II Semester M Sc Mathematics

Advanced Abstract Algebra

MCQ

1. A field E is an extension field of a field F if
   A) $E = F$ B) $F \leq E$ C) $E \leq F$ D) None of these

2. Which of the following is an example for transcendental number?
   A) $\sqrt{2}$ B) π C) i D) 2

3. Find the dimension of $\mathbb{Q}(\sqrt{2})$ over $\mathbb{Q}$.
   A) 2 B) 3 C) 5 D) 1

4. Find the dimension of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.
   A) 1 B) 4 C) ∞ D) 2

5. Select the number which is not an element of $\mathbb{Q}(\sqrt{2})$.
   A) 1 B) 4 C) $\sqrt[3]{2}$ D) 2

6. Which of these are not constructible numbers?
   A) $\sqrt{2}$ B) π C) $\sqrt{3}$ D) 4

7. Find an algebraic element over $\mathbb{Q}$.
   A) $\sqrt{2}$ B) π C) e D) $\pi + 1$

8. Let E be a finite extension of degree n over a finite field F. If F has q elements, then E has ______ elements.
   A) q B) nq C) $n + q$ D) q^n

9. Find a generator of $\mathbb{Z}_{11}^*$.
   A) 2 B) 4 C) 5 D) 3

10. Which of the following angle is not constructible?
    A) 72 B) 20 C) 36 D) 18

11. If E is a finite extension of a field F, then $[E:F] = 1$ if and only if
    A) $E = F$ B) $F \leq E$ C) $E \leq F$ D) None of these

12. An extension field E of F is called algebraic if
    A) Some elements of E is algebraic over F B) E and F are same
    C) Every element of E is algebraic over F D) E is a subfield of F

13. If a field F is algebraically closed, then every polynomial $f(x)$ of positive degree over F
    A) does not have a zero in F B) has at least one zero in F
    C) is irreducible over F D) cannot decide

14. If $\mathbb{C}$ is the field of complex numbers, then
A) algebraic closure of $\mathbb{C}$ is $\mathbb{C}$ itself B) algebraic closure of $\mathbb{C}$ does not exist
C) algebraic closure of $\mathbb{C}$ is countable D) none of these

15. The algebraic closure of the field of real numbers $\mathbb{R}$ is
A) $\mathbb{R}$ B) $\mathbb{C}$ C) $\mathbb{Q}$ D) does not exist

16. The degree of $\mathbb{Q}(\sqrt{3}+\sqrt{2})$ over $\mathbb{Q}$ is
A) 1 B) 2 C) 3 D) 4

17. If $E = \mathbb{Q}(\sqrt{2}, \sqrt{5})$ and $F = \mathbb{Q}(\sqrt{2}+\sqrt{5})$, then
A) $E = F$ B) $F \leq E$ C) $E \leq F$ D) E and F are not comparable

18. The degree of $\mathbb{Q}(\sqrt{19})$ over $\mathbb{Q}$ is
A) 1 B) 2 C) 3 D) 0

19. If $E = \mathbb{Q}(\sqrt{2})$ and $F = \mathbb{Q}(\sqrt{2}+\sqrt{5})$, then
A) E is proper subfield of F B) F is proper subfield of E
C) E is a vector space over F D) E and F are not comparable

20. The multiplicative group of nonzero elements of a finite field is
A) non abelian B) non cyclic C) cyclic D) not defined

21. If $[E: F] = n$ and $[K: E] = m$, then $[K: F]$ is
A) m^n B) n^m C) $m + n$ D) mn

22. Find $\deg(\sqrt{2}, \mathbb{Q})$.
A) 1 B) 2 C) 3 D) 0

23. Find $[\mathbb{Q}(\sqrt[3]{2}): \mathbb{Q}]$.
A) 1 B) 2 C) 3 D) 0

24. Find $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}) : \mathbb{Q}]$.
A) 6 B) 4 C) 2 D) 8

25. Find the number of primitive 8$^{\text{th}}$ root of unity in $GF(9)$.
A) 1 B) 2 C) 3 D) 4

26. Find the number of primitive 15$^{\text{th}}$ root of unity in $GF(31)$.
A) 1 B) 2 C) 3 D) 8

27. Which of the following is a PID?
A) $\mathbb{Z} \times \mathbb{Z}$ B) $\mathbb{Z}[x]$ C) $\mathbb{Z}$ D) $\mathbb{Z}[\sqrt{-5}]$

