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Abstract

The real effects of an imperfectly credible disinflation depend critically on the extent of price rigidity. In this paper, we examine how credibility affects the outcome of a disinflation in a model with endogenous time-dependent pricing rules. Both the endogenous initial degree of price rigidity and changes in the duration of price spells during disinflation play an important role in explaining the effects of imperfect credibility. We initially consider the costs of disinflation when the degree of credibility is fixed, and then allow agents to use Bayes' rule to update beliefs about the "type" of monetary authority that they face. In both cases, the interaction between the endogeneity of time-dependent rules and imperfect credibility increases the output costs of disinflation. The pattern of the output response is more realistic in the case with learning.

Key words: disinflation, optimal price setting, endogenous time-dependent pricing

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1 Introduction

Lack of credibility has, for a long time, been pointed out as an important ingredient in explaining real effects of disinflation (e.g. Sargent, 1983). It arises when a monetary authority that is serious about disinflating faces distrust from the private sector. The existence of nominal price (and wage) rigidities is a main reason why an imperfectly-credible disinflation may have meaningful output costs (Ball 1995).

Additionally, the *extent* of price rigidity matters for the effect of imperfect credibility. Consider an economy during an imperfectly-credible disinflation in which individual prices are fixed for extremely short periods of time. Then, the price optimally set by each firm tends to be very similar to the price that would be set under full credibility, since there is relatively little uncertainty about the monetary policy regime in the very short run. In this case the existence of imperfect credibility has little impact on the real effects that the disinflation might produce due to nominal rigidities. The same is not true of an economy where prices are fixed for long periods of time. Since policy uncertainty tends to build up with time, in that case agents perceive a much higher probability of a policy reversal between price adjustments. This uncertainty affects pricing decisions, leading to substantial differences between the individual prices set during an imperfectly-credible and a perfectly-credible disinflation.[1]

Because the role of credibility depends on the frequency of price changes, conclusions about the effect of imperfect credibility that are based on models where this frequency is chosen arbitrarily will reflect this arbitrary choice. In addition, since a disinflation typically involves a policy regime change, analyses based on such models are inherently subject to the Lucas critique. Not only should the degree of price rigidity respond to the change in regime: the response should also depend on its credibility. For those reasons, the study of the role of credibility in disinflation episodes should not be dissociated from the analysis of the determinants of the frequency of price changes.

In this paper we analyze how a policymaker's credibility affects the outcome of a disinflation in a model in which the extent of price rigidity is endogenous. In our model firms face frictions that make it optimal to choose ex-ante the time of the next price change. As a result, the time period between price adjustments - the duration of the spells of price rigidity - responds to changes in the economic environment.

[1] This applies to models where not all firms have the *option* to react instantaneously and with full information to an eventual policy reversal. It applies both to time-dependent models with nominal rigidity, as in Taylor (1979, 1980) and Calvo (1983), and sticky-information models, as in Mankiw and Reis (2002), and Reis (2006). It does not apply to state-dependent pricing models (e.g. Caplin and Spulber 1987), where information is continuously available.

Credibility affects the output costs of disinflation through a direct and an indirect effect on prices. The *direct* effect is through the expectation of the path of marginal costs until the time of the next price change, given the frequency of price changes. It appears in models based on exogenous time-dependent pricing rules (e.g. Ball 1995, and Erceg and Levin 2003). As we argued above, the magnitude of this effect hinges on the duration of price spells. Our framework naturally brings discipline to the analysis, since such spells are determined endogenously.[2] The *indirect* effect arises in our model with endogenous pricing rules because changes in the frequency of price changes during the disinflation also affect the individual prices chosen. With policy regime shifts, as it happens when a new disinflationary policy is put in place, this effect becomes important.

In Section 2 we derive the optimal pricing rule under the assumption that firms cannot obtain, process and react to new information *nor* adjust prices based on their old information unless they incur a real lump-sum cost, as in Bonomo and Carvalho (2004). We provide more explicit foundations to our earlier approach, and extend it to derive the optimal pricing rule during an imperfectly-credible disinflation. The resulting pricing strategy is an endogenous time-dependent rule, where each time a firm incurs the information/adjustment cost, it sets a price and chooses ex-ante when next to gather and process information to decide on a new price. We refer to such chosen times as *pricing dates*.[3]

We view the assumption of a single information/adjustment cost as a tractable way to incorporate information frictions and adjustment costs that appear to be present in price-setting decisions, as documented by Zbaracki et al. (2004). The resulting pricing rule displays time-dependency that resembles the "pricing seasons" described by those authors, and nominal rigidities that are consistent with microeconomic evidence on individual prices (e.g. Bils and Klenow 2004 for recent evidence for the U.S. economy).

Our main interest is to analyze the mechanism through which an imperfectly-credible disinflation affects output in a setting in which price-setting decisions are optimal. We take the initial level of credibility as exogenous, and model imperfect credibility as a discrepancy be-

[2] One could argue that the arbitrariness in specifying (exogenously) the duration of price spells could be avoided by calibrating the frequency of price changes to the microeconomic evidence. However, this would restrict the scope of analysis to economic environments similar to the ones that produced the evidence used in the calibration. In contrast, our approach allows us to calibrate the primitive parameters of the model to the available evidence, and compute the frequency of price changes for different economic environments.

[3] A pure adjustment cost ("menu cost") would give rise to state-dependent pricing (e.g. Barro 1972, Sheshinski and Weiss 1977), whereas a pure information cost would lead to the choice of price paths between optimally chosen information-gathering dates, as in Caballero (1989) and Reis (2006). Ball, Mankiw, and Romer (1988) analyze a model with endogenous contract lengths in inflationary steady states as an approximation to state-dependent pricing. In a related paper, Romer (1990) proposes an optimally chosen frequency of price adjustment in a Calvo-type model as a tractable (albeit suboptimal) alternative to state-dependent policies. For a recent application of Romer's model, see Levin and Yun (2007).

tween private agents' beliefs about the likelihood that the monetary authority abandons the disinflation and the objective likelihood. Such beliefs affect aggregate outcomes through their effect on the choice of the time interval between pricing dates, and prices set by firms.[4] For tractability, we model disinflation as a policy shift that changes the growth rate of nominal aggregate demand instantaneously, without making explicit the details of the transmission mechanism.[5]

In Section 3 we examine the case where the degree of credibility is fixed, so that price setters' beliefs do not change, despite the fact that the disinflation policy is never abandoned. For a given frequency of price changes, imperfect credibility increases the costs of disinflation because agents believe that there is some probability that the stabilization will be abandoned before their next pricing date, and therefore set prices higher than in the case of full credibility. To properly measure this *direct* effect, we set the exogenous frequency of price changes equal to the one that would be optimal for the inflationary environment that prevailed prior to the disinflation.

