Gian-Carlo Rota

1932–1999

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Joseph P. S. Kung

NATIONAL ACADEMY OF SCIENCES

GIAN-CARLO **ROTA**

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By Joseph P. S. Kung

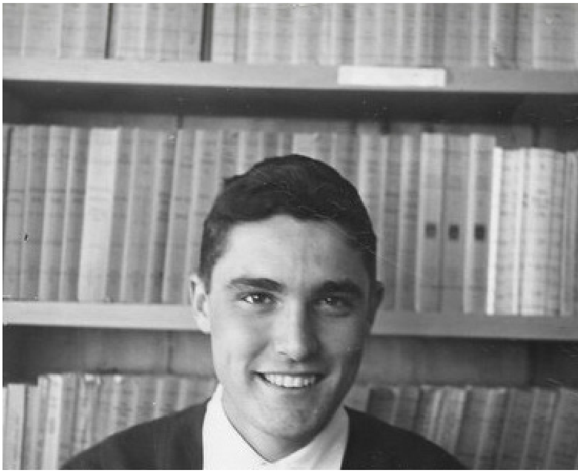
Gian-Carlo Rota was born, in Vigevano, Italy, a small town on the Ticino River about 40 kilometers from Milan. Growing up in a well-to-do family responsive to his intelligence and curiosity, Gian-Carlo—the son of Giovanni Rota (a civil engineer and architect) and Luigia Facoetti Rota—would likely have enjoyed an idyllic childhood. But this was not to be under the Fascist government of Mussolini, especially as Giovanni Rota was an open and active opponent of the Fascist regime.

In July 1943, Giovanni moved the family for safety to the mountain village of Rimella, in the province of Vercelli, but remained in Vigevano. When in September he managed to avoid arrest but was put on a "blacklist" of wanted subversives, Giovanni temporarily joined his family but soon had to escape into Switzerland, leaving the family to spend the remainder of the war at the farm of an uncle in Roncallo, by Lake Orta. During this time, Gian-Carlo's education was maintained, first at home with his father and a priest, Don Giuseppe, and later at a high school in Orta. To attend the school the young Rota had to cross the lake in a boat, but because the Partisans and the Fascists occupied opposite shores, this was done at considerable risk, sometimes under fire. In contrast to events in his later life, on which he wrote candidly and at length, Rota never discussed this wartime period in print and remained reticent in private. Nonetheless, he kept a reproduction of the blacklist framed in his living room.[1]

The family moved to Quito, Ecuador, in 1946. Speaking not a word of Spanish or English when he first arrived there, Rota attended high school at the Colegio Americano de Quito (1947–1950). The principal of the school was a Princeton University graduate,

1. Rota's younger sister, Ester Rota Gasperoni, gave an account of this period of his life in her article "Gianco my brother' in E. Damiani, O. D'Antona, O. Marra, F. Palombi (editors), *From Combinatorics to Philosophy*, New York: Springer US, 2009. The events in this period form the factual core of her novels *Orage sur le lac* and *L'arbre de capulíes*, Paris: L'école des loisir, 1995 and 1996.

and following his recommendation, Rota applied to Princeton and was accepted. As he wryly recalled, "actually I received a small scholarship."



Gian-Carlo Rota, 1943.
(Photograph from Ester Rota Gasperoni, used by permission.)

Rota later wrote that he was "strongly influenced at Princeton by William Feller, D. C. Spencer, A. W. Tucker, Solomon Lefschetz, Emil Artin, and Salomon Bochner," but it was not their specific mathematical skills or interests that he assimilated. Indeed, it was at Princeton that he rebelled against the conventional wisdom that "a piece of mathematics was the more highly thought of, the closer it came to Germanic number theory." And it was there as well that he acquired his distaste for "personality cults" to which the mathematical community is so irrationally prone. Rota wrote an excruciatingly honest essay about this experience in his 1989 article "Fine Hall in Its Golden Age" (Rota 1997, Chapter 1).

Graduating from Princeton summa cum laude in 1953, he chose Yale University for graduate work, at the forceful suggestion of Tucker ("You are not going to Chicago..."). There Rota joined the group of graduate students in functional analysis laboring on what would become the field's canonical work, often referred to as "Dunford-Schwartz."[2] Thus did he find the area of mathematics in which he felt most "at home," and there would always be some functional analysis in all of his mathematical work.

