Eric T. Bell

1883–1960

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Judith R. Goodstein
and Donald Babbitt

NATIONAL ACADEMY OF SCIENCES

ERIC TEMPLE BELL

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Elected to the NAS, 1927

E. T. (Eric Temple) Bell—number theorist, science-fiction novelist (as John Taine), and poet—joined the faculty of the California Institute of Technology (Caltech) in 1926 as a professor of mathematics. At age 43, "he [had] become a very hot property in mathematics" (Reid 2001), having spent 14 years on the faculty of the University of Washington, along with prestigious teaching stints at Harvard University and The University of Chicago. Two years before his arrival in Pasadena, he had received the American Mathematical Society's coveted Bôcher Memorial Prize for outstanding work appearing in the society's *Transactions*[1] His swift election to the National Academy of Sciences was expected.

E.T. Bell.

By Judith R. Goodstein and Donald Babbitt

Early life

Bell was born in Peterhead, a fishing village in northern Scotland, in 1883, the son of Helen Lyall, who was the daughter of a parish schoolmaster, and James Bell, a fish curer. When the boy was 15 months old, the family came to the United States and settled in San Jose, California, where his father bought an orchard and raised fruit, probably apricots and prunes. Bell grew up in San Jose, but for some reason he never revealed this fact in *Twentieth Century Authors* or any other biographical work. However, memories of that boyhood are recounted in an unpublished autobiographical poem ("A California Valley"), unearthed by his biographer Constance Reid, who described it as "a veritable hymn in praise of the Santa Clara Valley," which brought him "as close to paradise as he was ever to come" (Reid 1993). A sampling: "Behold! The later noon/ Ripens the fruits of gold, and slackens a creek that flows/All year round the orchard's garden to a sluggish, emerald pool."

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[1] The theory that Bell advanced in his long and fundamental paper Arithmetical Paraphrases I, II (Bell 1921) provided many useful applications to the theory of numbers. (Cowinner Solomon Lefschetz's paper offered essentially a complete topological theory of algebraic surfaces.)

Otherwise, little is known about Bell's childhood, or about the circumstances of his father's untimely death in early 1896. Soon after becoming a widow, Bell's mother sold the property and returned to England with Eric and his two siblings, James and Enid. She settled the family in Bedford, a small town 50 miles south of London, and enrolled Eric in a local preparatory school that focused on preparing its students for entrance examinations at the country's illustrious public schools.

In 1898, Bell entered the Bedford Modern School, where he encountered Edward Mann Langley, the teacher who would inspire him to become a mathematician. "Whatever success I have had as a mathematician I attribute entirely to Mr. Langley's coaching," Bell later wrote to the boys at his old school. A student's innate ability to do scientific research, he added,

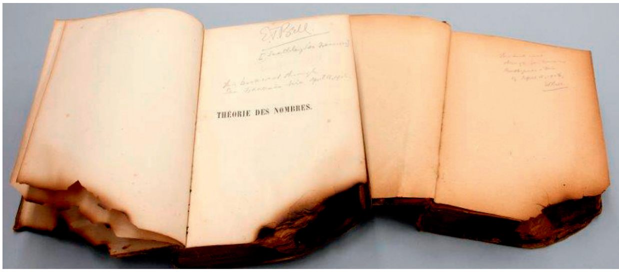
> *usually needs a flick at the critical moment, by some similarly constituted mind, to start it off in the right direction. By a great stroke of good luck I fell in with Mr. Langley at the most impressionable period—when I was about 15. He administrated the necessary flick—as painlessly as was possible under the circumstances. Being a mathematician himself, as well as an extraordinary teacher, he knew how to create mathematicians, not mere examination-passers (Reid 1993).*

After Bell's education at the Modern School ended in 1900, when he turned 17, Langley continued to privately tutor him in mathematics during the next two years, introducing him to topics ranging from analytical geometry and algebra to elementary mechanics, geometrical conics, and the calculus. Bell later attributed the beginnings of his lifelong interest in number theory to his tutor, who showed him "an ingenious use of [John] Wilson's theorem [that gave] a very elegant proof of the determination of the quadratic character of two" (Reid 1993). Langley also left the budding mathematician with a taste for elliptic functions, another favorite subject of Bell's in later life. A desire to read Homer and other ancient writers in their original language led Bell to master Greek around this time as well, with the help of a language instructor.

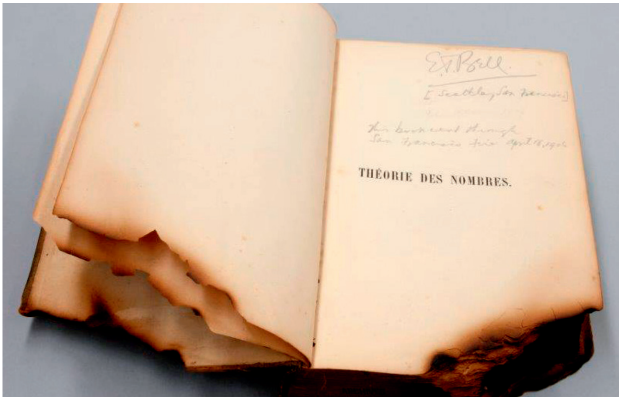
Although Bell passed the entrance exam for the University of London, there is no evidence he ever took any classes there. Instead, in summer 1902, he boarded a ship at Liverpool bound for Montreal, traveled by train across Canada, crossed into the United States north of Seattle, and made his way down the coast to his boyhood home of San Jose. He never ventured across the Atlantic again, nor does it appear that he ever saw his family again (Reid 1993).

That autumn, Bell enrolled at Stanford University with advanced standing; while tutoring students for examinations on the side, he earned a bachelor's degree in mathematics in two years. After spending the summer as a muleskinner in Nevada, Bell returned to Stanford as a graduate student, combining one course in group theory with a full-time job at a local telephone company; he left after six months, however, without earning a master's degree.

