NATIONAL ACADEMY OF SCIENCES

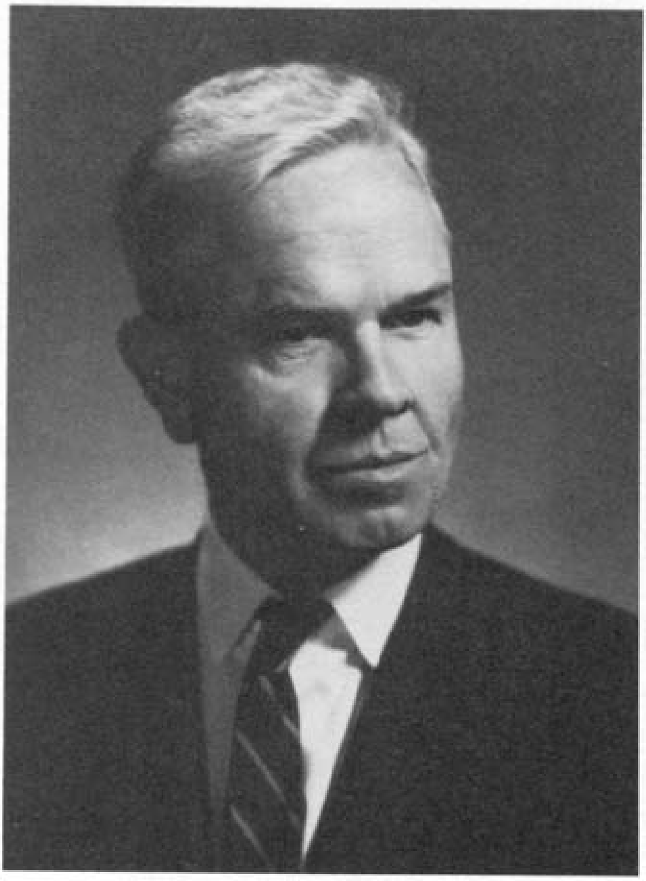
NORMAN EARL STEENROD

1910—1971

A Biographical Memoir by

GEORGE W. WHITEHEAD

Biographical Memoir



N. E. Steenrod

NORMAN EARL STEENROD

April 22, 1910–October 14, 1971

BY GEORGE W. WHITEHEAD

THE SUBJECT OF ALGEBRAIC TOPOLOGY has undergone a spectacular development in the years since World War II. From a position of minor importance, as compared with the traditional areas of analysis and algebra, its concepts and methods have come to exert a profound influence over the older fields, and it is now commonplace that a mathematical problem is "solved" by reducing it to a homotopy-theoretic one. And, to a great extent, the success of this development can be attributed to the influence of Norman Steenrod.

Norman Earl Steenrod was born in Dayton, Ohio, April 22, 1910, the youngest of three surviving children of Earl Lindsay Steenrod and his wife Sarah (née Rutledge). The Steenrods, reputedly of Norwegian origin, came to this country by way of Holland before the Revolutionary War, and Norman's great-great-great-grandfather, Cornelius Steenrod, raised a company of soldiers who fought in that war. Both his parents were teachers—his mother for two years before her marriage, his father for some forty years as a high school instructor in manual training and mechanical drawing (and occasionally other subjects). Neither parent had any special interest in mathematics, though Earl Steenrod had a keen interest in astronomy, which he communicated to his son. From his mother Norman acquired a lifelong interest in music, to which he devoted much of his spare time. Other interests included tennis, golf, chess, and bridge.

Norman attended the public schools in Dayton, finishing the twelve-year course in nine years. After graduation from high school he worked for two years or so as a tool designer, having learned the trade from his elder brother. In this way he earned enough to help with his college expenses. That these were a severe problem throughout his student days is made clear in the following excerpt from a letter from Professor R. L. Wilder.

