MATH 270
SPRING 2003
HOMEWORK 7

Due Friday March 14, 2003.

1. (10 pt) In the definition of *well-ordering* your book says that a well-ordered set is a totally ordered set, A with the property that every subset of A has a least element. Prove that the assumption "A is totally ordered" is not needed. That is, show that any partially ordered set, A, with the property that every subset of A has a least element is automatically totally ordered.

2. (3 pt) Show that a finite set is well-ordered if and only if it is totally ordered. Give an example of a finite set that is not well-ordered.

3. Let $(A, \leq)$ be a poset. We now define a new relation on A (we call it $\preceq$) by declaring that for all $x, y \in A$, $x \preceq y$ if $y \leq x$. Show the following:

a) (5 pt) Show $(A, \preceq)$ is a poset.
b) (5 pt) Show that $(A, \leq)$ is well-ordered if and only if $(A, \preceq)$ has the property that every subset of A has a greatest element.

4. Give examples (and verify that they work) of:

a) (3 pt) A partially ordered set that is not totally ordered.
b) (3 pt) A totally ordered set that is not well-ordered.