Rota did his doctoral research under the direction of Jacob T. Schwartz and defended his thesis, "Extension Theory of Differential Operators," in 1956. His account of the time he spent at Yale ("Light Shadows," Chapter 2 of Rota 1997) was uncharacteristically mellow, even nostalgic: "In New Haven...the light shadows are softened in a silky white haze, which encloses the colleges in a cozy aura of unreality."

2 Dunford, N, and J. T. Schwartz. 1958; 1963. *Linear Operators I. General Theory; Linear Operators II. Spectral Theory. Self-adjoint operators in Hilbert Space.* New York: Interscience. In addition to being part of a team that checked the exercises, Rota was acknowledged as the author of the final two sections of Chapter XIII.

After Yale, Rota was a temporary member of the Courant Institute of Mathematical Sciences (1956–1957) and a Benjamin Pierce Instructor at Harvard University (1957–59). He joined the faculty of the Massachusetts Institute of Technology in 1959 and, except for an unhappy interlude from 1965 to 1967 at Rockefeller University, remained at M.I.T. for the rest of his career, becoming professor of mathematics in 1967 and professor of applied mathematics and philosophy in 1975. He won the Killian Faculty Achievement Award in 1996.

Rota prepared his lectures by writing a draft, in a spidery longhand, in quarto-size leather-bound notebooks—his signature black books. Except for philosophy lectures, in which he followed the custom of "reading a paper," the draft was never completed.

Although M.I.T. lacked the discreet charm of the Ivy League universities, in many ways it was the ideal institution for Rota. Students at the Institute, mostly practical minded, appreciated that when Rota taught he got directly to the bottom line and never confused, in his words, "telling the truth [with] giving a logically correct presentation." His probability course (18.313) was not only popular with mathematics majors but also attracted students from all disciplines. Rota made the astute choice to be in the applied mathematics section, thereby insulating himself from the pettiness among the pure mathematicians, which he described in merciless but amusing detail in Rota 1997, Chapter 3.

Rota was able to build at M.I.T. an outstanding group in combinatorics. In addition, as the Institute required its students to take humanities classes, he had a ready-made audience for his philosophy courses. There was a classic academic dispute (which he mischievously kindled) about whether his students could take humanities credits and what "number" his course should have, parallel to the question, settled in 1975, whether he should be a professor of natural philosophy or of philosophy.

Rota used his philosophy courses as a testing ground for his thoughts on phenomenology. As those he taught were not professional philosophers in training or disciples, his philosophy was always "grounded"—and in many senses, including the electrical, as his students joked. Some of his philosophical papers were collected in Rota 1997. Moreover, Fabrizio Palombi has written a lucid account of Rota's achievements as a philosopher.[3]

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3 Palombi, F. 2011. *The star and the whole.* Boca Raton FL: CRC Press.

Gian-Carlo Rota, 1949.
(Photograph from Ester Rota Gasperoni, used by permission.)

His aim was to use it as a prompt, to make sure that the lecture followed the intended dramatic trajectory, with the right examples at the right time, leading to the one main point, all the while taking care that everyone in the audience had something to take home and that he never ran overtime. Rota hardly ever gave a major lecture from an earlier draft (so that one could find various versions of a lecture in his notebooks); instead, he used successive lectures on a given subject to refine his ideas. When he felt that his thoughts were ready for print, he would write the paper, almost always with coauthors to correct the many technical errors and typos. What Rota wrote of lectures by William Feller would also be true of his own: "During a Feller lecture, the hearer was made to feel privy to some wondrous secret, one that often vanished by magic as he walked out… . Like many great teachers, Feller was a bit of a con man" (Rota 1997, Chapter 2).

Among his numerous honors, Rota was a senior fellow at the Los Alamos National Laboratory from 1971 on, spending several weeks there every year. He was elected to the National Academy of Sciences in 1982. He gave many lectures, including the Hedrick Lectures of the Mathematical Association of America (Toronto, 1967), invited addresses at the International Congress of Mathematicians (Nice, 1970 and Helsinki, 1978), the Hardy Lectures of the London Mathematical Society (1972), the Lezioni Lincee (Rome, 1979 and Pisa, 1986), the Lezioni Leonardesche (Milan, 1990), the Colloquium Lectures of the American Mathematical Society (1997, Baltimore), and the Lezioni Fubini (Turin, 1998).

Gian-Carlo Rota, 1955.
(Photograph from Ester Rota Gasperoni, used by permission.)

Rota received honorary degrees from the Université de Strasbourg (1984), the Università degli Studi dell'Aquila (1990), the Università di Bologna (1996), and the Polytechnic University in Brooklyn, NY (1997). In 1988, the American Mathematical Society awarded him a Steele Prize for his fundamental paper "Foundations of Combinatorial Theory. I. Theory of Möbius Functions." The National Security Agency awarded him the Medal for Distinguished Service in 1992, though what he did to deserve the honor was classified.