> Norman's undergraduate days were filled with frustration, most of which seems to have been due to lack of funds. He attended Miami University in Oxford, Ohio, from 1927 to 1929, leaving there with twenty-six hours in mathematics, and with "Honors in Mathematics and High Distinction." I gather that he found a job for the year 1929–1930. He then came to [the University of] Michigan for the summer session of 1930. He was able to stay on for the first semester of the year 1930–1931, but withdrew in February, 1931 (no doubt for financial reasons; his grades were all A's). He came back to Michigan in the fall of 1931. My course in topology was the only mathematics course that he enrolled in, all the others being in physics, philosophy and economics. By the beginning of the second semester, I knew I had in him the makings of a real mathematician, and before he left in June, 1932, I started him on a problem.
>
> The year 1932–1933 was a hard one for him. He couldn't get a fellowship, so he went back home to Dayton, Ohio. However, he couldn't find steady employment; he wrote me "Misfortune is dogging my family." But mathematically he made up for the lack of other employment. He was so capable that it was possible to direct his work by correspondence, and by the end of the year he had finished his first paper *Finite arc sums* . . . and also started on the problem which became his second paper *Characterization of certain curve sums*. . . . And in February, 1933, he could renew applications for a fellowship, this time accompanying them with manuscript copies of his first paper. Harvard, Princeton and Duke all offered him fellowships, and he accepted the Harvard offer. In the late summer of 1933 he did land a job at the Flint Chevrolet plant as a die designer and saved $60 by the time he had to quit to go to Cambridge.

He took a heavy load at Harvard (counting the "unofficial" courses), but continued to polish off his second paper cited above. He also had worked out a plan to solve the Four Color Problem (never pursued), and for generalizing the theory of cyclic elements to higher dimensions (later done by Whyburn).

In the spring of 1934, he was offered fellowships at both Harvard and Duke, but turned both down when an offer of a fellowship came from Michigan. I was at Princeton that year (1933–1934); it was the first year of the Institute [for Advanced Study], and I had made up my mind that Steenrod should come to Princeton. Lefschetz arranged for me to talk to the fellowship committee, which kindly concurred in my views and made an offer to Steenrod. It took some persuasion on my part to get him to accept; he liked Michigan and seemed happy in the thought of going on working with me.

By this time Norman's financial problem had eased. At Princeton he worked with Solomon Lefschetz, obtaining his Ph.D. in two years. He remained at Princeton as an instructor for three more years.

Norman was married to Carolyn Witter in Petoskey, Michigan, August 20, 1938. In 1939 he came to the University of Chicago as an assistant professor. The Steenrods' first child, Katherine Anne, was born in Chicago in 1942. In that year he left Chicago to return to the University of Michigan. As Wilder has written, he had a strong attachment to Michigan. Moreover, his decision to move was influenced by his reluctance to raise a family in a large city. His other child, Charles Lindsay, was born in Ann Arbor in 1947. It was during his stay in Ann Arbor that he began his collaboration with Eilenberg, which was to result in their influential work, *Foundations of Algebraic Topology*.

In 1947 Steenrod returned to Princeton, where he was to spend the remainder of his career. During this period his book with Eilenberg was published, as was his book on fibre bundles. In 1956 he was elected to the National Academy, and in 1957 he gave the Colloquium Lectures before the American Mathematical Society.

Steenrod's work in algebraic topology is probably best known for the algebra of operators that bears his name. There are, however, at least two other aspects of his work which have had a profound and lasting influence on the development of the subject.

The first of these is concerned with the foundations. The fifty years following the appearance of Poincaré's fundamental memoir saw great progress in the development of algebraic topology. The fundamental theorems of the subject—the invariance theorem, the duality theorems of Poincaré and Alexander, the universal coefficient and Künneth theorems, the Lefschetz fixed point theorem—had all been proved, at least for finite complexes. The intersection theory for algebraic varieties had been extended, first to manifolds, then, with the invention of cohomology groups, to arbitrary complexes. Nevertheless, by the early forties the subject was in a chaotic state. Partly in a quest for greater insight, partly in order to extend the range of validity of the basic theorems, there had arisen a plethora of homology theories—the singular theories due to Alexander and Veblen and to Lefschetz, the Vietoris and Čech theories, as well as many minor variants. Thus, while there were many *homology theories*, there was not yet a *theory of homology*. Indeed, many of the concepts that are routine today, while appearing implicitly in much of the literature, had never been explicitly formulated. The time was ripe to find a framework in which the above-mentioned results could be placed in order to determine their interconnections, as well as to ascertain their relative importance.

