# Bandpass communication and the Complex Envelope

EELE445-14
Lecture 25

MONTANA STATE UNIVERSITY
Mountains & Minds

## Complex Envelope

- Review sections 4-1 to 4-4 in the text
- look at the complex envelope representations in table 4-1
- be able to calculate or find:
  - complex envelope of a waveform expression
  - calculate the signal power
  - calculate the PEP
  - define Modulation

MONTANA STATE UNIVERSITY
Mountains & Minds

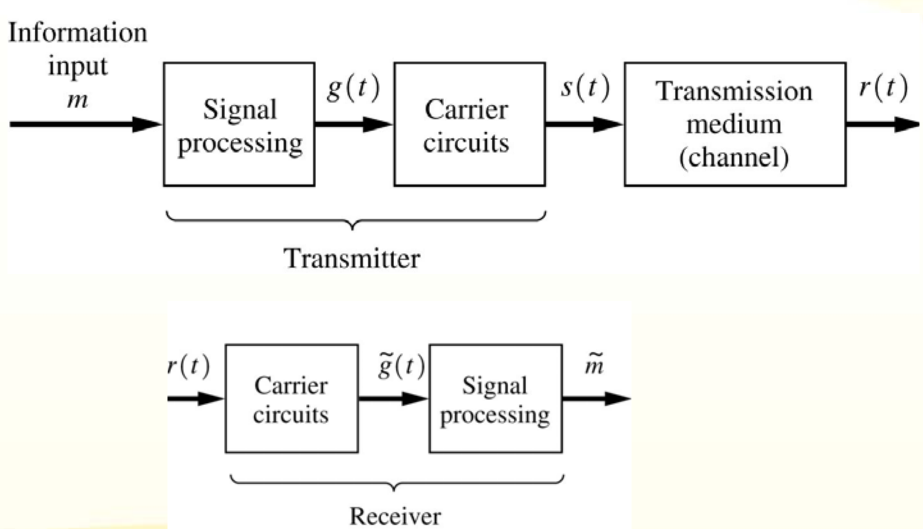
## Some Definitions

A ***baseband*** waveform has a spectral magnitude that in nonzero for frequencies in the vicinity of the origin (i.e. f=0) and negligible elsewhere.

A ***bandpass*** waveform has a spectral magnitude that is nonzero for frequencies in some band concentrated about a frequency f=+/- $f_c$ where $f_c$ is much greater than zero.

***Modulation*** is the process if imparting the source information onto a bandpass signal with a carrier frequency $f_c$ by the introduction of amplitude or phase perturbations or both. The resulting bandpass signal is called the modulated signal s(t), and the baseband source signal is called the modulating signal m(t)

**Figure 4–1 Bandpass** Communication system.



## Complex Envelope

Theorem: *Any physical bandpass waveform can be represented by:*

$$s(t) = \text{Re}\{g(t)e^{j\omega_c t}\} \quad \text{Re}\{\bullet\}\, denotes\ real\ \ part$$
$$g(t) \equiv complex\ envelope\ of\ \ s(t)$$
$$f_c \equiv carrier\ \ frequency,\ w_c \equiv 2\pi f_c$$

$$g(t) = x(t) + jy(t) = |g(t)|e^{\angle g(t)} = R(t)e^{j\theta(t)}$$

MONTANA STATE UNIVERSITY

Mountains & Minds

## Complex Envelope

$$g(t) = x(t) + jy(t) = |g(t)|e^{\angle g(t)} = R(t)e^{j\theta(t)}$$
$$x(t) = \text{Re}\{g(t)\} \equiv R(t)\cos(\theta(t))$$
$$y(t) = \text{Im}\{g(t)\} \equiv R(t)\sin(\theta(t))$$
$$R(t) = \left|g(t) \equiv \sqrt{x(t)^2 + y(t)^2}\right|$$
$$\theta(t) = \angle g(t) \equiv \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$
$$s(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t) = \text{Re}\{g(t)e^{j\omega_c t}\}$$

MONTANA STATE UNIVERSITY

Mountains & Minds

## Complex Envelope

g(t) = x(t) + jy(t)

x(t) is the "I" or In phase modulation

y(t) is the "Q" or Quadrature modulation

R(t) is >=0 and determines the signal power

$$g(t) = g[m(t)]$$

$$g[\bullet] \textit{ is the mapping operator for } m(t)$$

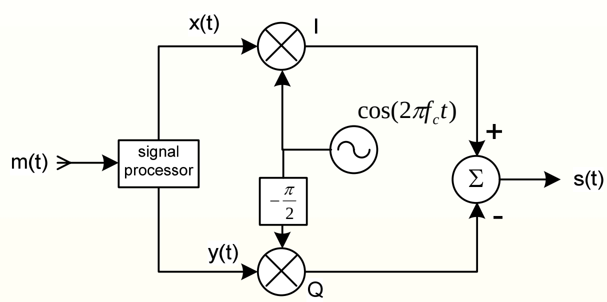
g[ ] function determines the bandpass modulation type

MONTANA STATE UNIVERSITY

Mountains & Minds

## Complex Envelope- Vector Modulator

$$s(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t) = \text{Re}\{g(t)e^{j\omega_c t}\}$$




