	2.8570	2.8564
0.500E 02	3.3547	3.3547	3.3547	3.3547	3.3547	3.3547	3.3547	3.3546	3.3536
0.700E 02	3.6855	3.6855	3.6855	3.6855	3.6855	3.6855	3.6855	3.6854	3.6839
0.100E 03	4.0379	4.0379	4.0379	4.0379	4.0379	4.0379	4.0379	4.0377	4.0356
0.150E 03	4.4401	4.4401	4.4401	4.4401	4.4401	4.4401	4.4400	4.4397	4.4365
0.200E 03	4.7261	4.7261	4.7261	4.7261	4.7261	4.7261	4.7260	4.7257	4.7212
0.300E 03	5.1299	5.1299	5.1299	5.1299	5.1299	5.1299	5.1298	5.1292	5.1226
0.500E 03	5.6394	5.6394	5.6394	5.6394	5.6394	5.6394	5.6393	5.6383	5.6271
0.700E 03	5.9753	5.9753	5.9753	5.9753	5.9753	5.9753	5.9751	5.9737	5.9580
u.100E 04	6.3315	6.3315	6.3315	6.3315	6.3315	6.3315	6.3313	6.3293	6.3064
0.150E 04	6.7367	6.7367	6.7367	6.7367	6.7367	6.7366	6.7363	6.7333	6.6497
0.200E 04	7.0242	7.0242	7.0242	7.0242	7.0242	7.0241	7.0237	7.0197	6.9750
0.300E 04	7.4295	7.4295	7.4295	7.4295	7.4295	7.4294	7.4287	7.4228	7.3561
0.500E 04	7.9402	7.9402	7.9402	7.9402	7.9402	7.9401	7.9389	7.9290	7.8192
0.700E 04	8.2766	8.2766	8.2766	8.2766	8.2766	8.2764	8.2748	8.2609	H.1092
0.100E 05	8.6332	8.6332	8.6332	8.6332	8.6332	8.6330	8.6307	8.6109	н.3983

FIGURE 4.4.—Example of output from program for computing drawdown due to constant discharge from a well in a leaky artesian aquifer.

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### Solution 5: Constant discharge from a well in a leaky aquifer with storage of water in the confining beds

Assumptions:

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- 1. Well discharges at a constant rate, Q.
- 2. Well is of infinitesimal diameter and fully penetrates the aquifer.
- 3. Aquifer is overlain and underlain everywhere by confining beds having hydraulic conductivities K' and K'', thicknesses b' and b'', and storage coefficients S' and S'', respectively, which are constant in space and time.
- 4. Flow in the aquifer is two dimensional and radial in the horizontal profiand flow in confining beds is vertical. This assumption is approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining beds.
- 5. Conditions at the far surfaces of the confining beds are (fig. 5.1):
  - Case 1. Constant-head plane sources above and below.
  - Case 2. Impermeable beds above and below.
  - Case 3. Constant-head plane source above and impermeable bed below.

#### Differential equations:

For the upper confining bed

$$\frac{\partial^2 s_1}{\partial z^2} = \frac{S'}{K'b'} \frac{\partial s_1}{\partial t}$$
(1)

For the aquifer

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + \frac{K'}{T} \frac{\partial}{\partial z} s_1(r, b', t) - \frac{K''}{T} \frac{\partial}{\partial z} s_2(r, b' + b, t) = \frac{S}{T} \frac{\partial s}{\partial t}$$
(2)

For the lower confining bed

$$\frac{\partial^2 s_2}{\partial z^2} = \frac{S''}{K''b''} \frac{\partial s_2}{\partial t}$$
(3)

Equations 1 and 3 are, respectively, the differential equations for nonsteady vertical flow in the upper and lower semipervious beds. Equation 2 is the differential equation for nonsteady two-dimensional radial flow in an aquifer with leakage at its upper and lower boundaries.

Boundary and initial conditions:

Case 1: For the upper confining bed

$$s_1(r,z,0) = 0$$
 (4)

$$s_1(r,0,t) = 0$$
 (5)

$$s_1(r,b',t) = s(r,t)$$
 (6)

For the aquifer

$$s(r,0) = 0 \tag{7}$$

$$s(\infty,t) = 0 \tag{8}$$

$$\lim_{r \to 0} r \frac{\partial s(r,t)}{\partial r} = -\frac{Q}{2\pi T}$$
(9)

For the lower confining bed

$$s_2(r,z,0)=0$$
 (10)

$$s_2(r,b'+b+b'',t)=0$$
 (11)

$$s_2(r,b'+b,t) = s(r,t)$$
 (12)

Case 2: Same as case 1, with conditions 5 and 11 being replaced, respectively, by

$$\frac{\partial s_1(r,0,t)}{\partial z} = 0 \tag{13}$$

$$\frac{\partial s_2(r,b'+b+b'')}{\partial z} = 0$$
(14)

Case 3: Same as case 1, with condition 11 being replaced by condition 14.

Equations 4, 7, and 10 state that initially the drawdown is zero in the aquifer and within each confining bed. Equation 5 states that a plane of zero drawdown occurs at the top of the upper confining bed. Equations 6 and 12 state that, at the upper and lower boundaries of the aquifer, drawdown in the aquifer is equal to drawdown in the confining beds. Equation 8 states that drawdown is small at a large distance from the pumping well. Equation 9 states that, near the pumping well, the flow is equal to the discharge rate. Equation 11 states that a plane of zero drawdown is at the base of the lower confining bed. Equation 13 states that there is no flow across the top of the upper confining bed. Equation 14 states that no flow occurs across the base of the lower confining bed.

#### Solutions (Hantush, 1960, p. 3716):

I. For small values of time (t less than both b'S'/10K' and b''S''/10K''):

 $u = \frac{r^2 S}{\Delta T t}$ 

 $\beta = \frac{r}{4} \left( \sqrt{\frac{K'S'}{h'TS}} + \sqrt{\frac{K''S''}{h''TS}} \right)$ 

$$s = \frac{Q}{4\pi T} H(u,\beta) , \qquad (15)$$

where

and

$$H(u,\beta) = \int_{u}^{\infty} \frac{e^{-u}}{y} \operatorname{erfc} \frac{\beta \sqrt{u}}{\sqrt{y(y-u)}} dy$$
$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{\chi}^{\infty} e^{-u^{2}} dy .$$

#### II. For large values of time:

A. Case 1, t greater than both 5b'S'/K'and 5b''S''/K''

$$s = \frac{Q}{4\pi T} W(u\delta_1, \alpha) , \qquad (16)$$

(01 01)00

where u is as defined previously

and

$$\sigma_1 = 1 + (S + S)/3S,$$
$$\alpha = r \sqrt{\frac{K'/b'}{T} + \frac{K''/b''}{T}}$$

$$W(u,x) = \int_u^\infty \frac{\exp\left(-y - x^2/4y\right)}{y} \, dy \; .$$

B. Case 2, t greater than both 10b'S'/K' and 10b'S''/K''

$$s = \frac{Q}{4\pi T} W(u\delta_2) , \qquad (17)$$

where

$$W(u) = \int_{u}^{\infty} \frac{e^{-u}}{y} \, dy \; .$$

 $\delta_2 = 1 + (S' + S'')/S$ 

C. Case 3, t greater than both 5b'S'/K' and 10b"S"/K"

$$s = \frac{Q}{4\pi T} W\left(u \,\delta_3, \ r \ \sqrt{\frac{K'/b'}{T}}\right), \qquad (18)$$

where

$$\delta_3 = 1 + (S'' + S'/3)/S$$

and W(u,x) is as defined in case 1.

Comments:

A cross section through the discharging well is shown in figure 5.1. The flow system is actually three-dimensional in such a geometric configuration. However, as stated by Hantush (1960, p. 3713), if the hydraulic conductivity in the aquifer is sufficiently greater than the hydraulic conductivity of the confining beds, flow will be approximately radial in the aquifer and approximately vertical in the confining beds. A complete solution to this flow problem has not been published. Neuman and Witherspoon (1971, p. 250, eq. II-161) developed a complete solution for case 1 but did not tabulate it. Hantush's solutions, which have been tabulated, are solutions that are applicable for small and large values of time but not for intermediate times.

The "early" data (data collected for small values of t) can be analyzed using equation 15. Figure 5.2 on plate 1 shows plots of  $H(u,\beta)$  from Lohman (1972, pl. 4). Hantush (1961d) has an extensive tabulation of  $H(u,\beta)$ , a part of which is given in table 5.1. The corresponding data curves would consist of observed drawdown versus  $t/r^2$ . Superposing the data curves on the type curves and matching the two, with graph axes parallel, so that the data curves lie on or between members of the type-curve family and choosing a convenient match point  $(H(u,\beta), 1/u), T$  and S are computed by

$$T = \frac{Q}{4\pi s} H(u,\beta) ,$$
$$S = 4T \frac{t}{r^2} / \frac{1}{u} .$$

If simplifying conditions are applicable, it is possible to compute the product K'S' from the  $\beta$  value. If K''S''=0,  $K'S'=16\beta^2b'TS/r^2$ , and if K''S''=K'S',



CASE 1



FIGURE 5.1.—Cross sections through discharging wells in leaky aquifers with storage of water in the confining beds, illustrating three different cases of boundary conditions.

$$K'S' = \frac{16\beta^2}{r^2} TS \frac{b'b''}{b'+b''+2\sqrt{b'b''}} .$$

The curves in figure 5.2 are very similar from  $\beta = 0$  to about  $\beta = 0.5$ . Therefore, the  $\beta$  val-

ues in this range are indeterminate. There is also uncertainty in curve matching for all  $\beta$ values because of the fact that it is a family of curves whose shapes change gradually with  $\beta$ . This uncertainty will be increased if the data covers a small range of t values. The problem e. a. S.
#### TABLE 5.1.—Values of $H(u,\beta)$ for selected values of u and $\beta$

[From Hantush (1961d). Numbers in parentheses are powers of 10 by which the other numbers are multiplied; for example 963(-4) = 0.0963]

					β			
u =	0.03	0.1	0.3	1	3	10	30	100
$1 \times 10^{-9}$	12.3088	11.1051	10.0066	8.8030	7.7051	6.5033	5.4101	4.2221
2	11.9622	10.7585	9.6602	8.4566	7.3590	6.1579	5.0666	3.8839
3	11.7593	10.5558	9.4575	8.2540	7.1565	5.9561	4.8661	3.6874
5	11.5038	10.3003	9.2021	7.9987	6.9016	5.7020	4.6142	3.4413
7	11.3354	10.1321	9.0339	7.8306	6.7337	5.5348	4.4487	3.2804
$1 \times 10^{-8}$	11.1569	9.9538	8.8556	7.6525	6.5558	5.3578	4.2737	3.1110
2	10.8100	9.6071	8.5091	7.3063	6.2104	5.0145	3.9352	2.7858
3	10.6070	9.4044	8.3065	7.1039	6.0085	4.8141	3.7383	2.5985
5	10.3511	9.1489	8.0512	6.8490	5.7544	4.5623	3.4919	2.3662
7	10.1825	8.9806	7.8830	6.6811	5.5872	4.3969	3.3307	2.2159
$1 \times 10^{-7}$	10.0037	8.8021	7.7048	6.5032	5.4101	4.2221	3.1609	2.0591
2	9.6560	8.4554	7.3585	6.1578	5.0666	3,8839	2.8348	1.7633
3	9.4524	6.2525	1.1560	5.9559	4.6001	3.0074	2.0409	1.5966
5 7	9.1955	7.9900	6.9009	0.7010 5 5946	4.0141	3.4413	2.4137	1.3944
/	9.0201	7.0203	0.7329	0.0040 5.0575	4.4400	3.2004	2.2627	1.2000
1 × 10 °	0.0403	7 2024	6 2001	0.0070 5.0141	4.2/30	3.1110	2.1051	1.1301
2	0.4900	7.0024	6.2091	0.0141	3.9300	2.1001	1.6074	.0990
ა ჳ	8.0304	6 8497	5 7599	4.0100	3.1304	2.0004	1.0090	.1120
5	7 8584	6 6737	5 5947	4.3017	3 3304	2.0001	1 3061	.0200
1 × 10-5	7 6754	6 4 9 4 4	5 4071	4.0302	3 1606	2.2100	1.5001	.0070
2 2 10	7 3170	6 1453	5.0624	3 8827	2 8344	1 7632	0330	3091
4	7 1051	5 9406	4 8610	3 6858	2.0344	1.5965	8046	2402
5	6 8353	5 6821	4.6075	3 4394	2 4 1 3 1	1 3943	6546	1685
7	6 6553	5 5113	4 4408	3 2781	2 2619	1 2664	5643	1300
1 × 10-4	6 4623	5 3297	4 2643	3 1082	2 1042	1 1359	4763	963(-4)
2	6.0787	4.9747	3.9220	2.7819	1.8062	8992	3287	494(-4)
3	5.8479	4.7655	3.7222	2,5937	1.6380	7721	.2570	315(-4)
5	5.5488	4.4996	3.4711	2.3601	1.4335	.6252	.1818	166(-4)
7	5.3458	4.3228	3.3062	2.2087	1.3039	.5370	.1412	103(-4)
$1 \times 10^{-3}$	5.1247	4.1337	3.1317	2.0506	1.1715	.4513	.1055	390(-5)
2	4.6753	3.7598	2.7938	1.7516	.9305	.3084	551(-4)	169(-5)
3	4.3993	3.5363	2.5969	1.5825	.8006	.2394	355(-4)	713(-6)
5	4.0369	3.2483	2.3499	1.3767	.6498	.1677	190(-4)	205(-6)
7	3.7893	3.0542	2.1877	1.2460	.5589	.1292	120(-4)	821(-7)
$1 \times 10^{-2}$	3.5195	2.8443	2.0164	1.1122	.4702	955(-4)	695(-5)	274(-7)
2	2.9759	2.4227	1.6853	.8677	.3214	487(-4)	205(-5)	226(-8)
3	2.6487	2.1680	1.4932	.7353	.2491	308(-4)	888(-6)	
5	2.2312	1.8401	1.2535	.5812	.1733	160(-4)	261(-6)	
1	1.9558	1.6213	1.0979	.4880	.1325	982(-5)	106(-6)	
$1 \times 10^{-1}$	1.6667	1.3893	.9358	.3970	966(-4)	552(~5)	365(-1)	
2	1.12/8	.9497	.6352	.2452	468(-4)	149(-5)	307(-8)	
3	.0309	.7103	.4740	.1729	281(-4)	392(-0)		
0 7	.0207	.4430	.2900	.1000 646( 4)	130(-4) 714(-5)	131(-0) 524(-7)		
1 1 1	.3400	.2300	1170	265(-4)	714(-3) 997(-5)	151(-7)		
2 1	458(-4)	395(-4)	264(-4)	760(-5)	487(-6)	101( 1)		
3	199(-4)	106(-4)	204(-4) 707(-5)	100(-5)	102(-6)			
5	108(-5)	934(-6)	624(-6)	167(-6)	672(-8)			
7	109(-6)	941(-7)	629(-7)	165(-7)	012( 0)			
i x 10	391(-8)	339(-8)	227(-8)	100( 1)				
$\hat{2}$		500( 0)						
3								
5								
7								

can be avoided, if data from more than one observation well are available, by preparing a composite data plot of s versus  $t/r^2$ . This data plot would be matched by adding the constraint that the r values for the different data curves representing each well fall on proportional  $\beta$ curves. The "late" data (for large values of t) can be analyzed using equations 16, 17, and 18; these equations are forms of summaries 1, W(u), and 4, L(u, v). However, for cases 1 and 3, the late data fall on the flat part of the L(u,v) curves and a time-drawdown plot match would be indeterminate. Thus, only a distance-drawdown match could be used. Drawdown predictions, however, could be made using the L(u, v) curves.

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Assumption 5, that no drawdown occurs in the source beds, has been examined by Neuman and Witherspoon (1969a, p. 810, 811) for the situation in which two aquifers are separated by a less permeable bed. This is equivalent to case 3 with K''=0 and S''=0. They concluded that (1)  $H(u,\beta)$ , in the asymptotic solution for early times, would not be affected appreciably because the properties of the source bed have a negligible effect on the solution for  $Tt/r^2S \leq 1.6 \beta^2/(r/B)^4$ , which is equivalent to  $t \leq S'b'/10K'$ , where  $B = \sqrt{Tb'/K'}$ ; and (2) if  $T_s > 100T$ , where  $T_s$  represents the transmissivity of the source bed, it is probably justified to neglect drawdown in the unpumped aquifer.

Table 5.2 is a listing of a FORTRAN program for computing values of  $H(u,\beta)$  for  $u \ge 10^{-60}$ using a procedure devised and programed by S. S. Papadopulos. Input data for this program consists of three cards. The first card contains the beginning value of 1/u, coded in columns 1-10, in format E10.5, and the ending (largest) value of 1/u, coded in columns 11-20, in format E10.5. The next two cards contain 12 values of  $\beta$ , coded in columns 1-10, 11-20, ..., and 71-80 on the first card and columns 1-10, 11-20, ..., 31-40 on the second card, all in format E10.5. The function is evaluated as follows (S. S. Papadopulos, written commun., 1975):

$$H(u,\beta) = \int_{u}^{\infty} (e^{-u}/y) \ erfc \ (\beta \sqrt{u}/\sqrt{y(y-u)}) \ dy$$
$$= \int_{u}^{\infty} f \ dy ,$$

where f represents the integrand. For  $\beta = 0$ ,  $H(u,\beta) = W(u)$ , where W(u) is the well function of Theis. Because  $erfc(x) \le 1$  for  $x \ge 0$ , it follows that  $H(u,\beta) \le W(u)$ , and for u > 10,  $W(u) \approx 0$  and therefore for u > 10,  $H(u,\beta) \approx 0$ . The tables of  $H(u,\beta)$  indicate that  $H(u,\beta) \approx 0$  for  $\beta > 1$  and  $\beta^2 u > 300$ . For an arbitrarily small value of u, the integral can be considered as the sum of three integrals

$$\int_{u}^{\infty} f \, dy = \int_{u}^{u_{1}} f \, dy + \int_{u_{1}}^{u_{2}} f \, dy + \int_{u_{2}}^{\infty} f \, dy ,$$

where  $u_2 = (u/2)(1 + \sqrt{1 + 10^{20}\beta^2/u}),$ 

and 
$$u_1 = (u/2)(1 + \sqrt{1 + 0.025 \beta^2/u}).$$

The significance of  $u_2$  and  $u_1$  is that

erfc  $(\beta \sqrt{u}/\sqrt{y(y-u)}) \approx 1$  for  $u > u_2$ and

erfc 
$$(\beta \sqrt{u}/\sqrt{y(y-u)}) \approx 0$$
 for  $u < u_1$ .

Therefore,

and

$$\int_{u}^{u_{1}} f \, dy \approx 0 ,$$

$$\int_{u}^{\infty} f \, dy \approx W(u_{2}),$$

where  $W(u_2)$  is the well function of Theis. The function can be evaluated as

$$H(u,\beta) \approx W(u) \text{ for } u > u_2$$

$$H(u,\beta) \approx \int_u^{u_2} f \, dy + W(u_2) \text{ for } u_1 < u < u_2$$
and 
$$H(u,\beta) \approx \int_{u_1}^{u_2} f \, dy + W(u_2) \text{ for } u < u_1.$$

If  $u_2 > 10$ , then

$$\int_{u_1}^{u_2} f \, dy = \int_{u_1}^{10} f \, dy, \, W(u_2) \approx 0$$

An example of output from this program is shown in figure 5.3.

## Solution 6: Constant discharge from a partially penetrating well in a leaky aquifer

Assumptions:

- 1. Well discharges at a constant rate, Q.
- 2. Well is of infinitesimal diameter and is screened in only part of the aquifer.
- 3. Aquifer has radial-vertical anisotropy.

H (U+RETA)

	1	RETA				
17	∕u i	0.30E-01	0.10E 00	0.30F UN	0.105 01	0.30E 01
0.100E	07	1.6667	1.3894	0.9358	0.3970	0.0965
0.150E	02	1.9953	1.6531	1.1203	0.5010	0.1374
0.200E	02	2.2308	1.8401	1.2536	0.5812	0.1733
0.300E	02	2.5626	2.1010	1.4435	0.7023	0.2320
0.500E	02	2.9759	2.4228	1.6853	0.8677	0.3214
0.700E	02	3.2428	2.6296	1.8457	0.9836	0.3897
0.100E	03	3.5196	2.8443	2.0164	1,1122	0.4702
0.150E	03	3.8256	3.0826	2.2112	1.2647	0.5717
0.200E	0.3	4.0369	3.2483	2.3499	1.3767	0.6498
0.300F	03	4.3259	3.4775	2.5459	1.5394	0.7683
0.500F	03	4.6754	3.7598	2.7938	1.7516	0.4305
0.700F	03	4.8969	3,9425	2.9576	1.8953	1.0447
0.100F	04	5.1247	4,1338	3,1317	2.0507	1,1715
0 150E	04	5 3756	4 3486	3 3301	2 2306	1 3225
0.200E	04	5.5488	4.4996	3.4712	2.3602	1.4335
0.300E	<u>64</u>	5,7971	4.7109	3.6704	2.5452	1.5451
0.500E	04	6.0787	4.9747	3,9220	2.7819	1.5062
0.700E	04	6.2565	5,1474	4.0880	2.9396	1.4444
0.100F	05	6.4623	5.3297	4.2643	3.1082	2.1042
0.150E	05	6.6816	5.5361	4.4650	3.3014	2.2837
0.200F	05	6.8353	5.6821	4.6076	3.4394	2.4131
0.300E	05	7.0498	5.8874	4.8087	3.6349	2.5474
0.500F	05	7.3170	6.1454	5.0624	3.6827	2.8344
0.700E	05	7.4915	6.3149	5.2297	4.0467	2.9921
0.100E	06	7.6754	6.4944	5.4072	4.2212	3.1606
0.150E	06	7.8834	6.6983	5.6090	4.4202	3.3535
0.200E	06	8.0304	6.8427	5.7523	4.5617	3.4917
0.300E	06	8.2369	7.0462	5.9544	4.7616	3.6472
0.500E	06	8.4960	7.3024	6.2091	-5.0141	3.9351
0.700E	0.6	8.6662	7.4710	6.3770	5.1807	4.(1991
0.100E	07	8.8463	7.6497	6.5549	5.3576	4.273n
0.150E	07	9.0507	7.8528	6.7573	5.5589	4.4726
0.200E	07	9.1955	7.9968	6.9010	5.7018	4.5141
0.300E	07	9.3995	8.1998	7.1034	5.4035	4.8141
0.500E	07	9.6560	8.4554	7.3586	6.1578	5.0666
0.700E	07	9.8249	8.6237	7.5267	6.3255	5.2332
0.100E	80	10.0038	8.802S	7.7049	h.5033	5.4101
0.150E	80	10.2070	9.0050	7.9075	6.7055	5.6114
0.200E	08	10.3512	9.1489	8.0512	6.8490	5.7544
0.300E	08	10.5543	9.3517	8.2539	7.0513	5.9561
0.500E	0.8	10.8101	9.6072	8.5092	7.3063	5.2104
0.700E	08	10.9785	9.7754	8.6773	7.4744	6.3781
0•100E	09	11.1570	9.9538	8.8556	7.6525	6.5554
0.150E	09	11.3599	10.1566	9.0583	7.8550	6.7581
0.200E	09	11.5039	10.3004	9.2021	7.9988	6.9016
0.300E	09	11.7067	10.5032	9.4048	°.2014	7.1040
0.500E	09	11.9625	10.7586	9.6602	F.4556	7.3590
0.700E	09	12.1305	10.9269	9.8264	8.6248	7.5270
0.100E	10	12.3089	11.1052	10.0067	8.8031	7.7052

FIGURE 5.3.—Example of output from program for computing drawdown due to constant discharge from a well in a leaky aquifer with storage of water in the confining beds.

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- 4. Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').
- 5. Confining bed is overlain, or underlain, by an infinite constant-head plane source.
- 6. Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
- 7. Flow is vertical in the confining bed.
- 8. The leakage from the confining bed is assumed to be generated within the aquifer so that in the aquifer no vertical flow results from leakage alone.

Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + \frac{a^2 \partial^2 s}{\partial z^2} - \frac{sK'}{Tb'}$$
$$= \frac{S}{T} \frac{\partial s}{\partial t}$$

 $a^2 = K_z/K_r$ 

This is the differential equation describing nonsteady radial and vertical flow in a homogeneous aquifer with radial-vertical anisotropy and leakage proportional to drawdown.

Boundary and initial conditions:

$$s(r,z,0) = 0, r \ge 0, 0 \le z \le b$$
(1)  

$$s(x,z,t) = 0, 0 \le z \le b, t \ge 0$$
(2)  

$$\partial s(r,0,t)/\partial z = 0, r \ge 0, t \ge 0$$
(3)  

$$\partial s(r,b,t)/\partial z = 0, r \ge 0, t \ge 0$$
(4)

$$\lim_{r \to 0} r \frac{\partial s}{\partial r} = \begin{cases} 0, & \text{for } 0 < z < d \\ -Q/(2\pi K_r(l-d)), & \text{for } d < z < l \\ 0, & \text{for } l < z < b \end{cases}$$

Equation 1 states that, initially, drawdown is zero. Equation 2 states that drawdown is small at a large distance from the pumping well. Equations 3 and 4 state that there is no vertical flow at the upper and lower boundaries of the aquifer. This means that vertical head gradients in the aquifer are caused by the geometric placement of the pumping well screen and not by leakage. Equation 5 states that near the pumping well the discharge is distributed uniformly over the well screen and that no radial flow occurs above and below the screen.

Solution:

I. For the drawdown in a piezometer, a solution by Hantush (1964a, p. 350) is given by

$$s = Q/4\pi T \{W(u,\beta) + f(u,ar/b,\beta,d/b,l/b,z/b)\},\$$

where

$$W(u,\beta) = \int_{u}^{\infty} \frac{e^{-y - \frac{\beta^{2}}{4y^{2}}}}{y} dy$$
$$u = \frac{r^{2}S}{4Tt}$$
$$\beta = \sqrt{\frac{r^{2}K'}{Tb'}}$$
$$a = \sqrt{K_{z}/K_{r}}$$

$$= 2b/\pi(l-d) \sum_{n=1}^{\infty} l/n(\sin n\pi l/b - \sin n\pi d/b) \\ \cdot \cos(n\pi z/b) W\left(u, \sqrt{\beta^2 + (n\pi ar/b)^2}\right)$$

II. For the drawdown in an observation well

$$s = Q/4\pi T \{ W(u,\beta) + \overline{f}(u,ar/b,\beta,d/b,l/b,d'/b,l'/b) \},$$

where

$$\begin{cases} f(u,ar/b,\beta,d/b,l/b,d'/b,l'/b) \\ = 2b^2/\pi^2(l-d)(l'-d') \\ &\cdot \sum_{n=1}^{\infty} 1/n^2(\sin n\pi l/b - \sin n\pi d/b) \end{cases}$$

$$(\sin n\pi l'/b - \sin n\pi d'/b)W(u,\sqrt{\beta^2 + (n\pi ar/b)^2})$$

#### Comments:

The geometry is shown in figure 6.1. The differential equation and boundary conditions are based on the assumption that vertical flow in the aquifer is caused by partial penetration of the pumping well and not by leakage. Hantush (1967, p. 587) concluded that this assumption is correct if  $b\sqrt{K'/Tb'} < 0.1$ . The solutions are based on a uniform distribution of flow over the screen of the pumped well. Depending on friction losses within the well, a more realistic assumption might be constant drawdown over



FIGURE 6.1.-Cross section through a discharging well that is screened in part of a leaky aquifer.

the screen of the pumped well; this assumption would imply nonuniform distribution of flow. Hantush (1964a, p. 351) postulates that the actual drawdown at the face of the pumping well will have a value between these two extremes. The solutions should be applied with caution at locations very near the pumped well. The effects of partial penetration are insignificant for  $r>1.5 \ b/a$  (Hantush, 1964a, p. 350), and the solution is the same for the solution 4.

Because of the large number of variables involved, presentation of a complete set of type curves is impractical. An example, consisting of curves for selected values of the parameters, is shown in figure 6.2 on plate 1. This figure is based on function values generated by a FOR-TRAN program.

The computer program formulated to compute drawdowns due to pumping a partially penetrating well in a leaky aquifer is listed in table 6.1. Input data to this program consists of cards coded in specific FORTRAN formats. Readers unfamiliar with FORTRAN format items should consult a FORTRAN language manual. The first card contains: aquifer thickness (b), coded in format F5.1 in columns 1-5; depth, below top of aquifer, to bottom of pumping well screen (l), coded in format F5.1 in columns 6-10; depth, below top of aquifer, to top of pumping well screen (d), coded in format F5.1 in columns 11–15; number of observation wells and piezometers, coded in format I5 in columns 16-20; smallest value of 1/u for which computation is desired, coded in format E10.4 in columns 21-30; largest value of 1/u for which computation is desired, coded in format E10.4 in columns 31-40. The next two cards contain 12 values of r/B, all coded in format E10.5, in columns 1-10, 11-20, 21-30, 31-40, 41-50, 51-60, 61-70, and 71-80 of the first card and columns 1-10, 11-20, 21-30, and 31-40 of the second card. Computation will terminate with the first zero (or blank) value coded. Next is a series of cards, one card per observation well or piezometer, containing: radial distance from the pumped well multiplied

by the square root of the ratio of vertical to horizontal conductivity  $(r\sqrt{K_z/K_r})$ , coded in format F5.1 in columns 1-5; depth, below top of aquifer, to bottom of observation well screen (code blank for piezometer), coded in format F5.1, in columns 6-10; depth, below top of aquifer, to top of observation well screen (total depth for a piezometer), coded in format F5.1, | W(U+R/BR)+F(U+R/B+R/BR+L/B+D/B+Z/B)+ Z/B= 0+50+ SORT(KZ/KR)+R/B= 0+10+ L/R= 0+70+ D/B= 0+30

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in columns 11-15. Output from this program is a table of function values. An example of the output is shown in figure 6.3.

Because most aquifers are anisotropic in the r-z plane, it is generally impractical to use this solution to analyze for the parameters. However, it can be used to predict drawdown if the parameters are determined independently.

I	R/BR							
1/0 1	0.10E-05	0.10E-04	0.102-03	0.10E-02	0.10E-01	0.10E 00	0.10E 01	0.10E 05
0.100E 01	0.5478	0.5478	0.5478	0.5478	0.5478	0.5468	0.4631	0.0001
0.150E 01	0.9901	0.9901	0.9901	0.9901	0.9900	0.9878	0.7872	0.0001
0.200E 01	1.3804	1.3804	1.3804	1.3804	1.3803	1.3764	1.0398	0.0001
0.300E 01	2.0043	2.0043	2.0043	2.0043	2.0042	1.9964	1.3767	0.0001
0.500E 01	2.8381	2.8381	2.8381	2.8381	2.8379	2.8221	1.6931	0.0001
0.700E 01	3.3737	3.3737	3.3737	3,3737	3.3735	3.3499	1.8159	0.0001
0.100E 02	3.9049	3.9049	3.9049	3,9049	3.9046	3-8700	1.8826	0.0001
0.150E 02	4.4488	4.4488	4.4488	4.4488	4.4483	4.3975	1.9094	0.0001
0.200E 02	4.7951	4.7951	4.7951	4.7951	4.7944	4.7291	1.9143	0.0001
0.300E 02	5.2379	5,2370	5.2370	5.2379	5,2269	5.1455	1.9155	0.0001
0-500E 02	5.7539	5.7529	5.7539	5 75 30	5 75 25	5 6135	1 0165	0.0001
0.7005 02	6 0864	6 0964	6 0964	5 1 5 5 7	6 0964	E 0001	1.9155	0.0001
0.1005 02	6 4 3 9 0	6 6 200	6 6 300	6 4 2 9 0	6 4 3 4 3	2 1050	1.9100	0.0001
0.1000 03	0+4370	0.4390	6 • 4 3 9 0	0.4307	0.4303	0+1859	1.9155	0+0001
0.1205 03	0.0411	0+0411		D+8411	0.03/2	5+4010	1+9155	0+0001
0.2000 03	7.12/1	7.12/1	7.12/1	1.12/1	7.1220	0.0009	1.9155	0.0001
0.300E 03	7.5309	7.5309	7+5309	7.5309	1.5233	6.8854	1.9155	0.0001
0.500E 03	8.0404	8.0404	8.0404	8.0403	8.0278	7+0788	1.9155	0.0001
0.700E 03	8.3763	8.3763	8.3763	8.3762	8.3588	7.1556	1.9155	0.0001
0+100E 04	8.7326	8.7326	8.7326	8.7323	8.7076	7.2002	1.9155	0.0001
0.150E 04	9.1377	9.1377	9.1377	9.1373	9.1005	7.2199	1.9155	0.0001
0.200E 04	9.4252	9.4252	9.4252	9.4247	9.3758	7.2239	1.9155	0.0001
0.300E 04	9.8305	9.8305	9.8305	9.8298	9.7568	7.2250	1.9155	0.0001
0.500E 04	10.3412	10.3412	10.3412	10.3400	10.2199	7.2251	1.9155	0.0001
0.700E 04	10.6776	10.6776	10.6776	10.6759	10.5099	7.2251	1.9155	0.0001
0.100E 05	11.0343	11.0343	11.0343	11.0318	10.7990	7.2251	1.9155	0.0001
W(U.R/BR)	F(II.R/R.R/	AR.1 /8.0/8		3 . I * / B= 0	51. D1/8=	0.49. 508	T (K7/KR) 48	/R= 0.10.
1/8= 0.1	70. D/H= 0	20			• 51 • 0 • 7 0 =	0 • • 7 • Jun		/
		30						
	R/9R	0 105 04	0 105-03	0 105 03	0 105 01		0 ) 0F 0)	
1/0 1	0.102-05	0.102-04	0.102-03	0.101-02	0.100-01	0.102 00	0.100 01	0.100 07
0.1000 01	0.54//	0.5477	0+5477	0.5477	0.5477	0.5468	0.4031	0.0001
0.1502 01	0.9899	0.9899	0.9899	0.9899	0.9894	0.9875	0.7871	0.0001
0.200E 01	1.3801	1.3801	1.3801	1.3801	1.3801	1.3761	1.0396	0.0001
0.300E 01	2.0038	2.0038	2.0038	2.0038	2.0037	1.9959	1.3764	0.0001
0.500E 01	2.8372	2.8372	2.8372	2.8372	2.8371	2.8213	1.6927	0.0001
0.700E 01	3.3727	3.3727	3.3727	3.3727	3.3725	3.3488	1.8153	0.0001
0.100E 02	3.9037	3.9037	3.9037	3.9037	3.9034	3.8688	1.8821	0.0001
0.150E 02	4.4475	4.4475	4.4475	4.4475	4.4470	4.3962	1.9089	0.0001
0.300E 05	4.7937	4.7937	4.7937	4.7937	4.7930	4.7277	1.9138	0.0001
0.300E 02	5.2365	5.2365	5.2365	5.2365	5.2356	5+1441	1.9150	0.0001
0.500E 02	5.7525	5.7525	5.7525	5.7525	5.7511	5+6122	1.9150	0.0001
0.700E 02	6.0850	6.0850	6.0850	6.0849	6.0830	5.8987	1.9150	0.0001
0.100E 03	6.4376	6.4376	6.4376	6.4375	6.4349	6.1845	1.9150	0.0001
0.150E 03	6.8397	6.8397	6.8397	6.8397	6.8358	6.4802	1.9150	0.0001
0.200E 03	7.1257	7.1257	7.1257	7.1257	7.1206	6.6655	1.9150	0.0001
0.300E 03	7.5295	7.5295	7.5295	7.5295	7.5219	6.8840	1.9150	0.0001
0.500E 03	8.0390	8.0390	8.0390	8.0389	8.0264	7.0775	1.9150	0.0001
0.700E 03	8.3749	8.3749	8.3749	8.3748	8.3574	7.1542	1.9150	0.0001
0.100E 04	8.7312	8.7312	8.7312	8,7309	8.7062	7.1988	1.9150	0.0001
0.150F 04	9.1363	9.1363	9.1363	9,1359	9.0991	7.2185	1.9150	0.0001
0.200F 04	9.4238	9.4238	9.4238	9.4233	9.3743	7.2225	1.9150	0.0001
0.300F 04	9,8291	9.8291	9.8291	9.8284	9 7554	7.2236	1.9150	0.0001
0.500F 04	10.3398	10.3309	10.3398	10.3386	10.2185	7.2237	1.9150	0.0001
0.7005 04	10.6762	10.6762	10.6762	10.4745	10.5085	7.2237	1.0160	0.0001
	10.0702	10.0702	10.0102	11 0304	10 7074	7 2227	10150	0.0001
0.100F 05	11.0424		1 1 4 11 1 2 2 2					

FIGURE 6.3.—Example of output from program for partial penetration in a leaky artesian aquifer.

## Solution 7: Constant drawdown in a well in a leaky aquifer

#### Assumptions:

- 1. Water level in well is changed instantaneously by  $s_x$  at t=0.
- 2. Well is of finite diameter and fully penetrates the aquifer.
- 3. Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').
- 4. Confining bed is overlain, or underlain, by an infinite constant-head plane source.
- 5. Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
- 6. Flow in the aquifer is two dimensional and radial in the horizontal plane and flow in the confining bed is vertical. This assumption is approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining bed.

Differential equation:

$$\partial^2 s / \partial r^2 + (1/r) \partial s / \partial r - s K' / T b' = (S/T) \partial s / \partial t$$

This differential equation describes nonsteady radial flow in a homogeneous isotropic confined aquifer with leakage proportional to drawdown.

Boundary and initial conditions:

$$s(r,0)=0, r \ge 0$$
 (1)

$$s(r_w,t) = s_w, t \ge 0 \tag{2}$$

$$s(\infty,t) = 0, \ t \ge 0 \tag{3}$$

Equation 1 states that, initially, drawdown is zero. Equation 2 states that at the wall or screen of the discharging well, drawdown in the aquifer is equal to the constant drawdown in the well, which assumes that there is no entrance loss to the discharging well. Equation 3 states that the drawdown approaches zero as distance from the discharging well approaches infinity. Solutions (Hantush, 1959): I. For the discharge rate of the well,

$$Q = 2\pi T s_w G(\alpha, r_w/B),$$

where

$$G(\alpha, r_w/B) = (r_w/B)K_1(r_w/B)/K_0(r_w/B) + (4/\pi^2) \exp \left[-\alpha(r_w/B)^2\right] \\ \cdot \int_0^\infty \left\{ u \exp(-\alpha u^2) / \left[J_0^2(u) + Y_0^2(u)\right] \right\} \\ \cdot du / \left[u^2 + (r_w/B)^2\right],$$

$$\alpha = Tt/Sr_w^2,$$

$$B = \sqrt{Tb'/K'}.$$

 $K_0$  and  $K_1$  are zero-order and first-order, respectively, modified Bessel functions of the second kind.  $J_0$  and  $Y_0$  are the zero-order Bessel functions of the first and second kind, respectively.

II. For the drawdown in water level

$$s = s_{w}(K_{0}(r/B)/K_{0}(r_{w}/B) + (2/\pi)\exp(-\alpha r_{w}^{2}/B^{2})\int_{0}^{\infty} \frac{\exp(-\alpha u^{2})}{u^{2} + (r_{w}/B)^{2}} \cdot \frac{J_{0}(ur/r_{w})Y_{0}(u) - Y_{0}(ur/r_{w})J_{0}(u)}{J_{0}^{2}(u) + Y_{0}^{2}(u)} u du$$
(4)

with  $\alpha$ , B,  $K_0$ ,  $J_0$ , and  $Y_0$  as defined previously. Comments:

A cross section through the discharging well is shown in figure 7.1. The boundary conditions most commonly apply to a flowing artesian well, as is shown in this illustration.

Figure 7.2 on plate 1 is a plot of dimensionless discharge  $(G(\alpha, r_w/B))$  versus dimensionless time  $(\alpha)$  from data of Hantush (1959, table 1) and Dudley (1970, table 2). Selected values of  $G(\alpha, r_w/B)$  are given in table 7.1. The corresponding data curve should be a plot of observed discharge versus time. The data curve is matched to figure 7.2 and from match points  $(\alpha, G(\alpha, r_w/B))$  and (t, Q), T and S are computed from the equations TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS



FIGURE 7.1.-Cross section through a well with constant drawdown in a leaky aquifer.

$$T = Q/(2\pi s_w G(\alpha, r_w/B))$$

and

 $S = Tt/(\alpha r_w^2).$ 

Figure 7.3 on plate 1 contains plots of dimensionless drawdown  $(s/s_w)$  versus dimensionless time  $(\alpha r_w^2/r^2)$ . The corresponding data plot would be observed drawdown versus observation time. Matching the data and type curves by superposition and choosing convenient match points  $(s/s_w, \alpha r_w^2/r^2)$  and (s,t), the ratio of transmissivity to storage coefficient can be computed from the relation

$$T/S = (\alpha r_w^2/r^2)(r^2/t).$$

Figure 7.3 was plotted from function values generated by a FORTRAN program. This program is listed in table 7.2. The input data for this program consist of three cards coded in specific formats. Readers unfamiliar with

FORTRAN format items should consult a FORTRAN language manual. The first card contains: the smallest value of alpha for which computation is desired, coded in format E10.5 in columns 1-10; the largest value of alpha for which computation is desired, coded in format E10.5 in columns 11-20. The output table will include a range in alpha spanning these two values up to a limiting range of nine log cycles. The second card contains 13 values of  $r_u/B$ . These coded values are the significant figures only and should be greater or equal to 1 and less than 10. The power of 10 by which each of these coded values is multiplied is calculated by the program. Zero (or blank) coding is permissible, but the first zero (or blank) value will terminate the list. The 13 values, all coded in format F5.0, are coded in columns 1-5, 6-10, 11-15, 16-20, 21-25, 26-30, 31-35, 36-40, 41-45, 46-50, 51-55, 56-60, and 61-65. The third card contains the radius of the control well and distances to the observation wells.