In spring 1905, Bell moved to a boarding house on the edge of San Francisco's red-light district, and he taught at the Lyceum, a private preparatory school. He also joined a group of friends in starting the Central Energy Secret Telephone Company, which laid the telephone lines for the Fairmont Hotel; and pursued his interest in number theory, working his way through the classic texts of Paul Bachmann, Édouard Lucas, and G. B. Mathews. Tossed out of bed before dawn by the 1906 earthquake, Bell hurriedly buried his math books in the garden behind the house, hoping in vain to preserve them as the fires, triggered by fallen electric wires, approached. But he had not buried them deeply enough. He would later donate two of those scorched volumes to the Caltech library.



Bell was living in a boarding house in San Francisco at the time of the 1906 earthquake and buried these two math books in the backyard. (Photos Jim Staub.)



Several days later, Bell rescued his copy of *Théorie des Nombres*, Édouard Lucas's 1891 classic text in number theory, from the scorched earth. Beneath his name and the names of two cities, Seattle and San Francisco, Bell wrote the following inscription: "This book went through the San Francisco Fire April 16, 1906."

The following fall, Bell moved up the coast to Seattle, to join the University of Washington's Mathematics-Astronomy Department as a graduate assistant. There he taught an advanced course in number theory and instructed engineering students in plane and solid geometry and higher algebra. He took only two courses in the department— education courses, possibly to meet the requirements for teaching mathematics in a public high school—but Bell satisfied the requirements of a thesis, which remained unpublished until 1920; he had received the master's degree in June 1908. To this day, school officials cannot explain how or in what field he could have earned it.

Bell moved on to the Siskiyou County High School in Yreka, a small mountain community in Northern California, where he taught a variety of science courses, continued to write poetry, and spent time in UC Berkeley's library reading math journals. In Yreka, he met and married Jessie Lillian Brown, a widow who taught art and commercial subjects at the high school. In 1911, the 28-year-old aspiring doctoral student and his wife left Yreka for New York's Columbia University, where he confidently told Cassius J. Keyser, the chairman of the Mathematics Department, that he had funds for exactly one year of study to get a Ph.D. Bell kept to that timetable: he submitted his dissertation, a topic in number theory ("The Cyclotomic Quinary Quintic"), in fall 1911; it was accepted in the spring; and he received his doctoral degree in June 1912.

Teaching career

Returning to Seattle that fall, Bell taught mathematics at the University of Washington. There his son, Taine Temple Bell was born in 1917, as Bell slowly climbed the academic ladder over the course of fourteen years, from instructor to full professor of mathematics.

In 1919, Bell began writing science fiction novels under the pen name John Taine. His first novel *The Purple Sapphire*, published in 1924, revolves around a mad scientist and an exotic ancient civilization that discovers the secret of atomic power and makes a fatal mistake, a theme Bell returned to often in his novels. One reviewer described *Sapphire* as "exhibiting a marked technical virtuosity in combining a number of disparate Victorian story formulas," but conceded that Bell was "skilled enough as a romancer to make this composite structure succeed" (Reid 1993). *The Time Stream*, a story about time travel, and the first of the John Taine novels to have a biological theme, began as a four-part serial in the magazine *Wonder Stories* before its publication by the Buffalo Book Company in 1946. Bell always insisted that *The Time Stream* was his best piece of prose writing. He published *G.O.G. 666* ("*General Order of Genetics*"), a story about human beings forced to mate with gorillas in the Soviet Union, in 1954, bringing his literary output to 13 science-fiction books.

In 1926, the Bells left Washington for Pasadena, CA, where Bell joined the Caltech faculty as a professor of mathematics. He remained there until 1953, when he retired.

Bell had been lured to Caltech by the renowned experimental physicist Robert A. Millikan, who was in the process of transforming what had been a modest technical school into one of the country's foremost scientific institutes. Caltech had started out in 1891 as Throop University (later, Throop Polytechnic Institute), named for its founder, philanthropist Amos G. Throop. At the end of World War I, Throop underwent a radical transformation, and by 1921 it had a new name, a handsome endowment, and, under Millikan, a new educational philosophy (Goodstein 1991). Understanding that mathematics could be applied to many other academic fields, Millikan began to expand the department at Caltech.

Catalog descriptions of Caltech's program of advanced study and research in pure mathematics in the 1920s were intended to interest "students specializing in mathematics…to devote some of their attention to the modern applications of mathematics" and promised "to provide definitely for such a liaison between pure and applied mathematics by the addition of instructors whose training and interests have been in both fields" (CIT 1928). Indeed, the mathematical-physics faculty at Caltech at that time was probably as good as anywhere in the country.

Millikan had apparently given Bell some indication that he might build up pure mathematics at the institute, now that matrices had come into fashion in physics, thanks to the recent invention of matrix mechanics as a way to formulate quantum theory. But Bell came to understand even before setting foot on campus that Millikan's "conversion" was something of a delusion. As he told Aristotle Michal, a job-hunting postdoc in mathematics whom he had met at Harvard, "Millikan thinks that he believes in pure mathematics, but he doesn't. He hasn't the least conception of what it is all about" (Bell 1927b). In a letter to Harvard mathematician George Birkhoff, he noted that the institute's stars were the theoretical physicist Paul Epstein, an expert on quantum theory; the "inexhaustible" Harry Bateman, who wore many scientific hats; and Richard Chace Tolman, who was "a mathematician gone wrong on chemistry" (Bell 1926e).

