Ray Aberrations

Rays represent the direction of wave-front propagation. Therefore, rays point in the direction of the wave-front surface normal and can be calculated as the wave-front gradient.

The “transverse ray aberration” (TRA) is the distance, orthogonal to the optical axis, between a paraxial ray and its corresponding real ray (i.e., the transverse distance between ideal and real ray locations). The TRA can be calculated as a derivative of the wave front:

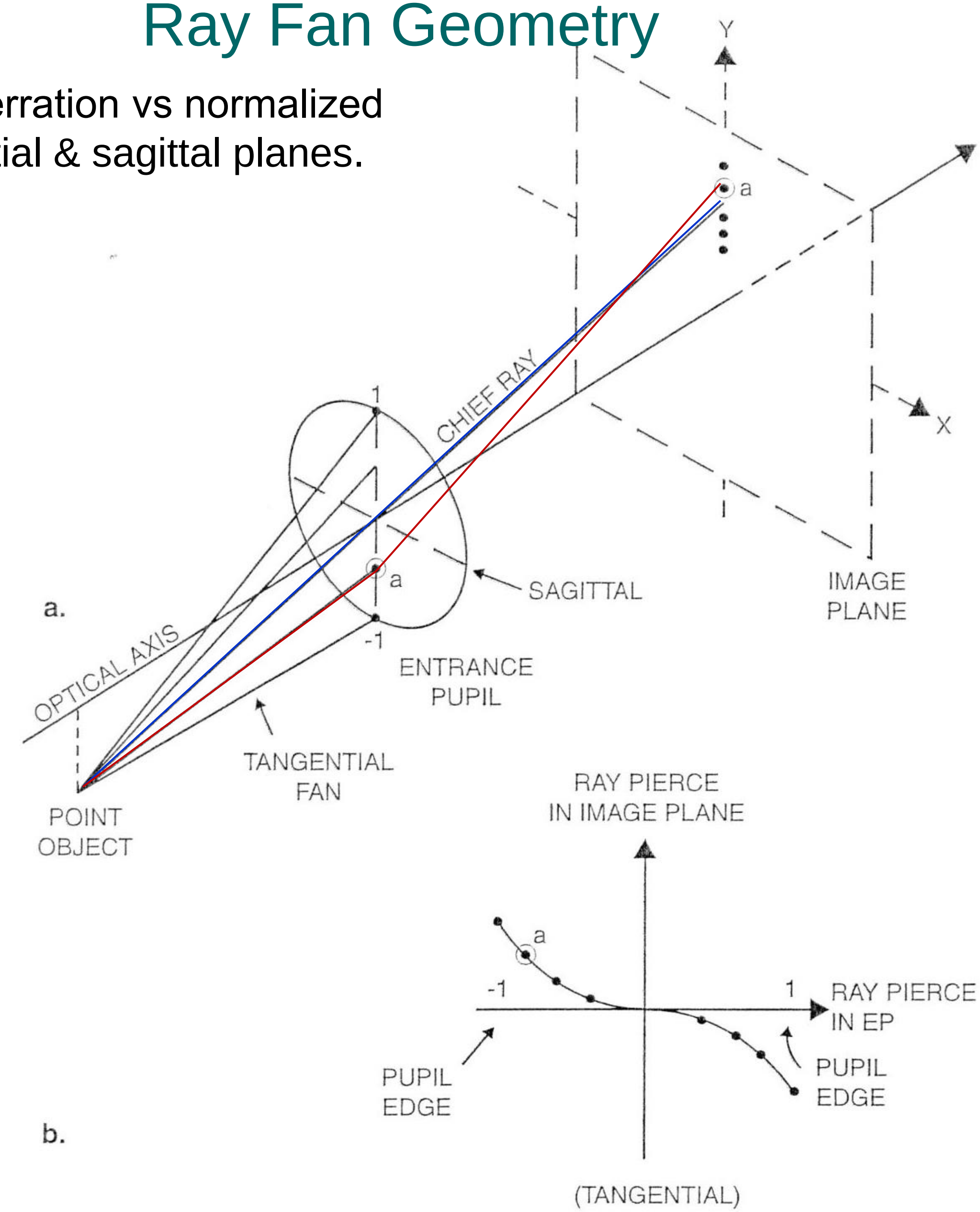
$$TRA(y) = -\left(\frac{R}{nr}\right)\frac{\partial W}{\partial y}$$

R = radius of curvature of reference sphere
r = exit pupil height
n = index of refraction in image space
W = wave-front aberration function (OPD)
y = meridional-plane (vertical) coordinate in exit pupil

References
1. J. Sasian, *Introduction to aberrations in optical imaging systems,* Cambridge Press, 2013.
2. W. T. Welford, *Aberrations of optical systems,* Adam Hilger Press (Bristol and Philadelphia), 1986.
3. R. R. Shannon, *The art and science of optical design*, Cambridge Press, 1997.
4. P. Mouroulis and J. Macdonald, *Geometrical optics and optical design,* Oxford Press, 1997.

Ray Fan Geometry

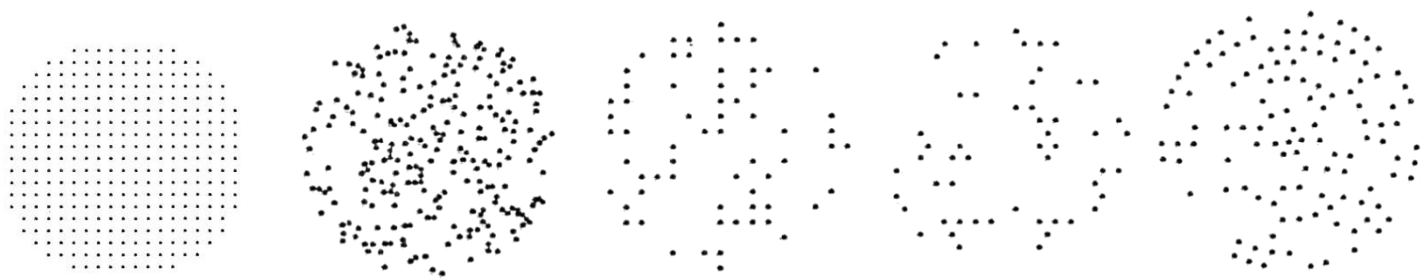
"Ray fans" plot the ray aberration vs normalized pupil coordinate in tangential & sagittal planes.



J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.1

Ray Patterns in the Entrance Pupil

In Zemax and other optical design codes, ray aberrations are determined by tracing many rays from a single object point, through many locations in the entrance pupil, to the image plane. Here are some of the possible "pupil grids" for determining where the rays intersect the pupil.

