Orthogonal Polynomials and Special Functions

SIAM Activity Group on Orthogonal Polynomials and Special Functions

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Contents

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The News

By the time this edition reaches your mailbox, a one-day symposium on Special Functions and Orthogonal Polynomials will have been held (on May 26) at the Delft Technical University in The Netherlands. This was in honor of prof. Henk Gerrit Meijer's 25 years of service at that university. The speakers on this special occasion were Jaap Korevaar, Adhemar Bultheel, Aad Dijksma, Roelof Koekkoek, and Rene Swarttouw. The organizer was Marcel de Bruin.

Also at around this time Charles Dunkl had planned to be in Holland, in Leiden, to continue discussions with Eric Opdam. These discussions began here in January of 1992 and they involve analysis on root systems, or special functions of several variables with harmonic analysis overtones.

Now that summer is approaching this is a good time to remind you that any announcements of visitors staying longer than, say, $n + 1$ weeks are suitable material for this Newsletter.

Donna DiLisi at SIAM has informed us that the business meeting of our group, which is also referred to as the SIAG/OS, is scheduled for Wednesday, 14 July at 3:30 pm, during the SIAM Annual Meeting in Philadelphia. The location will be Salon 7, situated on the mezzanine level of the Wyndham Franklin Plaza Hotel. This information should appear in the final conference program.

Electronic communication issues have been of some interest lately and in this edition we talk about a new development by Waleed Al-Salam involving the Internet.

Here are a couple of address corrections to the member list. The e-mail address of Johannes Boersma should read: wstanal@win.tue.nl. Then Galliano Valent's e-mail should be valent@lpthe.jussieu.fr, and his phone number is 44 27 39 90. Also, Henry Thacher now has e-mail: thacher@netcom.com. Incidentally, we will print additions and corrections as they are received but you should also inform SIAM directly because they prepare the member list from their own in-house database.

Finally, and especially for those who wish to contribute, the Newsletter schedule is laid out in the last section **About the Newsletter**.

Electronic Communication

Waleed Al-Salam has set up an "ANONYMOUS-FTP SITE" devoted to "Orthogonal Polynomials and Related Subjects." This is an electronic depository which allows anyone who has access to INTERNET and FTP (File Transfer Protocol) to log in with the name "anonymous", and then to use FTP to transfer files back and forth between their computer and certain directories on Waleed's computer. Although his FTP site has only a few TEX or LATEX files (of preprints that he has received so far), it could expand rapidly as more files are received, and it could be extended to include public domain software as well as databases on special functions.

The procedure for downloading files from Waleed's machine is as follows:

1. Type "ftp euler.math.ualberta.ca" (but without the quotation marks).
2. When prompted for your Name, type "anonymous".
3. When prompted for your Password, type your e-mail address or anything else.
4. When the ftp> prompt appears, type "cd pub" to switch to the /pub directory.
5. Examine the /pub directory by typing "ls" or "ls -l".
6. After the ftp> prompt you may move to any subdirectory by typing "cd dirname" where "dirname" is the name of the subdirectory. To return to the previous directory, type "cd..".
7. To download a file, type "get filename" where "filename" is the name of the file.
8. To exit, type "quit" after the ftp> prompt.

If you wish to make your unpublished manuscripts available to the Orthogonal Polynomials and Special Functions community, they should first of all be written in TEX, LATEX, AMS-TEX or AMS-LATEX. Then they can be submitted to Waleed's FTP site by one of these two methods:

- Via e-mail to waleed@euler.math.ualberta.ca
- Via FTP by doing steps 1, 2, 3 above and then, after the ftp> prompt appears, type "cd submissions" to change to the submissions directory, and finally type "put filename" where "filename" is the name of the file you wish to submit.

Large files and manuscripts with one or more related macro/style files will be tarred and compressed to create a single *.tar.Z file for UNIX machines and a *.zip file for DOS machines. These can be untarred and uncompressed, using a standard UNIX facility, or unzipped using any one of many unzipping programs available for DOS machines. If you wish to receive periodic information regarding recent additions to this electronic depository, just send a message to waleed@euler.math.ualberta.ca and remember to give your e-mail address.

SIAM Activity Group
on
Orthogonal Polynomials and Special Functions

CHARLES DUNKL, *Chair*
GEORGE GASPER, *Vice Chair*
MARTIN E. MULDOON, *Program Director*
TOM KOORNWINDER, *Secretary*

◇

THE PURPOSE *of the Activity Group*
is to promote basic research in orthogonal polynomials and special functions; to further the application of this subject in other parts of mathematics, and in science and industry; and to encourage and support the exchange of information, ideas, and techniques between workers in this field, and other mathematicians and scientists.