We assess the *indirect* effect of credibility by examining the case in which pricing rules are endogenous. We find the output costs of disinflation to be higher in this case. With endogenous rules, when faced with lower expected trend inflation after the disinflation is launched, firms optimally choose to change prices less frequently. This raises the probability of a policy reversal occurring between pricing dates, amplifying the difference between individual prices set under perfect and imperfect credibility, and thus raising the output costs of disinflation.

In Section 4 we introduce learning. The assumption that agents do not update their beliefs, despite useful for gaining insight, is not realistic. It generates the unappealing result that after disinflation output remains permanently below potential. One should expect the monetary authority to gain credibility through time, as agents observe that disinflation continues and update their beliefs about its resolve to disinflate. We model the evolution of agents' beliefs through Bayesian learning. The result is a more realistic output path in which the monetary authority gains credibility, and the recession is gradually eliminated. Moreover, the main result of the paper, that endogeneity of pricing rules and lack of credibility interact to generate higher disinflation costs, continues to hold.

Finally, we use the imperfectly-credible disinflation under learning to illustrate a strik-

[4] There is another line of investigation that focuses on explaining credibility. Those models usually have a simple aggregate supply structure, and rely on the discretionary nature of monetary policy. Recent examples are Siu (2008) and Westelius (2005). Backus and Driffill (1985a,b) provided earlier contributions.

[5] For other purposes it might be worthwhile to embed our endogenous time-dependent pricing rule in a model with an explicit transmission mechanism, and study the effects of disinflation when it is implemented in alternative ways (e.g. lowering an inflation target, adopting a currency peg etc).

ing implication of endogenous time-dependent pricing rules at the microeconomic level. In our model, the frequency of price changes depends on expectations about the evolution of the underlying state of the economy between pricing dates, rather than on its actual evolution. This follows from the fact that each pricing date is predetermined as of the previous such date.[6] As illustrated by our simulations, this difference becomes apparent during an imperfectly-credible disinflation under learning, since there is a persistent discrepancy between the expected stance of monetary policy, and its actual stance. As a result, the evolution of the optimal frequency of price changes during the disinflation is unrelated to the actual monetary policy stance, but strongly related to expectations about the future monetary policy stance. We conclude with a discussion of how this implication of endogenous time-dependent pricing might be useful to discriminate empirically between alternative models of price setting.

The literature that links imperfect credibility and price rigidity explicitly starts with Ball (1995), who argues that both ingredients are necessary to explain the output costs of disinflation. He focuses on average effects of disinflation when agents' beliefs are in fact correct (i.e. they know the distribution of abandonment times). Erceg and Levin (2003) explain the output costs during the Volcker disinflation with a model where agents have to learn about a structural change in the interest rate rule. Both papers use exogenous pricing rules. Nicolae and Nolan (2006) model a credibility problem similar to ours, but assume simple learning schemes instead of Bayesian updating. Moreover, they limit the choice of pricing rules: prices are adjusted either every period or every other period. Finally, Almeida and Bonomo (2002) analyze the output costs of disinflation under imperfect credibility and endogenous state-dependent pricing.[7] In that model, price setters observe monetary policy and reconsider their pricing decisions continuously, under full information. As a result, imperfect credibility has only a small effect through its impact on the optimal pricing rule.

2 The model of optimal time-dependent pricing

We start from a model with a representative consumer who derives utility from a Dixit-Stiglitz composite of different varieties of a consumption good. She incurs disutility from supplying labor in a competitive market to a continuum of monopolistically competitive firms. Each firm hires labor to produce its variety of the consumption good. For simplicity

[6] In constrast, in standard state-dependent pricing models all information is perfectly and continuously observable, so that past expectations of the current state of the economy have no effect on the timing of price changes.

[7] Danziger (1988) and Ireland (1997) also analyze disinflation in models in which pricing is (at least partially) state-dependent.

we abstract from real shocks. Firms face frictions that make it optimal to undertake pricing decisions infrequently, as we discuss extensively below.

In Appendix A we develop the model from fundamentals, and derive the following log-linear expression for the frictionless optimal price that a firm would charge if it did not face pricing frictions, p_t^*:[8]

$$p_t^* = p_t + y_t, \tag{1}$$

where p_t is the aggregate price level, y_t is aggregate output. Letting $\mathcal{Y}_t = p_t + y_t$ denote (log) nominal output, aggregate output will be given by:

$$y_t = \mathcal{Y}_t - p_t = p_t^* - p_t.$$

Thus, output fluctuations will be caused only by the interaction between monetary developments and the frictions that make infrequent pricing decisions optimal.

The microeconomic evidence on nominal price rigidity has usually been rationalized by the existence of menu costs of changing prices. As it is well known, this leads to pricing decisions that are state dependent. However, available evidence based on interview studies (Blinder et al. 1998) and direct measurement through field work (Zbaracki et al. 2004) shows the importance of other types of costs associated with price-setting decisions, such as information-gathering, decision-making, and internal-communication costs. Those costs prevent the continuous information gathering and processing that are necessary for the implementation of purely state-dependent pricing strategies.

Non-convex information and decision-making costs lead to infrequent pricing decisions, and time-dependency (Reis 2006). However, in the absence of adjustment costs, the optimal pricing rule calls for the choice of a price path at each decision date. This implication is at odds with the microeconomic evidence on nominal price rigidity.

A model endowed with both information and adjustment costs should capture the time-dependency uncovered in recent work (e.g. the pricing seasons documented in Zbaracki et al. 2004) and at the same time generate nominal price rigidity. A tractable model with these features is proposed by Bonomo and Carvalho (2004), who assume that firms cannot obtain, process and react to new information *nor* adjust prices based on their old information unless they incur a lump-sum cost. Here we provide better foundations for our earlier approach, and extend it to obtain the optimal pricing rule during an imperfectly-credible disinflation.

[8]Throughout the paper, lowercase variables denote log-deviations of the respective quantity from the deterministic steady state, as detailed in Appendix A. For expositional simplicity, we omit the expression "log-deviation from the steady state", refering directly to the names of the corresponding variables.

2.1 The pricing problem

Every time a firm decides to gather and process information *and/or* adjust its price it incurs a real fixed cost, which we refer to as the *pricing cost*. Therefore, information collection and processing, and price adjustments are undertaken infrequently, and the optimal pricing policy amounts to choosing a sequence of pricing dates. At each such date the firm decides on the next pricing date and sets a price that will be fixed until then.[9] The choice of the optimal time interval between pricing dates weights the benefits of updating information and changing prices frequently against the pricing cost.