In short, Bell was well aware of the lesser status of pure mathematics at Caltech, but he had hopes of turning Millikan around. "To my way of thinking," he wrote Michal in another letter, "Pasadena has the most interesting possibilities of anyplace in the United States" (Bell 1926a). In the same letter, Bell spoke of the institute's need "to build up the library and, by prevailing on Millikan to take on a few good *young* men, to make the place as strong in mathematics (pure) as it is in applied." Millikan, according to Bell, had given him a free hand "to work out my own problems," with the understanding that Bell was to "help with the math. physics," a task Bell thought more suitable for H. P. Robertson, who had been his student at the University of Washington.

Bell had reasons for optimism. Pure mathematics had blossomed into an active and expanding discipline in the United States in the decade following the end of World War I. In 1926, his own publishing record, which had begun at Washington in 1915, ran to sixty-eight titles, a substantial number for a ten-year period (and just a fraction of his career total—more than three hundred, not counting his fictional output as John Taine). It was a number probably matched only by Leonard Dickson, who specialized in algebra and number theory at Chicago. Nevertheless, before hiring Bell, Millikan had polled Dickson, Birkhoff, and Princeton's Oswald Veblen—research-minded mathematicians who had put The University of Chicago, Harvard, and Princeton on the map as outstanding centers in mathematics. More important from Millikan's point of view, all three were members in good standing of the National Academy of Sciences. As Millikan wrote to Veblen in late December 1924, "From the standpoint of physics and mathematical physics we are fairly competent here at the Institute to form judgments in which we have some confidence, but from the standpoint of mathematics I feel keenly my own incompetence" (Reid 1993). From the start, Millikan had insisted on inviting only scientists of National Academy caliber to join him in Pasadena, which helps explain why the recently anointed Nobel Prize winner asked his fellow academicians what Bell's chances were of election to the National Academy.

Veblen volunteered only that Dickson thought highly of Bell's work. Birkhoff, winner of the first Bôcher Prize, gave Bell high marks for his specialty ("[H]e is great in his field, theory of numbers") and his publishing record ("very prolific"), but questioned whether Bell's long list of papers told the whole story ("[W]hen it comes to papers outside his specialty, his work is not always of high order") (Birkhoff 1925). Dickson, however, lavished praise on Bell, describing him as "an A1 mathematician of very exceptional ability in research of high order on fundamental subjects....I have long been strongly impressed by his unusual originality and his success in research of fundamental character" (Dickson 1925). He predicted that Bell would be elected to the academy before Harry Bateman was, which proved correct: Bell would be elected in 1927; Bateman's election followed in 1930, two years after he became a fellow of the Royal Society of London. If Millikan wanted to hire Bell, added Dickson, "you could hardly...get a better man," and he cautioned that Chicago also had its eye on Bell. Nor would the Seattle mathematician likely leave the Pacific Northwest without an annual salary of $5,000 or better, a figure the famously penny-pinching Millikan stored away for future reference.

As Dickson noted, Bell did not lack for other academic suitors. Fresh from sharing the Bôcher Prize, he spent the summer of 1925 lecturing on his own material at The University of Chicago and the fall semester at Harvard. He also had offers of professorships from the University of Michigan, Bryn Mawr, and Columbia University in his pocket, while Chicago began to woo him. Millikan, in discussing the need for expansion of the mathematics faculty before Caltech's Executive Council in early 1926, concluded that everything short of kidnapping should be done to acquire Bell. Bell, in turn, used Columbia's tantalizing offer of $7,500 to secure a quick response from Millikan. Within a month, he and Millikan had reached an agreement for an annual salary of $6,000.

Mathematics on the West Coast

Bell accepted Millikan's invitation to come to Caltech because the institute had already acquired a certain cachet in scientific circles and, perhaps just as importantly, because it was on the West Coast. Bell liked to say that the West Coast, while underdeveloped, had the potential to equal anything the East Coast establishment had to offer.

Bell chose Caltech over other institutions in part just to prove his point. He railed against the stuffy traditions at schools such as Harvard; indeed, he would later warn at least one job-seeker that "the Eastern places" were "not the whole cheese," adding that even Washington was "better than some potty Eastern college" (Bell 1926c). In the West, one could at least breathe fresh air. Indeed, during the 1920s and 1930s, Berkeley, Stanford, and UCLA would all develop outstanding mathematics departments.

Unlike most American institutions of higher learning, Caltech lacked traditional departments and department chairs. Bell became a member of the Physics, Mathematics, and Electrical Engineering Division, which reported to Millikan. Bell would be responsible for the graduate work in mathematics, and the mathematicians there looked to him to speak up for the field and take the lead in dealing with Millikan. Bell's plans to build up mathematics in Pasadena included both Howard Percy (Bob) Robertson, who would shortly cap a two-year fellowship in Germany with a fellowship year at Princeton; and Michal, a specialist in functional analysis, differential equations in abstract spaces, and integral invariants. "I shall first try for Robertson (Millikan wants him), and then for you," Bell told Michal. "You and he will not conflict; he is primarily an applied mathematician" (Bell 1926b). Bell planned to "ditch" (his word) on Robertson the task of teaching the applied side of mathematics. “I have so many things I really want to do that I can’t take time to be expert in math. physics,” he wrote to Robertson, just before leaving Seattle. “For instance, I have recently opened up a whole new field in ‘General Arith.,’ where there are hundreds of things to be done, and where the job can be best executed by one man working with a lot of able students” (Bell 1926d). Michal would sign on in 1929, but it took almost two more decades to snare Robertson.

As it turned out, the story of mathematics at Caltech in the interwar years was marked by a tangle of personalities and rivalries among Bell’s small group of mathematicians. Relations between Michal and Bell would eventually degenerate into name-calling and shouting.[2] Michal also seemed to be getting the most graduate students, which upset Bell. Above all, there was Bell’s tumultuous love/hate relationship with Robertson. Berkeley mathematician Abraham Haskel Taub, a longtime friend and colleague of Robertson’s, analyzed it this way: “Bell and Bob were both strong people. Bell taught Bob, learned from him, fought with him in fun and sometimes not in fun, and both cared deeply for each other” (Taub 1962).