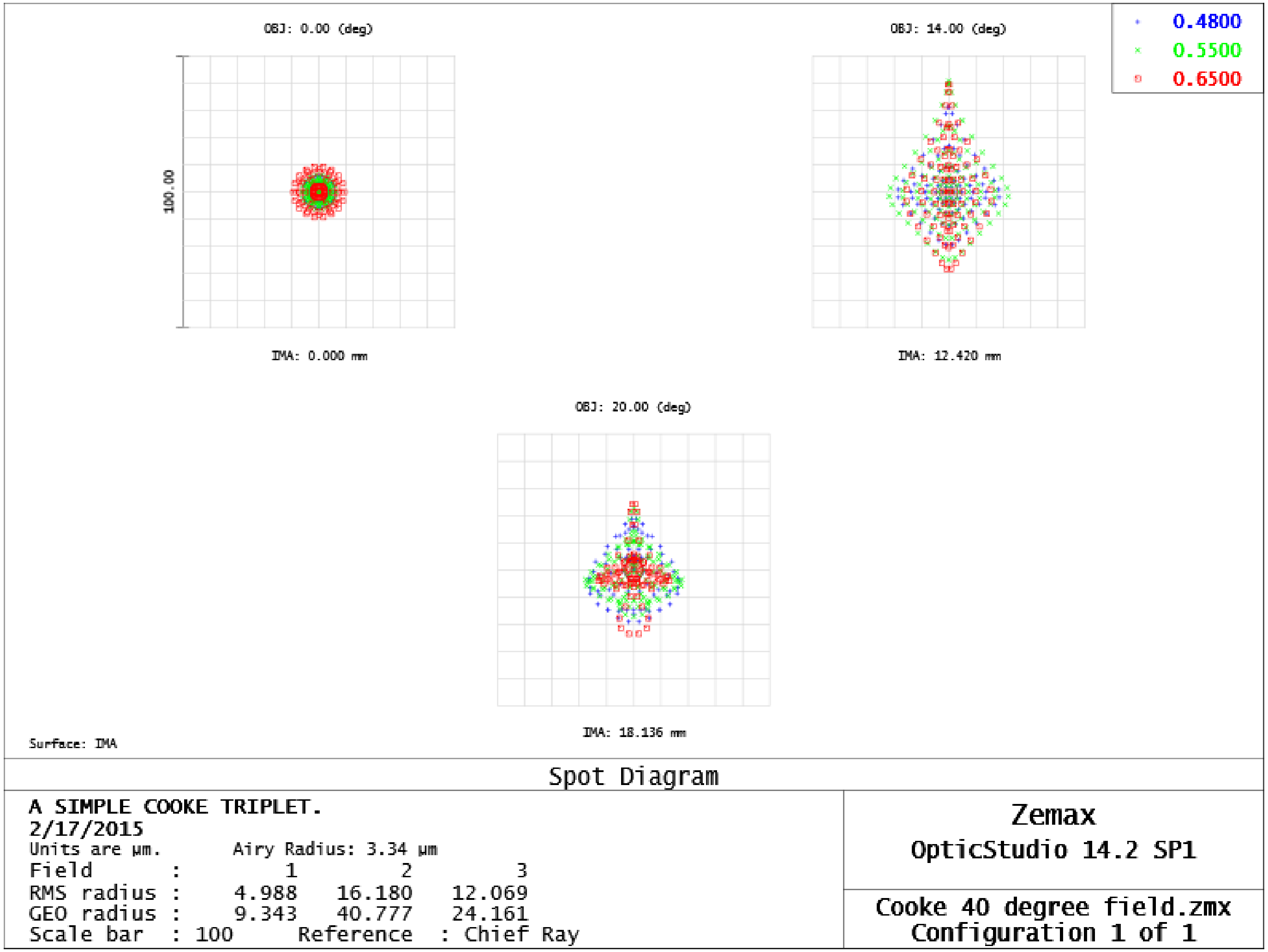


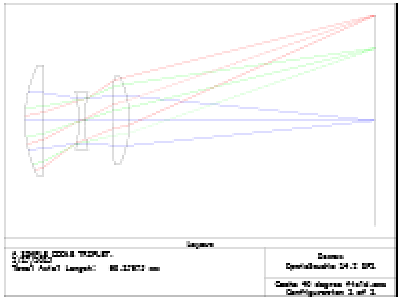
Uniform   random   square   triangular   polar

J. Geary, *Introduction to Lens Design with practical Zemax examples,* Figs. 7.2, 7.4

Spot Diagrams

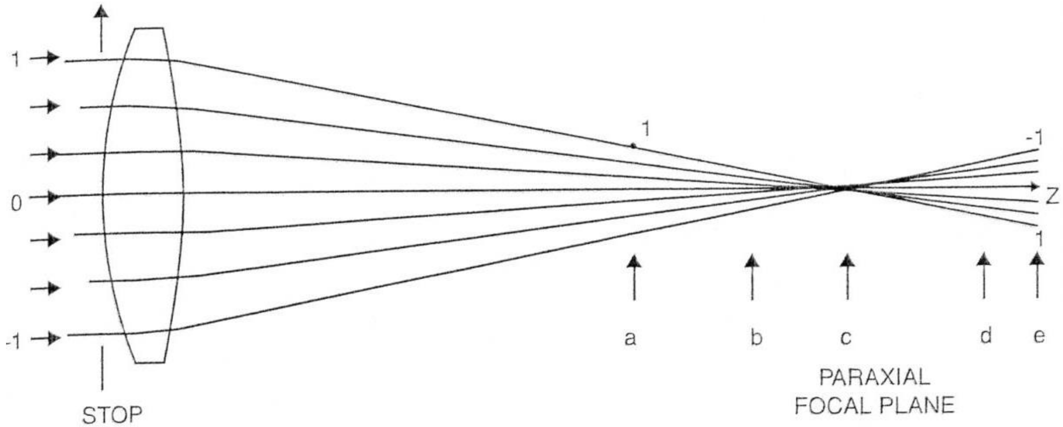
In Zemax and other optical design codes, spot diagrams are maps of where rays intersect the image plane after passing through the pupil with a chosen grid pattern. A spot diagram can be considered to be an image of a point source. Here is one example …