This task was accomplished by Steenrod and Eilenberg, who announced their system of axioms for homology theory in 1945. What was especially impressive was that a subject as complicated as homology theory could be characterized by properties of such beauty and simplicity. But the first impact of their work was not so much in the explicit results as in their whole philosophy. The conscious recognition of the functorial properties of the concepts involved, as well as the explicit use of diagrams as a tool in constructing proofs, had thoroughly permeated the subject by the appearance of their book in 1952.

Inspection of their axioms reveals that the seventh "Dimension Axiom" has an entirely different character from the first six, the latter being of a very general nature, while the former is very specific. That it is accorded the same status as the others is no doubt because no interesting examples of nonstandard theories were known at that time. In any case, a great deal of the work does not depend on the Dimension Axiom.

With the great advances in homotopy theory in the fifties and sixties, there arose numerous examples of extraordinary theories—theories satisfying only the first six axioms. Among these were the stable homotopy and cohomotopy groups of Spanier and Whitehead; the K-theories of Atiyah and Hirzebruch; and the bordism theories of Atiyah and of Conner and Floyd, as well as their more recent generalizations. It is a tribute to the insight of Eilenberg and Steenrod that these theories, whose existence was undreamt of in 1945, fit so beautifully into their framework.

Another elegant application was the theorem of Dold and Thom. The Symmetric group $S(n)$ acts on the n-fold Cartesian power X^n of a space X by permuting the factors; the orbit space is the n-fold symmetric power $SP^n(X)$. There are imbeddings of $SP^n(X)$ into $SP^{n+1}(X)$; thus one can form the infinite symmetric product $SP\infty(X)$. Dold and Thom showed that the homotopy groups $\pi_q(SP\infty(X))$ satisfy the axioms, and hence:

$$\pi_q(SP\infty(X)) \approx H_q(X;Z).$$

For example, $SP\infty(S^n)$ is an Eilenberg-Mac Lane space $K(Z,n)$.

The other aspect of Steenrod's work mentioned above is the theory of fibre bundles. Steenrod always had a strong interest in differential geometry; indeed, one of his earliest papers, written jointly with S. B. Myers, established one of the fundamental results of global differential geometry—the group of isometries of a Riemannian manifold is a Lie group. Thus it was to be expected that he would have something to say about the young and rapidly growing subject of fibre bundles.

One of the first problems in the subject is that of the existence of a cross section. This problem is attacked by a stepwise extension process, parallel to that used in ordinary obstruction theory. However, the coefficient groups for the obstruction vary from point to point, the various groups are isomorphic, but the isomorphism between the groups at two different points depends on a homotopy class of paths joining them. In other words, the coefficient groups form a *bundle of groups*. During the 1940s Steenrod introduced the notion of homology and cohomology with coefficients in a bundle of groups ("local coefficients"). This theory, which extended and clarified the work of Reidemeister on "homotopy chains," provided the proper setting for obstructions, not only in bundle theory, but also for mappings into nonsimple spaces. Moreover, it gave the first satisfactory formulation of Poincaré duality for nonorientable manifolds.

