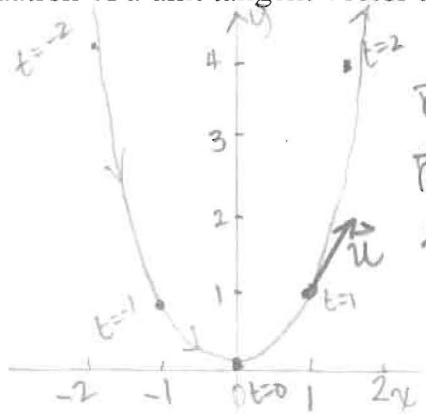
Derivatives of Vector Functions (Section 3.7)

EXAMPLE 1
Sketch the following vector equations and include the direction of the curve. Find the equation of a unit tangent vector to the curve at the given value of t.

$\boldsymbol{r}(t) = \langle t, t^2 \rangle, \quad t = 1$

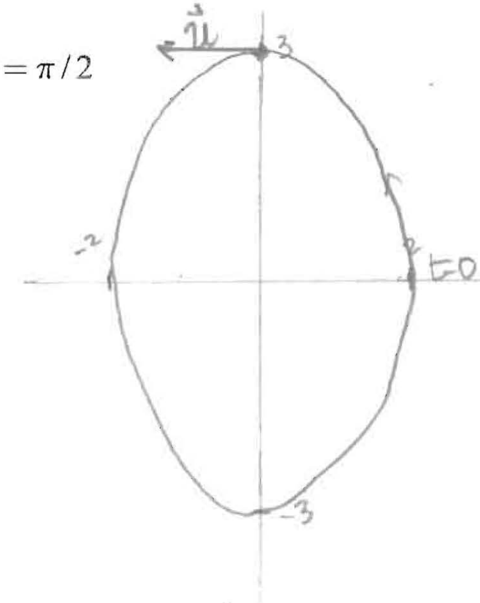
t	$x=t$	$y=t^2$
-2	-2	4
-1	-1	1
0	0	0
1	1	1



$\vec{r}\,'(t) = \langle 1, 2t \rangle$
$\vec{r}\,'(1) = \langle 1, 2 \rangle$
$\vec{u} = \dfrac{\langle 1, 2 \rangle}{\sqrt{3}}$

$\boldsymbol{r}(t) = \langle 2\cos t, 3\sin t \rangle, \quad t = \pi/2$

t	x $2\cos t$	y $3\sin t$
0	2	0
$\pi/6$	$2(\sqrt{3}/2)$	$3(1/2)$
$\pi/4$	$2(\sqrt{2}/2)$	$3(\sqrt{2}/2)$
$\pi/3$	$2(1/2)$	$3(\sqrt{3}/2)$
$\pi/2$	$2(0)$	$3(1)$

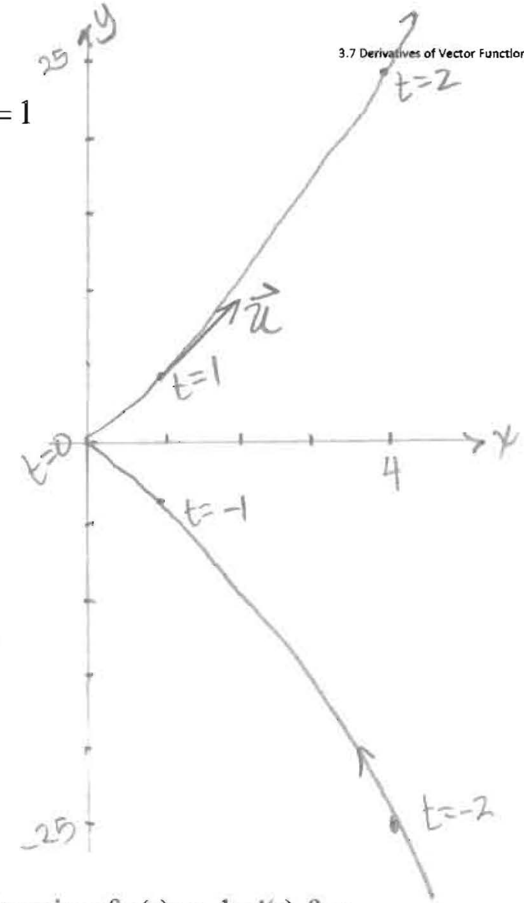


$\vec{r}\,'(t) = \langle -2\sin t, 3\cos t \rangle$
$\vec{r}\,'(\pi/2) = \langle -2, 0 \rangle$
$\vec{u} = \dfrac{\langle -2, 0 \rangle}{\sqrt{2^2+0^2}} = \langle -1, 0 \rangle$

