Prof. Dr. Reiner Lauterbach
Jan Henrik Sylvester

Summer term 2016

Functional analysis exercises
Problems – Sheet 10

Problem 37. Let K be the operator from Exercise 12 or 36. Show:

1. For $T = \mathbb{1} - K$, $\ker T$ is closed.
2. $\ker T$ is finite dimensional.
3. The image of $T = \mathbb{1} - K$ is closed and has finite codimension.

Problem 38. For $m > \frac{1}{p}$, let $i_{m,p} : w^{m,p} \to \ell^p$ be the embedding $x \mapsto x$. Show that $i_{m,p}$ is well defined and $i_{m,p}(B_1^{w^{m,p}}(0)) \subseteq \ell^p$ is relatively compact.

Problem 39. Let $O(n)$ be the group of orthogonal matrices in $\mathbb{R}^{n\times n}$. For $f \in L^p(\mathbb{R}^n;\mathbb{R})$, set

$$A.f(x) = f(A^{-1}x).$$

Show that the set $\{A.f \mid A \in O(n)\}$ is compact in $L^p(\mathbb{R}^n)$.

Problem 40. Let X be a B-space over $\mathbb{K}$. Show:

1. X is reflexive, iff X' is reflexive.
2. If X is reflexive, any closed subspace $Y \subseteq X$ is reflexive.

Exercises are due on June 21, 2016.

Prof. Dr. Reiner Lauterbach
Jan Henrik Sylvester Summer term 2016

Space of the week

Name:	Orlicz spaces $L^\varphi(\Omega; X)$, $(\Omega, \mathcal{A}, \mu)$ σ finite measure space, X B-algebra, $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ convex, $\lim_{x\to 0} \frac{\varphi(x)}{x} = 0$, $\lim_{x\to\infty} \frac{\varphi(x)}{x} = \infty$
Definition:	$\left\{ f : \Omega \to X \;\middle|\; f \text{ measurable, } \int_\Omega \varphi(\|f(\omega)\|_X)\, \mathrm{d}\mu < \infty \right\}$
Norm:	$\|f\|_{L^\varphi(\Omega;X)} = \sup \left\{ \|fg\|_1 \;\middle|\; \int_\Omega \varphi(\|g(\omega)\|_X)\, \mathrm{d}\mu \le 1 \right\}$
Dual space:	
Dual space of:	
Reflexive:	
Criterion for compactness:	
Criterion for weak convergence:	
Additional aspects:	generalization of L^p spaces