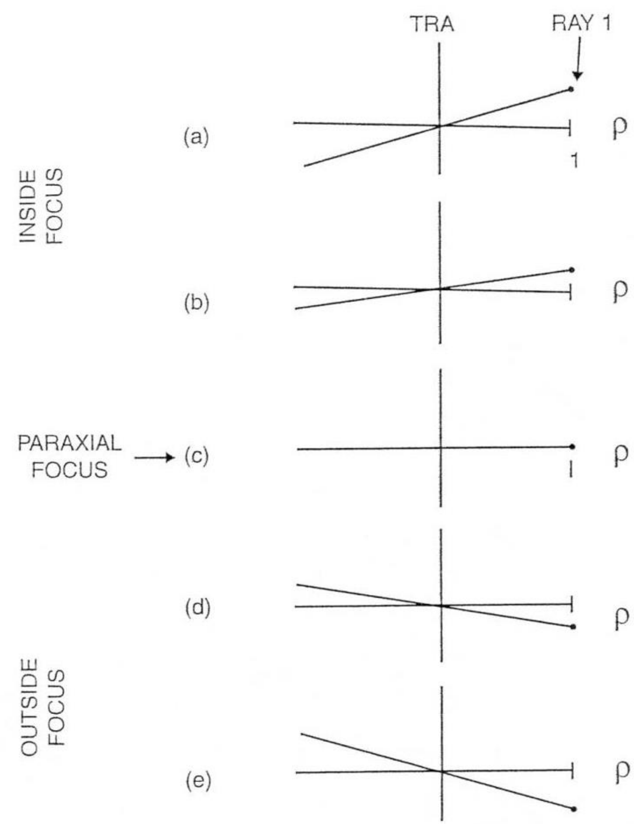


Defocus (W_{020})

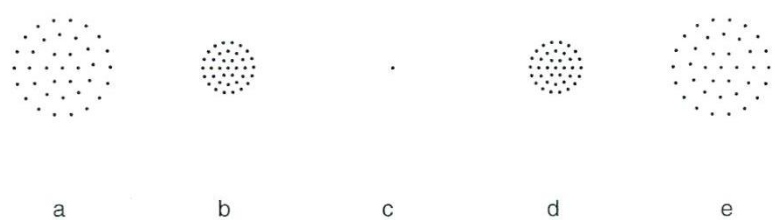
The aberration called "defocus" is more of a user-controlled variable than an actual aberration. It varies quadratically with aperture in wave front form and linearly with aperture in ray form.



Ray fans



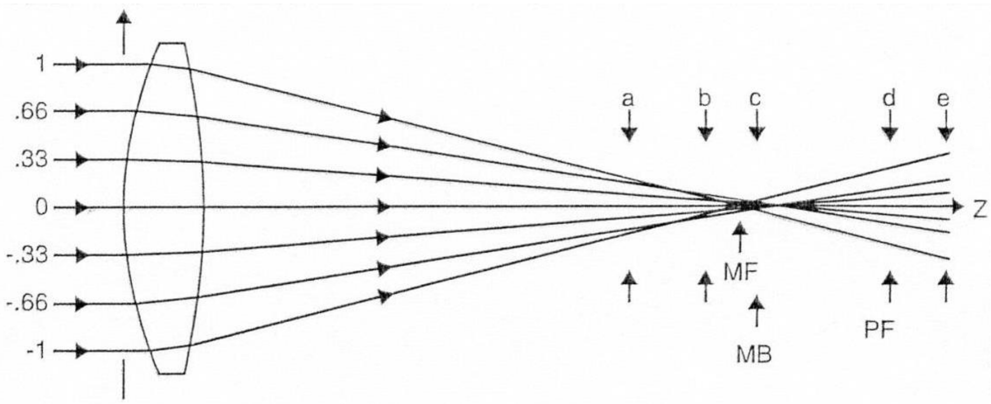
Spot diagrams



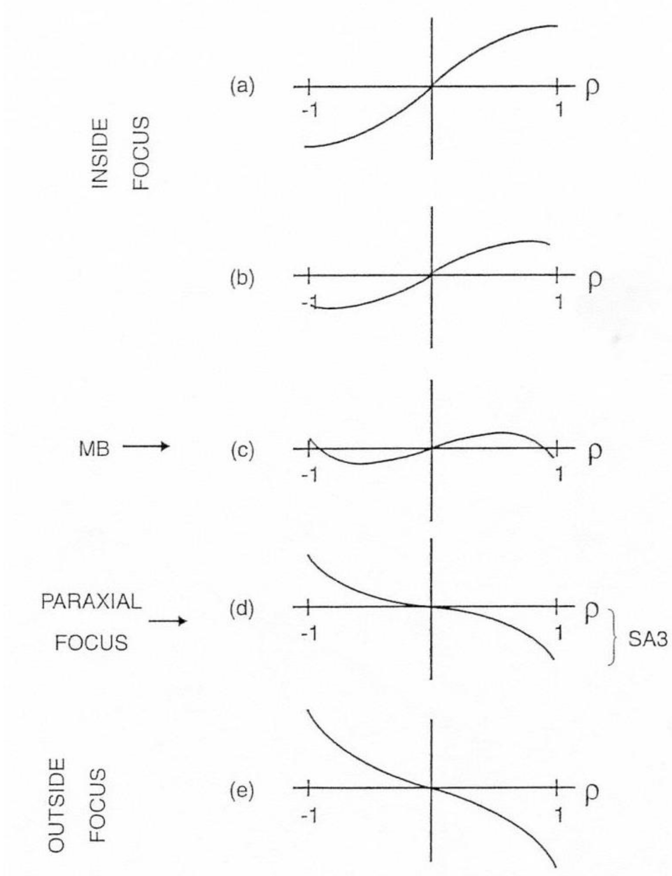
J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.5

Spherical (W_{040})

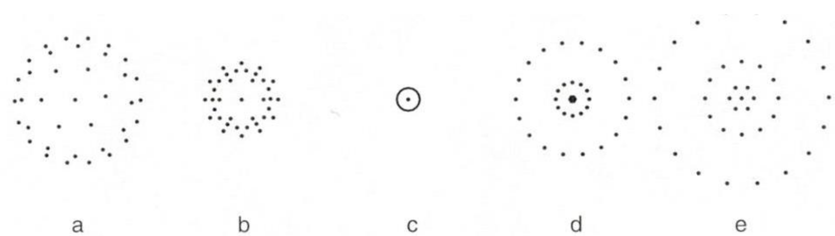
Spherical aberration is an on-axis aberration that varies as the 4th power of the aperture in the wave front form and as the 3rd power of the aperture in the ray aberration form.