“A kid named Robertson”

Their story began in 1922, when Bell, who was offering a course in mechanics at the University of Washington, encountered “a kid named H. P. Robertson” in his class who breezed through the assignments. In a letter to Harold Hotelling, a former student who later became a distinguished statistician, Bell marveled: “He was just 19 last month, and he goes through the most difficult problems and theory like a shot. Even complicated setups in problems by Lagrange’s equations don’t bother him in the least.…Robertson is a prize” (Bell 1922).

Bell took Robertson under his wing—much to the consternation of Washington’s conservative mathematicians, who believed that research did not go hand in hand with good teaching—and got him interested in the theory of relativity. “If I were 15 years younger,” Bell confided to Hotelling, “I would go into relativity; as it is, I hate to scrap the detailed knowledge of the theory of numbers which has taken so long to acquire” (Bell 1922). Bell taught a relativity course in the department, which Robertson also took before receiving a bachelor’s degree in 1922 from Washington. Bell also persuaded Robertson

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[2] One of Michal’s main goals in the 1930s was to generalize the analysis and geometry on finite-dimensional manifolds to abstract infinite-dimensional manifolds, although he did not seem to have proven any new deep theorems. Other mathematicians at the time, including Bell, viewed these efforts as abstraction for its own sake, with no sense of problem. Subsequently, however, the topics became important and active fields in pure and applied mathematics.

to stay another year at the university to continue studying mathematics, electricity, and relativity—using, for the last, Hermann Weyl's *Raum-Zeit-Materie* as his textbook.


A gathering of H. P. Robertson's friends on the steps of the Athenaeum, 1936. Standing, from left: Virginia Thomas, Tracy Thomas, Howard Percy Robertson, Ethel Bateman, Pipo von Karman, Angus Taylor, Patsy Taylor, Toby Bell, Eric Temple Bell, Mary Bowen, and Aristotle Michal. Seated from left: Hazel Mewborn and Luddye Michal.

At one of Caltech's weekly physics research conferences in 1924, Robertson, now a Caltech graduate student studying with Bateman and Epstein, gave a talk on relativity in which he credited Bell with sparking his interest in the subject. Millikan and Epstein, who were in the audience, congratulated Robertson afterward on a fine talk; Robertson dodged the compliment by heaping praise on Bell. Millikan filed the name away. "Thanks for tooting my horn," Bell later wrote to Robertson. "I know what they want and they've got him—Bateman" (Bell 1924). Still, Bell appeared ready to pack his bags and head south. As he told Robertson in closing, "I'd sell my left foot to get into a job in California."

Before Bell managed to plant both feet in the Southland, Robertson had obtained a Caltech Ph.D., with a major in mathematical physics and a minor in mathematics. He completed his dissertation in 1925 on a topic in relativity—"on the dynamical space-times which contain a conformal Euclidean 3-space"—and then crossed the Atlantic for a year's study in Germany as a National Research Council Fellow in mathematics. The fellowship was renewed for a second year with the promise of a third either at Harvard or Princeton. Robertson spoke of permanently locating in Pasadena, "but that is as yet unsettled," he wrote in spring 1927 to a family friend" (Abel 1927). That June, Millikan wrote to Robertson, who was still in Germany and expecting to continue his NRC Fellowship at Princeton, and offered him a faculty position, effective September 1928, with the status of assistant professor of mathematics on leave until that time. Robertson quickly sent back his acceptance letter.

The Contretemps with Robertson

On March 20, 1929, Bell wrote an uncharacteristically icy letter to Robertson, who had chosen to continue his NRC Fellowship at Princeton and had then remained an additional year. It was a letter—totally out of keeping with the usually informal, sometimes bawdy, tone of the correspondence between the two friends—in which he intimated that Robertson might be better off teaching somewhere other than at Caltech. "This is rather an official letter, written at the request of [Millikan's assistant Earnest] Watson and Dr. Millikan," Bell began by way of explanation.

> *They want to know whether you are definitely planning to return here next year....As you know, the rule here is no advancement in either rank or salary unless a man is productive. So if you see anything that you consider more attractive, please let us know as soon as possible. We must know definitely by April 1st.*

While "I personally hope you will accept, and fight it out on the lines proposed," Bell wrote in closing, he emphasized again that there was "absolutely no chance here for a man who is not productive" (Bell 1929). On April 1st, Robertson sent Millikan a telegram saying that "under present circumstances" he felt obliged to request release from his appointment. Marietta Fay, Robertson's daughter, recalled that "both my parents were deeply hurt by Bell's letter. They never *forgave* [emphasis in the original] him" (Reid 1993).

Bell continued hounding his protégé to publish more often. He wrote to Oswald Veblen in winter 1931, "If you happen to think of it, would you mind jogging Robertson up to get out some of his stuff on mathematical physics? As it is, other people are running away with bits of it under his nose" (Bell 1931a). Even Robertson's election to the National Academy of Sciences in 1951 provided Bell with an excuse to nag him. "This is something many years overdue, mainly your own fault, for straddling the fence between two sciences and not letting your fellows know on which side your balls hung," he wrote (Bell 1951a). Robertson, no stranger to such repartee, replied in kind.

> *Thank you for your obscene congratulations....You attribute the delay in me sitting on the fence with one orchid hanging over on either side; I say they were only waiting for them to shrivel up and drop off before I would be ripe for the honor (Robertson 1951).*

Caltech mathematics faculty and teaching fellows, 1932. Front row, from left: Aristotle Michal, Harry Bateman, Eric T. Bell, and Harry C. Van Buskirk. Rear row: William Birchby, James H. Wayland, Carlton C. Worth, Luther E. Wear, Robert S. Martin, [unknown], and J. Lawrence Botsford.

