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OF

# ELIAKIM HASTINGS MOORE

1862–1932

BY

G. A. BLISS and L. E. DICKSON

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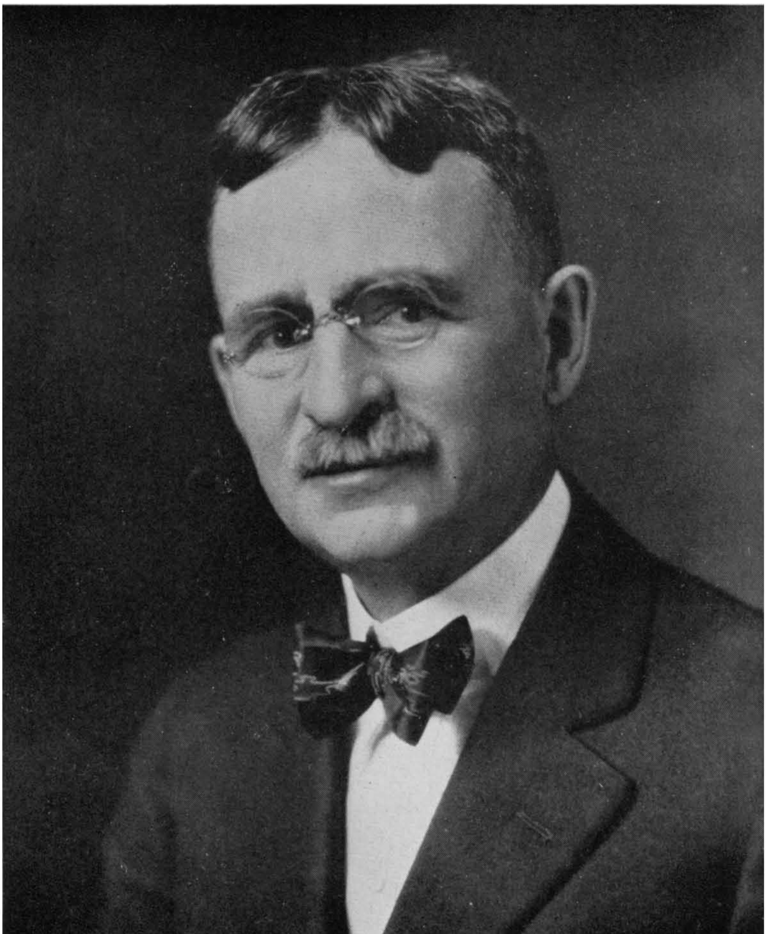


E H Moore

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The great development which has taken place in our American mathematical school during and since the last decade of the last century has been in large part due to the activities of a relatively small group of men whose names and devoted interest are well known to all of us. In our memories and our histories their achievements will be indelibly recorded with grateful appreciation and esteem. One of the leaders among these men, in enthusiasm and scholarship and clearness of vision for the future, was Eliakim Hastings Moore.

He was born in Marietta, Ohio, on January 26, 1862, and died on December 30, 1932, in Chicago, where he was professor and head of the department of mathematics at the University of Chicago. It is interesting to note that the environment in which Moore grew to manhood was a most suitable nursery for the distinction which he afterward attained in so great a measure. His grandfather was an earlier Eliakim Hastings Moore, a banker and treasurer of Ohio University at Athens, Ohio, a county officer and collector of internal revenue, and a Congressman. Eliakim Hastings the younger served as messenger in Congress during one summer vacation while his grandfather was there. His father was a Methodist minister, David Hastings Moore, and his mother was Julia Carpenter Moore of Athens. The family moved from place to place while E. H. Moore was young, as necessitated by the profession of his father, but a considerable part of his childhood was spent in Athens, where one