TABLE	7.1.	Values	of $G(\alpha, r)$	r/B)
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[Values for  $r_{s}/B \le 1 \times 10^{-2}$  and  $\alpha \ge 1 \times 10^{2}$  are from Hantush (1959, table 1), others are from Dudley (1970, table 2)]

					r <sub>v</sub> /B				
α	0	6×10 <sup>-3</sup>	1×10-7	2×10-2	6×10-2	1×10-1	2×10-1	6×10-1	1×10°
$1 \times 10^{-1}$ 2 5	2.24 1.71 1.23	2.24 1.71 1.23	2.24 1.71 1.23	2.25 1.71 1.23	2.25 1.72 1.23	2.25 1.72 1.24	2.26 1.73 1.25	2.31 1.81 1.38	2.43 1.96 1.61
$1 \times 10^{\circ}$	.983 .800 628	.983 .800 628	.983 .800 628	.984 .801 629	.986 .804 633	.990 .809 642	1.01 .834 682	1.18 1.07 1.01	1.49 1.44 1.43
$1 \times 10^{1}$ 2	.534 .461 389	.534 .461 389	.534 .461 389	.535 .462 390	.541 .472 407	.554 .491 438	.611 .569 548	2.01	
$1 \times 10^{2}$	.346 .311	.346 .311	.346 .312	.349 .316	.374 .353 241	.417 .408	.545		
$\frac{3}{1} \times 10^{3}$	.274 .251 .232	.252	.276 .255 .239	.264 .266 .255	.339	.400			
$\frac{5}{1} \times 10^{4}$	.210 .196 .185	.215 .204 .197	.222 .216 .213	.249 .248					
$     \begin{array}{c}       3 \\       1 \times 10^{5} \\       2 \\       5     \end{array} $	.161 .152	.192	.212						
$\frac{3}{1} \times 10^{6}$	.143 .136 .130	101	212	242		40.0			1.40
5	.123	. 191	.212	.248	.339	.406	.545	1.01	1.43
					r <sub>s</sub> /B				~
α	0	1×10-5	2×10 <sup>-3</sup>	6×10-3	$\frac{r_{\kappa}/B}{1\times10^{-4}}$	2×10 <sup>-4</sup>	6×10-4	1×10-3	2×10-3
$\frac{\alpha}{1 \times 10^4}$	0	1×10 <sup>-3</sup> 0.196	2×10 <sup>-3</sup>	6×10 <sup>-3</sup>	$r_{\rm r}/B$ 1×10 <sup>-4</sup> 0.196	2×10 <sup>-4</sup> 0.196	6×10-4 0.196	1×10 <sup>-3</sup> 0.196	2×10 <sup>-3</sup> 0.197
$\alpha$ $1 \times 10^4$ $2$ $5$	0 0.196 .185 170	1×10 <sup>-3</sup> 0.196 .185 .170	2×10 <sup>-3</sup> 0.196 .185 170	6×10 <sup>-3</sup> 0.196 .185 170	r <sub>*</sub> /B 1×10 <sup>-4</sup> 0.196 .185 170	2×10 <sup>-4</sup> 0.196 .185 170	6×10 <sup>-1</sup> 0.196 .185 170	1×10 <sup>-3</sup> 0.196 .185 170	2×10 <sup>-3</sup> 0.197 .185 173
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \end{array} $	0 0.196 .185 .170 .161	1×10 <sup>-3</sup> 0.196 .185 .170 .161	2×10 <sup>-3</sup> 0.196 .185 .170 .161	6×10 <sup>-3</sup> 0.196 .185 .170 .161	r <sub>к</sub> /B 1×10 <sup>-4</sup> 0.196 .185 .170 161	2×10 <sup>-4</sup> 0.196 .185 .170 .161	6×10 <sup>-4</sup> 0.196 .185 .170 .162	1×10 <sup>-3</sup> 0.196 .185 .170 .162	2×10 <sup>-3</sup> 0.197 .185 .173 .167
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \end{array} $	0 0.196 .185 .170 .161 .152	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152	r <sub>e</sub> /B 1×10 <sup>-4</sup> 0.196 .185 .170 161 .152	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152	6×10-4 0.196 .185 .170 .162 .153	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163
$\begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \\ 5 \\ 1 \\ 1 \\ 0 \\ 5 \\ 1 \\ 0 \\ 5 \\ 1 \\ 0 \\ 5 \\ 1 \\ 0 \\ 5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	0 0.196 .185 .170 .161 .152 .143	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143	r_/B 1×10 <sup>-4</sup> 0.196 185 .170 161 .152 .143 143	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143	6×10-4 0.196 .185 .170 .162 .153 .144	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161
$\alpha$ $1 \times 10^4$ 2 $5 \times 10^5$ 2 $5 \times 10^6$	0 0.196 .185 .170 .161 .152 .143 .136 .130	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 120	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .120	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130	r <sub>x</sub> /B 1×10 <sup>-4</sup> 0.196 185 .170 161 .152 .143 .136 130	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139	$     \begin{array}{r} 1 \times 10^{-3} \\         0.196 \\         .185 \\         .170 \\         .162 \\         .155 \\         .148 \\         .144 \\         .142     \end{array} $	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 5 \\ 1 \\ 2 \\ 5 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	0 0.196 .185 .170 .161 .152 .143 .136 .130 123	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 123	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 123	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 123	r <sub>x</sub> /B 1×10 <sup>-4</sup> 0.196 .185 .170 161 .152 .143 .136 .130 123	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 124	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	$     \begin{array}{r}       1 \times 10^{-3} \\       0.196 \\       .185 \\       .170 \\       .162 \\       .155 \\       .148 \\       .144 \\       .143 \\       142     \end{array} $	2×10 <sup>-3</sup> 0.197 .185 .173 .163 .161 .159 .159
$ \begin{array}{c} \alpha \\ 1 \times 10^{4} \\ 2 \\ 5 \\ 1 \times 10^{5} \\ 2 \\ 5 \\ 1 \times 10^{6} \\ 2 \\ 5 \\ 1 \times 10^{7} \end{array} $	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118	r <sub>*</sub> /B 1×10 <sup>-4</sup> 0.196 185 170 161 152 143 136 130 123 118	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	$\begin{array}{c} 1\times10^{-3} \\ 0.196 \\ .185 \\ .170 \\ .162 \\ .155 \\ .148 \\ .144 \\ .143 \\ .142 \end{array}$	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^{4} \\ 2 \\ 5 \\ 1 \times 10^{5} \\ 2 \\ 5 \\ 1 \times 10^{6} \\ 2 \\ 5 \\ 1 \times 10^{7} \\ 2 \end{array} $	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114	r_/B 1×10 <sup>-4</sup> 0.196 185 170 161 152 143 136 130 123 118 114	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	$\begin{array}{c} 1\times10^{-3} \\ 0.196 \\ .185 \\ .170 \\ .162 \\ .155 \\ .148 \\ .144 \\ .143 \\ .142 \end{array}$	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^{4} \\ 2 \\ 5 \\ 1 \times 10^{5} \\ 2 \\ 5 \\ 1 \times 10^{6} \\ 2 \\ 5 \\ 1 \times 10^{7} \\ 2 \\ 5 \\ \end{array} $	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108	r_/B 1×10 <sup>-4</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .110	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	$     \begin{array}{r} 1 \times 10^{-3} \\         0.196 \\         .185 \\         .170 \\         .162 \\         .155 \\         .148 \\         .144 \\         .143 \\         .142     \end{array} $	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 5 \\ 1 \times 10^8 \\ \end{array} $	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105	<i>r_∕B</i> 1×10 <sup>-4</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .110 .108	2×10 <sup>-+</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-+</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	2×10 <sup>-3</sup> 0.197 .185 .167 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 5 \\ 1 \times 10^8 \\ 2 \\ 7 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .101	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105 .103	<i>r_∕B</i> 1×10 <sup>-+</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .110 .108 .107	2×10 <sup>-+</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-+</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	$     \begin{array}{r} 1 \times 10^{-3} \\         0.196 \\         .185 \\         .170 \\         162 \\         .155 \\         .148 \\         .144 \\         .143 \\         .142     \end{array} $	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 1 \\ 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0958	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0020	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .101 .0966 .0942	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105 .103 .102	r_/B 1×10 <sup>-+</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .108 .107	2×10-4 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-+</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 5 \\ 1 \times 10^8 \\ 5 \\ 2 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 3 \\ 5 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .104 .100 .0958 .0927 .0899	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .104 .100 .0958 .0930 .0906	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .104 .101 .0966 .0943 .0927	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .130 .123 .118 .114 .108 .105 .103 .102	r <sub>*</sub> /B 1×10 <sup>-+</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .108 .107	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10-4 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 5 \\ 1 \times 10^8 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0927 .0899 0864	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0930 .0906 .0880	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .101 .0966 .0943 .0927 .0916	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105 .103 .102	r <sub>x</sub> /B 1×10 <sup>-4</sup> 0.196 185 170 161 152 143 136 130 123 118 114 108 107	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	2×10 <sup>-3</sup> 0.197 .185 .173 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 5 \\ 1 \times 10^8 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \times 10^{10} \end{array} $	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .104 .104 .100 .0958 .0927 .0899 .0864 .0838	1×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0930 .0906 .0880 .0867	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .101 .0966 .0943 .0927 .0916	6×10-3 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105 .103 .102	r <sub>x</sub> /B 1×10 <sup>-4</sup> 0.196 185 170 161 152 143 136 130 123 118 114 108 107	2×10 <sup>-4</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	$\begin{array}{c} 2\times10^{-3} \\ 0.197 \\ .185 \\ .173 \\ .163 \\ .161 \\ .159 \\ .159 \\ .158 \end{array}$
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 5 \\ 1 \times 10^8 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \times 10^{10} \\ 2 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0927 .0899 .0864 .0838 .0814	$\begin{array}{c} 1\times10^{-3} \\ \hline 0.196 \\ .185 \\ .170 \\ .161 \\ .152 \\ .143 \\ .136 \\ .130 \\ .123 \\ .118 \\ .114 \\ .108 \\ .104 \\ .100 \\ .0958 \\ .0930 \\ .0906 \\ .0880 \\ .0880 \\ .0867 \\ .0862 \end{array}$	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .104 .104 .101 .0966 .0943 .0927 .0916 .0914	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105 .103 .102	r <sub>x</sub> /B 1×10 <sup>-4</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .110 .108 .107	2×10-4 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10 <sup>-4</sup> 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	2×10 <sup>-3</sup> 0.197 .185 .173 .163 .161 .159 .159 .158
$ \begin{array}{c} \alpha \\ 1 \times 10^4 \\ 2 \\ 5 \\ 1 \times 10^5 \\ 2 \\ 5 \\ 1 \times 10^6 \\ 2 \\ 5 \\ 1 \times 10^7 \\ 2 \\ 5 \\ 1 \times 10^8 \\ 2 \\ 5 \\ 1 \times 10^9 \\ 2 \\ 5 \\ 1 \times 10^{10} \\ 2 \\ 5 \\ 1 \\ 2 \\ 5 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0927 .0899 .0864 .0838 .0814 .0785	$\begin{array}{c} \hline 1\times10^{-3} \\ \hline 0.196 \\ .185 \\ .170 \\ .161 \\ .152 \\ .143 \\ .136 \\ .130 \\ .123 \\ .118 \\ .114 \\ .108 \\ .104 \\ .100 \\ .0958 \\ .0930 \\ .0906 \\ .0880 \\ .0867 \\ .0862 \\ .0860 \\ \end{array}$	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .101 .0966 .0943 .0927 .0916 .0914	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105 .103 .102	r_/B 1×10 <sup>-+</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .110 .108 .107	2×10 <sup>-+</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10-1 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .159 .158
$\begin{array}{c} \alpha \\ 1 \times 10^{4} \\ 2 \\ 5 \\ 1 \times 10^{5} \\ 2 \\ 5 \\ 1 \times 10^{5} \\ 2 \\ 5 \\ 1 \times 10^{6} \\ 2 \\ 5 \\ 1 \times 10^{7} \\ 2 \\ 5 \\ 1 \times 10^{8} \\ 2 \\ 5 \\ 1 \times 10^{10} \\ 5 \\ 1 \times 10^{10} \\ 5 \\ 1 \times 10^{11} \\ 2 \\ 5 \end{array}$	0 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .100 .0958 .0927 .0899 .0864 .0838 .0814 .0785 .0764	$\begin{array}{c} 1\times10^{-3} \\ \hline 0.196 \\ .185 \\ .170 \\ .161 \\ .152 \\ .143 \\ .136 \\ .130 \\ .123 \\ .118 \\ .114 \\ .108 \\ .104 \\ .100 \\ .0958 \\ .0930 \\ .0930 \\ .0930 \\ .0930 \\ .0930 \\ .0930 \\ .0930 \\ .0880 \\ .0880 \\ .0860 \\ .0860 \\ .0860 \\ .0860 \end{array}$	2×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .104 .104 .0943 .0927 .0916 .0914	6×10 <sup>-3</sup> 0.196 .185 .170 .161 .152 .143 .136 .130 .123 .118 .114 .108 .105 .103 .102	<i>r</i> _∕B 1×10 <sup>-+</sup> 0.196 .185 .170 161 .152 .143 .136 .130 .123 .118 .114 .110 .108 .107	2×10 <sup>-+</sup> 0.196 .185 .170 .161 .152 .143 .137 .131 .124 .120 .116	6×10-1 0.196 .185 .170 .162 .153 .144 .139 .135 .133	1×10 <sup>-3</sup> 0.196 .185 .170 .162 .155 .148 .144 .143 .142	2×10 <sup>-3</sup> 0.197 .185 .173 .167 .163 .161 .159 .159 .158

The control well radius  $(r_w)$  is coded first, in columns 1-8 in format F8.2. The distances (r)to the observation wells (maximum of nine) are coded next, in monotonic increasing order (smallest r first, largest r last), in columns 9-16, 17-24, 25-32, 33-40, 41-48, 49-56, 57-64, 65-72, and 73-80, all in format F8.2. If two or more observation wells have the same distance, this common distance should be coded only once, the function values will apply to all wells at the same distance from the control well. If the number of observation wells is less than nine, the remaining columns on the card should be left blank.

The integral in equation 4 is approximated by

$$\int_0^\infty f(u,\alpha,r_w/B) \, du \doteq \\ \frac{8000}{\sum\limits_{i=1}^\infty f(-\Delta u/2 + i\Delta u,\alpha,r_w/B) \, \Delta u} \, .$$

This expression is a composite quadrature with equally spaced abscissas. The abscissas are chosen at the midpoints of the intervals instead of the ends because the integrand is singular at u = 0. The value of  $\Delta u$  used is related to  $\alpha$  and is  $\Delta u \leq 10^{-3}/\sqrt{\alpha}$ . The  $r_w/B$  values then selected by the program satisfy  $r_w/B \geq 10 \Delta u$ . These two constraints, though empirical, are related to the behavior of the integrand; the first constraint is related to the term  $e^{-\alpha u^2}$  as u becomes large, and the second to  $u/(u^2 + (r_w/B)^2)$  as u becomes small.

The Bessel functions  $K_0(r/B)$ ,  $K_0(r_w/B)$  are evaluated by the IBM subroutine BESK. A description of this subroutine may be found in the IBM Scientific Subroutine Package.

The Bessel functions of the second kind in the integrand,  $Y_0(u)$  and  $Y_0(ur/r_w)$ , are evaluated using IBM subroutine BESY, which is discussed in IBM SSP manual. The Bessel functions  $J_0(u)$  and  $J_0(ur/r_w)$  are evaluated for arguments less than four by a polynomial approximation consisting of the first 10 terms of the series expansion

$$J_0(x) = \sum_{n=0}^{\infty} (-1)^n (x^2/2)^n / (n!)^2.$$

For arguments greater than or equal to four, the asymptotic expansion is used

$$J_0(x) = P \cos (x - \pi/4) + Q \sin (x - \pi/4).$$

P and Q are calculated by the algorithm used in IBM subroutine BESY.

The output from this program consists of tables of function values, an example of which is shown in figure 7.4.

## Solution 8: Constant discharge from a fully penetrating well of finite diameter in a nonleaky aquifer

Assumptions:

- 1. Well discharges at a constant rate, Q.
- 2. Well is of finite diameter and fully penetrates the aquifer.
- 3. Aquifer is not leaky.
- Discharge from the well is derived from a depletion of storage in the aquifer and inside the well bore.

Differential equation:

$$\partial^2 s / \partial r^2 + (1/r) \partial s / \partial r = (S/T) \partial s / \partial t, r \ge r_w$$

This differential equation describes nonsteady radial flow in a homogeneous isotropic aquifer in the region outside the pumped well.

Boundary and initial conditions:

$$s(r_w, t) = s_w(t), t > 0$$
 (1)

$$s(x,t) = 0, t \ge 0$$

$$s(r, 0) = 0, r \ge r$$
(2)

$$(r, 0) = 0, r \ge r_w$$
(3)  
$$s_w(0) = 0$$
(4)

$$(2\pi r_w T)\partial s(r_w, t)/\partial r - (\pi r_c^2)\partial s_w(t)/\partial t$$
  
= -Q, t>0 (5)

Equation 1 states that the drawdown at the well bore is equal to the drawdown inside the well, assuming that there is no entrance loss at the well face. Equation 2 states that drawdown is small at a large distance from the pumping well. Equations 3 and 4 state that, initially, drawdown in the aquifer and inside the well is zero. Equation 5 states that the discharge of the well is equal to the sum of the flow into the well and the rate of decrease in storage inside the well.

Solution (Papadopulos and Cooper, 1967; Papadopulos, 1967):

$$s = (Q/4\pi T) F(u,\alpha,\rho),$$

where

$$F(u,\alpha,\rho) = (8\alpha/\pi) \int_0^\infty \frac{\left[(1 - \exp(-\beta^2 \rho^2/4u)\right] \left[J_0(\beta\rho)A(\beta) - Y_0(\beta\rho)B(\beta)\right]}{\left[A(\beta)\right]^2 + \left[B(\beta)\right]^{2+}\beta^2} d\beta$$

and

and

$$B(\beta) = \beta J_0(\beta) - 2\alpha J_1(\beta),$$
  

$$A(\beta) = \beta Y_0(\beta) - 2\alpha Y_1(\beta),$$
  

$$u = r^2 S/4Tt,$$
  

$$\alpha = r_w^2 S/r_c^2,$$
  

$$\rho = r/r_w,$$

 $J_0$  and  $Y_0$ ,  $J_1$  and  $Y_1$ , are zero-order and first-order Bessel functions of the first and second kind, respectively.

ø

Z(ALPHA,R/RW,RW/B), R/RW= 100.

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1	R₩/8												
ALPHA I	0.10E-03	0.15E-03	0.20E-03	0.30E-03	0.50E-03	0.708-03	0.10E-02	0.15E-02	0.20E-02	0.30E-02	0.50E-02	0.70E-02	0.10E-01
0.100E 05	0.114	0.114	0.114	0.114	0.113	0.113	0.113	0.112	0.112	0.109	0.102	0.091	J•074
0.150E 05	0.142	0.142	0.142	0.141	0.141	0.141	0.141	0.140	0.138	0.134	0.122	0.107	0.082
0.200E 05	0.161	0.161	0.161	0.161	0.161	0.161	0.160	0.159	0.157	0.151	0.135	0.115	0.086
0.300E 05	0.188	0.188	0.188	0.188	0.188	0.188	0.187	0.184	0.181	0.173	0.150	0.123	6.088
0.500E 05	0.221	0.221	0.221	0.221	0.220	0.220	0.218	0.214	0.209	0.196	0.162	0.128	0.089
0.700E 05	0.242	0.242	0.242	0.241	0.241	0.240	0.237	0.232	0.225	0.208	0.167	0.130	0.089
0.100E 06	0.263	0.262	0.262	0.262	0.261	0.260	0.257	0.250	0.240	0.218	0.169	0.130	0.089
0.150E 06	0.285	0.285	0.285	0.284	0.283	0.281	0.277	0.267	0.254	0.225	0.170	0.130	0.089
0.200E 06	0.300	0.300	0.300	0.299	0.298	0.295	0.289	0.277	0.262	0.228	0.171	0.130	0.089
0.300E 06	0.321	0.321	0.320	0.319	0.317	0.313	0.305	0.289	0.269	0.231	0.171	0.130	0.089
0.500E 06	0.345	0.345	0.344	0.343	0.339	0.333	0.322	0.299	0.275	0.232	0.171	0.130	0.089
0.700E 06	0.360	0.360	0.359	0.357	0.352	0.344	0.330	0.303	0.276	0.232	0.171	0.130	0.089
0.100E 07	0.375	0.375	0.374	0.371	0.364	0.355	0.337	0.305	0.277	0.232	0.171	0.130	0.089
0.150E 07	0.391	0.391	0.389	0.386	0.376	0.364	0.342	0.306	0.277	0.232	0.171	0.130	0.089
0.200E 07	0.402	0.401	0.400	0.396	0.384	0.368	0.344	0.307	0.277	0.232	0.171	0.130	0.089
0.300E 07	0.417	0.416	0.414	0.408	0.392	V.373	0.345	0.307	0.277	0.232	0.171	0.130	0.089
0.500E 07	0.435	0.432	0.429	0.421	0.399	0.376	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.700E U7	0.445	0.442	0.438	0.427	0.401	0.376	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.100E 08	0.456	0.452	0.446	0.43	0.403	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.150E 08	0.467	0.461	0.454	0.437	0.403	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.200E 08	0.474	0.467	0.458	0.439	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.300E 08	0.483	0.473	0.462	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.500E 08	0.492	0.479	0.465	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.700E 08	0.497	0.482	0.466	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089
0.100E 09	0.501	0.483	0.467	0.440	0.404	0.377	0.346	0.307	0.277	0.232	0.171	0.130	0.089

FIGURE 7.4.—Example of output from program for constant drawdown in a well in a leaky artesian aquifer.

The drawdown inside the pumped well is obtained at  $r = r_w$  and can be expressed as (Papadopulos and Cooper, 1967, p. 242):

$$s_w = (Q/4\pi T) F(u_w,\alpha),$$

where

and

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 $u_{\mu}$ 

$$u_w = r_w^2 S/4tT.$$

 $F(\mathbf{u}_w,\alpha) = F(u,\alpha,1),$ 

Comments: A cross section through the discharging well is shown in figure 8.1. The geometry, except for the region of the well bore, is the same as for solution 1 (Theis solution). It is apparent from figure 8.2 and 8.3 (on plate 1) that  $F(u,\alpha,\rho)$  approaches W(u), the Theis solution, as time becomes large. Papadopulos (1967, p. 161) stated that for  $t > 2.5 \times 10^3 r_c/T$ , or  $\alpha \rho^2/u > 10^4$ , the function  $F(u,\alpha,\rho)$  can be closely approximated by  $F(u,\alpha,\rho) = W(u)$ . Papadopulos and Cooper (1967, p. 242) stated that for  $t > 2.5 \times 10^2 r_c^2/T$ , or  $\alpha/u_w > 10^3$ , the function  $F(u_w,\alpha) = W(u_w)$ . An examination of the type curves and function values indicates that  $F(u_w,\alpha) \approx W(u_w)$  (less than 5-percent error) for  $\alpha/u_w > 10^2$ , and hence t should only be greater than 25  $r_c^2/T$  for drawdown in the pumped well.

Figures 8.2 and 8.3 were prepared from function values given in Papadopulos and Cooper (1967) and Papadopulos (1967), which are reproduced in table 8.1. For drawdown observations in the pumped well, the method of analysis is to plot drawdown versus time and



FIGURE 8.1.—Cross section through a discharging well of finite diameter.



FIGURE 8.2.—Five selected type curves of  $F(u_w, \alpha)$ , and the Theis solution, versus  $1/u_w$ .

then superimpose the plot on figure 8.2. After match points of (s,t) and  $(F(u_w,\alpha), 1/u_w)$  are chosen, the transmissivity can be computed from the relation  $T = (Q/4\pi s) F(u_w,\alpha)$ . Then, the storage coefficient can be determined from  $S = (4 Tt/r_w^2)/(1/u_w)$ .

For observations not in the pumped well, two procedures are available for analyzing the data. To analyze the data from a single observation well, a family of type curves of  $F(u,\alpha,\rho)$ versus 1/u for different values of  $\alpha$  can be plotted for the  $\rho$  value appropriate for the observation well, using values in table 8.1. This procedure produces a family of type curves similar to that shown for  $\rho = 1$  in figure 8.2. If  $\rho$  for the observation well is between  $\rho$  values in table 8.1, function values can be interpolated. Using this approach, the data for the observation well are plotted as drawdown versus time and matched to the best-fitting member of the plotted type curves. Transmissivity and storage coefficient can be calculated from  $T = (Q/4\pi s)$  $F(u, \alpha, \rho)$  and  $S = (4Tt/r^2)/(1/u)$ .

Drawdowns at more than one observation point may be combined by preparing a composite plot of the drawdowns at each observation well versus  $t/r^2$ . This composite plot would be analyzed by matching it to a family of type curves of  $F(u,\alpha,\rho)$  versus 1/u for constant  $\alpha$ . An example of such a type-curve family for  $\alpha = 10^{-4}$ is shown in figure 8.3. This method requires multiple sheets of type curves, one sheet for each value of  $\alpha$ . When the data curves are matched to the type-curve family, care should be taken to insure that the data for each well fall on the type curve having the appropriate  $\rho$ value. This will be possible for all the data for only one value of  $\alpha$ . Transmissivity and storage coefficient are calculated from  $T = (Q/4\pi s)$  $F(u,\alpha,\rho)$  and  $S = 4T(t/r^2)/(1/u)$ .

In both of these methods of plotting and comparing data, an alternate computation of storage coefficient is  $S = r_c^2 \alpha / r_w^2$ . However, as pointed out by Papadopulos and Cooper (1967, p. 244), the shapes of type curves differ only slightly when  $\alpha$  changes by an order of magnitude, therefore the determination of S is sensitive to choosing the "correct" curve. Papadopulos and Cooper (1967, p. 244) suggest that if S can be estimated within an order of magnitude, the value of  $\alpha$  to be used for matching the data can be decided.

### **TABLE** 8.1.—Values of the function $F(u,\alpha,\rho)$

[Values for  $\rho = 1$  from Papadopulos and Cooper, 1967. Other values from Papadopulos, 1967]

				ρ				
u	1	2	5	10	20	50	100	200
			1	For $\alpha = 10^{-1}$				
$2 \times 10^{\circ}$	$4.88 \times 10^{-2}$	$1.96 \times 10^{-2}$	$1.75 \times 10^{-2}$	$2.41 \times 10^{-2}$	$3.48 \times 10^{-2}$	$4.24 \times 10^{-2}$	$4.48 \times 10^{-2}$	4.50 × 10 <sup>-1</sup>
1	9.19	7.01	9.55	1.41 × 10⁻¹	1.85 × 10 <sup>-1</sup>	$2.09 \times 10^{-1}$	$2.14 \times 10^{-1}$	2.15 × 10-
י־10 × 10	$1.77 \times 10^{-1}$	1.95 × 10⁻¹	3.21 × 10⁻¹	4.44	5.20	5.49	5.55	5.59
2	4.06	5.78	9.42	$1.13 \times 10^{0}$	$1.19 \times 10^{\circ}$	$1.22 \times 10^{a}$		
1	7.34	$1.11 \times 10^{0}$	$1.60 \times 10^{\circ}$	1.76	1.80			
$5 \times 10^{-2}$	$1.26 \times 10^{\circ}$	1.84	2.33	2.43	2.46			
2	2.30	2.97	3.28	3.34	3.35			
1	3.28	3.81	4.00	4.03				
$5 \times 10^{-3}$	4.26	4.60	4.70	4.72				
2	5.42	5.58	5.63	5.64				
ĩ	6.21	6.30	6.33					
$5 \times 10^{-4}$	6.96	7.01						
2	7 87	7.93						
ĩ	8.57	8 63						
5 × 10-5	9.32	0.00						
2	10.24						-	
-	<b>101</b>							
			· · · · · · · · · · · · · · · · · · ·	For $\alpha = 10^{-2}$	.,			
2 × 10°	$4.99 \times 10^{-3}$	$2.13 \times 10^{-3}$	$2.11 \times 10^{-3}$	$3.52 \times 10^{-3}$	$7.47 \times 10^{-3}$	$2.03 \times 10^{-2}$	$3.44 \times 10^{-2}$	4 35 × 10 <sup>-1</sup>
1	9.91	7 99	$1.32 \times 10^{-2}$	$2.69 \times 10^{-2}$	6 12 × 10 <sup>-2</sup>	1 42 × 10 <sup>-1</sup>	$1.91 \times 10^{-1}$	2 11 × 10-
$5 \times 10^{-1}$	$1.97 \times 10^{-2}$	$2.40 \times 10^{-2}$	5 40	$1.21 \times 10^{-1}$	$2.63 \times 10^{-1}$	4 65	5.31	5.51
2	4 89	8.34	$2.33 \times 10^{-1}$	5.12	9.15	$1.16 \times 10^{\circ}$	1 20 × 10°	$1.22 \times 10^{\circ}$
1	9.67	$1.93 \times 10^{-1}$	5.67	$1.12 \times 10^{9}$	$1.58 \times 10^{9}$	1 78	1.20 / 10	1.22 . 10
$5 \times 10^{-2}$	$1.90 \times 10^{-1}$	4 16	$1.18 \times 10^{10}$	1 95	2.32	2.44	2 46	
2	4 53	$1.03 \times 10^{9}$	2.42	3 11	3 29	3 34	3.35	
1	8.52	1.87	3.48	3.90	4 00	4 03	0.00	
5 x 10-3	$1.54 \times 10^{9}$	3.05	4 43	4 65	471	4 72		
2	3.04	4 78	5.52	5.61	5.63	5.64		
ĩ	4 55	5.00	6.97	6 31	6.33	0.04		
5 x 10-1	6.03	6.81	6.99	7.01	0.00			
9	7.56	7.95	7 09	7.04				
ĩ	8 44	8.50	8.63	1.34				
5 x 10-3	0.93	0.00	0.00					
3 ^ 10 ·	9.20 10.20	9.00 10.93						
1	10.40	10.20						
5 10-6	11.01	10.30						
0 / 10 -	11.02	11.00						
4	12.04							
T	13.24							

TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

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				ρ				
<u>u</u>	<u> </u>	2	5	10	20	50	100	200
		,		For $\alpha = 10^{-3}$				_
2 × 10°	5.00 × 10 <sup>-4</sup>	$2.15 \times 10^{-4}$	$2.15 \times 10^{-4}$	3.70 × 10 <sup>-4</sup>	$8.35 \times 10^{-3}$	$3.05 \times 10^{-3}$	$8.38 \times 10^{-3}$	$1.50 \times 10^{-3}$
1	9.99	8.11	$1.37 \times 10^{-3}$	$2.95 \times 10^{-3}$	$7.58 \times 10^{-3}$	$2.81  imes 10^{-2}$	$7.56 \times 10^{-2}$	$1.47 \times 10^{-1}$
5 × 10 <sup>-1</sup>	$2.00 \times 10^{-3}$	$2.45 \times 10^{-3}$	5.77	$1.42 \times 10^{-2}$	$3.90 \times 10^{-2}$	$1.54 \times -1$	$3.23 \times 10^{-1}$	4.78
2	4.99	8.71	$2.67 \times 10^{-2}$	7.24	$2.03 \times 10^{-1}$	6.59	$1.02 \times 10^{\circ}$	$1.17 \times 10^{\circ}$
	9.97	$2.07 \times 10^{-2}$	7.16	$2.01 \times 10^{-1}$	5.41	$1.38 \times 10^{\circ}$	1.70	1.79
$5 \times 10^{-2}$	$1.99 \times 10^{-2}$	4.66	$1.74 \times 10^{-1}$	4.87	$1.19 \times 10^{\circ}$	2.27	2.40	2.45
2	4.95	$1.29 \times 10^{-1}$	5.05	$1.31 \times 10^{\circ}$	2.52	3.22	3.32	3.35
l	9.83	2.70	$1.04 \times 10^{\circ}$	2.38	3.59	3.96	4.02	
$5 \times 10^{-3}$	$1.95 \times 10^{-1}$	5.47	1.96	3.68	4.50	4.69	4.72	
2	4.73	$1.31 \times 10^{\circ}$	3.81	5.23	5.55	5.63	5.64	
L	9.07	2.39	5.34	6.13	6.28	6.32		
5 × 10-4	$1.69 \times 10^{\circ}$	3.98	6.57	6.92	7.00	7.02		
2	3.52	6.44	7.77	7.90	7.93			
1	5.53	7.95	8.55	8.61	8.63			
5 × 10 <sup>-5</sup>	7.63	9.02	9.28	9.31				
2	9.68	10.12	10.22	10.24				
L	10.68	10.88	10. <b>9</b> 3					
5 × 10-6	11.50	11.59	11.62					
2	12.49	12.53	12.54					
1	13.21	13.23	13.24					
$5 \times 10^{-7}$	13.92	13.93						
2	14.84							
l 	15.54							
				For $\alpha = 10^{-4}$				
2 × 10°	$5.00 \times 10^{-5}$	$2.17 \times 10^{-5}$	$2.18 \times 10^{-5}$	3.73 × 10 <sup>-3</sup>	$8.46 \times 10^{-5}$	3.16 × 10 <sup>-4</sup>	9.56 × 10 <sup>-4</sup>	3.83 × 10 <sup>-:</sup>
1	$1.00 \times 10^{-4}$	8.15	$1.38 \times 10^{-4}$	$2.98 \times 10^{-4}$	$7.77 \times 10^{-4}$	$3.23 \times 10^{-3}$	$1.01 \times 10^{-2}$	$3.42 \times 10^{-3}$
5 × 10⁻¹	2.00	2.47 × 10 <sup>-4</sup>	5.81	1.45 × 10⁻³	$4.10 \times 10^{-3}$	$1.80 \times 10^{-2}$	5.62	י-1.75 × 10
2	5.00	8. <b>76</b>	$2.71  imes 10^{-3}$	7.54	$2.27 \times 10^{-2}$	$1.03 \times 10^{-1}$	$3.04 \times 10^{-1}$	7.10
l	$1.00 \times 10^{-3}$	$2.09 \times 10^{-3}$	7.34	2.16 × 10 <sup>-2</sup>	6.69	2.97	7.92	1.43 × 10º
5 × 10 <sup>-2</sup>	2.00	4.72	$1.82 \times 10^{-2}$	5.55	$1.74 \times 10^{-1}$	7.30	$1.62 \times 10^{\circ}$	2.24
2	5.00	$1.32 \times 10^{-2}$	5.56	$1.74 \times 10^{-1}$	5.36	$1.87 \times 10^{-0}$	2.95	3.28
1	9.98	2.81	1.23 × 10⁻¹	3.86	$1.14 \times 10^{\circ}$	3.08	3.84	4.02
5 × 10 <sup>-3</sup>	$1.99 \times 10^{-2}$	5.88	2.64	8.13	2.17	4.25	4.63	4.71
2	4.97	1.53 × 10 <sup>-1</sup>	6.89	1.97 × 10°	4.14	5.47	5.60	5. <b>63</b>
1	9.90	3.10	1.36 × 10º	3.44	5.61	6.24	6.31	6.33
5 × 10⁻⁴	$1.97 \times 10^{-1}$	6.18	2.53	5.26	6.71	6.98	7.01	
2	4.81	$1.48 \times 10^{\circ}$	4.95	7.33	7.82	7.92	7.94	
1	9.34	2.72	7.03	8.37	8.57	8.62		
5 × 10 <sup>-5</sup>	$1.77 \times 10^{\circ}$	4.65	8.65	9.20	9.29	9.32		
2	3.83	7.87	10.02	10.19	10.23	10.24		
1	6.25	9.92	10.83	10.91	10.93			
5 × 10-6	8. <b>99</b>	11.23	11.57	11.62	11.63			

**TABLE** 8.1.—Values of the function  $F(u,\alpha,\rho)$ —Continued

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$2 \\ 1 \\ 5 \times 10^{-7} \\ 2 \\ 1 \\ 5 \times 10^{-8} \\ 2 \\ 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$	11.74 12.91 13.78 14.79 15.51 16.22 17.14 17.84	12.40 13.17 13.90 14.83 15.53 16.23	12.52 13.23 13.93	12.54 13.24
1	17.84			

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\*

For	α	=	10 <sup>-</sup>	5
roi	a	_	10	

9 × 100	5 00 × 10-6	0.07 × 10-6	9.49 × 10.6	4 10 × 10-6	$0.00 \times 10^{16}$	2 01 × 10 5	0.77 × 10-5	215 × 10-4
2 × 10"	$3.00 \times 10^{-5}$	$2.27 \times 10^{\circ}$	$2.40 \times 10^{-5}$	4.19 × 10 °	$9.00 \times 10^{-5}$	$3.21 \times 10^{-4}$	$9.77 \times 10^{-3}$	$3.13 \times 10^{-3}$
1	1.00 × 10 *	0.00	1.44 × 10 °	3.07 × 10 °	7.89 × 10 °	$3.27 \times 10^{-3}$	$1.04 \times 10^{-6}$	3.44 × 10 "
5 × 10 ·	2.00	2.51 × 10 °	0.94	$1.47 \times 10^{-1}$	$4.14 \times 10^{-3}$	1.84 × 10 "	6.02 6.02	2.00 × 10 -
2	5.00	8.87	$2.74 \times 10^{-1}$	1.61	$2.31 \times 10^{-3}$	$1.08 \times 10^{-5}$	$3.61 \times 10^{-1}$	$1.19 \times 10^{-1}$
1	$1.00 \times 10^{-7}$	$2.11 \times 10^{-4}$	7.42	$2.18 \times 10^{-3}$	6.85	3.30	$1.10 \times 10^{-1}$	3.50
$5 \times 10^{-2}$	2.00	4.77	$1.84 \times 10^{-3}$	5.65	$1.82 \times 10^{-2}$	8.90	2.92	8.57
2	5.00	$1.34  imes 10^{-3}$	5.64	$1.80 \times 10^{-2}$	5.92	$2.89 \times 10^{-1}$	8.91	$2.12 \times 10^{\circ}$
1	$1.00 \times 10^{-3}$	2.84	$1.26  imes 10^{-2}$	4.09	$1.36 \times 10^{-1}$	6.49	$1.80 \times 10^{0}$	3.34
$5 \times 10^{-3}$	2.00	5.96	2.74	9.03	3.01	$1.35 imes10^{\circ}$	3.14	4.40
2	5.00	$1.56  imes 10^{-2}$	7.43	$2.47 \times 10^{-1}$	8.06	3.03	5.01	5.52
1	9.99	3.20	$1.55 \times 10^{-1}$	5.15	$1.60  imes 10^{\circ}$	4.75	6.06	6.27
$5 \times 10^{-4}$	$2.00  imes 10^{-2}$	6.54	3.20	$1.04 \times 10^{\circ}$	2.96	6.31	6.90	6.99
2	4.98	$1.66 \times 10^{-1}$	8.08	2.45	5.58	7.71	7.89	7.93
1	9.93	3.34	$1.58 \times 10^{\circ}$	4.28	7.54	8.52	8.61	8.63
$5 \times 10^{-5}$	$1.98 \times 10^{-1}$	6.62	2.93	6.63	8.90	9.21	9.31	
2	4.86	$1.59 \times 10^{\circ}$	5.86	9.36	10.10	10.22	10.24	
1	9.49	2.95	8.53	10.60	10.86	10.92		
$5 \times 10^{-6}$	$1.82 \times 10^{\circ}$	5.15	10.67	11.48	11.59	11.62		
2	4.03	9.08	12.28	12 49	12.53	12.54		
ī	6.78	11 76	13 12	13 21	13 23	13.24		
$5 \times 10^{-7}$	10.13	13 41	13.88	13.92	13.93			
2	13.71	14.68	14.83	14.85	2010-			
ī	15 13	15 46	15 54					
$5 \times 10^{-8}$	16.05	16.20	10101					
2	17.08	17 14						
1	17.81	17.84						
$\hat{5} \times 10^{-9}$	18 51							
9	19 40							
1	20 15							
±						·		

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The early parts (short time) of the curves in figure 8.2 are straight lines. According to Papadopulos and Cooper (1967, p. 244), these represent conditions under which all the water pumped is derived from storage within the well. The straight lines approached by the curves satisfy the equations

$$F(u_w,\alpha) = \alpha/u_w$$

and

$$s_w = Qt/\pi r_c^2 = \frac{\text{volume of water discharged}}{\text{area of well}}$$

Therefore, as pointed out by Papadopulos and Cooper (1067, p. 244), data that fall on this straight part of the type curves do not indicate information about the aquifer characteristics.

Table 8.2 is a listing of two FORTRAN programs by S. S. Papadopulos that evaluate

F(UW+ALPHA) FOR ALPHA= 1+00000	E-04
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 $F(u_w,\alpha)$  and  $F(u,\alpha,\rho)$ . The input data to both programs consists of cards coded in specified format (readers unfamiliar with FORTRAN language format should refer to a FORTRAN language manual). Input to the programs is one or more groups of data, each group of data consisting of two cards. The first card contains one value of alpha in columns 1-10, coded in format E10.5. The program to evaluate  $F(u,\alpha,\rho)$  also requires a value of rho on this card in columns 11-20. This value of rho, which must be greater than one, is also coded in format E10.5. The second card contains 16 values of u coded in columns 1-5, 6-10, ..., 75-80 in format 16F5.0. The  $F(u_w,\alpha)$  or  $F(u,\alpha,\rho)$  values will be printed in the order that the u values are coded. If less than 16 values of u are desired, the remaining columns on the card may be left blank. Outputs from these two programs are shown in figures 8.4 and 8.5.

UW	INTEGRAL	INTEGRAL ERROR	F(UW+ALPHA)	X (PEAK)	Y (PEAK)
2.00000E 00	1.54210E 03	-6.98844E-02	4.99991E-05	5.96561E-03	5.55886E 05
1.00000E 00	3.08412F 03	-1.39817E-01	9,999566-05	5.96561E-03	1-11177E 06
5.00000E=01	6.16789E 03	-2.74775E-01	1.99980E-04	5.96561E-03	2.22353E 06
2.00000E-01	1.54184E 04	-6.97533E-01	4.99907E-04	5.965612-03	5.55875E 06
1.00000E-01	3.08331E 04	-1.39715E 00	9.99695E-04	5.96560E-03	1.11173E 07
5-00000E-02	6-16529E 04	-2.71364E 00	1.99896E-03	5.96559E-03	2.223358 07
2.00000E-02	1.54061E 05	-6.97112E 00	4.99507E-03	5.96559E-03	5.55764E 07
1.00000E-02	3.07919E 05	-1.39383E 01	9.98359E-03	5.96554E-03	1.11128E 08
5-00000E-03	6.15138E 05	-2.78767E 01	1.99445E-02	5.96549E-03	2.22157E 08
2.00000E-03	1.53334E 06	-6.82757E 01	4.97152E-02	5.96527E-03	5.54652E 08
1.00000E-03	3.05367E 06	-1.38658E 02	9.90083E-02	5.96493E-03	1.10684E 09
5-00000E-04	6.06085E 06	-2.76458E 02	1.96509E-01	5.96425E-03	2.20389E 09
2.00000E-04	1.48475F 07	-6.79220F 02	4.81397E-01	5.96223F-03	5.43712F 09
1.00000F-04	2.88072F 07	-1.30780E 03	9.34008E-01	5.95886E-03	1.06380E 10
5.00000E-05	5.45352E 07	-2.50960E 03	1.76818E 00	5.95237E-03	2.03734E 10
2.00000E-05	1.18065E 08	-5.40026E 03	3.82800E 00	5.93415E-03	4.49196E 10

FIGURE 8.4.—Example of output from program for drawdown inside a well of finite diameter due to constant discharge.