As mentioned above, Waleed already has a few papers on file including, for example, works by Askey and Suslov, Gautschi, and Van Assche.

The Minisymposium in Philadelphia

At the upcoming SIAM annual meeting in Philadelphia July 12-16, 1993, we will again sponsor a Minisymposium. This one will be called "Applications of Special Functions". Here, in alphabetical order, is the official list of speakers along with the titles of their talks.

Adel Faridani, Oregon State University: "Quasiregular Sampling and Computed Tomography". faridani@math.orst.edu

Hans J. Haubold, United Nations, New York: "Special Functions Applied in Astrophysics and Cosmology". hjh2@aip.org
(This involves joint work with A.M. Mathai.)

Peter A. McCoy, U.S. Naval Academy: "Special Functions, Boundary Value Problems, and Linear Elliptic Partial Differential Equations". pam@math2.sma.usna.navy.mil

Dennis Stanton, University of Minnesota: "Basic Hypergeometric Orthogonal Polynomials and Combinatorics". stanton@math.umn.edu

The organizer is Charles F. Dunkl, cfd5z@virginia.edu.

Meetings and Conferences

Some of these items have appeared in previous editions so you may be reading them here for the second time.

1. Nico Temme has just sent this announcement of a Special Day at CWI, Amsterdam, in honor of Arjeh M. Cohen and Tom H. Koornwinder. There will be a symposium with the title "Groups: algebra, combinatorics, and analysis". The date is Monday 21 June 1993 and here is the lecture program:

Dick Askey (University of Wisconsin):
"Variations on the binomial theorem"

Ton Springer (University of Utrecht):
"A generalization of the orthogonality of Green functions"

Francis Buekenhout (University of Brussels):
"Finite flag-transitive incidence geometries and buildings in a historical perspective"

Hans-Peter Kraft (University of Basel): "Embedding and cancellation of affine varieties"

Gerrit van Dijk (University of Leiden):
"Harmonic analysis on symmetric spaces"

Jacques Faraut (Université Pierre et Marie Curie, Paris): "Hardy spaces on a hyperboloid"

Cor Baayen (CWI scientific director):
"Farewell speech to Arjeh and Tom"

For information please contact

Michiel Hazewinkel mich@cwi.nl.
Tel: 31 20 592-4204 Fax: 31 20 592-4199

2. An International Workshop on "Symmetry Methods in Physics" is being organized in Dubna, July 6 to 9, 1993, in memory of Prof. Ya. A. Smorodinsky. The workshop will be devoted to current theoretical investigations of symmetry methods in physics, and the following topics will be discussed: dynamical symmetry and super-integrable systems, quantum algebras and groups, special functions and group representation theory, geometric methods in quantum theory, quantum optics and coherent states, the symmetry method in nuclei, Lie symmetries and the few-body problem, new applications.

The program will consist of the invited talks of 30 minutes duration, some shorter contributions of 20 minutes, as well as discussions.

The registration fee for the Workshop will amount to U.S.$ 200-250. This will cover the cost of the Proceedings, the hotel accomodations at Dubna, plus transportation from Moscow to Dubna and return.

Additional information can be obtained from

George S. Pogosyan
Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
141980 Dubna, Moscow Region, Russia
Telex: 911621 Dubna SU
Fax: 7 (095) 975-2381
e-mail: pogos@theor.jinrc.dubna.su

3. The 1993 session of the European School of Group Theory will be held from July 19 to July 30, 1993 in Trento, Italy. The main courses will be given by C. De Concini, M. Duflo, T.H. Koornwinder, N. Wallach and A. Valette. Koornwinder's course will be on "Quantum groups and q-special functions". The organizers are M.W. Baldoni Silva and M. Picardello (University of Rome "Tor Vergata"). The registration is already closed according to Koornwinder.

4. A conference on "Applications of Hypergroups and Related Measure Algebras" will be held in Seattle from 31 July to 6 August 1993. For more information please contact

Alan L. Schwartz hypergrp@arch.umsl.edu
Department of Mathematics & C.S.
University of Missouri-St. Louis
St. Louis, MO 63121-4499
Tel: (314) 553-6719 Fax: (314) 553-5415

There is one significant deadline for housing at the University of Washington dormitories, namely July 9. We also should mention that Bill Connett is a co-organizer. He will be able to handle inquiries about the conference. You may contact him at (314) 553-6343.

5. A conference on "Nonlinear Numerical Methods and Rational Approximation" will be held at the University of Antwerp from September 5 to 11, 1993. Registration will cover housing, participation, and a copy of the conference proceedings—which will be similar to the 1987 proceedings. Special attention will be paid to housing and to social events for the participants.