In Appendix B we formulate this problem from first principles and show that under certain conditions it can be approximated by the following dynamic programing problem:

$$V(s_t) = \min_{z,\tau} E_t \left[\int_0^\tau e^{-\rho r} \left[z - \left(p^*_{t+r} - p^*_t \right) \right]^2 dr + e^{-\rho\tau} \left(F + V(s_{t+\tau}) \right) \right], \tag{2}$$

where V is the present value of profit losses due to existence of pricing costs, F is the (normalized) pricing cost as a share of steady-state profits, τ denotes the time until the next pricing date, and $z \equiv x_t - p^*_t$ denotes the discrepancy between the price set at t, x_t, and its frictionless optimal level p^*_t. The term $\left(z - \left(p^*_{t+r} - p^*_t \right) \right)^2$ is proportional - to a second-order approximation - to the flow of profits foregone due to price rigidity. The relevant state of the economy is denoted by s_t, with jth component s_{jt}, and its law of motion is described by $s_{t+\Delta t} = \Omega\left(s_t, \eta_{t,t+\Delta t}\right)$, where $\eta_{t,t+\Delta t}$ is the set of innovations that hit the economy between t and $t + \Delta t$. The state of the economy matters for flow values through its effect on the distribution of future frictionless optimal prices before the next pricing date (p^*_{t+r}, for $0 < r < \tau$) conditional on the information available at time t.

The first-order conditions for problem (2) are:

$$z^*(s_t) = \frac{\rho}{1 - e^{-\rho\tau^*(s_t)}} \int_0^{\tau^*(s_t)} e^{-\rho r} E_t \left(p^*_{t+r} - p^*_t \right) dr, \tag{3}$$

$$E_t \left[\left(z^*(s_t) - \left(p^*_{t+\tau^*(s_t)} - p^*_t \right) \right)^2 \right] = \rho F + \rho E_t V\left(s_{t+\tau^*(s_t)} \right) - \frac{\partial}{\partial\tau} E_t V\left(s_{t+\tau^*(s_t)} \right), \tag{4}$$

[9] This behavior is consistent with the evidence in Zbaracki et al. (2004). One should note that in this setting any new information that becomes available to the firm is not taken into account until its next pricing date. This is also a feature of the inattention model of Reis (2006). In contrast, when inattention arises due to information-processing constraints in the spirit of Sims (2003), as in Woodford's (2009) model, firms continuously incur costs to entertain (imperfect) information about the underlying state of the economy.

and the envelope conditions with respect to the components of s_t are:

$$\frac{\partial V(s_t)}{\partial s_{jt}} = \left[\int_0^{\tau^*(s_t)} \frac{\partial E_t \left[z_t^* - \left(p_{t+r}^* - p_t^*\right)\right]^2}{\partial s_{jt}} e^{-\rho r} dr\right] + e^{-\rho\tau^*(s_t)} \frac{\partial}{\partial s_{jt}} E_t V\left(s_{t+\tau^*(s_t)}\right).$$

Equations (2), (3), and (4) together with the envelope conditions fully characterize the optimal pricing rule, as long as the second-order conditions are satisfied. Equation (3) gives the optimal discrepancy. It should be set equal to a weighted average of expected increments in the frictionless optimal price until the next pricing date. Equation (4) characterizes the optimal time interval until the next pricing date. It states that the expected marginal profit loss from postponing the next pricing decision (left-hand side) should be equal to the expected marginal benefit of doing so (right-hand side).

2.2 Inflationary steady state

In analyzing disinflation we start from an inflationary steady state characterized by a constant average growth rate of nominal aggregate demand. Throughout, we also assume that the latter is subject to permanent shocks and follows a Brownian motion with drift:

$$d\mathcal{Y}_t = \pi dt + \sigma d\widetilde{W}_t, \tag{5}$$

where $\widetilde{W}_t$ is a standard Brownian motion.

Given that $p_t^* = \mathcal{Y}_t$, this assumption implies that the conditional distribution of $z - \left(p_{t+r}^* - p_t^*\right)$ given information at t is Gaussian with mean $z - \pi r$ and variance $\sigma^2 r$. It depends only on the time elapsed since time t, and is the same for all firms. As a result, the dynamic problem (2) in the inflationary steady state can be parameterized by π and written as:

$$V_\pi = \min_{z,\tau} E_t \left[\int_0^{\tau} \left(z - \left(p_{t+r}^* - p_t^*\right)\right)^2 e^{-\rho r} dr + e^{-\rho\tau}\left(F + V_\pi\right)\right], \tag{6}$$

where V_π represents the (constant) value function for the steady state problem with nominal aggregate demand growth rate equal to π.

The first-order conditions are:

$$z^* = \frac{\rho}{1 - e^{-\rho\tau^*}} \int_0^{\tau^*} E_t\left(p_{t+r}^* - p_t^*\right) e^{-\rho r} dr, \tag{7}$$

$$E_t\left[z^* - \left(p_{t+\tau^*}^* - p_t^*\right)\right]^2 - \rho\left(V_\pi + F\right) = 0. \tag{8}$$

From (5), (6), (7), and (8) we arrive at a non-linear equation which defines τ^* implicitly.

Then we obtain z^* by using (5) and (7). In Bonomo and Carvalho (2004) we show that the optimal time interval between pricing dates is decreasing in $|\pi|$ and σ, and increasing in F. In addition, higher uncertainty makes such time interval less sensitive to trend inflation.

In our simulations, we set $\sigma = 3\%$ and calibrate F so that with $\pi = 3\%$, $\sigma = 3\%$ and $\rho = 2.5\%$ a year, firms make pricing decisions once a year. As a result we set $F = 0.000595$. This frequency of price changes seems to be a reasonable characterization of price-setting behavior in low inflation environments. It is consistent with the findings of Dhyne et al. (2006) for the Euro area, and with earlier evidence for the U.S. economy (e.g. Carlton, 1986 and Blinder et al., 1998), although it is lower than the frequency of price changes reported by Bils and Klenow (2004) for the U.S. economy.

In order to check the robustness of our calibration, we also compute the optimal time between pricing dates for high and very-high inflation rates. The model performs well when confronted with the Israeli experience reported by Lach and Tsiddon (1992), and it also fits the Brazilian hyperinflation experience of the 80's (Ferreira, 1994). With inflation rates of 77% per year the model predicts spells of price rigidity of 2.6 months, against 2.2 months reported by Lach and Tsiddon (1992). With annual inflation of 210% the spells implied by the model go down to 1.68 months, against 1.38 months reported by Ferreira (1994).

In accounting for the effects of average inflation on the frequency of price changes, the performance of our endogenous time-dependent model is comparable to that of the menu-cost model analyzed by Golosov and Lucas (2007). This has important implications for the literature that aims to discriminate between alternative models of price setting based on microeconomic data (e.g. Klenow and Kryvtsov 2008). In particular, the empirical finding that the frequency of price changes responds to the economic environment cannot be taken as evidence in favor of purely state-dependent pricing behavior.[10] Thus, making progress in this area will require exploring alternative dimensions along which these models can be distinguished.[11] We return to this important open question at the end of Section 4.

2.3 Optimal pricing rule under imperfectly-credible disinflation

In this subsection we derive the optimal pricing rule during disinflation. The dynamic program formulated in (2) encompasses policy uncertainty in general, which enters the problem through the expectations operator. In modeling uncertainty about the disinflation policy, we assume that agents believe that the new policy will be abandoned with some (non-zero)

[10] In the simple version of the model that we use in this paper to study an imperfectly-credible disinflation, the optimal time between pricing dates in the inflationary steady state is constant. More generally, however, the optimal time interval until the next pricing date depends on the *state* of the economy on the current pricing date (see Appendix B, and the discussion in the previous subsection).

