ME 209
Basic Thermodynamics
Analysis of Open Systems

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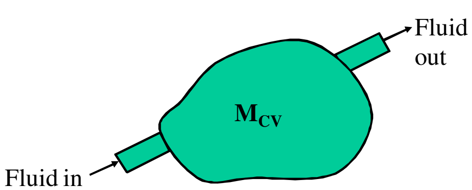
Analysis of Open Systems

- Our previous discussions were focussing on closed system generally confined to a chamber with rigid walls or piston cylinder where a fixed mass of fluid trapped.
- Many engineering systems have fluid flowing through them and the mass of fluid inside the device need not be constant
- Systems through which mass passes by are called open systems and we shall derive the governing relations. This kind of analysis is also called control volume analysis
- We will show later that the two forms are equivalent. However, one form makes the problem easier to solve.

Conservation of Mass-I

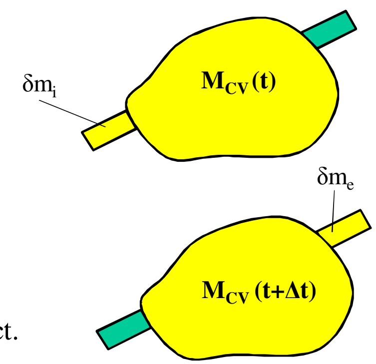
- Consider an arbitrary control volume as shown through which mass crosses (flowing from ducts)



- The aim shall be to use the equations derived for constant mass systems previously
- We shall now look at two snapshots one at t and other at t+Δt

Conservation of Mass-II

- Let us consider the same mass of fluid as shown in yellow
- At time, t, the fluid fills the control volume and a portion of inlet duct
- The same fluid at t+Δt fills the control volume and a portion of exit duct.



$$\Rightarrow M_{CV}(t+\Delta t)+\delta m_e = M_{CV}(t)+\delta m_i$$
$$\Rightarrow M_{CV}(t+\Delta t)-M_{CV}(t) = \delta m_i - \delta m_e$$

Conservation of Mass-III

- Dividing both sides by Δt and shrinking the Δt to 0, we can write,

$$\left.\frac{M_{CV}(t+\Delta t)-M_{CV}(t)}{\Delta t}\right|_{\Delta t\to 0}=\left.\frac{\delta m_i}{\Delta t}\right|_{\Delta t\to 0}-\left.\frac{\delta m_e}{\Delta t}\right|_{\Delta t\to 0}$$

$$\frac{dM_{CV}}{dt}=\dot{m}_i-\dot{m}_e$$

- If there are several inlets and exits, the mass balace implies

$$\frac{dM_{CV}}{dt}=\sum\dot{m}_i-\sum\dot{m}_e$$

Conservation of Mass-IV

- If we put the above equation in words, we can write

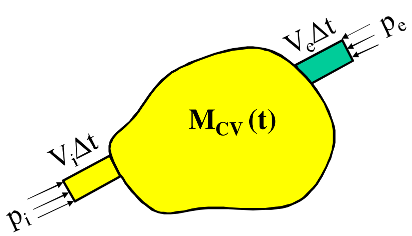
Rate of mass accumulated in a control volume	=	Total mass flow rate into the CV	-	Total mass flow rate exiting the CV

- In terms of uniform velocities in ducts, we can write

$$\frac{dM_{CV}}{dt}=\sum\rho_i A_i V_i-\sum\rho_e A_e V_e$$

Concept of Flow Work-I

- During the time Δt, at the inlet duct, the boundary of the control mass is pushed inwards by a distance equal to $V_i\,\Delta t$ and at the same time at outlet duct, the boundary is pushed outward by $V_e\,\Delta t$



- Work for the system at inlet is $-p_iA_iV_i\,\Delta t$ and at outlet is $+p_eA_eV_e\,\Delta t$

- It implies that rate of Work for the system at inlet is $-p_iA_iV_i$ and at outlet is $+p_eA_eV_e$

Concept of Flow Work-II

- Or the same can respectively be written as

$$-\frac{p_i}{\rho_i}\dot{m}_i \text{ and } \frac{p_e}{\rho_e}\dot{m}_e$$

$$\text{Or } -p_i v_i \dot{m}_i \text{ and } p_e v_e \dot{m}_e$$

- Thus the rate of work for the control volume at inlet and outlet are respectively $-p_i v_i \dot{m}_i$ and $p_e v_e \dot{m}_e$