28. Which of the following statement is true?

A) $\mathbb{Z}[x]$ is a PID
B) $\mathbb{Z}[\sqrt{-3}]$ is a UFD
C) $\mathbb{Z}[i]$ is a Euclidean domain
D) $\mathbb{Q}[x]$ is not a Euclidean domain

29. Which of the following polynomial is primitive in $\mathbb{Z}[x]$?

A) $4x^2 + 6x + 2$
B) $6x^2 + 2x + 3$
C) $3x^3 + 6x^2 - 9x + 3$
D) $2x^2 + 12x + 14$

30. Which of the following statement is not true?

A) If D is a UFD, then D[x] is a UFD
B) Every PID is a UFD
C) $\mathbb{Z}$ is a UFD
D) Every PID is a Euclidean domain

31. If D is a UFD, then which of the following statement is true?

A) D[x] is not a UFD
B) Product of two primitive polynomials in D[x] is primitive
C) D is a PID
D) D is a Euclidean domain

32. Which of the following element is an irreducible of the indicated domain?

A) 14 in $\mathbb{Z}$
B) $2x - 10$ in $\mathbb{Z}[x]$
C) $2x - 10$ in $\mathbb{Q}[x]$
D) $x^2 - x - 6$ in $\mathbb{Q}[x]$

33. Let D be a Euclidean domain. Then which of the following statement is true?

A) D is not a PID
B) D is a PID, but not a UFD
C) D is a PID
D) D is a UFD, but not a PID

34. The gcd of 49349 and 15555 in $\mathbb{Z}$ is

A) 62
B) 51
C) 52
D) 61

35. Which of the following statement is true?

A) There exists a Euclidean domain, which is not a PID
B) Every PID is a Euclidean domain
C) If ν is a Euclidean norm on a Euclidean domain D, then $\nu(1) \geq \nu(a)$ for nonzero $a \in D$
D) If ν is a Euclidean norm on a Euclidean domain D, then $\nu(a) = \nu(1)$ for a unit $a \in D$

36. Which of the following is a UFD but not a PID?

A) $\mathbb{R}[x]$
B) $\mathbb{C}[x]$
C) $\mathbb{Q}[x]$
D) $\mathbb{Z}[x]$

37. Let N be the norm function on $\mathbb{Z}[i]$. Then which of the following is not true?

A) $N(\alpha) \geq 0$
B) $N(\alpha) < 0$
C) $N(\alpha) = 0$ if and only if $\alpha = 0$
D) $N(\alpha\beta) = N(\alpha)\,N(\beta)$

38. Which one of the following is the factorization of the Gaussian integer $4 + 3i$ into a product of irreducibles in $\mathbb{Z}[i]$?

A) $(1 + 2i)(1 - 2i)$ B) $(1 + 2i)(2 - i)$

C) $(1 + 2i)(2 + i)$ D) $(1 - 2i)(2 - i)$

39. The gcd of $8 + 6i$ and $5 - 15i$ in $\mathbb{Z}[i]$ is

A) $7 - i$ B) $5 + i$ C) $7 + i$ D) $5 - i$

40. Which of the following statement is not true?

A) $\mathbb{Z}[i]$ is a PID B) $\mathbb{Z}[i]$ is a Euclidean Domain

C) $\mathbb{Z}[i]$ is an integral domain D) Every complex number is a Gaussian integer

41. Which of the following statement is true about $\mathbb{Z}[\sqrt{-5}]$?

A) $\mathbb{Z}[\sqrt{-5}]$ is an integral domain B) $\mathbb{Z}[\sqrt{-5}]$ is a unique factorization domain