* The material in this biography is taken from two papers by G. A. Bliss entitled "Eliakim Hastings Moore" and "The Scientific Work of Eliakim Hastings Moore," Bulletin of the American Mathematical Society, vol. 39 (1933), pp. 831-838, and vol. 40 (1934), pp. 501-514. See also the biographical notes by L. E. Dickson, Science, vol. 77 (1933), pp. 79-80; H. E. Slaught, The American Mathematical Monthly, vol. 40 (1933), pp. 191-195; and G. A. Bliss, The University Record of the Unisity of Chicago, vol. 19 (1933), pp. 130-134.

of his playmates was Martha Morris Young, who was afterward to become his wife. His father, D. H. Moore, besides being a preacher, was successively a captain, major, and lieutenant colonel in the Civil War; president of Cincinnati Wesleyan College; an organizer and first chancellor of the University of Denver; editor of the *Western Christian Advocate;* and Bishop of the Methodist Episcopal Church in Shanghai with jurisdiction in China, Japan, and Korea. His was the distinguished career of a man much beloved. Our E. H. Moore, the son of D. H. Moore, was married in Columbus, Ohio, on June 21, 1892, to Martha Morris Young, who survives him. She is a sister of John Wesley Young, late professor of mathematics at Dartmouth College, the memories of whose friendship and achievements are cherished possessions of mathematicians in this country. Her father, William Henry Young of Athens, was a professor at Ohio University, a colonel in the Civil War, and the son of a Congressman. Mrs. Moore herself was before her marriage an instructor of Romance languages at the University of Ohio, and also at the University of Denver during the chancellorship of D. H. Moore. Shortly after their marriage the young couple went to live in Chicago where Mr. Moore had just been appointed professor and acting head of the department of mathematics in the new University of Chicago which opened its doors in the autumn of 1892. Professor and Mrs. Moore have one son, also named Eliakim Hastings Moore, who was graduated from the University of Chicago and who now lives in Texas.

While E. H. Moore was still in high school, Ormond Stone, director of the Cincinnati Observatory, secured him one summer in an emergency as an assistant. Professor Stone was afterward director of the Leander McCormick Observatory of the University of Virginia, and a founder of the Annals of Mathematics which began its career at Virginia, later moved to Harvard, and finally to Princeton. Though primarily an astronomer, Professor Stone had a high appreciation for mathematics, and he inspired his young assistant with a first interest in that science. This interest was later confirmed at Yale University which the student, Moore, had been persuaded to enter by two of his friends, Horace Taft and Sherman Thatcher. The former is the brother of the late President William Howard Taft, and the latter is a son of a professor at Yale. These two men were Moore's best friends in college, and life long friends thereafter. Curiously enough both of them founded famous schools for boys, one in Watertown, Connecticut, and the other in California. But the man at Yale who had the most profound influence upon E. H. Moore, and who first inspired in him the spirit of research, was Herbert Anson Newton, professor of mathematics and a scientist of distinction. That Moore responded ably to the personal encouragement of Newton, as well as that of Stone, is indicated by his career as an undergraduate. During his college course he took three prizes in mathematics and one each in Latin, English, and astronomy. In his junior year he won the "philosophical oration appointment" and second prize at "junior exhibition," and in his senior year he held the Foote Scholarship and was valedictorian of his class. His nickname was "Plus" Moore. He took his A.B. at Yale in 1883 and his Ph.D. in 1885. Professor Newton, deeply impressed with the ability of the young mathematician, financed for him a year of study at Göttingen and Berlin in return for a promise to pay at some future time.

We have been able to find only meager information concerning the year which Professor Moore spent in Germany. He went first to Göttingen, in the summer of 1885, where he studied the German language and prepared himself for the winter of 1885-6 in Berlin. The professors of mathematics most prominent in Göttingen at that time were Weber, Schwarz, and Klein. At Berlin, Weierstrass and Kronecker were lecturing. We know that Moore was received in friendly fashion and greatly influenced by these distinguished men. It seems that the work of Kronecker made the most lasting impression upon him, but in his habits of mathematical thought and in his later work there are many indications of influences which might be traced to Weierstrass and Klein. There is no doubt that the year abroad affected greatly Professor Moore's career as a scholar. It established his confidence in his ability to take an honorable place in the international as well as our national circle of mathematicians, acquainted him at first hand with the activities of European scientists, and established in him a respect and friendly interest for German scholarship which lasted throughout his life.