All modulation types may be generated using a vector modulator

MONTANA STATE UNIVERSITY

Mountains & Minds

## Complex Envelope: time and frequency domain

Bandpass waveform :

$$v(t) = \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$

Spectrum :

$$V(f) = \frac{1}{2}\left[G(f - f_c) + G^*(-f - f_c)\right]$$

PSD :

$$P_v(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c)\right]$$

Power :

$$P_v = \left\langle v^2(t)\right\rangle = \int_{-\infty}^{\infty} P_v(f)df = R_v(0) = \frac{1}{2}\left\langle |g(t)|^2\right\rangle$$



## Complex Envelope, of various modulations

TABLE 4–1 COMPLEX ENVELOPE FUNCTIONS FOR VARIOUS TYPES OF MODULATION[a]

| Type of Modulation | Mapping Functions $g(m)$ | Corresponding Quadrature Modulation $x(t)$ | $y(t)$ |
|---|---|---|---|
| AM | $A_c[1 + m(t)]$ | $A_c[1 + m(t)]$ | 0 |
| DSB-SC | $A_c m(t)$ | $A_c m(t)$ | 0 |
| PM | $A_c e^{jD_p m(t)}$ | $A_c \cos[D_p m(t)]$ | $A_c \sin[D_p m(t)]$ |
| FM | $A_c e^{jD_f \int_{-\infty}^{t} m(\sigma)\, d\sigma}$ | $A_c \cos\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$ | $A_c \sin\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$ |
| SSB-AM-SC[b] | $A_c[m(t) \pm j\hat{m}(t)]$ | $A_c m(t)$ | $\pm A_c \hat{m}(t)$ |
| SSB-PM[b] | $A_c e^{jD_p[m(t) \pm j\hat{m}(t)]}$ | $A_c e^{\mp D_p \hat{m}(t)} \cos[D_p m(t)]$ | $A_c e^{\mp D_p \hat{m}(t)} \sin[D_p m(t)]$ |
| SSB-FM[b] | $A_c e^{jD_f \int_{-\infty}^{t}[m(\sigma) \pm j\hat{m}(\sigma)]d\sigma}$ | $A_c e^{\mp D_f \int_{-\infty}^{t} \hat{m}(\sigma)d\sigma} \cos\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$ | $A_c e^{\mp D_f \int_{-\infty}^{t} \hat{m}(\sigma)d\sigma} \sin\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$ |
| SSB-EV[b] | $A_c e^{\{\ln[1+m(t)] \pm j\hat{\ln}[1+m(t)]\}}$ | $A_c[1 + m(t)] \cos\{\hat{\ln}[1 + m(t)]\}$ | $\pm A_c[1 + m(t)] \sin\{\hat{\ln}[1 + m(t)]\}$ |
| SSB-SQ[b] | $A_c e^{(1/2)\{\ln[1+m(t)] \pm j\hat{\ln}[1+m(t)]\}}$ | $A_c\sqrt{1 + m(t)} \cos\{\frac{1}{2} \hat{\ln}[1 + m(t)]\}$ | $\pm A_c\sqrt{1 + m(t)} \sin\{\frac{1}{2} \hat{\ln}[1 + m(t)]\}$ |
| QM | $A_c[m_1(t) + jm_2(t)]$ | $A_c m_1(t)$ | $A_c m_2(t)$ |

## Complex Envelope, (R θ), of various modulations

TABLE 4–1 COMPLEX ENVELOPE FUNCTIONS FOR VARIOUS TYPES OF MODULATION (*cont.*)

| Type of Modulation | Corresponding Amplitude and Phase Modulation: $R(t)$ | Corresponding Amplitude and Phase Modulation: $\theta(t)$ | Linearity | Remarks |
|---|---|---|---|---|
| **AM** | $A_c\lvert 1 + m(t)\rvert$ | $\begin{cases} 0, & m(t) > -1 \\ 180°, & m(t) < -1 \end{cases}$ | L[c] | $m(t) > -1$ required for envelope detection |
| DSB-SC | $A_c\lvert m(t)\rvert$ | $\begin{cases} 0, & m(t) > 0 \\ 180°, & m(t) < 0 \end{cases}$ | L | Coherent detection required |
| PM | $A_c$ | $D_p m(t)$ | NL | $D_p$ is the phase deviation constant (rad/volt) |
| FM | $A_c$ | $D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma$ | NL | $D_f$ is the frequency deviation constant (rad/volt-sec) |
| SSB-AM-SC[b] | $A_c\sqrt{[m(t)]^2 + [\hat{m}(t)]^2}$ | $\tan^{-1}[\pm\hat{m}(t)/m(t)]$ | L | Coherent detection required |
| SSB-PM[b] | $A_c e^{\pm D_p \hat{m}(t)}$ | $D_p m(t)$ | NL | |
| SSB-FM[b] | $A_c e^{\pm D_f \int_{-\infty}^{t} \hat{m}(\sigma) d\sigma}$ | $D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma$ | NL | |
| SSB-EV[b] | $A_c\lvert 1 + m(t)\rvert$ | $\pm \hat{\ln}[1 + m(t)]$ | NL | $m(t) > -1$ is required so that $\ln(\cdot)$ will have a real value |
| SSB-SQ[b] | $A_c\sqrt{1 + m(t)}$ | $\pm\frac{1}{2}\, \hat{\ln}[1 + m(t)]$ | NL | $m(t) > -1$ is required so that $\ln(\cdot)$ will have a real value |
| QM | $A_c\sqrt{m_1^2(t) + m_2^2(t)}$ | $\tan^{-1}[m_2(t)/m_1(t)]$ | L | Used in NTSC color television; requires coherent detection |