F(U,ALPHA,RHO) FOR ALPHA= 1.00000E-05, RHO= 2.00000E 00

U	INTEGRAL	INTEGRAL ERROR	F(U+ALPHA+RHO)
9.99999900E-04	6.29273600E 02	5.45096700E-01	3.20486300E-02
5.0000000E-04	1.28359500E 03	1.11649700E 00	6-53728800F-02
1•99999900E-04	3.26376700E 03	2.47402200E 00	1.66222200E-01
1.0000000E-04	6.55423000E 03	3.31468400E 00	3-33803700F-01
5.0000000E-05	1.30015800E 04	3.53750700E 00	6-62164900F-01
2.0000000E-05	3.11692500E 04	3.54940500E 00	1.58743500F 00
9.99999900E-06	5.79505700E 04	3.54602200E 00	2,95139600F 00
4.99999900E-06	1.01023500E 05	3.53222000E 00	5-14508300E 00
1.99999900E-06	1.78237100E 05	3.62180400E 00	9.07753300E 00
1.00000000E-06	2.30897600E 05	3.66347000E 00	1.17595100F 01
4.99999900E-07	2.63222100E 05	3.68847000E 00	1-340578006 01
1.99999900E-07	2.88201800E 05	3.52180300F 00	1.46779900F 01

FIGURE 8.5.-Example of output from program for drawdown outside a well of finite diameter due to constant discharge.

Solution 9: Slug test for a finite-diameter well in a nonleaky aquifer

Assumptions:

- 1. A volume of water, V, is injected into, or is discharged from, the well instantaneously at t=0.
- 2. Well is of finite diameter and fully penetrates the aquifer.
- 3. Aquifer is not leaky, and flow is in radial direction only.

Differential equation:

 $\frac{\partial^2 h}{\partial r^2} + (1/r) \frac{\partial h}{\partial r} = (S/T) \frac{\partial h}{\partial t}, r > r_w$ 

This differential equation describes nonsteady radial flow in a homogeneous isotropic aquifer beyond the radius of the injected well.

Boundary and initial conditions:

$$\begin{aligned} h(r_w,t) = H(t), t > 0 & (1) \\ h(\infty,t) = 0, t > 0 & (2) \end{aligned}$$

$$2\pi r_w T \,\frac{\partial h(r_w,t)}{\partial r} = \pi r_c^2 \,\frac{\partial H(t)}{\partial t} \,, \, t > 0 \tag{3}$$

 $h(r,0) = 0, r > r_w$  (4)

$$H(0) = H_0 = V/\pi r_c^2$$
 (5)

Equation 1 states that the head change in the aquifer at the face of the well is equal to that inside the well; one assumes that there is no exit loss at the well face. Equation 2 states that the head change approaches zero as distance from the discharging well approaches infinity, a condition which will be approximated if boundaries of the aquifer are sufficiently distant from the discharging well. Equation 3 states that near the well the radial flow is equal to the rate of change in volume of water inside the well. Equations 4 and 5 state that initially the head change is zero in the aquifer, and the head increase or decrease inside the well is equal to  $H_{0}$ .

Solution (Cooper and others, 1967):

$$h = (2H_0/\pi) \int_0^\infty \left( \exp(-\beta u^2/\alpha) \left\{ J_0(ur/r_w) \right. \\ \left. \left. \left[ uY_0(u) - 2\alpha Y_1(u) \right] - Y_0(ur/r_w) \right. \\ \left. \left. \left[ uJ_0(u) - 2\alpha J_1(u) \right] \right\} \right] \left. \left( \Delta(u) \right) du,$$
(6)

and

$$\beta = Tt/r_c^2,$$
  

$$\Delta(u) = \left[ uJ_0(u) - 2\alpha J_1(u) \right]^2 + \left[ uY_0(u) - 2\alpha Y_1(u) \right]^2$$

 $\alpha = r^2 S/r^2$ 

 $J_0$  and  $Y_0$ ,  $J_1$  and  $Y_1$ , are zero-order and firstorder Bessel functions of the first and second kind, respectively.

The head, H, inside the well, obtained by substituting  $r = r_w$  in equation (6) is

$$H/H_0 = F(\beta,\alpha),$$

where

$$F(\beta,\alpha) = (8\alpha/\pi^2) \int_0^\infty (\exp(-\beta u^2/\alpha)/u \,\Delta(u)) \, du$$

and where  $\alpha$ ,  $\beta$ ,  $\Delta(u)$  are as defined previously. Comments: Figure 9.1 is a cross section showing geometric configuration along the well bore. The volume of water injected into or discharged from the well is  $\pi r_c^2 H_0$ . The waterlevel data in the injected well, expressed as a fraction of  $H_0$ , is plotted versus time on semilogarithmic graph paper. This plot is superimposed on figure 9.2, keeping the baselines the same and sliding horizontally until a match or interpolated fit is made. A match point for  $\beta$ , t, and  $\alpha$  is picked from the two graphs. Transmissivity is calculated from  $T = \beta r_c^2 / t$  and storage coefficient from  $S = \alpha r_c^2 / r_w^2$ . As pointed out by Cooper, Bredehoeft, and Papadopulos (1967, p. 267), the determination of S by this method has questionable reliability because of the similar shape of the curves, whereas the determination of T is not as sensitive to choosing the correct curve. Figure 9.2 on plate 1 is plotted from data in table 9.1, which contains original material from two sources (Cooper and others, 1967; and Papadopulos and others, 1973).

Table 9.2 is a listing of a FORTRAN program by S. S. Papadopulos that evaluates  $F(\beta,\alpha)$ . Input to the program consists of cards coded in a specific format (readers unfamiliar with FORTRAN formats should refer to a FOR-TRAN language manual). Input consists of two or more cards, each containing a single value of



FIGURE 9.1.—Cross section through a well in which a slug of water is suddenly injected.

 $\alpha$  coded in format F16.5. The first  $\alpha \leq 0$  will signal program termination. Output from the program is shown in figure 9.3.

## Solution 10: Constant discharge from a fully penetrating well in an aquifer that is anisotropic in the horizontal plane

### Assumptions:

- 1. Well discharges at a constant rate, Q.
- 2. Well is of infinitesimal diameter and fully penetrates the aquifer.
- 3. Aquifer is anisotropic in the horizontal plane.
- 4. Aquifer is not leaky.
- 5. The transmissivity of the aquifer, T, is a two-dimensional symmetric tensor.

Differential equation:

$$T_{xx} \frac{\partial^2 s}{\partial x^2} + 2T_{xy} \frac{\partial^2 s}{\partial x \partial y} + T_{yy} \frac{\partial^2 s}{\partial y^2} + Q \delta(x) \delta(y) = S \frac{\partial s}{\partial t}.$$

This differential equation describes nonsteady flow in a homogeneous anisotropic aquifer with a constantly discharging well at x=y=0. The Dirac delta function is represented as  $\delta(z)$  and has the following properties:  $\delta(z)=0$ if  $z\neq 0$  and  $\int_{-x}^{x} \delta(z)dz = 1$ .

Boundary and initial conditions:

$$s(x, y, 0) = 0$$
 (1)

- $s(\pm\infty, y, t) = 0 \tag{2}$
- $s(x, \pm \infty, t) = 0 \tag{3}$

### TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

.....

From Cooper, Bredehoeft, and Papadopulos, 1967										
Tt/r, <sup>2</sup>	α	10-1	10-2	10-3	10 *	10-5				
	1.00	0.9771	0.9920	0.9969	0.9985	0.9992				
10-3	2 15	9658	9876	9949	9974	9985				
	4 64	9490	9807	9914	9954	9970				
	1.00	9238	9693	9853	9915	9942				
10-2	2 15	8860	9505	9744	9841	9883				
10	4 6A	8293	0187	9545	9701	9781				
	1.01	7460	9655	0193	9434	0579				
10-1	1.00	.1400	.0000	.5100	2025	0167				
10	2.10	.0203	.1102	.0000	.0900	.9107				
	4.04	.4702	.0430	.7430	.0031	.0410				
1.00	1.00	.3117	.4098	.0729	.0520	.7000				
10.	2.15	.1005	.2597	.3043	.43.64	.5038				
	4.04	.07415	.1086	.1554	.2082	.2620				
	7.00	.04625	.06204	.08519	.1161	.1521				
	1.00	.03065	.037,80	.04821	.06355	.08378				
	1.40	.02092	.02414	.02844	.03492	.04426				
10'	2.15	.01297	.01414	.01545	.01723	.01999				
	3.00	.009070	.009615	.0	.01083	.01169				
	4.64	.005711	.004919	.006111	.006319	.006554				
	7:00	.003722	.003809	.003884	.003962	.004046				
	1.00	.002577	.002618	.002653	.002688	.002725				
10 <sup>2</sup>	2.15	.001179	.001187	.001194	.001201	.001208				
		From Pap	adopulos, Bredeh	oeft, and Cooper, 1	973					
Tt/re <sup>2</sup>	α	10-*	10-7	10-*	10-9	10-10				
	1	0 9994	0.9996	0.9996	0 9997	0 9997				
	$\tilde{2}$	9989	9992	9993	9994	9995				
10-3	4	9980	9985	9987	9989	9991				
10	Â	9972	9978	9982	9984	9986				
	Ř	9964	9971	9976	9980	9982				
	1	0056	.5571	0071	.5500	0078				
	1 0	.5500	.5500	.5571	.5510	.5510				
10-2	4	.3313	.9904	.9944	.9902	.9900				
10 -	4	.9040	.9070	.9094	.9908	.9919				
	<b>b</b>	.9782	.9819	.9846	.9866	.9881				
	8	.9718	.9765	.9799	.9824	.9844				
	1	.9655	.9712	.9753	.9784	.9807				
	2	.9361	.9459	.9532	.9587	.9631				
10-1	4	.8828	.8995	.9122	.9220	.9298				
	6	.8345	.8569	.8741	.8875	.8984				
	8	.7901	.8173	.8383	.8550	.8686				
	1	.7489	.7801	.8045	.8240	.8401				
	2	.5800	.6235	.6591	.6889	.7139				
	3	.4554	.5033	.5442	.5792	.6096				
	4	.3613	.4093	.4517	.4891	.5222				
10°	5	2893	3351	3768	4146	.4487				
	Ğ	2337	2759	3157	3525	3865				
	7	1903	2285	2655	3007	3337				
	Å	1562	1903	2243	2573	2888				
	ă	1902	1504	1009	22010	2505				
	1	1078	1242	1690	1900	.2000				
	1 0	1070 09790	.1040	.1020	.1300	.6170 06170				
	4	.02720	.03343	.04129	.00071	.00149				
101	3	.01286	.01448	.01667	.01956	.02320				
10,	4	.008337	.008898	.009637	.01062	.01190				
	5	.006209	.006470	.006789	.007192	.007709				
	6	.004961	.005111	.005283	.005487	.005735				
	8	.003547	.003617	.003691	.003773	.003863				
	1	.002763	.002803	.002845	.002890	.002938				
10 <sup>2</sup>	2	.001313	.001322	.001330	.001339	.001348				

#### TABLE 9.1.—Values of H/H<sub>o</sub>

i i HZH0

5	(BETA.	AL DHAL	FOR		1.000-01
۳.	UCLA		r ur	ALPHA-	1.000-01

BE	ΞŤ	Α	

1.000-03	0.9769
2.000-03	0.9670
4.000-03	0.9528
6.000-03	0.9417
8.000-03	0.9322
1.000-02	0.9238
2.000-02	0.8904
4.000-02	0.8421
6.000-02	0.8048
8.000-02	0.7734
$1 \cdot 000 - 01$	0.7459
2.000-01	0.6418
4.000-01	0.5095
6.000-01	0.4227
8.000-01	0.3598
1.000 00	0.3117
5.000 00	0.1786
3.000 00	0.1196
4.000 00	0.0876
5.000 00	0.0681
6.000 00	0.0553
7.000 00	0.0463
8.000 00	0.0396
9.001) 00	0.0346
1.000 01	0.0306
5°001 01	0.0141
3.000 01	0.0091
4.00D 01	0.0067
5.000 01	0.0053
6.000 01	0.0044
7.000 01	0.0037
8.000 01	0.0032
9.000 01	0.0059
1.000 02	0.0056
5.000 05	0.0013
4.000 02	0.0006
6.000 02	0.0004
8.000 05	0.0003
1.000 03	0.0003

FIGURE 9.3.—Example of output from program to compute change in water level due to sudden injection of a slug of water into a well.

Equation 1 states that, initially, drawdown is zero. Equations 2 and 3 state that the drawdown approaches zero as distance from the discharging well approaches infinity, a condition which will be approximated if boundaries of the aquifer are sufficiently distant from the discharging well. Solution (Papadopulos, 1965, p. 23):

$$s = (Q/4\pi\sqrt{T_{xx}T_{yy}-T_{xy}^2}) W(u_{xy}), \qquad (4)$$

where

and

 $u_{xu}$ 

$$W(u) = \int_{u}^{\infty} (e^{-v}/v) \, dv$$

$$= (S/4t)(T_{rr}y^2 + T_{uu}x^2)$$

$$\frac{115}{-2T_{xy}xy}/(T_{xx}T_{yy}-T_{xy}^2).$$
 (5)

If the coordinate axes x and y are the same as the principal axes  $\epsilon$  and  $\eta$  (fig. 10.1) of the transmissivity tensor, the preceding equation for drawdown becomes

$$s = (Q/4\pi\sqrt{T_{\epsilon\epsilon}} T_{\eta\eta}) W(u_{\epsilon\eta}),$$
  
where  
$$u_{\epsilon\eta} = (S/4t)(T_{\epsilon\epsilon} n^2 + T_{\eta\eta} \epsilon^2)/T_{\epsilon\epsilon} T_{\eta\eta}.$$

Comments: The method of type-curve solution as outlined by Papadopulos (1965, p. 26) requires observation of drawdown in at least three observation wells. First, choose a convenient rectangular coordinate system with the pumped well at the origin. Then, plot the observed drawdown versus t on logarithmic paper. Match these plots to the W(u) type curve given in solution 1. Choose a match point of (t,s) and  $(1/u_{xy}, W(u_{xy}))$  for each well and compute  $T_{xx}T_{yy}-T_{xy}^2 = (QW(u_{xy})/4\pi s)^2$  for each well. Match points for all observation wells should yield approximately the same value of  $(T_{xx}T_{yy}-T_{xy}^2)$ . Usually they will not and judgment must be used to obtain an "average" value. Substituting this value and the three values of (x,y) in equation 5 gives three equations in three unknowns  $ST_{xx}$ ,  $ST_{yy}$ , and  $ST_{xy}$ . These equations are of the form

$$y^{2}(ST_{xx}) + x^{2}(ST_{yy}) - 2xy(ST_{xy})$$
  
=  $4tu_{xy}(T_{xx}T_{yy} - T_{xy}^{2})$ .

Solve these three equations to determine  $T_{xx}$ ,  $T_{xy}$ , and  $T_{yy}$  in terms of S, and S may be determined from

#### TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS



FIGURE 10.1.—Plan view showing coordinate axes.

$$S = \sqrt{(ST_{xx}ST_{yy} - (ST_{xy})^2)/(T_{xx}T_{yy} - T_{xy}^2)}.$$

Then, compute  $T_{xx}$ ,  $T_{yy}$ , and  $T_{xy}$  from  $ST_{xx}$ ,  $ST_{yy}$ , and  $ST_{xy}$ .  $T_{\epsilon\epsilon}$ ,  $T_{\eta\eta}$ , and  $\Theta$  (the angle between the x and the  $\epsilon$  axis) may be calculated from the relations (Papadopulos, 1965, p. 28)

$$\begin{split} T_{\epsilon\epsilon} &= 1/2(T_{xx} + T_{yy} + ((T_{xx} - T_{yy})^2 \\ &+ 4T_{xy}^2)^{1/2}) \\ T_{\eta\eta} &= 1/2(T_{xx} + T_{yy} - ((T_{xx} - T_{yy})^2 \\ &+ 4T_{xy}^2)^{1/2}) \\ \Theta &= \arctan\left((T_{\epsilon\epsilon} - T_{xx})/T_{xy}\right). \end{split}$$

## Solution 11: Variable discharge from a fully penetrating well in a leaky aquifer

Assumptions:

- 1. Well discharge changes as a specified function of time.
- 2. Well is of infinitesimal diameter and fully penetrates the aquifer.
- 3. Aquifer is overlain, or underlain, everywhere by a confining bed having uniform hydraulic conductivity (K') and thickness (b').

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- 4. Confining bed is overlain, or underlain, by an infinite constanthead plane source.
- Hydraulic gradient across confining bed changes instantaneously with a change in head in the aquifer (no release of water from storage in the confining bed).
- 6. Flow in the aquifer is two-dimensional and radial in the horizontal plane and flow in the confining bed is vertical. This assumption will be approximated closely where the hydraulic conductivity of the aquifer is sufficiently greater than that of the confining bed.

#### Differential equation:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} - \frac{sK'}{Tb'} = \frac{S}{T} \frac{\partial s}{\partial t}$$

This is the differential equation describing nonsteady radial flow in a homogeneous isotropic aquifer with leakage proportional to drawdown.

Boundary and initial conditions:

$$s(r,0) = 0 \tag{1}$$

$$s(\infty,t) = 0 \tag{2}$$

$$\lim_{r \to 0} r \frac{\partial s}{\partial r} = -\frac{Q(t)}{2\pi T}, t \ge 0$$
 (3)

Equation 1 states that, initially, drawdown is zero. Equation 2 states that drawdown is zero at large distances from the pumped well. Equation 3 states that near the pumped well the radial flow is equal to the discharge of the pumped well, which is a function of time.

#### Solution:

Solutions for certain discharge functions have been published by Abu-Zied and Scott (1963), and Werner (1946) for a nonleaky aquifer, and by Hantush (1964a) for both leaky and nonleaky aquifers. For arbitrary discharge functions for leaky aquifers, a solution using the convolution integral has been presented by Moench (1971, eq. 3):

$$s = (1/4\pi T) \int_0^t (Q(t')/(t-t')) \\ \cdot \exp(-A/(t-t') - (t-t')K'/Sb')dt', \quad (4)$$

where Q(t) is the discharge function of time and  $A = r^2 S/4T$ . A numerical integration scheme is generally necessary to evaluate the above equation.

For type curves, a more useful form of equation 4 is

$$s = (Q_r/4\pi T) \int_0^t \left[ Q(t')/Q_r (t-t') \right] \\ \cdot \exp\left[ -A/(t-t') - (t-t')K'/Sb' \right] dt', \quad (5)$$

or

$$s = (Q_r/4\pi T) SO(t), \qquad (6)$$

where SO(t), read "system output function," represents the integral expression in equation 5, and  $Q_r$  is an arbitrary discharge that eliminates dimension from the integral expression. For example,  $Q_r$  could be the initial, final, or average discharge, according to the needs of the user.

Comments: Figure 11.1 is a cross section through the discharging well. This situation is the same as for solution 4, except for the varying discharge of the well. The effect of finite well radius  $(r_w)$  was investigated by Hantush (1964b, p. 4224), who concluded that for  $t>25r_w^2S/T$  and  $r_w/\sqrt{Tb'/K'} < 0.1$  the drawdown could be represented closely by the convolution integral.

Figure 11.2 on plate 1 shows a selected set of type curves for linear change in discharge in a nonleaky aquifer. The solution for this type of discharge function has been presented by Werner (1946, p. 706). The discharge function for figure 12.2 is  $Q(t) = Q_0(1+ct)$ , and the resulting drawdown is

$$s = (Q_0/4\pi T)W(u) \{1 + ct [u + 1 - e^{-u}/W(u)]\},\$$

where W(u) is the well function of Theis. Substituting A/u for t in the above expression gives

$$s = (Q_0/4\pi T) W(u) \cdot (1 + cA \{ 1 + (1/u) [1 - e^{-u}/W(u)] \} ),$$

or

$$s = (Q_0/4\pi T) SO(t),$$

where SO(t) represents

$$W(u) (1 + cA \{ 1 + (1/u) [1 - e^{-u}/W(u)] \} ).$$



FIGURE 11.1.—Cross section through a well with variable discharge.

This substitution permits the plotting of a family of type curves, each curve specified by a value of cA.

Statistics -

Table 11.1 is the listing of a FORTRAN program designed to evaluate the above convolution integral for five different discharge functions. Three of these discharge functions are those devised by Hantush (1964a, p. 343, 344), who presented solutions for drawdown resulting from these functions. These three discharge functions are:

(a) 
$$Q(t) = Q_s [1 + \delta \exp(-t/t^*)],$$
  
(b)  $Q(t) = Q_s [1 + \delta/(1 + t/t^*)],$   
and (c)  $Q(t) = Q_s [1 + \delta/\sqrt{1 + t/t^*}],$ 

where  $Q_s$  is the ultimate steady discharge and  $\delta$ and  $t^*$  are parameters defining a particular function. The first discharge function, for an exponentially decreasing discharge (case "a" of Hantush, 1964a) is virtually the same as the discharge function of Abu-Zied and Scott (1963). Besides the three functions of Hantush, the program also includes discharge as a fifthdegree polynomial of time,  $Q(t) = \sum_{i=0}^{5} a_i t^i$ , where the  $a_i$  are the coefficients of the polynomial, and as a piecewise linear function of time with eight segments,

for

$$t_{j-1} < t \le t_j, j = 1, 2, \ldots, 8,$$

 $Q(t) = a_i + b_i(t - t_{i-i})$ 

where  $a_j$  and  $b_j$  are parameters defining the  $j^{\text{th}}$  line segment. The program uses a different, but equivalent to equation 4, expression for the convolution integral

$$s = (1/4\pi T) \int_0^t (Q(t-t')/t') \\ \cdot \exp(-A/t' - t'K'/Sb') dt'.$$

The program uses a sum to approximate the convolution integral. It chooses a starting value of t' that satisfies  $r^2S/4Tt' + K't'/Sb' =$  100. If such a value of t' does not exist, that is,  $(r^2S/4T) (K'/Sb') > 2500$ , then a value of zero is assigned for the integral value. The ending point of the interval is picked as 10 times the

for

starting point. The integral over this interval is approximated by a trapezoidal sum using 500 subdivisions of the interval. A new interval is then constructed using the previous end point as a new starting point and a new ending point equal to 10 times the new starting point. This new interval is again evaluated by a trapezoidal sum of 500 segments. This summation procedure over intervals that are successively an order of magnitude larger continues until either t'=t or  $(r^2S/4Tt') + (K't/Sb')$ >101. Input to this program consists of cards coded in specific formats. Readers unfamiliar with FORTRAN formats should refer to a FORTRAN language manual. Input consists of one or more groups of data, each group consisting of the following. First, one card containing the beginning time of the period of analysis in columns 1-10, coded in format E10.3; the ending time coded in columns 1311–20, in format E10.3; and a discharge index (a number from 1 through 5) coded in column 25, in format I1; and a reference discharge, QR, coded in columns 31–40, in format E10.3. The discharge index, IQ, selects a discharge function, Q(t), in the following manner. If IQ = 1, the discharge function is exponentially decreasing,

$$Q(t) = Q_s \left[ 1 + \delta \exp(-t/t^*) \right].$$

This is case (a) of Hantush (1964a, p. 343). If IQ = 2, the discharge function is hyperbolically decreasing,

$$Q(t) = Q_s [1 + \delta/(1 + t/t^*)].$$

This is case (b) of Hantush (1964a, p. 344). If IQ=3, the discharge function is the same as case (c) of Hantush (1964a, p. 344),

$$Q(t) = Q_s \left[ 1 + \delta / \sqrt{1 + t/t^*} \right].$$

If IQ = 4, the discharge function is a fifthdegree polynomial of time,

$$Q(t) = \sum_{i=0}^{5} a_i t^i$$

If IQ = 5, the discharge function is a piecewiselinear function of time with eight or less segments,

$$Q(t) = a_j + b_j(t - t_{j-1})$$
  
 $t_{j-1} < t \le t_j, j = 1, 2, ..., 8.$ 

The reference discharge, QR, is used to determine the form of the output from the program: If QR is coded as zero (or blank), the output shows t, s (as defined by eq. 4), and Q(t). If a value greater than zero is coded for QR, the output shows 1/u, SO(t) (as defined by eq. 6), and Q(t)/QR.

Second, there are one or more cards containing parameters of the discharge function. If IQ = 1, 2, or 3, then it consists of one card containing: QST, the ultimate steady discharge, coded in columns 1–10, in format E10.3; DE-LTA, a rate parameter, coded in columns 11-20, in format E10.3; TSTAR, a time parameter, coded in columns 21-30, in format E10.3. If IQ = 4, it is one card containing the six polynomial coefficients. They are coded in the order  $a_0, a_1, \ldots, a_5$ , in columns 1-10; 11-20,  $\ldots$ , 51-60 all in format E10.3. If IQ = 5, then the program requires four cards, each card containing  $t_i$ ,  $a_i$ ,  $b_j$ ,  $t_{j+1}$ ,  $a_{j+1}$ ,  $b_{j+1}$ ; the four cards representing j = 1, 3, 5, 7. The last part of each set of data consists of two or more cards containing coded values for: distance from pumped well, in columns 1-10; storage coefficient, in columns 11-20; transmissivity, in columns 21-30; and ratio of hydraulic conductivity to thickness for the confining bed, in columns 31-40, all in format E10.3. A blank card is used to signal the end of each set of data. Output from this program is shown in figure 11.3.

# References

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### TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

R##2#	S/(4+TRA	NS)= 1.000E-	-04, K!/	(S*B*)= ;	2.500E 03.	QR= 1.257E	05							
170														
11 -	SO(T)		50(T)		$s = s_0(t)$		solution							
1.0	0.185	1.000E 00	0.819	1.000E	00 0.842	1.000E 00	0.842	1.000E 00						
1.5	0.317	1.000E 00	0.837	1.000E	00 0.842	1.000E 00	0.842	1.000E 00						
2.0	0.421	1.000E 00	0.841	1.000E	00 0.842	1.000E 00	0.842	1.000E 00						
3.0	0.566	1.000E 00	0.842	1.000F	00 0.842	1.000E 00	0.842	1.000F 00						
5.0	0.715	1.000E 00	0.842	1.000E	0 0.842	1.000E 00	0.842	1.000E 00						
7.0	0.780	1.000E 00	0.842	1.000E	00 0.842	1.000E 00	0.842	1.000E 00						
R##2#	S/(4#TRA	NS)= 1.000E-	•04, K1/	(S*B+)= ;	2.500E 01,	QR= 1.257E	05							
1711	171141	044 0	1/1101	000 1	1 /118	1044 2	17641	0443						
170	SO(T)		SO(T)			0(1)/08	SO(T)							
1.0	0.219		1.805	1.000F	no 3.815	1.000F 00	4.829	1.000F 00						
1.5	0.397	1.000E 00	2.167	1.000E	10 4.111	1.000E 00	4.849	1.000E 00						
2.0	0.558	1.000E 00	2.427	1.000E	00 4.296	1.000E 00	4-853	1.000E 00						
3.0	0.826	1.000E 00	2.793	1.000E		1.000E 00	4.854	1.000E 00						
5.0	1.216	1.000E 00	3.244	1.000E	00 4.708	1.000E 00	4.854	1.000E 00						
7.0	1.495	1.000E 00	3.530	1.000E	00 4.785	1.000E 00	4.854	1.000E 00						
					•••••									
R##2#	R##2#S/(4#TRANS)= 1.000E-04, K+/(S*B+)= 2.500E-01, QR= 1.257E 05													
1/0	1/0#1	0** 0	1/U#1	0** 1	1/U*	10** 2	1/041	0** 3						
	SO(T)	Q(T)/QR	SO(T)	Q(T)/QI	R SO(T)	Q(T)/QR	SO(T)	Q(T)/QR						
1.0	0.219	1.000E 00	1.823	1.000E	00 4.036	1.000E 00	6.307	1.000E 00						
1.5	0.398	1.000E 00	2.196	1.000E	00 4.437	1.000E 00	6.700	1.000E 00						
2.0	0.560	1.000E 00	2.468	1.000E	00 4.721	1.000E 00	6.975	1.000E 00						
3.0	0.829	1.000E 00	2.857	1.000E 4	00 5.123	1.000E 00	7.356	1.000E 00						
5.0	1.223	1.000E 00	3.354	1.000E	00 5.627	1.000E 00	7.820	1.000E 00						
7.0	1.507	1.000E 00	3.684	1.000E	00 5.958	1.000E 00	8.110	1.000E 00						
R##2#	S/(4#TRA	NS)= 1.000E-	•04, K*/	(S*B+)= ;	2.500F-03.	QR= 1.257E	05							
1.711	1 21161	0.8.8.0	1 /1101		1 7118	1044 3	1 /001	0.8.8. 3						
170	1/U*1 50(T)		50/7)		2 SO(T)	0(1)/00	50(1)	0/1)/00						
1 0	0 210	1 000E 00	30(17				6 2 2 2							
1.0	0.209		2 107			1 000E 00	6 - 3 3 2	1.0000 00						
2 0	0.540		2 . 1 7 /	1.0000		1 0000 00	7 0 2 4	1.0000 00						
2.0	0.000	1.000E 00	2 400		JU 4+720		7 4 20	1.000E 00						
3.0	0.029	1.0000 00	2.051		0 5+130		7 0 2 0	1.0000 00						
7 0	1.623	1.0000 00	3.355			1.0000 00	1.737	1.0000 00						
1.0	1.507		3.000	1.0000			0.215							
	FIGURE II	.3.—Example of d	utput from	program to	compute the co	nvolution integra	if for a leaky	aquiler.						
wit	thin a thicl	k aquitard, in G	eological S	urvey <b>r</b> e-	Hydrauli	cs Div. Jour., Am	. Soc. Civil E	ngineers Proc.						
sea	arch 1970: U	J.S. Geol. Survey	Prof. Paper	700-C, p.	p. 171–195.									
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Ferris.	J. G., Know	les, D. B., Brown	R. H., and	Stallman.	Inst. Min	ing and Technol	ogy Prof. Pan	er 102, 34 p.						
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# SUPPLEMENTAL DATA

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TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer

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TABLE 2.1.-Listing of program for partial penetration in a nonleaky artesian aquifer-Continued

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PPN
2 WRITE (IPT,7) DPB,RB,LB,DB
                                                                            61
                                                                       PPN
  GO TO 4
                                                                            62
  WRITE (IPT,8) LPB,DPB,RB,LB,DB
                                                                       PPN
                                                                            63
                                                                       PPN
4 WRITE (IPT,9) (A(I),C(I),IARG(I),I=1,JLIMIT)
                                                                            64
                                                                       PPN
                                                                            65
  DO 5 I=1,13
                                                                       PPN
  WRITE (IPT,10) ARG(I), (ARRAY(I,J), J=1, JLIMIT)
                                                                            66
5 CONTINUE
                                                                       PPN
                                                                            67
  STOP
                                                                       PPN
                                                                            68
                                                                       PPN
                                                                            69
                                                                       PPN
                                                                            70
6 FORMAT (3F5,1,15,2E10,4)
                                                                       PPN
                                                                            71
7 FORMAT (111, 1W(U)+F(U,R/8,L/8,D/8,Z/8), Z/8=1,F5,2,1, SQRT(KZ/KR)*PPN
                                                                            72
 1R/Ba1, F5, 2, 1, L/Ba1, F5, 2, 1, D/Ba1, F5, 2, 1, U#1/N1)
                                                                       PPN
                                                                            73
8 FORMAT (111, 1W(U)+F(U, R/8, L/8, 0/8, L11/8, D11/8), L11/8=1, F5, 2, 1, D1PPN
                                                                            74
 11/B=1,F5,2,1, SQRT(KZ/KR)*R/B=1,F5,2,1, L/B=1,F5,2,1, D/B=1,F5,2, PPN
                                                                            75
 2, U=1/N')
                                                                       DDN
                                                                            76
9 FORMAT (101,2X, 1N1, 1X, 12(2A4, 12))
                                                                       PPN
                                                                            77
10 FORMAT ((! 1,F4,1,12(F9,4,1X)))
                                                                       PPN
                                                                            78
                                                                       PPN
  END
                                                                            79-
                                                                         F
  REAL FUNCTION F+4(U,RB,LB,DB,LPB,DPB)
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                                                                         P
  FUNCTION F
                                                                             4
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                                                                         P
  PURPOSE
                                                                             6
      TO COMPUTE DEPARTURES FROM THEIS CURVE CAUSED BY PARTIAL
                                                                         F
                                                                             7
                                                                         F
      PENETRATION OF PUMPED WELL.
                                                                             8
                                                                         F
  USAGE
                                                                             0
                                                                         F
      F(U,R8,L8,08,LP8,0P8)
                                                                            10
                                                                         F
  DESCRIPTION OF PARAMETERS
                                                                            11
                                                                         F
      ALL REAL, U DOUBLE PRECISION
                                                                            12
                                                                         F
      U - R*+2+8/4+T+TIME (RADIAL DISTANCE SQUARED * STORAGE
                                                                            13
                                                                         F
          COEFFICIENT / 4*TRANSMISSIVITY * TIME
                                                                            14
                                                                         F
      RB - R/B ( RADIAL DISTANCE / AQUIFER THICKNESS )
                                                                            15
      LB - L/B ( FRACTION OF AQUIFER PENETRATED BY PUMPED WELL)
                                                                         F
                                                                            16
      DB - D/B ( FRACTION OF AQUIFER ABOVE PUMPED WELL SCREEN)
                                                                         F
                                                                            17
      LPB - LIVB (FRACTION OF AQUIFER PENETRATED BY OBS. WELL, ZERO
                                                                         F
                                                                            18
                                                                         F
            FOR PIEZOMETER)
                                                                            19
      UPB - DI/B (FRACTION OF AQUIFER ABOVE OBS, WELL SCREEN, TUTAL
                                                                         F
                                                                            20
                                                                         F
            DEPTH FOR PIEZOMETER)
                                                                            21
                                                                         F
  SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                                                                            22
                                                                         F
                                                                            53
      DQL12, SERIES, BESK, FCT, L
                                                                         F
                                                                            24
  METHOD
                                                                         F
      SUMS THE SERIES THROUGH N+PI+R/8 EQ 20
                                                                            25
                                                                         F
                                                                            26
                                                                         F
   *****************
                                                                            27
   REAL+8 U.V
                                                                         F
                                                                            28
                                                                         F
                                                                            29
   REAL+4 L, N, LB, LPB
                                                                         F
   SUM=0.
                                                                            30
                                                                         F
   N=0.
                                                                            31
                                                                         F
   PIR8=3,141593*R8
                                                                            32
                                                                         ø
                                                                            33
   PIL8=3,141593+LB
                                                                         F
                                                                            34
   PID8=3,141593+DB
   IF (LPB=0.) 1,1,4
                                                                         F
                                                                            35
   CHECKS FOR WELL OR PIEZOMETER
                                                                         F
                                                                            36
                                                                         F
 1 PIZB=3,141593+DPB
                                                                            37
                                                                         F
                                                                            38
 2 N=N+1.
                                                                         F
   VEN#PIRB/2.
                                                                            39
                                                                         F
   IF (V.GT.10.) GD TO 3
                                                                            40
   TRUNCATES SERIES WHEN V>10
                                                                         F
                                                                            41
                                                                         F
                                                                            42
   X=L(U,V)/N
```

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#### TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

 TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

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ALC: NO

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SUM#SUM+(SIN(N*PILE)=SIN(N*PIDE))*CUS(N*PIZE)*X
                                                                        F
                                                                           43
  GO TO 2
                                                                        5
                                                                           44
                                                                        F
3 F=.6366198+SUM/(LB=DB)
                                                                           45
                                                                        F
  GO TO 7
                                                                           46
                                                                        F
4 PILP8=3,141593*LP8
                                                                           47
                                                                        F
  PIDP8=3_141593*DP8
                                                                           48
                                                                        F
5 N#N+1
                                                                           49
  V=N*PIRB/2.
                                                                        F
                                                                           50
  IF (V.GT.10.) GD TO 6
                                                                        F
                                                                           51
  TRUNCATES SERIES WHEN VOID
                                                                        F
                                                                           52
                                                                           53
  X=L(U,V)/N
                                                                        F
  SUM#SUM+(SIN(N*PILB)=SIN(N*PIDB))*(SIN(N*PILPB)=SIN(N*PIDPB))*X/N
                                                                        F
                                                                           54
  GO TO S
                                                                        F
                                                                           55
                                                                        F
6 F=_2026424*$UM/((LB=DB)*(LPB=DPB))
                                                                           56
                                                                        F
7 RETURN
                                                                           57
  END
                                                                        F
                                                                           58-
  REAL FUNCTION L+4(U.V)
                                                                        L
                                                                            1
  ******************
                                                                        L
                                                                            2
                                                                        L
                                                                            ٦
  FUNCTION L
                                                                        L
                                                                            4
                                                                        L
                                                                            S
 PURPOSE
                                                                        L
                                                                            6
     TO CUMPUTE THE INTEGRAL( EXP(=Y=V=+2/Y)/Y) SUMMED OVER Y FROM
                                                                            7
                                                                        L
     U TO INFINITY (WELL FUNCTION FOR LEAKY AQUIFERS).
                                                                        L
                                                                            8
 DESCRIPTION OF PARAMETERS
                                                                            q
                                                                        Ł
     BOTH DOUBLE PRECISION
                                                                        L
                                                                           10
     U = R**2*S/4*T*TIME (RADIAL DISTANCE SQUARED * STORAGE
                                                                        L
                                                                           11
         COEFFICIENT / 4+TRANSMISSIVITY + TIME
                                                                        L
                                                                           12
     V = R/2*SORT(K!/(T+8'))==ONE=HALF RADIAL DISTANCE*SQUARE RUDT
                                                                        L
                                                                           13
         (HYD, COND, OF CONFINING BED/TRANSHISSIVITY*THICKNESS
                                                                        L
                                                                           14
         OF CONFINING BED)
                                                                           15
                                                                        L
 SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                                                                        L
                                                                           16
    DGL12, SERIES, BESK, FCT
                                                                           17
                                                                        L
 METHOD
                                                                        L
                                                                           18
     IN THE FULLOWING FREXP(-Y+V**2/Y)/Y
                                                                           19
                                                                        L
  (1) U>=1, USES & GAUSSIAN-LAGUERRE QUADRATURE FORMULA TO
                                                                           20
                                                                        L
     EVALUATE INTEGRAL (F) FROM U TO INF.
                                                                        F
                                                                           21
  (2) V**2<U<1, USES THE G=L QUADRATURE TO EVALUATE INTEGRAL(F)
                                                                        L
                                                                           22
    FROM ONE TO INF AND A SERIES EXPANSION TO EVALUATE INTEGRAL(F)
                                                                        L
                                                                           23
    FROM U TO ONE.
                                                                           24
                                                                        L
   (3) U<1, U<=v++2, USES THE REPRESENTATION INTEGRAL(F) FROM U
                                                                        L
                                                                           25
     TO INF. = 2*KO(2*V) +INTEGRAL(F) FROM V**2/U TO INF.
                                                                        L
                                                                           26
     EVALUATES THE ZERO ORDER MODIFIED BESSEL FUNCTION OF SECOND
                                                                        L
                                                                           27
     KIND WITH IBM SUBROUTINE, EVALUATES INTEGRAL BY GOL QUAD.
                                                                        L
                                                                           28
                                                                        L
                                                                           29
  *********************
                                                                        L
                                                                           30
 EXTERNAL FCT
                                                                        L
                                                                           31
 REAL+8 U,V,Z,F,VV,SERIES
                                                                        L
                                                                           32
  COMMON /C1/ VV.Z
                                                                        L
                                                                           33
  \nabla \nabla \equiv \nabla
                                                                        L
                                                                           34
  IF (U-1,) 1,2,2
                                                                        L
                                                                           35
                                                                        L
  CHECKS IF UK1
                                                                           36
1 Z=V+V/U
                                                                        L
                                                                           37
 IF (Z-1.) 3,4,4
                                                                        L
                                                                           38
  CHECKS IF V*+2/U < 1
                                                                        L
                                                                           39
                                                                        L
                                                                           40
2 Z=U
  CALL DOL12(FCT.F)
                                                                        L
                                                                           41
 L=F
                                                                        L
                                                                           42
  INTEGRAL U TO INF. EVALUATED BY GAUSS-LAGUERRE GUADRATURE
                                                                        L
                                                                           43
                                                                           44
 60 10 5
                                                                        L
3 Z=1.
                                                                        L
                                                                           45
```

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 $\sim p^{*}$ 

TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

c c	4	CALL DGL12(FCT,F) L=F+SERIES(U,V) INTEGRAL 1 TO INF, BY G=L QUAD., INTEGRAL U TO 1 BY SERIES EXP, GO TO 5 TWOVE2.*V CALL BESK(TWOV,0,8K,IER) CALL DGL12(FCT,F) L=2.*BK=F 2K0(2V)=INTEGRAL V**2/U TO INF, RETURN END		467 489 55555555555555555555555555555555555
		REAL FUNCTION SERIES+8(U,V)	SER	1
Ç		***************************************	SER SFD	2
Ċ		FUNCTION SERIES	SER	4
C		DUPDOSE	SEN SED	5
č		TO EVALUATE S(1)=S(U), WHERE S IS A SERIES EXPANSION OF	SER	7
C		INTEGRAL(EXP(-Y-V++2/Y)DY/Y) GIVEN BY: S= SUM, M=0 TO INFINITY,	SER	8
Ç		(F(M)+SUM, N=0 TO INF,,(V**(2*N)/((N1)*(M+N)1)) WHERE F(M)=	SER	9
ç		LUG(U) IF MEO AND E ((#1)**M/M)*(U**M=(V**2/U)*#H) IF HDU;	GED JEK	10
č		BOTH DOUBLE PRECISION	SER	12
č		U = R##2#S/4+T#TIME (RADIAL DISTANCE SQUARED # STURAGE	SER	13
C		CUEFFICIENT / 4*TRANSMISSIVITY * TIME	SER	14
ç		V + R/2+SQRT(KI/(T+B'))-+ONE-HALF RADIAL DISTANCE+SQUARE ROOT	SER	15
C		(HTD, CUND, OF CUNFINING BEDVIRANSMISSIVITTATHICKNESS OF CONSTINING RED)	36 K 8 F D	10
č		SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	SER	18
Č		NONE	SER	19
C		METHOD	SER	20
ç		SUMMATION IS TERMINATED FOR THE INNER SERIES WHEN A TERM Recomed i for than a firm and for outer gerter when a term	SER SER	21
c		BECOMES LESS THAN SEEVITH AND FOR DUTER SERIES HOLD A TEND	SER	23
č			SER	24
C		**************	SER	25
		REAL+8 DLUG, DABS, 8(2), VUM, UU	SER	26
		KERERO ICSIJUJUMJENJENJSUMIJSUMJSIGNJVJVSGUJKNUGJIEKNJIEKMI TESTES.De07	OCK SFR	28
		VSQEV+V	SER	29
		UU=U	SER	30
-		DU 6 I=1,2	SER	31
C		EVALUATES SERIES FOR LOWER LIMIT # U AND UPPER LIMIT # 1	95 P	36
		UM#1.	SER	34
		EM=1.	SER	35
		SUM1=0.	SER	36
		SIGN=1,	SER	37
		A CULTER & C VII	SFR	30 20
	1	EMBEM+1.	SER	40
	•	IF (EH=,1) 2,3,3	SER	41
C	~	CHECKS FOR MED	SER	42
	2	HTULEULUG(U) TEDM(=(	35 K 85 D	<b>43</b> 1111
			SER	45
	3	UM#UM#U	SER	46
		IF (VUM.LT.1.D=30) VUM=0.	SER	47
			SER	48
		TERMIZTERMI/EM	SER	47
		n perfect g = n perfect g # teg IT	· ·	~ v

### TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

TABLE 2.1.-Listing of program for partial penetration in a nonleaky artesian aquifer-Continued

4	STGNESTCN	980	
-		JER	21
		SER	52
	TERMETERM1	SER	53
	EN=0.	SFR	64
5	ENREN-1	OC D	
		<b>OEX</b>	22
	TERM#TERM#VSQ/(EN+(EN+EM))	SER	56
			21
	IF (IES), LE + DABS(RHULKENNIERM)) GU TU 5	JER	5.8
	TRUNCATES INNER SERIES IF DUTER TERMANAINNER TERM < 5_E=7	SER	59
	SUM1#SUM1+STGN#PMUL+SUM	960	4.0
		SER	0.0
	IF SEMANIANTS OF IN S	SER	61
	IF (TEST.LE.DABS(RMUL+SUM)) GO TO 1	SÉR	62
	TRUNCATES OUTED SERVES TE OUTED TEDMATNARD BUM & S. F.T.	860	4.1
4	all maint	JER	03
0	attleanw1	SER	64
		SÉR	65
	SERIES=S(2)=S(1)	8E D	
	DETILDA		00
		3EN	67
	END	SER	68-
	REAL FUNCTIÓN FCT+8(X)	FCT	1.
	*****		-
			2
		FCT	3
	FUNCTION FCT	FCT	4
		Fer	
	DUPPOSE	P C I	2
	PURPUSE	FCT	6
	TO CUMPUTE FCT(X)=EXP(=Z=V++2/(X+Z))/(X+Z)	FCT	7
	DESCRIPTION OF PARAMETERS	FOT	á
		FUT	
	A THE DOUBLE PRECISION VALUE OF X FOR WHICH FCI IS COMPUTED	PCT	9
	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	FCT	10
	NONE	FCT	
	METHOD	r C r	11
		P C T	12
	FORTHAN EVALUATION OF FUNCTION	FCT	13
		FCT	14
	**************	. For	
		HRPCI	15
		FGT	16
	COMMON /C1/ V.Z	FCT	17
	TF (X) 1.2.2	ECT	
	al shi afufu	PUT	10
*		FCT	19
	GO TU 4	FCT	20
2		607	51
-		P 6 1	61
-		PCT	22
3	アビリアリセスアリックノイズナズ)	FCT	23
4	RETURN	FOT	24
	END	E C F	
	WAR AND		27-
	SOBKONITAE DAFTS(LC()1)	0615	380
		DL 12	10
		01.12	20
		ULIE	50
	SUBROUTINE DOL12	DL12	40
		01 12	50
	Phi Phi RE		
	FURFUSE	ULIC	60
	IU COMPUTE INTEGRAL(EXP(+X)+FCT(X), SUMMED OVER X	DF15	70
	FROM 0 TO INFINITY).	0112	80
		01 4 3	00
	1104.05	VLIC	40
		DL12	100
	CALL DQL12 (FCT,Y)	0L12	110
	PARAMETER FOT REBUTRES AN EXTERNAL STATEMENT	n + 2	120
	en unalte fas naaduma un fulfuid" Alulfuit.		150
		0115	130
	DESCHIPTION OF PARAMETERS	DL12	140
	FCT - THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION	0112	150
		ULIE	100
	Y 👘 THE REBULTING DOUBLE PRECISION INTEGRAL VALUE,	DL12	170

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 TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

	01.12	180
	DI 12	190
	DI 12	200
	0112	210
SUBPORTINES AND FUNCTION SUBPORCEAMS PERITEED	DI 12	220
THE EVEDNAL FORBLE DEFINITION SUNCTION SURPRISEDAM FOT(Y)	0112	210
INTE EXTERNAL UDUBLE FREETUIDA FUNCTION SUBFREENAN FETTAF	Di 12	2/10
MUST DE FURNIANED DI THE USER.	0113	380
	DI 13	230
METHUM Evaluation to done by means of 43-botht cansstand acuerde	0112	200
EVALUATION IS DUNE OF MEANS OF IEPOINT GAUGGANALAUGARE	DITE	270
GUADRAIURE FURMULA, MILM INICHAILS EARLILT,	DLIC	200
WHENEVER PLI(X) IS A POLYNUMIAL OP TO DEGREE <3,	DLIZ	290
FUR REFERENCE, BEE Build and a realized of terms and callertan weithing of	DLIE	300
SHAUTCHENTFRANK, TABLES UP ZENUS AND GAUSSIAN HEIGHIS UP	DLIE	310
CERTAIN ASSOCIATED LAGUERRE PULYNOMIALS AND THE RELATED		320
GENERALIZED HERMITE PULTNUMIALS, IBM TECHNICAL REPURT	ULIE	330
1800,1100 (MARCH 1964), PP.24=25.	OLIZ	340
	PLIZ	350
	PLIZ	560
·		370
	DL12	390
	DLIZ	400
DUBLE PRECISION X, Y, PCT	DL12	410
	OLIZ	420
x=,3709912104446692 D2	DL12	430
Y=, 814807746742624 D=15*FCT(X)	DL12	440
x=,2848796725098400 D2	DL12	450
Y=Y+.3061601635035021 D=11+FCT(X)	OL12	460
x=,2215104037939701 02	DL12	470
Y=Y+_1342391030515004 D=8+FCT(X)	DL12	480
x=,1711685518746226 D2	DL12	490
Y=Y+,1668493876540910_D=6*FCT(X)	DL12	500
x=,1300605499330635 D2	DL12	510
Y#Y+_836505585681980 D=5*FCT(X)	DL12	520
x=,962131664245687 D1	DL12	530
Y=Y+,2032315926629994 D=3+FCT(X)	DL12	540
X=_6844525453115177 01	DL12	550
Y=Y+,2663973541865316 D=2*FCT(X)	DL12	560
x=,4599227639418348 D1	DL12	570
Y=Y+_2010238115463410 D=1+FCT(X)	DL12	580
X=,2833751337743507 D1	DL12	590
Y#Y+ 904492222116809 D=1*FCT(X)	PLIZ	600
x=,1512610269776419 D1	DLIZ	610
Y#Y+ 2440820113198776 DOAPCI(X)	ULIK	674
x=,6117574845151307 D0		630
Y#Y+ \$777592758731380 DO#FCI(X)	DLIZ	040
x=,1157221173580207 00		050
Y#Y+ 2647313710554432 DO*FCT(X)	PLIC	000
RETURN		07U
END	ULIX	00*
SUBROUTINE BESK(X,N,8K,IER)	BESK	410
	BESK	10
	BESK	50
	BESK	30
SUBROUTINE BESK	BESK	40
	BESK	50
COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDE	RBESK	60
	BESK	70
USAGE	BE8K	80
CALL BESK(X,N,BK,IER)	BESK	90

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TABLE 2.1.-Listing of program for partial penetration in a nonleaky artesian aquifer-Continued

```
BESK 100
      DESCRIPTION OF PARAMETERS
                                                                               BESK 110
                                                                               BESK 120
          X .THE ARGUMENT OF THE K BESSEL FUNCTION DESIRED
                                                                              8ESK 130
            •THE ORDER OF THE K BESSEL FUNCTION DESIRED
          N
          BK -THE RESULTANT & BESSEL FUNCTION
                                                                               BESK 140
                                                                               8E8K 150
          IER-RESULTANT ERROR CODE WHERE
                                                                               BESK 160
             IER#0 NO ERROR
                     N IS NEGATIVE
                                                                               888K 170
             IER=1
                     X IS ZERD OR NEGATIVE
                                                                              8ESK 180
             IER=2
                     X .GT. 170, MACHINE RANGE EXCEEDED
                                                                              BE8K 190
             IER#3
                                                                              BESK 200
             IER#4 BK .GT. 10**70
                                                                              BE8K 210
     REMARKS
                                                                               BESK 220
                                                                               BESK 230
          N MUST BE GREATER THAN OR EQUAL TO ZERO
                                                                               BESK 240
      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                                                                               BESK 250
         NUNE 540

NUNE 560

HOD

CUMPUTES ZERO ORDER AND FIRST URDER BESSEL FUNCTIONS USING 558 290

SERIES APPROXIMATIONS AND THEN COMPUTES N TH ORDER FUNCTION 558 310

USING RECURRENCE RELATION. 558 310

RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE 558 320
     METHOD
          AS DESCRIBED BY A, J, M, HITCHCOCK, POLYNOMIAL APPROXIMATIONS
                                                                               BESK 330
          TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TU RELATED
                                                                               BESK 340
         FUNCTIONS!, M.T.A.C., V.11, 1957, PP.86-88, AND G.N. WATSON,
'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE
                                                                               BE8K 350
                                                                              -BESK 360
          UNIVERSITY PRESS, 1958, P. 62
                                                                               8ESK 370
                                                                               BESK 380
   BESK 400
                                                                               BESK 420
   DIMENSION T(12)
                                                                               BESK 430
   8K#.0
   IF(N)10,11,11
                                                                               BESK 440
                                                                               BESK 450
10 IER=1
   RETURN
                                                                               BESK 460
                                                                               BESK 470
11 IF(X)12,12,20
12 IER=2
                                                                               BESK 480
                                                                               BESK 490
   RETURN
20
   IF(X=170.0)22,22,21
                                                                               BE8K 500
                                                                               BESK 510
21 IER#3
                                                                               BESK 520
   RETURN
                                                                               BE8K 530
55 IEB=0
   IF(X=1,)36,36,25
                                                                               BESK 540
25 ABEXP(=X)
                                                                               BESK 550
   8=1./X
                                                                               BESK 560
   C=SQRT(B)
                                                                               868K 570
                                                                               8ESK 580
   T(1)=8
   DO 26 L#2,12
                                                                               BESK 590
                                                                               BE8K 600
26 T(L)=T(L=1)+B
                                                                               BESK 610
   IF(N=1)27,29,27
                                                                               8ESK 620
   COMPUTE KO USING POLYNOMIAL APPROXIMATION
                                                                               BESK 630
                                                                               BESK 640
27 GO#A*(1,2533141=,1566642*T(1)+,08811128*T(2)=,09139095*T(3)
                                                                               BESK 650
  2+,1344596*T(4)=,2299850*T(5)+,3792410*T(6)=,5247277*T(7)
                                                                               BESK 660
  3+,5575368+T(8)+,4262633+T(9)+,2184518+T(10)+,06680977+T(11)
                                                                               BESK 670
                                                                               BESK 680
  4+_009189383+T(12))*C
                                                                               BESK 690
   IF(N)20,28,29
                                                                               BESK 700
28 BK=G0
                                                                               BESK 710
   RETURN
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TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

BESK 720 C C BESK 730 COMPUTE K1 USING POLYNOMIAL APPROXIMATION BESK 740 C BESK 750 29 G1=A\*(1,2533141+,4699927\*T(1)=,1468583\*T(2)+,1280427\*T(3) 2=,1736432\*T(4)+,2847618\*T(5)=,4594342\*T(6)+,6283381\*T(7) BESK 760 3-,6632295\*T(8)+,5050239\*T(9)-,2581304\*T(10)+,07880001\*T(11) **BESK 770** BESK 780 4=\_01082418\*T(12))\*C BESK 790 IF(N=1)20,30,31 BESK 800 30 8K=G1 BESK 810 RETURN BESK 820 C BESK 830 FROM KO,KI COMPUTE KN USING RECURRENCE RELATION С BESK 840 C **BESK 850** 31 DO 35 J=2,N BESK 860 GJ=2.\*(FLDAT(J)=1.)\*G1/X+G0 **BESK 870** IF(GJ=1,0E70)33,33,32 BESK 880 32 IER=4 BESK 890 GO TU 34 **BESK 900** 33 G0=G1 **BESK 910** 35 G1=GJ BESK 920 34 BK#GJ BESK 930 RETURN BESK 940 36 B=X/2, **BESK 950** A= 5772157+ALOG(8) CaB+B BESK 960 BESK 970 IF(N+1)37,43,37 BESK 980 ¢ BESK 990 Ċ COMPUTE KO USING SERIES EXPANSION **BESK1000** BESK1010 37 G0==A X2J=1. BESK1020 FACTE1. BESK1030 **BESK1040** HJ#.0 BESK1050 DO 40 J=1,6 BESK1060 RJ=1\_/FLOAT(J) **BESK1061** IF(X2J\_LT\_1\_E=40) X2J=0. PREVIDUS STATEMENT ADDED TO IBM SUBROUTINE TO CURRECT UNDERFLOW BESK1062 Ç Ĉ 8ESK1063 PROBLEM ON WATFOR COMPILER BESK1070 X2J=X2J\*C FACTEFACT#RJ#RJ BESK1080 BESK1090 HJ=HJ+RJ **BESK1100** 40 G0=G0+X2J\*FACT\*(HJ=A) BESK1110 IF(N)43,42,43 42 BK=G0 BESK1120 BESK1130 RETURN C C BE8K1140 BESK1150 COMPUTE KI USING SERIES EXPANSION C **BESK1160** 43 X2J=8 BESK1170 BESK1180 FACT=1. BESK1190 HJ=1. BESK1200 G1=1, /X+X2J+(, 5+A=HJ)DO 50 J=2,8 8E3K1210 **BESK1220** X2J=X2J+C BESK1230 RJ#1./FLOAT(J) FACT#FACT#RJ#RJ BESK1240 BESK1250 HJ#HJ+RJ BESK1260 50 G1=G1+X2J+FACT+(+5+(A=HJ)+FLOAT(J)) 8E\$K1270 IF(N=1)31,52,31 BESK1280 52 BK=G1 BESK1290 RETURN BESK130. END

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 TABLE 2.1.—Listing of program for partial penetration in a nonleaky artesian aquifer—Continued

EAPL 1         SUBRUUTINE EXPI       EAPL 2         PURPOSE       EXPI 4         COMPUTES THE EXPONENTIAL INTEGRAL *EI(=X)       EXPI 4         VSAGE       EXPI 5         CALL EXPI(X,RES)       EXPI 10         CALL EXPI(X,RES)       EXPI 10         UESCRIPTION OF PARAMETERS       EXPI 11         X       A REQUENT VALUE       EXPI 12         AUX       RES       HESULT VALUE       EXPI 14         AUX       RES       HESULT VALUE       EXPI 14         AUX       RES       HESULT VALUE       EXPI 14         AUX       RESULT VALUE       EXPI 14       EXPI 14         AUX       RESULT VALUE       EXPI 14       EXPI 14         AUX       RESULT VALUE       EXPI 14       EXPI 14         AUX       RESULT VALUE       EXPI 16       EXPI 16         REMARKS       EXPI 16       EXPI 17       EXPI 16         SUBRUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 28       EXPI 28         NONE       EXPI 28       EXPI 28       EXPI 28         METHOD       EXPI 28       EXPI 28       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATION ARE USED IN THE EXPI 28       IF(X=1,)2,1,1       EXPI 28
SUBRUUTINE EXPI       EXPI 3         SUBRUUTINE EXPI       EXPI 4         PURPOSE       EXPI 4         COMPUTES THE EXPONENTIAL INTEGRAL *EI(=X)       EXPI 5         USAGE       EXPI 4         CALL EXPI(X,RES)       EXPI 10         DESCRIPTION OF PARAMETERS       EXPI 11         X = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI 12         X = ARGULT XAUE       EXPI 14         AUX = RESULTANT AUXILIARY VALUE       EXPI 14         AUX = RESULTANT AUXILIARY VALUE       EXPI 12         REMARKS       EXPI 12         Y X GT 170 (X LT =174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXPI 14         AUX = RESULT VALUE       EXPI 12         WITH THE EXPONENTIAL FUNCTION       SUBROUTINES AND FUNCTION SUBPRUGRAMS REQUIRED       EXPI 24         METHOD       EXPI 23       EXPI 24         METHOD       EXPI 24       EXPI 24         METHOD       EXPI 24       EXPI 24         MANGES 1 LE X, X LE =9 AND 9 LT X LE = 3 RESPECTIVELY, EXPI 30       EXPI 32         IF(x=1,)2,1,1       EXPI 35       EXPI 36         1 y=1,XX       A PULYNOMIAL APPROXIMATION IS USED IN -3 LT X LT 1, EXPI 31       EXPI 36         I y=1,XX       EXPI 3239E0)*Y+2,709479E=1)/((((Y EXPI 327)55319E)*Y+2,593686E0)*Y+2,709479E=1)/(((Y EXPI 328)*E)*Y+2,593
SUBRUUTINE EXPI       EXPI 4         PURPOSE       EXPI 5         COMPUTES THE EXPONENTIAL INTEGRAL *EI(=x)       EXPI 5         USAGE       EXPI 6         CALL EXPI(x,RE3)       EXPI 10         UESCRIPTION OF PARAMETERS       EXPI 11         UESCRIPTION OF PARAMETERS       EXPI 12         X       = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI 13         RES       = HESULT VALUE       EXPI 14         AUX       = RESULTANT AUXILIARY VALUE       EXPI 14         RES       = HESULT VALUE       EXPI 14         MUX       = RESULTANT AUXILIARY VALUE       EXPI 16         REMARKS       EXPI 170       EXPE 17         Y       X GT 170 (X LT = 174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXPI 16         PURPOSE       EXPI 16       EXPI 16         WITH THE EXPONENTIAL FUNCTION       EXPI 20       EXPI 21         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 21         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 22         NONE       EXPI 23       EXPI 24         METHOD       EXPI 24       EXPI 24         DEFINITION       EXPI 24       EXPI 24         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE UBED IN THE       EXPI 24
EXPI 5       EXPI 6         COMPUTES THE EXPONENTIAL INTEGRAL *EI(=x)       EXPI 5         USAGE       EXPI 7         CALL EXPI(X,RES)       EXPI 10         DESCRIPTION OF PARAMETERS       EXPI 11         X = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI 12         X = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI 13         RES = NESULT VALUE       EXPI 14         AUX = RESULTANT AUXILIARY VALUE       EXPI 16         REMARKS       EXPE 17         X GT 170 (X LT =174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXPI 16         WITH THE EXPONENTIAL FUNCTION       EXPI 16         WITH THE EXPONENTIAL FUNCTION       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 23         NONE       EXPI 24       EXPI 24         METHOD       EXPI 25       EXPI 26         DEFINITION       SUBROUTINES AND FUNCTION SUBPROXIMATIONS ANE USED IN THE EXPI 26       EXPI 26         NONE       EXPI 28       EXPI 28       EXPI 28         I (100       EXPI 28       EXPI 28       EXPI 28         I (110)
PURPOSE       EXPI 0         COMPUTES THE EXPONENTIAL INTEGRAL *EI(*X)       EXPI 7         CALL EXPI(X,RES)       EXPI 10         CALL EXPI(X,RES)       EXPI 11         DESCRIPTION OF PARAMETERS       EXPI 12         X = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI 12         RES = RESULT VALUE       EXPI 14         AUX = RESULT VALUE       EXPI 15         REMARKS       EXPI 17         X GT 170 (X LT =174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXPI 18         WITH THE EXPONENTIAL FUNCTION       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 22         NONE       EXPI 22       EXPI 22         NONE       EXPI 22       EXPI 22         METHOD       EXPI 22       EXPI 23         DEFINITION       EXPI 24       EXPI 24         METHOD       EXPI 25       EXPI 26         DEFINITION       EXPI 25       EXPI 26         RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY, EXPI 30       EXPI 32         IF (X=1,)2,1,1       EXPI 32       EXPI 32         If (X=1,)2,1,1 <t< td=""></t<>
LUMPUTES THE EXPONENTIAL INTEGRAL *EI(*X)       EXPI ************************************
USAGE       EXPI 9         CALL EXPI(X,RES)       EXPI 10         DESCRIPTION OF PARAMETERS       EXPI 12         X = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI 13         RES = RESULT VALUE       EXPI 14         AUX = RESULTANT AUXILIARY VALUE       EXPI 15         REMARKS       EXPI 16         WITH THE EXPONENTIAL FUNCTION       EXPE 16         WITH THE EXPONENTIAL FUNCTION       EXPI 12         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 22         NONE       EXPI 23         METHOD       EXPI 24         METHOD       EXPI 25         DEFINITION       RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).         EVALUATION       EXPI 24         METHOD       EXPI 25         OFFINITION       EXPI 26         RANGES 1 LE X, X LE =9 AND -9 LT X LE -3 RESPECTIVELY.       EXPI 30         IF(x=1,)2.1,1       EXPI 36         I Y=1,/X       EXPI 36         A PULYNOMIAL APPROXIMATION IS USED IN -3 LT X LT 1.       EXPI 37         IF(UNN       EXPI 37         IF(x+1,)2.1,1       EXPI 36         I Y=1,/X       EXPI 37         AUX=((((('1,122452E=7*X=1,766345E=6)*X+2,709479E=1)/(((('4* EXPI 36         I Y=1,X       EX
CALL EXPI(X,RES)       ÉXPÍ 10         CALL EXPI(X,RES)       ÉXPÍ 10         UESCRIPTION OF PARAMETERS       EXPÍ 11         X       = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPÍ 13         RES       = RESULT VALUE       EXPÍ 14         AUX       = RESULTANT AUXILIARY VALUE       EXPÍ 14         REMARKS       EXPÍ 17         X GT 170 (X LT =174) MAY GAUSE UNDERFLOW (OVERFLOW)       EXPÍ 18         WITH THE EXPONENTIAL FUNCTION       EXPÍ 121         SUBRDUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPÍ 24         METHOD       EXPÍ 24         NONE       EXPÍ 22         NONE       EXPÍ 24         METHOD       EXPÍ 24         DEFINITION       SUBPROXIMATIONS ANE USED IN THE         RES=INTEGRAL (EXP( <t) dver="" from="" infinity).<="" summed="" t="" t,="" td="" tú="" x="">       EXPÍ 24         METHOD       EXPÍ 25         DEFINITION       AND =9 LT X LE =3 RESPECTIVELY,         RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY,       EXPÍ 30         IF(X=1,)2,1,1       IF(X=3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPÍ 36         1,07255E0+5,716943E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPÍ 36         RETÚNN       EXPÍ 40         I,1,07255E0+5,716943E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPÍ 36</t)>
EXPI110       OF PARAMETERS       EXPI112         X       = ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI122         RES       = RESULT VALUE       EXPI14         AUX       = RESULTANT AUXILIARY VALUE       EXPI14         AUX       = RESULTANT AUXILIARY VALUE       EXP114         REMARKS       EXP116       EXP116         REMARKS       EXP117       EXP116         WITH THE EXPONENTIAL FUNCTION       EXP1121       EXP121         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXP122         NONE       EXP124       EXP124         METHOD       EXP124       EXP124         DEFINITION       SUBPROXIMATIONAL APPROXIMATIONS ARE USED IN THE       EXP127         EVALUATION       ILEX, X LE =0 AND =0 LT X LE =3 RESPECTIVELY, EXP130       EXP132         IF(x=1,)2,1,1       IX X LE =0 AND =0 LT X LE =3 RESPECTIVELY, EXP130       EXP134         IF(x=1,)2,1,1       IX X LE =0 SADE OIN THE EXP138       EXP149         IF(x=1,)2,1,1       EXP135560)*Y+2,05215660)*Y+2,709470E=1)/((((Y* EXP136       EXP140         I Y=1,X       AUX=1,=YE((Y+3,377356E0)*Y+2,05215660)*Y+2,709470E=1)/((((Y* EXP136       EXP140         I Y=1,X       AUX=1,=YE(4)       EXP141       EXP141         I Y=1,X       IIIIIIIIIIIIIIIIIIIIIIIIII
DESCRIPTION OF PARAMETERS       EXPI 12         X       - ARGUMENT OF EXPONENTIAL INTEGRAL       EXPI 13         RES       - RESULT VALUE       EXPI 14         AUX       - RESULT VALUE       EXPI 14         AUX       - RESULT VALUE       EXPI 14         AUX       - RESULT VALUE       EXPI 16         REMARKS       EXPI 17         X GT 170 (X LT = 174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXPI 16         WITH THE EXPONENTIAL FUNCTION       EXPI 21         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 22         NONE       EXPI 23         METHOD       EXPI 24         METHOD       EXPI 25         DEFINITION       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED         RESSINTEGRAL(EXP(-T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 26         RESSINTEGRAL(EXP(-T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 26         RESSINTEGRAL(EXP(-T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 27         RESSINTEGRAL(EXP(-T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 28         IF (X=1,)2,1,1       EXPI 2052156E0)*Y+2,709479E=1)/((((Y*       EXPI 30         I, 07255160+5,7169435E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y*       EXPI 30         I, 07255160+5,7169435E0)*Y+6,945239E0)*Y+2,593888860)*Y+2,709496E=1)       EXPI 40
X       = ARGUMENT VALUE       EXP1 14         RES       = RESULT VALUE       EXP1 14         AUX       = RESULTANT AUXILIARY VALUE       EXP1 14         AUX       = RESULTANT AUXILIARY VALUE       EXP1 15         REMARKS       EXP1 15         X GT 170 (X LT = 174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXP1 16         WITH THE EXPONENTIAL FUNCTION       EXP1 21         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXP1 22         NONE       EXP1 24         METHOD       EXP1 24         DEFINITION       EXP1 24         RESSINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXP1 26         RESSINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXP1 26         RESSINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXP1 26         RESSINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXP1 26         RESSINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXP1 27         RANGES 1 LE X, X LE = 9 AND = 9 LT X LE = 3 RESPECTIVELY.       EXP1 30         I 1 Y=1, /X       EXP1 34       EXP1 34         I 1
AUX       = RESULTANT AUXILIARY VALUE       EXPI 15         AUX       = RESULTANT AUXILIARY VALUE       EXPI 15         REMARKS       EXPI 16       EXPI 17         X GT 170 (X LT =174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXPI 18         WITH THE EXPONENTIAL FUNCTION       EXPI 12         WITH THE EXPONENTIAL FUNCTION       EXPI 20         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 21         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 22         NONE       EXPI 22         METHOD       EXPI 22         DEFINITION       EXPI 24         METHOD       EXPI 24         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 24         RANGES 1 LE X, X LE =9 AND -9 LT X LE =3 RESPECTIVELY, EXPI 30       EXPI 32         IF (x=1,)2,1,1       EXPI 34       EXPI 34         I Y=1,/X       AUX=1,=Y+(((Y+3, 377358E0)+Y+2, 052156E0)+Y+2, 709479E=1)/((((Y+ EXPI 38       EXPI 34         I Y=1,/X       EXPI 34       EXPI 34       EXPI 34         I Y=1,/X       EXPI 44
REMARKS       EXPI 16         REMARKS       EXPI 17         X GT 170 (X LT =174) MAY GAUSE UNDERFLOW (OVERFLOW)       EXPI 18         WITH THE EXPONENTIAL FUNCTION       EXPI 18         WITH THE EXPONENTIAL FUNCTION       EXPI 21         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 22         NONE       EXPI 23         METHOD       EXPI 24         DEFINITION       EXPI 24         RESSINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 24         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 24         RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY.       EXPI 34         IF(x=1,)2,1,1       EXPI 34         I *=1,X       EXPI 34         I *=1,X       EXPI 34         I *=1,X       EXPI 35060)*Y+2.052156E0)*Y+2.709479E=1)/((((Y* EXPI 36         I *=1,X       EXPI 34         I *=1,X       EXPI 34         I *=1,X       EXPI 34         I *=1,X       EXPI 3553500)*Y+2.052156E0)*Y+2.709479E=1)/((((Y* EXPI 36         I *=1,X       EXPI 34         I *=1,X       EXPI 34         I *=1,X       EXPI 35         I *=1,X       EXPI 36         I *=1,X       EXPI 37         I *=1,X
REMARKS       EXPEND: X GT 170 (X LT = 174) MAY CAUSE UNDERFLOW (OVERFLOW)       EXPI 18 EXPI 18 WITH THE EXPONENTIAL FUNCTION         WITH THE EXPONENTIAL FUNCTION       EXPET 20 EXPI 20 EXPI 21         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 21 EXPI 22 NONE         METHOD       EXPI 23 EXPI 24         METHOD       EXPI 24 EXPI 25         DEFINITION       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED         RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 24 EXPI 26         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE EXPI 28       EXPI 24         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE EXPI 28       EXPI 24         If (x=1,)2,1,1       EXPI 24.         I Y=1,/X       EXPI 24.         AUX=1,=Y=(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y*       EXPI 34         EXPI 34       EXPI 34         If (x=1,)2,1,1       EXPI 34         I Y=1,/X       EXPI 34         AUX=1,=Y=(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y*       EXPI 34         I Y=1,/X       EXPI 34         AUX=1,=Y=(((Y,1,22452E=7*X=1,7653260)*Y+2,709479E=1)/((((Y*       EXPI 34         1 Y=1,+(X       EXPI 44         2 AUX=((((((((7,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X=2,535379E=4EXPI 43         3 AUX=((((((((7,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X=
<pre>X GT 170 (x LT =174) MAY GAUSE UNDERFLOW (OVERFLOW) EXPI 16 WITH THE EXPONENTIAL FUNCTION EXPI 10 FUR x = 0 THE RESULT VALUE IS SET TO =1,ETS EXPI 20 SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED EXPI 22 NONE EXPI 24 METHOD EXPI 25 DEFINITION EXPI 26 RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY). EXPI 27 EVALUATION EXPI 26 RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY, EXPI 30 A PULYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1, EXPI 31 IF(x=1,)2,1,1 EXPI 26 NETURN EXPI 26 SUBROUX=**EXP(=X) AUX=1,=**E(((*+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((** EXPI 36 1 Y=1,/X AUX=1,=**E((*+3,377358E0)*Y+2,052156E0)*Y+2,593888E0)*Y+2,709496E=1) EXPI 36 1 Y=1,/X AUX=1,=**E(((*,122452E=7*x=1,766345E=6)*X+2,928433E=5)*X=2,535379E=4EXPI 43 I)***1,664156E=3)**=1,041576E=2)*X+5,555602E=2)*X=2,500001E=1)*X EXPI 43 PF(X=3,64,5,4 AUX=1(=C(((4,45,4))=5,772157E=1) EXPI 46 RES=1,E75 FFURN EXP(=X) EXPI 45 RES=1,E75 FFURN EXPI 45 RES=</pre>
WITH THE EXPONENTIAL FUNCTION       EXP: 19         FUR X = 0 THE RESULT VALUE IS SET TO =1.675       EXPI 20         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED       EXPI 23         NONE       EXPI 23         METHOD       EXPI 23         DEFINITION       SUBPROGRAMS REQUIRED       EXPI 23         METHOD       EXPI 24         METHOD       EXPI 25         DEFINITION       EXPI 26         RANGES 1 LE X, X LE =9 AND +9 LT X LE =3 RESPECTIVELY, EXPI 30       EXPI 34         A PULYNOMIAL APPROXIMATION IS USED IN -3 LT X LT 1, EXPI 31       EXPI 34         IF (X-1,)2,1,1       EXPI 34         If (X+1,)2,1,1       EXPI 34         If (X+1,)2,1,1       EXPI 34         If (X+3,)5,6,3       EXPI 34         I y=1,/X       EXPI 34         AUX=1(((((1,122452E=7*x×1,766345E=6)*x+2,928433E=5)*x=2,353579E=4EXPI 43         1)*X*1,664156E=3)*X=1,041576E=2)*X*5,555682E=2)*X=2,500001E=1)*X       EXPI 44         2+0,99999E=1       EXPI 44         RESE=1,E75       EXPI 44         RESE=1,E75       EXPI 45         RESE=1,E75       EXPI 46         RESE=1,E75       EXPI 46         RETURN       EXPI 46         RETURN       EXPI 46         RESE=1,E
FUR X = 0 (HE RESULT VALUE IS SET 10 =1,E75)       EXPI 20         SUBROUTINES AND FUNCTION SUBPRUGRAMS REQUIRED       EXPI 21         NONE       EXPI 23         METHOD       EXPI 25         DEFINITION       EXPI 25         DEFINITION       EXPI 26         RESEINTEGRAL (EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 26         RESEINTEGRAL (EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 27         EVALUATION       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 28         RANGES 1 LE X, X LE =9 AND =9 LT X LE = 3 RESPECTIVELY, EXPI 30       EXPI 32         A PULYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1, EXPI 31       EXPI 34         IF(X=1,)2,1,1       EXPI 34       EXPI 34         IF(X=1,)2,1,1       EXPI 37       EXPI 34         IF(X=1,2,1,1       EXPI 34       EXPI 34         IF(X=1,2,1,1       EXPI 34       EXPI 34         IF(X=1,2,2,1,1       EXPI 34       EXPI 34         IF(X=1,2,1,1       EXPI 34       EXPI 34         IF(X=1,2,2,1,1       EXPI 34       EXPI 34         IF(X=1,2,2,1,1       EXPI 34       EXPI 34         IF(X=1,2,2,1,1       EXPI 34       EXPI 34         IF(X=1,2,2,1,2,1,1       EXPI 34
SUBROUTINES AND FUNCTION SUBPRUGRAMS REQUIRED       EXPI 22         NONE       EXPI 23         METHOD       EXPI 24         DEFINITION       EXPI 24         RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY), EXPI 26         RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY), EXPI 26         RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY), EXPI 27         EVALUATION       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 29         RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY, EXPI 30       EXPI 32         A PULYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1, EXPI 31       EXPI 32         IF(X=1,)2,1,1       EXPI 34         I Y=1,/X       EXPI 34         A UX=((((Y+3, 377358E0)*Y+2, 052156E0)*Y+2, 709479E=1)/((((Y* EXPI 38         11,072553E0+5,716943E0)*Y+2,052156E0)*Y+2,593886E0)*Y+2,709496E=1) EXPI 39         HESBAUX*Y=EXP(=X)       EXPI 40         RETURN       EXPI 41         21 F(X+3,)6,6,3       EXPI 41         31 \*X^41,664156E=3)*X=1,041576E=2)*X+2,928433E=5)*X=2,335379E=4EXPI 43         1)*X^41,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 44         2+0,909909E=1       EXPI 44         2+0,90909E=1       EXPI 45       EXPI 44         2+0,90909E=1       EXPI 44
NUNE       EXPI 23         METHOD       EXPI 24         DEFINITION       EXPI 25         RES=INTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY), EXPI 27         EVALUATION       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 28         A PULYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1,       EXPI 32         IF(x=1,)2,1,1       EXPI 36         IF(x=1,)2,1,1       EXPI 38         IF(x=1,)2,1,1       EXPI 38         IF(x=1,)2,1,1       EXPI 34         EXPI 34       EXPI 34         IF(x=1,)2,1,1       EXPI 34         EXPI 34       EXPI 34         II, 072553E0+5,716943E0)*V+6,945239E0)*V+2,709479E=1)/((((V*       EXPI 36         I1, 072553E0+5,716943E0)*V+6,945239E0)*V+2,593888E0)*V+2,709496E=1) EXPI 38       EXPI 40         RETURN       EXPI 41       EXPI 44         IF(X+3,)6,6,3       EXPI 44       EXPI 45         I)*X+1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 44
METHOD       EXPI 24         DEFINITION       EXPI 25         DEFINITION       EXPI 26         RESSINTEGRAL(EXP(-T)/T, SUMMED OVER T FROM X TO INFINITY).       EXPI 27         EVALUATION       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 29         RANGES 1 LE X, X LE = 9 AND =9 LT X LE = 3 RESPECTIVELY,       EXPI 31         EXPI 32       EXPI 32         A PULYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1.       EXPI 32         IF(x=1,)2,1,1       EXPI 32         IF(x=1,)2,1,1       EXPI 38         1.072553E0+5.716943E0)*Y+2.052156E0)*Y+2.709470E=1)/((((Y* EXPI 38         1.072553E0+5.716943E0)*Y+6.945239E0)*Y+2.5938886E0)*Y+2.709496E=1) EXPI 39         METÚRN       EXPI 40         RETÚRN       EXPI 41         2 IF(X+3,)6,6,3       EXPI 42         3 AUX=((((((((((((((((((((((((((((((((((((
METHOD       EXPI 25         DEFINITION       EXPI 26         RESEINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY), EXPI 27         EVALUATION       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 28         RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY, EXPI 30       A PULYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1, EXPI 32         IF(X=1,)2,1,1       EXPI 34         IF(X=1,)2
DEFINITION       EXPI 20         RESSINTEGRAL(EXP(=T)/T, SUMMED OVER T FROM X TO INFINITY), EXPI 27         EVALUATION       EXPI 28         THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE       EXPI 29         RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY, EXPI 30         A PULYNOMIAL APPROXIMATION IS USED IN =3 LT X LT 1, EXPI 31         EXPI 34         IF(X=1,)2,1,1       EXPI 36         1 Y=1,/X       EXPI 38         AUX=1,=Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPI 38         11,072553E0+5,716943E0)*Y+2,052156E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 39         11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 40         RETURN       EXPI 43         11,07253E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 40         RETURN       EXPI 41         2 IF(X+3,16,6,3       EXPI 42         3 AUX=((((((17,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X=2,335379E=4EXPI 43         1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 42         3 AUX=((((((17,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X=2,335379E=4EXPI 43       I)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X         2 RES==1,E75       EXPI 45       EXPI 45       IF(X)4,5,4       IF(X)4,5,4       IF(X)4,5,4       IF(X)4,5,4       IF(X)4,5,4
Evaluation       Explore
THREE DIFFERENT RATIONAL APPROXIMATIONS ARE USED IN THE EXPI 29         RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY, EXPI 30         A PULYNOMIAL APPRDXIMATION IS USED IN =3 LT X LT 1, EXPI 31         EXPI 32         IF(x=1,)2,1,1         Y=1,/X         AUX=1,=Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPI 36         1,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)         EXPI 30         RES=AUX*Y*EXP(=X)         RETURN         2 IF(X+3,)6,6,3         3 AUX=((((((7,122452E=7*x×1,766345E=6)*X+2,928433E=5)*X=2,335379E=4EXPI 43         1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X         EXPI 42         RES==1,E75         IF(X)4,5,4         4 RES=XX+AUX=ALDG(AB3(X))=5,772157E=1         5 RETURN       EXPI 46         6 IF(XY+9,)8,8,7
RANGES 1 LE X, X LE =9 AND =9 LT X LE =3 RESPECTIVELY,       EXPI 30         A PULYNOMIAL APPRDXIMATION IS USED IN =3 LT X LT 1,       EXPI 31         EXPI 32       EXPI 33         IF(x=1,)2,1,1       EXPI 30         1 Y=1,/X       EXPI 31         AUX=1,=Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPI 38         11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 30         RESEAUX*Y*EXP(=X)       EXPI 30         RETURN       EXPI 40         2 IF(X+3,)6,6,3       EXPI 43         1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 43         1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 44         2 PO9090E=1       EXPI 45         RES=1,E75       EXPI 45         IF(X)4,5,4       EXPI 45         4 RES=X+AUX=ALDG(ABS(X))=5,772157E=1       EXPI 45         5 RETURN       EXPI 46         6 IF(X+9,)8,8,7       EXPI 40
A PULYNOMIAL APPROXIMATION IS USED IN -3 LT X LT 1, EXPI 31 EXPI 32 EXPI 33 EXPI 34 EXPI 34 EXPI 36 EXPI 36 EXPI 36 EXPI 37 AUX=1,-Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E-1)/((((Y* EXPI 38 11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E-1) EXPI 39 RES=AUX*Y*EXP(-x) RETURN 2 IF(X+3,)6,6,3 3 AUX=(((((((7,122452E=7*x=1,766345E=6)*x+2,928433E=5)*x=2,335379E=4EXPI 43 1)*x+1,664156E=3)*x=1,041576E=2)*x+5,555682E=2)*x=2,500001E=1)*X 2+9,999999E=1 RES=-1,E75 IF(X)4,5,4 4 RES=x*AUX=ALDG(ABS(X))=5,772157E=1 5 RETURN 6 IF(X)49,)8,8,7 EXPI 40 EXPI 40
EXPI 32 IF (X=1,)2,1,1 I Y=1,/X AUX=1,-Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPI 36 11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,5938886E0)*Y+2,709496E=1) EXPI 36 RES=AUX*Y*EXP(=X) RETURN 2 IF (X+3,)6,6,3 3 AUX=(((((((7,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X+2,335379E=4EXPI 43 1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X 2+9,999999E=1 RES=1,E75 IF (X)4,5,4 4 RES=X*AUX*ALDG(AB8(X))=5,772157E=1 5 RETURN 6 IF (X+9,)8,8,7 EXPI 40 EXPI
IF (x=1,)2,1,1       EXPI 34         1 Y=1,/X       EXPI 35         AUX=1,-Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPI 36         11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 36         11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 36         RES=AUX*Y*EXP(=X)       EXPI 40         RETURN       EXPI 41         2 IF (X+3,)6,6,3       EXPI 43         3 AUX=(((((((((7,122452E=7*x=1,766345E=6)*X+2,928433E=5)*X=2,535379E=4EXPI 43         1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 44         2+9,999999E=1       EXPI 44         RES==1,E75       EXPI 46         IF (X)4,5,4       EXPI 47         4 RES=X*AUX=ALDG(ABS(X))=5,772157E=1       EXPI 46         5 RETURN       EXPI 46         6 IF (X+9,)8,8,7       EXPI 40
IF (x=1,)2,1,1       EXPI 36         1 Y=1,/X       EXPI 37         AUx=1,=Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPI 38)         11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 39         RES=AUX*Y*EXP(=X)       EXPI 40         RETURN       EXPI 41         2 IF (X+3,)6,6,3       EXPI 43         3 AUX=((((((((7,122452E=7*x=1,766345E=6)*X+2,928433E=5)*X=2,335379E=4EXPI 43)         1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 44         2+9,999999E=1       EXPI 44         RES=1,E75       EXPI 45         IF (X)4,5,4       EXPI 45         4 RES=x*AUX*ALUG(AB8(X))=5,772157E=1       EXPI 46         5 RETURN       EXPI 46         6 IF (X)+9,)8,8,7       EXPI 49
1 Y=1,/X EXPI 37 AUX=1,=Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y* EXPI 38 11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1) EXPI 39 RES=AUX*Y*EXP(=X) EXPI 40 RETURN EXPI 40 2 IF(X+3,)6,6,3 EXPI 41 2 IF(X+3,)6,6,3 EXPI 43 1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,335379E=4EXPI 43 1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X EXPI 44 2+9,999999E=1 EXPI 43 RES==1,E75 EXPI 44 RES==1,E75 EXPI 44 RES==1,E75 EXPI 45 IF(X)4,5,4 EXPI 45 5 RETURN EXPI 49,08,8,7 EXPI 44 2 RES=X*AUX=ALDG(ABS(X))=5,772157E=1 EXPI 46 EXPI 49 EXPI
AUX=1,=Y*(((Y+3,377358E0)*Y+2,052156E0)*Y+2,709479E=1)/((((Y*       EXPI 38         11.072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1)       EXPI 39         RES=AUX*Y*EXP(=X)       EXPI 40         RETURN       EXPI 41         2 IF(X+3,)6,6,3       EXPI 43         3 AUX=((((((((7,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X=2,335379E=4EXPI 43         1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 44         2+9,99999E=1       EXPI 44         RES=1,E75       EXPI 46         IF(X)4,5,4       EXPI 47         4 RES=X*AUX=ALDG(ABS(X))=5,772157E=1       EXPI 48         5 RETURN       EXPI 49         6 IF(X)49,38,8,7       EXPI 49
11,072553E0+5,716943E0)*Y+6,945239E0)*Y+2,593888E0)*Y+2,709496E=1) EXPI 39 RES#AUX*Y*EXP(=X) RETURN 2 IF(X+3,)6,6,3 3 AUX#(((((((7,122452E=7*X=1,766345E=6)*X+2,928433E=5)*X=2,335379E=4EXPI 43 1)*X*1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X EXPI 43 2+9,99999E=1 RES==1,E75 IF(X)4,5,4 4 RES#X*AUX=ALDG(ABS(X))=5,772157E=1 5 RETURN 6 IF(X+9,)8,8,7 EXPI 49 EXPI 4
RES=AUX+T+EXP(=x)       EXPI 40         RETURN       EXPI 41         2 IF(X+3,)6,6,3       EXPI 42         3 AUX=(((((((7,122452E=7+X=1,766345E=6)*X+2,928433E=5)*X+2,335379E=4EXPI 43       1)*X+1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 43         1)*X+1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X       EXPI 44         2+9,999999E=1       EXPI 44         RES==1,E75       EXPI 45         IF(X)4,5,4       EXPI 47         4 RES=X+AUX=ALDG(ABS(X))=5,772157E=1       EXPI 48         5 RETURN       EXPI 48         6 IF(X+9,)8,8,7       EXPI 49
2       IF(X+3,)6,6,3       EXPI 42         3       AUX#((((((7,122452E=7*x=1,766345E=6)*X+2,928433E=5)*X+2,335379E=4EXPI 43         1)*x+1,664156E=3)*x=1,041576E=2)*X+5,555682E=2)*x=2,500001E=1)*X       EXPI 43         2+9,99999E=1       EXPI 45         RES==1,E75       EXPI 44         1F(X)4,5,4       EXPI 47         4       RES=x*AUX=ALDG(ABS(X))=5,772157E=1       EXPI 48         5       RETURN       EXPI 49         6       IF(X+9,)8,8,7       EXPI 50
3 AUX#((((((((7,122452E=7+x=1,766345E=6)*X+2,928433E=5)*X=2,335379E=4EXPI 43 1)*X+1,664156E=3)*X=1,041576E=2)*X+5,555682E=2)*X=2,500001E=1)*X EXPI 44 2+9,999999E=1 EXPI 45 RES==1,E75 EXPI 45 IF(X)4,5,4 EXPI 45 4 RES=X*AUX=ALDG(ABS(X))=5,772157E=1 EXPI 48 5 RETURN EXPI 49 6 IF(X+9,)8,8,7 EXPI 45
1) *X*1,664156E=3) *X=1,041576E=2) *X+5,555682E=2) *X=2,500001E=1) *X 2+9,999999E=1 RES==1,E75 IF(X)4,5,4 4 RES=X*AUX=ALDG(ABS(X))=5,772157E=1 5 RETURN 6 IF(X+9,)8,8,7 EXPI 49 EXPI 49
2+9.999999E=1       EXPI 45         RES=0.E75       EXPI 46         IF(X)4,5,4       EXPI 47         4 RES=X*AUX=ALDG(ABS(X))=5.772157E=1       EXPI 48         5 RETURN       EXPI 49         6 IF(X+9.)8.8.7       EXPI 50
KES=01,275       EAPI 40         IF(X)4,5,4       EXPI 47         4 RES=X*AUX=ALDG(ABS(X))=5,772157E=1       EXPI 48         5 RETURN       EXPI 49         6 IF(X+9,)8,8,7       EXPI 50
4       RES#X#AUX=ALDG(ABS(X))=5.772157E=1       EXPI 48         5       RETURN       EXPI 49         6       1F(X+9.)8.8.7       EXPI 50
5 RETURN EXPI 49 6 IF(X+9.)8.8.7 EXPI 50
6 IF(X+9.)8.8.7 Exp1 50
7 AUX=1,=((((5,176245E-2*X+3,061037E0)*X+3,243665E1)*X+2,244234E2)*XEXPI 51
1+C+40007/C2)/((((X+5+740101CU)*X+5+84374441)*X+C+C0301811*X EXPL 52 244 807837531 EXPL 52
AUX#1.=Y*(((Y+7.659824E=1)*Y=7.271015E=1)*Y=1.080693E0)/((((Y EXPI 56
AUX#1,=Y*(((Y+7,659824E=1)*Y=7,271015E=1)*Y=1,080693E0)/((((Y EXPI 56 1*2,518750E0+1,122927E1)*Y+5,921405E0)*Y=8,666702E0)*Y=9,724216E0) EXPI 57
AUX#1.#Y*(((Y+7.659824E=1)*Y=7.271015E=1)*Y=1.080693E0)/((((Y EXPI 56 1*2.518750E0+1.122927E1)*Y+5.921405E0)*Y=8.666702E0)*Y=9.724216E0) EXPI 57 9 RES=AUX*EXP(=X)/X EXPI 58 0 FT(DA

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TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer

*****	***UB	1
	WUB	2
PURPOSE	WUB	3
TO COMPUTE A TABLE OF VALUES OF THE LEAKY AQUIFER WELL	WUB	4
FUNCTION - $W(U, R/B)$ - HANTUSH, M.S., AND JACOB, C.E., 1955,	WUB	5
NON-STEADY RADIAL FLOW IN AN INFINITE LEAKY AQUIFER: AM.	WUB	6
GEDPHYS. UNION TRANS V. 36. NO. 14 P. 95-100.	WUB	7
TNPHT DATA	WUR	Å
1  (ADD) = Ecommutizein Ex	WUB	ă
I CARD - FURMAILEEINSJJ Homail - Maile of Value de 470 ero wetre rumbutation 199		
USMALL & SMALLEST VALUE OF IVU FUR WHICH COMPUTATION IS	<b>H</b> U B	10
DESIRED,	WUB	11
ULARGE - LARGEST VALUE UP 1/U FUR WHICH CUMPUTATION IS	WUB	12
DESIRED,	WUB	13
2 CARDS - FORMAT(8E10,5)	WUB	14
BDAT - 12 VALUES OF R/B FOR TABLE,	WUB	15
SUBROUTINES AND FUNCTION SUBPRUGRAMS REQUIRED	WU8	16
L,SERIES,FCT,BESK,DQL12	WUB	17
	WUB	18
	****	10
	W1/8	20
		24
NEALHO UIY		61
DIMENSIUN ARRAT(73,12), T(73), BDAT(12), TNUM(6)	HUB	22
DATA YNUM/1.,1.5,2.,5.,5.,7./	WUB	52
IRD#5	WUB	24
IPT#6	WU8	25
READ (IRD,6) USMALL,ULARGE	WU8	26
READ (IRD,6) BDAT	WUB	27
IBEGIN=ALOG10(USMALL)	WUB	85
IENDBALDG10(ULARGE)+.99999	WU8	29
TI TMITE(TEND-TREGIN)+6+1	WUR	30
TE CILINIT.GT 73) TI THITA73	WUB	ίΪ.
	WUR	12
	410 B	22
	+ UD	33
	FUB	34
	MOR	32
GO 10 3	MUR	30
2 NB=1-1	WUB	37
3 II=0	WUB	38
DO 4 INIFICIMIT	WUB	39
II=II+1	WU8	40
IF (II.GT.6) II=1	WUB	41
IEXP=IBEGIN+(I=1)/6	WUS	42
Y(I)=YNUM(II)+10_++IEXP	WUB	43
	WUR	44
DD 4 Jel.NA	WUR	<u>л</u> щ
		40
T TE TITT TANTATIN TAL NEN		47
TRAIE (AFT)// (BUAI(A//AFA))		40
	WU8	49
S WHITE (LPT)8) Y(I)#(ARRAY(I)J)J=1,NB)	MUR	50
STOP	WUB	51
	WUB	52
	WUB	53
6 FORMAT (8E10,5)	WUB	54
7 FORMAT (111, W(U,R/B) //101, 101, 11 R/B1/1 1,6X, 11/U 11, 12E10.2)	WUB	55
8 FORMAT (1 1,E10.3,12F10.4)	WUB	56
END	WUR	57
DEAL FUNCTION LANZING		
neme rungilun L#4(U,Y)	L	1
***********************	1** L	2
	L	3
FUNCTION L	L	- 4
	L	5

 TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

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1. 1. J. 1.

	PURPOSE	<b>.</b>	
	TO COMPUTE THE INTEGRAL ( EXP(-Y-VA+2/Y)/Y) SUMMED OVER Y PROM	<u> </u>	1
	U TO INFINITY(WELL FUNCTION FOR LEAKY AQUIFERS).	- <b>L</b>	8
	DESCRIPTION OF PARAMETERS	L.	9
	BOTH DOUBLE PRECISION	L.	10
	U = R*+2*8/4*T*TIME (RADIAL DISTANCE SQUARED * STORAGE	Ļ	11
	COEFFICIENT / 4#TRAN8MISSIVITY # TIME	Ļ	11
	V = R/2+8QRT(K1/(T+B1))==QNE=HALF RADIAL DISTANCE+8QUARE RODT	L	-13
	(HYD. COND. OF CONFINING BED/TRANSMISSIVITY+THICKNE8S	L	14
	OF CONFINING BED)	L	15
	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	Ē.	16
	OQL12, SERIES, BESK, FCT	Ē	17
	METHOD	Ē	18
	IN THE FOLLOWING FREXP(-Y-V++2/Y)/Y	Ē	15
	(1) USE1, USES A GAUSSTAN-LAGUERRE DUADRATURE FORMULA TU	ī	20
	EVALUATE INTEGRALIES FORM IL TO THE	Ē	21
	(2) VARSE INTEGRATIC FROM O TO TO APP.	- E	22
	ED THE VULLY USED THE OF BURNEHINE TO ETHERNE INTEGRALITY	1	- 51
	FROM UNE IN INF AND A GENIES EARANSIUN ID ETALDATE INTEGNALITY		5
	TRUTH U TO UNE.	5	
	(3) UKI, UKEVWEZ, USES INE REPRESENTATION INTEGRALIPT PRUM U	5	2
	IN INF, E SAKU(SAY) THIERALLY PROM VARE/V IN INF.	L.	2
	EVALUATES THE ZERU UNDER HUDTPIED BESSEL FUNCTION OF SECUND	Ļ.	2
	KIND WITH IBM SUBROUTINE, EVALUATES INTEGRAL BY GOL GUAD.	Ļ	SI
		Ę.	Š
	***********	L	30
	EXTERNAL FCT	L.	3
	REAL+B U,V,Z,F,VV,SERIES	L	3
	COMMON /C1/ VV/Z	L	31
		L	- 34
	IF (U=1.) 1,2,2	L	35
	CHECKS IF UK1	L	30
1	2=V*V/U	Ļ	31
	IF (Z=1.) 3,4,4	L	3(
	CHECKS IF VA+2/U < 1	Ĺ	39
2		Ē	4
-	CALL DOLL2(FCT.F)	ĩ	4
		ĩ	4
	THTEGRAL II TO THE EVALUATED BY GAUSSELAGUERRE QUADRATURE	Ē	4
		ī	4
1		Ĩ	
		ĩ	
		-	- 11
	LEFTGEREGIUJVJ Thteedal I to the av cal dhan . Thteedal II to 1 av redter evo	5	- 14
	INTEGRAL I TO INF. BY GWL WORD, INTEGRAL O TO I DY SERIES CAP.	Г	
		- <b>L</b> a	4
4		<u>ь</u>	2
	CALL BESK(TWUV,0,BK,IEK)	Ļ.	2
	CALL DULI2(FCT/F)	Ŀ	5
	LE2. + BR-F	Ę.	53
_	2KO(2V)=INTEGRAL V#+2/U TU INF,	Ŀ	54
5	RETURN	Ŀ	-5!
	END	L.	5
	REAL FUNCTION SERIES+8(U,V) S	ER	
	***************************************	ER	i
	3	ER	1
	FUNCTION SERIES S	ER	- (
	3	ER	
	PURPOSE S	ER	(
	TO EVALUATE S(1)-S(U), WHERE S IS A SERIES EXPANSION OF S	ER	1
	INTEGRAL(EXP(-Y=V*+2/Y)DY/Y) GIVEN BY: SE SUM, MEO TU INFINITY,S	ER	- (
	(F(M)+8UM, NED TO INF , (V++(2+N)/((N1)+(M+N)1)) WHERE F(M) S	ER	•
	LOG(U) IF MED AND = ((=1)***/M)*(U***=(V**2/U)***) IF M>0. 8	ER	10
	DESCRIPTION OF PARAMETERS	ER	1
	BOTH DOUBLE PRECISION	ER	1
	U = R+2+2+5/4+T+TIME (RADIAL DISTANCE SQUARED + STORAGE 9	FR	1
			4
	V = R/24SODT/KI/(TAHI))=OUTOFILE DANTAL OTGIAUFEAGUAGE DUNY - G	50	1.4
	(HVD. COND. OF CONFINING REDITORNEY TO AN OWIGETUTE WATER OWDERG RUUL O	6 R 6 D	17
	UE CONETATION DE CONTRANSMISSITTITITICALES - 3	11.R 11.D	10
	or Powktwide Droi 2	2.7(	17

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TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

С		SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	SER	18
C		NONE	95 7 95 9	14
Ç		METHOD	95 A	20
ç		SUMMATIUN IS TERMINALED FUR THE INNER SERIES HEEN A TERM	SFR	22
C		BELIMES LESS THAN 3.EVIN AND FUR DUTER SERIES HOLD A TENN	SFR	23
C A		DECUMES LESS THAN J.L.	SER	24
C			SER	25
6			SER	26
		DEAL AR TE AT HIM. FM. FN. SUMISSUMS SIGN, V. VSQ, VSQU, RMUL, TERM, TERMI	SER .	27
			SER	28
			SER	29
			SER	30
		D0 6 I=1,2	SER	31
С		EVALUATES SERIES FOR LOWER LIMIT = U AND UPPER LIMIT = 1	SER	32
-		IF (I,EQ,2) U=1,	SER	33
		UM#1,	SER	34
		EMB=1.	SER	35
		SUM1=0,	SEN	36
		SIGN==1.	SER	37
		VUM#1.	SER .	30
		A80/= A80/n	35 K 9 F D	74
	1	EMBEM+1.	QCR	/11
•		IF (EMmal) 2/3/3	SFR	41
C	•		SER	43
	2	₩₩UL=ULULULULULULULULULULULULULULULULULU	SER	44
			SER	45
	7		SER	46
	3	TE (VIM.LT.1.N-RA) VIMED.	SER	47
			SER	48
			SER	49
		TERM1=TERM1/EM	SER	50
	4	SIGN==SIGN	SER	51
		SUM=TERM1	SER	52
		TERMETERMI	SER	53
		EN=0,	SER	54
	5	EN#EN+1.	SER	55
		TERM#TERM#VSQ/(EN*(EN+EM))	32 M	20
		SUM#SUM+TERM	SER SED	- 2/ EA
		IF (TEST,LE,DABS(RHULRENWICKH)) GU TO D	SER	59
Ç		TRUNÇATES INNER SERIES IF OUTER TERMANAINNEN TERM < 34001	SER	60
			869	61
		IF (Emelige) 90 10 1 Te (Teot ) 5 de ardan ( Arima) 60 10 1	SER	62
•		IF LIESISEESUADSERMULASUNJ, 60 10 1 Tounsites Outeo Serves IF NUTED TERMETINER RUM & S.E.7	SER	63
6	*	ALLERINA	SER	64
	9		SER	65
•		SERIES=S(2)=S(1)	SER	66
		RETURN	SER	- 67
		END	SER	68
		REAL FUNCTION FCT+8(X)	FCT	1
C		*************************	*FCT	2
C			FCT	3
C		FUNCTION FCT	FCT	4
C			FCT	5
C		PURPOSE	PCT	6
Ç		TO COMPUTE FCT(X)#EXP(=Z=V++d/(X+Z))/(X+Z)	P G T	7

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

1234	DESCRIPTION OF PARAMETERS X = THE DOUBLE PRECISION VALUE OF X FOR WHICH FCT IS COMPUTED SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED NONE METHOD FORTRAN EVALUATION OF FUNCTION ************************************	FCTTFFCTTT FCTTFFCCTTTT FFCTTFFCCTTTT FFCTTTTT	89011234567 111234567 1112322234 2550
		DL12	10
		12 12	20
	SUBROUTINE DOL12	DL12	40
		DL12	50
	PURPUSE	DL12	60
	IU LUMPIJIE INTEGRAL(EAP(DA)PETTIA), SUMMED UVER A FROM 0 TO INFINITY).	DL12	80
		DLIZ	90
	USAGE	DL12	100
	CALL DQL12 (FCT,Y)	DL12	110
	PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT	DL12	120
			1.00
	FCT - THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION	DL12	150
	BUBPRUGRAM USED,	DLIZ	160
	Y . THE RESULTING DOUBLE PRECISION INTEGRAL VALUE.	DL12	170
		DL12	180
	REMARKS		200
		DL12	210
	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	DLIZ	220
	THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPROGRAM FCT(X)	DL12	230
	MUST BE FURNISHED BY THE USER.	PL12	240
	METH(ID /	0112	220
	EVALUATION IS DONE BY MEANS OF 12-POINT GAUSSIAN-LAGUERRE	DL12	270
	QUADRATURE FORMULA, WHICH INTEGRATES EXACTLY,	0L12	280
	WHENEVER FCT(X) IS A POLYNOMIAL UP TO DEGREE 23.	DL12	290
	FUR REFERENCE, SEE	0115	300
	TEPTATE ACCOLTATED LAGIEDEE DOLENONTALS AND THE RELATED	0112	320
	GENERALIZED HERMITE PULYNOMIALS, IBM TECHNICAL REPORT	DL12	330
	TR00,1100 (MARCH 1964), PP,24-25,	DL12	340
		DL12	350
		0112	300
		OL12	390
		DLIZ	400
	DOUBLE PRECISION X, Y, FCT	DL12	410
		DL12	420
	X8,5704412104446692 D2	0L12	430
	1=401404/14014C0C4 U=13×FU1741	VUIE	

C

TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

Y= 2848796725098400 D2	DL12	450
	0112	1160
4844 200100102002051 Daltaceicv)	PLIE -	
XE,2215109037939701 D2	DLIE	470
Y=Y+,1342391030515004 D=8+FCT(X)	DL12	480
- 1711685518746226 D2	DI 12	490
	0112	EAA
AEA4*10084A3010240A10 De0#brify1	VLIE	500
Xm.1300605499330635 D2	DL12	510
VEVA ATASASSAFABIORA DESEFET(X)	DI 12	520
	01.4.3	5 1 4
X=, 462131684245687 D1	ULIE	230
Y=Y+,2032315926629994 D=3*FCT(X)	DL12	540
X# 6844525453115177 D1	DL12	550
	DI 12	5.6.0
TETT,20037/3341803310 UHERFUI(A)		500
X=,4599227639418348 D1	0715	570
Y=Y+,2010238115463410 D=1*FCT(X)	DL12	580
V- 2811751117741507 D1	DL 12	590
	0112	600
AE44 404445555110004 D=1HLCI(Y)	VLIE	000
x=,1512610269776419 D1	DL12	610
Y#Y+,2440820113198776 D0*FCT(X)	DL12	620
Y = 411757/8/6151307 D0	DI 12	630
	0.40	4 4 6
AEA+*21/12451281280 DO#LETEX1	ULIE	040
x=,1157221173580207 D0	0L12	650
YEY+ 2647313710554432 DO+FCT(X)	0L12	660
	DI 12	470
RETURN	VLIC	070
END	0115	60-
SUBROUTINE BESK(X.N.BK.IER)	BESK	410
	RESK	10
	BEBY	30
	DEON	<b>E</b> U
	REAK	30
SUBROUTINE BESK	BESK	40
	BESK	50
COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER	RBESK	60
	BESK	70
	DEOV	
	DEGN	00
CALL BESK(X,N,BK,IER)	REAK	90
	BESK	100
DESCRIPTION OF PARAMETERS	BESK	110
Y THE ADDIMENT OF THE K BESSEL FUNCTION DESIDED	AFSK	120
$\mathbf{x} = \{\mathbf{x}_{i} \in \{\mathbf{x}_{i}\}, \mathbf{y}_{i} \in \{$	BLOV	170
N THE UNDER UP THE & BESSEL FUNCTION DESTRED	05.30	130
BK #THE RESULTANT K BESSEL FUNCTION	BESK	140
IER-RESULTANT ERROR CODE WHERE	BESK	150
TEPRO NO EPROP	BESK	160
	RCBV	170
IEREI N IS NEGALIVE	OCON	170
IER=2 X IS ZERU OR NEGATIVE	BESK	180
IER=3 X .GT. 170, MACHINE RANGE EXCEEDED	BESK	190
IER#4 BK .GT. 10##70	BESK	200
	DEOK	310
		E10
REMARKS	BEBK	220
N MUST BE GREATER THAN OR EQUAL TO ZERO	BESK	230
	BESK	240
SUBRUNTINES AND FUNCTION SUBPROCEAMS REQUIRED	BESK	250
	REAK	34.0
NUNE	DESA	200
	DESK	270
METHOD	BESK	280
CUMPUTES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING	BESK	290
SERIES APPROXIMATIONS AND THEN COMPLITES N TH UNDER PUNCTION		100
USING BECHDELEE DELATION HAV HIEN LUMPUIGG N IN GAVER FUNGLIUM	BESK	
LISTING RECINKENCE RELATIONS	BESK	300
	BESK	310
RECURRENCE RELATION AND PULYNOMIAL APPROXIMATION TECHNIQUE	BESK BESK BESK	310 320
RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE As described by A.J.M.HITCHCOCK. POLYNOMIAL APPROXIMATIONS	BESK BESK BESK BESK	310 320 330
RECURRENCE RELATION AND POLYNOMIAL APPROXIMATION TECHNIQUE As described by A,J,M,HITCHCOCK, POLYNOMIAL APPROXIMATIONS To bessel functions of order 7500 and one and to related	BESK BESK BESK BESK BESK	310 320 330 340
RECURRENCE RELATION AND PULYNOMIAL APPROXIMATION TECHNIQUE As described by A,J,M,HITCHCOCK, POLYNOMIAL APPROXIMATIONS To bessel functions of order zero and one and to related functions of order zero and one and to related	BESK BESK BESK BESK BESK	310 320 330 340
RECURRENCE RELATION AND PULYNOMIAL APPROXIMATION TECHNIQUE AS DESCRIBED BY A, J, M, HITCHCOCK, 'POLYNOMIAL APPROXIMATIONS TO BESSEL FUNCTIONS OF URDER ZERD AND ONE AND TO RELATED FUNCTIONS!, M, T, A, C, , V, 11, 1957, PP, 86-88, AND G, N, WATSON,	BESK BESK BESK BESK BESK	310 320 330 340 350
RECURRENCE RELATION AND PULYNOMIAL APPROXIMATION TECHNIQUE AS DESCRIBED BY A,J,M,HITCHCOCK,'POLYNOMIAL APPROXIMATIONS TO BESSEL FUNCTIONS OF ORDER ZERO AND ONE AND TO RELATED FUNCTIONS!, M,T,A,C,, V,11,1957,PP,86-88, AND G,N, WATSON, 'A TREATISE ON THE THEORY OF BESSEL FUNCTIONS', CAMBRIDGE	BESK BESK BESK BESK BESK BESK	310 320 330 340 350 360

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TABLE 4.3—Listing of program for radial flow in a leaky artesian aquifer—Continued

~		AFRK TA	0
		HEAK TO	ň
C		95 6K /A	Ň
C		DESK 40	0
	DIMENSION T(12)	0535 42 8504 42	0
	BK=•0	DESK 45	0
	IF(N)10,11,11	863K 44	U
	10 IER#1	BESK 45	0
	RETURN	BESK 46	0
	11 IF(X)12,12,20	BESK 47	0
	12 IER#2	BESK 48	0
	RETURN	BESK 49	0
	20 IF(X=170,0)22,22,21	BESK SO	0
	21 IER=3	BESK SA	0
	RETURN	BESK 52	.0
	22 IER=0	BESK 53	0
	IF(X=1,)36,36,25	BESK 54	0
	25 A=EXP(~X)	BESK 55	0
	8=1./X	BESK 56	Q.
	C=SGRT(B)	BESK ST	0
	T(1)=0	BESK 58	0
	00 26 L=2,12	858K 59	0
	26 T(L)aT(L+1)+H	BESK 60	0
	TE (N=1)27.29.27	BESK 61	0
r	1	BESK 62	20
2	CONDUTE KO URTNE DOLYNONTAL APPROXIMATION	BESK 63	0
	COMPOSE NO DEING POLINGMINE APPROXIMATION	BESK 64	10
L.	57 CARA4() 3527(4)- (544443+T(1)+ 08811138+T(3)+.09139095#T(3)	BESK 65	0
	$ \begin{array}{c} \mathcal{L} \\ \mathcal$	BESK 66	0
	$z + \frac{1}{2} + $	BESK AT	0
	74 22/22021/01/01/01/1/22/22/21/41/22/22/22/22/22/22/22/22/22/22/22/22/22	BESK 68	۱ñ
		REAK TO	10
		859x 70	50
		BERK 71	~
	RETURN	0E0K 71	10
C		858× 73	10
C	COMPUTE KI USING PULYNUMIAL APPRUATMATIUN	DEGN 75	
C		BEEK VE	10
	29  G 1 = A = (1 + 2) = (1 + 2	BEEK 74	
	24,1/304324(4)4,204/8104(3)4,40743424(0)7,020300(1)47/11)	BEOK 70	
	3+ 99355424 (8)+ 20205244 (4)+ 52212044 (10)+ 0/0000014 (11)	DEGN //	10
	4-01082418*1(12))*C	OLON /D Afak 70	
	IF(N=1)20,30,31	DEGN /Y	50
	30 BK=61	DESK OU	0
_	RETURN	050× 01	
С		DEGN OF	
С	FROM KO,K1 COMPUTE KN USING RECURHENCE RELATION	DESK 83	0
C		DESK 84	10
	31 DO 35 J=2,N	5238 85	10
	GJ=2,*(FLUAT(J)=1,)*G1/X+G0	DESK 80	90
	IF(GJ=1,0E70)33,33,32	BESK 87	0
	32 IER=4	BESK 88	0
	GQ TO 34	BESK 89	10
	33 G0=G1	HESK 90	0
	35 G14GJ	BESK 91	0
	34 BK=GJ	BESK 92	0
	RETURN	BESK 93	0
	36 B=x/2,	BESK 94	0
	A=,5772157+ALOG(8)	BESK 95	50
	C≖8×8	BESK 96	0
	1F(N=1)37,43,37	BESK 97	0
C		BESK 98	10
C	COMPUTE KO USING SERIES EXPANSION	BESK 99	0

### TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS

TABLE 4.3-Listing of program for radial flow in a leaky artesian aquifer-Continued

C			BESK1000
-	37	GQ=-A	BESK1010
		x2J=1.	8E9K1020
		FACTEL.	8ESK1030
		HJE.0	BESK1040
		DU 40 J=1,6	BESK1050
		RJ=1./FLOAT(J)	BESK1060
		IF(X2J.LT.1.E.40) X2J=0.	8E3K1061
C		PREVIOUS STATEMENT ADDED TO IBM SUBROUTINE TO CORRECT UNDERFLOW	BESK1062
Ċ		PROBLEM ON WATFOR COMPILER	BESK1063
			<b>BESK1070</b>
		FACT#FACT*RJ*RJ	BESK1080
		L9+LM=CH	BESK1090
	40	$Go=Go+X^2J+FACT+(HJ=A)$	BESK1100
		IF (N)43,42,43	BESKIIIO
	42	BK≡G0	8ESK1120
		RETURN	BESK1130
C			BESK1140
C		COMPUTE KI USING SERIES EXPANSION	BESK1150
C			BESK1160
	43	X2J=8	BESK1170
		FACTEL	BESK1180
		HJ=1,	8ESK1190
		G1=1,/X+X2J*(,5+A-HJ)	8E3K1200
		DD 50 J#2,8	BESK1210
		23#22#22X=0	BESK1220
		RJ=1,/FLUAT(J)	BESK1230
		FACT#FACT+RJ+RJ	BESK1240
		HJEHJ+RJ	BESK1250
	50	G1=G1+X2J+FACT+(,5+(A=HJ)+FLOAT(J))	BESK1260
		IF(N=1)31,52,31	BESK1270
	52	BK=G1	BE8K1280
		RETURN	BF2K1540
		END	RF24130=

TABLE 5.2—Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds

C LST 2 Ċ LST 3 C PURPOSE TO CUMPUTE TYPE CURVE FUNCTION VALUES FOR H(U, BETA) --4 C LST 5 C MANTUSH, M.S., 1960, MODIFICATION OF THE THEORY OF LEAKY LST AGUIFERSE JOUR, GEOPHYS. RES., V. 65, ND. 11, P. 3713-3725. The computational algurithm has devised and phughammed by C LST 6 č LST 7 S.S.PAPADOPULUS. LST 8 ¢ 9 LST C INPUT DATA 1 CARD - FORMAT(2E10,5) LST 10 С USMALL - SMALLEST (BEGINNING) VALUE OF 170. LST с с 11 ULARGE - LARGEST(ENDING) VALUE OF 1/U. LST 12 2 CARDS + FURMAT(8E10,5) С LST 13 BOAT - 12 VALUES OF BETA (ZERO DR BLANK VALUES ARE C LST 14 Ċ PERMISSIBLE IF LESS THAN 12 DESIRED, WILL TERMINATE LST 15 AT FIRST ZERO OR BLANK VALUE). Subroutines and function subprograms required LST 16 LST 17 C H,DGG32,HUH,W = MUST BE INCLUDED IN DECK. DSGRT,DEXP,DERFC,DLUG = MUST BE IN COMPUTEN LIBRARY. LST 18 ¢ LST 19 C 20 Ç LST Ç 21 REAL+8 U, BETA, H LST 55 DIMENSIUN ARRAY(73,12), Y(73), BDAT(12), YNUM(6) LST 23 DATA YNUM/1.1.5,2.,5.,5.,7./ LST 24 IRU#5 LST 25 ÎPT≢6 LST 26 READ (IRD,6) USMALL,ULARGE READ (IRD,6) BDAT LST 27 LST 28 29 IBEGINBALUGIO(USMALL) LST IENDEALUGID(ULARGE)+,99999 LST 30 ILIMIT=(IEND=IBEGIN)+6+1 LST 31 LST 32 IF (ILIMIT+GT.73) ILIMIT=73 DU 1 1=1,12 LST 33 IF (BDAT(I),E0,0,) G0 TU 2 LST 34 LST 35 1 CONTINUE LST NR#15 36

	60.10.3	1.9.1
;		LST
	T T B C	LST
5	DG 4 I=1. ILIMIT	LST
	TEXP#IBEGIN+(T=1)/6	LST
	TIMIT+1	LST
	IF (II.67.6) 11m1	LST
	Y(1)#YNUM(11)+10.++TEXP	LST
	U=1./Y(1)	LST
		LST
	BETABUDAT(J)	LST
	ARRAY(1,J)=H(U,BETA)	LST
	WRITE (IP1.7) (BDAT(I).I=1.NB)	LST
	D(+ 5 I#1+1LIMIT	LST
	WRITE (1PT,8) Y(1), (ARRAY(1,1), J=1, N8)	LST
	STCP	LST
		LST
	FDRMAT (8E10,5)	LST
1	FURMAT (111, H(U, BETA) 1/101, 10X, 11 BETA1/1 1,6X, 11/U 11, 12E10,2)	LST
	FURMAT (1 1,E10,3,12F10,4)	LST
	END	LST
	DOUBLE PRECISION FUNCTION H(U,B)	н
	******	h H
		н
	FUNCTION H	H
	PURPOSE	Ħ
	TO CUMPUTE THE INTEGRAL OF	н
	EXP(=Y)+ERFC(B+SQRT(U)/SQRT(Y+(Y+U)})/Y SUMMED UVER Y	H
	FROM U TO INFINITY (FUNCTION H(U,BETA) OF HANTUSH).	н
	DESCRIPTION OF PARAMETERS	н
	BOTH DOUBLE PRECISION	H
	U = R++2+S/(4+T+TIME), (RADIAL DISTANCE SQUARED + STURAGE	<b>H</b>
	CDEFFICIENT / (4 + THANSMISSIVITY + TIME), U MUST BE > 1.0-60.	
	$B = (R/4) + (SQRT(K'+S'/(S'+T+S)+K'+S')/(S'+T+S))_{0}$	
	R', S', B' = HYD, CUND,, STURAGE CUEPP,, THICKNESS UP	M
	UPPER CONFINING BED,	
	K''', B'', B'' = HYD, CUND, J SIBRAGE CUEFF, J INICKNESS DP	
	PRAME CONFINING BEN <sup>4</sup>	
	T 5/2 1 7 1 5-40 MI COMBUTATION 18 MARS	2
	A, FUR O X 1,00000 NU COPPOTATION 18 MADE, TT EGG Raa, Man Alemanda (Trefs well Endetion)	
	III HURANIA	
		н.
	TV. FRECADONED FOR ARG > 40 AND HEILAN = HEIR.HN	н
	FOR U C Y C UB WHERE UB 14 THE U CODERRONNING TO AND # 40	х н н
	SINCE HIGHES C WORN THEN FOD OR SINCE HUAN TO AND TO	- II
	ERECTARGES A FOR ARG C 2. FULL AND HOURS HIS HOURS	н
	WHERE DUB IS THE U CORRESPONDING TU ARG # 2-F=10-	H
	IF BUR S 10. H(U.B) = INTEGRAL FROM UR TO 10.	H
	IF USE 2 10 HUBY T THERE FROM US TO DUR + WOULD	н
	The second state and a supported to an an and a second state of the second state of th	н
	***************************************	ь н
	THELICIT REAL AGAMA, DAZ)	н
		н
	EXTERNAL HUB	н

,

TABLE 5.2 nfining beds—