One of the most vital notions in bundle theory is that of universal bundle (equivalently, of classifying space). The importance of the Grassmann manifold $G(k,m)$ of k-planes in R^{k+m} was first recognized by Whitney, who proved that every $(k\text{-}1)$-sphere bundle over a complex of dimension at most m is induced by a map of its base space into $G(k,m)$. In 1944 Steenrod proved the decisive result in this direction: the space $G(k,m)$ is a *classifying space* for $(k\text{-}1)$-sphere bundles over a base complex of dimension $\leq$m, that is, for such a space X, the homotopy classes of maps of X into $G(k,m)$ are in one-to-one correspondence with the isomorphism classes of S^{k-1}-bundles over X. There is a standard bundle $B(k,m)$ over $G(k,m)$, whose total space is the set of all pairs (π,x), where π is a k-plane in R^{k+m} and x a unit vector in π. The above correspondence associates with each map $f\colon X\to G(k,m)$ the induced bundle $f^*B(k,m)$. The total space of the *principal associated bundle* is the Stiefel manifold $V_{k+m,k}$ of oriented k-frames in R^{k+m}; the crucial fact used in the proof is that $V_{k+m,k}$ is $(k\text{-}1)$-connected. Once this was realized, the generalization to bundles whose group is an arbitrary compact Lie group was not difficult, and it was found by Steenrod and several other authors independently. Later developments included the construction by Milnor of a classifying space for an arbitrary topological group; while in recent years the classifying spaces BO, BU, . . . for *stable vector bundles* have been of enormous importance.

These noteworthy contributions to bundle theory occurred during an era in which the subject was growing with great rapidity, and was in a sadly confused state. A systematic account of the subject was badly needed, and this need was amply met with the appearance in 1951 of Steenrod's book, *The Topology of Fibre Bundles*. But the importance of the book was not limited to its treatment of bundle theory. It must be remembered that, while homotopy groups had been in existence for sixteen years and obstruction theory for twelve, neither topic had received a treatment in book form. Steenrod's book gave a very clear, if succinct, treatment of both topics and thus served as an introduction to homotopy theory for a whole generation of young topologists.

The notion of fibre bundle has been a most important one for the applications of topology to other fields. The concept is an intricate one, however, and for the purposes of homotopy theory, it is entirely too rigid. Indeed, its importance in homotopy theory is due primarily to the *homotopy lifting property* (HLP). It was this fact that led Steenrod and Hurewicz in 1941 to the notion of *fibre space*. In the intervening years, the accepted definition of fibre space has undergone a number of modifications; but all definitions have been directed toward proving the HLP. In fact, the modern approach is to define a fibre map to be a continuous map that has the HLP for arbitrary spaces. In any case, the notion is a crucial one in homotopy theory (for example, in my recent treatise it occupies a major portion of the first chapter).

And now the time has come to turn our attention to the Steenrod algebra. The problem of classifying the maps of an m-complex K into the n-sphere S^n had long occupied topologists. In 1933 Hopf gave the solution for $m=n$; these results were reformulated in terms of cohomology, and thus greatly simplified, by Whitney in 1937. In 1931 Hopf showed that $\pi_3(S^2)$ is nonzero; that it is infinite cyclic was established by Hurewicz in 1935. In 1937 Freudenthal proved his fundamental *suspension theorem* and showed that, for $n \geq 3$, $\pi_{n+1}(S^n)$ is a cyclic group of order two generated by the suspension of the Hopf map. In 1941 Pontryagin classified the maps of K^3 into S^2; his classification involved the relatively new cup products of Alexander-Čech-Whitney.

The next outstanding problem was the classification of the maps of K^{n+1} into S^n for $n \geq 3$. That this problem was quite subtle is amply demonstrated by the announcement of solutions by two very distinguished mathematicians—both of which turned out to be incorrect.

In 1947 Steenrod solved this problem. His solution was interesting, not only per se, but by virtue of the new operations in terms of which the solution was expressed. These were the celebrated Steenrod squares. They were presented as generalizations of the cup square; Steenrod's first construction was by cochain formulas generalizing the Alexander-Čech-Whitney formula for the cup product. While this gave a simple and effective procedure for calculating the squares, it was not at all clear how to generalize the construction to obtain reduced n^{th} powers. It was not long before Steenrod realized that it was the Lefschetz approach to cup products by chain approximations to the diagonal, rather than that of Alexander-Čech-Whitney, that yielded a fruitful generalization, and he soon succeeded in constructing the higher reduced powers.