C) $\mathbb{Z}[\sqrt{-5}]$ is a principal ideal domain D) $\mathbb{Z}[\sqrt{-5}]$ is a Euclidean domain

42. Let p be an odd prime in $\mathbb{Z}$. Then $p = a^2 + b^2$ for integers a and b in $\mathbb{Z}$ if and only if

A) $p \equiv 1 (\mathrm{mod}\ 4)$ B) $p \equiv 2 (\mathrm{mod}\ 4)$

C) $p \equiv 3 (\mathrm{mod}\ 4)$ D) $p \equiv 0 (\mathrm{mod}\ 4)$

43. Let D be an integral domain with a multiplicative norm N. Then which of the following is true?

A) $N(1) < 1$ B) $|N(u)| = 1$ for every unit $u \in D$

C) $N(1) > 1$ D) There exists a unit $u \in D$ such that $|N(u)| \neq 1$

44. Which of the following is a multiplicative norm on $\mathbb{Z}[\sqrt{-5}]$?

A) $N(a + b\sqrt{-5}) = a^2 + 5b^2$ B) $N(a + b\sqrt{-5}) = a^2 - 5b^2$

C) $N(a + b\sqrt{-5}) = a^2 + \sqrt{5}b^2$ D) $N(a + b\sqrt{-5}) = a^2 - \sqrt{5}b^2$

45. Which of the following is an irreducible in $\mathbb{Z}[i]$?

A) $2 - 4i$ B) 5 C) $1 - 2i$ D) 2

46. Let p be an odd prime in $\mathbb{Z}$. Then p is irreducible in $\mathbb{Z}[i]$ if and only if

A) $p \equiv 1 (\mathrm{mod}\ 4)$ B) $p \equiv 2 (\mathrm{mod}\ 4)$

C) $p \equiv 3 (\mathrm{mod}\ 4)$ D) $p \equiv 0 (\mathrm{mod}\ 4)$

47. Let $R = \mathbb{Z}[\sqrt{-5}]$ and $\alpha = 3 + \sqrt{-5}$, then which of the following is true?

A) α is a prime B) α is an irreducible

C) R is a UFD D) R is not an integral domain

48. Which one of the given functions ϑ is not a Euclidean norm for the given integral domain
   A) $\vartheta(n) = n^2$ for nonzero $n \in \mathbb{Z}$
   B) $\vartheta(f(x)) =$ (degree of f(x)) for nonzero $f(x) \in \mathbb{Z}[x]$
   C) $\vartheta(f(x)) =$ (the absolute value of the coefficient of the highest degree nonzero term of f(x)) for nonzero $f(x) \in \mathbb{Z}[x]$
   D) $\vartheta(n) = |n|$ for nonzero $n \in \mathbb{Z}$

49. The content of the polynomial $18x^2 - 12x + 48$ in $\mathbb{Z}[x]$ is
   A) 2 B) 3 C) 6 D) 12

50. Which of the following statement is not true?
   A) In a PID, every nonzero element that is not a unit is a product of irreducibles.
   B) An ideal $< p >$ in a PID is maximal if and only if p is an irreducible
   C) In a PID, if an irreducible p divides ab, then either p | a or p | b
   D) If D is a PID, then D[x] is a PID

51. Find all conjugates of $\sqrt{2}$ over $\mathbb{Q}$ in the field $\mathbb{C}$.
   A) 1 and $\sqrt{2}$ B) $-\sqrt{2}$ and $\sqrt{2}$ C) 1 and $-\sqrt{2}$ D) 1, $\sqrt{2}$ and $-\sqrt{2}$

52. Find all conjugates of $3 + \sqrt{2}$ over $\mathbb{Q}$ in the field $\mathbb{C}$.
   A) $3 - \sqrt{2}$ only B) 1 and $3 - \sqrt{2}$ C) $3 - \sqrt{2}$ and $3 + \sqrt{2}$ D) $3 + \sqrt{2}$ only