Bringing up the next generation

Morgan Ward, Bell's first graduate student, entered Caltech in 1924 (where he was one of only 48 graduate students altogether), after earning his bachelor's degree at UC Berkeley. He became Caltech's first Ph.D. in mathematics, receiving his degree summa cum laude in 1928 with a dissertation on the foundations of general arithmetic. In 1929, Ward joined the faculty as an assistant professor of mathematics and, aside from a year at Princeton in 1934–1935, remained at Caltech until his death in 1963. Like Bell, he had a deep interest in the theory of numbers and a "great contempt for those who proliferate easy empty generalizations of the great classic ideas of mathematics," according to Derrick H. Lehmer, who got to know Ward while spending the 1930–1931 academic year at Caltech on an NRC Fellowship in mathematics (Lehmer 1993).

The onset of the Great Depression in 1929 may have dampened Bell's spirits a bit. In response to a 1931 letter from Veblen hinting that he would welcome an invitation to visit Pasadena while Albert Einstein was in residence, Bell wrote:

> *I fear it is out of the question. The financial stringency has hit us hard. The mathematicians never did have any funds available to pay outside lecturers. The one time when we did pay a lecturer, namely Harald*

> *Bohr, [funding] was provided by a crumb dropped from the physicists' banquet....I was to have got a new man this year, but the money wasn't forthcoming. In the past they have usually paid railway fare to essential meetings; this year that also is cut out, so I shall have to pay my way to New Orleans. However, this depression can't last forever (Bell 1931b).*

In fact, the Depression dragged on, and no new Caltech faculty appointments were made in mathematics until the early 1940s.

Angus Taylor, Aristotle Michal's first student and one of his best, was another promising star. Taylor and the other graduate students in mathematics at Caltech in the 1930s were pushed to the research frontier as quickly as possible. Because he already knew the theory of functions of a complex variable, Taylor skipped Harry Bateman's course, which leaned heavily on Whittaker & Watson's *A Course of Modern Analysis.* (Taylor later described Bateman as "a very gentle and nice man…in a little rut all by himself" [Taylor 1981].) Instead, under Michal's guidance, Taylor became proficient in Lebesgue measure and integration, the theory of abstract spaces and functional analysis, and Riemannian and non-Riemannian geometry. "Michal did some lecturing, but made the students do quite a bit of it themselves," Taylor recalled (Taylor 1981).

Taylor studied abstract algebra under Bell, who, he said, "didn't lecture. He had all the students tell the class what was in the books, so the students did all the lecturing; Bell commented and criticized." In a memoir for the Mathematical Association of America, Taylor characterized Bell as "a stimulating person, given to expressing strong opinions, [but] I don't think he spent much time preparing what he was going to say in class" (Taylor 1984). The curriculum had gaps, particularly in combinatorial topology and point-set topology—subjects offered at Princeton, Texas, Virginia, and Michigan. "There really was not a mathematics department in an administrative sense," Taylor remembered. "I don't think there was much planning of curricula. There was very little guidance of graduate students" (Taylor 1981).

In 1937, Taylor turned down an instructorship at Caltech and went to Princeton as an NRC Fellow, where he worked with Salomon Bochner. Taylor's fellowship was renewed for 1938–1939, and then, unexpectedly, he received an appointment offer from UCLA for that academic year. Keen to return to California, he tried to find out whether Caltech planned to ask him back in 1939. "So there I was, in April or May of 1938, a bit up in the air about my future," he later recalled. "Then unexpectedly, I received word from E. T. Bell urging me to accept the UCLA offer. That told me what I hadn't known for sure about Caltech's interest in me. After discussing the matter with people at Princeton, and against their advice, I resigned the second year of the fellowship and took the job at UCLA. I've never regretted it" (Taylor 1981). Morgan Ward, who was hoping Taylor would return to Pasadena, blamed Bell. Taylor, he wrote Robertson, would have done "his fair share of the work, something which A. D. Michal and E. T. Bell avoid successfully." Bell, he added, "seems down [on] him, but I cannot find out exactly why—he [Taylor] seems to have made the wrong remark to him some time ago, and Bell has treasured it" (Ward 1938).

In 1939, Ward received an offer from Johns Hopkins. Bell told Millikan that if Hopkins wanted Ward, Caltech "should do the utmost we can to make it worth his while to stay" in Pasadena (Bell 1939). Ward was promoted to professor the following year.

The first substantial revamping of Caltech's mathematics faculty in more than a decade came during World War II. In 1942, a new assistant professor in mathematics was needed to replace the retiring Harry Clark Van Buskirk. Ward's former student Robert Dilworth and Michal's former student Angus Taylor were the candidates. The screening took place "in house." When Millikan turned to several physicists at the institute for advice, they assured him that Dilworth's research was potentially of greater use to physicists—although the spectroscopist William Houston questioned the criterion Millikan appeared to be using in deciding between the two mathematicians ("I was rather surprised to have it mentioned [by you] that he [Dilworth] seemed to be more inclined toward the applications of mathematics than many other mathematicians") (Houston 1942). In a letter to one of us [JRG] many years later, Taylor wrote that he knew

> *that Bell and Michal were at odds over getting me to go back to Caltech from Princeton....Millikan made it quite clear, it seemed to me, that he was only interested in what Dilworth or I might be useful for in teaching the physics graduate and undergraduate students, and not at all in our scholarly potential in research (Taylor 1991).*

Taylor had no idea that he was a candidate again, along with Dilworth, for a tenure-track position at the school several years later. "I would not have left UCLA for Caltech then, in any event," he later told John Greenberg, a Pasadena historian of mathematics (Taylor 1981).