According to the Mathematical Genealogy Project, Rota had 49 doctoral students—an underestimate, as this count left out many of his unofficial Italian students. Each student had a different experience, but a common element was Rota's insistence that the student learn to "read and write." That is, they were to read a paper for its ideas, not the details, and to write so that the ideas were clear—for example, by including an informative introduction.

Rota passed away unexpectedly, shortly before his 67th birthday, on April 18, 1999, in his apartment in Cambridge. To use a cliché, he died in harness; among the duties on his calendar at the time, he was scheduled to give the Emil Grosswald Lecture the next day at Temple University in Philadelphia.

During what turned out to be his later years, Rota was haunted by the fear that, like two of his closest friends—Stanislaw Ulam and Nick Metropolis—he would succumb to the infirmities of old age—"go gaga", as he put it. He wrote that "the etiquette of old age does not seem to have been written up, and we have to learn it the hard way" (Rota 1997, p. 203). He was spared that lesson.

Rota's mathematical work may be divided into three periods. In the first period, from the late 1950s through the 1960s, his interests centered on operator theory, differential equations, and ergodic theory. His first paper (Rota 1958), based on his thesis, was about "the problem of determining sufficient conditions on non-self-adjoint ordinary differential operators, which will insure that the operators be 'spectral' in the sense of Dunford." He commented that in this paper, "there is perhaps what is historically the first occurrence of an 'invariance of index' theorem."A later paper (Rota 1961) gave natural examples of singular operators (such as Riemann differential operators) on L^p-spaces whose essential spectra depend on p and, more unexpectedly, change from discrete to continuous as p changes. Other papers from this period were on models of operators and invariant subspaces, positive operators, Reynold operators, Frobenius reciprocity and Abel limits in ergodic theory, the joint spectral radius, an inequality among derivatives of a function, and other topics.

It was in this period that Rota wrote what he considered his two best research papers. The earlier one (Rota 1962) showed that if P_i is a sequence of doubly stochastic operators on $L^1(\mu)$ over a probability space, and f is a function in $L^p(\mu)$, with $p > 1$, then the sequence

$$P_1 P_2 \cdots P_n P_n{}^* P_{n-1}{}^* \cdots P_1{}^* f$$

converges almost everywhere. His proof was elegant, based on the analogy that "doubly stochastic operators are related to conditional expectation operators in much the same way as contraction operators in Hilbert spaces are related to orthogonal projections." He first constructed an abstract L-space, obtained a dilation of the sequence, and finished by using a martingale convergence theorem. This paper's reception was disappointing, however, because the literature was entangled with previously known special cases and an independent discovery of the result itself using different methods. It is not surprising that Rota gave up work on a continuous version and moved to combinatorics soon afterward.

The later paper (Rota 1969), arbitrarily divided into a pair, linked his first two periods. In Rota 1995, he described how, starting from analysis with averaging and Reynolds operators, he came to the insight that it is easier to study Baxter operators without the analytically motivated assumption that they are idempotent. He discovered the surprising and deep property that the free Baxter algebra is isomorphic to the algebra of symmetric functions with a shift operator; this allowed him to prove identities in probability, q-series, and other areas by proving *one* overarching identity involving symmetric functions. In this way, he came to the striking conclusion that "algebra dictates the analysis." Baxter algebras also illuminate the combinatorics of the function $x^+ = \max\{0, x\}$. Rota loved identities, and his favorite, for real numbers x and y,

$$\{(x^+ + y)^+, (x + y^+)^+\} = \{(x + y)^+, x^+ + y^+\}$$

as multisets, underlies a Baxter algebra occurring in fluctuation theory.

In the early 1960s, the start of his second period, Rota's interest shifted to combinatorics, a subject he was fond of describing as "putting balls into boxes." He became "one of the many who unknotted themselves from the tentacles of the Continuum and joined the then Rebel Army of the Discrete." In 1963 he taught his first combinatorics course, and in 1964 he published the paper "Foundations I," for which he received the Steele Prize. In the first part of his response to receiving the prize, he spoke about his realization, on reading the work of Philip Hall and Louis Weisner, that the Möbius function, defined as a two-variable function on a partially ordered set, unified much of enumerative combinatorics.