The emphasis will be on Padé approximation, rational interpolation, rational approximation, continued fractions, and orthogonal polynomials. Each of these topics will be introduced by a one hour survey lecture. Also welcome are contributions on multivariate or multidimensional problems, error analysis, software development, and applications. Participants are invited to present a 20 minute research talk.

The invited speakers will be A. Gonchar (Moscow), M. Gutknecht (Zürich), W.B. Jones (Boulder, USA), D. Lubinsky (Witwatersrand, South Africa), and E. Saff (Tampa, USA).

For more information contact the organizer, who is

Annie Cuyt cuyt@wins.uia.ac.be
Department of Mathematics & C.S.
University of Antwerp
B-2610 Wilrijk-Antwerp, Belgium
Tel: (32) 3 820-2407 Fax: (32) 3 820-2244

6. "Symbolic Computation in Combinatorics" is the title of a workshop to be held from 21 to 24 September 1993 at the Mathematical Sciences Institute of Cornell University in Ithaca, N.Y. Lectures will be given at the Center for Symbolic Methods in Algorithmic Mathematics, often called ACSyAM. This center is supported by the U.S. Army Research Office.

The focus of the workshop will be on the role of computer algebra in solving problems concerning symbolic manipulation of combinatorial formulae, and on the formal treatment of analytical problems in combinatorics. For example, q-binomial sums, q-hypergeometric series, and recurrences will be considered. The intended scope also includes nontrivial applications as well as new algorithms, systems aspects, and demonstrations. As for the main speakers, G.E. Andrews will speak on "AXIOM and the Borwein Conjecture", P. Flajolet will give a talk on "Statistical Combinatorics", and D. Zeilberger will talk about "The Holonomic Paradigm and beyond".

The workshop is being co-organized by

P. Paule ppaule@risc.uni-linz.ac.at, and
V. Strehl strehl@informatik.uni-erlangen.de

7. A session on the topic of special functions will be held during the March 25-26, 1994 meeting of the AMS in Manhattan, Kansas. Anyone who might be interested in presenting a paper at this meeting should please contact

Robert Gustafson
Department of Mathematics
Texas A&M University
College Station, TX 77843,
Tel: (409) 764-8933 and (409) 845-3950
Fax: (409) 845-6028 rgustaf@math.tamu.edu

8. In Leiden, and also in Toulouse, the 100th anniversary of T.J. Stieltjes' premature death will be commemorated on 31 December 1994. A number of activities are planned at both locations. Thomas Jan Stieltjes was born in 1856, in Zwolle, Holland. An astronomer who became a mathematician, he developed interests in analysis and in number theory. T.J. Stieltjes eventually became a professor at the University of Toulouse and he was a friend of Charles Hermite.

First of all, in April 1994 the Congress of Wiskundig Genootschap (Dutch Mathematical Society) will be held in Leiden, where Richard Askey will give the Stieltjes Lecture. Contact:

Prof. G. van Dijk
Rijksuniversiteit Leiden
Afdeling Wiskunde en Informatica
P.O. Box 9512
2300 RA Leiden, The Netherlands

Then in Toulouse there will be a colloquium, of perhaps three days, in the spring of 1995, where it has been agreed that the academic year 1994-1995 will be known as "l'année Stieltjes". The focus will be on continued fractions and moment problems, orthogonal polynomials, Laplace transforms, the Riemann hypothesis, and other topics. This will have a somewhat historical character. There will also be an emphasis on the work of Stieltjes in both the graduate and undergraduate seminars.

Contact:

Prof. J.-B. Hiriart-Urruty
Groupe d'Histoire des Mathématiques
de l'Université Paul Sabatier
118, Route de Narbonne
31062 Toulouse, France

Preceding these activities in Leiden and Toulouse, a new edition of the collected works of Stieltjes will appear at the beginning of 1993. It will be edited by G. van Dijk and published by Springer-Verlag. In addition to containing all the Stieltjes papers (as in the first edition published by Noordhoff in 1914/1918), there will be a short biography of Stieltjes, an English translation of his important paper "Recherches sur les fractions continues", and four other contributions:

"The Impact of Stieltjes' Work on Continued Fractions and Orthogonal Polynomials", by W. Van Assche

"The Stieltjes Integral and Its Influence on Analysis", by W.A.J. Luxemburg

"The Contributions of Stieltjes to Number Theory", by F. Beukers

"Stieltjes and The Riemann Hypothesis", by H.J. te Riele

Also, The University of Leiden has taken the initiative to create a "Stieltjes Institute" of mathematics. This will be a joint effort involving several Dutch Universities.

Finally, G. van Dijk is preparing an article on the life and work of Stieltjes for The Mathematical Intelligencer.