Ray fans



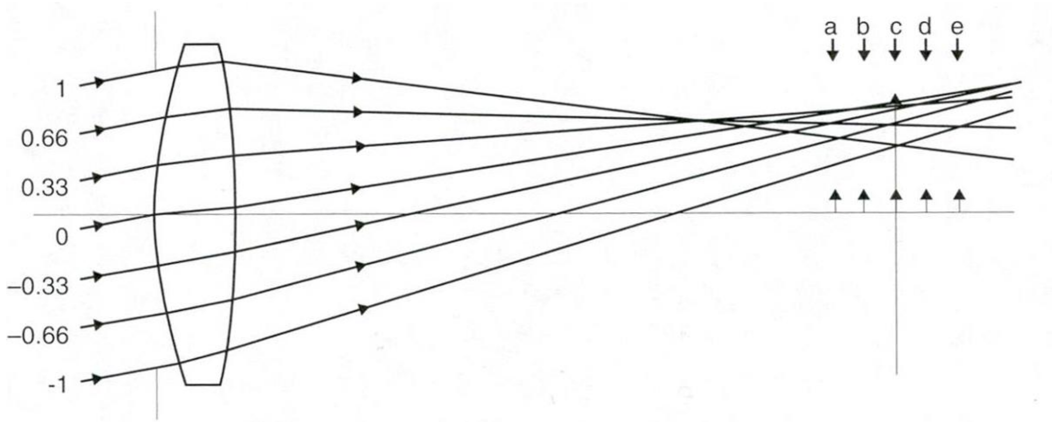
Spot diagrams



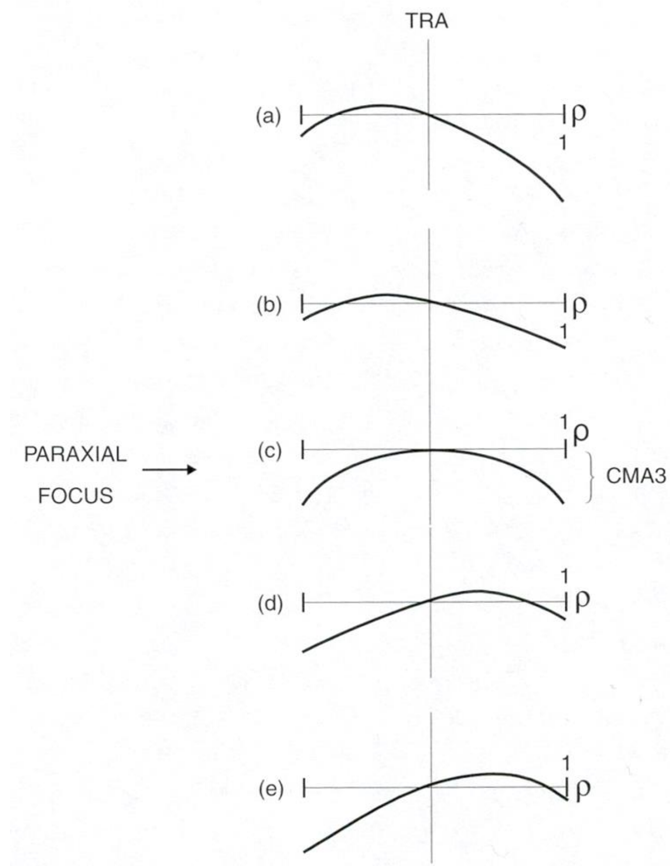
J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.6

Coma (W_{131})

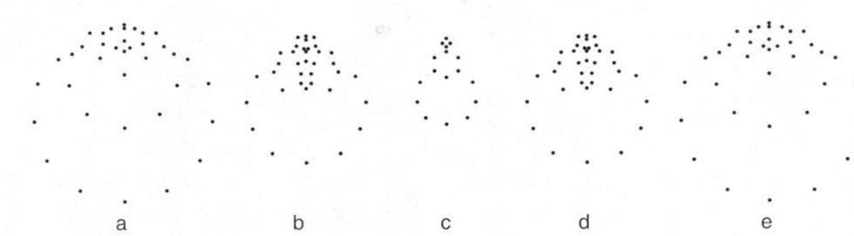
Coma is an off-axis aberration that varies as the 3rd power of the aperture in the wave front form and as the 2nd power of the aperture in the ray aberration form.