When Moore returned to the United States from his sojourn in Europe he entered at once upon his career as teacher, scholar, and independent investigator. His first position was an instructorship in the Academy at Northwestern University in 1886-7. For the next two years he was a tutor at Yale University. In 1889 he returned to Northwestern as assistant professor, and in 1891 he was advanced to an associate professorship. Meanwhile he had published four papers in the field of geometry, and one concerning elliptic functions, and his aggressive genius as a rising young scholar was recognized by President William R. Harper of the newly founded University of Chicago. When the University first opened in the autumn of 1892 Moore was appointed professor and acting head of the department of mathematics. In 1896, after four years of unusual success in organizing the new department, he was made its permanent head, and he held this position until his partial retirement from active service in 1931.

As a leader in his department, Professor Moore was devotedly unsparing of his own energies and remarkably successful. He persuaded President Harper to associate with him two unusually fine scholars, Oskar Bolza and Heinrich Maschke, both former students at Berlin and Ph.D.'s of the University of Göttingen. The three of them supplemented each other perfectly. Moore was brilliant and aggressive in his scholarship, Bolza rapid and thorough, and Maschke more deliberate but sagacious and one of the most delightful lecturers on geometry of all time. They early organized the Mathematical Club of the University of Chicago whose meetings are devoted to research papers, and which continues to meet bi-weekly to the present day. Those of us who were students in those early years remember well the tensely alert interest of these three men in the papers which they themselves and others read before the Club. They were enthusiasts devoted to the study of mathematics, and aggressively acquainted with the activities of mathematicians in a wide variety of domains. The speaker before the Club knew well that the excellencies of his paper would be fully appreciated, but also that its weaknesses would be discovered and thoroughly discussed. Mathematics, as accurate as our powers of logic permit us to make it, came first in the minds of these leaders in the youthful department at Chicago, but it was accompanied by a friendship for others having serious mathematical interests which many who experienced their encouragement will never forget. With no local precedent or history of any sort to guide them, Moore and Bolza and Maschke, with Moore as the guiding spirit, created a mathematical department which promptly took its place among the group of active centers from which have flowed the influences creative of our present American mathematical school. Their success is recorded not only in published papers, but also in the activities of their students, who are distributed widely in the colleges and universities of this country.

In the lecture room Professor Moore's methods defied most established rules of pedagogy. Such rules, indeed, meant nothing to him in the conduct of his advanced courses. He was absorbed in the mathematics under discussion to the exclusion of everything else, and neither clock time nor meal time brought the discussion to a close. His discourse ended when some instinct told him that his topic for the day was exhausted. Frequently he came to his class with ideas imperfectly developed, and he and his students studied through successfully or failed together in the study of some question in which he was at the moment interested. He was appreciative of rapid understanding, and sometimes impatient when comprehension came more slowly. No one could have been more surprised than he, or more gentle in his expressions of regret, when someone called to his attention the fact that feelings had been hurt by such impatience. It is easy to understand under these circumstances, however, that poor students often shunned his courses, and that good students whose principal interests were in other fields sometimes could not afford the time to take them. But it was a proud moment when one who was ambitious and interested found himself in the relatively small group of those who could stand the pace. It is no wonder that among the ablest mathematicians of our country at the present time those who drew their chief inspiration from Professor Moore are numerous. He was essentially a teacher of those who teach teachers.

Unless we pause to make a computation we often fail to comprehend the rapidity of spread and the magnitude of the influence of such a man.