Announcement

George Polya Prize
Call for Nominations

This $20,000 prize is given every two years, honoring the memory of George Polya (1887-1985). It is the largest award given by SIAM and it will next be presented at the 1994 SIAM Annual Meeting in San Diego. The prize will be given this time for a notable contributon in one of the following six areas: approximation theory, complex analysis, number theory, orthogonal polynomials, probability theory, or mathematical discovery and learning.

The award will consist of an engraved medal and a $20,000 cash prize. There are no eligibility restrictions except that the prize is broadly intended to recognize specific work.

A letter of nomination, including a description of achievement(s), should be sent by 1 August 1993 to:

Professor Richard A. Askey
Chair, Polya Prize Committee

c/o Allison Bogardo
Society for Industrial and Applied Mathematics
3600 University City Science Center
Philadelphia, PA 19104-2688

Tel: (215) 382-9800 Fax: (215) 386-7999
e-mail: bogardo@siam.org

The prize alternates between combinatorics and the six areas mentioned above. Thus, the 1996 prize will be for work in combinatorics, the 1998 prize will again be given for work in one of the six areas, and so on.

A Short Biography of Ramanujan

In December last year your editor had the pleasure of visiting the Ramanujan Institute for Advanced Study in Mathematics at the University of Madras, India. There he met G. Rangan, M.S. Rangachari and others. He was given three little booklets, of about 50 pages each, of material culled from Ramanujan's notebooks. The booklets were written in celebration of the centenary of Ramanujan's birth, and should "help teachers to present him at appropriate contexts in the classroom". Although these booklets were intended for the schools in India, they might also serve as an inspiration for university students here. In any case they make good reading. There are sections on prime numbers, magic squares, 1729 and allied themes, plus others.

These booklets are available for U.S. $8.00 (for all three including sea mail postage) from

R. Athmaraman
Administrative Secretary
Association of Mathematics Teachers of India
35, Venkatesa Agraharam, Mylapore
Madras 600 004, India

This publication goes by the name of *An Introduction to Creativity of Ramanujan*. The three booklets are subtitled Primary Schools, Middle Schools, and High Schools.

The following biographical sketch is contained in each of the booklets and it is reprinted with permission from Prof. G. Rangan representing The Association of Mathematics Teachers of India. Two photos from Ragami's Collections in Madras have been inserted here but the biography is otherwise reproduced without any changes.

Srinivasa Ramanujan
A Brief Biographical Sketch

by G. RANGAN, University of Madras

Srinivasa Ramanujan occupies a unique position in the history of mathematics. Though blocked by the system from getting the formal university exposure in mathematics education, he rose to be one of the most outstanding creative mathematicians of modern times. His work continues to engage the attention of top researchers in mathematics all over the world, even when the world is celebrating the Centenary of his birth.

He was born on 22nd December 1887 in an orthodox Vaishnavite family at Erode, a town in Tamil Nadu State of South India. He grew up at Kumbakonam where his father was an accountant in a cloth merchant's shop. His schooling started in Kumbakonam when he was five years old. He passed the Government Primary Examination in November 1897 in First Class. He went to Town High School and became a Matriculate of the Madras University in 1903. Securing Junior Subramaniam Scholarship for his proficiency, he joined the F.A. Class of the Government College, Kumbakonam.

He found himself absorbed in mathematics and no other subject gave him as much thrill as mathematics. He became a researcher in mathematics right from his school days, raising questions and seeking new directions. In a system that demanded 'equal' attention to all subjects and consequent passing in all subjects in the final examination, Ramanujan's natural indifference to subjects other than mathemtics landed him in trouble and saw him fail F.A. But his mathematical studies and investigations continued with unabated vigour. He sought access to advanced books in mathematics. His later attempts too to pass F.A. did not succeed though he continued to score centum in mathematics.

At nineteen he had no university qualifications but a burning desire to explore and enjoy discovering new ideas in mathematics. What with the books he could get from Government College, Kumbakonam and the encouragement he got from his friends and a few discerning teachers, he was finding a lot of new things and jotting them down in his notebooks. He was endowed with an extraordinarily intuitive mental make up and a remarkably diligent attitude for hard work. Loney's Trigonometry Part II exposed him to *infinite series, complex quantities, expansions of logarithmic, circular and hyperbolic functions, logarithms of complex quantities* etc. He had the gift to do a lot by himself once given a start. Carr's Synopsis of Elementary Results in Pure Mathematics gave him hours of delightful verification, justification and research. It contained propositions, formulae and methods of analysis with abridged demonstrations in various branches of mathematics.

His note books preserved for posterity in fascimile edi-

tion since 1957 reveal the workings of his creative mind in mathematics and the style of his jottings bear the influence of Carr.