```
EXTERNAL HUB
  ບບບ⊭ບ
  888*8
  IF (U.GT.1.0+60) GO TU 1
WRITE (6,7)
  STOP
1 IF (B.EQ.0.0) GO TO 5
IF (U.GT.10.0) GO TO 5
  80=8+8+0
  IF (B.GT.1.0.AND.BU.GE.300.0) GU TU 6
 H1=+(UU8)
  UPEUUB
5 H2=0.0
  XL≖UB
3 XU=10,*XL
  IF (XU,GE,UP) XUBUP
Call Dug32(XL,XU,HUB,AREA)
H28H2+AREA
XL=XU
IF (XL_LG,UP) GU TO 4
GU TU 3
4 MH11+H2
  RETURN
5 H=H(U)
  RETURN
```

H 36

H

н 38 39

н 40

н 41 н 42

н 43 H H

н 51

н 52

н 53

м 54

н 55

н 56 57 н н 58

н 59 н 60 н 61 н 62

н

н 63

64 65 H

44

45 46 н н 48 н Ħ 49 н 50  $\mathbb{C} \subset \mathcal{A}_{\mu}$ 

### TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS

 TABLE 5.2—Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds—

 Continued

	6	M#0.0 Return	H R	66 67
C	7	FORMAT ('0','U TUO SMALL FOR COMPUTATION') End		68 69 70=
c		SUBROUTINE DAG32(XL,XU,FCT,Y)	DuG Dug	1 2
ç			DQG	3
с с		SUBROUTINE DUG32	000	5
ç			DUG	<b>b</b> .
ç		PURPUSE TO COMPUTE INTEGRAL(FCT(X), SUMMED UVER X FROM XL TO XU)	DUG	8
č			DWG	9
ç		USAGE CALL DOCTO (XL.XII.ECT.V)	DUG Dug	10
č		PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT	DUG	12
ç			000	13
č		XL - DOUBLE PRECISION LOWER BOUND OF THE INTERVAL.	DwiG	15
Č		XU = DOUBLE PRECISION UPPER BOUND OF THE INTERVAL.	DaG	16
C C		SUBPROGRAM USED.	DUG	17
č		<ul> <li>THE RESULTING DOUBLE PRECISION INTEGRAL VALUE,</li> </ul>	DGG	19
ç		HFMARK &	006	20
č		NONE	Dug	22
ç		SUMPOUTTNES AND SHAFTTON SUBSDUCSAME DESULTED	000 000	23
č		THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPRUGRAM FCT(X)	DUG	25
ç		MUST BE FURNISHED BY THE USER,	DOG	26
ĉ		METHUD	Dag	28
č		EVALUATION IS DUNE BY MEANS OF 32-POINT GAUSS GUADRATURE	DQG	29
C C		FURMULA, NHICH INTEGRATES POLYNOMIALS UP TU DEGREE 63 FXACTLY, FOR REFERENCE, SEF	DOGG	30
č		V.I.KRYLOV, APPROXIMATE CALCULATION OF INTEGRALS,	DUG	32
ç		MACMILLAN, NEW YORK/LUNDON, 1962, PP.100=111 AND 337=340.	000	33
č			066	35
C		0/11HIE BRECTRENN VI. VI. V.A.H.F.ECT	000	36
		A#,500*(XU+XL)	DWG	38
			DUG	39
		y=,3509305004735048D=2+(FCT(A+C)+FCT(A=C))	DQG	41
		C#, 4928057557726342D0*8	DUG	42
		Y#Y* e 013/14/363452040*2*(FU!(A*U)*FU!(A#U)) C#.482361127793753200*8	Duig	43
		Y=Y+,1269603265463103D=1+(FCT(A+C)+FCT(A=C))	DOG	45
		C=_4674530379688698D0+8 Y=Y+_1713693145651072D+1+(FCT(A+C)+FCT(A=C))	Duig Duig	40 47
		C=,4481605778830261D0AB	DWG	48
		Y=Y+,2141794901111334D=1+(FCT(A+C)+FCT(A=C)) C= 42468380688608068	DeG	49 50
		Y=Y+ 2549902963116609D=1+(FCT(A+C)+FCT(A=C))	DOG	51
		Ce, 397241897983971200+8	000	52
		C=,3660910593701448D0+B	DUG	54
		Y=Y+_3291111138818092D=1*(FCT(A+C)+FCT(A=C)) C=	000 000	55
		Y=Y+,3617289705442425D=1+(FCT(A+C)+FCT(A=C))	UGG	57
		C#,293857878620381200+B	000	58
		C=,2534499544661147D0+B	DUG	60
		Y=Y+,4165596211347338D=1+(FCT(A+C)+FCT(A=C))	DAG	61
		u=,2108/383608351/700+8 y=y+,43826046502201910+1+(FCT(A+C)+FCT(A=C))	DwG	63
		C#,1659343011410638D0+6	DUG	64
		C=,1196436811260685D0+H	Dec	66
		YBY+,46922199540402280-1+(FCT(A+C)+FCT(A-C))	DOG	67
		C=,/223340074134020#1#8 Y#Y+,47819360039637430=1#(FCT(A+C)+FCT(A=C))	DUG	69
		C=,2415383284386916D=1*8	DUG	70
		Y=B+(Y+,4827004425736390D+1+(FCT(A+C)+FCT(A=C))) PFTUPN	DUG Dug	71 72
		END	Dug	73-
_		DOUBLE PRECISION FUNCTION MUB(X)	нив	1
C C		***************************************	*HU8	2
ç		FUNCTION HUB	HUB	4
ç		PURPUSE	NUB	5
č		DESCRIPTION OF PARAMETER	HUB	7
C		X - DUUBLE PRECISION, POINT AT WHICH INTEGRAND IS EVALUATED.	HUB	8

 TABLE 5.2—Listing of program for radial flow in a leaky artesian aquifer with storage of water in the confining beds—

 Continued

ç	METMOD	нин	q
c	FURTHAN EVALUATION OF FUNCTION.	нив	10
С		MUB	11
С	****	HUB	15
	IMPLICIT KEAL+8(A++,0+2)	HUB	13
	COMMON UDU, BHB	HuB	14
	ARG#DSGKT((BHB+BHB+BHB+DDD)/(X+X+X+X+UUD))	ньв	15
	HUB#DEXP(+x)+DERFC(ARG)/x	HUB	16
	RETURN	Muß	17
	END	#118	18-
	DOUBLE PRECISION FUNCTION W(U)	۴U	1
с	****	<b>≁</b> U	5
С		≁U	3
С	FUNCTION -	ъU	4
C .	PURPUSE	₩U	5
C .	TO EVALUATE THE WELL FUNCTION OF THEIS,	ъU	6
C	DESCRIPTION OF PARAMETER	۳U	7
C	U - DOUBLE PRECISION, ARGUMENT FUR WELL FUNCTION.	ΝU	8
C		•0	9
C	*******************	₩U	10
	IMPLICIT REAL+B (A+H;U+2)	wυ	11
	IF (U,LE,0,0) GO TO 2	τŲ	12
	IF (U.GT.100.) GO TO 3	6. E I	13
	IF (U,GE,1,0) GU TG 1	ΨU	14
	w==,57721566+U+(,99999193+U+(=,24991055+U+(,05519968+U+(=,00976004	жÜ	15
	1+,00107857+0))))=0L(G(U)	×υ	16
	GR TU 4	×U	17
	1 ENUM#DEXP(+U)*(,2677737343+U*(8,6347608925+U*(18,0590169730+U*(8,5	wU	18
	1733287401+0))))	*U	19
	DEN#U#(3,9584969228+U#(21,0996530827+U#(25,6329561486+U#(4,5753223	. wD	50
	1454+03)))	wit	51
	WEENUM/DEN	*1)	22
	GU TU 4	×U	23
	2 WRITE (6,5) U	₩U	24
	STOP	WLF	25
	5 WEU.0	*U	20
	4 RETURN	NU.	21
C		* U	20
	S FURMAT ('U')SX,'*(U) WIT DEFINED FUR UE',1PD35,8)	• U	29
		- C	20.

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer

*****	PPL 1
	PPL 2
PURPOSE	PPL 3
TO COMPUTE TYPE CURVE FUNCTION VALUES FOR PARTIAL PENETRATION	PPL 4
IN A LEAKY AQUIFER USING EQ. 73 OF HANTUSH, M.S., 1964,	PPL 5
HYDRAULICS OF WELLS IN CHOW, VEN TE, ADVANCES IN HYDRUSCIENCE.	PPL 6
VUL. 11 ACADEMIC PRESS, NEW YURK, P. 281-442.	PPL 7
INPUT DATA	PPI A
1 CARD - FORMAT (365.1.15.2F10.4)	PPI 9
A . AQUITER THICKNESS	PPI 10
F DEPTH, RELOW TOP OF ADUTEER, TO BOTTOM OF PUMPING	PPI 11
WELL ODEN	PPI 12
D = DEPTH, AFINE TOP OF ADUTFER, TO TOP NE PUMPING WELL	PPI 13
SCREEN	PPI 14
NIN - NUMBED OF ORSEDVATION WELLS OF DIFICMFTEDS TIMES	PPI 15
NUMBER OF VALUES OF VEZZO A FILLONCIERO TICES	
CONDER OF TREES OF RETRIE OF USE FOR WHITE COMPLICATION IS	DD 14
LARGE - LARGERT VALUE DE 1711 EDD WHITH COMPLITATION ID	PPL 10
LARGE - LARGEST THERE OF 170 FOR HOLEN CONFOLMTION 13	
2 FARE - FORMATIRE (A E)	
E GARDA Y FURNAI (DELU,3) Holt (3) White of Dird (00, 7000 Values Should St	FFL 21
ELAST - 12 VALUES OF RYDRY NON ZERU VALUES SHOULD BE	PPL 22
FIRST, WILL TERMINATE AT FIRST ZERU (UR BLANK) VALUE,	PPL 25
NUM CARDS (ONE FUR EACH UDS, WELL OF PIEZUMETER AND FUR EACH	PPL 24
VALUE OF R#SURT(RZ/RR) - FURMAT (SPS.1)	PPL 25
R = RADIAL DISTANCE FROM PUMPED WELL TIMES SGRT(KZ/KR).	PPL 26
LPRIME - DEPTH, BELUW TOP DF AQUIFER, TO BOTTOM OF UBS.	PPL 27
WELL SCREEN (ZERO FOR PIEZOMETER)	PPL 28
DPRIME - DEPTH, BELOW TOP OF AQUIFER, TO TUP OF OBS, WELL	PPL 29

1995 B.

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

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Ĉ
               SCREEN (TOTAL DEPTH FOR PIEZUMETER)
                                                                              PPL
                                                                                   30
      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                                                                              PPL
C
                                                                                   31
                                                                              PPL
Ĉ
         DQL12, SERIES, BESK, FCT, L, FL
                                                                                   32
C
                                                                              PPI
                                                                                   33
C
      ******************
                                                                                   34
      REAL+8 U.V
                                                                              PPL
                                                                                   35
      REAL+4 LILBILPBILPRIMEILARGE
                                                                              PPL
                                                                                   36
      DIMENSION ARRAY(55,12), ARG(6), BDAT(12), Y(55)
                                                                              PPL
                                                                                   37
                                                                              PPL
      DATA ARG/1.,1.5,2.,3.,5.,7./
                                                                                   38
      DATA ARRAY/660+0./. Y/55+0./
                                                                              PPL
                                                                                   39
                                                                              PPL
      IRD#5
                                                                                   40
      IPT#6
                                                                              PPL
                                                                                   41
      READ (IRD,9) B,E,D,NUM, SMALL, LARGE
                                                                              PPL
                                                                                   42
      READ (IRD,14) BDAT
                                                                              PPL
                                                                                   43
      00 1 1=1,12
                                                                              PPL
                                                                                   44
      IF (BDAT(I),EQ.0.) GD TO 2
                                                                              PPL
                                                                                   45
    1 CONTINUE
                                                                              PPL
                                                                                   46
                                                                              PPL
      NB=12
                                                                                   47
      GO TO 3
                                                                              PPL
                                                                                   48
    2 N8=I=1
                                                                              PPL
                                                                                   49
    3 L8=E/8
                                                                              PPL
                                                                                   50
      DB=D/8
                                                                              PPL
                                                                                   51
      IBEGIN=ALOG10(SMALL)
                                                                              PPL
                                                                                   52
      IENDEALUG10(LARGE)+,1
                                                                              PPL
                                                                                   53
                                                                              PPL
      JLIMIT=IEND=IBEGIN
                                                                                   54
      IF (JLIMIT,GT,9) JLIMIT=9
                                                                              PPL
                                                                                   55
      ILIMIT=6+JLIMIT+1
                                                                              PPL
                                                                                   56
      DD 8 K=1, NUM
                                                                              PPL
                                                                                   57
      READ (IRD,9) R, LPRIME, DPRIME
                                                                              PPL
                                                                                   58
      RB=R/8
                                                                              PPL
                                                                                   59
      LPB=LPRIME/B
                                                                              PPL
                                                                                   60
      OPB#OPRIME/6
                                                                              PPL
                                                                                   61
      DD 4 I=1,ILIMIT
                                                                              PPL
                                                                                   62
      INDEX=(I=1)/6
                                                                              PPL
                                                                                   63
      IEXP#IBEGIN+INDEX
                                                                              PPL
                                                                                   64
      II=I=INDEX+6
                                                                              PPL
                                                                                   65
      Y(I)=ARG(II)+10.**IEXP
                                                                              PPL
                                                                                   66
      U=1./Y(I)
                                                                              PPL
                                                                                   67
      DD 4 J=1,NB
                                                                              PPL
                                                                                   68
      BETA#BDAT(J)
                                                                              PPL
                                                                                   69
      VEBETA/2.
                                                                              PPL
                                                                                   70
    4 ARRAY(I, J)=L(U, V)+FL(U, R8, BETA, L8, D8, LP8, DP8)
                                                                              PPL
                                                                                   71
                                                                              PPL
      IF (LP8=0.) 5,5,6
                                                                                   72
    5 WRITE (IPT, 10) DPB, RB, LB, DB
                                                                              PPL
                                                                                   73
      GO TO 7
                                                                              PPL
                                                                                   74
    O WRITE (IPT, 11) LPB, DPB, RB, LB, DB
                                                                              PPL
                                                                                   75
    7 WRITE (1PT, 12) (BDAT(1), I=1, NB)
                                                                              PPL
                                                                                   76
      DU 8 I#1,ILIMIT
                                                                              PPL
                                                                                   77
      WRITE (IPT,13) V(1), (ARRAY(1,J), J=1, NB)
                                                                              PPL
                                                                                   78
    8 CUNTINUE
                                                                              PPL
                                                                                   79
      STUP
                                                                              PPL
                                                                                   80
Ĉ
                                                                              PPL
                                                                                   81
Ĉ
                                                                              PPL
                                                                                   82
    9 FORMAT (3F5.1, 15, 2E10.4)
                                                                              PPL
                                                                                   83
   10. FORMAT (111, 1W(U,R/BR)+F(U,R/B,R/BR,L/B,D/B,Z/B), Z/B=1,F5,2,1, SQPPL
                                                                                   84
     1RT(KZ/KR) +R/8=',F5.2,', L/8=',F5.2,', D/8=',F5.2)
                                                                              PPL
                                                                                   85
   11 FORMAT ('11', 'W(U,R/BR)+F(U,R/B,R/BR,L/B,D/B,L+'/B,D'+/B), L+'/B=+,PPL
                                                                                   86
     1F5.2,1, D11/B=1,F5.2,1, SQRT(KZ/KR)*R/B=1,F5.2,1, L/B=1,F5.2,1, D/PPL
                                                                                   87
     28=1,F5,2)
                                                                              PPL
                                                                                   88
   12 FURMAT ('0',9x,') R/BR'/! ',5x,'1/U 1',12E10,2)
13 FURMAT (' ',E10,3,12F10,4)
                                                                              PPL
                                                                                   89
                                                                              PPL
                                                                                   90
   14 FORMAT (8E10.5)
                                                                              PPL
                                                                                   91
      END
                                                                              PPL
                                                                                   92.
```