Bell was disappointed by the turn of events. If Millikan wanted a mathematician more tuned to applied mathematics, then Robertson was the man. Others on the campus, including Ward and aeronautical engineer Theodore von Kármán, who had turned to Robertson for help on a turbulence problem, also plugged for him. But Millikan, as Bell wrote to Robertson, was unwilling to "spend any real money," and Bell "frankly" saw "no chance until, if ever, Pa [Millikan] retires and a more enlightened financial policy is adopted" (Bell 1942).

In 1945, Millikan retired as Caltech's head, and the following year physicist Lee A. DuBridge became the institute's president. Bell's original band of mathematicians also passed into history: Harry Bateman, the AMS's Gibbs lecturer in 1943, died three years later, en route to New York to receive an award from the Institute of Aeronautical Sciences.[3] Aristotle Michal died of heart disease in 1953. That year, E. T. Bell retired from Caltech, 27 years after coming to Pasadena. The transformation of the mathematics faculty, which Millikan had largely regarded as a service department, into a first-class research group began under the inspired leadership of H. Frederic Bohnenblust, who arrived as a full professor of mathematics in 1946. Applied mathematics as a distinct discipline came to Caltech in the mid-1960s. To avoid friction with the pure mathematicians, it was organized as a research school within Caltech's engineering division. Bell died in Watsonville, California, on December 21, 1960.

Bell's legacy

Whereas Bell was a prolific and well-received writer of poetry and science fiction under his pseudonym of John Taine, he approached rock-star recognition among mathematicians and non-mathematicians alike for his 1937 book, *Men of Mathematics.* Still popular today, *Men of Mathematics* discusses the personalities and mathematics of a host of great mathematicians, including Niels Henrik Abel, Carl Friedrich Gauss, David Hilbert, and Bernhard Riemann, concentrating on those born from the 18th century on (Bell 1937). Bell followed this account of men (and the occasional woman) in mathematics with *The Development of Mathematics*, issued by McGraw-Hill in 1940, with a revised edition five years later (Bell 1945). This book was a sweeping account of the history of mathematics, starting with its beginnings in ancient Babylonia and Egypt and charting its progress to 1945.

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[3] The Bateman Manuscript Project, based in part on notes he left, started up several years later, under the direction of Arthur Erdélyi and aided by three research associates, Wilhelm Magnus, Fritz Oberhettinger, and Francesco G. Tricomi. The project culminated in three volumes of *Higher Transcendental Functions*, supplemented by two volumes of *Tables of Integral Transforms*.

Readers of *The Development of Mathematics* require some mathematical sophistication to fully appreciate it, however. In his review in *Isis*, I. Bernard Cohen, the dean of American historians of science at that time, wrote, "Within one restriction, the present book is excellent; that restriction consists in the fact that it really begins on p. 99, with Ch. 7: 'The Beginning of Modern Mathematics, 1637–1687.' In the first 98 pages, there are many statements that one will take exception to" (Cohen 1941). Cohen's objections included Bell's breezy description of al-Khwarizmi's algebraic methods ("a psychiatrist might say it was the death instinct having its way") and "the grand manner" in which Bell demolished Plato's detractors ("Of all changes that mathematical thought has suffered in the past 2,300 years, the profoundest is the 20th-century conviction, apparently final, that Plato's conception of mathematics was and is fantastic nonsense of no possible value to anyone").

Like Cohen, most of the other reviewers gave the book high praise, while also calling attention to Bell's unorthodox style. D. R. Curtiss, writing in *National Mathematics Magazine*, noted, "After a few drier pages there is always a pungent remark on human frailties, a bit of grim humor, sometimes an aside of a half-page or more on what dictators are doing to mathematics, and what philosophers or theologians would do if they could" (Curtiss 1941). Rudolph Langer, who reviewed the book for *Science*, probably spoke for many when he declared, "The presentation of the whole is admirable. It is flowing and graceful and often characterized by a genuine and delightful humour" (Langer 1941).

Bell also had his detractors, some of whom accused him of being flippant. Reviewers of *Men of Mathematics* complained of his inclination to sacrifice historical accuracy for a more colorful story. The most blatant example is his exaggerated account of the life of Évariste Galois, who died at 20, following a celebrated duel. Did Galois really create group theory in its entirety during the last night before the duel? Not according to Tony Rothman, who in *Genius and Biographers: The Fictionalization of Évariste Galois* (Rothman 1982) accuses Bell of inventing history. Based on his research on Galois, Rothman concludes that Bell "consciously or unconsciously saw his opportunity to create a legend.…Unfortunately, if this was Bell's intent, he succeeded."

Today's historians of mathematics are even less generous in their praise of Bell as a historian. Ann Hibner Koblitz, who has written about Sofia Kovalevskaya, thinks that Bell "might well become known to future generations of mathematicians and historians as the legend maker of the history of mathematics. It is to him that mathematicians are largely indebted for distorted impressions of their predecessors" (O'Connor and Robertson 2017). Koblitz's tart remarks are echoed by the University of Vermont mathematician Roger Cooke, who deems Bell's treatment of Kovalevskaya an "infuriatingly patronizing, innuendo-laden mistreatment" (O'Connor and Robertson 2017).

Bell's mathematical contributions

E. T. Bell was both a world-class number theorist and a wonderful expositor in many areas of pure mathematics. To support his first interest, he was a lifelong member of the American Mathematical Society (AMS), an organization devoted to advancing mathematical research. To support the second, Bell was a lifelong member and sometime president (1931–1932) of the Mathematical Association of America (MAA), a society espousing mathematical exposition and pedagogy.