53. Find all conjugates of $\sqrt{3} + \sqrt{2}$ over $\mathbb{Q}$ in the field $\mathbb{C}$.
   A) $\sqrt{3} + \sqrt{2}$ and $\sqrt{3} - \sqrt{2}$ B) $\sqrt{3} - \sqrt{2}$ and $-\sqrt{3} - \sqrt{2}$
   C) $\sqrt{3} - \sqrt{2}$ and $-\sqrt{3} + \sqrt{2}$ D) $\sqrt{3} + \sqrt{2}$, $\sqrt{3} - \sqrt{2}$, $-\sqrt{3} + \sqrt{2}$ and $-\sqrt{3} - \sqrt{2}$

54. Find all conjugates of $i + \sqrt{2}$ over $\mathbb{Q}$ in the field $\mathbb{C}$.
   A) $i + \sqrt{2}$ and $i - \sqrt{2}$ B) $i + \sqrt{2}$ and $-i + \sqrt{2}$
   C) $i + \sqrt{2}$, $i - \sqrt{2}$, $-i - \sqrt{2}$ and $-i + \sqrt{2}$ D) $i - \sqrt{2}$ and $-i + \sqrt{2}$

55. Find all conjugates of $i + \sqrt{2}$ over $\mathbb{R}$ in the field $\mathbb{C}$.
   A) $i + \sqrt{2}$ and $i - \sqrt{2}$ B) $i + \sqrt{2}$ and $-i + \sqrt{2}$
   C) $i + \sqrt{2}$, $i - \sqrt{2}$, $-i - \sqrt{2}$ and $-i + \sqrt{2}$ D) $i - \sqrt{2}$ and $-i + \sqrt{2}$

56. Find all conjugates of $\sqrt{1 + \sqrt{2}}$ over $\mathbb{Q}(\sqrt{2})$ in the field $\mathbb{C}$.
   A) $\sqrt{1 + \sqrt{2}}$ and $-\sqrt{1 + \sqrt{2}}$ B) $\sqrt{1 + \sqrt{2}}$ and $\sqrt{1 - \sqrt{2}}$
   C) $-\sqrt{1 + \sqrt{2}}$ and $\sqrt{1 - \sqrt{2}}$ D) $\sqrt{1 + \sqrt{2}}$, $-\sqrt{1 + \sqrt{2}}$ and $\sqrt{1 - \sqrt{2}}$

57. Consider the conjugation isomorphism $\Gamma_2 = \Psi_{\sqrt{2},-\sqrt{2}} : (Q\,(\sqrt{5},\sqrt{3}))\,(\sqrt{2}) \rightarrow (Q\,(\sqrt{5},\sqrt{3}))(-\sqrt{2})$. Then the value of $\Gamma_2(\sqrt{3}\,)$ in $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$ is

A) $\sqrt{3}$ B) $-\sqrt{3}$ C) $\sqrt{2}$ D) $\sqrt{5}$

58. Consider the conjugation isomorphism $\Gamma_2 = \Psi_{\sqrt{2},-\sqrt{2}} : (\mathbb{Q}\,(\sqrt{5},\sqrt{3}))\,(\sqrt{2}) \rightarrow (\mathbb{Q}\,(\sqrt{5},\sqrt{3}))(-\sqrt{2})$. Then the value of $\Gamma_2(\sqrt{2} + \sqrt{5}\,)$ in $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$ is

A) $\sqrt{2} + \sqrt{5}$ B) $\sqrt{2} - \sqrt{5}$ C) $-\sqrt{2} + \sqrt{5}$ D) $-\sqrt{2} - \sqrt{5}$

59. Consider the following conjugation isomorphisms

$\Gamma_2 = \Psi_{\sqrt{2},-\sqrt{2}} : (\mathbb{Q}\,(\sqrt{5},\sqrt{3}))\,(\sqrt{2}) \rightarrow (\mathbb{Q}\,(\sqrt{5},\sqrt{3}))(-\sqrt{2})$

$\Gamma_3 = \Psi_{\sqrt{3},-\sqrt{3}} : (\mathbb{Q}\,(\sqrt{2},\sqrt{5}))\,(\sqrt{3}) \rightarrow (\mathbb{Q}\,(\sqrt{2},\sqrt{5}))(-\sqrt{3})$.