MONTANA STATE UNIVERSITY

Mountains & Minds

# DSB-SC and AM

EELE445-14

Lecture 26

MONTANA STATE UNIVERSITY

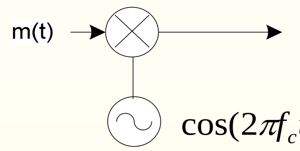
Mountains & Minds

# DSB-SC – section 5-1 in the text

- This is AM without the Carrier
- All of the power in the PSD is dependent on the information → m(t)
- The modulation efficiency is 100%
- Synchronous demodulation is required

$|m(t)| \le 1$

$\langle m(t)^2 \rangle \le 1$




$\cos(2\pi f_c t)$

$s(t) = A_c m(t) \cos(2\pi f_c t)$

$g(t) = A_c m(t)$ pre - envelope

MONTANA STATE UNIVERSITY

Mountains & Minds

# DSB-SC – Spectrum

$s(t) = A_c m(t) \cos(2\pi f_c t)$

$g(t) = A_c m(t)$ pre - envelope $\quad |m(t) \le 1|$, (normalize d m(t))

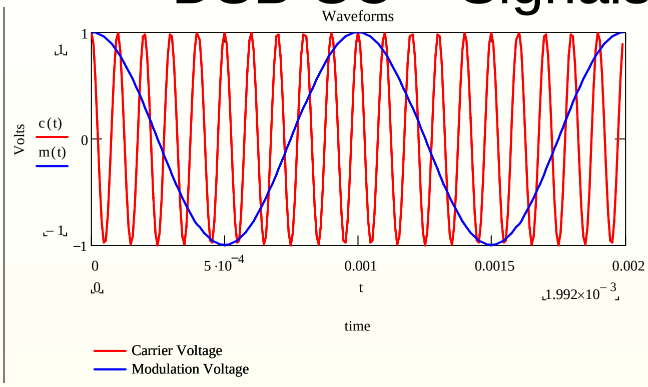
$G(f) = A_c M(f)$

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

MONTANA STATE UNIVERSITY

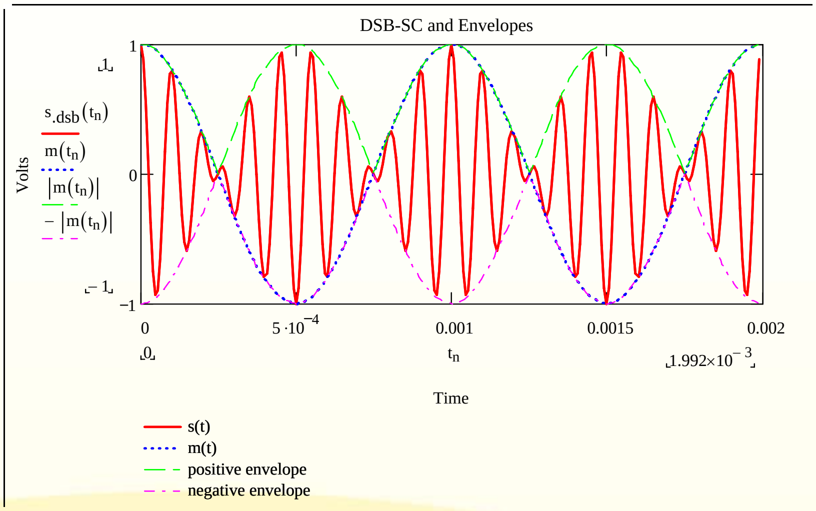
Mountains & Minds

# DSB-SC – Signals

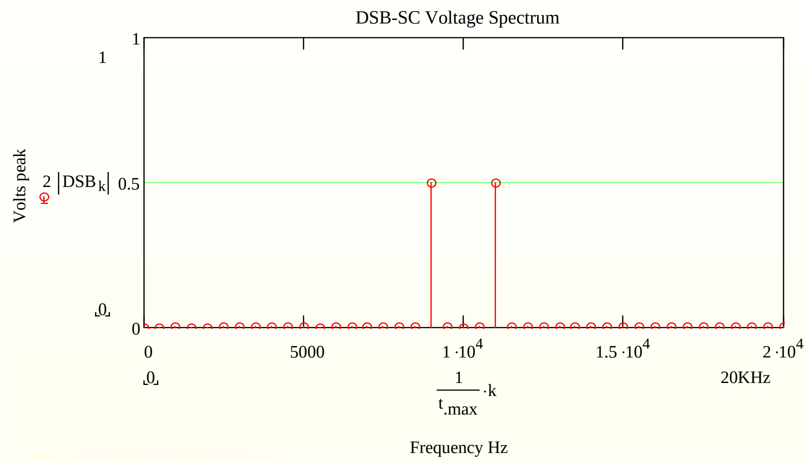





# DSB-SC – Signal



# DSB-SC – Signal




MONTANA STATE UNIVERSITY

Mountains & Minds

# AM – section 5-1

$g(t) = A_c[1 + m(t)]$ pre - envelope

$s(t) = A_c[1 + m(t)]\cos(2\pi f_c t)$

$|m(t)| \leq 1 \quad \langle m(t)^2 \rangle \leq 1$

$$\%\text{modulation} = \frac{A_{max} - A_{min}}{2A_c}100$$

$$= \frac{\max[m(t)] - \min[m(t)}{2}100$$

MONTANA STATE UNIVERSITY

Mountains & Minds