[11] One such dimension, explored by Midrigan (2008), is the response *conditional* on identified shocks.

probability. A credibility problem arises when the probability of a policy reversal perceived by agents is higher than the objective probability.

For simplicity, we assume that the objective probability of a policy reversal is zero: the monetary authority never reneges on the promise to disinflate. Therefore, after the stabilization policy is launched at $t = 0$, the actual process for nominal aggregate demand, $\mathcal{Y}_t$, is given by:

$$\begin{aligned} d\mathcal{Y}_t &= \pi' dt + \sigma d\widetilde{W}_t, \\ \mathcal{Y}_0 &= 0, \end{aligned}$$

where π' is the targeted average growth rate for nominal output, and where we introduce the normalization $\mathcal{Y}_0 = 0$. We refer to the case of $\pi' = 0$ as "full disinflation," while $0 < \pi' < \pi$ corresponds to a "partial disinflation." We abstract from the details of the transmission mechanism of monetary policy, and implicitly assume that the monetary policy instrument is used to generate the postulated disinflation path for nominal aggregate demand.

However, agents believe there are multiple possible "types" of monetary authority, characterized by the probability with which they abandon the disinflation policy. Given a finite time interval, each such type has a constant probability of aborting the disinflation.[12] In case the disinflation is abandoned, agents believe that the old policy is resumed and maintained forever. Abandonment is modeled as being triggered by the first arrival of a Poisson counting process with a constant hazard h.

For simplicity, we model agents beliefs as a probability distribution involving two types of policymakers, one being necessarily the true type - the one that never reneges on the promise to disinflate. Imperfect credibility, in this context, is a situation where agents attribute a non-zero probability to a policymaker of type $h > 0$. The higher this probability and h are, the larger the credibility problem is.

In this section we make the credibility problem simpler by assuming that agents are *sure* to be facing a monetary authority of type $h > 0$.[13] Thus, from the agents' perspective the average growth rate of nominal aggregate demand after the new policy is implemented evolves according to:

$$\begin{aligned} d\mathcal{Y}_t^b &= (\pi' + (\pi - \pi')\, \mathbb{1}_{\{N_t \geqslant 1\}}) dt + \sigma d\widetilde{W}_t, \\ \mathcal{Y}_t^b &= 0, \end{aligned}$$

[12] Ball (1995) uses a constant-probability assumption to model policy uncertainty in the context of a monetary disinflation that may be abandoned. In his model agents know the true distribution of abandonment times, and set prices accordingly.

[13] We relax this assumption in Section 4.

where $\mathcal{Y}_t^b$ denotes agents' beliefs about the evolution of nominal aggregate demand, N_t is a Poisson counting process with constant arrival rate h, and $\mathbb{1}_{\{\cdot\}}$ is the indicator function. With this notation, in agents' minds $N_t = 0$ if the disinflation has been maintained up to time t, and $N_t \geq 1$ otherwise.

Given agents' certainty that they are facing a monetary authority of type $h > 0$, this parameter becomes the relevant measure of credibility. If h is high agents believe the monetary authority will renege with high probability, and vice versa if h is low. The subjective probability that stabilization will last until time t is given by e^{-ht}. If $h = 0.5$ (at an annual rate), the subjective probability that the stabilization will last at least one year is 61%. The polar cases of perfect and no credibility correspond to $h = 0$ and $h = \infty$, respectively.

In general, solving for the optimal pricing rule requires solving an optimization and an aggregation problem simultaneously: the optimal pricing rule depends on the expected path for the aggregate price level and other aggregate variables, which in turn result from the aggregation of agents' behavior in equilibrium. However, if the optimal pricing problem can be expressed solely as a function of exogenous variables, the optimization and aggregation problems can be solved sequentially, in that order.

Our model economy satisfies that condition, due to the absence of strategic complementarity or substitutability in price setting, and of any other dependence of the optimal pricing problem on endogenous variables.[14] When making pricing decisions firms only care about the evolution of nominal aggregate demand, and therefore we can solve for the optimal pricing rule independently of equilibrium considerations.[15] Moreover, the fact that we model the frictionless optimal price as a random walk, combined with the assumptions that (eventual) policy shifts involve instantaneous jumps between regimes, and that policy reversals arrive according to a constant-hazard process, simplify the pricing problem substantially.

The relevant state of the economy after the disinflation is launched can be summarized by the Poisson counting process N_t, which indicates whether disinflation has been abandoned up to time t ($N_t \geq 1$) or not ($N_t = 0$).[16] If a policy reversal has occurred before time t, the pricing problem becomes identical to that of the original inflationary steady state. Otherwise, the problem of a firm on a pricing date incorporates the possibility of the disinflation being

[14] This follows from our assumptions on preferences and technology, which are spelled out in the appendices.

[15] The absence of interactions in pricing decisions is common in state-dependent pricing models, where aggregation can be cumbersome (e.g. Caplin and Leahy 1991, Almeida and Bonomo 2002, and Golosov and Lucas 2007). Caplin and Leahy (1997), Gertler and Leahy (2008), and Nakamura and Steinsson (2010) are noticeable exceptions.

[16] Note that policy reversals are a possibility that agents take into account when setting prices; but they never materialize.

abandoned sometime in the future:

$$V_h(N_t) = \begin{cases} \min_{z,\tau} \mathcal{V}_h(z,\tau), \text{ if } N_t = 0 \\ V_\pi, \text{ if } N_t \geq 1, \end{cases} \tag{9}$$

where

$$\mathcal{V}_h(z,\tau) \equiv G_h(z,\tau) + e^{-\rho\tau}\left(F + e^{-h\tau}V_h(0) + \left(1 - e^{-h\tau}\right)V_\pi\right), \text{ and} \tag{10}$$

$$\begin{aligned} G_h(z,\tau) \equiv\ & e^{-h\tau}\left[\int_0^\tau \left((z - \pi' r)^2 + \sigma^2 r\right) e^{-\rho r} dr\right] \\ & + \int_0^\tau \begin{bmatrix} \int_0^r \left((z - \pi' s)^2 + \sigma^2 s\right) e^{-\rho s} ds + \\ \int_r^\tau \left((z - \pi(s - r) - \pi' r)^2 + \sigma^2 s\right) e^{-\rho s} ds \end{bmatrix} h e^{-hr} dr. \end{aligned} \tag{11}$$

In (9), $\mathcal{V}_h$ is the full present value of the expected costs due to pricing frictions when the perceived hazard rate of abandonment is h, the starting discrepancy is z, and the time interval until the next pricing date is τ (assuming that subsequent choices are made optimally). $G_h(z,\tau)$ is the expected cost due to deviations from the frictionless optimal price during the next interval of length τ, starting with the discrepancy z. If a policy reversal occurs in the near future (i.e. before $t+\tau$), agents will account for it on their *next* pricing date.[17] Then, the new pricing decision will be made under conditions identical to the original inflationary steady state. This results in the value function V_π. In (11), the first line of the expression refers to the subjective probability that the stabilization will be maintained during the next interval of length τ multiplied by the cost in this case. The second line gives the cost if abandonment occurs before the next pricing date. It considers each possible abandonment time $t+r$, and adds the resulting costs weighted by the (subjective) likelihood of each event.