The potency of the new operations soon became apparent. The Cartan formulas for the squares of a cup product allowed one to calculate the squares in truncated projective spaces. This had deep consequences for the old problem: how many tangent vector fields can be found on S^n that are linearly independent at each point? Using the above results, Steenrod and J. H. C. Whitehead were able to show that, if k is the exponent of the largest power of two dividing $n + 1$, then S^n does not admit a tangent 2^k-frame. This was a tremendous step forward; previously it was known to be true only for $k = 0$ or 1.

The reduced powers are examples of cohomology operations, i.e., natural transformations of one cohomology functor into another. Moreover, they are stable operations, in the sense that they are defined in every dimension and commute with suspension. The set of all stable operations in mod p cohomology forms an algebra $\mathcal{A} = \mathcal{A}_p$, which was soon to be known as the *Steenrod algebra.* In 1952 Serre showed that $\mathcal{A}_2$ is generated by the squares and exhibited an additive basis composed of certain iterated squares. In 1954 Cartan proved an analogous result for odd primes; besides the reduced p^{th} powers $\mathcal{P}^i$, one additional operation, the Bockstein operator β_p, is needed.

In the meantime, Adem used Steenrod's approach to find relations among the iterated squares. In particular, he proved that Sq^i is decomposable if i is not a power of two. This had a most important application: there is no map of S^{2n-1} into S^n of Hopf invariant one, unless n is a power of two. This again was a great step forward; previously it was known only that n (if > 2) had to be divisible by 4.

Adem also showed that his relations gave rise to *secondary cohomology operations*; these differed from the old ones in that they were not everywhere defined (the domain was the kernel of a certain primary operation) and not single-valued (the range was the cokernel of another primary operation). Nevertheless, they were sufficiently powerful to prove that the iterated Hopf maps η^2, ν^2, σ^2 are stably nontrivial.

The analogous relations among the reduced p^{th} powers were found independently by Adem and Cartan a year or so later.

Meanwhile, Steenrod had not been idle. His new approach to the subject revealed deep connections with the Eilenberg-Mac Lane homology of groups. Specifically, he showed that each element of $H_q(T;G)$, where T is a subgroup of the symmetric group $S(n)$ of degree n, gives rise to an operation. These operations included the old ones (which arose from a transitive cyclic subgroup of the symmetric group) and more—the generalized Pontryagin powers $\mathfrak{P}_p$ of Thomas were also included. Moreover, Steenrod and Thomas showed that all operations derived from permutation groups by Steenrod's procedure were generated by the $\mathcal{P}^i$ and the $\mathfrak{P}_p$, with the aid of the primitive operations of addition, cup product, coefficient group homomorphisms, and Bocksteins. Later, Moore, Dold, and Nakamura showed that *all* operations are obtained in this way (at least if the coefficient groups are finitely generated).

Further applications now followed thick and fast. Only a few examples will evince the extent to which the Steenrod algebra has permeated the field of algebraic topology in recent years. These are: (1) the structure of the Thom spectrum $M(G)$ as an $\mathcal{A}$-module was crucial in the determination of the various bordism rings by Thom, Wall, Milnor, Anderson-Brown-Peterson; (2) Milnor's observation that the Cartan formulas make $\mathcal{A}$ into a Hopf algebra and his determination of its structure have greatly deepened our insight; (3) the introduction of homological-algebraic methods by Adams has revealed the crucial importance of the cohomology of $\mathcal{A}$ in stable homotopy; and (4) Steenrod's own work on unstable $\mathcal{A}$-modules led Massey and Peterson to their unstable version of the Adams spectral sequence, one of the most promising of our tools in studying unstable homotopy theory.

Some other aspects of Steenrod's work which deserve mention are:

1) While the notion of inverse limit of a system of abelian groups had been known for some time, that of direct limit is due to Steenrod and made its first appearance in his thesis;

2) His 1940 paper on regular cycles in compact metric spaces was a forerunner of Borel-Moore homology theory;

3) His calculation, in the same paper, of the cohomology groups of the complement of a solenoid in R^3 led Eilenberg and Mac Lane to study the relation between group extensions and homology, thereby inaugurating a long and fruitful collaboration.