# AM – section 5-1 in the text

$$s(t) = \frac{1}{2}\left\langle |g(t)|^2 \right\rangle = \frac{1}{2}A_c^2\left\langle [1+m(t)]^2 \right\rangle$$

$$= \frac{1}{2}A_c^2 + A_c^2\left\langle m(t)\right\rangle + \frac{1}{2}A_c^2\left\langle m(t)^2\right\rangle$$

$$= \text{Carrier} + \text{Carrier shift} + \text{Sideband}$$

---

for 0 mean, $\left\langle m(t)\right\rangle = 0$:

$$\left\langle s(t)^2\right\rangle = \text{Carrier power} + \text{sideband power}$$

$$= \frac{1}{2}A_c^2 + \frac{1}{2}A_c^2\left\langle m(t)^2\right\rangle$$

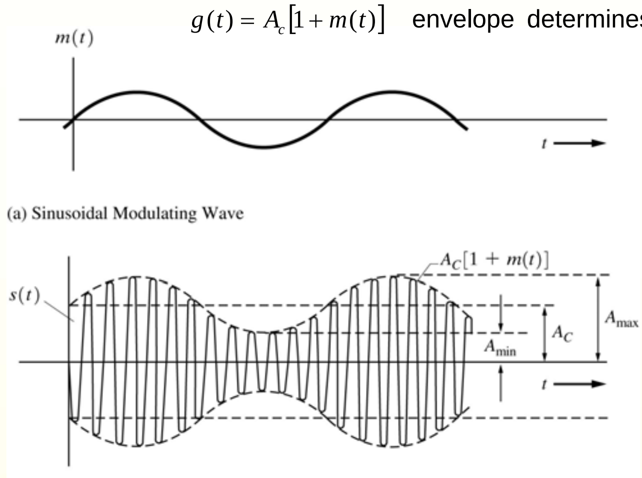
MONTANA STATE UNIVERSITY

Mountains & Minds

**Figure 5–1** AM signal waveform.

$s(t) = A_c[1+m(t)]\cos(\omega t)$ physical wavform

$g(t) = A_c[1+m(t)]$ envelope determines the power




(b) Resulting AM Signal

MONTANA STATE UNIVERSITY

Couch, Digital and Analog Communication Systems, Seventh Edition



Mountains & Minds

## Mathcad AM signal waveform, $\mu$=50%

$f_m := 1\text{KHz}$ Modulating frequency

$A_m := 1$ Modulating amplitude

$f_c := 10\text{KHz}$ Carrier frequency

$A_c := 1$ Carrier amplitude

$\mu_{am} := 0.5$ AM modulation index

$m(t) := A_m \cdot \cos(2 \cdot \pi \cdot f_m \cdot t)$

$c(t) := A_c \cdot \cos(2 \cdot \pi \cdot f_c \cdot t)$

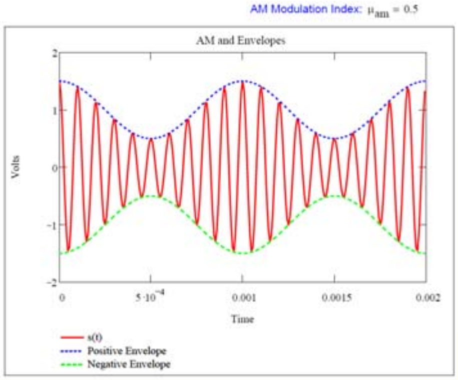
$g(t) := A_c \cdot (1 + \mu_{am} \cdot m(t))$

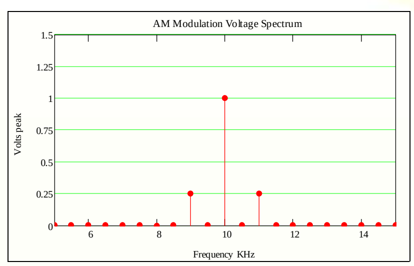
$s_{AM}(t) := (1 + \mu_{am} \cdot m(t)) \cdot c(t)$