Rota then proceeded to write an "expository paper." As he commented in another context, "it is the paper by which [he is] most likely to be remembered." This paper literally opened a new world, revealing deep connections with algebraic topology, algebraic geometry, commutative algebra, and representation theory. It also inspired the development of systematic methods in combinatorics. Two important milestones were his 1967 Hedrick lectures and the National Science Foundation's 1970 Summer Research Institute in Combinatorial Geometry—where he served as principal lecturer —at Bowdoin College. His slogan during that event, that "combinatorics do not need more theorems, but more theories" inspired a generation of combinatorial theorists.

The focus of the Steele Prize on a single paper oversimplified Rota's work in combinatorics. In addition to Möbius functions, he worked on the theory of generating function, the umbral calculus and its categorical counterpart, the theory of species, Witt vectors and necklaces, counting in finite vector spaces as a way to interpret q-identities, cubical lattices, significance arithmetic, cyclic derivatives, and many other topics. His abstract (Rota 1976) on coalgebras, or "the combinatorics of assemblage," initiated the area of combinatorial Hopf algebras in mathematics and theoretical physics. Rota's proposed research program, to develop a combinatorial theory of the representation of the symmetric group, in the spirit of Alfred Young, remains unrealized.

The second part of Rota's response to receiving the Steele Prize involved critical problems. Starting from the remark by Garrett Birkhoff that the chromatic polynomial for graphs can be computed by Möbius inversion on the lattice of contractions of the graph, he saw that the problem of coloring a graph is "only one instance (which, by chance, happened to be historically the first) of a wide class of combinatorial problems, old and new, all of them presenting difficulties of the same kind."

In contrast to the prominence given to them in his response to the Steele prize, Rota wrote only twice on critical problems, as in Crapo and Rota 1970, Chapter 16. However, critical problems have had enormous influence, along with his many conjectures, in matroid theory. For example, one conjecture—that there exists a finite number of obstructions or forbidden minors for representability of a matroid over a given finite field—led to the "Matroid Minors Project." Another conjecture, that the coefficients of the characteristic polynomial of a matroid form a log-concave sequence—led to a deep infusion of algebraic geometry methods into matroid theory.

One might also mention Rota's observation, in the 1967 Hedrick lectures, that the "ring in graph theory" of W. T. Tutte is a Grothendieck ring of a category of graphs under the deletion-contraction recursion. This statement gave a new direction to the study of what are now called Tutte polynomials of graphs and matroids.

Rota studied matroids through their geometric lattices of flats, the approach initiated by Birkhoff. Alone among the older mathematicians with whom Rota had close contact, Birkhoff was never the subject of a Rota sketch. Perhaps the reason was that he did not want fuel added to "the visceral and widespread hatred of lattice theory from around 1940 to 1979," a dislike partly attributable to Birkhoff 's personality. But I would hazard that the hidden reason was that Birkhoff, with similar interests, was the only mathematician for whom Rota felt any anxiety of influence.

At the 1970 Bowdoin summer institute, Rota startled the participants by beginning his lectures on combinatorial geometries (or matroids) with symmetrization operators, by following up with an extended exposition of classical (or 19th-century) invariant theory, and ending with an argument that matroid theory is a combinatorial abstraction of classical invariant theory. Rota treated invariant theory from several perspectives. He defined the letter-place algebra to be the algebra of polynomials in the variables $(x_i|u_j)$, with each variable standing for a generic coordinate or an inner product between a vector and dual vector. Letter-place algebras provide the syntax, with the grammar given by the straightening formula—a generalization of the Laplace expansion for determinants. The straightening formula yields a proof of the fundamental theorems of classical invariant theory, which hold over any infinite field; it is one of very few results in algebra that are truly characteristic-free. The semantics are provided by Cayley or Peano spaces—vector spaces with a determinant or volume element. Operations on subspaces are performed with meets and joins of extensors. Cayley spaces are more intuitive and computationally friendly versions of Grassmann's exterior algebras. In later work, "superalgebras" were defined by giving the variables in letter-place algebras positive and negative signs (making them analogs of distinguishable and indistinguishable particles). Much of Rota's work in this area can be found in (Doubilet, Rota, and Stein 1974) and (Grosshans, Rota, and Stein 1987).