In the last years of his life, Steenrod devoted his attention to the *realization problem*. To understand this problem, it is necessary to observe that, while the homology and cohomology groups of a space can be assigned almost at will, the cohomology of a space admits additional structure, that is, an algebra structure due to the existence of cup products, as well as a module structure over the Steenrod algebra. Indeed, the latter two structures are linked by the condition that $H^*(X;Z_p)$ is an *algebra over* $\mathcal{A}_p$. Two questions then naturally arise: 1) which graded algebras over Z_p admit the structure of an $\mathcal{A}_p$-algebra and 2) which $\mathcal{A}$-algebras are the cohomology algebras of some space?

Steenrod gave a preliminary account of his work on these questions before the Conference on H-spaces at Neuchâtel in August, 1970. He continued to work on the problem during his sabbatical leave at Cambridge University during the ensuing year. In the spring of 1971 he suffered an attack of phlebitis, and, after his return to Princeton that fall, a stroke. In the following weeks he appeared to be making a good recovery, but then suffered a succession of strokes, to which he succumbed on October 14, 1971.

Steenrod will long be remembered, not only for his mathematical work, but also for his patience and care in dealing with his students. My own case is illustrative. In 1939, when he came to Chicago, algebraic topology was still a young subject, not even a regular part of the curriculum in many schools, and homotopy theory was in its infancy. My acquaintance with the subject was limited to a one-quarter course taken in the summer of 1939. When I became Steenrod's student, he arranged that I see him once a week, whether or not I had any progress to report; it was in these weekly conferences that I really learned the subject. This system of weekly conferences was continued by Steenrod with his other students, and many of them in turn continued the tradition with their own students.

Steenrod was a gifted expositor; he believed that a result worth proving was equally worthy of a clear exposition. Those of us who were his students remember well the great patience he showed in reading one version after another of our maiden efforts, and his often devastating comments. In retrospect, we agreed that it was time well spent. And his interest continued well beyond our student days. As an editor of the *Annals of Mathematics*, which published most of the significant work in algebraic topology, he was in an excellent position to continue his interest and help.

Another notable contribution of Steenrod's was his compilation of reviews of all papers in algebraic topology and related areas. This involved a painstaking process of selection of the relevant articles from *Mathematical Reviews* and arranging, classifying, and cross-referencing them. This labor of love took several years, and the result has been extremely useful to anyone interested in the subject. So successful has it been that the American Mathematical Society, besides publishing this work, has followed his lead with similar compilations in several other areas. It is not often that a mathematician of Steenrod's stature would take the trouble to carry out such an onerous task; but it is typical of Steenrod to envisage the value of such a work and not to shrink from the labor involved.

Norman was a gifted raconteur with an endless fund of humorous anecdotes. And he loved an argument for its own sake; often he would enliven a gathering by presenting a totally outrageous proposition and defending it against all comers with dexterity and wit. The demands on his time by visits of his former students to Princeton must have been large; but he always seemed glad to see us, and his and Carolyn's hospitality was warm and open-handed.

I AM INDEBTED to Norman's widow, Carolyn, for much personal information; to his sister, Miss Virginia Steenrod, for information on his early life; and to Professor R. L. Wilder, for the story of his early mathematical development.

In April, 1970 a conference to celebrate Steenrod's sixtieth birthday was held at the Battelle Memorial Institute, Columbus, Ohio. The proceedings of the conference were published by Springer-Verlag in their series, *Lecture Notes in Mathematics*. At the conference I spoke on Steenrod's mathematical work. As his death occurred within a year and a half of the conference, this account is almost complete, and is included here, without substantial change, by permission of Springer-Verlag.

BIBLIOGRAPHY

1934

Finite arc-sums. Fund. Math., 23:38–53.

Characterizations of certain finite curve-sums. Am. J. Math., 56:558–68.

1935

On universal homology groups. Proc. Natl. Acad. Sci. USA, 21:482–84.

1936

Universal homology groups. Am. J. Math., 58:661–701.

1938

Remark on weakly convergent cycles. Fund. Math., 31:135–36.

With J. H. Roberts. Monotone transformations of two-dimensional manifolds. Ann. Math., 39:851–62.