There is not space here to trace the achievements of the men who were influenced primarily during their student years by Professor Moore. The list of those whose thesis work for the doctor's degree was done under his supervision is, however, a distinguished one. We give it here in the order in which the degrees were taken: L. E. Dickson, H. E. Slaught, D. N. Lehmer, W. Findlay, O. Veblen, T. E. McKinney, R. L. Moore, G. D. Birkhoff, N. J. Lennes, F. W. Owens, H. F. MacNeish, R. P. Baker, T. H. Hildebrandt, Anna J. Pell (Mrs. A. L. Wheeler), A. D. Pitcher, R. E. Root, E. W. Chittenden, M. G. Gaba, C. R. Dines, Mary E. Wells, A. R. Schweitzer, V. D. Gokhale, E. B. Zeisler, J. P. Ballantine, C. E. Van Horn, R. E. Wilson, M. H. Ingraham, R. W. Barnard, H. L. Smith, F. D. Perez.

Professor Moore's success as an educator was due to his profound interest in mathematics and his faculty for inspiring his colleagues, and especially the strongest graduate students, with some of his own enthusiasm. With students not so far advanced he was less successful. His own comprehension was so rapid, and his concentration on the mathematics at hand was so absorbing to him, that he found it hard to comprehend or await the slower development of understanding in the less experienced. But he appreciated the importance of the problems of elementary instruction and at times participated actively in their solution. Not many people remember that in 1897 he edited an arithmetic for use in elementary schools. In 1903-4 and following years he modified radically the methods of undergraduate instruction in mathematics at the University of Chicago, and he himself gave courses in beginning calculus. With characteristic independence he cast aside the text books and concentrated on fundamentals and their graphical interpretations. The courses were so-called laboratory courses, meeting two hours each day, and requiring no outside work from the students. It might be added parenthetically that, as with many such new plans, the amount of work required of the instructor was exceedingly great. The two hour period was the feature which later caused the abandonment of the plan because of the very practical difficulty in finding hours on schedules which would not interfere with the offerings of other departments. At the present time we are facing serious criticisms of the teaching of mathematics in colleges and high schools. If we are to find a satisfactory answer we must perhaps consider again the deletion of the irrelevant and concentration on fundamentals. The laboratory method too has distinct advantages. It has appealed to many, and has in one form or another been made a part of numerous new plans for the teaching of mathematics. In these educational experiments which Professor Moore undertook, as at every other stage of his leadership in his department, he had one permanent characteristic. He believed in the exercise of individuality in class room instruction, and he gave his colleagues unlimited freedom in the development of their class room methods. He expected and insisted on success, and he was always sympathetically interested in a new proposal or procedure, but so far as is known to us he never prescribed a textbook.

The foundations of Professor Moore's leadership lay undoubtedly in his scholarship. In this biography no adequate description of his investigations can be given. The reader will find an analysis of his more important research activities in the second of the papers to which reference was made above. In his earlier years he was a prolific writer, and his published papers promptly established him as a mathematician of resourcefulness and power. Two of the characteristic qualities of his research were accuracy and generality. He was a master of mathematical logic, and his originality in making one or more theories appear as special instances of a new and more general one was remarkable. We remember a number of meetings of the Mathematical Club of the University of Chicago at which this interest in generalization was characteristically exhibited. At one of them an economist was struggling with the old problem of the selection of a mean for the proper interpretation of certain statistical data. At the next meeting Professor Moore summarized the postulates implied in the paper of the economist, and exhibited the infinite totality of generalized means which they characterized. At another meeting Professor Bolza described some of the properties of a family of cycloid arches which are important for the brachistochrone problem of the calculus of variations. At the following meeting Professor Moore showed that the class of families of arches with the same properties is indeed a much more general one, as is now well recognized. His success in these and much more important generalizations, especially in the domain of integral equations, culminated in a theory which he called General Analysis and which became his principal interest. In 1906 when he lectured on this theory at the New Haven Colloquium of the American Mathematical Society, he was ahead of the times. In recent years, however, many mathematicians have continued his ideas or have encountered them in independent approaches from other standpoints. Professor Moore's enthusiasm for mathematical research never waned, but in his later years his interest in formal writing declined. This was due primarily, we think, to two reasons. In 1899 he became one of the chief editors of the *Transactions of the American Mathematical Society* and for eight years thereafter he devoted himself unstintingly to the affairs of the journal and of the Society. The value of this work to our mathematical community, then still in its youthful and formative stage, cannot be overestimated. But for Professor Moore himself it had the effect of decreasing markedly the number of his published papers. Later, after others had assumed the responsibilities which he had so long courageously shouldered, he adopted a logical symbolism, largely of his own creation, for the expression of his mathematical ideas to himself and his students. It was not well understood by mathematicians in general, and not well suited for publication in journals. In those days, when research assistants for mathematicians were almost unknown, the translation of his writings from his convenient symbolisms to conventional mathematical language was far less interesting to him than the continuation of his own investigations. The result has been that he has left in symbolic form a great legacy of unpublished research material concerning General Analysis.