The first-order conditions for the problem (9) are presented in Appendix C. They can be combined with (6) and (9) to obtain a nonlinear equation in τ_h^*, which can be solved numerically. Then, with τ_h^* we can compute z_h^* using the first-order condition taken with respect to z.

Figure 1 shows the optimal time interval between pricing dates as a function of the level of credibility in a full disinflation, for two levels of initial inflation ($\pi = 0.1$ and $\pi = 0.2$). It shows that the lower the credibility is (the higher h), the shorter the duration of price spells is. A lower level of credibility implies higher expected trend inflation, increasing the expected profit loss from having a fixed price during a spell of a given duration. Thus, if the

[17] In Bonomo and Carvalho (2004), firms are allowed to reevaluate their pricing policies when the disinflation is announced. This leads to important changes in the results for high - but not for low - inflation environments. We conjecture that a similar conclusion obtains with respect to abandonment under imperfect credibility, for the same reasons outlined in that paper.

duration is unchanged, the expected marginal loss at the next pricing date will exceed the expected marginal benefit of postponing the pricing date. Therefore, firms are led to reduce the time interval between pricing dates in order to restore the balance between the marginal benefit and cost of postponing a price change.

3 Aggregate results

3.1 Aggregation methodology

We assume that, prior to disinflation, pricing dates are distributed uniformly over time. Having solved for the optimal pricing rule before and after the disinflation is announced, we can compute the sequence of pricing dates chosen by firms that change prices at any given time. Thus, to obtain the aggregate price level at any point in time after $t = 0$, we can trace back the last pricing date of all firms, and aggregate the corresponding prices.[18] For brevity we present the details of the aggregation algorithm in Appendix D.

3.2 Results

In general, the results of the disinflation will depend on the realization of the stochastic component of nominal output. We focus on the average result across all possible realizations.[19]

In Figure 2 we illustrate why taking into account the optimality of pricing rules might be relevant for assessing the direct effect of imperfect credibility appropriately. We compare the output effects of perfectly- and imperfectly-credible disinflations, fixing the same arbitrary duration of price spells for two different initial inflation rates.[20] It is apparent that the direct effect is more important for higher inflation rates. The reason is that, given the same time interval between pricing dates, agents set higher prices because of the risk of facing higher trend inflation in case the stabilization is abandoned before their next pricing date. In Figure 3, on the other hand, the duration of price spells is fixed at the optimal level for each initial inflation rate. The relation between inflation and the direct effect of imperfect credibility is now unclear. The reason is that the spells of price rigidity are shorter for higher initial

[18] If the time interval between pricing dates remained constant despite the disinflation, any future time would be a pricing date for some firms, given that we start from a uniformly staggered distribution of pricing dates. This would simplify aggregation tremendously. However, because the optimal spell of price rigidity changes after the disinflation, this is not the case in our model, and we need to keep track of when firms choose to adjust.

[19] Note that this is *not* the same as simply assuming that there are no shocks to nominal aggregate demand, since uncertainty about the path of the latter affects the optimal time between pricing dates.

[20] The durations of spells are fixed at the level corresponding to the optimum for $\pi = 3\%$.

inflation rates and so, despite the fact that inflation would be higher in the case of a policy reversal, the probability that this event happens before the next pricing date is now smaller.

These results illustrate the importance of the extent of price rigidity to the assessment of the direct effect of imperfect credibility. Therefore, in all of our subsequent experiments, we fix the duration of price spells under exogenous rules at the optimal level implied by our model for the initial inflationary steady state. This is a suitable assumption for the experiments we analyze, which are unexpected disinflations. We start from an inflationary steady state which is expected to last, and so it makes sense to use spells of price rigidity which are compatible with that steady state. This allows us to properly assess the indirect effect of imperfect credibility, by appropriately taking the direct effect into account.

Figure 4 depicts the output effects of a full disinflation with our baseline calibration for two levels of credibility ($h = 0.5$, and $h = 2$), with both endogenous and exogenous pricing rules. The case of perfect credibility ($h = 0$) is presented for comparison purposes. As expected, with imperfect credibility the recession generated is larger. It is clear that endogeneity of pricing rules reinforces this result. This happens because the time interval between pricing dates increases after the disinflation begins, as firms optimally respond to lower expected inflation. With perfect credibility, as shown in Bonomo and Carvalho (2004), in the case of full disinflation and no strategic complementarity in price setting, the output costs of disinflation are the same with endogenous or exogenous pricing rules. The reason is that every firm that adjusts after the disinflation is announced knows that on average its frictionless optimal price will remain constant. As a result, the systematic effects of the new policy only depend on the timing of price changes, which is determined by the distribution of pricing dates that was in place before the announcement (assumed to be the same under exogenous and endogenous pricing rules). With imperfect credibility this result ceases to be true, since agents attribute some probability that the monetary authority will abandon the stabilization before their next pricing date, in which case inflation will resume. With endogenous pricing rules, prices are optimally set for a longer interval when compared with exogenous rules, which implies a higher (subjective) probability of abandonment before the next pricing date. Therefore, prices are set at higher levels and the recession is larger. This is a result of the interaction between imperfect credibility and the optimally chosen frequency of repricing.

If credibility is lower, the duration of price spells increases less after the disinflation is announced, and so the differences between endogenous and exogenous pricing rules are attenuated. On the other hand, the differences relative to the case of perfect credibility are amplified due to the direct effect of imperfect credibility, as can be noted in Figure 4.

In Figure 5 we explore the role of the uncertainty that stems from the shocks to nominal aggregate demand. The lower σ is, the more responsive is the frequency of price changes to trend inflation. So, when $\sigma = 0$ the differences between endogenous and exogenous pricing rules are amplified.

These results on the effects of different levels of credibility and uncertainty illustrate important general features of the interaction between imperfect credibility and the optimal pricing rule, which also apply to the other results that we present. To avoid having too many simulations, however, we illustrate them only through the previous experiments.

A partial disinflation presents some qualitative differences when compared to a full disinflation. The reason is that, with nominal rigidity in individual prices, the expected discrepancy while there is no individual price adjustment only remains constant when the inflation drift is zero. So, in contrast with the full-disinflation case, in a partial disinflation a longer time interval between pricing dates will induce firms to set higher prices even with full credibility. With partial disinflation and imperfect credibility, continuing inflation and the probability of a policy reversal interact with the time interval between pricing dates, and affect pricing decisions. Given the optimally chosen longer spell of price rigidity, firms incorporate both the (higher) probability of abandonment and ongoing inflation when setting their prices. As a consequence, the recession tends to be larger.