Then the value of $(\Gamma_3\,\Gamma_{2)}\,(\sqrt{2} + 3\,\sqrt{5}\,)$ in $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$ is

A) $-\sqrt{2} + 3\sqrt{5}$ B) $\sqrt{2} + 3\sqrt{5}$ C) $\sqrt{2} - 3\sqrt{5}$ D) $-\sqrt{2} - 3\sqrt{5}$

60. Consider the automorphism $\Gamma_3 = \Psi_{\sqrt{3},-\sqrt{3}}$ of $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$. Then the fixed field of Γ_3 is

A) $\mathbb{Q}\,(\sqrt{2},\sqrt{3})$ B) $\mathbb{Q}\,(\sqrt{2},\sqrt{5})$ C) $\mathbb{Q}\,(\sqrt{3},\sqrt{5})$ D) $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$

61. Consider the automorphism $\Gamma_3 = \Psi_{\sqrt{3},-\sqrt{3}}$ of $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$. Then the fixed field of the automorphism $\Gamma_3^{\,2}$ is

A) $\mathbb{Q}\,(\sqrt{2},\sqrt{3})$ B) $\mathbb{Q}\,(\sqrt{2},\sqrt{5})$ C) $\mathbb{Q}\,(\sqrt{5},\sqrt{3})$ D) $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$

62. Consider the automorphisms $\Gamma_2 = \Psi_{\sqrt{2},-\sqrt{2}}$ and $\Gamma_3 = \Psi_{\sqrt{3},-\sqrt{3}}$ of $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$. Then the fixed field of $\{\,\Gamma_2, \Gamma_3\}$ is

A) $\mathbb{Q}\,(\sqrt{2},\sqrt{3})$ B) $\mathbb{Q}\,(\sqrt{5})$ C) $\mathbb{Q}\,(\sqrt{3})$ D) $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$

63. Consider automorphisms $\Gamma_2 = \Psi_{\sqrt{2},-\sqrt{2}}$ and $\Gamma_5 = \Psi_{\sqrt{5},-\sqrt{5}}$ of $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$. Then the fixed field of the automorphism $\Gamma_5\,\Gamma_2$ is

A) $\mathbb{Q}\,(\sqrt{2},\sqrt{3})$ B) $\mathbb{Q}\,(\sqrt{3},\sqrt{10})$ C) $\mathbb{Q}\,(\sqrt{5},\sqrt{10})$ D) $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$

64. Consider automorphisms $\Gamma_2 = \Psi_{\sqrt{2},-\sqrt{2}}$, $\Gamma_3 = \Psi_{\sqrt{3},-\sqrt{3}}$ and $\Gamma_5 = \Psi_{\sqrt{5},-\sqrt{5}}$ of $\mathbb{Q}(\sqrt{2},\sqrt{3},\sqrt{5})$. Then the fixed field of the automorphism $\Gamma_5\,\Gamma_3\,\Gamma_2$ is

A) $\mathbb{Q}\,(\sqrt{6},\sqrt{10})$ B) $\mathbb{Q}\,(\sqrt{3},\sqrt{10})$ C) $\mathbb{Q}\,(\sqrt{5},\sqrt{10})$ D) $\mathbb{Q}\,(\sqrt{2},\sqrt{3},\sqrt{5})$

65. Let F be a finite field of characteristic p. Then the Frobenius automorphism $\sigma_p : F \rightarrow F$ is defined as

A) $\sigma_p\,(a) = a^p$ for $a \in F$ B) $\sigma_p\,(a) = a^p - a$ for $a \in F$