It is too early to attempt a judgment of the significance for mathematicians in general of Professor Moore's notations. He was a specialist in symbolisms, every detail of which meant something to him. In thinking or lecturing about mathematics, others as well as himself have found his notations not only convenient but also a potent aid in the formulation and testing of sequences of logical steps. They are especially effective in the development of theories involving limiting processes. It is true that the important things in mathematics are ideas rather than the symbols by means of which we represent them, but it is evident also that the structure of our science as we know it today would be impossible without the increasingly convenient notations which mathematicians through the ages have successively developed. That Professor Moore was fully conscious of this, and that he regarded notational problems as among the most important and difficult ones which mathematicians have to face, is clearly indicated by his correspondence with the late Professor Florian Cajori in 1919. It was their exchange of letters at that time which led to the preparation and publication in 1928 and 1929 of Cajori's two volumes on *The History of Mathematical Notations.*

It was to be expected that a man so highly regarded as a scientist should become a leader in his university and in the associations of workers in his field. Professor Moore was one of the youngest, but also one of the most spirited, of the notable group of scholars who in the nineties of the last century first shaped the character of the new University of Chicago and gave it great distinction. From the opening day of the University he devoted himself unselfishly to its interests, and his counsel through the years had great influence. At all times he stood unequivocally for the highest ideals of scholarship. His services to the University were signalized in 1929 by the establishment of the Eliakim Hastings Moore Distinguished Service Professorship, one among the few of these professorships which have been named in honor of members of the faculty of the University. The first and present incumbent is Professor Leonard Eugene Dickson. Professor Moore was a moving spirit in the organization of the scientific congress at the World's Columbian Exposition of 1893, and in the first colloquium of American mathematicians held shortly thereafter in Evanston with Klein as the principal speaker. He was influential in the transformation of the local New York Mathematical Society into the American Mathematical Society in 1894, and in the foundation of the first so-called section of the Society whose meetings were held in or near Chicago and of which he was the first presiding officer in 1897. The formation of the Chicago Section was an outgrowth of the Evanston colloquium. After that meeting a number of mathematicians from universities in and near Chicago occasionally met informally for the exchange of mathematical ideas. After the organization of the American Mathematical Society they applied for and were granted recognition as a section of the Society. It was the success of this first section which led to the establishment, in various parts of the country, of other similar meeting places which have added greatly to the influence and value of the Society. Professor Moore was vice-president of the Society from 1898 to 1900, and president from 1900 to 1902. In 1921 he was president of the American Association for the Advancement of Science. In 1899 he and other aggressive members induced the Society to found the *Transactions of the American Mathematical Society*, now our leading mathematical journal. The first editors were E. H. Moore, E. W. Brown of Yale, and T. S. Fiske of Columbia. These men set standards of editorial supervision which have endured to this day. Professor Moore retired from his editorship in 1907. From 1908 to 1932 he was a non-resident member of the council of the Circolo Matematico di Palermo and of the editorial board of its *Rendiconti*. From 1914 to 1929 he was the chairman of the editorial board of the University of Chicago Science Series. Nineteen volumes were published in the *Series* during that period, two of them, by H. F. Blichfeldt and L. E. Dickson, in the domain of mathematics. From 1915 to 1920 Professor Moore was a member of the editorial board of the *Proceedings of the National Academy of Sciences*. In 1916, by his advice and encouragement, he gave great assistance to Professor H. E. Slaught, who was a moving spirit in the formation of the Mathematical Association of America. In the decades preceding 1890 research scholars in mathematics in America were few and scattered, with limited opportunities for scientific intercourse. At the present time we have a well-populated and aggressive American mathematical school, with frequent opportunities for meetings, one of the world's great centers for the encouragement of scientific genius. From the record of Professor Moore's activities described above, it is clear that at every important stage in the development of this school he was one of the progressive and influential leaders.