Figure 6 shows the result of a partial disinflation under imperfect credibility for both exogenous and endogenous pricing rules. As expected, the latter generate a larger recession, but also output cycles. These cycles result from gaps in the new distribution of pricing dates, which are generated by the sudden increase in the optimal time interval between pricing decisions.[21]

4 Disinflation with learning

The results analyzed so far correspond to a situation in which the monetary authority never reneges on the announced disinflation, but nevertheless agents continue to believe that there is always the same probability of a policy reversal. Thus, on average the recession continues indefinitely, which is clearly unrealistic. This result arises from the conjunction of two assumptions: initial beliefs that do not correspond to the "true type" of the monetary authority, and lack of updating of such beliefs as disinflation evolves, due to a degenerate prior belief that puts zero probability on the true type of the monetary authority.

As we argued previously, discrepancies between agents' beliefs and the true type of the monetary authority capture the essence of the problem faced by a monetary authority that

[21] Note that those gaps also occur in the case of full disinflation. However, they cause no output oscillation because on average firms keep their prices constant.

is serious about disinflating, but has low credibility. Lack of updating of beliefs, on the other hand, is clearly an extreme and unrealistic assumption, which we drop in this section.

We analyze how credibility evolves during disinflation, and how this interacts with optimal price setting to determine the output costs of disinflation. Initially, all agents hold the same beliefs about the types of the monetary authority that they might be facing. After the disinflation is launched, on every pricing date firms update their beliefs, taking into account whether or not disinflation has been abandoned. Updating is done according to Bayes' rule.

We first present the framework with learning, and derive the optimal pricing rule. We then compare the costs of disinflation under endogenous and exogenous pricing rules.

4.1 Optimal pricing rule

We continue to assume that agents believe there are two possible types for the monetary authority, characterized by the constant hazard rate for the Poisson process according to which it reneges on the promise to disinflate: the true type that never reneges on the promise (which we refer to as "type 0"), and a type with $h > 0$. When the disinflation policy is launched at $t = 0$, agents have the same belief about the type of monetary authority they face. In what follows μ_0 denotes the prior probability of the monetary authority being of type 0. Imperfect credibility amounts to prior beliefs that assign positive probability to type h (i.e. $\mu_0 < 1$). Moreover, for any given belief μ_0 a higher value of h worsens the credibility problem, as in the previous sections. The setup analyzed earlier can be seen as a special case of the model with learning, with a degenerate prior that assigns zero probability to the true type of the monetary authority (i.e. $\mu_0 = 0$).

At any time $t > 0$, whenever firms incur the pricing cost to gather and process information and make pricing decisions, they observe whether disinflation has been abandoned and, conditional on no abandonment, form the posterior μ_t, according to Bayes' rule:[22]

$$\begin{aligned}
\mu_t &\equiv \Pr\{type = 0 | N_t = 0\} \\
&= \frac{\Pr\{type = 0, N_t = 0\}}{\Pr\{type = 0, N_t = 0\} + \Pr\{type = h, N_t = 0\}} \\
&= \frac{\mu_0}{\mu_0 + (1 - \mu_0)\, e^{-ht}}. \qquad (12)
\end{aligned}$$

Now the set of state variables for the pricing problem is augmented by the posterior belief μ_t, given by (12). Given the parameter h and the initial belief μ_0, the posterior is a function

[22] Agents would also update their beliefs if they observed that the disinflation had been abandoned. However, this never happens since we consider the problem of a monetary authority that in reality never reneges on its promise to disinflate.

only of the time elapsed since disinflation was launched. If a policy reversal has occurred before time t, the pricing problem becomes identical to that of the original inflationary steady state. Otherwise, the problem of a firm on a pricing date incorporates the possibility of the disinflation being abandoned sometime in the future, according to its beliefs:

$$V(N_t, \mu_t) = \begin{cases} \min\limits_{z,\tau} \left[\begin{array}{c} \mu_t G_0(z,\tau) + (1-\mu_t) G_h(z,\tau) \\ +e^{-\rho\tau} \left(\begin{array}{c} F + \left(\mu_t + (1-\mu_t) e^{-h\tau}\right) V\left(0, \mu_{t+\tau}\right) \\ + \left(1 - \left(\mu_t + (1-\mu_t) e^{-h\tau}\right)\right) V_\pi \end{array} \right) \end{array} \right], \text{ if } N_t = 0 \\ \\ V_\pi, \text{ if } N_t \geq 1. \end{cases}, \tag{13}$$

where $G_h(z,\tau)$ is as defined in (11).

The first-order conditions for this problem are presented in Appendix C. Combined with (13), they characterize the solution conditional on no abandonment, which we denote by z_t^*, τ_t^* and $V(0,\mu_t)$. To find the solution, we first pick a large $\bar{t}$ so that for $t > \bar{t}$, $\mu_t \approx 1$. This is justified: conditional on no abandonment, the subjective probability that the monetary authority is of type 0 keeps increasing, and approaches one asymptotically. As a result, for $t > \bar{t}$, $V(0,\mu_t) \approx V_0(0)$. Then, we proceed by moving backwards in time: for each t, we find z_t^*, τ_t^* and use them to compute $V(0,\mu_t)$, which is then used to find z^*, τ^* at earlier times. Additional details are provided in Appendix C.

4.2 Results

Figure 7 presents the path for the optimal time interval between pricing dates during a full disinflation. When the disinflation begins at $t = 0$, firms who are on a pricing date choose to fix prices for longer periods when compared to the inflationary steady state. The initial jump is a reaction to the announcement of the new policy, which lowers expected trend inflation. As the disinflation evolves, the monetary authority gains credibility and firms who make pricing decisions subsequently choose progressively longer spells of price rigidity. In the limit, as $t \to \infty$, agents end up believing that the monetary authority is actually not going to renege, and so the optimal frequency of price changes approaches the new steady state.

The paths for output under both endogenous and exogenous pricing rules are presented in Figure 8. They share the general features of the full-disinflation case without learning (Figure 4), with one noticeable exception: now, as credibility builds up, output reverts towards the steady state level. Once more, the recession is larger under endogenous pricing rules.

The differences between those results and the ones for a full disinflation without learning hinge on the process of updating of beliefs. According with our assumptions, on pricing dates firms update their beliefs about the type of the monetary authority they face. Because firms have different pricing dates, at each point in time there is a distribution of beliefs among price-setters, which can be represented by $\{\mu_{it}\}_{i\in[0,1]}$, where $\mu_{it} \equiv \Pr\{type = 0|N_{t_i} = 0\}$, and $t_i \leq t$ represents firm i's last pricing date.