Jottings in his notebooks cover Bernoulli numbers, definite integrals, continued fractions, infinite series including divergent series, analytical theory of numbers, expressions for numbers and functions. They start with magic squares, his first passion begun possibly in his school days.

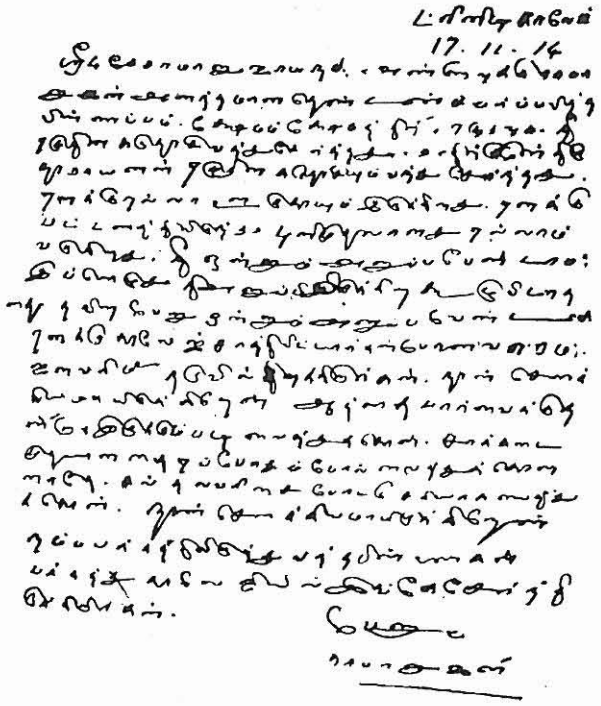
At 22, he was married to Janaki Ammal (now happily awaiting the centenary celebrations) and there arose the need to run a family and that made him go in search of a job. With no university qualification to support him, getting a decent job was not easy. He tried to maintain himself by giving private tuitions but it was not a success as catering to examination needs alone was painful to him. He believed he could interest some well-placed persons with mathematical attainments and secure a job. Ramaswamy Iyer, though not serving as a Mathematics Professor, was called a professor for his contributions to mathematics and he was working as a Deputy Collector in Tirukoilur. He had founded in 1907 the Indian Mathematical Society. Ramachandra Rao with a reputation for his interest in mathematics was working as Collector at Nellore. Ramanujan approached both of them for help in getting employment. They understood his research strivings and so advised him to stay in Madras and recommended him to Seshu Iyer of Presidency College, Madras, his former teacher at Kumbakonam, for getting employment at Madras itself and pursuing his work in mathematical research. For some time Ramachandra Rao supported him with a monthly allowance. Ramanujan's first research article on Bernoulli numbers apeared in the Journal of the Indian Mathematical Society in 1911. He started contributing several problems to the journal from then on.

It was 1912; he was 25 and he succeeded in getting a temporary job as an officiating clerk in Accountant General's Office and worked there nearly for a month and a half. He made an application to Madras Port Trust for a clerkship and it was fortunately accepted. Even in his application letter, he stated that he was devoting all his time to mathematics and developing the subject, revealing his great faith in his capabilities in mathematical research. Providence was kind as he had in Narayana Iyer, the Manager of the Port Trust, a mathematician who was formerly on the Staff of St. Joseph's College, Tiruchirapalli and then a Treasurer of the Indian Mathematical Society and in Francis Spring, the Chairman, an Engineer who could appreciate and encourage mathematical gifts. No wonder he soon attracted their notice and attempts were made to get his contributions better known and introduced and assessed. Gilbert T. Walker, the then Head of the Indian Meteorological Department, Shimla visited Madras Harbour and when he was shown Ramanujan's work, he was so impressed that he recommended strongly granting Ramanujan facilities to engage himself in mathematical research for at least a few years with no anxiety for livelihood.

In the meantime Ramanujan read Prof. Hardy's tract on Orders of Infinity and therein the statement that there was yet no formula giving the number of primes less than a given number attracted him as he had done some work in that regard. He was encouraged to write to Prof. Hardy, enclosing his numerous findings. Prof. Hardy, took no time to realize the need for Ramanujan to work in a research climate that was not then available in India. Hardy's response proved a turning point in Ramanujan's life.

Soon steps were taken to admit Ramanujan as a research scholar for a couple of years in the Madras University. Though there was some opposition to the move, he was accepted and he became the first research scholar in Mathematics in the University. He was granted leave on loss of pay from his position by the Port Trust and the stipend was two and a half times the amount he was earning in the Port Trust. It is worthy noting that there was then no Department of Mathematics in the University. Terms of his engagement saw him submitting quarterly reports of his research work. They deal with integrals, interpolation formulae, fractional composition of functions and fractional differentiation. Refereeing his work did not prove easy. Unlike the jottings in his notebooks, he gave proofs.