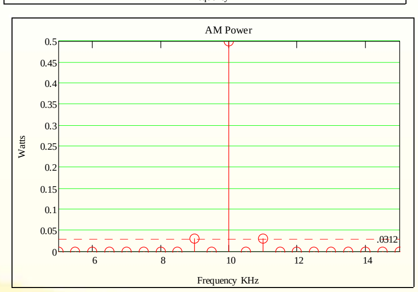



## Mathcad AM signal waveform $\mu$=0.5

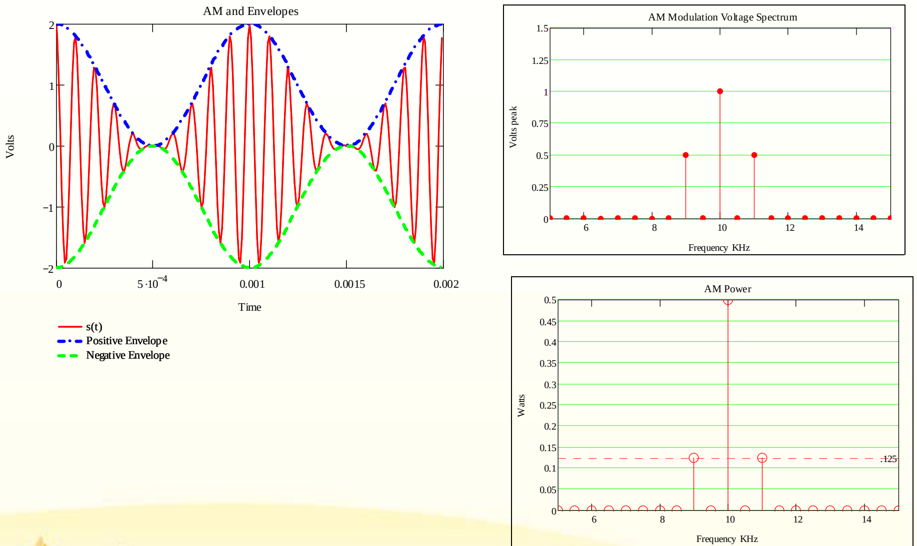
AM Modulation Index: $\mu_{am} = 0.5$



## Mathcad AM signal waveform $\mu$=1







## AM – Power and modulation efficiency

$$\langle s^2(t)\rangle = \frac{1}{2}\langle g^2(t)\rangle = \frac{A_c^2}{2}\langle [1+m(t)]^2\rangle$$

$$= \frac{A_c^2}{2}\langle 1+2m(t)+m^2(t)\rangle$$

$$= \frac{A_c^2}{2} + A_c^2\langle m(t)\rangle + \frac{A_c^2}{2}\langle m^2(t)\rangle$$

= Carrier power + power shift in carrier + sideband power

# AM – section 5-1

$$s(t) = A_c[1 + m(t)]\cos(2\pi f_c t)$$

real envelope : $R(t) = g(t) = A_c[1 + m(t)]$,

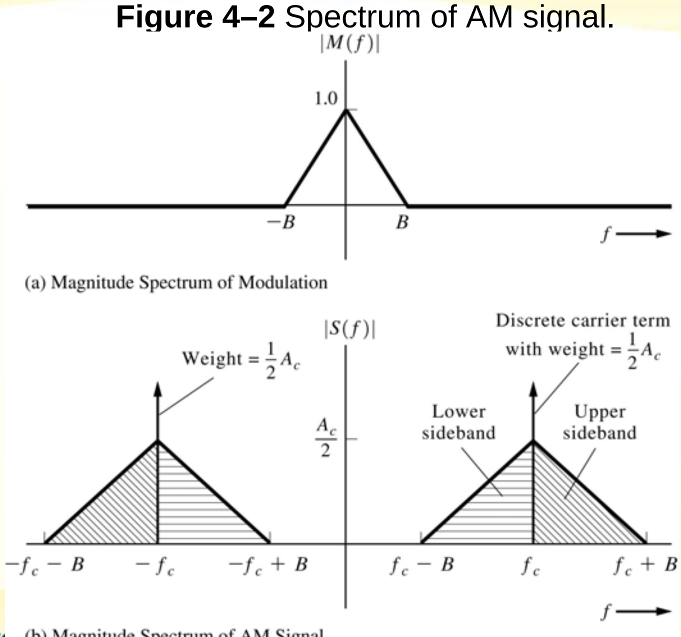
$$|m(t)| \leq 1 \quad \langle m(t)^2 \rangle \leq 1$$

%modulation $= \dfrac{A_{max} - A_{min}}{2A_c}100$

$$= \frac{\max[m(t)] - \min[m(t)}{2}100$$



**Figure 4–2** Spectrum of AM signal.







MONTANA STATE UNIVERSITY
Mountains & Minds

# AM – section 5-1 in the text

- use your class notes and review section 5-1 in the text.
- also review chapter 4 for AM detection and the superheterodyne receiver.
- AM is used for commercial broadcasting in order that low cost envelope detection may be used.