We summarize the evolution of this distribution of beliefs during disinflation by its mean ($\overline{\mu}_t \equiv \int_0^1 \mu_{it}di$) and standard deviation ($\sigma_t^\mu \equiv \sqrt{\int_0^1 (\mu_{it} - \overline{\mu}_t)^2 di}$). When the disinflation is launched all agents hold the same belief, given by the common prior μ_0. As disinflation evolves, firms that undertake price revisions update their beliefs μ_{it} upwards, and therefore the average belief $\overline{\mu}_t$ increases, at the same time as σ_t^μ starts to indicate dispersion in the corresponding distribution. This process continues for a while, with beliefs becoming more dispersed as firms choose to reprice less often and make decisions on different pricing dates, until a point where the tendency reverts and beliefs start to converge. Meanwhile, the average belief $\overline{\mu}_t$ increases steadily towards unity.

4.3 Understanding the frequency of price changes

During the imperfectly-credible disinflation under learning, actual inflation drops faster than expected inflation, as can be seen in Figure 9.[23] At the same time, the optimal interval between pricing dates increases slowly (Figure 7). Thus, in light of the steady-state comparative statics that relate the frequency of price changes to the inflation drift (Subsection 2.2), an observer trying to obtain a relationship between actual inflation and the frequency of price changes might be puzzled: despite only minor changes in inflation, the duration of price spells increases systematically after the disinflation; as a result, the observed ex-post variation in the duration of price spells is essentially unrelated to actual inflation. A starker but qualitatively similar result obtains if we use actual and expected nominal output growth instead of inflation.

The reason for this disconnect is that in our optimal time-dependent pricing model the frequency of price changes depends on expectations about the evolution of nominal output until the next pricing date, rather than on the actual evolution of nominal output. This follows from the fact that each pricing date is predetermined as of the previous pricing date. With imperfect credibility, the expected evolution of nominal output can differ significantly

[23] For each time t, actual and expected inflation correspond to the subsequent one-year period. The latter is obtained by integrating the possible values over all realizations of nominal output under the distribution implied by the posterior belief μ_t.

from the actual evolution for a substantial period of time. One should thus expect to find a more clear-cut relationship between the frequency of price changes and expected - rather than actual - nominal output growth. This is indeed the case. Figure 10 shows a scatter plot of the optimal durations of price spells during the disinflation against expected nominal output growth. To construct the plots we use the value of the optimal duration of price spells at each time t (same as in Figure 7), and the expected one-year-ahead nominal output growth obtained by integrating the possible values over all realizations of nominal output under the distribution implied by the posterior belief μ_t. In contrast, a scatter plot of the observed price spells against *actual* nominal output growth shows no relationship between these variables.

Before taking this implication of our model at face value, it is important to realize that its extreme form depends on some of our simplifying assumptions. In particular, the result that the optimal duration of price spells is completely independent of the realization of nominal output depends on the assumption that its random component is unpredictable, and on the absence of pricing complementarity or substitutability. In a more general version of the model, the optimal time between two pricing dates depends on the actual underlying state of the economy as of the earlier pricing date, as well as on expectations about the evolution of the underlying state of the economy (see Section 2; in particular equation (2)). Moreover, the complete disconnect between the frequency of price changes and the actual evolution of nominal output also requires that firms not entertain *any* information between pricing dates. If firms assess imperfect information about the underlying state of the economy between pricing dates - as opposed to no information - the frequency of price changes should depend on both its evolution, and expectations thereof.[24]

On the other hand, there is a sense in which the implication of our model that we just analyzed holds more generally: it applies to any situation in which realizations of the variables that determine the optimal pricing rule differ from their expected values - not only because of imperfect credibility. We believe that this insight, if combined with a more general model of price setting, can be used to assess the extent of time-dependency in price setting.

5 Conclusion

The role of credibility in monetary disinflations depends critically on the extent of price rigidity. This paper evaluates the effect of imperfect credibility of a disinflation policy in a

[24]Woodford (2009) studies a model in which agents have imperfect information about the underlying state of the economy between pricing dates, due to flow information-processing costs. However, he assumes that accessing previously known information is equally costly, and as a result firms optimally choose not to do so. This shuts down the effect of past expectations about the current underlying state of the economy.

model in which the time period between individual price adjustments is chosen optimally based on the information available as of the earlier price-adjustment date. As a result we are able to evaluate both the direct effect of credibility, for a given frequency of price adjustments, and the indirect effect, which is engendered by the optimality of the pricing rule.

We find that imperfect credibility and endogeneity of pricing rules interact to generate larger output costs of disinflation. For a given frequency of price changes, imperfect credibility increases the costs of disinflation because agents assign some probability that the stabilization will be abandoned before their next pricing date, and therefore set prices higher than in the case of full credibility. With endogenous pricing rules, when faced with lower expected trend inflation after the disinflation is announced, firms optimally choose to change prices less frequently. This raises the perceived probability of a policy reversal occurring between pricing dates, and amplifies the difference between individual prices set under perfect and imperfect credibility.

We find the results to be encouraging enough to justify further research, both theoretical and empirical, based on endogenous time-dependent pricing models. From a normative perspective our results point to the importance of analyzing the welfare implications of alternative disinflation strategies under optimal pricing rules. Danziger (1988) and Ireland (1997) perform such an analysis in models in which pricing is (at least partially) state-dependent. While our results show that state- and endogenous time-dependent pricing models share many similarities in terms of the behavior predicted at the microeconomic level, their aggregate implications can differ dramatically. Thus it seems worthwhile to analyze the welfare implications of disinflation policies under endogenous time-dependent pricing. Finally, in empirical terms an important remaining challenge is to find ways to distinguish between these two as well as alternative models of price setting.

References

[1] Almeida, H. and M. Bonomo (2002), "Optimal State-Dependent Rules, Credibility and Inflation Inertia," *Journal of Monetary Economics* 49: 1317-1336.

[2] Backus, D. and J. Driffill (1985a), "Rational Expectations and Policy Credibility Following a Change in Regime," *Review of Economic Studies* 52: 211-221.

[3] ____________ (1985b), "Inflation and Reputation," *American Economic Review* 75: 530-538.

[4] Ball, L. (1995), "Disinflation with Imperfect Credibility," *Journal of Monetary Economics* 35: 5-23.

[5] Ball, L., G. Mankiw and D. Romer (1988), "The New Keynesian Economics and the Output-Inflation Trade-off," *Brookings Papers on Economic Activity* 1: 1-65.

[6] Barro, R. (1972), "A Theory of Monopolistic Price Adjustment," *Review of Economic Studies* 39: 17-26.

[7] Bils, M. and P. Klenow (2004), "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy* 112: 947-985.

[8] Blinder, A., E. Canetti, D. Lebow, and J. Rudd (1998), *Asking about Prices: A New Approach to Understanding Price Stickiness*, Russel Sage Foundation.

[9] Bonomo, M. and C. Carvalho (2004), "Endogenous Time-Dependent Rules and Inflation Inertia," *Journal of Money, Credit and Banking* 36: 1015-1041.