Ray fans



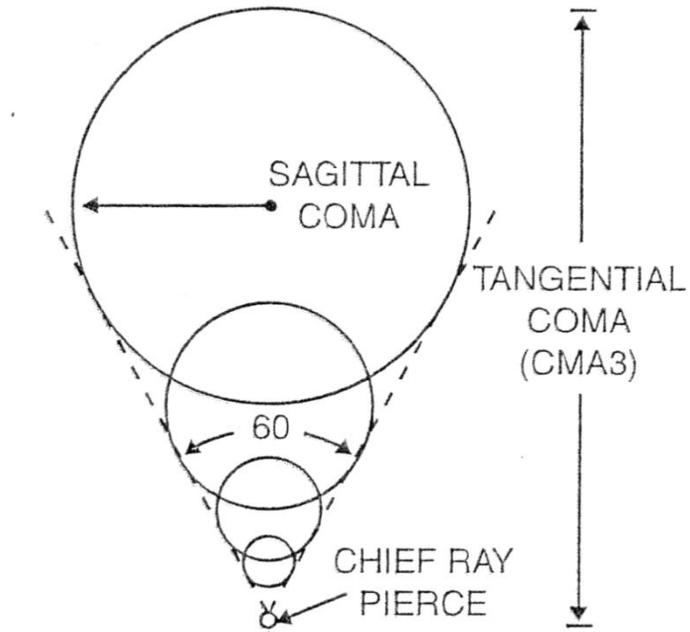
Spot diagrams



J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.7

Tangential and Sagittal Coma

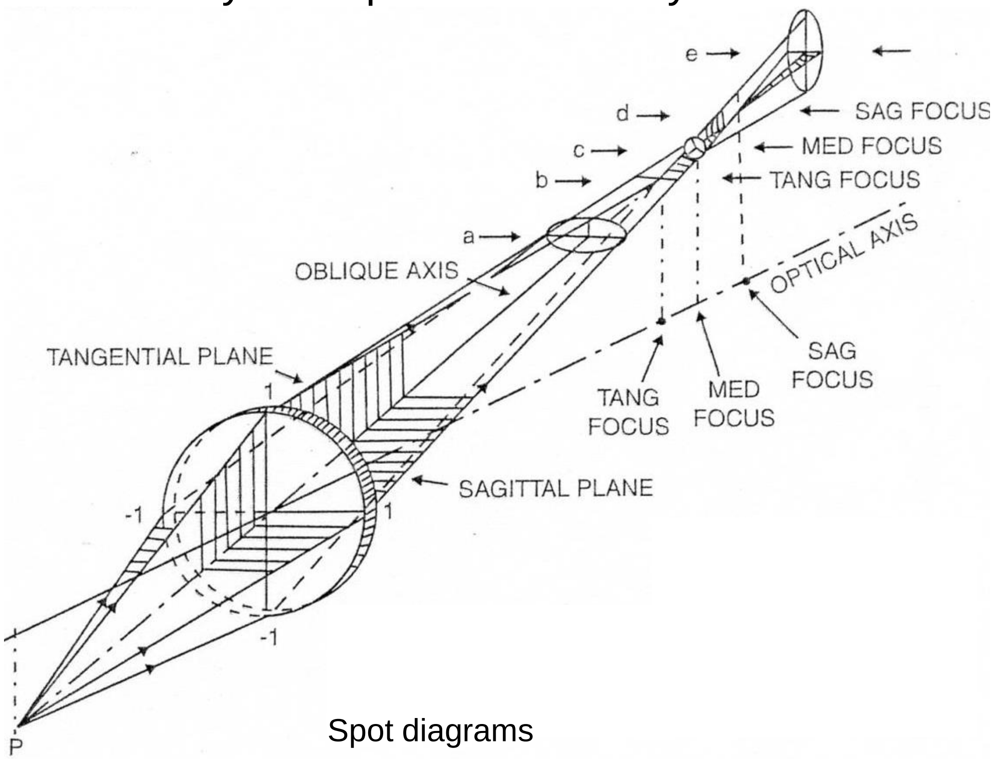
Coma forms a comet-shaped flare of ray intersections in the image plane, spread away from the chief-ray intersection (where the Gaussian image is located). Its magnitude can be expressed as either tangential coma (CMA3 = 3rd-order coma ray ab) or as sagittal coma.



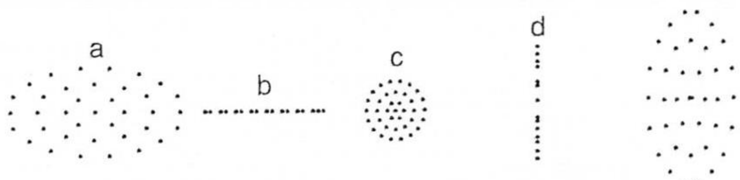
J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.8

Astigmatism (W_{222})

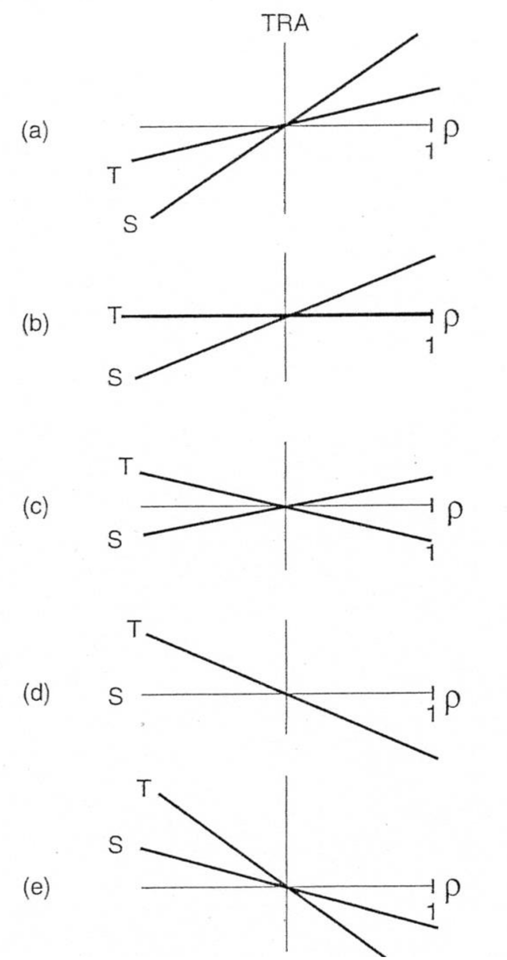
Astigmatism is an off-axis aberration that varies quadratically with aperture in the wave front form and linearly with aperture in the ray aberration form.



Spot diagrams



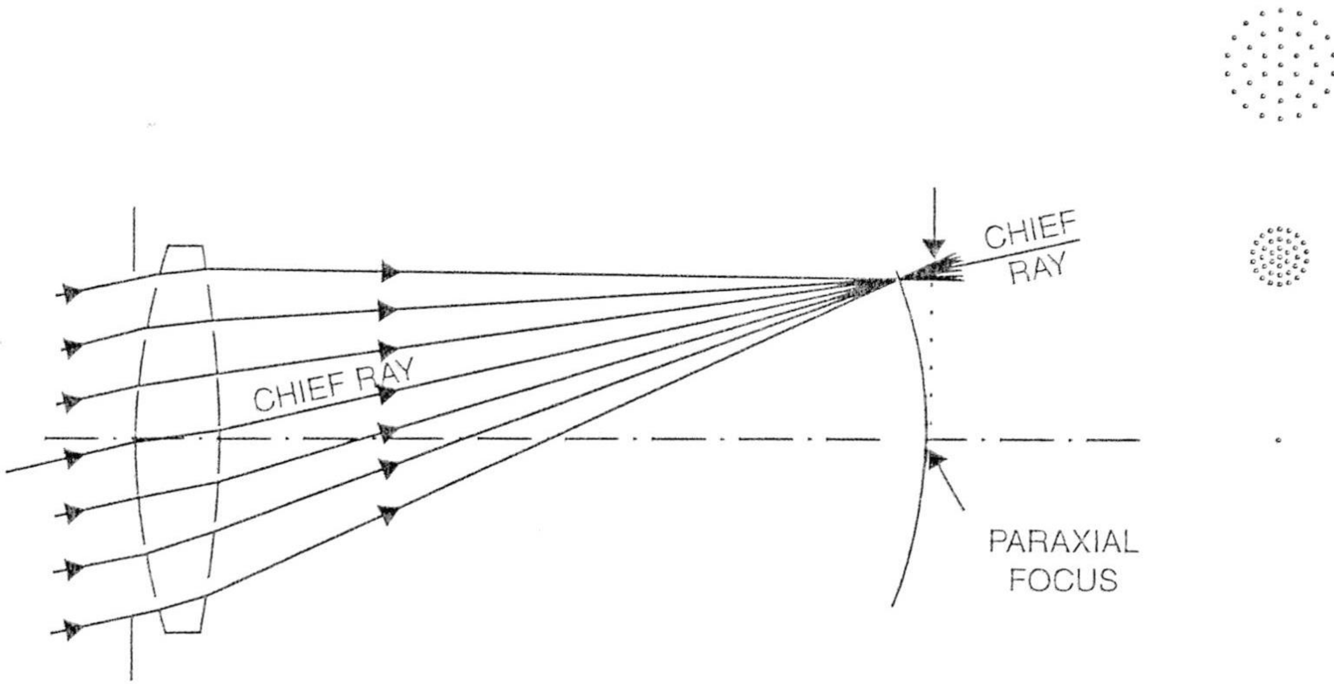
Ray fans



J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.9

Field Curvature (W_{220})