$\boldsymbol{r}(t) = \langle t^2, 3t^3 \rangle, \quad t = 1$

t	x	y
-2	4	-24
-1	1	-3
0	0	0
1	1	3
2	4	24

$\vec{r}\,'(t) = \langle 2t, 9t^2 \rangle$
$\vec{r}\,'(1) = \langle 2, 9 \rangle$
$\vec{u} = \langle 2, 9 \rangle / \sqrt{85}$
$\approx \langle .2, 1 \rangle$



EXAMPLE 2
Find $\boldsymbol{r}'(t)$ and the domain of $\boldsymbol{r}(t)$ and $\boldsymbol{r}'(t)$ for $\boldsymbol{r}(t) = \langle t^2 - 4, \sqrt{9-t} \rangle$

domain of $\vec{r}(t) \Rightarrow t \le 9$

$\vec{r}\,'(t) = \langle 2t, \frac{-1}{2\sqrt{9-t}} \rangle$

domain $t < 9$

EXAMPLE 3

The position (in feet) of an object at time t (in seconds) is given by $\boldsymbol{r}(t)=\langle t, 25t-5t^2\rangle$, $\vec{r}\,'(t)=\langle 1, 25-10t\rangle=\vec{v}(t)$

(a) Find the position, velocity, and speed at the time t=1

$\vec{r}(1)=\langle 1, 25-5\rangle=\langle 1, 20\rangle$

$\vec{v}(1)=\langle 1, 25-10\rangle=\langle 1, 15\rangle$

$|\vec{v}(1)|=\sqrt{1^2+15^2}=\sqrt{226}$

(b) When does the item strike the ground and with what speed?

strikes ground when $y=0$

$y=25t-5t^2=0$

$\Rightarrow 5t(5-t)=0$

$\Rightarrow t=5$

$\vec{v}(5)=\langle 1, 25-10(5)\rangle=\langle 1, -25\rangle$

$|\vec{v}(5)|=\sqrt{1^2+(-25)^2}=\sqrt{626}$

EXAMPLE 4

Find the angle of intersection of the curves $\boldsymbol{r}(t)=\langle 1-t, 3+t^2\rangle$ and $\boldsymbol{s}(u)=\langle u-2, u^2\rangle$.

① intersect when $x=x$ and $y=y$

$\Rightarrow 1-t=u-2$ and $3+t^2=u^2$

$\hookrightarrow u=3-t$

$3+t^2=(3-t)^2$

$3+t^2=3-6t+t^2$

$\Rightarrow 6t=6 \Rightarrow t=1$

and $u=3-1=2$

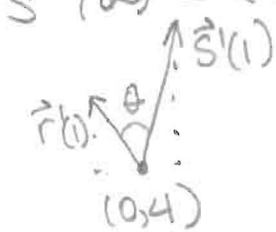
② check $t=1, u=2$

$\vec{r}(1)=\langle 0,4\rangle$ ✓

$\vec{s}(1)=\langle 0,4\rangle$

③ $\vec{r}\,'(t)=\langle -1, 2t\rangle$, $\vec{r}\,'(1)=\langle -1, 2\rangle$

$\vec{s}\,'(u)=\langle 1, 2u\rangle$, $\vec{s}\,'(2)=\langle 1, 4\rangle$



$\cos\theta=\vec{r}_1'(1)\cdot\vec{s}\,'(2)/|\vec{r}\,'(1)||\vec{r}\,'(2)|$

$=\langle -1,2\rangle\langle 1,4\rangle/\sqrt{(-1)^2+2^2}\cdot\sqrt{1^2+4^2}$

$=-1+8/\sqrt{85}=7/\sqrt{85}$

$\theta=\cos^{-1}(7/\sqrt{85})$

$\approx 41°$