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

	REAL FUNCTION FL+4(U,RB,BETA,LB,DB,LPB,DPB)	FL	1
	*************************	FL	2
		FL	3
	FUNCTION FL	FL	4
		FL	5
	PURPUSE	FL	6
	TO COMPUTE DEPARTURES FROM HANTUSH-JACOB LEAKY AQUIFER CURVE	FL	7
	CAUSED BY PARTIAL PENETRATION OF PUMPED WELL.	FL	8
	USAGE	FL	9
	FL(U,RB,BETA,LB,DB,LPB,OPB)	FL	10
	DESCRIPTION OF PARAMETERS	₹L	11
	ÁLL REAL, U DOUBLE PRECISION	FL	12
	U = R**2*S/4*T*TIME (RADIAL DISTANCE SQUARED * STURAGE	FL	13
	COEFFICIENT / 4ATRANSMISSIVITY & TIME	FL	14
	RB = R/H ( RADIAL DISTANCE / AQUIFER THICKNESS )	FL	15
	BETA - R*SQRT(K!/B'T) - (RADIAL DISTANCE * SQUARE ROUT	FL	16
	(HYD. COND. OF CONFINING BED/THICKNESS OF CONFINING	FL	17
	BED + TRANSMISSIVITY OF AQUIFER))	FL	18
	LB - L/B ( FRACTION OF AQUIFER PENETRATED BY PUMPED WELL)	FL	19
	DB - D/B ( FRACTION OF ADUIFER ABOVE PUMPED WELL SCREEN)	FL	20
	LPB - LIVE (FRACTION OF AGUIFER PENETRATED BY USS. WELL, ZERO	FL	21
	FOR PIFZOMETER)	FL.	22
	DP5 - DI/B (FRACTION OF AQUIFER ABOVE OBS. WELL SCREEN, TOTAL	FL	23
	DEPTH FOR PIEZOMETER)	FL	24
	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	FL	25
	DOL12.SERTES.BESK.FCT.L	FL	26
	METHOD	FL	27
	SUMS THE SERIES THROUGH N+PT+PZH FO 20	FI	28
	cone the period torgon out this ed to	FI	20
		FL	30
		FI	31
		-	12
		FI	22
		FI	34
	RETSOBRETA+RETA	F L	15
		FE	36
		FI	37
		FL	18
		FL	20
	IF TERMEND WELL AD DIEZAMETED	FI	40
	CITERS IN REL ON FIELDMEICH	6 G	41
•		5	~~~
2	N-ROTIE Subscripting the tension of tension	F LL	113
		ты. Ба	<u> </u>
	TO INFO (61979) VO IO J TO INFO (61979) VO IO J	гц. Бі	<u>4</u> 5
	THE TILL ALL OF THE TYLE	гц. 231	
	AFE 197779 Gund Gund / Sta / Maditu & N = Sta / Maditus / N = Sta / Maditus / T = Sta /	ть. Сі	117
	en to 5 en to 5	r L. Li	
1		ГЦ 20	40
3		F L	<b>E</b> 0
"		F 6.	50
4		7 L.	21
e	Navet	гц. с,	72
2	N=0(DT/9510)	Г Ц. В 1	23
	YRGURILDEIGUPNENEFIERDGUJ/C.	гц. с)	34 EE
	IF TYSTEITS GEDTED WEN VN10	гц. с.	22
	INDICATES SERTES AUEN ANTA	F 6.	70
	X=L_LUYY}/N 010400042/0781020200000000000000000000000000000000	РЦ. С)	51
	ov 40 s Johegomi/giu/wakifolagiw/wakind)la/giu/wakifedlagiu/wakinkolladla/waki	г Ц. с)	20
		F 6	<b>77</b>
0	rL=;{vc94443U7/([Lt90)]*([r000r0])	гЦ. с/	5U 44
7		76	01
	ENU	PL.	_ 0 < ■

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TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

		REAL FUNCTION L+4(U,V)	L	1
C		***********************	L	2
Ç			Ļ	3
C C		PUNCTION L	ь 1	4
č		PURPOSE	Ľ	6
č		TO COMPUTE THE INTEGRAL( EXP(+Y+V++2/Y)/Y) SUMMED OVER Y FROM	ĩ.	7
C		U TO INFINITY(WELL FUNCTION FOR LEAKY AQUIFERS),	L	8
Ç		DESCRIPTION OF PARAMETERS	L	9
C		BUTH DOUBLE PRECISION	<u> </u>	10
C		U = R##2#3/4#T#TIME (RADIAL DISTANCE SQUARED # SIUHAGE	<b>.</b>	11
C		LUCFFICIENT / HRIKANOMIJOIVITY R FIME V _ D/JARODT/VI//TARTYANOMIJOIVITY R FIME	- La 1	16
ř		(HVD. COND. OF CONFINING AFD/TRANSMISSIVITYATHICKNESS	ĩ	14
č		OF CONFINING BED)	ū	15
Č		SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	L	16
C		DQL12, SERIES, BESK, FCT	۰ <b>L</b>	17
C		METHOD	Ļ.	18
Ç		IN THE FULLOWING FEEXP(=Y=V*Z/Y)/Y	L.	14
5		(1) UP-1, USES A GAUSSIANWEAGUERRE GUADRATURE FURNULA TU Evaluate tateodal dei egon 11 to tae	140 1	21
č		(2) V++2 <u<1. evaluate="" ghl="" integral(f)<="" quadrature="" td="" the="" to="" uses=""><td>Ľ</td><td>22</td></u<1.>	Ľ	22
č		FROM DNE TO INF AND A SERIES EXPANSION TO EVALUATE INTEGRAL(F)	ũ	23
Ĉ		FROM U TU ONE,	L	24
C		(3) U<1, U<=v++2, USES THE REPRESENTATION INTEGRAL(F) FROM U	L.	25
C		TO INF. = 2*KO(2*V)+INTEGRAL(F) FROM V**2/U TO INF.	Ļ	26
C		EVALUATES THE ZERU URDER MUDIFIED BESSEL FUNCTION OF SECOND	E.	21
C C		VIND WILL TOW SUBKONITHE' EAVENWIED INTERNET DI ARE ANAN'	- <b>L</b>	20
č		*********	Ľ	30
•		EXTERNAL FCT	Ē.	31
		REAL+B U, V, Z, F, VV, SERIES	L	32
		CUMMON /C1/ VV,Z	L	33
			L.	34
r		ir (Umlej l <i>icic</i> Checke te inci	i i	37
•	1		ĩ	37
	•	IF (Z=1,) 3,4,4	Ē	38
C		CHECKS IF V#+2/U < 1	L	39
	5	Zeu	L	40
		CALL DQL12(FCT,F)	L.	41
~		LEP Thtechal in the evaluated by called a cheque onaduating	Г	42
L		INTEGRAL U ID INF. EVALUATED DI GRUSSALAGUERRE GUADRATURE	- G	43
	3	Z=1.	Ē	45
		CALL DQL12(FCT,F)	Ũ	45
		L=F+SERIES(U,V)	L	47
¢		INTEGRAL 1 TO INF, BY G-L QUAD,, INTEGRAL U TO 1 BY SERIES EXP,	L	48
			<u> </u>	49
	4	THUYERSTAN TALI REGKTTUDV.A. RK. TERN	1 1	51
		CALL DOL12(FCT.F)	ĩ	52
		L=2,+BK=F	Ē	53
C		2KO(2V)=INTEGRAL V**2/U TO INF.	Ē	54
	5	RETURN	Ļ	55
			Ļ	56=
r		REAL FUNCTION SERIES#S(U/V)	SER SED	1
č			SFD	2 1
č		FUNCTION SERIES	SER	4
C			SER	5
Ç		PURPOSE	SER	6

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 TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

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	TO EVALUATE 9/11-9/01. WHERE S IS A SERIES EXPANSION OF	SER	7
	THE CALL AND	SFR	Å
	INTEGRAL (EXPLOYER ACTION AND A STATE OF STATE AND A STATE	8£ D	ă
	$(F(M)+SUM, N=0 TD INF_{+}(V+*(Z+N)/((N))*(M+N)T))$ where runs		
	LOG(U) IF MHO AND B ((=1)**M/M)*(U**M=(V**Z/U)**M) IF M>0.	SEN	10
	DESCRIPTION OF PARAMETERS	8ER	11
		SER	12
	DUTH DUDDE PRECIDION	950	11
	U + REALASTATINE (RADIAL DISTANCE SQUARED + STORAGE		
	CUEFFICIENT / 4×TRANSMISSIVITY + TIME	SER	14
	V - R/2+SORT(K!/(T+B!))-=ONE=HALF RADIAL DISTANCE+SQUARE ROOT	SER	15
	(HYD. COND. OF CONFINING BED/TRANSMISSIVITY+THICKNESS	SER	16
		8FR	17
		GE D	
	SUBROUTINES AND FUNCTION SUBPRUGRAMS REQUIRED	954	10
	NONE	SEX	14
	METHOD	SER	20
	SUBMATION 19 TERMINATED FOR THE INNER SERIES WHEN A TERM	SER	21
	PERMANENT TO THANK E E-TAN AND FOR OUTED GEDTES WHEN A TERM	SER	2.2
	DECUMES LESS THAN SECTION AND FOR DUTER SEATED WHEN H SECTION	900	31
	BECOMES LESS THAN 5.E=7	OLC P	6.2
			24
	· ************************************		25
	PEAL +A DI MG. DARS. S(2). VIM. IN		26
	REAL & TEAT IN THE EN EN STALL OTHER TON N. VON. VONLOHIH TRAN TRANT TO	015-75	27
	KEVTAO IESIIN'NULEUIEVIOUIIJOUIJIOUIAIAOMIAOODIUURE AMBARASE A	660	38
	TESTES UP07	OL H	20
		SAM	29
	1.1.8.1. ·	SER	30
		SER	31
	DU G IGIE Evaluate acade to love livit - 1 and 10060 itmit - 1	0.0	12
	EVALUATES SERJES FOR LUMER LIMIT # U AND OPPER LIMIT # 1		36
	IF (I_EQ_2) U=1.	JER	دد
	UM=1.	SER	34
	FME#1	SER	35
		SFR	36
		859	17
	STCN==1.	Or o	31
	VUM=1.	SER	30
	VSQUEVSQ/U	SER	39
•	EMBENel.	SER	40
		SFR	41
		020	113
	CHECKS FUR Ma0	0 E R	46
2	RMUL≥DLŪG(U)	SER	45
	TERM1=1.	SER	44
	60 10 4	8ER	45
		SER	46
2		850	47
	IF (VUM+LI+1+D=30) VUM#0+	JER Den	NH /
	VUM#VUM#VSQU	SER	48
	RMUL=(UM=VUM)/EM	SER	49
	TERM1=TERM1/FM	SER	50
"	γματομ=γματογγγγγγγγγγγγγγγγγγγγγγγγγγγγγγγγγγγ	SER	51
4	2 TAUR - 2 T	000	é s
	SUMAICKUI	OCR OF T	26
	TERMATERM1	SER	22
	EN#0.	SER	54
E	FNEEN+1	SER	55
3		SEP	54
			27
	SUMESUM+TERM	JER.	21
	IF (TEST,LE,DABS(RMUL*EN+TERM)) GU TU 5	SER	20
	TRUNCATES INNER SERIES IF DUTER TERM*N*INNER TERM < 5.E=7	SER	59
	SUM12SUM1+SIGNERMULESUM	SER	60
		SFD	61
			4.7
	IF (TEST, LE, DABS(RMUL+SUM)) GU IU I	JEN	0 ¢
	TRUNCATES DUTER SERIES IF DUTER TERM*INNER SUM < 5.4-7	SER	63
6	S(I)=SUM1	SER	64
		SER	65
		SFP	66
	328423-3(2)-3(1)		47
	RETURN	JCK	0/
	END	SER	68-

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 TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

	REAL FUNCTION FCT+8(X)	FCT	1
	***************************************	*r61 507	÷
	FUNCTION FOT	F 6 1	2
		FOT	4
	DINDORS	FCT	2
	TU FUNDUIS FFT(V)-FV0/-7-V2/(V-7))//-27)	FUI MCT	5
	TO COMPANY TO CAPENY AND TARESTANDING TO COMPANY TA COMPANY TA COMPANY TO COMPANY TA COM	F (L) 807	
	Y THE DURIE DESCRITION VALUE DE V ERO MATCH EST TO COMPACE.	F 6 7	0
	A THE DUDDE PRESIDIN TRUE OF A FUR WHICH FUI IS SUMPOIED.	FOT	
	NONE	FUI FOT	10
		FG1	11
	EDEDAN EVALUATION OF FUNCTION	FOI	12
	FURTHER EVALUATION OF FURGIION	FCT	13
		- FCT	14
		FCT	12
		F G T	10
		PCT	11
1		FCT	10
•	FD TD 4	FCT	20
2		FCT	24
-	TF (P=5-01) 3-3-1	FCT	22
3		FCT	26
ŭ	RETURN	Frt	24
-	END	FCT	25-
	SUBROUTINE DOI 12(FCT.Y)	DI 12	180
		0112	10
		. DL 12	20
		DL12	30
	SUBROUTINE DOL12	DL12	40
		DL12	50
	PURPOSE	PLIZ	60
	TÚ COMPUTE INTEGRAL(EXP(=X)+FCT(X), SUMMED OVER X	DL12	70
	FROM O TO INFINITY).	DL12	80
		DL12	90
	USAGÉ	0L12	100
	CALL DUL12 (FCT,Y)	DL12	110
	PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT	OLIS	120
		DL12	130
	DESCRIPTION OF PARAMETERS	DL12	140
	FCT - THE NAME OF AN EXTERNAL DOUBLE PRECISION FUNCTION	DL12	150
	SUBPROGRAM USED.	DL12	160
	Y - THE RESULTING DOUBLE PRECISION INTEGRAL VALUE.	0112	170
		UL12	180
	KEMAKKS	DL12	190
	NUNZ	DL12	500
		DLIS	210
	SUDRUUTINES AND FUNCTION SUBPRUGRAMS REQUIRED	UL12	220
	THE EXTERNAL DUUBLE PRECISION FUNCTION SUBPRUGRAM FCT(X)	0112	230
	MUDI DE FUKNISHEU DY INE USER,	0112	240
		UL12	250
	TETHUN	UL12	260
	EVALUATION IS DUNE BY MEANS OF 120POINT GAUSSIAN & LAGUERRE	UL12	270
	WURVERIUME FURMULA, MALUA INTEGRATES EXACTLY,	0115	280
	HIGHEVER FEILX) IN A FULTHUMIAL UP TU DEGREE KI.	0112	540
	FUR REFERENCES JEE Rhamarchiensen and tabler de Jeron and Causatan Metouro of	ULIZ	300
	JERUTURENTERARET TADLEJ UF ZERUS AND GAUSSIAN WEIGHTS UF		510
	CENERAL TORN HERMITE DOLYNOMIALS AND THE RELATED		320
	TRAD (100 (MARCH 1044)) BR 34-35		220
	INAN'IION (WAKPU JAOHI) LL <sup>8</sup> CH+G2*		340
			740
		PLIC .	201

DL12 370 DL12 390 DL12 400 DOUBLE PRECISION X, Y, FCT 0L12 410 DL12 420 DL12 430 X=.3709912104446692 D2 DL12 440 Y#.814807746742624 D=15\*FCT(X) DL12 450 X=.2848796725098400 D2 Y=Y+, 3061601635035021 D=11\*FCT(X) DL12 460 DL12 470 x=,2215109037939701 D2 Y#Y+,1342391030515004 D=8+FCT(X) DL12 480 X#,1711685518746226 D2 DL12 490 Y=Y+,1668493876540910 D=6\*FCT(X) DL12 500 DL12 510 X=,1300605499330635 D2 DL12 520 Y=Y+\_836505585681980 D=5\*FCT(X) DL12 530 X=,962131684245687 D1 DL12 540 DL12 550 DL12 560 DL12 570 Y=Y+,2032315926629994 D=3\*FCT(X) X=,6844525453145177 D1 Y=Y+,2663973541865316 D=2+FCT(X) x=,4599227639418348 D1 DL12 580 Y=Y+,2010238115463410 D=1+FCT(X) DL12 590 x=,2853751337743507 D1 DL12 600 Y=Y+,904492222116809 D=1+FCT(X) UL12 610 x=,1512610269776419 D1 DL12 620 Y=Y+,2440820113198776 D0\*FCT(X) DL12 630 X=,6117574845151307 D0 Y=Y+.3777592758731380 D0+FCT(X) DL12 640 DL12 650 x=,1157221173580207 D0 DL12 660 Y=Y+\_2647313710554432 D0\*FCT(X) DL12 070 RETURN DL12 68-END SUBROUTINE BESK(X, N, BK, IER) BESK 410 BESK 10 50 BESK 30 BESK 40 SUBRUUTINE BESK BESK 50 COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDERBESK 60 BESK 70 BESK 80 USAGE BESK 90 CALL BESK(X,N,BK,IER) **BESK 100** BESK 110 DESCRIPTION OF PARAMETERS X •THE ARGUMENT OF THE K HESSEL FUNCTION DESIRED BESK 120 -THE ORDER OF THE K BESSEL FUNCTION DESIRED BESK 130 BK -THE RESULTANT & BESSEL FUNCTION BESK 140 BESK 150 IER-RESULTANT ERROR CODE WHERE IER#0 ND ERRUR BESK 160 IER#1 N IS NEGATIVE BESK 170 X IS ZERO OR NEGATIVE IER=2 . BESK 180 IER#3 X .GT. 170, MACHINE RANGE EXCEEDED IER#4 BK .GT. 10\*\*70 BESK 190 BESK 200 BESK 210 REMARKS BESK 220 N MUST BE GREATER THAN OR EQUAL TO ZERO BESK 230 BESK 240 SUBROUTINES AND FUNCTION SUBPRUGRAMS REQUIRED BESK 250 BESK 260 NONE BESK 270 BESK 280 METHUD COMPUTES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING BESK 290 SERIES APPROXIMATIONS AND THEN COMPUTES N TH UNDER FUNCTION BESK 300 USING RECURRENCE RELATION. BESK 310

TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

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 TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer.—Continued

			SC OF	7 3 0
C .		RECURRENCE RELATION AND PULTNUMIAL APPRUXIMATION TECHNINUE	DESN	JEV
С		AS DESCRIBED BY A.J.M.HITCHCUCK, POLYNOMIAL APPROXIMATIONS	BESK	330
ċ		TO BESSEL FUNCTIONS OF URDER ZERO AND ONE AND TO RELATED	BESK	340
ř		FUNCTIONS! . M. T.A.C V. 11. 1957. PP.86-88. AND G.N. #ATSUN.	BESK	350
ž		TA TACATTOC ON THE THEORY OF REARES FUNCTIONS . CAMBRIDGE	BESK	360
5		TA TACATINE ON THE THEORY OF DEDUCE TONETIONS F AN ENDEDUC	RESK	170
Ç		UNIVERSITA PRESS, 1430, P. 02	LCOV	370
Ċ			0530	300
Ç			BE SK	390
Ċ			BESK	400
•		DIMENSION T(12)	BESK	420
			BESK	430
			BESK	440
			Ares	150
	10		DEDK	4.50
		RETURN	DEDA	400
	11	IF(X)12,12,20	DESK	470
	12	IER=2	BESK	480
	• -	RETURN	BESK	490
	20	TE (X=170,0)22,22,21	BESK	500
	51		BESK	510
	<b>c</b> 1	LERTJ	BESK	520
			AL GK	510
	25	I EKEO	DCON	230
		IF(X=1+)30,36,25	DESN	340
	25	i ABEXP(#X)	BESK	550
		Bel./X	BESK	560
		C=SQRT(B)	BESK	570
			BESK	580
			BESK	590
	34		AFSK	600
	20		0.00	410
		IF(N=1)27,29,27		010
C			05.3K	620
C		COMPUTE KO USING POLYNOMIAL APPROXIMATION	86 SK	630
ċ			BESK	640
-	27	/ G08A+(1,2533141+,1566642+T(1)+,08811128+T(2)+,09139095*T(3)	BESK	650
		24 1344596+T(4)= 2299850+T(5)+ 3792410+T(6)= 5247277*T(7)	BESK	660
		$z_{+} = z_{+} = z_{+$	BESK	670
		31 131 330 4 1 (3) 4 4 C C C 3 4 1 ( 7) ( 1 ( 1 ( 1 ( 1 ( 1 ( 1 ( 1 ( 1 ( 1 (	REGK	680
		44.004104303#1(12))*6	9C BK	100
		IF (N) 20, 28, 24	DEON	070
	28	3 BK≡G0	OF SK	700
		RETURN	BESK	710
С			8ESK	720
Ĉ		COMPUTE KI USING POLYNOMIAL APPROXIMATION	BESK	730
ř			BESK	740
•	20		BESK	750
	<b>E</b> 7	/ 01/0/m/(11/2)/////////////////////////////////	BESK	760
		$\sum_{i=1}^{n} \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2} \sum_{i$	BCOK	770
		2=1003554241(9)+202052441(4)="520120441(10)+"0.000001#1(11)		770
		4-,01082418+T(12))+C	0530	700
		IF(N=1)20,30,31	BESK	790
	30	) 8K#G1	BESK	800
		RETURN	BESK	810
C			BESK	820
č		FROM KO.KI COMPUTE KN USING RECURPENCE RELATION	BESK	830
ř		LKCH HOMM - ANALY - KA BOTHA REARRANCE REPAILON	BESK	840
Ŀ			REGE	450
	21		DEGN	0.00
		$GJ=Z_{q} + (PLUAT(J) + 1_{q}) + G1/X + G0$	OCON	000
		IF(6J=1,0E70)33,33,32	OLSK	0/0
	32	2 IER#4	BESK	880
		GO TO 34	BESK	890
	33	3 G0#61	BESK	900
	39	5 G1=GJ	BESK	910
	1	AKAGI	BESK	920
	34	DEFIN	BESK	
	- 26		OCON	

 TABLE 6.1.—Listing of program for partial penetration in a leaky artesian aquifer—Continued

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	A=,5772157+ALDG(B)	BESK 950
	C=B+B	BESK 960
	IF(N=1)37,43,37	BESK 970
		BESK 980
	COMPUTE KO USING SERIES EXPANSIÓN	665K 990
		BESKIDDO
17		BESKIDIO
31		86941020
		OF SKINUSU
		DESKIUAU
	DD 40 J#1,6	BESK1050
	RJ=1./FLOAT(J)	8E\$K1060
	IF(X2J,LT,1,E=40) X2J=0,	BESK1061
	PREVIOUS STATEMENT ADDED TO IBM SUBROUTINE TO CORRECT UNDERFLOW	BESK1062
	PROBLEM ON WATFOR COMPILER	BESK1063
	X2J=X2J*C	BESK1070
	FACTRFACTORJOD	BESK1080
		BE 5K 1090
		B5961100
•••		96941110
	TL (1) 431 461 43	B BBBA 1 1 3 A
42		DOGNIIZV
	RETURN	DESKIISU
		863K1140
	CUMPUTE K1 USING SERIES EXPANSIUN	BESK1150
		BESK1160
43	Balanta	BESK1170
	FACT=1.	8ESK1180
	HJS1.	BESK1190
	G1=1./X+X2J+(.5+A=H.1)	BESK1200
		BE8K1210
		BESK1220
		BESK1230
		BESK1240
		REGEISEN
		95941530
50	G1=01+x2J#rALT#(*5+(A#HJ)#rLUA!(JJ)	
	IF (N=1)31,32,31	DEGNIE/U
-52	BK=G1	DESK1280
	RETURN	BE3K1290
	END	BESK130=

TABLE 7.2.-Listing of program for constant drawdown in a well in an infinite leaky aquifer

***************************************	2	
POSE	ź	
TO COMPUTE A TABLE OF FUNCTION VALUES FOR DRAWDOWN IN A	Z	
LEAKY ARTESIAN AQUIFER IN RESPONSE TO A STEP CHANGE IN	Z	
WATER LEVEL IN THE CONTROL WELL, FUNCTION VALUES ARE	Z	
EXPRESSED AS A FRACTION OF DRAHDOWN IN CONTROL WELL (S/SH),	Z	
REFERENCE - HANTUSH, M.S., 1959, NONSTEADY FLOW TU FLUWING	Z	
WELLS IN LEAKY AQUIFERS: JOUR, GEOPHYS, RESEARCH, V. 64,	Z	
ND. 8, P. 1043-1052.	Z	
INPUT DATA	Z	
$1 \text{ CARD } \bullet \text{ FORMAT(2E10.5)}$	Z	
TSMALL - SMALLEST VALUE OF ALPHA FOR WHICH COMPUTATION	Z	
IS DESIRED.	Z	
TLARGE - LARGEST VALUE OF ALPHA FOR WHICH COMPUTATION	Z	
IS DESIRED.	Z	
1 CARD - FORMAT(13F5.0)	Z	
BDAT - 13 VALUES OF RW/B. NON ZERO VALUES SHOULD BE GE 1	Z	
AND LT 10. FIRST ZERD (OR BLANK) WILL TERMINATE THE	Z	
LIST. AT LEAST ONE NON ZERO VALUE MUST BE CUDED. INPUT	ž	
VALUES ARE MULTIPLIED BY POWER OF TEN DETERMINED BY	ž	
PROGRAM FROM ALPHA.	ž	

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TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```
1 CARD - FORMAT(10F8.2)
                                                                             Z
                                                                                23
          RW - RADIUS OF CONTROL WELL.
                                                                             Z
                                                                                24
          RDAT . 9 VALUES OF RADIAL DISTANCE OF OBSERVATION PUINTS
                                                                             7
                                                                                25
          FROM CONTROL WELL, SHOULD BE CODED WITH SMALLEST NUMBER
                                                                             Z
                                                                                26
          FIRST, THEN BY INCREASING DISTANCE, THE FIRST ZERU
                                                                             Z
                                                                                 27
           (UR BLANK) VALUE WILL TERMINATE COMPUTATION.
                                                                             7
                                                                                 85
 METHOD
                                                                             2
                                                                                 29
     EVALUATES EQ, 13 OF HANTUSH, EVALUATION OF BESSEL FUNCTIONS
BY SUBRUUTINES BESK AND BESY AND FUNCTION JO, EVALUATES
                                                                             Z
                                                                                 30
                                                                             Z
                                                                                 31
     INTEGRAL BY SUM, I=1 TO BOOO, F((DELTA U)*(I=.5))*(DELTA U).
                                                                             Z
                                                                                 32
     CHOOSES INITIAL DELTA U = .001/SORT(SMALLEST ALPHA) AND USES
This value for all RW/B GE 10+(Delta U). For smaller RW/B,
                                                                             Z
                                                                                 33
                                                                             Z
                                                                                 34
     DIVIDES DELTA U BY 10 AND MULTIPLIES SMALLEST ALPHA BY 100.
                                                                                 35
                                                                             Z
 REMARKS
                                                                             7
                                                                                 36
     SMALLEST RW/B GE ,01/SURT(SMALLEST ALPHA)
                                                                             Z
                                                                                 37
  SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                                                                             Z
                                                                                 38
                                                                             Z
                                                                                 39
     BESK, BESY, JO
                                                                             Z
                                                                                 μn
  ******************
                                                                             Z
                                                                                 41
                                                                              Z
  REAL+8 SUM1, SUM2
                                                                                 42
  REAL+4 KOBP,KOB,J0,J0PU,J0U,Y(8000),J(8000),F(8000),FT(8000),
                                                                             Z
                                                                                 43
   FB(8000),RDAT(9),TDAT(6),BDAT(13),ARRAY(25,9,13),B(13),T(25)
                                                                             Z
                                                                                 44
 1
                                                                             Z
                                                                                 45
  DATA FT/8000+0./.F8/8000+0./
 DATA HOAT/9+1.
                                                                             Z
                                                                                 46
                                                                             Z
  DATA ARRAY/2925+0,/,TDAT/1,,1,5,2,,3,,5,,7,/
                                                                                 47
  IRD=5
                                                                             Z
                                                                                 48
                                                                             Z
                                                                                 40
  IPT=6
                                                                             Z
                                                                                 50
  READ (IRD, 24) TSMALL, TLARGE
  READ (IRD,23) BDAT
                                                                             Z
                                                                                 51
  READ (IRU,22) RW, RDAT
                                                                              Z
                                                                                 52
  IBEGIN=ALOG10(TSMALL)
                                                                              Z
                                                                                 53
  IEND=ALOG10(TLARGE)+.99999
                                                                              Z
                                                                                 54
  IF ((IBEGIN/2+2),LT,IBEGIN) IBEGIN=18EGIN=1
                                                                              Z
                                                                                 55
  ISPAN#IEND=IBEGIN
                                                                              Z
                                                                                 56
  MLIMIT=(ISPAN+1)/2
                                                                              Z
                                                                                 57
  COMPUTES INITIAL DELTA U (DU) # .001/SGRT(SMALLEST ALPHA)
                                                                              Z
                                                                                 58
                                                                                 59
                                                                              Z
  DU= +001/SGRT(TDAT(1)+10+++IBEGIN)
                                                                             Z
  EXPONENT (JBEGIN) OF SMALLEST RW/B IS COMPUTED FROM EXPONENT
                                                                                 60
                                                                              Z
  (IBEGIN) OF SMALLEST ALPHA.
                                                                                 61
                                                                              Z
  JBEGIN==IHEGIN/2=2
                                                                                 62
  DO 1 I=1,13
                                                                              Z
                                                                                 63
  IF (BDAT(I),EG,0,) GO TO 2
                                                                              Z
                                                                                 64
                                                                              Z
                                                                                 65
1 CONTINUE
                                                                              Z
  N8=13
                                                                                 66
                                                                                 67
  GO TO 3
                                                                              Z
2 NB=I=1
                                                                              Z
                                                                                 68
3 CONTINUE
                                                                              Z
                                                                                 69
                                                                              2
                                                                                 70
  DO 4 1=1,9
                                                                              Z
  IF (RDAT(I),EQ.0.) GO TO 5
                                                                                 71
4 RDAT(I)=ROAT(I)/RW
                                                                              Z
                                                                                 72
  NR=9
                                                                              Z
                                                                                 73
  GU TU 6
                                                                              Z
                                                                                 74
5 NR#1+1
                                                                              Z
                                                                                 75
6 DO 21 M#1, MLIMIT
                                                                              Z
                                                                                 76
  NUMBB000
                                                                              Z
                                                                                 77
  START==DU/2.
                                                                              Z
                                                                                 78
  UESTART
                                                                              Z
                                                                                 79
                                                                              Z
                                                                                 80
  DO 7 I=1,NUM
  U¤U+DU
                                                                              Z
                                                                                 81
  CALL BESY(U, 0, Y(I), IDUMY)
                                                                              Z
                                                                                 82
7 J(I)=J0(U)
                                                                              7
                                                                                 83
  DU 19 IR=1,NR
                                                                              7
                                                                                 84
```

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TABLE 7.2.-Listing of program for constant drawdown in a well in an infinite leaky aquifer-Continued

```
RHO#RUAT(IR)
  UESTART
  00 8 I=1.NUM
  U=U+DU
  CALL BESY(RHO+U,0,YOPU,IDUMY)
  JOPU#J0(RHO*U)
   JOU#J(I)
  YOU#Y(1)
8 F(I)=(JUPU+YOU=YOPU+JUU)/(JUU+JOU+YOU+YOU)
  DO 19 IT=1,25
  INDEX=(IT=1)/6
   IEXP=IBEGIN+INDEX
  II#IT#INDEX+6
  TAU=TDAT(II) +10, ++IEXP
  T(IT)=TAU
  USTART
  NUMTENUM
  DD 9 IF1,NUMT
  U≡U+DU
  FTEST=F(I)
  IF (ABS(FTEST) LT. 1. E-30) GO TO 10
  XTEST==TAU+U+U
  IF (XTEST+69.) 10,10,9
9 FT(I)=FTEST+EXP(XTEST)
  GO TO 11
10 NUMTRI#1
  FT(1)=0.
11 DO 19 I8=1,13
  JNDEX=(IB=1)/NB
   JEXP=JBEGIN+JNDEX
   JJ#I8=JNDEX*NB
  BETA=BDAT(JJ)+10+++JEXP
  B(IB)=BETA
  UESTART
  BSQ=BETA*BETA
   NUMBENUMT
  DO 12 I=1,NUMB
   U=U+DU
   FTEST=FT(I)
  IF (ABS(FTEST), LT, 1, E+30) GD TO 13
12 FB(I)=FTEST/(U+BSG/U)
  GD TU 14
13 NUMBEI=1
   FB(I)=0,
14 SUM1=0.
   SUM2=0.
   00 15 I=1,NUMB,2
   SUM1=SUM1+FB(I)
15 SUM2=8UM2+F8(1+1)
   XINT#(SUM1+SUM2)*DU
   CALL BESK(RHO+BETA,0,KOBP,IDUMY)
   CALL BESK(BETA, 0, KOB, IDUMY)
   RATIO=0.
   IF (KUBP.GT.0.) RATIO=KUBP/KUB
   XTEST==TAU+BSQ
   IF (XTEST+30,) 16,17,17
16 XPT=0.
   GO TO 18
17 XPT=EXP(XTEST)
18 Z=RATIO+ 6366198*XPT*XINT
   IF ((Z,LT,0,),AND, (Z,GT,=5,E=5)) Z=0,E0
19 ARRAY(IT, IR, IB)=Z
```

۶

Z 85

Z 86

Z 87

Z 88

Z

Z

Z 91

2 92

Z 94

Z 95

Z 96

Z 97

Z 98

Z 99

Z 100

Z 101

Z 102

Z 103

Z 10,4

Z 105

Z 106

Z 107 Z 108

Z 109

Z 110

Z 111 Z 112

Z 113

Z 114

Z 115

Z 116

Z 117

Z 118

Z 119

Z 120

Z 121

Z 122 Z 123 Z 124

Z 125

Z 126

Z 127

Z 128

Z 129

Z 130

Z 131

Z 132 Z 133

Z 134

Z 135

Z 136

Z 137

Z 138

Z 139

Z 140

Z 141 Z 142

2 143

Z 144

Z 145 Z 146

89

90

Z 93

° ∙<sub>256</sub>,\*' TABLE 7.2.-Listing of program for constant drawdown in a well in an infinite leaky aquifer-Continued