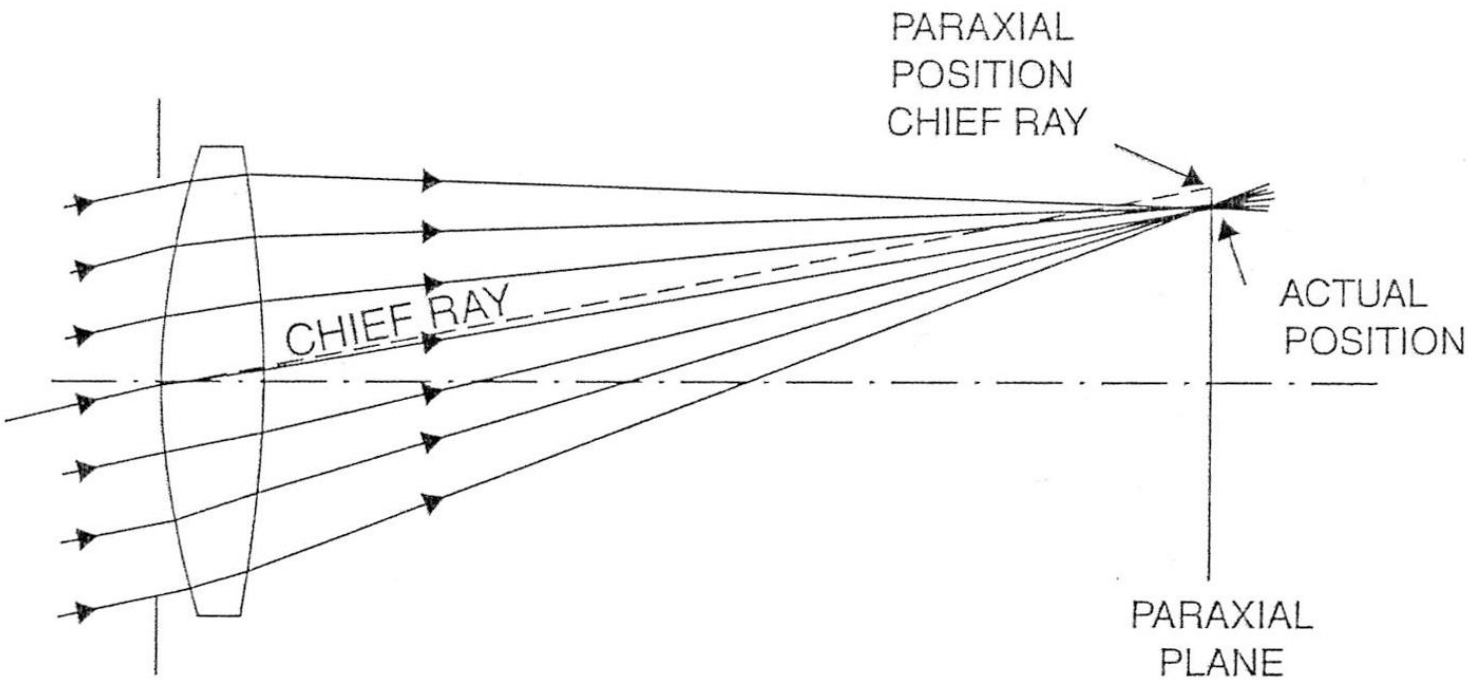
Field curvature is an off-axis aberration that affects the axial position of the point spread function (psf) but not its shape (for every chief ray there is a location of ‘ideal’ focus).



J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.11

Distortion(W_{311})

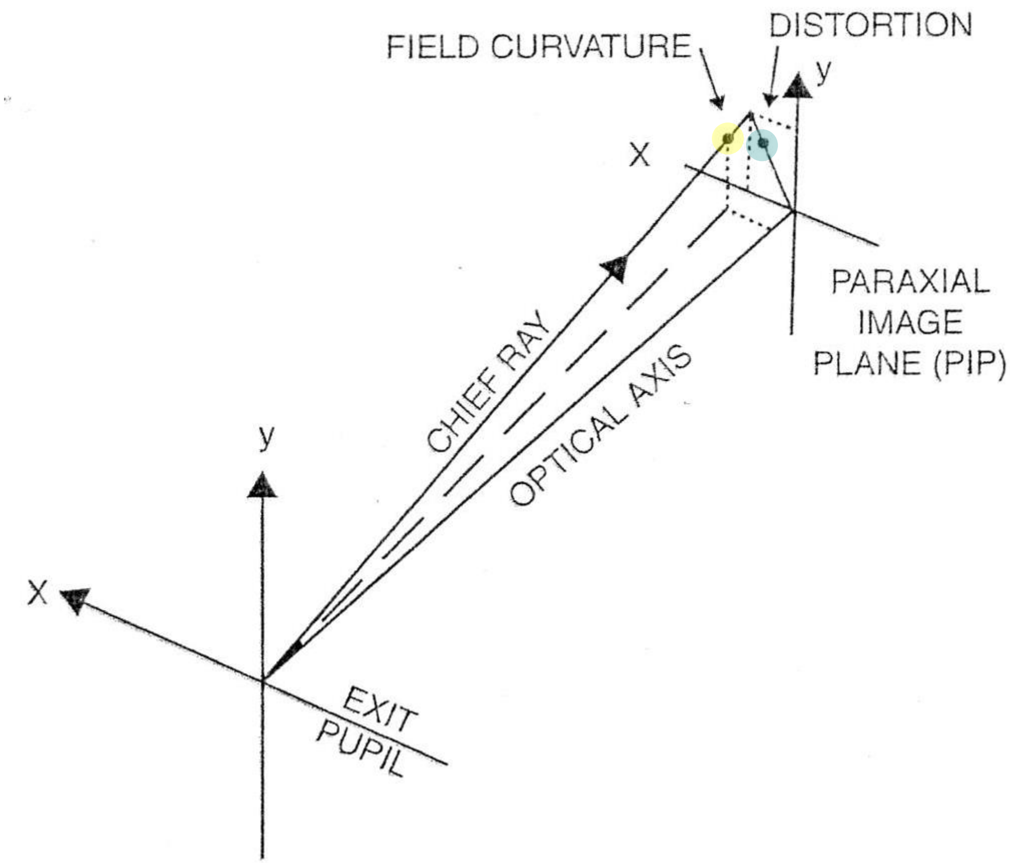
Distortion is an off-axis aberration that affects the transverse position of the psf but not its shape (the rays still focus tightly, but at a point shifted in a transverse plane).