There are 312 items in Bell's bibliography: Roughly two-thirds are primarily research articles in number theory and closely related areas; the rest are largely expository articles and books encompassing a somewhat broader area of subjects. His research articles almost always appeared in the world's top mathematics journals, including *Annals of Mathematics* (23), *American Journal of Mathematics* (20), and *Transactions of the AMS* (19). *The American Mathematical Monthly* (*TAMM*) was a popular forum for his expository articles (25). One section of TAMM, "Problems and Discussion," drew Bell's attention throughout his career. Between 1912 and 1948, he either proposed or solved a problem for this section 15 times, a practice that seemed to capture one aspect of Bell's mathematical personality.

We follow the practice of Lincoln Durst, an E. T. Bell scholar, by organizing Bell's research into four main areas: arithmetical functions, arithmetical paraphrases, Bell numbers and Bell polynomials, and multiplicative Diophantine equations (Durst 2001). To these we added a fifth area, algebraic arithmetic.

Arithmetical functions and Liouville formulas, 1912–1920

As mentioned earlier, Bell's mathematical interests were primarily in number theory. But his early work involved arithmetical functions, which are functions from the positive integers n into the real or complex numbers that express some arithmetical property of n. During these early years, Bell became especially interested in a collection of arithmetic formulas that had been announced without proofs in 1857 by Joseph Liouville; and although Liouville claimed he had a simple and direct way of proving them, he never did publish it. Bell, however, apparently was able to produce proofs by himself.

Bell's first real research paper appeared in 1915 (Bell 1915), but because the journal in which he published it was not widely distributed, the depth and importance of his results were not recognized until later. Starting sometime in the period 1915–17, Bell became aware of another series of papers by Liouville between 1857 and 1865, announcing, again without proof, an additional series of arithmetical formulas and again claiming he had proofs (which he never published). From the time of the publication of these assertions up to 1920, various mathematicians gave proofs of all but one. This was subsequently proved by Bell himself in 1936 (Bell 1936), and the result later became known as Liouville's Last Theorem (Andrews 1999).

It was also during this period that Bell began working on a process he called "arithmetical paraphrasing," and he presented some of his ideas on the subject at an AMS meeting in 1918. Not long after, Bell developed at least one important new tool for studying arithmetical functions:

Definition (Apostol 1976): Given an arithmetical function f and a prime p, the *Bell series mod p of f* is the formal power series:

$$f_p(x) =: \sum_{n=0}^{\infty} f\left(p^n\right) x^n$$

According to Durst, Bell wrote 35 papers on arithmetical functions between 1915 and 1951 (Durst 2001).

Arithmetical paraphrases

Once all but one of the latter of Liouville's arithmetical formulas were proven, Bell worked on developing a general technique for creating new formulas; in that way, interesting new number-theory theorems could be proven. Bell called this technique *arithmetical paraphrasing*. It used elliptic functions and the parity properties of the Liouville formulas in an essential way to produce new arithmetical formulas (Bell 1921). As a reward for this outstanding work, Bell shared the 1924 Bôcher Memorial Prize of the American Mathematical Society with Solomon Lefschetz and was appointed a member of that society. This series of papers also no doubt played an important role in Bell's 1926 appointment as a full professor of mathematics at Caltech. Between 1917 and 1947, he published more than 80 papers on quadratic forms and other questions arising from, or related to, his study of paraphrases (Durst 2001).

Algebraic arithmetic, AMS Colloquium Series

In 1926, Bell was selected to give the American Mathematical Society Colloquium Lectures, which led to the publication of his *Algebraic Arithmetic* in the AMS Colloquium Series, one of the most prestigious publishing venues in the world of mathematics (Bell 1927a). The goal of the book was, among other things, to place his work on arithmetical paraphrases and Euler algebra (Bell 1923) into a more general and natural context. As Bell explained in the book's introduction:

> *Intermediate between the modern analytic theory of numbers and classic arithmetic, as developed by the school of Gauss, is an extensive region of the theory of numbers where the methods of algebra and analysis are freely used to yield relations between integers expressed wholly in finite terms and without reference, in the final propositions, to the operations or concepts of limiting processes. This part of the theory of numbers we shall call algebraic arithmetic....It is the purpose of the following chapters to outline a few promising directions in which progress may be made toward classifying, extending, and generalizing the methods and results of algebraic arithmetic.*

In his review in the *Bulletin of the American Mathematical Society*, the number theorist Leonard Dickson, one of America's leading mathematicians, writes:

> *This book of marked originality is of vital interest to advanced students in various branches of mathematics, including the theory of numbers, abstract algebra, elliptic and theta functions, Bernoullian numbers and functions, and the foundation of mathematics....A leading feature of the book seems to the reviewer to be its success in a systematic attempt to find a unified theory for each of the various classes of related problems in the theory of numbers, including its interrelations with algebra and analysis.*

Unfortunately, other prominent mathematicians did not understand just what Bell was trying to convey. One of them was the number theorist Tom M. Apostol, who was Bell's successor at Caltech when he retired in 1953. Apostol and his number-theorist student Basil Gordon tried on and off over several years to make sense of *Algebraic Arithmetic* and never succeeded to their satisfaction (Apostol 2012). Finally, in 1998, the MIT applied mathematician Gian-Carlo Rota gave Bell's book its due, claiming that "*Algebraic Arithmetic* remains to this day the book of seven seals" (Crapo and Senato 2001).

Bell numbers and Bell polynomials

The Bell numbers and Bell polynomials are powerful tools in the field of combinatorics, with important applications to probability and statistics, as noted in the comments of John Greenberg below. Bell polynomials were initially called exponential polynomials by Bell (Bell 1934). He was not particularly interested in their applications, though they helped establish him as one of the leading combinatorialists of the first half of the 20th century.