1939

With S. B. Myers. The group of isometries of a Riemannian manifold. Ann. Math., 40:400–416.

1940

Regular cycles of compact metric spaces. Ann. Math., 41:833–51.

1941

Regular cycles of compact metric spaces. In: *Lectures in Topology,* ed. Raymond L. Wilder and William L. Ayres, pp. 43–55. Ann Arbor: University of Michigan Press.

With W. Hurewicz. Homotopy relations in fibre spaces. Proc. Natl. Acad. Sci. USA, 27:60–64.

1942

Topological methods for the construction of tensor functions. Ann. Math., 43:116–31.

1943

Homology with local coefficients. Ann. Math., 44:610–27.

1944

The classification of sphere bundles. Ann. Math., 45:294–311.

1945

With S. Eilenberg. Axiomatic approach to homology theory. Proc. Natl. Acad. Sci. USA, 31:117–20.

1947

Products of cocycles and extensions of mappings. Ann. Math., 48:290–320.

Cohomology invariants of mappings. Proc. Natl. Acad. Sci. USA, 33:124–28.

1949

Cohomology invariants of mappings. Ann. Math., 50:954–88.

1951

With J. H. C. Whitehead. Vector fields on the n-sphere. Proc. Natl. Acad. Sci. USA, 37:58–63.

The Topology of Fibre Bundles. Princeton: Princeton University Press. viii + 240 pp.

1952

Reduced powers of cohomology classes. Ann. Math., 56:47–67.

With S. Eilenberg. *Foundations of Algebraic Topology*. Princeton: Princeton University Press. xv + 328 pp.

1953

Homology groups of symmetric groups and reduced power operations. Proc. Natl. Acad. Sci. USA, 39:213–17.

Cyclic reduced powers of cohomology classes. Proc. Natl. Acad. Sci. USA, 39:217–33.

1957

The work and influence of Professor S. Lefschetz in algebraic topology. In: *Algebraic Geometry and Topology, A Symposium in Honor of S. Lefschetz*, pp. 24–43. Princeton: Princeton University Press.

Cohomology operations derived from the symmetric group. Comment. Math. Helv., 31:195–218.

With Emery Thomas. Cohomology operations derived from cyclic groups. Comment. Math. Helv., 32:129–52.

1958

Cohomology operations. In: *Symposium Internacional de Topología Algebraica*, pp. 165–85. Mexico City: Universidad Nacional Autónoma de México and UNESCO.

1961

The cohomology algebra of a space. Enseign. Math. 2(7):153–78.

1962

Cohomology Operations. Lectures by N. E. Steenrod, ed. D. B. A. Epstein. Ann. Math. Stud., No. 50. Princeton: Princeton University Press. vii + 139 pp.

1965

With M. Rothenburg. The cohomology of classifying spaces of H-spaces. Bull. Am. Math. Soc., 71:872–5.

1966

With W. G. Chinn. *First Concepts of Topology. The Geometry of Mappings of Segments, Curves, Circles, and Disks.* New Philosophical Library, vol. 18. New York: Random House; Syracuse, N.Y.: The L. W. Singer Co. viii + 160 pp.

1967

A convenient category of topological spaces. Mich. Math. J., 14:133–52.

Homology of Cell Complexes, by George E. Cooke and Ross L. Finney (based on lectures by Norman E. Steenrod). Princeton: Princeton University Press; Tokyo: University of Tokyo Press. xv + 256 pp.

1968

Milgram's classifying space of a topological group. Topology, 7:349–68.

Reviews of Papers in Algebraic and Differential Topology, Topological Groups, and Homological Algebra (classified by N. E. Steenrod). Providence, R.I.: American Mathematical Society.

1971

Polynomial algebras over the algebra of cohomology operations. In: *H-spaces*, Lecture Notes in Mathematics, No. 196, pp. 85–99. Berlin-Heidelberg-New York: Springer-Verlag.

1972

Cohomology operations, and obstructions to extending continuous functions. Adv. Math., 8:371–416.