Prof. Hardy invited him to work with him in Cambridge University, England. Going abroad was not a common thing and there was therefore some hesitation as his parents and parents-in-law were not quite in favour of it. But his well-wishers encouraged him and he made up his mind to go. With a special grant from the Government he sailed to England in March 1914, cropping his hair and wearing a western outfit. It took him a month to reach the shores of England.



Ramanujan's letter from Trinity College to his father Srinivasa Iyengar (at Kumbakonam), dated 17 November 1914. Photo: Ragami's Collections, Madras, South India.

His life as a professional full time mathematician started in a congenial atmosphere and it lasted for about five years. He published twenty papers some in collaboration with Hardy and some by himself. Hardy, with a rare sense of sensitivity and consideration, guided Ramanujan in getting familiar with accepted standards of presenting proofs and notational usages and providing him access to developments in research areas of interest to him. Hardy was himself a leading research worker and he found in Ramanujan a very competent collaborator. But Ramanujan's capabilities were much wider. His papers published in England centered around arithmetical functions, theory of partitions, hypergeometric series and Prof. Hardy marvelled at his intuitive flashes and uncanny findings. In his fourth year abroad he fell ill and the treatment that was given in Sanatoria at Wells, Matlock and London was not of much help. However he continued his research activity. He was strict in keeping to his cultural roots, in what he ate and how he moved with others and spent his time. He read some general books and kept himself aware of happenings in the world and in a sense he was unlucky as the first world war was raging.

Honours came to him in quick succession. B.A. degree which he could have got only by passing the examination in India was conferred upon him for research in England. He was awarded Trinity College Fellowship and elected the first Indian fellow of the Royal Society on February 28, 1918 by virtue of scholarship (unlike Corsetji who preceded him through subscription). He is compared to Euler and Jacobi in his calibre.

By early 1919 there were signs of some improvement in his health and he left for home. A rousing reception was given to him at Madras Central when he arrived from Bombay where he landed. Prof. Neville's prophecy made in his letter written in July 1914 to Dewsbury, the Registrar of Madras University that 'his (Ramanujan's) name will become one of the greatest in the history of mathematics and the University and city of Madras, will be proud to have assisted in his passage from obscurity to fame' turned out to be absolutely true. His ailment however continued and he was given the best medical attention and care that was available in his day. Mathematicians in France and Germany were awaiting his recovery and opportunities of collaborating with him. The treatment saw no improvement and he joined the immortals on April 20, 1920. He was staying at Sethupattu, Madras and even during illness he worked on Mock Theta Functions and sent his papers to Prof. Hardy. This contribution of his, unearthed last year and considered his lost note book is acclaimed today as the crowning piece of his creative endeavour in mathematics.

Lacking access to developments in mathematical thinking in his day, Ramanujan could not help making many rediscoveries. Ironically enough, incomplete study of his work finds even today many researchers in the world facing the unpleasant surprise of their discoveries turning into rediscoveries.

The Post and Telegraph Department of The Government of India brought out a special stamp on 22 December 1962, commemorating the 75th anniversary of Ramanujan's birth. Photo: Ragami's Collections, Madras, South India.

A conjecture made by Ramanujan in 1916 was proved only in the year 1974, by the French mathematician Deligne using very sophisticated methods of Algebraic Geometry. Some of Ramanujan's contributions are finding application in Statistical Mechanics, String Theory in Physics and other fields. A recently discovered computer algorithm to calculate π to 17.5 million digits is claimed to have its origin in a formula for π given by Ramanujan. Professors Watson and Wilson started editing his notebooks (which are proving to be a mine of research possibilities more than his Collected Works) and could not complete them. Prof. Bruce Berndt of Illinois University, USA has emerged as a missionary of Ramanujan and he has taken upon himself the task of bringing out all his notebooks fully edited.

At a time when it was fashionable to exercise one's intellectual gifts to seek lucrative position in civil service, Ramanujan started the tradition of getting recognition through strivings and spectacular achievements in the world of science and he was the morning star in the awakening of India. The tradition was kept up by a galaxy of great men beginning with Sir C.V. Raman.

With the passage of time, world wide appreciation of his work is increasing, his fame spreading and his extraordinary devotion and dedication regardless of handicaps continuing to inspire mankind.

Madras, India
November, 1987

Problems

1. Prove or disprove

$$\prod_{n=1}^{\infty} \frac{\cos(t/2^n) - \cos(\pi 3^n/5)}{1 - \cos(\pi 3^n/5)} = \frac{4\cos(t)}{\sqrt{5}+5} + \frac{1}{\sqrt{5}}.$$

Submitted by R. William Gosper, August 13, 1992.