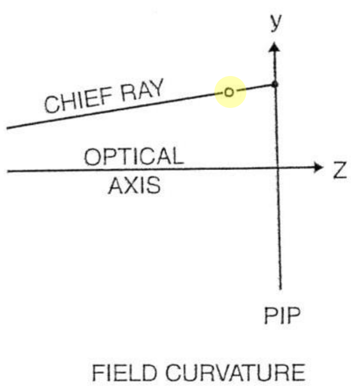


J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.12

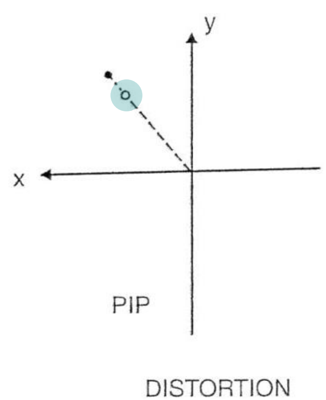
Field Curvature (W_{220}) and Distortion (W_{311})

These are off-axis aberrations that do not affect the shape of the point spread function (psf), but instead alters its position.





Field Curvature: height-dependent axial focus shift



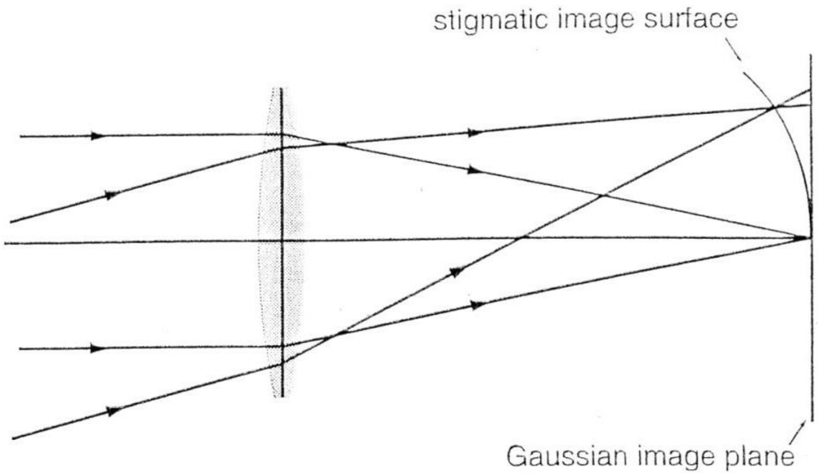
Distortion: height-dependent transverse focus shift

J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.10

Field Curvature (W_{220}) and Distortion (W_{311})

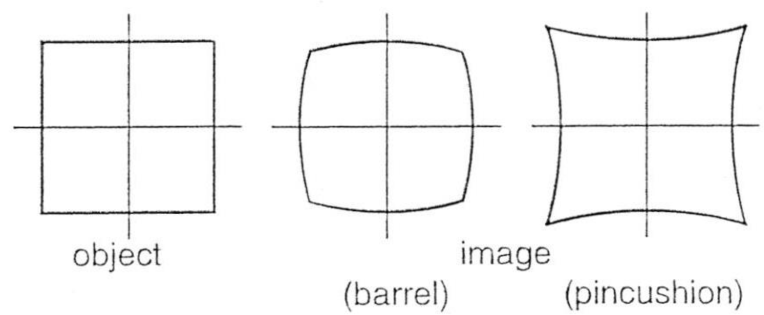
These are off-axis aberrations that do not affect the shape of the point spread function (psf), but instead alters its position.

"stigmatic" = point-like



Field curvature ←

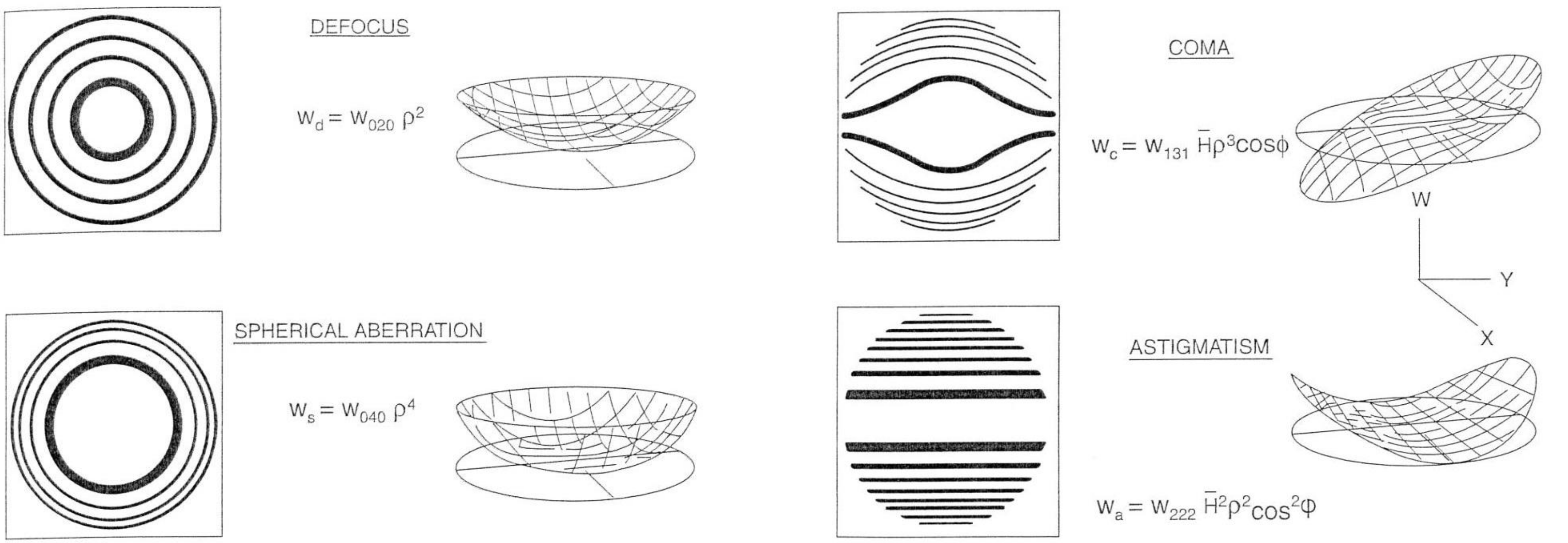
Distortion ↓



W. Smith, *Modern Optical Engineering,* Figs. 7.12, 7.13

Seidel aberrations that alter the psf shape

These are the Seidel aberrations that alter the shape of the point spread function (psf).