Specifically the *Bell numbers* are concerned with *partitions* of the sets of positive integers $\{1,2,\ldots,n\}, n = 1,2\ldots,$ where a *partition* of $\{1,2,\ldots,n\}$ is a finite union $A_1 \cup A_2 \cup \ldots A_n$ of disjoint subsets of $\{1,2,\ldots,n\}$. For a discussion of partitions see Andrews (1984).

Definition (*Bell numbers*): The *n*th Bell number B_n is the number of partitions of the positive integer *n*.

Let *n* be a positive integer and

$(k) \Rightarrow n =: \{k_1,\ldots,k_n$ non-negative integers such that $\sum_{jkj} = n\}$

Definition (n^{th} Bell polynomials B_n) (Andrews 1984):

$$B_n(x_1,\ldots,x_n) =: \sum_{(k)\Rightarrow n}(n!/k_{1!}\ldots k_{n!})(x_1/1!)^{k_1}(x_2/2!)^{k_2}\ldots(x_n/n!)^{k_n}$$

Between 1928 and 1949, Bell published nearly 30 papers on these special numbers and polynomials (Durst 2001).

Sidebar: A comment on Bell's influence on the field of combinatorics

(Excerpted from John Greenberg's appendix to Goodstein and Babbitt [2013]).

E. T. Bell would have been surprised to see where his real influence on mathematics lay, at least in recent times. The study of networks for simulating the central nervous system helped to motivate graph theory (a branch of combinatorics) and, in general, to stimulate a revival of combinatorics after World War II. Bell's work on what are now called Bell numbers and Bell polynomials was much cited in the literature of combinatorics in the 1950s and '60s. It struck two mathematicians as odd that it took so long for the followups to what they deemed Bell's "classic paper" (Bell 1934) on exponential polynomials to appear (Gould and Harper 1962). Gian-Carlo Rota acknowledged the widespread role that Bell's numbers played in a great many problems of enumeration and of probability (Rota 1964). The Bell polynomials also manifested themselves as an important feature of many combinatorial and statistical problems (Riordan 1958).

There is some irony that the revival of combinatorial analysis was brought about in large part by the advent of the programmable computer and the wholesale creation of branches of mathematics to minister to the associated computer science, for if mathematics was a science that stood on its own, as Bell seemed to think, his own work with the greatest impact was that which was pressed into the service of other sciences. If Bell abhorred applied mathematics, he nevertheless helped to open up rich new areas of research in applied mathematics in spite of himself!

Bell's theorem on multiplicative Diophantine equations

Definition: Let $A_i, B_i, I = 1,2,\ldots M$ be nonzero integers and $ai1, ai2, \ldots, aiK, bi1, bi2, \ldots, biL$ positive integers. Then a multiplicative Diophantine equation is a Diophantine equation of the form:

$$A_i\left(x_1\right)^{ai1}\left(x_2\right)^{ai2}\ldots\left(x_1\right)^{aiK} = B_i\left(y_1\right)^{bi1}\left(y_2\right)^{bi2}\ldots\left(y_L\right)^{biL}$$

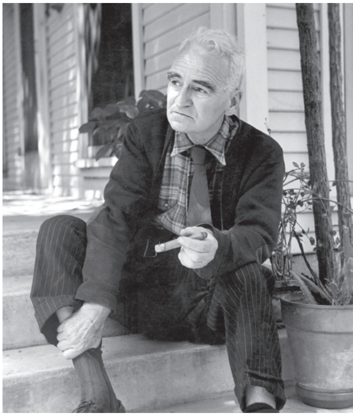
$I = 1,2,\ldots,M.$

Bell's Theorem (Bell 1933): All multiplicative Diophantine equations are completely solvable in integers.

Bell first reduced the problem to solving seven different and much simpler special cases, which he went on to solve. In a series of later papers, "Bell showed how to reduce a large variety of Diophantine equations to multiplicative form, from which complete solutions could then be found" (Durst 2001). According to Durst, Bell's work in this general area extended from 1928 to 1949 and included approximately 30 papers.

Bell's later years

Between 1940 and 1950, Bell published some 4–5 papers a year on number theory, history and biography, and other topics. In 1951, his final research paper, "Solution of a Functional Equation in the Multiplicative Theory of Numbers," appeared


Eric Temple Bell (1883-1960), ca. 1951.

(Bell 1951b). The legendary Hungarian mathematician Paul Erdős, known for solving intractable number-theory problems, reviewed it in *Mathematical Reviews*. He limited himself to treating Bell's solution to a specific functional equation in five lines without comment.

Bell owned a copy of the 1670 edition of Diophanatus's *Arithmetica*, which included Fermat's claim that he had proven what became known as Fermat's Last Theorem; but alas, the author cited a lack of space to show Fermat's proof. Shortly after his 65th birthday, Bell set out to discuss that omission and generally recount the history of this famous theorem; he began work on a book called *The Last Problem*. Published posthumously in 1961, it was, appropriately, Bell's last book.

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Although he wrote at length about the lives of other mathematicians, E. T. Bell glossed over many details of his own biography. Moreover, he systematically destroyed much of his professional correspondence before he retired. H. P. Robertson, his friend and colleague, was invited to write Bell's biographical memoir for the National Academy of Sciences, but he declined; and two of Bell's former students passed away before completing their biographical memoir on him. It took until 1993 for the first authoritative portrait of the Scottish-born mathematician to be published: Constance Reid's *The Search for E. T. Bell, Also Known as John Taine.*

Portions of an earlier version of this article—Goodstein, J. R., and D. Babbitt. 2013. E. T. Bell and mathematics at Caltech between the wars. *Notices of the American Mathematical Society* 60(6):686–698—were used here with permission.

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