C) $\sigma_p\,(a) = a^2$ for $a \in F$ D) $\sigma_p\,(a) = pa$ for $a \in F$

66. Let E be a finite extension of a field F. Then the number of isomorphisms of E onto a subfield of $\bar{F}$ leaving F fixed is called

A) the dimension [E : F] of E over F
B) the index {E : F} of E over F
C) the cardinality of E over F
D) none of the above

67. The degree (over $\mathbb{Q}$) of the splitting field over $\mathbb{Q}$ of the polynomial $x^p - 1$, for a prime p is

A) p B) p - 1 C) p^2 D) p!

68. The degree (over $\mathbb{Q}$) of the splitting field over $\mathbb{Q}$ of the polynomial $x^3 - 1$ in $\mathbb{Q}[x]$ is

A) 1 B) 2 C) 3 D) 0

69. The degree (over $\mathbb{Q}$) of the splitting field over $\mathbb{Q}$ of the polynomial $x^2 + 3$ in $\mathbb{Q}[x]$ is

A) 0 B) 1 C) 2 D) None of the above

70. The degree (over $\mathbb{Q}$) of the splitting field over $\mathbb{Q}$ of the polynomial $x^4 - 1$ in $\mathbb{Q}[x]$ is

A) 3 B) 1 C) 2 D) 4

71. The splitting field of the polynomial $x^4 - 1$ in $\mathbb{Q}[x]$ is

A) $\mathbb{Q}$ B) $\mathbb{Q}(\sqrt{2})$ C) $\mathbb{Q}(\sqrt{3})$ D) $\mathbb{Q}(i)$

72. The degree (over $\mathbb{Q}$) of the splitting field over $\mathbb{Q}$ of the polynomial $(x^2 - 2)(x^2 - 3)$ in Q[x] is

A) 1 B) 2 C) 3 D) 4

73. The splitting field of the polynomial $(x^2 - 2)(x^2 - 3)$ in $\mathbb{Q}[x]$ is

A) $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ B) $\mathbb{Q}(\sqrt{2})$ C) $\mathbb{Q}(\sqrt{3})$ D) $\mathbb{Q}$

74. The splitting field of the polynomial $x^3 - 2$ in $\mathbb{Q}[x]$ is

A) $\mathbb{Q}(\sqrt[3]{2}, i\sqrt{3})$ B) $\mathbb{Q}(\sqrt[3]{2})$ C) $\mathbb{Q}(\sqrt[3]{2}, i, \sqrt{3})$ D) $\mathbb{Q}(\sqrt{2})$

75. The degree (over $\mathbb{Q}$) of the splitting field over $\mathbb{Q}$ of the polynomial $(x^2 - 2)(x^3 - 2)$ in $\mathbb{Q}[x]$ is

A) 1 B) 6 C) 12 D) 2

76. The order of $G(\mathbb{Q}(\sqrt[3]{2}) / \mathbb{Q})$ is

A) 1 B) 2 C) 3 D) 6

77. The order of $G(\mathbb{Q}(\sqrt{2}, \sqrt{3}) / \mathbb{Q})$ is

A) 1 B) 2 C) 4 D) 6

78. Let F be a field and $\bar{F}$ be the algebraic closure of F. If $\alpha \in \bar{F}$, then {F(α) : F} is equal to

A) [F(α) : F]
B) deg(α, F)
C) the number of distinct zeros of irr(α, F)
D) can be infinite

79. Let f(x) be irreducible in $\mathbb{R}[x]$. If $\alpha \in \mathbb{C}$ is a zero of f(x), then the multiplicity of α is
A) always one B) can be two
C) can be three D) can be any positive integer

80. Let E be an extension of a field F such that [E : F] = 10. Then which of the following is not a possible value for {E : F}?
A) 1 B) 2 C) 3 D) 5

81. Let E be an extension of a field F such that [E : F] = 12. Then which of the following is not a possible value for {E : F}?
A) 1 B) 2 C) 3 D) 5