2. Is it true that

$$x^2 t^x \, {}_2F_1(x+1,\, x+1;\, 2;\, 1-t)$$

is a convex function of x whenever $-\infty < x < \infty$ and $0 < t < 1$?

Submitted by George Gasper, August 19, 1992.

3. The following Toeplitz matrix arises in several applications. Define for $i \neq j$

$$A_{ij}(\alpha) = \frac{\sin \alpha\pi(i-j)}{\pi(i-j)},$$

and set $A_{ii} = \alpha$. Conjecture: the matrix

$$M = (I - A)^{-1}$$

has positive entries. A proof is known for $0 < \alpha \leq 1/2$. Can one extend this to $0 < \alpha < 1$?

Submitted by Alberto Grünbaum, November 3, 1992.

4. Prove that

$$\sum_{n=0}^{\infty} \frac{(-)^n (6n+1/2)!}{\sqrt{\pi}\,(6n+1)!} = \frac{\sin\frac{\pi}{8}}{3\sqrt[4]{2}} + \sqrt{\frac{\sqrt{6}-\sqrt{3}-\sqrt{2}+2}{18}}.$$

Submitted by R. William Gosper, May 10, 1993.

Solutions to Problem #1

Two solutions were received and they appear to follow the same line.

Solution to Problem #1
by MIZAN RAHMAN

Department of Mathematics and Statistics
Carleton University, Ottawa, Ontario K1S 5B6

Gosper's identity is

$$\prod_{n=1}^{\infty} \frac{\cos(t/2^n) - \cos(\pi 3^n/5)}{1 - \cos(\pi 3^n/5)} = \frac{4\cos t}{\sqrt{5}+5} + \frac{1}{\sqrt{5}}. \tag{1}$$

First observe that

$$\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}, \qquad \cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4},$$

and

$$\cos\frac{\pi 3^n}{5} = \begin{cases} \cos\frac{\pi}{5}, & n = 2,4,6,\ \ldots \\ -\cos\frac{2\pi}{5}, & n = 1,3,5,\ \ldots\ . \end{cases}$$

The right hand side of (1) is

$$\frac{4}{\sqrt{5}+5}\left(\cos t + \frac{\sqrt{5}+1}{4}\right) = \tag{2}$$

$$= \frac{4}{\sqrt{5}+5}\left(\cos t - \cos\frac{4\pi}{5}\right)$$

$$= \frac{8(\cos^2\frac{t}{2} - \cos^2\frac{2\pi}{5})}{\sqrt{5}+5}$$

$$= \frac{\cos\frac{t}{2} - \cos\frac{3\pi}{5}}{1-\cos\frac{3\pi}{5}} \cdot \frac{2(\sqrt{5}+3)}{\sqrt{5}+5}\left(\cos\frac{t}{2} - \cos\frac{2\pi}{5}\right)$$

$$= \frac{\cos\frac{t}{2} - \cos\frac{3\pi}{5}}{1-\cos\frac{3\pi}{5}} \cdot \frac{\cos\frac{t}{4} - \cos\frac{3^2\pi}{5}}{1-\cos\frac{3^2\pi}{5}} \cdot \frac{4(\cos\frac{t}{2^2} + \frac{\sqrt{5}+1}{4})}{\sqrt{5}+5}.$$

The last factor on the right hand side of (2) is the same as we started with, except that t is replaced by $t/2^2$. So, iterating it n times we get

$$\frac{4}{\sqrt{5}+5}\left(\cos t + \frac{\sqrt{5}+1}{4}\right) = \tag{3}$$

$$= \left[\prod_{k=1}^{2n} \frac{(\cos\frac{t}{2^k} - \cos\frac{3^k\pi}{5})}{1-\cos\frac{3^k\pi}{5}}\right]\left[\frac{4(\cos\frac{t}{2^{2n}} + \frac{\sqrt{5}+1}{4})}{\sqrt{5}+5}\right].$$

Taking the limit $n \to \infty$ on the right gives (1).

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Solution to Problem #1
by JOHANNES BOERSMA

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A proof of Gosper's identity

$$\prod_{n=1}^{\infty} \frac{\cos(t/2^n) - \cos(\pi 3^n/5)}{1 - \cos(\pi 3^n/5)} = \frac{4\cos(t)}{\sqrt{5}+5} + \frac{1}{\sqrt{5}}$$

can be based on the following observations:

$$\frac{1}{1-\cos(\pi 3^n/5)} = \begin{cases} 3-\sqrt{5}, & \text{if } n \text{ is odd} \\ 3+\sqrt{5}, & \text{if } n \text{ is even} \end{cases} \tag{1}$$

and

$$[\cos(t/2^n) - \cos(\pi 3^n/5)]\,[\cos(t/2^n) + \cos(\pi 3^n/5)] =$$

$$= \frac{1}{2}[\cos(t/2^{n-1}) + \cos(\pi 3^{n-1}/5)] \tag{2}$$

which are both easily verified.