# Bandpass Digital Signal Examples on board…

EELE445-14

MONTANA STATE UNIVERSITY
Mountains & Minds

Asymmetric Sideband Signals
USSB, LSSB, VSB

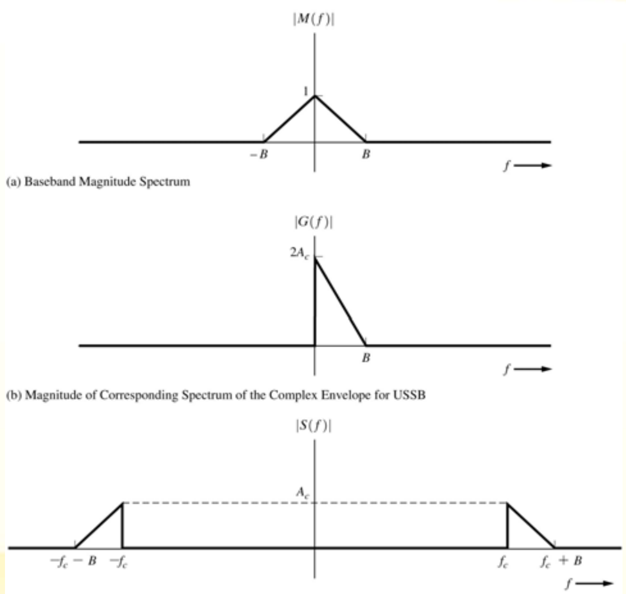
EELE445-13

Lecture 27

MONTANA STATE UNIVERSITY

Mountains & Minds

**Figure 5–4** Spectrum for a USSB signal.




Couch, *Digital and Analog Communication Systems*, Seventh Edition



MONTANA STATE UNIVERSITY

Mountains & Minds

**Figure 5–5** Generation of SSB.

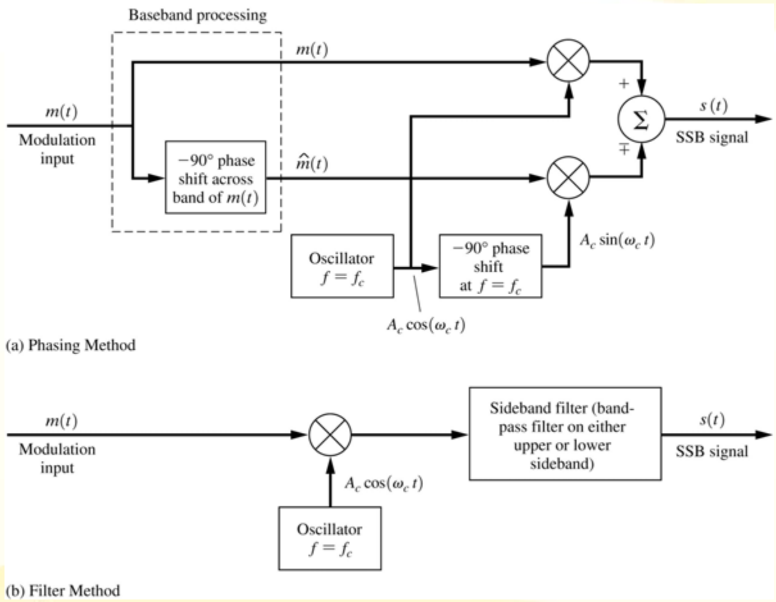
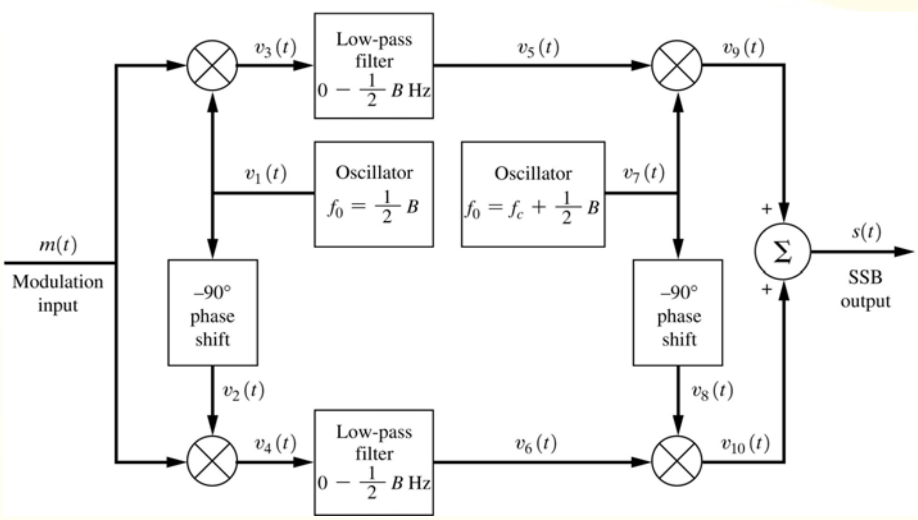