First Law for Open System-I

- We can now derive the first law for open system.
- Let the kinetic, potential and internal energy per unit mass be denoted by $V^2/2$, u, gz. Collectively they can be clubbed together at inlet as

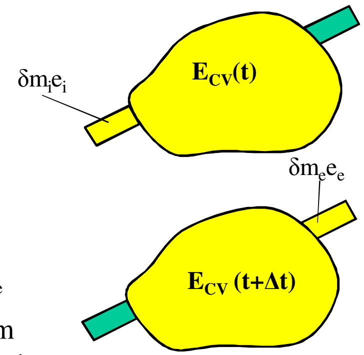
$$e_i = u_i + \frac{V_i^2}{2} + gz_i$$

- The same at outlet shall be

$$e_e = u_e + \frac{V_e^2}{2} + gz_e$$

First Law for Open System-II

- Now let us consider the arbitrary system again
- At time, t, the total energy of the control mass is $E_{CV}(t) + \delta m_i e_i$
- At time, t+ Δt, the total energy of the control mass is $E_{CV}(t+\Delta t) + \delta m_e e_e$
- During this Δt, the system has heat and work interaction of δQ and δW. Usual Sign convention applies



First Law for Open System-III

- The first law for the control mass can be written as,

$$E_{CV}(t+\Delta t) + \delta m_e e_e - (E_{CV}(t) + \delta m_i e_i) = \delta Q - \delta W$$

- Rearranging and dividing both sides by Δt and shrinking the Δt to 0, we can write,

$$\left.\frac{E_{CV}(t+\Delta t) - E_{CV}(t)}{\Delta t}\right|_{\Delta t\to 0} = \left.\frac{\delta Q}{\Delta t}\right|_{\Delta t\to 0} - \left.\frac{\delta W}{\Delta t}\right|_{\Delta t\to 0} + e_i \left.\frac{\delta m_i}{\Delta t}\right|_{\Delta t\to 0} - e_e \left.\frac{\delta m_e}{\Delta t}\right|_{\Delta t\to 0}$$

$$\text{or, } \frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W} + \dot{m}_i e_i - \dot{m}_e e_e$$

First Law for Open System-IV

- Splitting the flow work out of W_{CV} and accounting it separately

$$\dot{W} = \dot{W}_{Flow\ work} + \dot{W}_{CV}$$

$$= -p_i v_i \dot{m}_i + p_i v_i \dot{m}_i + \dot{W}_{CV}$$

$$\therefore \frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}_i (e_i + p_i v_i) - \dot{m}_e (e_e + p_e v_e)$$

$$\dot{E}_{CV} = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}_i \left(u_i + \frac{V_i^2}{2} + gz_i + p_i v_i \right) - \dot{m}_e \left(u_e + \frac{V_e^2}{2} + gz_e + p_e v_e \right)$$

First Law for Open System-V

- For multiple inlets and outlets, we can write,

$$\dot{E}_{CV} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum \dot{m}_i \left(u_i + \frac{V_i^2}{2} + gz_i + p_i v_i \right) - \sum \dot{m}_e \left(u_e + \frac{V_e^2}{2} + gz_e + p_e v_e \right)$$

- Steady state implies that the state of the fluid in Control volume does not change with time

First Law for Open System-VI

- For Steady flow,

$$\frac{dM_{CV}}{dt} = 0 = \sum \rho_i A_i V_i - \sum \rho_e A_e V_e$$

$$\Rightarrow \sum \dot{m}_i = \sum \dot{m}_e$$

- For Single inlet and outlet

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

First Law for Open System-VII

- For Steady flow, Energy equation

$$\dot{E}_{CV} = 0 = \dot{Q}_{CV} - \dot{W}_{CV} + \sum \dot{m}_i \left(u_i + \frac{V_i^2}{2} + gz_i + p_i v_i \right) - \sum \dot{m}_e \left(u_e + \frac{V_e^2}{2} + gz_e + p_e v_e \right)$$

- For Single inlet and outlet

$$0 = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m} \left((h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e) \right)$$

Steady Flow Energy Equation