```
Z 147
     D0 20 K#1,NR
      WRITE (IPT,25) RDAT(K),8
                                                                          Z 148
      WRITE (1PT,26) (T(1), (ARRAY(1,K,L),L#1,13),1#1,25)
                                                                          Z 149
                                                                          Z 150
   20 CUNTINUE
                                                                          Z 151
      EXPUNENT OF SMALLEST RW/B DECREASED BY ONE EACH TIME THROUGH LOOP
Ĉ
                                                                          Z 152
      JBEGIN=JBEGIN=1
      EXPONENT OF SMALLEST ALPHA INCREASED BY TWO EACH TIME THROUGH LOOP
                                                                          Z 153
C
                                                                          Z 154
      IBEGIN#IBEGIN+2
      DELTA U (DU) IS DIVIDED BY 10 EACH TIME THROUGH THE LUUP
C
                                                                          Z 155
   21 DU=,1+DU
                                                                          Z 156
      STUP
                                                                          2 157
                                                                          2 158
Ć
   22 FURMAT (10F8,2)
                                                                          Z 159
   23 FORMAT (13F5.0)
                                                                           Z 160
                                                                           Z 161
   24 FORMAT (2E10.5)
   25 FURMAT (111, 2(ALPHA, R/RW, RW/B), R/RW=1, F6, 0/101, 9%, 11 RW/B1/(1 1,
                                                                          Z 162
                                                                          2 163
     13X, ALPHA 11, 13E9, 2))
   26 FURMAT (1 1, E10, 3, 1389.3)
                                                                          Z 164
      END
                                                                          Z 165-
      REAL FUNCTION JO+4(X)
                                                                         10
                                                                              1
                                                                               2
C
      **************
                                                                         70
                                                                         JO
                                                                               3
С
C
C
                                                                          J 0
                                                                               4
      FUNCTION JO
                                                                               5
                                                                          .10
C
                                                                          JO
                                                                               6
      PURPOSE
         TO COMPUTE THE ZERO UNDER J BESSEL FUNCTION FOR A GIVEN
C
                                                                               7
                                                                          J٥
         ARGUMENT
C
                                                                          J0
                                                                               8
                                                                               a
C
      USAGE
                                                                          J0
                                                                          10
                                                                             10
Ċ
         J0(X)
      DESCRIPTION OF PARAMETER
C
                                                                          J0
                                                                             11
         X - REAL+4, ARGUMENT OF JO BESSEL FUNCTION DESIRED.
C
                                                                          10
                                                                             15
C
      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                                                                          J0
                                                                             13
                                                                          J0
                                                                             14
         NONE .
C
C
                                                                             15
                                                                          J0
      METHOD
Ċ
         POLYNUMIAL APPROXIMATION FOR X44 AND ASYMPTOTIC SERIES FUR
                                                                          10
                                                                             16
C
         X GE 4. THE POLYNOMIAL APPROXIMATION IS THE FIRST 10 TERMS OF
                                                                          J0
                                                                             17
С
         THE POWER SERIES FOR JO(X) (MILLER, K.S., 1957,
                                                                          J 0.
                                                                              18
                                                                             19
         ENGINEERING MATHEMATICS: RINEHART AND CO., INC., NEW YORK,
                                                                          10
C
         P. 120), THE ASYMPTOTIC EXPANSION OF JO(X) IS GIVEN ON P. 82
C
                                                                          J0
                                                                             20
         OF BUWMAN, FRANK, 1958, INTRODUCTION TO BESSEL FUNCTIONS:
C
                                                                          J0
                                                                             21
         DOVER PUBLICATIONS INC., NEW YORK, THE TERMS P ('A+PO') AND
                                                                          J0
                                                                             22
С
                                                                          JO
         Q (!=B+Q0!) OF THE ASYMPTOTIC EXPANSION ARE COMPUTED BY AN
C
                                                                              23
                                                                          J0
                                                                              24
C
         ALGORITHM FROM IBM SUBROUTINE BESY.
۵
                                                                          J0
                                                                             25
C
      26
      IF (X=4.) 1,3,3
                                                                              27
                                                                          J0
С
      COMPUTE JO BY FIRST 10 TERMS OF POWER SERIES
                                                                          J0
                                                                              28
    1 A==X+X/4.
                                                                          70
                                                                              29
                                                                          J0
                                                                              30
      8=1.
      00 2 I=1,10
                                                                          J 0
                                                                              31
      C≈11.+I
                                                                          JO
                                                                              35
    2 B=1,+B*(A/(C+C))
                                                                          J0
                                                                              33
                                                                          JO
      J0≢B
                                                                              34
                                                                              35
                                                                          J0
      GU TO 4
C
      COMPUTE JO BY ASYMPTOTIC SERIES
                                                                          J0
                                                                              36
    3 T1=4./X
                                                                              37
                                                                          J0
                                                                          0L
                                                                              38
      12#11+11
      P0=((((-=0000037043+T2+.0000173565)+T2=.0000487613)+T2+.00017343)+ J0
                                                                              39
     1T2=,001753062)*T2+,3989423
                                                                              40
                                                                          1 O L
      Q0=((((,0000032312+12=,0000142078)+12+,0000342468)+12=,0000869791) J0
                                                                              41
     1*T2+,0004564324)*T2=,01246694
                                                                          J0
                                                                              42
                                                                          JO
                                                                              43
      A=2.0/SURT(X)
```

		JO	44
	C=X+,7853982	JO	45
	J0=A+P0+C19(C)=B+Q0+9IN(C)	30	46
4	RETURN	70	47
	END	J O L	48-
	SUBBOUTINE BEAVEN. N.BY. TED	HERY	410
	SOUNDOTTHE BESTLATASTICA	BEOV	10
		DEDI	20
			20
		0001	30
	SUBRUUTINE BEST	BEST	40
		BEST	50
	PURPUSE	BESY	60
	COMPUTE THE Y BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER	88ESY	70
		BESY	80
	USAGE	BESY	90
	CALL BESY(X,N,BY,IER)	BESY	100
		86'SY	110
	DESCRIPTION OF PARAMETERS	BESY	120
	X THE ARGUMENT OF THE Y BESSEL FUNCTION DESIRED	BESY	130
	N THE ORDER OF THE Y BESSEL FUNCTION DESIRED	BESY	140
	BY THE RESULTANT Y BESSEL FUNCTION	BESY	150
	IER+RESID TANT ERROR CODE WHERE	BESY	160
	IEREO NO FRROR	BESY	170
	TEREL N TA NEGATIVE	BESY	180
	TERED X TO NEGATIVE OR ZERO	BESY	190
	TERE BY HAR EXCEPTED MAGNITUDE OF 104470	AFSV	200
	TENES BY HAS EXCEPTED HASKINGE OF INAMA	REQU	210
	DEMADK &	UC OV	230
	NEPARAS	DEDI	220
	FUNCTION A COLOR A MAT LAUSE INC RANGE OF THE LIDHART	DC OT	230
	MULT DE COELEGE HAN SEDO	DCOV	240
	A RUJI BE GREATER THAN ZERU N MURT BE GREATER THAN DO EQUAL TO TERR	0001	230
	N HUSI BE GREATER THAN UR EWUAL IU ZERD	DEAT	200
		BEST	270
	SUBRUUTINES AND FUNCTION SUBPROGRAMS REQUIRED	BESY	280
	NONE	BESY	290
		BESY	300
	METHOD	BESY	310
	RECURRENCE RELATION AND PULYNOMIAL APPROXIMATION TECHNIQUE	BESY	320
	AS DESCRIBED BY A.J.,M.HITCHCOCK, POLYNOMIAL APPRUXIMATIONS	BESY	330
	TO BESSEL FUNCTIONS OF ORDER ZERD AND ONE AND TO RELATED	BESY	340
	FUNCTIONSI, M.T.A.C., V.11,1957,PP.86-88, AND G.N. WATSON,	BESY	350
	A TREATISE ON THE THEORY OF BESSEL FUNCTIONS , CAMBRIDGE	BESY	360
	UNIVERSITY PRESS, 1956, P. 62	BESY	370
		BESY	380
		BESY	390
		BESY	400
		BESY	420
	CHECK FOR ERRORS IN N AND X	BESY	430
		BESV	440
	TE(N)180-10-10	RESV	450
10		BESV	460
10		REGV	400
	41 VU147VE47VEEV	AFRY	440
	RRANCH TE Y LERG THAN OR FOULL 4	ALAN	404
	DRANUN JE A LEGG INAN UN EMNAL 4	PE JY	470
		DEOT	500
20	TL FX### 01m0% 40% 20	DESY	210
		DESY	520
	CUMPUTE YO AND YI FUR X GREATER THAN 4	DESY	530
_		BESY	540
30	T1=4.0/X	BESY	550
	T2#T1+T1	BESY	560
	P0#((((=,0000037043*T2+,0000173565)*T2=,0000487613)*T2	BESY	570

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

```
+,00017343) +T2+,001753062) +T2+,3989423
                                                                         BESY 580
  1
   Q0#((((,0000032312*T2-,0000142078)*T2+,0000342468)*T2
                                                                         BESY 590
  1 =,0000869791) *T2+,0004564324) *T2=,01246694
                                                                         BE8Y 600
   P1=((((,0000042414*12=,0000200920)*12+,0000580759)*12
                                                                         BESY 610
                                                                         BESY 620
    -.000223203) * 12+.002921826) * 12+.3989423
  1
                                                                         BESY 630
   Q1=((((-,0000036594+T2+,00001622)*T2-,0000398708)*T2
                                                                         BESY 640
    +,0001064741)*T2=,0006390400)*T2+,03740084
                                                                          BE8Y 650
   A=2.0/SURT(X)
                                                                          BESY 660
   BEA*T1
                                                                          BESY 670
   C=X=,7853982
                                                                          BESY 680
   Y0=A+P0+SIN(C)+B+G0+CUS(C)
                                                                          8ESY 690
   Y1==A+P1+COS(C)+B+Q1+SIN(C)
                                                                          BESY 700
   GU TU 90
                                                                          BESY 710
                                                                          8ESY 720
     CUMPUTE YO AND YI FOR X LESS THAN OR EQUAL TO 4
                                                                          8ESY 730
                                                                          8ESY 740
40 XX=X/2.
                                                                          BESY 750
   X2=XX+XX
                                                                          BESY 760
   T=ALOG(XX)+.5772157
                                                                          BESY 770
   SUME0.
                                                                          BESY 780
   TERMET
                                                                          BESY 790
   Y0=T
                                                                          BESY BOO
   DD 70 L#1,15
                                                                          BESY 810
   IF(L=1)50,60,50
                                                                          BESY 820
50 SUM#SUM+1./FLOAT(L=1)
                                                                          BESY 830
60 FL=L
                                                                          BESY 840
    TS=T=SUM
   IF(ABS(TERM), LE, 1, E=40) TERMEO.
                                                                          BE8Y 841
                                                                          BESY 850
    TERMa(TERMa(=x2)/FLaa2)*(1,-1,/(FL*T8))
                                                                          BESY 860
70 YOMYO+TERM
                                                                          BESY 870
    TERM = XX+(I+.5)
                                                                          BESY 880
    SUME0.
                                                                          8ESY 890
    Y1=TERM
                                                                          BESY 900
   DO 80 L#2,16
                                                                          BESY 910
    SUM=SUM+1 /FLOAT(L=1)
                                                                          BESY 920
   FL=L
                                                                          BESY 930
   FL1=FL=1.
                                                                          BESY 940
    TS=T=SUM
                                                                          BESY 941
    IF (ABS(TERM) .LE.1.E=40) TERMED.
                                                                          BESY 950
    TERM#(TERM*(*X2)/(FL1*FL))*((TS=,5/FL)/(TS+,5/FL1))
                                                                          BESY 960
BU Y1=Y1+TERM
                                                                          BESY 970
    PI2=,6366198
                                                                          BESY 980
    Y0=PI2+Y0
                                                                          BESY 990
    Y1==PI2/X+PI2+Y1
                                                                          8ESY1000
    CHECK IF UNLY YO DR Y1 IS DESIRED
                                                                          BESY1010
                                                                          BESY1020
                                                                          BESY1030
90 IF(N=1)100,100,130
                                                                          BESY1040
                                                                          8ESY1050
    RETURN EITHER YO UR YI AS REQUIRED
                                                                          8E 8Y1060
                                                                          BESY1070
100 IF(N)110,120,110
                                                                          BESY1080
110 BY=Y1
                                                                          BESY1090
    GO TO 170
                                                                          BE SY1100
120 BYEY0
                                                                          BESY1110
    GO TO 170
                                                                          BESY1120
   PERFORM RECURRENCE OPERATIONS TO FIND YN(X)
                                                                          BESY1130
                                                                          8ESY1140
                                                                          BESY1150
130 YASYD
                                                                          BESY1160
    YB=Y1
                                                                          BESY1170
    K#1
```

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TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

140	T#FLQAT(2+K)/X	BESY1	180
• • •	YCBT#YB#YA	BESY1	190
	IF(ABS(YC)+1,0F70)145,145,141	BESY1	200
141	TFRa3	BESY1	210
	DETURN	BESYI	220
145		BESYS	230
142	N=73 TELEAN1150.160.160	BFSYI	240
. 5 ^	T C M M 1 2 0 1 2 0 1 2 0	BESYS	250
130		RESVI	260
		ACOVA	270
	GE TO 140	DEDIT	300
160	BAEAC	95911	200
170	RETURN	DESTI	290
180	IER#1	DESTI	300
	RETURN	02311	310
190	IER#2	DESTI	320
	RETURN	BESYI	330
	END	BESYI	34=
	SUBROUTINE BESK(X,N,BK,IER)	BESK	410
		BESK	10
		BESK	20
		BESK	30
	SUBRUUTINE BESK	BESK	40
		BESK	50
	COMPUTE THE K BESSEL FUNCTION FOR A GIVEN ARGUMENT AND ORDER	BESK	60
		BE 8K	70
	USAGE	BESK	80
	(ALL BESK(X.N.BK.TER)	BESK	90
		BESK	100
	DESCRIPTION OF PARAMETERS	BESK	110
	X THE ADDITION OF THE K BESSEL FUNCTION DESIRED	BESK	120
	N THE RODED OF THE KAESSEL FUNCTION DESTRED	BESK	130
	ar the druck of the K begget starter begand	BESK	140
	DR WINE RESULTARI R DESCE FORETOR	RESK	150
	IERARESULIANI ERROR CODE MERE	AFRE	160
	IERBU NU ERRUR	BEEK	170
	ICREI N IS NEGRITE	DEGN	1/0
	TERES X IS ZERU UR NEGATIVE ENDEE ENCEPTED	BCON	100
	LERES X GI 1/0, MACHINE RANGE EAGEEDED		200
	IEH#4 BK sGi 10##70	DE ON	200
		DESN	210
	REMARKS	DESK	220
	N MUST BE GREATER THAN OR EQUAL TO ZERU	DESK	230
		BESK	240
	SUBROUTINES AND FUNCTION SUBPROGRAMS REGUIRED	DESK	250
	NONE	BESK	260
		BESK	270
	METHOD	HESK	280
	COMPUTES ZERO ORDER AND FIRST ORDER BESSEL FUNCTIONS USING	BESK	290
	SERIES APPROXIMATIONS AND THEN CUMPUTES N TH URDER FUNCTION	BESK	300
	USING RECURRENCE RELATION.	BESK	310
	RECURRENCE RELATION AND PULYNOMIAL APPRUXIMATION TECHNIQUE	BESK	320
	AS DESCRIBED BY A.J.M.HITCHCOCK, 'POLYNOMIAL APPROXIMATIONS	BESK	330
	TU BESSEL FUNCTIONS OF URDER ZERO AND ONE AND TO RELATED	BESK	340
	FUNCTIONS', M.T.A.C., V.11, 1957, PP.86=88, AND G.N. WATSON,	BESK	350
	IA TREATISE ON THE THEORY OF BESSEL FUNCTIONS!, CAMBRIDGE	BESK	360
	UNIVERSITY PRESS, 1958, P. 62	BESK	370
		BESK	380
		BESK	390
	•••••	BESK	400
	DIMENSIUN T(12)	BESK	420
	RKELD	BESK	430
	TE(N)10.11.11	BESK	440
• •	10 5 1 2 5 2 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	BFSK	450
10	1 C 7 = 1		- 20

 $= \left\{ \hat{\boldsymbol{\theta}}_{t,\boldsymbol{y}_{t},\boldsymbol{y}_{t}}^{T} \right\}_{t \in \mathcal{F}_{t},\boldsymbol{y}_{t},\boldsymbol{y}_{t}}$ 

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TABLE 7.2.-Listing of program for constant drawdown in a well in an infinite leaky aquifer-Continued

			BESK	460
	11	TF(X)12,12,20	BESK	470
	12		BESK	480
		RETURN	BESK	490
	20	IF(X+170,0)22,22,21	BESK	500
	21	IER=3	BESK	510
		RETURN	DESK	520
	55	IEREO	DESA	530
		IF(X=1,)30,36,25	BERK	550
	25	AEEXP(=X)	RESK	560
		Bale/X	BESK	570
			BESK	580
		1(1)#0	BESK	590
	34	UU EG 1-2112 T(1)=T(1=1)=B	BESK	600
	¢0	TE(N=1)27.29.27	BESK	610
r			BESK	620
ř		COMPUTE KU USING POLYNOMIAL APPRUXIMATION	BESK	630
č			BESK	640
•	27	G0=A*(1,2533141=,1566642*T(1)+,08811128*T(2)=,09139095*T(3)	BESK	650
		2+,1344596*T(4)=,2299850*T(5)+,3792410*T(6)=,5247277*T(7)	BESK	660
		3+,5575368*T(8)=,4262633*T(9)+,2184518*T(10)=,06680977*T(11)	BESK	670
		4+,009189383*T(12))*C	BESK	600
		IF(N)20,28,29	DEGN	300
	28	BKEGO	REGK	710
		RETURN	RESK	720
C		THE TO SERVE BOUNDARY ARRANGES	BESK	730
C		COMPUTE RI USING PULTNUMIAL APPROATBATION	BESK	740
С	20		BESK	750
	24	$G_1 = 47\pi (1_0 + 35_1) + (1_0 + 7(1_0) + (1_0 + 1_0) + ($	BESK	760
		T	BESK	770
		=_01082418+T(12))*C	BESK	780
		IF(N=1)20,30,31	BESK	790
	30	BK=G1	BESK	800
		RETURN	BESK	810
С			DESA	820
С		FROM KO,K1 COMPUTE KN USING RECURRENCE RELATION	DC 3M	010
С			RESK	850
	31		BESK	860
		GJ324*(FLUA (J)#]#J#G1/X760 #F/C1_4 AC70\73 73 73	BESK	870
	7 3	150m/ Th (07m/*05/0123*23*25	BESK	880
	25		BESK	890
	11	60#61	BESK	900
	- 23	G1#GJ	BESK	910
	34	BKİÇJ	BESK	920
		RETURN	BESK	930
	36	B=X/2,	BESK	940
		A#,5772157+AL0G(B)	BESK	950
		Ç=8*8	DESK	960
		IF(N=1)37,43,37	0534	080
C		ACT DUTT HAT HARNO AFOTER EVOLUTION	BFSK	990
C		CUMPUTE KU USING SERIES EXPANSION	BESK	1000
C			BESK	1010
	57	vo 1≊1.	BESH	1020
		Kavia Factel	BESP	1030
		HJW.O	BES	1040
		DO 40 J=1,6	BES	1050
		RJ=1,/FLDAT(J)	BES	1060
		IF(X2J,LT,1,E-40) X2J=0.	BESP	1061
C		PREVIOUS STATEMENT ADDED TO IBM SUBROUTINE TO CORRECT UNDERFLOW	OF 31	1062

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TABLE 7.2.—Listing of program for constant drawdown in a well in an infinite leaky aquifer—Continued

-		85 8K 1 A 6 3
C	PRUBLEM UN WATPUR CUMPILER	DEGRIAMA
	X2J#X2J*C	BESKIU/U
	FACT#FACT*RJ*RJ	BE3K1080
	HJEHJ+RJ	BESK1090
4	IO GORGO+X2J+FACT+(HJ+A)	BESK1100
	IF(N)43,42,43	BESK1110
4	2 BK=GO	BESK1120
	RETURN	BESK1130
r		BESK1140
č.	COMPHTE KI USING SERIES EXPANSION	BESK1150
č	foundle ut office douted fully star	BESK1160
U //	IT V2.TER	BESK1170
-	FACTE1.	BESK1180
		BESK1190
		HESKIZOO
	00 20 J=5'8	DESTIN
	X2J=X2J+C	DE341220
	RJ=1./FLOAT(J)	BESK1230
	FACT#FACT+RJ+RJ	BESK1240
	HJAHJ4RJ	BESK1250
5	G GIRGI+X2J+FACT+/_S+(A+HJ)+FL(AT(J))	8ESK1260
-	12/N=4114.50.94	BFSK1270
	17 (N=1)21/26/31	AFSKIZAN
3	)C DREUI	956×(300
	RETURN	DEGNIZAN
	END	BESK130-

 TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter

C + + 1	*********	* F A H	1
ř		FAR	5
ř	PURPOSE	FAR	3
ř	COMPUTES FUNCTION VALUES OF F(U, ALPHA, RHU) FUR RHO > 1 =	FAR	4
ř	PAPADOPHINS, T.S. AND CODPER.H.H., JR., 1967, DRAWDUWN IN	FAR	5
ř	A WELL OF LADGE DIAMETERI WATER RESOURCES RESEARCH, V. 3,	FAR	6
r		FAR	7
5	DONGAN RY S C DADADADIII ()S.	FAR	8
2	TARDIT DATA - ONE OF MORE GROUPS, FACH GROUP CODED AS FOLLOWS	FAR	9
L C	I CAD - FORMATION 53	FAR	10
5	I CARD - FURNETLE DUBAS AND	FAR	11
5	ALTIN DO NOR HAD TAN ADULTER SOMALD A STORAGE	FAR	12
C C	CORECTORENT / PADTING OF CASTNG (OVER INTERVAL OF	FAR	13
ί. Γ	COEFFICIENT / MARGES SOLADED.	FAR	14
5	WALER LEVEL CHARGE GOURAGED WELL / RADIUS OF	FAR	15
C	REU • RYRE • DIGIANCE FROM FORFED ALLE / ALLEGO G	FAR	16
C	WELL ISLREEN ON GENERAL IN RUITERT TOT OF	FAR	17
C	GREATER THAN UNES	FAR	18
Ç	1 CARD & FURMAILIOES,UJ . CAADARAKAATATMEN - OTSTANDE ERUM	FAR	19
G	UN 16 VALUES UP UN REASTANTATIONES DISTANCE FROM	FAD	20
C	PUMPED WELL SWUARED * STURAGE LIEFFICIENT /	EAD	21
C	4 * TRANSMISSIVILY * TIME, IF LESS THAN TO DESTRED,	C A D	21
C	BLANK OR ZERO VALUES MAY BE CUDED FOR THE REST.	E A R	22
C	SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED	C AR	23
C	PEAK,SIMP,APEKE,EXBSL1,JY0,JY1,RUMTS - MUST BE IN DECK.	FAR	24
C		FAR	25
C**:	****	*PAR	- 59
	DIMENSIUN V(40,40),U(16)	FAR	27
	COMMON XPK,YPK	FAR	28
	COMMON/PBLK/A, B, RHO	FAR	- 59
	EXTERNAL EXBSL1	FAR	30
	1 RFAD (5.16.END#15) ALPHA, RHO	FAR	- 31
	TE (ALPHA) 15,15,2	FAR	32

91

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

		F A R	33
5		FAR	34
	WRITE (0)10)	FAR .	15
3	READ (5,19) U	1.0	16
	DO 14 II=1,16	- <u>A</u> R	20
	TF (U(II)) 1.1.4	PAH	31
		FAR	38
- 4	ARALPHATALTHA	FAR	39
	B#0,25/U(II)	E A D	
	CALL APEKE(EXBSL1)	- <b></b>	N U
	CALL PEAK(EXBSL1)	PAR	41
		FAR	42
_		FAR	43
- 5	WRITE (6,20) XPK,U	E A G	
	GO TÚ 3	<b>F A R</b>	44
	TE (XPK#3.0) 8.7.7	PAH	45
ŭ		FAR	46
- (	HRITE (DJET) APRIO	FAR	47
	GO TO 5	EAD	- , / <b>A</b>
- 8	EPS=0.000001		40
	HBAREO, 007+XPK	PAR	49
	CALL STMPSIC S. VPK. FPS. HBAR. SUM. DEL . EXBSL 1)	FAR	50
		FAR	51
	XWI=((2*14124502*1*01)(0*0*(KHD=1*))+1*c=0)*(40))==	FAD	62
	DX1=XM1=(1,0E=6)=RHD	E 4 0	52
	DXN#(2_0+3_14159265*RH0)/(5_*(RH0=1,))	PAK	23
	DI = 3 14159265+PH0/(PH0=1.)	FAR	54
		FAR	55
	CALL RUUIS(AMI, DAI, RII) CAUGEI)	FAR	56
	HBAR#0,007+(RT1=XPK)	f an	
	CALL SIMPS(XPK,RT1,EPS,HBAR,TRM1,ERR1,EXBSL1)	PAR	51
	allwaglim+TPM1	FAR	58
		FAR	59
	DELADELTERRI	FAR	6.0
	X1=RT1		00
	Isl	FAR	61
		FAR	62
	/ ANTERFICE	FAR	63
	CALL RUDISLAM, UNIX 2/ LAUGLI	E A D	6.0
	HBAR#0,007*(X2+X1)	T AR	04
	CALL SIMPS(X1,X2,EPS,HBAR,TRM,ERR,EXBSL1)	PAR	62
	V(1.T)EABS(TRM)	FAR	66
		FAR	67
		FAH	6.8
	1=1+1	<b>F</b> . <b>C</b>	
	IF (I=40) 10,10,11	PAR	04
1	0 x1=x2	₹AR	70
•		FAR	71
		FAR	72
1		E . D	
	DO 12 K=2,40	FAR	13
	M=41=K	FAR	74
		FAR	75
		FAR	76
1		FAD	47
	DD 13 N#1,40	E A D	
	L=N=1	PAR	78
		FAR	79
		FAR	80
•		FAD	
	SUMESUMEEST	P A IS	
	PUARa4,04A#RH()+SUM/3,14159265	PAR	0¢
	WRITE (6,22) U(II),SUM,DEL,PUAR	FAR	83
	a continue	FAH	84
+		FAR	AS
			0 J 6 A
1	5 STUP	FAR	60
		FAR	87
1	A FORMAT (2F10.5)	FAR	88
	T FORMAT THE FEATURE ALOUA DUAL FOR ALOUALT FORTE S. F. RUDEL FETT 51	FAR	дO
1	/ FURMAL (11); TELUJALPHAJKHUJ FUK ALFHAB; JIFCIJJJ]; J HOUM (JICIJJJ)	FAO	60
1	8 FURMAT (1H0,12X,1HU,16X,8HINTEGRAL,9X,14HINTEGRAL ERRUR,6X,14HFU)	7 F AR	40
	1ALPHA,RHU)/IH )	FAR	91
1	9 FORMAT (1665-0)	FAR	92
	and a second s		

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TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

	20	FORMAT (5H XPK#,E15,8,3X,16HT00 SMALL FOR U=,E10,3)	FAR	93
	21	FURMAL (5H XFK#,513,0,3X,10HIUU LARGE FUR U=,510,3) Furmat (14 - 10,520, 3)	F A K	94
	22	END	FAR	95 96=
		FUNCTION EXESLICE)	E81	1
C+	***	****	*EB1	2
Ċ			E81	3
Ċ		PURPOSE	EB1	4
С		COMPUTES VALUES OF THE INTEGRAND FOR F(U,ALPHA,RHO)	E81	5
C		DESCRIPTIUN OF PARAMETER	EBI	6
C		X- REAL - ARGUMENT DF INTEGRAND	EB1	7
C			E81	8
C *	***	*****	*EB1	9
		CUMMON/PBLK/A,B,R	EB1	10
		IF (x) 1,1,2	E81	11
	1	Exesting and the second s	E81	15
	_	GO TO 8	201	15
	2		E 0 1	14
		IF (W-1,0E7) 4,4,3	EB1	15
	3	FNU=A+CUS(W+(R-1,0))-W+SIN(W+(R-1,0))	E81	16
		DEE(w + w + SQRT(R)) + (w + w + A)	EB1	17
		EXBSL1=FNU/DE	281	18
		GO_1D_8	EB1	14
	4	Y=B+X+X	281	50
		IF (Y=0,01) 5,5,6	EB1	21
	5	E X P U = Y + (1, 0 = Y + (0, 5 = Y + ((1, 0/6, 0) = Y + (1, 0/24, 0))))	281	22
		GO TU 7	2131	23
	6	EXPU=1,0+EXP(=Y)	281	24
	7	CALL JYO(W,WJO,WYO)		27
		CALL JY1(W,WJ1,WY1)	681	20
		Y MEM + MÁ Ó mÝ + MÁ I	281	21
			E01	20
		CALL JYO(X, BJO, BYO)	5.01	24
		FNUM#EXPO*(Aw+BJ0=Bw+BY0)	5.51	30
			201	51
	-	EXBSL1#FNUM/DEN	E 81	32
	8	RETURN		33
		END	C D 1	34-
		SUBROUTINE ROOTS(XM,DX,ROOT,F)	RUD	1
C # 1	***	************	*R00	5
C			RUD	3
С		PURPOSE	ROD	4
C		SEARCHES FUR ROOT OF F IN THE INTERVAL XM+DX TO XM+DX.	RUO	5
C		DESCRIPTION OF PARAMETERS . ALL REAL	800	6
C		XM • CENTER OF INTERVAL SEARCHED.	RDU	7
C		DX - HALF WIDTH OF INTERVAL SEARCHED.	RDU	8
C		ROOT - RETURNED ROOT LOCATION.	RUU	9
C		F • FUNCTION REFERENCE.	ROU	10
C			R00	11
Ç×	***	*************************	*R00	12
		XL#XM=DX	RUD	13
		XR=XM+DX	ROO	14
		YL=F(XL)	ROD	15
		YR®F(XR)	RDO	16
		EP=0,000001+ABS(YL)	RUU	17
		DD 9 1=1,200	KÜQ	18
		YM#F(XM)	RUD	19
		UPTAUS(TM)	RUU	50
		IF (UP,LT, EP, AND, UP, LT, 1, 0D+7) GO TO 1	K00	21
		IF (YM) 20102	ROO	22

 TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

	1	ROUTEXM	RUU	23
		GO TO 10	ROU	24
	2	TE (VM+VI) 7.3.4	ROU	25
			PUG	26
	2		Provide State	20
		GU TU 10	RUU	61
	- 4	IF (YM*YR) 8,5,6	RUD	58
	5	ROOT=XR	RDO	29
	-	GD TO 10	RUO	30
			Pun	21
	0	REAL COLLI ALLAN	800	
		STUP	<b>RUU</b>	26
	7	XRaxM	ROO	35
		YREYM	ROO	34
		9 OT 03	RUD	35
	я	AT 3 X W	800	36
	ų		900	17
	•		800	31
	¥		RUU	20
		ROOTEXM	RUD	39
	10	RETURN	ROO	40
Ĉ			ROO	41
-	11	FORMAT (1H .10%.27HND ROOT IN INTERVAL MHEDX #.1PE20.8.5%.11HAND	XROO	42
	••	MANY C. (DESA DA)	800	41
		mtua mpirtev (0/)	200	
		END	RUU	44*
		SUBRUUTINE APEKE(EXBSL)	APE	1
C+	***	*****	*APE	2
C			APE	3
ē.		DURPASE	APE	ù
ž		CETE ETERT ADDONIMATION TO DEAK DORITION	APE	Ś
5		dera rikal Approximation to reak position		
C			AFE	
Ç*	***	***************************************	ANE	7
		COMMON XPK,YPK	APE	ð
		XPK=0_0	APE	9
		ABK#0 0	APE	10
		00 2 1#1-17	APE	11
			ADE	12
			100	1.4
		TELADOL(X)	AFC	13
		IF (Y#YPK) 5,3,1	APE	14
	1	XPK=X	APE	15
		YPKEY	APE	16
	2	CONTINUE	APE	17
	ī	DE TIION	APE	18
			ADE	
			0.0.0	1.1-
_		SUDMULITNE PEAK(EXB3L)	PLA	1
Ç *	***	***************************************	*PEA	2
Ç			PEA	3
C		PURPOSE	PEA	4
č		ATTEMPTS TO FIND POSITION OF MAXIMUM FOR INTEGRAND	PEA	S.
ř		arrent for the transfer of the transferred of the setter and	DEA	ĩ
¥.				
U *	***	***************************************	TRPEA	
		COMMON XPK,YPK	PEA	8
		YPK#EXBSL(XPK)	PEA	9
		DD 13 L=1,200	PEA	10
			PEA	11
			PEA	12
			DEA	1 7
			7 E A	13
		AKRAPAT DA	PLA	14
		YREEXBSL(XR)	PEA	15
		DEN=YR+YL=YPK=YPK	PEA	16
		IF (DEN) 1.9.1	PEA	17
	1	XEXPK=0.5+(YR=YI)+DX/DEN	PEA	1 A
			DEA	ia
	- <b>-</b>			4 T

Ś

And a state of the second

C

3 6 7 9 10 11 12 13	<pre>X=0.0 Y=EX8SL(X) IF (YH=Y) 6,6,5 Y=YR X=xR IF (YL=Y) 8,8,7 Y=YL X=xL IF (Y=YPK) 14,14,12 IF (Y=YPK) 14,14,12 IF (Y=YPK) 11,10,10 X=xPK+Dx+Dx GO TO 2 X=xPK=Dx=DX GO TO 2 YPK=Y XPK=X CONTINUE RETURN END</pre>	<b>PPPPPPPPPPPPPPPPP</b> <b>EEEEEEEEEEEEEEEEEEE</b>	201223456789012334567890123345678901233456789012333455678901233455678901233455678
	END	FQA	20-
	SUBHOUTINE SIMPS(Q,R,EPS,HBAR,AREA,DEL;F)	SIM	1 7
***	***************************************	8IM	ž
	PURPOSE	SIM	4
	TO DETERMINE THE INTEGRAL OF A FUNCTION, F, FROM Q TU R,	SIM	5
	USING SIMPSON'S RULE,	SIM	6
	DESCRIPTION OF PARAMETERS	SIM	7
	ALL REAL	31 M	8
	U = LUNER LIMIT OF INTEGRAL	91M	10
	FPS - DESTDED ACCURACY	SIM	11
	HHAR - MINIMUM DIVISION OF THE INTERVAL	81M	12
	AREA - COMPUTED VALUE OF INTEGRAL BETWEEN Q AND R	SIM	13
	DEL . COMPUTED ESTIMATE OF ERROR	SIM	14
	F. THE INTEGRAND (FUNCTION REFERENCE)	SIM	15
	HETHOD	SIM	16
	USES SIMPSON'S RULE TO COMPUTE A SUM APPROXIMATING THE INTEGRAL	LSIM	17
	USES INITIAL HE (R+G)/2, COMPUTES A SEQUENCE OF SUMS BY HALVING	SIM	18
	H EACH TIME, CUMPUTES ESTIMATE UP ERRUR (DEL) AS (PREVIOUS	61M 91M	14
	SUM & LUKKENI SUMJ/13, LUMPUIATIUN SIUPS MEEN 17 NAMBAR;	STW.	20
	ZJ ADGLUELJKADGLEPSKUNKENT GUMJE IF HDAK IG LE VA Tuen maade avgeband	STM	22
		SIM	23
* * * *	**********	*SIM	24
	H2R+Q	SIM	25
	IF (H) 1,1,2	SIM	59
1	AREA=0,0	SIM	27
	DEL=0,0	SIM	58
	GO TO 10	SIM	29
RM	UST BE GREATER THAN Q	SIM	30
5	SP#1,0E35	SIM	51
	33=V.V 61#5(1).55(2)	81M 91M	3€ 31
	оцисцијусској ТЕ (нвар) 3.3.4	SIM	34
۲	HBAK=0.007#H	SIM	35
4	S2¤0,0	SIM	36
	X=u+0,5+H	SIM	37
5	92#92+4,0*F(X)	SIM	38
	X#X+H	SIM	39
	IF (X+R) 5,5,6	SIM	40
6	3C#(31+32+35)*H*0,16666667	SIM	41

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

```
42
     DEL=0,066666667*(SP=SC)
                                                                 SIM
                                                                 SIM
     IF (ABS(DEL)=ABS(EPS*SC)) 7,8,8
                                                                      43
                                                                 SIM
                                                                      44
   7 AREA=SC=DEL
                                                                 SIM
                                                                      45
     GO TO 10
   8 93=93+0,5+92
                                                                 SIM
                                                                      46
     H=0.5+H
                                                                 SIM
                                                                      47
     IF (H=HBAR) 7,9,9
                                                                 SIM
                                                                      48
   9 SP=SC
                                                                 SIM
                                                                      49
     GU TO 4
                                                                 SIM
                                                                      50
  10 RETURN
                                                                 SIM
                                                                      51
                                                                 SIM
                                                                      52•
     END
     SUBRUUTINE JYC(X, JO, YO)
                                                                 JY0
                                                                      1
2
C
                                                                 JY0
                                                                       3
C
     PURPOSE
                                                                 JY0
                                                                       4
        COMPUTES BESSEL FUNCTIONS OF THE FIRST AND SECOND KIND,
C
                                                                 JY0
                                                                       5
C
        ZERO ORDER, FOR PUSITIVE ARGUMENTS.
                                                                 JY0
                                                                       6
        SEE NBS AMS 55, P. 369=370.
¢
                                                                       7
                                                                 JY0
C
     DESCRIPTION OF PARAMETERS - ALL REAL
                                                                 JYO
                                                                       8
C
        X= ARGUMENT, MUST BE >0
                                                                 JY0
                                                                       Q
С
        JO - RETURNED FUNCTION VALUE, JO(X)
                                                                 JY0
                                                                      10
C
        YO . RETURNED FUNCTION VALUE, YO(X)
                                                                 JY0
                                                                      11
C
                                                                 JY0
                                                                      15
13
     REAL JO
                                                                 JY0
                                                                      14
     IF (X=3,0) 1,2,3
                                                                 JY0
                                                                      15
   1 IF (X) 4,4,2
                                                                 JY0
                                                                      16
   2 Z=(0,33333333+X)++2
                                                                 JY0
                                                                      17
     J0=1,0=Z*(2,2499997=Z*(1,2656208=Z*(0,3163866=Z*(0,0444479=Z*(0,00JY0
                                                                      18
                                                                      19
    139444#0,00021*Z))))
                                                                 JY0
     Y0=0,63661977*AL0G(0,5*X)*J0+0,36746691+Z*(0,60559366=Z*(0,7435038JY0
                                                                      20
    14=Z*(0,25300117=Z*(0,04261214=Z*(0,00427916=0,00024846*Z)))))
                                                                 1 ¥ 0
                                                                      21
     RETURN
                                                                 JY0
                                                                      22
   3 Z=3,0/X
                                                                 JY0
                                                                      23
     f=0,79788456-z*(0,77E-6+2*(0,0055274+z*(0,0009512=Z*(0,00137237=ZJY0
                                                                      24
    1+(0,00072805=0,00014476+Z)))))
                                                                 JY0
                                                                      25
     P=0,78539816+Z*(0,04166397+Z*(0,00003954=Z*(0,00262573=Z*(0,000541JY0
                                                                      26
    125+Z*(0.00029333=0.00013558*Z)))))
                                                                 JY0
                                                                      27
     G=SURT(1_0/X)
                                                                 JY0
                                                                      28
     JOEU*F*CUS(X+P)
                                                                 JY0
                                                                      29
     Y0=G+F+SIN(X+P)
                                                                      30
                                                                 JY0
   4 RETURN
                                                                 JY0
                                                                      31
     END
                                                                 JY0
                                                                      32-
     SUBROUTINE JY1(X, J1, Y1)
                                                                 JY1
                                                                       1
2
C
                                                                 JY1
                                                                       3
Ç
     PURPUSE
                                                                 JY1
                                                                       4
Ĉ
        COMPUTES BESSEL FUNCTIONS OF THE FIRST AND SECOND KIND,
                                                                 JY1
                                                                       5
¢
        FIRST URDER, FOR POSITIVE ARGUMENTS,
                                                                 JY1
                                                                       6
С
        SEE NBS AMS 55, P. 370.
                                                                 JY1
                                                                       7
C
     DESCRIPTION OF PARAMETERS - ALL REAL
                                                                 JY1
                                                                       8
C
        X. ARGUMENT, MUST BE >0
                                                                 JY1
                                                                       ٥
        J1 = RETURNED FUNCTION VALUE, J1(X)
C
                                                                 JY1
                                                                      10
С
        Y1 - RETURNED FUNCTION VALUE, Y1(X)
                                                                 JY1
                                                                      11
Ć
                                                                 JY1
                                                                      12
13
     REAL J1
                                                                 JY1
                                                                      14
     IF (X=3.0) 1,2,3
                                                                 JY1
                                                                      15
   1 IF (X) 4,4,2
                                                                 JY1
                                                                      16
   2 Z=(0+33333333*X)**2
                                                                 JY1
                                                                      17
```

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

	11-14/0 E-14/0 F4-74000F-74/0 34007E73. 74/0 070F430-74/0	
	17+(0,00031761_0,00041403=2×(0,6210735/3424(0,03434207=2×(0	104453140J11 10
	V1=0.63661977+A106005100+CJJJJJJ	12.1682709.IV1 20
	1=7*(1,3164827=7*(0,3123951=7*(0,0400976=0,0027873*71))))	1/X JV1 21
	RETURN	JY1 22
	3 Z=3.0/X	JY1 23
	F=0.79788456+7+(0.156E+5+Z+(0.01659667+7+(0.00017105+Z+(	0.00249511341 24
	1=Z*(0.00113653=0.00020033+Z))))	JY1 25
	P=0,78539816=Z*(0,12499612+Z*(0,0000565=Z*(0.00637879=Z*	(0.0007434JY1 26
	18+Z*(0,00079824=0.00029166+Z)))))	JY1 27
	Q#SQRT(1,0/X)	JY1 28
	J1=Q+F+8IN(X+P)	JY1 29
	Y1≅⇒Q★F★COS(X⇔P)	JY1 30
	4 RETURN	JY1 31
	END	JY1 32=
C**	******************	***********************
C		S AUR
C	PURPOSE	FUA 3
C	COMPUTES FUNCTION VALUES OF F(UW,ALPHA) -	FUA 4
C	PAPADOPULOS, 1, 3. AND COOPER, H.H., JR., 1967, DRAWDOWN	IN FUA 5
Ç	A WELL OF LARGE DIAMETERS WATER RESOURCES RESEARCH, V	. 3/ FUA 6
C	ND. 1, P. 241-244.	FUA 7
C	PROGRAM BY S.S.PAPADOPULOS.	FUA B
C	INPUT DATA - ONE OR MORE GROUPS, EACH GROUP CODED AS FOL	LOWS FUA 9
C	1 CARD - FORMAT (E10,5)	FUA 10
Ç	S = (ALPHA) = RW##2#S/RC##2 = RADIUS OF WELL (SC	REEN FUA 11
C	OR OPEN BORE IN AQUIFER) SQUARED * STORAGE	FUA 12
Ç	COEFFICIENT / RADIUS OF CASING (OVER INTERV	AL UP FUA 13
C	WATER LEVEL CHANGE) SQUARED.	FUA 14
Ç	1 CARD P FURMAT(1665:0) He to Mattice US Him - Dwarder Addit Tottes - Dantie	0E EUA 15
C	DUMBED WELL SOUNDED & STORAGE COEFFICIENT /	
	A TOALOHIGOTVITY A TIME TE LEGG THAN 14	NERTREN EUA 14
	A A LEANGHIGGIATHI A THES IN FROM THE B	EST. FUA 10
2	DLANK UR LEKU VALUED "AT DE LUDEU FOR ERE R Dissurtinfo and finietion disodocdame ofoutded	EDIA - FUR 17
ř	DEAK.STMP.ADEKE.EYHSID. 1V0.1V1 - MUST BE INCLUDED IN	DECK. FUA 21
ř	LENNIGTH THEFTEREDEEDDEEDDIGTOTT - HOOT OF THEFOREN TH	Fila 22
č++		*********FUA 23
	COMMON XPK.YPK	FUA 24
	COMMON/PBLK/A,B	FUA 25
	EXTERNAL EXUSE2	FUA 26
	DIMENSION U(16)	FUA 27
	EPS=0.0001	FUA 28
	1 READ (5,13,END=12) 8	FUA 29
	IF (S) 1,1,2	FUA 30
	2 READ (5714) U	FUA 31
	WRITE (6,15) S	FUA 32
	DU 11 I=1,16	FUA 33
	Uweu(I)	FUA 34
	IF (UW) 1,1,3	FUA 35
	3 8¤0,25/U*	FUA 36
	A = 3 + 3	FUA 37
	CALL APEKE(EXBSL2)	FUA 38
	CALL PEAK(EXBSL2)	FUA 39
	IF (XPK=1.0E=8) 4,5,5	FUA 40
	4 WRITE (6,16) UW,S,XPK,YPK	FUA 41
	GO TU 11	FUA 42
	5 IF (XPK+1,0E8) 7,7,6	FUA 43
	6 WRITE (6,17) UN, S, XPK, YPK	FUA 44
	GU TO 11	FUA 45
	7 HBARBO.007*XPK	FUA 46

ł

 $\geq_{e_{i}} \leq e_{i}^{e_{i}}$ 

TABLE 8.2.—Listing of programs for constant discharge from a fully penetrating well of finite diameter—Continued

			<b>.</b>	
		CALL SIMPS(0,0,XPK,EPS,HBAR,SUM,DEL,EXBSL2)	FUA	47
		X5=XbK	FUA	48
		DXEXPK	FUA	49
	8	Dx=10,0*Dx	FUA	50
		x1=x2	FUA	51
		X2=X1+DX	FUA	52
		Y#EXBSL2(X2)	FUA	53
		HBAR=0.007+DX	FUA	54
		CALL STMPS(X1, X2, FPS, HBAR, TRM, ERR, EXBS(2)	FUA	55
			FILA	56
			FILA	57
			FUR	57
	_	IF (X2=1,0E4) 4,10,10	F U A	20
	9	YT=1,5707963/x2**4	PUA	54
		IF (ABS(Y=YT)/YT=0,5E=6) 10,8,8	FUA	60
	10	EST=0,52359878/X2**3	FUA	61
		SUM=SUM+EST	FUA	62
		FUWS=5,2422779+S+S+SUM	FUA	63
		WRITE (6,18) UW, SUM, DEL, FUWS, XPK, YPK	FUA	64
	11	CINTINUE	FUA	65
	• •	GO TO 1	<b>F</b> Ū <b>A</b>	66
	15		FUA	67
~	15	a lor	ELLA.	<u></u>
C,			FUA	4.0
	13	PURMAT (E10.5)	FUA	
	14	FORMAT (1665.0)	PUA	70
	15	FORMAT (11', F(UW, ALPHA) FOR ALPHAN', 1PE14, 57'0', 7X, 'UW', 12X, 'L	NTEPUA	71
		1GRAL',5X,'INTEGRAL ERHOR',5X,'F(UW,ALPHA)',8X,'X(PEAK)',10X,'Y(	PEAFUA	72
	i	26)1/1 1)	FUA	73
	16	FORMAT (1H .1PE14.7.9X.34HVALUES OF DUMMY VARIABLE TOD SMALL.1P	E25FUA	74
	•	1.7.1PE17.7)	FUA	75
	17	FORMAT (1H . 10E14.7.9X.34HVALUES OF DUMMY VARIABLE TOO LARGE, 1P	E25FUA	75
	• '	1.7.1PE17.71	FUA	77
	1.8	#004AT (14 .1051/ 5.105F17 5)	FUA	78
	40		FIIA	70.
				17-
		FUNCTION EXHST2(X)	EB2	1
C + 1			+++E82	2
c			FND	1
ř			683	د. //
ž		FORTUGE Completer userice of the Interran Ead Elim Album	E 10 2	2
		COMPUTES VALUES OF THE INTEGRAND FOR FOUNTALPHAY	505	~ ~
C		DESCRIPTION OF PARAMETER	202	0
C		, XO HEAL O ARGUMENT OF INTEGRAND	EBZ	1
C			E82	8
- C * ·	***	******	***EB2	9
		COMMON/PELK/A,B	E82	10
		IF (X) 1,1,2	E83	11
	1	EXBSL2=0.	E82	12
	-	GD 10 8	E82	13
	2	15 (Xm1, F+7) //// 1	FR2	14
	ī		£ H 2	15
	2		605	12
				10
	4		605	17
	-		C B Z	10
	5	PNUMET*(1,+Y*(+5+Y*((1,+/6,)+Y*(1,+/24,))))	F85	19
		GO TO 7	E83	20
	6	FNUM=1.=EXP(=Y)	EBS	21
	7	CALL JYO(X,HJO,BYO)	EB2	22
		CALL JY1(X, BJ1, BY1)	EB2	23
		DEN#((X+8J0=A+8J1)++2+(X+8Y0=A+8Y1)++>)+X++3	£82	24
		EX89L2#FNUM/DFN	E82	25
	8	RETURN	FRO	26
		END	883	27-
			C D K	

TABLE 9.2.-Listing of program to compute change in water level due to sudden injection of a slug of water into a well