82. The number of subgroups of the Galois group of $GF(p^n)$ over $\mathbb{Z}_p$ is
A) n B) 2n C) $\phi(n)$ D) d(n)

83. The number of subgroups of the Galois group of $GF(2^5)$ over $\mathbb{Z}_2$ is
A) 2 B) 3 C) 4 D) 5

84. The number of subgroups of the Galois group of $GF(2^6)$ over $\mathbb{Z}_2$ is
A) 2 B) 3 C) 4 D) 5

85. If E is a finite separable extension of F, then which of the following is not true?
A) $\{E : F\} \neq [E : F]$
B) Each α in E is separable over F
C) For every α in E, irr(α, F) has all zeros of multiplicity one
D) There exists α in E such that E = F(α)

86. Which of the following is not a perfect field?
A) $\mathbb{Q}$ B) $\mathbb{Z}_{101}$ C) $\mathbb{Z}_5(x)$ D) $\mathbb{R}$

87. Which of the following is not a perfect field?
A) $\mathbb{C}$ B) $\mathbb{Z}_{17}$ C) $\mathbb{Z}_7(x)$ D) $\mathbb{R}$

88. $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$ is same as which one of the following?
A) $\mathbb{Q}(\sqrt{6})$ B) $\mathbb{Q}(\sqrt{2})$ C) $\mathbb{Q}(\sqrt[3]{2})$ D) $\mathbb{Q}(2^{\frac{1}{6}})$

89. Which of the following is a primitive 4^{th} root of unity in $\mathbb{C}$?
A) -1 B) 1 C) i D) $\frac{-1+i\sqrt{3}}{2}$

90. Which of the following is a primitive 3^{rd} root of unity in $\mathbb{C}$?
A) 1 B) i C) $\frac{-1+i\sqrt{3}}{2}$ D) $\frac{1+i}{\sqrt{2}}$

91. Which of the following is a primitive 8^{th} root of unity in $\mathbb{C}$?

A) i B) 1+ i C) $\frac{-1+i\sqrt{3}}{2}$ D) $\frac{1+i}{\sqrt{2}}$

92. The order of the Galois group of the n^{th} cyclotomic extension of $\mathbb{Q}$ is

A) n B) $\pi(n)$ C) $\phi(n)$ D) d(n)

93. The order of the Galois group of the 10^{th} cyclotomic extension of $\mathbb{Q}$ is

A) 10! B) 10 C) 5 D) 4

94. The order of the Galois group of the p^{th} cyclotomic extension of $\mathbb{Q}$, for a prime p is

A) p-1 B) p C) p+1 D) p^2

95. The order of the Galois group of the 5^{th} cyclotomic extension of $\mathbb{Q}$ is

A) 5 B) 2 C) 4 D) 5!

96. The Galois group of the p^{th} cyclotomic extension of $\mathbb{Q}$, for a prime p is isomorphic to

A) $\mathbb{Z}_p$ B) $\mathbb{Z}_p \setminus \{0\}$ C) $\mathbb{Z}_{p-1}$ D) $\mathbb{Z}_{p-1} \setminus \{0\}$

97. The Galois group of the 5^{th} cyclotomic extension of $\mathbb{Q}$ is isomorphic to

A) $\mathbb{Z}_4$ B) $\mathbb{Z}_4 \setminus \{0\}$ C) $\mathbb{Z}_5$ D) $\mathbb{Z}_5 \setminus \{0\}$

98. The number of intermediate fields between $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ including both $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is

A) 2 B) 3 C) 4 D) 5

99. Let K be the splitting field of $x^3 - 2$ over $\mathbb{Q}$. Then the order of $G(K/\mathbb{Q})$ is

A) 2 B) 3 C) 4 D) 6

100. The Galois group of the 5^{th} cyclotomic extension of $\mathbb{Q}$ is

A) nonabelian B) abelian, but not cyclic

C) cyclic D) isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$