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In his third period, Rota sought a synthesis of the discrete and the continuous. He did this partly by making precise his intuition that "convexity is continuous combinatorics." His first paper (Rota 1971) on convexity explored how the Euler characteristic—the number of connected components—fused combinatorics with topology. From this, he developed a theory of finitely additive measures, or valuations, on distributive lattices, especially lattices of polyconvex sets. By means of invariant measures, Rota found a continuous version of Sperner's theorem and hoped to do the same for other results in extremal set theory. He and Daniel Klain pursued this goal in their book (Klain and Rota 1997), which was a model of exposition and vision. The Fubini lectures (Rota 2001), given in 1998, afforded Rota the opportunity to crystallize his thoughts on the foundations of probability. Among the new research areas he proposed was the use of (universal) algebra to clarify the structure of stochastic processes; this and related ideas await receptive readers. Rota's third period ended abruptly.

The 1960s was the decade when most of the first-generation specialized mathematics journals were founded, and Rota realized that combinatorics too could not flourish unless journals were dedicated to it. Working with Edwin Beschler at Academic Press, he created the *Journal of Combinatorial Theory* in 1966 and served on the editorial board from inception to when the journal split into two parts in 1970 (whereupon he continued serving Series A). But undoubtedly, Rota's greatest editorial achievement was *Advances in Mathematics*, which he took over in 1967 and transformed, in a remarkably short time, into the most interesting and open-minded journal in mathematics.[4]

In 1980, he founded *Advances in Applied Mathematics*, partly to relieve the backlog of *Advances in Mathematics*. Rota also began two book series, *Mathematicians of Our Time* (later renamed *Contemporary Mathematicians*) and *Encyclopedia of Mathematics and Its Applications.* One should note that these activities were by no means Rota's first experiences in publishing. He had founded a school newspaper *Sentite anche noi (Listen to us)* in Italy just after World War II and in 1949 served as editor-in-chief of *School Views*, the newspaper of the American School in Quito.

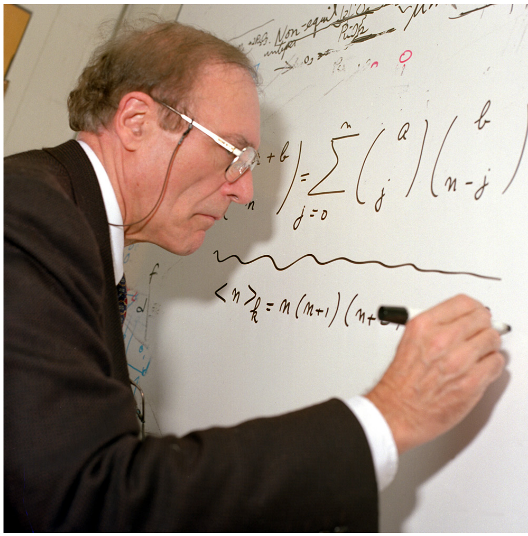
For Rota, journals and papers served many purposes (not least jobs and promotions), but he was motivated by their role in communicating knowledge and ideas, and he was sensitive to the needs of authors. For example, Rota realized that nothing can be more discouraging to a young author writing a first paper than a referee's report, obtained (typically) after an excessive delay, urging that the paper be rewritten in the reviewer's own way, using his or her notation and terminology. Rota was careful in choosing referees for younger authors, often reading the paper himself and making a decision on the spot. I will always remember that when the paper based on my thesis appeared in *Advances*, he said, with his boyish grin, that I "have made the *salon des refusés*." (The *salon* was the counter-exhibition, organized by impressionist painters in Paris in 1863 to display works rejected by the official *salon*.)

Any writer of a biography of Rota must confront his demand that "the greater a mathematician, the more important it is to bring out the contradictions in his or her personality" (Rota 1997, Chapter 2). It is fortunate that he himself had provided examples of how this can be done. I have tried (in sincere homage) to use as many of his own words as possible.

I end this memoir by comparing Rota to two great Italian poets, Ariosto and Dante. To me Rota's mathematics was more akin to the poetry of Ariosto than to that of Dante. For Ariosto, there were many heavens, not just one *paradiso*, and so too for Rota. He chose the title of his series of papers on combinatorial theory as "Foundations," in the plural, with the intellectual humility and generosity of the greatest thinkers. For him, there was

4 *Advances in Mathematics* began in 1961, under the editorship of Herbert Busemann, as a serial publication for survey papers with no fixed publication schedule. Volume I spanned the years 1961 to 1965. Rota took over *Advances* as a regular journal with Volume II, which appeared in 1968.

always the hope that others might work his beloved areas of mathematics and philosophy better than he did. Or, as Ariosto put it, *Forse altri canterà con miglior plettro* ("Perhaps another will sing with a better plectrum").[5]



Above and right: Gian-Carlo Rota, circa 1996.
(Photographs from the author, used by permission.)

5 Author's translation.

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