Consider now the partial product

$$P_{2N} = \prod_{n=1}^{2N} \frac{\cos(t/2^n) - \cos(\pi 3^n/5)}{1 - \cos(\pi 3^n/5)}.$$

Then, by repeated use of (2) one obtains

$$P_{2N} = \frac{\cos(t) + \cos(\pi/5)}{\cos(t/2^{2N}) + \cos(\pi 3^{2N}/5)}.$$

By taking limits as $N \to \infty$, it is readily found that

$$\lim_{N\to\infty} P_{2N} = \frac{\cos(t)+\cos(\pi/5)}{1+\cos(\pi/5)} = \frac{4\cos(t)}{\sqrt{5}+5} + \frac{1}{\sqrt{5}}$$

since $\cos(\pi/5) = (\sqrt{5}+1)/4$.

(Received 17 May 1993)

About the Newsletter

Like most newsletters, this one relies on input from the members it is supposed to serve. Here is the publication schedule. Material received on or before a *Draft* #1 date can be included in that particular edition. Material received after such a date will have to be considered for a later edition.

Edition	*Draft* #1	*SIAM Mailing*
Fall	10 August	28 August
Winter	10 November	28 November
Spring	10 February	28 February
Summer	10 May	28 May

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You may wish to note the Editor's new street address, new zip code, and separate fax line (his phone number and e-mail are the same).

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1993 SIAM ANNUAL MEETING

SELECTED THEMES

Computational Fluid Dynamics
Materials Industrial Problems
Optimization
Inverse Problems and Nondestructive Testing
Dynamical Systems
Discrete Mathematics, Combinatorics, and Applications
Finance
Biotechnology

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INVITED PRESENTATIONS

Mathematical Modeling for Quantitative Nondestructive Evaluation
Jan Achenbach, Northwestern University

The Creation and Motion of Interfaces During Phase Transitions
John Cahn, National Institute of Standards and Technology

Non-classical Effects Accompanying Diffusion in Polymers
Donald S. Cohen, California Institute of Technology

Solving Travelling Salesman Problems
William J. Cook, Bellcore

The Gibbs Phenomenon and Scientific Computing
David Gottlieb, Brown University

Numerical Methods and Analyses of Nonlinear Dispersive PDEs
James M. Hyman, Los Alamos National Laboratory

Pricing Options on Financial Securities Subject to Credit Risk
Robert Jarrow, Cornell University

Mathematics in Biotechnology
Polly Moore, Genentech, Inc

Mathematical Modeling in Electrophotographic Imaging Systems
John Spence, Eastman Kodak Company

The Many Faces of Optimization
Margaret H. Wright, AT&T Bell Laboratories

MINISYMPOSIA (Partial Listing)

Motion of Phase Boundaries
Aspects of Interface Motion
Nonlinear Diffusion in Polymers
Computational Aspects of Free Surface Flows
Viscoelastic Flows
High-Order Schemes for Shock Wave Calculation
Mathematics of Electrophotographic Imaging
Mathematical, Computational, and Integrated Models in Nondestructive Evaluation: I and II
The Mathematics of DNA: I and II
Optimization in Engineering: Multidisciplinary Optimization
Large-Scale Engineering Design
Time Stability in Large-Scale Computing
Mathematics of Finance: I and II
Combinatorial Optimization
Integer Linear Programming
Enumerative Combinatorics
Algebraic Combinatorics

Tutorial on Introduction to Wavelets and Applications
July 11, 1993
Charles K. Chui, Texas A&M University

Symposium on
INVERSE PROBLEMS AND OPTIMAL DESIGN IN INDUSTRY
July 8-10, 1993
Sponsored by ECMI and SIAM. In cooperation with IMA (Minnesota), INRIA, and SIMAI

Invited Participants

Kinji Baba, Mitsubishi Heavy Industries
Mario Bertero, Università di Genova
Christian Bischof, Argonne National Laboratory
Leonard J. Borucki, Motorola Advanced Technology Center
Catherine Chardaire-Rivière, Institut Français du Petrole
Tito Ciriani, IBM-SEMEA
J. Allen Cox, Honeywell Inc.
Ugo D'Elia, ALENIA
Richard E. Ewing, Institute for Scientific Computation, Texas A&M University
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Leonid Gurvits, Siemens Corporate Research, Inc.
Patrick S. Hagan, Los Alamos National Laboratory
Ichiro Hagiwara, Vehicle Research Laboratory, Nissan Motor Co., Ltd.
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