```
1
C
                                                                           FRA
                                                                                  2
C
      PURPOSE
                                                                           F8A
                                                                                  3
C
         COMPUTES FUNCTION VALUES OF F(BETA, ALPHA) - THE SLUG TEST
                                                                           FBA
                                                                                  4
C
         FUNCTION = COOPER, H.H., JR., BREDEHDEFT, J.D., AND PAPADOPULOS,
                                                                           FBA
                                                                                  5
C
         I.S., 1967, RESPONSE OF A FINITE-DIAMETER WELL TU AN
                                                                           FBA
                                                                                  6
C
         INSTANTANEOUS CHARGE OF WATER: WATER RESOURCES RESEARCH,
                                                                           FBA
                                                                                  7
C
         V. 3, NO. 1, P. 263-269.
                                                                           FBA
                                                                                  8
C
         PROGRAM BY S.S. PAPADOPULOS.
                                                                           FBA
                                                                                  9
C
     INPUT DATA
                                                                           FBA
                                                                                 10
Ċ
         1 OR MORE CARDS - FORMAT(F16.5)
                                                                           FBA
                                                                                 11
Č
               A = (ALPHA) = RW++2+S/HC++2 = RADIUS OF WELL (SCREEN OR
                                                                           FBA
                                                                                 12
Ĉ
                    OPEN BORE IN AQUIFER) SQUARED + STORAGE COEFFICIENT
                                                                           FBA
                                                                                 13
C
                    / RADIUS OF CASING (OVER INTERVAL OF WATER LEVEL
                                                                                 14
                                                                           FBA
C
                    CHANGE) SQUARED.
                                                                            FRA
                                                                                 15
C
      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                                                                           FBA
                                                                                 16
C
         PRX, DJY0, DJY1, DSIMPS - MUST BE INCLUDED IN DECK
                                                                           FBA
                                                                                 17
C
      METHOD
                                                                           FRA
                                                                                 18
C
         THIS PROGRAM CALCULATES THE SLUG TEST FUNCTION, F(BETA, ALPHA), FB*
                                                                                 19
č
         FOR VALUES OF BETA RANGING FROM 0,001 TO 1000,0 BY INCREMENTINGFBA
BETA ACCORDING TO DATA ARRAY BB(I), AVERAGE SOMPUTATION TIME FBA
                                                                                 20
С
                                                                                 21
C
         IS ABOUT 30 SECONDS PER VALUE OF ALPHA ON IBM 360/155.
                                                                           FRA
                                                                                 22
Ċ
                                                                           FRA
                                                                                23
FBA
                                                                                 24
      DOUBLE PRECISION A, B, PI, ZZ, EPS, Y, X1, X2, TERM, FAB, DATAN, DEL, HBAR
                                                                           FBA
                                                                                 25
      DIMENSIUN ZZ(40), BB(39)
                                                                           FBA
                                                                                 26
      COMMON A, 8, PI
                                                                           FBA
                                                                                 27
      EXTERNAL PRX
                                                                           FBA
                                                                                 28
      DATA ZZ/0.D+0,1.D=10,1.D=9,1.D=8,1.D=7,1.D=6,1.D=5,1.D=4,
                                                                           FBA
                                                                                 29
         1,D=3,1,D=2,1,D=1,2,D=1,3,D=1,4,D=1,5,D=1,6,D=1,7,D=1,8,D=1,
     1
                                                                           FBA
                                                                                30
         9, D=1,1, D+0,2, D+0, 3, D+0, 4, D+0, 5, D+0, 6, D+0, 7, D+0, 8, D+0,
     2
                                                                           FBA
                                                                                31
     ٦
         9,D+0,1,D+1,2,D+1,3,D+1,4,D+1,5,D+1,6,D+1,7,D+1,8,D+1,
                                                                           FBA
                                                                                 32
     4
         9,D+1,1,D+2,1,25D+2,1,5D+2/
                                                                           FBA
                                                                                 33
      DATA BB/,001,,002,,004,,006,,008,,01,,02,,04,,06,,08,,1,,2,,4,,6,,FBA
                                                                                 34
     18,1,,2,,3,,4,,5,,6,,7,,8,,9,,10,,20,,30,,40,,50,,60,,70,,80,,90,,1FHA
                                                                                35
     200,,200,,400,,600,,800,,1000,/
                                                                           FAA
                                                                                36
      PI=4,*DATAN(1,00+00)
                                                                           FBA
                                                                                37
      EPS=0.00001
                                                                           FBA
                                                                                38
    1 READ (5,6) A
                                                                           FBA
                                                                                39
      IF (A.LE.0.0) GO TO 5
                                                                           FBA
                                                                                40
      WRITE (6,7) A
                                                                           FBA
                                                                                41
      WRITE (6,8)
                                                                           FBA
                                                                                42
      DO 4 I=1,39
                                                                           FBA
                                                                                43
      8=88(I)
                                                                           FBA
                                                                                44
      Y=0.0
                                                                           FBA
                                                                                45
      DO 2 L=1,39
                                                                           FBA
                                                                                46
      X1=ZZ(L)
                                                                           FBA
                                                                                47
      X2=ZZ(L+1)
                                                                           FBA
                                                                                48
      HBARE0.
                                                                           FBA
                                                                                49
      CALL DSIMPS(X1,X2,EPS,HBAR,TERM,DEL,PRX)
                                                                           FBA
                                                                                50
      Y=Y+TERM
                                                                           FBA
                                                                                51
      IF (L.GT.20, AND, TERM, LT.EPS) GO TO 3
                                                                           FBA
                                                                                52
    2 CONTINUE
                                                                           FBA
                                                                                53
    3 FAB=4, *A*Y/(PI*PI)
                                                                           FBA
                                                                                54
    4 WRITE (6,9) B,FAB
                                                                           FBA
                                                                                55
      GO TO 1
                                                                           FBA
                                                                                56
    5 STUP
                                                                           FBA
                                                                                57
Ĉ
                                                                           FBA
                                                                                58
C
                                                                           FBA
                                                                                59
    6 FORMAT (F16.5)
                                                                           FHA
                                                                                60
    7 FORMAT (111,41X, F(BETA, ALPHA) FOR ALPHASI, 1909,2/)
                                                                           FUA
                                                                                61
    8 FORMAT (101,53%, 18ETA1,13%, 14/H01/)
                                                                           FBA
                                                                                62
    9 FORMAT (1 1,51%,1008,2,10%,0006,4)
                                                                           FBA
                                                                                63
                                                                                64=
      END
                                                                           FBA
```

.
TABLE 9.2.—Listing of program to compute change in water level due to sudden injection of a slug of water into a well— Continued

<b>.</b>		D	ou	B	LE		PI	26	C	I	6 1	0	N	F	U	NC	Ţ	10	)N	F	R	X	()	()		•			• •		<b>.</b>			• •		• •		- <b>-</b>		• •	<b>.</b>			• •	<b>.</b>	PI	RX P V	1	
0** C C	**	** Pi		* P	** D9	E E	* 1	* *	• #	*	<b>*</b> 1	¥ #	# 1	**	۲.	**	<b>*</b>		1 1	# 1	* *	Ħ	<del>4</del> 1	-	T TT	#1	* #		# #	T AT	#1	4 7	71	n W	#	- 7	17 T	1	1 <b>1</b> 7	ਜ ਸੋ	म्म 11 •		* #E 1	न म	<b>7 2</b>	Pi	RX RX	2 3 4	
Č C		D	ES	C		P	มา 10	TE I L	: JN	١		U	E ( P/	5 A R		F Më	T I T I	HE E F	2	I	I T	E	GF	R A	N	D	F	0	R	F	( 8	BE	Ţ/	١,	AI	. PI	HÅ	)								P I P I	RX RX	5	
C C				X		•	D	ינ	18	L	E	Ρ	R	ΞÇ	I	S I	0	N	•	4	NR	G	Ų	٩E	. N	Ţ	υ	F	I	N	T	EG	R	A N	D											PI PI	RX RX	7 8	
C * *	**	** Di	** DU	* B	* * L E	*	* I PF	* # R E	• # 2 C	* 1	* / 9 ]	** [0	* 1 N	* * Å	* •	** B,	* P	** I,	iπ X	* 1 X 1	ь # , Х	*	* * C /	* * , F	*	*1 , F	* *	*	** J0	:* );	** Y (	**	*: J:	**	*: Y:	**	* *	1	**:	* *	**		**	* *	**	PI PI	RX RX	9 10	
		Di	DU DM	M	LE DN	•	PF A	₹8 , 6	EC 3,	I P	91 I	0	N	0	L	ĴG	,	DS	90	R1	•	D	E	KP	)																					Pí Pí	RX RX	11 12	
		X	X= F	0	96 X)	R	T : 6 ;	( / , 1	\*   r	X 2	/8	3)																																		P   P	RX RX	13 14	
	1	P	rx O	# T	(P 0	1 6	<b>#</b> †	<b>P</b> ]	()	1	( )	6	• '	* Å	*	8)																														PI Pi	RX RX	15 16	
	2	I P	F R X	)	X.	L 0	Ţ	• 1	15	0	• 2	)	G	נ	T	0.	3																													P ( P (	RX RX	17 18	
	3	G	U F	Ť	U XX	6	G	٢,	0		0 (	0	1	)	G	n	Ŧ	0	4																											P	RX RX	19 20	
		C F	■ D 1 =	P	XP I#	י ( אי	5 *	, 7 ( 1	77   .	<b>.</b>	1 5 A ;	56 )	6	49	D	• 0	1	)/	2	•																										P	RX RX	21 22	
		F P	2∎ RX	X	₩C (8	)L  *	0( P)	G ( I •	(C	* 1	C ↓ # ()	A DE	*) Xi	X/ P(	8	)+ X)	4	•*	8 ( A	*	( F	1	*!	F 1	•	Fi	2*	F	2)	)																P	RX RX	23 24	
	4	G I	D F	1 (	0 X X	6	Ľ	۲.	, 5	0		)	G	כ	Ţ	Û	5																													P	RX RX	25	
		P G	R X U	i i i	(F U	6 1	*[	DE	X	P	( •	∎X	)	)/	(	2,	*	X	(*	()	K +	4	• 1	<b>*</b> A	*	8	))																			P	RX RX	27	
	5	C	AL	Ļ		)1 )1	YI	0 (	( X ( X	X X	, . , .	10 11	,	¥0 71	)																															P	RX RX	30	
		F	1#	; ( ; (	X	(* (*	J I Y I	0.	•2 •2	•	*/ */	\ * \ *	۲ ۲	1) 1)		-																														P	RX RX	31	
	6	P R	RX	ίΞ. Ú		i X	P	( 1	• X	)	/	( X	*	(F	1	* F	1	<b>+</b> P	2	*!		:)	)																							P	KX KX	33	
		Ł	NĽ	)	_		_		_		_	_			_			_	_																											-	<b>K</b> X	30	-
C * *	* * *	8 **	**	**	*1	)1 **	*	N   * 1	t. * *	U :*	۲. ۲	Υ0 ★ *	*	X, **	J t tt	0 ( *1	• ¥	0; *1	) * *	*	* 1	* *	*	* 1	• •	*	* *	k #	<b>±</b> 1	<b>k</b> 1	*	<b>*</b> 1	* *	* 1	1 #	* *	*1	h #	* *	* *	: # 1	<b>k (k</b> )	* *	*1	1 1 1	0 0 0 # 1	10 10	1	!
C		P	UF	٩P	0	SE		•					•	<b>.</b>				~ `		~		,	~	•			-	-	•					~	•					•.						D	10 10	3	) 
C		0	= (	Z	Ur Eł	1P 20			2 8 R () 7 1	E	R		F		? • •	P(	) S ) S	U I E	11	V	N 4 E	, A	0 8 1	r Gl	ו אן ח	E	r N 1	7 5 3	1) 1	4 3 0		م د (	• •	U 0 1	3	с.	U	NU.	n	¶ ₽	10.	,				0	J0 J0	5 6 7	) } 7
ĉ		U	5	X	•	۲. ۸	R	G		IE	N	, 1.	۳ ۲	An Mi	19	me T	8	E	<ul> <li></li> <li><td>0</td><td>•</td><td>~ </td><td>ы </td><td><b>6</b></td><td></td><td></td><td>06</td><td>, . , .</td><td>е ,</td><td>r</td><td></td><td></td><td>• 1</td><td>31</td><td>U</td><td>1.4</td><td></td><td></td><td></td><td></td><td></td><td></td><td>ι</td><td></td><td></td><td>D</td><td>10</td><td>8</td><td>j</td></li></ul>	0	•	~ 	ы 	<b>6</b>			06	, . , .	е ,	r			• 1	31	U	1.4							ι			D	10	8	j
č				Y	0	•	,	R	E	Ů,	R	NE	D	F	U	N(	; †	10	0		v i	1	U	Ē	,	Y	0 (	X	)																	D	J0	10	₽
Ç*1	***	**	*1 01	t★ IB	11 1 1 6	k *	* P	*: 81	* *	r# T	*: 9	**	*	**	*	#1 .10	**	*: ¥:	* *	*	* 1 . F	**	* ເລ	<b>* 1</b>	k #	*	19 19 - 19	**	#1 Di	* * r	:# 16	* I	• *	**	1 # 1 .	**	#1 .TP	* *	** D 8	** 12.6	: # 1 ) T	<b>k</b> # 1	* *	* 1	***	₩Ď	0 U	12	!
	1	Ĭ	F	) ( (	X	- - - - -	•	0	)	12	,	2,	3	•					.,	•	•	'	ľ	,.	.,		, ,	`'				,.			,,	~ •	•.				•••					D	J0 .10	14	ļ
	S	z J	121 ( 131	(X #1	/	Ś.	0 Z	) *	**	2	2	<b>4</b> 9	9	99	17	•7	*	ť	۱.	2	65	56	2	08	8.	2	* (	ta		τ,	6	3.6	36	6.	17	<b>*</b> {	0.	. 0	44	44	79	<b>)</b> = (	Z #	• •	). (	D 000	J0 J0	16	, ,
		13 W	94 = (	44	4	•0 5D	•	) ) ) ;	00	2	1	* Z	)	))	)	)		•	••	-			-	•		Ī		. •	•					•			•	, .	•••					•		D	0 U	18	•
		Y 1 (	0=	2	53	53 50	6	6 1	19 17	7	7 : Z :	*D * (	0	00	; (	₩ ) 2 (	) * >1	J ( 2 :	0+ 14	0	, 1 Z /	56 1 (	7 · 0	46	56 00	9 4	1 + 2 7	) Z 79	• 1	(0	•0	6(	)5 )0	59 02	)3 24	66 84	= / 61	Z #	() ))	•7	43	55	03	84	4 - 2	2+D D	J0 J0	20	1
	3	R	E 1 = 3	ru S.	R 1 0 /	N / X																,		-			•		-			•														D D	10 10	22	!
		F 1*	= ( ( (	).	79 00	7 0 0	8 7	8 21	45 80	i6 15	•;	Z #	0	0	7	7 ( 4 4	) <del>-</del>   7	6 • 6 •	+ Z + Z	*	(()))	).   ]	0 )	0 !	55	5	74	+	2 '	* (	0	• (	0	00	9	51	5	Z	* (	0.	00	01	37	23	57.	•ZD D	0 L 0 L	24	) j
		۹ 12	#( 5(	) . • Z	78 *(	35	3	9(	81 00	6	+ ; 9 ;	Z * 3 3	( 3	•	0	4 ) 0 (	6	6: 1:	39 35	7	+ 2 8 1	:# *Z	( )	0 );	, 0	0 )	0 0	)3	9	54	-	ZI	• (	0,	, 0	02	61	25	73	+2	*	0	• 0	00	)54	+10 D	10 10	26 27	)

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 TABLE 9.2.—Listing of program to compute change in water level due to sudden injection of a slug of water into a well— Continued

																							••			~~~																					
			U≢	(1	. (	0 Ç	0	) /	/ X																																				DJC	i	28
			Q=	0 9	0	RT	' (I	5																																					DJO	) i	20
			J0	= Q	*	F 🕯	D	ĈĊ	) 8	$\mathbf{O}$	(.	<b>P</b> )	)																																DJO	) :	3 (
			Y 0	ş (j	*	F 4	D	9 I	N	$\dot{o}$	(	P	)																																DJO	) 7	3 1
		4	RE	TU	R	N						•																																	DJO	) [	37
			ĒŇ	Ď																																									DJO	)	3 3
			911	ŘG	a a					n	īv	•	, v		1 4	. •	1	、																											0.11		-
e			••						-	24						* *	•. •	, 							<b>.</b> .								۰.			• •			• •		• •	• • •	• •	• •	0.11		÷
ເສາ ອ		-							Ŧ₩	<b>7</b> 7	• 7	-	* *		T M	* *	<b>H</b> 1	<b>.</b> .						-		i m			<b>W</b> 7				- 1			~ *							•		0.1	) I	
Ç			<b>n</b>		-																																								0.14		1
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 TABLE 9.2.—Listing of program to compute change in water level due to sudden injection of a slug of water into a well— Continued

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		Ha	8 • /	A																															DSI	2	8
		IF	- ()	H)	1.	1.	2																												DSI	5	9
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		DE	L#	0.0	)																														DSI	3	1
		GO	) 1(	0 1	0																														D81	- 3	2
	2	SP	=1.	.00	35	i -																													DSI	3	3
		83	=0	0																															DSI	3	4
		81	≡F.	(A)	+F	(8	)																												D <b>81</b>	3	5
		IF	° (1	HBA	R)	- 3	13	5,1	4																										D81	3	6
	3	HB	AR	<b>.</b> 0	00	7*	H																												DSI	3	7
	4	92	=0	.0												•																			DSI	3	8
		X	**	0.5	i#H	1																													D51	3	9
	5	92	= 3	2 + 4	.0	*F	0	()																											DSI	4	0
		X=	X+1	H																															DSI	- 4	1
		IF	( )	X = 8	)	5,	5,	6																											DSI	- 4	2
	6	SC	₩(;	914	82	+ 9	3	) <del>#</del>	1#1	5,1	66	56(	66	66	66	7																			DSI	- 4	3
		DE	L=(	0.0	66	66	6(	6	66'	7 * (	SF	<b>)</b> = {	SÇ	)																					DSI	4	4
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	8	-83	= 8)	3+0	, 5	i # 9	2																												DSI	4	8
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		IF	° (1	H+r	BA	R)		۱, ۱	Ρ, 9	9																									DSI	-5	0
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 TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer

C****	***************************************	HRT	1
Ċ		HRT	2
Č	PURPOSE	HRT	3
Č	COMPUTES CHANGES IN WATER LEVEL, H(R,T), IN RESPUNSE TO	HRT	4
Ċ	VARYING DISCHARGE USING THE CONVOLUTION INTEGRAL FUR	HRT	5
Ċ	LEAKY AQUIFERS - EW. 3 OF MOENCH, ALLEN, 1971, GROUND-WATER	HRT	6
Ċ	FLUCTUATIONS IN RESPONSE TO ARBITRARY PUMPAGES GROUND WATER,	HRT	7
Ç	V.9, NU.2, P.4-8.	HRT	8
C	INPUT DATA . ONE OR MORE GROUPS, EACH GROUP CODED AS FOLLOWS	HRT	9
C	1 CARD - FORMAT(2E10.5,4X,11,5X,E10.5)	HRT	10
C	TBEGIN - SMALLEST VALUE OF TIME FOR OUTPUT.	HRT	11
Ç	TEND - LARGEST VALUE OF TIME FOR OUTPUT.	HRT	12
C	IQ - INDICATES FORM OF DISCHARGE FUNCTION, Q(T),	HRT	13
C	IG=1,2,3 REFER TO DISCHARGE FUNCTIONS IN	HKT	14
¢	HANTUSH, M.S., 1964, HYDRAULICS OF WELLS IN CHOW,	HRT	15
C	VEN TE, ED,, ADVANCES IN HYDROSCIENCE, VUL. 11	HRT	16
C	ACADEMIC PRESS INC., NEW YORK, P. 281-442.	HRT	17
C	IGE1, G(T) IS AN EXPONENTIAL FUNCTION, CASE A,	HRT	18
C	P. 343 OF HANTUSH.	HRT	19
C	IQ=2, Q(T) IS A HYPERBOLIC FUNCTION, CASE B,	HRT	20
C	P. 344 DF HANTUSH.	HRT	21
C	19#3, Q(T) IS AN INVERŠE SQUARE ROOT FUNCTION,	HRT	22
C	CASE C, P. 344 UF HANTUSH.	HRT	23

## TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

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TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

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C									T	'HI	E	01	( 8	CF	14	RG	E	F	U	٩C	11	0	N	IN	ŧ	EA	C	H.	38	G	ME	NŢ	۲ 	IAS	Ţ	H	È.			H	RT	46
C									F	0	RM	1 6	9(	T)	1	8	A 1	[(	1	) +	81	C	1)	* (	( T)	• T	Ĩ	(1	•	)	).	I	F	LE	:55	1	ГH	AN	8	H	RT	47
Ç									9	IE (	Gм	E	NT.	3	A	RE	. I	NE	E	DE	0,		BL	A t	iΚ	S	C	<b>A</b> N	E	3E	C	OD	EC	) (	OR	t –				H	RT	48
C									5	ļU	C C	E	ΕD	I٨	łÇ	S	IE (	GΜ	E	V T	\$,	1																		H	RT	49
C			2	0	)R	M	OF	RE	C	: A	RD	8		F	0	R٢	IA'	T (	41	E 1	ΰ.	3	)																	H	RT	50
C						R			RA	D	IA	L	D	IS	37	AN	IC I	Ε	FI	٩Û	M	PI	ŲΜ	Pf	ΕD	-	E	LL		8	LA	NK	C	R	ZE	R	0			H	RT	51
Ċ										I	GN	AL	. 9	F	R	DG	R	A M		8 8	E	IN	D	TC	3	GR	20	UP	÷۵	)F	D	AT	Α.							H	RT	52
č						S			81	0	R A	G	2	ĈC	ĴĒ	FF	10	ĊĪ	E	νŤ					-	÷.														н	RŤ	53
č						Ť			T R		N S	M)	t s	š 1	ĪV	11	ŶŶ			•••																				н	RT	54
ř						þ	м	-	1	þ	17	м	ĬŇ			ĥч	'n		C (	١N	D .	. (	٦F	6	'n	NF	T	NT	NG	2	AF	D	01	VT	0F	'n				н	RT	55
ř						'	••	-		v.	· •	ы	ŕŕ	K N			ŭ	e hF	Č.		NE		ыт 1	N		9 F	n.					-	-		-					H	0 T	56
ř		• •	20	01	17	<b>T</b> NI			- C		- 2			7 1	in.	00 10	'a.	.)r IR		50	n n	• • • • •	<b>.</b> .	140		05		l De	n											н	0 <b>F</b>	50
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			ų	ייט		υĻ			•	m	Ug	• •	0	<u> </u>	4			90	5	<i>.</i>	1 1			6.5	•																n   n #	50
L.				•					•																	Ì.												•••				27
Ç I	****			<b>1</b> 1		**		* *	* *		**		# # 	* *	**	**				R W				<b>W</b> 1	н <b>П</b>	**	**	**		т <b>ж</b>	**	<b>N N</b>				1 198 1	<b>н н</b>	<b><b>HH</b></b>			<b>F</b> 1	DV.
		DI	ME	NB	51	UN				_	D	20	15	) (	1	ΕX		12	:)	, X	( )	)	, H	(C)	15	, 6	• >	<b>,</b> 6	33	(1	2,	6)		; P (	12	23	, C	16	151	н	RT	61
		DI	ME	NS	1	UN		41	(1	2	),	H	5(	14	2)	, G	11	(1	2	),	02	20	12	)																н	RT	62
		ÐI	ME	NS		UN	1	- 3	(1	2	),	H	4 (	12	2)	, G	13	(1	2	),	W (	+ (	15	)																H	RT	63
		CC	M M	UN	ł	AQ	10	ь)	• 1	I	(9	"	, A	1(	(9	),	8.	1(	9	),	0	31	, D	E	<u>, T</u>	A,	r T	<u>91</u>	<b>A</b>	2										H	RT –	64
		DA	TA	Ç	;P	11	21	<b>n</b> 1		Ť	* I	1	, C	Ŧ/	1	24	rt.	1/	ีปา	n f	11	D,	11	21	<b>F</b> †	10	)#	<b>#</b> I	1											н	RT	65
		DA	TA	H	11.	11	21	N 1		9	( '	1	۶H	51	11	5*	r † 1	R,	T	<b>)</b> †	1.	Q.	1/	12	2*	1			٠,	1.	95	11	21	r † G	111	)	11			H	RT	66
		DA	TA	Ħ	13.	11	2,	<b>₩</b> Ŧ			9 I	1	, H	4/	11	2+	11	0(	T	) 1	1.	G)	3/	12	2+	1	Q	(1	.+ ,	1.	64	11	21	1)	10	R	11			H	RT .	67
		DA	TA	X	(/	1.		1.	5,	2	.,	3.	. ,	5.		7.	1																							н	RT	68
		TI	(1	)=	0	•	•	•	•		•	_																												H	RT	69
		N	50	Ô.		-																																		н	RT .	70
	1	RF	AD	Ī	5	, 1	8 -	E	ND		17	•	T	8 F	G	IN	م ا	TE	N	ς.	Ī	1,1	00																	н	8T	71
	•	TF	1	10	1	- •   T	1	Ľ١	- 0	F	å n	í.	( -	. 1	0	• ''	6	9 T		36	1		- T	91	<b>7</b> A	٥														- 14 	DT.	74
		+ r + F	ì	10		Fo		4	0	Ē	* •		, J ( E	. 1	0	ί.	- A /	97 0			•		<b>*</b> '	9		n,														- F1	н I Ы Т	72
		17 15	~	44 11				₹7 2 \			4 U 4 A		いづ			!	41	9 7 1	,	• •		•	, ,	•	μ	÷ /		•	• -	• •	•	•									r   	73
		17	۱ ۲ ۳	Е. Т.А		ن ایا د		31	R	C I	×υ		. 3	11		1	r.	1		()	, 4	4	1	1	0	1 (	1	1.	1.4	• 2	, 4	1									K I	74
	-	WH DC	11	٢,	5		24	•1	~	~	-		•																											H	KT .	75
	2	RE	AU	ູ(	2	1	Å)	١.	ĸ,	3	<b>ا</b> ا		M																											H	RT	76
		ĮF	. (	K .	E		0	)	G	iU	ľ	U	1																											н	RŢ	77
		A 3	8*	Rŧ	3	/(	4	, *	T)																															H	RŤ	78
		Ba	PM	/9																																				H	RT	79
		Υs	AL	OG	1	0(	Ťθ	3E	GI	N)	)																													н	кт	80

18. <u>- - 1</u>

TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

	IF (V) 3.5.4	HRT	81
T		HPT	82
3			02
	GO TO 5	<b>PKI</b>	93
4	Y=Y+_001	HRT	84
5	TREGINEY	HRT	85
-	WALL DEAD (TENDA	HRT	86
		LIN •	00
	IF (Y) 6,8,7	TR I	87
6	Υ≖Υ∞_001	HRT	88
		HRT	89
7		HOT	00
8	I E ND#4	<b>HKI</b>	A 1
	M#IEND#IBEGIN+1	HRT	92
	1F (M.GT.12) ME12	HRT	93
		HRT	04
		HDT	0.5
	TEX(1)=10C@IN+1#1		77
	Y#10,**(IBEGIN+I⊕1)	HHT	96
	DU 10 J=1,6	HRT	9.7
	TIMERX(J)+Y	HRT	98
		HPT	00
		1111 <b>•</b>	
	CALL CONVUL(TIME,A,B,N,IG,SUM)	<b>DRI</b>	100
	IF (QR_GT_V) GO TO 9	HRT	101
	H(I,J)#SUM/(12,5664+T)	HRT	102
		HRT	103
			100
		TR I	104
9	H(I,J)=SUM/QR	HRT	105
	OS(I,J) = Q(TIME, IQ) / QR	HRT	106
10	FONTINIE	HRT	107
		HOT	104
		HOT	100
	IF (M <sub>8</sub> GT <sub>8</sub> 6) Ka6	<b>HKI</b>	104
	IF (QR_GT_0) GU TO 11	HRT	110
	WRITE (6,20) A,8,(CP(I),D(I),IEX(I),I=1,K)	HRT	111
	WRITE (6.21) (H1(T), H2(T), 01(T), 02(T), TH1, K)	HRT	112
		HD Y	1 4 2
			113
11	WRITE (6,25) A,8,0R,(CT(I),0(I),IEX(I),I#1,K)	MNI	114
	WRITE (6,21) (H3(I),H4(I),Q3(I),Q4(I),I=1,K)	HRT	115
12	DO 13 J8146	HRT	116
	$ \begin{array}{c} 0 & 1 \\ 0 & 1 $	HOT	117
	WRITE (DJEE) R(J); (H(I)); WO(I); J); = 1, H)		4.4.7
15	CUNITNUE	nr (	110
	IF (M <sub>8</sub> LE <sub>8</sub> 6) GD TO 2	HRT	119
	K1=K+1	HRT	120
	TE (08.61.0.) 60 TO 14	HRT	121
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		
	サガストロービジタビンス ししだしようまいしょうまえにかしようましたしまうがう		166
	WRITE (0,21) (H1(I),H2(I),Q1(I),Q2(I),I=K1,M)	HRT	123
	GO TO 15	HRT	124
14	WRITE (6,26) (CT(I).D(I),IEX(I),IEK(,M)	HRT	125
•	WETTE (A.21) (HILT), MULT), GULT), GULT), FRI, M)	HUT	124
	MATE (MET) (MS(T))MS(T))MS(T))MS(T))		120
13		HK I	151
	WRITE $(6,22) \times (J), (H(I,J), QS(I,J), I = K1, M)$	HRT	128
16	CONTINUE	HRT	129
		HRT	110
17		шüт	121
11	S ( Ur		131
		HRT	135
18	FORMAT (2210,5,4X,11,5X,E10,5)	HRT	133
19	FORMAT (6E10.3)	MRT	134
20	FORMAT (101, 10++2+8/(4+TRANS)=1, 10F10 3, 1, K11/(9+811)=1, F10.3/101	HRT	116
			133
	1/EAJ 1 - JAJO (EA4) [E, 7A] J	IN R I	120
21	PURMAT (' ',4X,6(2A4,2X,2A4,1X))	HRT	137

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## TYPE CURVES FOR FLOW TO WELLS IN CONFINED AQUIFERS

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 TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

	22	FORMAT (  ',Fu,1,6(0PF8.3,1PE11.3))	HRT	138
	23	FIRMAT (101.24.111.5X.6(244.12.9X))	HRT	139
	24	TOWAT ( U ) FERT : FJAPOLERAFIEF, MI)	HHT	140
	56	FORMAT (1917) FORMAT (1917)	HDT	4 4 4
	62			141
		10R#1,E10,3/101,1X,11/01,4X,6(2A4,12,9X))		142
	26	FURMAT (!0!,1x,!1/U!,4X,6(2A4,I2,9X))	MRT	143
		END	HRT	144=
_		SUCKOUTINE CONVOL(TIME, A, B, N, IG, SUM)	CUN	1
C * 1	***	***************************************	CON	2
C			CUN	3
0		PURPÓSE	CON	4
C		COMPUTES VALUES OF THE CONVOLUTION INTEGRAL FOR LEAKY	CUN	5
č		AQUIFERS, THE INTEGRAL IS, FROM 0 TO T. OF	CON	Ā
ř		$\Theta(T = T + T + T + T + T + T + T + T + T + $	CON.	7
ř		DESCRIPTION DE BADAFEES	CON	é
2		A BOW AND DEAL NO ADE INTEGED	CON	0
5	;	AJDJOUT ARE REALT NILW ARE INTEGER,	LUN	4
C		A = R**2*S/(4+T) = RADIAL DISTANCE SQUARED * STURAGE	CUN	10
C		COEFFICIENT / 4 · TRANSMISSIVITY.	CON	11
C		B • P'/(S*M') • HYD, COND, DF CONFINING BED DIVIDED BY	CÜN	12
C		AQUIFER STORAGE COEFFICIENT + THICKNESS OF CONF. BED.	CON	13
С		N - NUMBER OF INCREMENTS FOR EACH INTERVAL OF THE SUM.	CÚN	14
ē		IN . INDICATES FORM OF DISCHARGE FUNCTION.	CON	18
ř		SUBROUTING AND FUNCTION SUBPROBANG DECUTPED	CON.	14
ž			CON	10
L A			LUN	17
C .			CUN	18
Ç		APPROXIMATES INTEGRAL BY SUMMING THE TRAPEZOIDAL RULE APPLIED	CUN	19
C		TO A SEQUENCE OF SEGMENTS, LOWER LIMIT OF FIRST SEGMENT IS	CON	20
Ç		PICKED AT PUINT WHERE EXPONENT > =100 ,	ÇÜN	21
C		IF SUCH A PUINT DOES NOT EXIST (A+B > 2500) A FUNCTION VALUE	CON	22
C		UF 0 IS RETURNED, UPPER LIMIT = 10 + LOWER LIMIT FOR EACH	CON	23
Č		SEGMENT. USES INCREMENT OF DELTA TI & (U-L)/N WHERE N IS THE	CON	24
ř.		NUMBER OF INCREMENTS IN THE CALL, CEASES SUMMATION WHEN	CON	25
ř		FXPONENT C INCL	CON	24
ř			CON	20
č.,		· · · · · · · · · · · · · · · · · · ·	CON	21
6.43	***	***************************************	LUN	28
		REAL+8 DSUM	CUN	59
		REAL+4 NEWT,NEWTP,NEWX,NEWF	ÇÜN	30
		DSUM#0,D+0	CUN	31
		ISEO	CON	32
C		INITIAL TI COMPUTED FROM A,B	CON	33
-		ABEA+B	CUN	34
		TE (AB-GE-2500-) GO TO 7	CON	19
			CON.	14
	4		CUN	20
	1		CON	3/
	_		LUN	38
	2	ULDI=(1,-SQRT(1,-AB/2500,))*50*/8	CUN	39
		IF (ULDT_EQ_0,) GO TO 1	CON	40
C		INITIAL T=T	CÚN	41
	- 3	OLDTPETIME=ULDT	CON	42
		OLDX==A/OLDT=B+OLDT	CUN	43
		OLOF=G(OLDTP,IQ) + EXP(ULDX)/ULDT	CON	44
C		END OF SUMMATION SEGMENT IS 10 TIMES THE REGINNING	CUN	45
~	ц		CON	44
	-	TE (ENDT TTHEN CO TO S	CON .	40
		IF ACHINELINEL COLLON TO	CON	4/
		TL (OFD) 02 0 10 10 1	GUN A	48
		1981	CUN	49
		ENDTETIME	CUN	50

TABLE 11.1.—Listing of program to compute the convolution integral for a leaky aquifer—Continued

C		DELTA TI IS COMPUTED FROM LENGTH AND NUMBER OF INCREMENTS	CUN	51
	5	DEL TE(ENDT=OLDT)/N	CON	52
	-		004	55
-			CUN.	22
Ç		T' IS INCREMENTED BY DELTA I'	CON	54
		NEWT=OLDT+DELT	CON	55
		NEWX#=A/NEWT=B+NEWT	CON	56
C		TERMINATES SUMMATION WHEN EXPLORATIONATION C 1.375-444	CON	57
•			CON	31
			CUN	20
		NEWTPHTIMEHNEWT	CUN	59
		NEWF=Q(NEWTP,IQ)+EXP(NEWX)/NEWT	CUN	60
		DSUM#DSUM+(NEWF+ULDF)#DELT	CUN	61
		DI DTENEWT	CON	4.2
			60.0	02
			LUN	03
	0		CUN	64
		IF (IS,GT,O) GO TO 7	CUN	65
C		IF T' C T, BEGINS A NEW SEGMENT	CUN	66
		GD T() 4	CON .	67
	7		CON	40
	'			00
		RETORN	CON	69
		END	CUN	70-
		FUNCTION ACTIME.TAN	ß	1
<b>~</b>				
1.8.81	* * *	***************************************	U)	4
C			G	3
C		PURPOSE	Q	4
C		COMPUTES THE DISCHARGE FUNCTION, Q(T)	Q	5
ř		DESCRIPTION DE DADAMETERS	9	Ā
ž		THE ADDALL ADDER THE STACE DECIMITAD OF DEPENDED	Ā	,
ų,		THE A REAL OF ELAPSED THE STARE DEGINATED FOR ANDES	ur 43	
C		IQ = INTEGER + INDICATES FORM OF DISCHARGE FUNCTION,	ω.	8
C		IQ#1,2,3, CASES A,8,C, RESPECTIVELY, OF HANTUSH,M,S,,	G	9
C		1964, HYDRAULICS OF WELLS IN CHOW, VEN TE, ED.,	Q	10
ċ		ADVANCES IN HYDROSCIENCE, VOL. 1: ACADEMIC PRESS.	Q	11
ř			ō	12
~		NEW TURNE F. JEJEW PEARER ON WORTH OF THE		1.4
5		1044, DISCHARGE IS A FIFTH DEGREE POLTNUMIAE OF TIME,		13
C		IG=5. DISCHARGE IS A PIECEWISE LINEAR FUNCTION OF UP TO	ω	14
C		8 SEGMENTS.	Q	15
C		HETHOD	Q	16
č		FORTRAN EVALUATION OF BUNCTIONS.	ß	17
ž		- Oright ErtEphild of - Overland		4.0
			146 1	10
C**	* * 1	***************************************	u	14
		COMMON AQ(6),TI(9),AI(9),BI(9),QST,DELTA,TSTAR	G	20
		GO TO (1,2,3,4,5), IQ	G	21
	1	QEQ8T+(1,+DELTA+FXP(+TIMF/TSTAR))	G	22
	•	DETION	ā	21
	•		~	24
	2	WEWSTH(1,+DELTA/(1,+11ME/181AR))	u i	64
		RETURN	G	25
	3	QEQST*(1.+DELTA/SQRT(1.+TIME/TSTAR))	Q	26
		RETURN	Q	27
	μ	0 = A ( 1 ) + T 1 ME + ( A O ( 2 ) + T 1 ME + ( A O ( 3 ) + T 1 ME + ( A O ( 4 ) + T 1 ME + ( A O ( 5 ) + T 1 ME + A O ( A )	n ñ	2A
				20
			Ŵ	67
	_		Q	30
	5	00 6 1=2,9	G	31
		IF (TIME.LE.TI(I)) GO TO 7	Q	32
	6	CONTINUE	Q	33
			ō	14
	-			37
	1	WHAALIJTDILLJALTIME#[ILLAN]]	u	22
		RETURN	G	30
		END	Q	37+

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