That the distinction of Professor Moore's services to science and education was recognized in other universities as well as his own is indicated by the honors conferred upon him. He received an honorary Ph.D. from the University of Göttingen in 1899, and an LL.D. from Wisconsin in 1904. Since that time he has been awarded honorary doctorates of science or mathematics by Yale, Clark, Toronto, Kansas, and Northwestern. Besides his memberships in American, English, German, and Italian mathematical societies, he was a member of the American Academy of Arts and Sciences, the American Philosophical Society, and the National Academy of Sciences. Two funds have been established in his honor. The first is held by the American Mathematical Society for the purpose of assisting in the publication of his research and for the establishment of a permanent memorial to him in the activities of the Society. The second has been expended for a portrait of him which hangs in Bernard Albert Eckhart Hall for the mathematical sciences at the University of Chicago. The interest in these funds among the friends and admirers of Professor Moore was a remarkable tribute to him scientifically and personally.

The activities too concisely enumerated in the preceding paragraphs were the external evidences of a remarkable personality, a personality beyond the power of the writers of these pages adequately to describe. Professor Moore believed in mathematics, and his life was an unselfish and vigorous expression of his confidence in the importance of the opportunity of studying and teaching his chosen science, not only for himself but also for others who might have the interest and ability. He was sometimes misunderstood when he was impetuous or impatient, but his impatience was rarely personal. It was due almost always to the fact that someone was not understanding mathematics, and that someone might be either another person or himself. In the latter case he was likely to be for the moment unusually restless and irritable. In all of his activities he sought unceasingly for the truth, and for the words or symbols which might express truth accurately. He had at times a curious hesitation in his speech, characteristic of him, but unaccountable to those who recognized the unusual agility of his mind but who did not know him well. He would hesitate or stop completely in the midst of a sentence, searching among the wealth of words which presented themselves that particular one which would precisely express his meaning, just as in his mathematics he sought always the precisely suggestive symbol. In times of stress his patience with his colleagues was remarkable, and his friendship for them at all times was immovable. He believed in individuality and encouraged independence in their teaching, and he protected them in their research, often at great cost to himself. Outside, as well as in his own department, his enthusiasm, his scientific integrity, and his deep insight established an influence which will extend wherever mathematics is studied and truth is honored, beyond the confines of his country or his day.

## THE SCIENTIFIC WORK OF ELIAKIM HASTINGS MOORE

The preceding pages of this memoir are devoted to a biographical sketch of Eliakim Hastings Moore. No account of his life can approximate completeness, however, without a more detailed description of his scientfic activities than was given there. His enthusiasm for mathematical research was a dominant one, more characteristic of him than any other, in spite of the fact that he had many administrative and editorial responsibilities which often interfered seriously with his scientific work. He had a catholic interest in all domains of mathematics and a breadth of knowledge which was remarkable. There have been few men with so great an appreciation of the mathematical efforts of others, or so well qualified to discuss them in many different fields, qualities which were an important part of his insignia of leadership. If there were two characteristics of his research which could be distinguished above others, one could say that they would be rigor and generality. He strove for precision in thought and language at a time when vagueness and uncertainty were common in mathematical literature, and he profoundly influenced both students and colleagues in this respect by his teaching and example. He was furthermore among the very first to recognize the possibility and importance of the great generality in analysis which is now sought by many writers.