J. Geary, *Introduction to Lens Design with practical Zemax examples,* Fig. 7.19

Ray Fan Equation for Astigmatism & Defocus

$$W = W_{222}\bar{H}^2\rho^2\cos^2(\phi) + W_{020}\rho^2 \quad = W_{222}\bar{H}^2 y^2 + W_{020}(x^2 + y^2)$$

$$\mathrm{TRA}(x) = \frac{-R}{nr}\frac{\partial W}{\partial x} \qquad \frac{\partial W}{\partial x} = 0 + 2W_{020}x \quad = 2W_{020}\rho\sin(\phi)$$

$$\mathrm{TRA}(y) = \frac{-R}{nr}\frac{\partial W}{\partial y} \qquad \frac{\partial W}{\partial y} = 2W_{222}\bar{H}^2 y + 2W_{020}y \quad = 2\rho\cos(\phi)(W_{222}\bar{H}^2 + W_{020})$$

<u>Sagittal focus</u>

Sagittal fan extent ($\rho = 1, \phi = 90° \,\&\, 270°$): $\frac{\partial W}{\partial x} = \pm 2W_{020}$

But if $W_{220} = 0$, sagittal focus = paraxial focus, so $W_{020} = 0$ and the ‘sagittal fan extent’ = 0

Tangential fan extent ($\rho = 1, \phi = 0° \,\&\, 180°$): $\frac{\partial W}{\partial y} = \pm 2(W_{222}\bar{H}^2 + W_{020})$

Again, if $W_{220} = 0$, $W_{020} = 0$ and the ‘tangential fan extent’ = $\pm 2W_{222}\bar{H}^2$

Ray Fan Equation for Astigmatism & Defocus

Tangential focus

Tangential fan extent ≡ 0 ($\rho = 1, \phi = 0° \,\&\, 180°$): $\dfrac{\partial W}{\partial y} = \pm 2(W_{222}\bar{H}^2 + W_{020}) = 0$

∴ The amount of defocus required to reach the tangential focus is $W_{020} = -W_{222}\bar{H}^2$

Sagittal fan extent ($\rho = 1, \phi = 90° \,\&\, 270°$):

$$\frac{\partial W}{\partial x} = 2W_{020}\rho\sin(\phi) = -2W_{222}\bar{H}^2\rho\sin(\phi)$$

$$\boxed{\frac{\partial W}{\partial x} = \pm 2W_{222}\bar{H}^2}$$

We see that the sagittal focal line length = tangential focal line length = $\pm 2W_{222}\bar{H}^2$

(The 'medial focus' is halfway between the tangential & sagittal foci)

Seidel Aberration Coefficients

The Seidel aberration coefficients can be calculated from paraxial ray trace data.

Table 10.1				
Aberration Type	**Wavefront Coefficient**	**Relation to Seidel**	**Seidel Coefficient**	**Thin Lens and Mirror**
Spherical	W_{040}	$\frac{1}{8}S_I$	$S_I = -\Sigma A^2 y \Delta\left\{\frac{u}{n}\right\}$	$S_I = \frac{1}{4}y^4\phi^3\sigma_I$
Coma	W_{131}	$\frac{1}{2}S_{II}$	$S_{II} = -\Sigma A B y \Delta\left\{\frac{u}{n}\right\}$	$S_{II} = \frac{1}{2}Ly^2\phi^2\sigma_{II}$
Astigmatism	W_{222}	$\frac{1}{2}S_{III}$	$S_{III} = -\Sigma B^2 y \Delta\left\{\frac{u}{n}\right\}$	$S_{III} = L^2\phi\sigma_{III}$
Petzval Curv.	W_{220}	$\frac{1}{4}S_{IV}$	$S_{IV} = -L^2 \Sigma C \Delta\left\{\frac{1}{n}\right\}$	$S_{IV} = L^2\phi\sigma_{IV}$
Distortion	W_{311}	$\frac{1}{2}S_V$	$S_V = -\Sigma \frac{B}{A}\left[CL^2\Delta\left\{\frac{1}{n}\right\} - B^2 y \Delta\left\{\frac{u}{n}\right\}\right]$	$S_V = \sigma_V$

Where: $A = ni = n(u + yC)$ $B = n\bar{i} = n(\bar{u} + \bar{y}C)$ $L =$ Lagrange Invariant

$A = n'i' = n'(u' + yC)$ $B = n'\bar{i}' = n'(\bar{u}' + \bar{y}C)$ $L = n(\bar{u}y - u\bar{y})$

J. Geary, *Introduction to Lens Design with practical Zemax examples,*

Structural Aberration Coefficients

Structural Aberration Coefficient	Thin Lens	Spherical mirror
σ_I	$aX^2 - bXY + cY^2 + d$	Y^2
σ_{II}	$eX - fY$	$-Y$
σ_{III}	1	1
σ_{IV}	$\frac{1}{n}$	-1
σ_V	0	0

OBJ. at	Y
∞	1
Unit Mag.	0

Recall: $$X = \frac{C_1 + C_2}{C_1 - C_2}$$

$$Y = \frac{1 + M}{1 - M}$$

Where: $a = \frac{n+2}{n(n-1)^2}$ $\quad b = \frac{4(n+1)}{n(n-1)}$ $\quad c = \frac{3n+2}{n}$ $\quad d = \frac{n^2}{(n-1)^2}$

$e = \frac{n+1}{n(n-1)}$ $\quad f = \frac{2n+1}{n}$ $\quad \Delta\left\{\frac{u}{n}\right\} = \left[\frac{u'}{n'} - \frac{u}{n}\right]$

J. Geary, *Introduction to Lens Design with practical Zemax examples,*