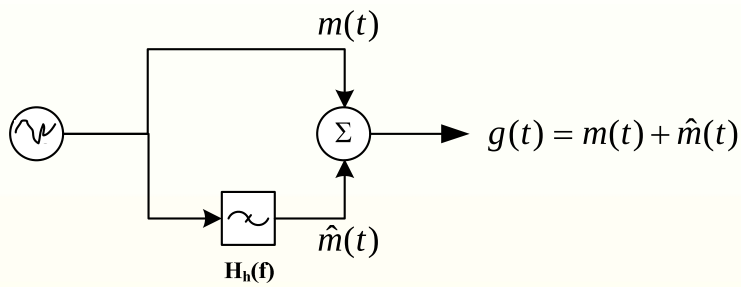




**Figure P5–12** Weaver's method for generating SSB.



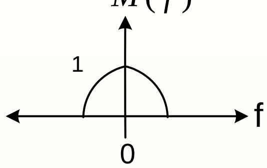
## Single Side Band- Phasing Method



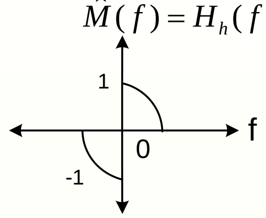

MONTANA STATE UNIVERSITY
Mountains & Minds

## Single Side Band- Phasing Method

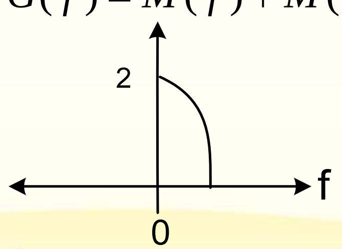
$M(f)$




$\hat{M}(f) = H_h(f) M(f)$




$$G(f) = M(f) + \hat{M}(f)$$

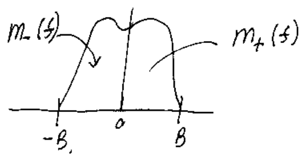



MONTANA STATE UNIVERSITY
Mountains & Minds

## Single Side Band

given $m(t)$ band Limited to $B$

$M(f) = \mathcal{F}(m(t))$




define $M_+(f) = M(f)u(f)$

$M_-(f) = M(f)\,U(-f)$

where $u(f)$ is the step function.

Let $m_+(t) = \mathcal{F}^{-1} M_+(f)$ and $m_-(t) = \mathcal{F}^{-1} M_-(f)$

(note that $2m_+(t)$ is called the preenvelope of $m(t)$ - we use it in signal analysis to simplify simulations)



## Single Side Band

$\Rightarrow$ since $|M_+(f)|$ and $|M_-(f)|$ are not even functions of $f$, $m_+(t)$ & $m_-(t)$ are not real, they are complex.

since $m(t)$ is from a real signal,

$$M_+(f) = M_-^*(f)$$

so it can be shown $m_+(t) = m_-^*(t)$

$m(t) = m_-(t) + m_+(t) \Rightarrow M(f) = M_+(f) + M_-(f)$

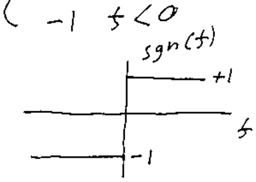
MONTANA STATE UNIVERSITY
Mountains & Minds

## Single Side Band

$m_+(f) = m(f)\,u(f)$

from p 36

$u(f) = \frac{1}{2}\left[1 + sgn(f)\right]$  $sgn(f) = \begin{cases} 1 & f > 0 \\ 0 & f = 0 \\ -1 & f < 0 \end{cases}$




$m_+(f) = \frac{m(f)}{2}\left[1 + sgn(f)\right]$

$m_+(f) = \frac{m(f)}{2} + \frac{m(f)}{2}\,sgn(f)$

└ origional spectrum

└ negative spectrum inverted.
pos " unchanged.



## Single Side Band

$m_+(t) = \mathcal{F}^{-1} m_+(f) = \frac{m(t)}{2} + \frac{j m_h(t)}{2}$ where $j m_h(t) \triangleq \mathcal{F}^{-1} m(f)\,sgn(f)$

also $M_h(f) \triangleq -j M(f)\,sgn(f)$

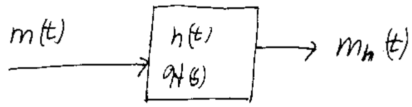
from table 2.1

$sgn(f) = \frac{+j}{\pi t}$

so $m_h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t - \alpha}\,d\alpha$

designed to be the Wilbert transform.

## Single Side Band




Hilbert transform filter.

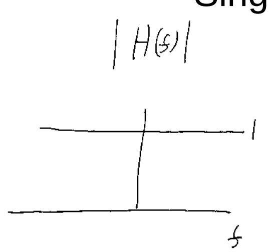
$$m_h(t) = m(t) * h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha}\, d\alpha$$

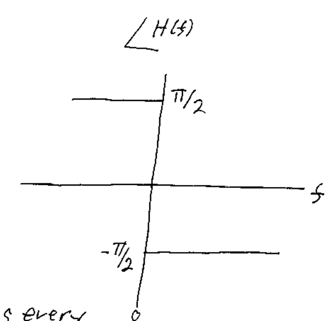
or $M_h(f) = M(f)\,\mathcal{H}(f)$ where $\mathcal{H}(f) = -j\,\mathrm{sgn}(f)$

$$\mathcal{H}(f) = -j\,\mathrm{sgn}(f) = \begin{cases} -j = e^{-j\pi/2} & f > 0 \\ j = e^{j\pi/2} & f < 0 \end{cases}$$



## Single Side Band



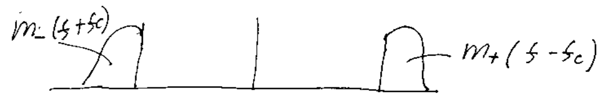




$m_h(f)$ is " created by delaying every component of $m(t)$ by $\frac{\pi}{2}$ without changing the amplitude!