Moore was a prolific thinker, though not throughout his lifetime a prolific writer. His papers, as given in the bibliography at the end of this article, fall roughly into the groups indicated in the following table which lists the numbers of the items in the bibliography belonging to each field and the dates of the first and last papers in each group:

I. Geometry; 1-4, 28, 41, 43-44, 47, 63; 1885-1913.
II. Groups, numbers, algebra; 6-9, 12, 13, 15-18, 20-27, 29, 32, 33, 42, 46, 48, 53, 60, 68, 69, 71; 1892-1922.
III. Theory of functions; 5, 10, 11, 14, 19, 30, 31, 35-40, 52, 59, 67, 73, 74; 1890-1926.
IV. Integral equations, general analysis; 50, 51, 54, 56, 58, 61, 62, 64-66, 70, 75; 1906-1922.
V. Miscellaneous; 34, 45, 49, 55, 57, 72; 1900-1922.

The table indicates fairly well, we think, the sequence of his major interests, though it does not represent adequately the relative enthusiasms with which he pursued them. The domains suggested in the second and fourth entries were the ones to which he gave most thought. His studies in algebra and the theory of groups fell in the period of his greatest activity as a writer, while integral equations and general analysis were his absorbing interest during the latter part of his life when he published least. For general analysis, in particular, he never lost his enthusiasm. He continued his speculations in that field into the last year of his life, as long as his strength permitted.

The comments on Moore's papers in these pages must necessarily be brief. For a more complete synopsis the reader is referred again to the second of the papers to which reference is made on the first page of this memoir. The papers on geometry fall into two groups, an early one concerned with algebraic geometry, which was Moore's first mathematical interest, and a later series of three papers on postulational foundations. The qualities exhibited by Moore in the earlier of these two groups of papers were in many ways characteristic of his research throughout his life. The theory of linear systems of plane curves, which he freely used, was at that time a central interest in algebraic geometry, as indicated, for example, in numerous papers which appeared between 1884 and 1887 in the *Palermo Rendiconti*. The skill which he showed in handling such systems, and the elegance of his results, are indicative of unusual power in so young a man, and the problems which he studied were fundamental ones for the algebraic geometry of that period. The papers on postulational foundations were inspired by Hilbert's book on the foundations of geometry of 1899 which attracted the attention of Moore and his students to postulational methods, including earlier work of Pasch and Peano as well as that of Hilbert. Moore analysed skillfully questions which had arisen concerning the independence of Hilbert's axioms, and gave a new formulation of a system of axioms for n-dimensional geometry, using points only as undefined elements instead of the points, lines, and planes of Hilbert in the 3-dimensional case.

Moore early became interested in the theory of abstract groups, one of the fields of research in which he was at various times most deeply engaged. An abstract of his first paper in this field, number (6) in the bibliography below, appeared in 1892, but his first published paper was the paper (8, 16) which he presented at the mathematical congress held at the World's Columbian Exposition in Chicago in 1893. It contained a generalization of the modular group, a statement and proof for the first time of the interesting and important theorem which says that every finite algebraic field is a Galois field, and a characterization of a doubly-infinite system of so-called simple groups, only a few of which had been known before. Although Moore's algebraic interests centered largely in groups and their applications to the theory of equations, he was nevertheless actively interested at times in a variety of other algebraic questions, notably the theory of numbers and modular systems. In each of these domains he made interesting and important contributions.

Moore's early interest in the theory of functions was indicated by the relatively simple papers (5) of 1890 and (11) of 1895. A much more important contribution, showing perhaps for the first time his full power in analysis, was his memoir (19) of 1897 concerning transcendentally transcendental functions. It is a model of clarity and elegance, and gives evidence of his increasing interest and ingenuity in mathematical generalizations. In 1900 mathematicians in this country were greatly interested in an extension by Goursat of a fundamental theorem in the theory of functions, and in the space-filling curves of Peano and Hilbert. Moore's papers (31) and (30) of that year illuminated these topics with an accuracy and clarity characteristic of him. His papers (35, 36, 37) of 1901 on improper definite integrals may with justice be regarded as the climax of the literature on this subject preceding the development of the later and more effective integration theories of Borel and Lebesgue.

But the domains which captured Moore's interest most effectively were the theories of integral equations and general analysis. In the years following 1900 the fundamental papers of Fredholm and Hilbert on integral equations attracted wide attention. Moore saw that the equations which they studied, as well as corresponding and more elementary ones well known in algebra, must be special instances of a much more general linear equation, and he set about the construction of a general theory which should include them all. His guiding principle, as often stated, was that "the existence of analogies between central features of various theories implies the existence of a general abstract theory which includes the particular theories and unifies them with respect to these central features." This principle was the dominant note of his Colloquium lectures (56) delivered before the American Mathematical Society at Yale University in 1906. The lectures were published in 1910, after the appearance of his related paper (54) presented at the International Congress of Mathematicians in Rome in 1909. In these papers and two later ones (58, 61) Moore gave in essential outline his first theory of general analysis and his generalization of preceding theories of linear equations. His attack was highly postulational, and original especially in the fact that the postulates applied to classes of functions rather than to individual ones. He was able to secure for his general theory most of the results which are of interest in the more special cases, but some of them eluded him. The attempts which he made in order to complete the theory in these respects led to such complexities that he finally turned from his first method to a second more constructive theory of similar character but with a much simpler basis. The results which he attained have appeared in part in his memoirs. They are now being revised by Professor R. W. Barnard and Dr. Max Coral and are being published by the American Philosophical Society.

The titles of the papers in the group designated as "miscellaneous" in the table are for the most part self-explanatory. Three of these should be mentioned more explicitly, however, Moore's addresses (45, 55, 72) as retiring president of the American Mathematical Society in 1902, at the 20th anniversary of Clark University in 1909, and as retiring president of the American Association for the Advancement of Science in 1922. The first contains in its earlier pages an illuminating description of Moore's conception of the logical structures of pure and applied mathematical sciences, the latter part being devoted to a discussion of the pedagogical methods by means of which one might hope to establish such concepts clearly in the minds of students in our schools, colleges and universities. It was written at a time when Moore himself was greatly interested in a laboratory method of instruction for college students of mathematics, and at the height of the so-called Perry movement in England which aroused great interest and discussion among those responsible for instruction in the mathematical sciences in our own country.

The second paper (55) was apparently unpublished and we have as yet found no manuscript. But there is a somewhat informally written paper with nearly the same title and date in the archives of the Department of Mathematics at the University of Chicago. There seems little doubt that it contains the material of the Clark address. It contains a non-technical description of the work of Pasch, Peano, and Hilbert on foundations of geometry, and of the contributions of Cantor, Russell, and Zermelo to the theory of classes.

His address (72) as retiring president of the American Association was also unpublished, but a typewritten copy is extant. It is a description of the historical development of the number systems of mathematics with the purpose of establishing the interesting thesis that mathematical theories, though well recognized as highly deductive in their ultimately sophisticated forms, are nevertheless the products of inductive developments similar to those well known in the laboratory sciences.

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The abbreviations used are explained at the end of the bibliography. Items marked with an asterisk are titles of more significant abstracts or unpublished addresses.


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7. *Concerning triple systems*, Annalen, vol. 43 (1893), pp. 271-285.

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LIST OF ABBREVIATIONS

Annalen = Mathematische Annalen
Annals = Annals of Mathematics
Bulletin = Bulletin of the American Mathematical Society
Journal = American Journal of Mathematics
Transactions = Transactions of the American Mathematical Society