The Hilbert filter - or Hilbert transformer does this.

## Single Side Band

$$USB(f) = M_+(f - f_c) + M_-(f - f_c)$$




① $usb(t) = \mathcal{F}^{-1} USB(f) = m_+(t)e^{j2\pi f_c t} + m_-(t)e^{-j2\pi f_c t}$

② but $m_+(t) = \dfrac{m(t)}{2} + \dfrac{j m_h(t)}{2}$ page 2

③ and similarly $m_-(t) = \dfrac{m(t)}{2} - \dfrac{j m_h(t)}{2}$.



## Single Side Band

combining ①,②,③

$$USB(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

can also show that

$$LSB(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

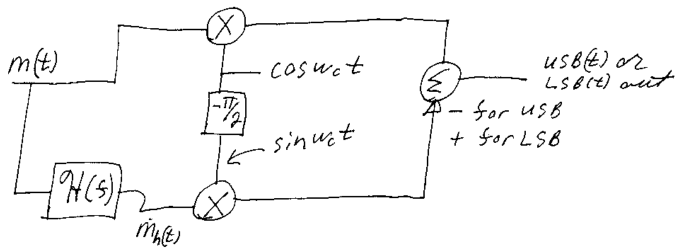
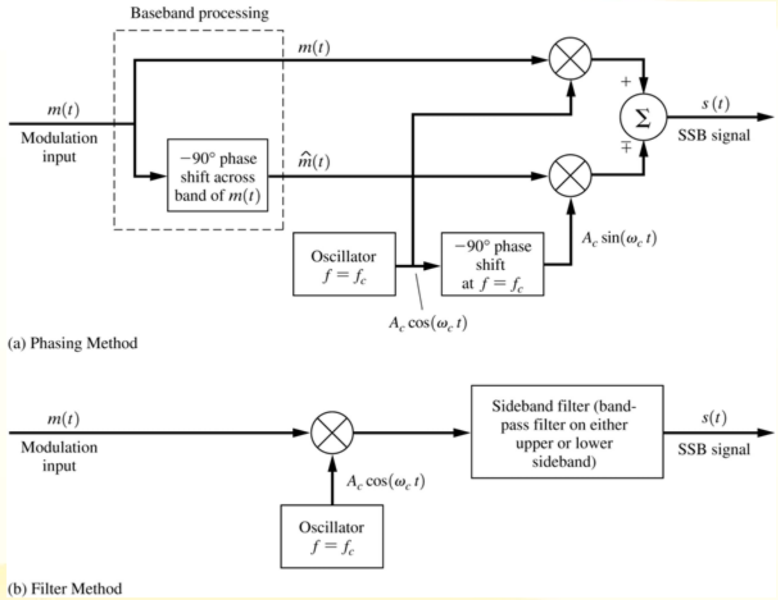


**Figure 5–5** Generation of SSB.



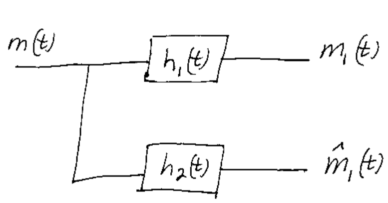





## Single Side Band

The problem is that $\mathcal{H}(s)$ is not real.

But all we need to do is shift every frequency in m(t) by 90 degrees relative to the original m(t), before we multiply by the carrier. Conider:



## Single Side Band

if we can find 2 functions $h_1$, $h_2$,

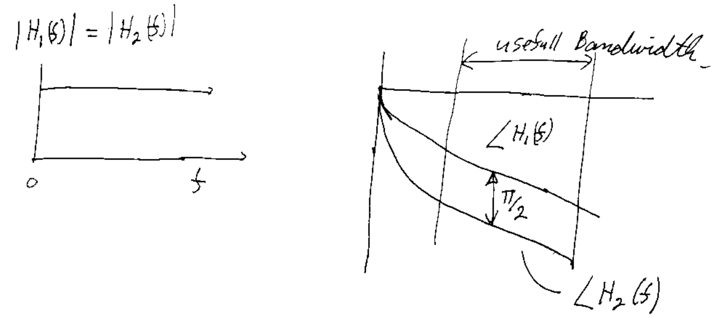
Such that $\frac{|H_1(f)|}{|H_2(f)|} = 1$, $\angle H_1(f) - \angle H_2(f) = \frac{\pi}{2}$

we have what we need.

This is what we do:



## Single Side Band




Hilbert filter implemented as a quadrature network as shown above.

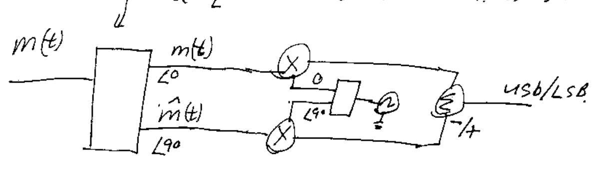


**Figure 5–6** VSB transmitter and spectra.