[10] Caballero, R. (1989), "Time-Dependent Rules, Aggregate Stickiness and Information Externalities," Columbia Working Paper 428.

[11] Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12: 383-98.

[12] Caplin, A. and J. Leahy (1991), "State-Dependent Pricing and the Dynamics of Money and Output," *Quarterly Journal of Economics* 106: 683-708.

[13] ____________ (1997), "Aggregation and Optimization with State-Dependent Pricing," *Econometrica* 65: 601-625.

[14] Caplin, A. and D. Spulber (1987), "Menu Costs and the Neutrality of Money," *Quarterly Journal of Economics* 102: 703-726.

[15] Carlton, D. (1986), "The Rigidity of Prices," *American Economic Review* 76: 637-658.

[16] Danziger, L. (1988), "Costs of Price Adjustment and the Welfare Economics of Inflation and Disinflation," *American Economic Review* 78: 633-646.

[17] ____________ (1999), "A Dynamic Economy with Costly Price Adjustments," *American Economic Review* 89: 878-901.

[18] Dhyne, E., L. Álvarez, H. Le Bihan, G. Veronese, D. Dias, J. Hoffman, N. Jonker, P. Lünnemann, F. Rumler and J. Vilmunen (2006), "Price Changes in the Euro Area and the United States: Some Facts from Individual Consumer Price Data," *Journal of Economic Perspectives* 20: 171-192.

[19] Erceg, C. and A. Levin (2003), "Imperfect Credibility and Inflation Persistence," *Journal of Monetary Economics* 50: 915-944.

[20] Ferreira, S. (1994), "Inflação, Regras de Reajuste e Busca Sequencial: Uma Abordagem sob a Ótica da Dispersão de Preços Relativos," M.A. Dissertation, PUC-Rio.

[21] Gertler, M. and J. Leahy (2008), "A Phillips Curve with an Ss Foundation," *Journal of Political Economy* 116: 533-572.

[22] Golosov, M. and R. Lucas (2007),"Menu Costs and Phillips Curves," *Journal of Political Economy* 115: 171-199.

[23] Ireland, P. (1997), "Stopping Inflations, Big and Small," *Journal of Money, Credit, and Banking* 29: 759-775.

[24] Klenow, P. and O. Kryvtsov (2008), "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?," *Quarterly Journal of Economics* 123: 863-904.

[25] Lach, S. and D. Tsiddon (1992), "The Behavior of Prices and Inflation: An Empirical Analysis of Disaggregated Price Data," *Journal of Political Economy* 100: 349-389.

[26] Levin, A. and T. Yun (2007), "Reconsidering the Natural Rate Hypothesis in a New Keynesian Framework," *Journal of Monetary Economics* 54: 1344-1365.

[27] Mankiw, G., and R. Reis (2002), "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics* 117: 1295-1328.

[28] Midrigan, V. (2008), "Is Firm Pricing State or Time-Dependent? Evidence from US Manufacturing," forthcoming in the *Review of Economics and Statistics.*

[29] Nakamura, E. and J. Steinsson (2010), "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model," forthcoming in the *Quarterly Journal of Economics.*

[30] Nicolae, A. and C. Nolan (2006), "The Impact of Imperfect Credibility in a Transition to Price Stability," *Journal of Money, Credit, and Banking* 38: 47-66.

[31] Reis, R. (2006), "Inattentive Producers," *Review of Economic Studies* 73: 793-821.

[32] Romer, D. (1990), "Staggered Price Setting With Endogenous Frequency of Adjustment," *Economics Letters* 32: 205-210.

[33] Sargent, T. (1983), "Stopping Moderate Inflations: The Methods of Poincare and Thatcher," in: Dornbusch, R. and M. Simonsen (eds.), *Inflation, Debt and Indexation*, MIT Press, Cambridge, MA.

[34] Sheshinski, E. and Y. Weiss (1977), "Inflation and Costs of Price Adjustment," *Review of Economic Studies* 44: 287-304.

[35] Sims, C. (2003), "Implications of Rational Inattention," *Journal of Monetary Economics* 50: 665-690.

[36] Siu, H. (2008), "Time Consistent Monetary Policy with Endogenous Price Rigidity," *Journal of Economic Theory* 138: 184-210.

[37] Taylor, J. (1979), "Staggered Wage Setting in a Macro Model," *American Economic Review* 69: 108-113.

[38] ____________ (1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* 88: 1-23.

[39] Westelius, N. (2005), "Discretionary Monetary Policy and Inflation Persistence," *Journal of Monetary Economics* 52: 477-496.

[40] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

[41] ____________ (2009), "Information-Constrained State-Dependent Pricing," *Journal of Monetary Economics* 56, Supplement: S100-S124.

[42] Zbaracki, M., M. Ritson, D. Levy, S. Dutta and M.J., M. Bergen (2004), "Managerial and Customer Dimensions of the Costs of Price Adjustment: Direct Evidence from Industrial Markets," *Review of Economics and Statistics* 86: 514-533.

Appendix A

Here we derive the frictionless optimal price in a general equilibrium framework. A representative consumer maximizes the following utility function:

$$E_{t_0} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left[\log(C_t) - H_t\right] dt,$$

subject to the budget constraints:

$$B_t = B_0 + \int_0^t W_r H_r dr - \int_0^t \left(\int_0^1 P_{ir} C_{ir} di \right) dr + \int_0^t T_r dr + \int_0^t \Lambda_r dQ_r + \int_0^t \Lambda_r dD_r, \text{ for } t \geq 0,$$

where the composite consumption good over which utility is defined is given by:

$$C_t \equiv \left[\int_0^1 C_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

with $\theta > 1$, and where C_{it} is the consumption of variety i, P_{it} is its price, H_t is the supply of labor, which is remunerated at wage W_t, B_t is total financial wealth, T_t denotes total net transfers, including any lump-sum flow transfer from the government, and profits received from the firms, which are owned by the representative consumer. Q_r is the vector of prices of traded assets, D_r is the corresponding vector of cumulative dividend processes, and Λ_r is the trading strategy, which we assume satisfies conditions that preclude Ponzi schemes. The price index associated with the composite consumption good, P_t, is given by:

$$P_t = \left[\int_0^1 P_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{14}$$

In this setting, the demand for an individual product has the following familiar relation with aggregate demand:

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\theta} C_t. \tag{15}$$

Each firm hires labor to produce its variety of the consumption good according to the following production function:

$$Y_{it} = H_{it}.$$

If producer i could adjust prices continuously, she would choose a price P_t^* to maximize profits according to the usual markup rule:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} Y_t, \tag{16}$$

where Y_t is the real marginal cost of producing Y_{it}. We refer to P_t^* as the firm's *frictionless optimal price*. Using $P_{it} = P_t^*$ and substituting the demand function (15) into (16) leads to:

$$\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\theta}} = \frac{\theta}{\theta - 1} Y_t, \tag{17}$$

where we have used the equilibrium condition $Y_{it} = C_{it}$. The latter, when applied for all i, also implies:

$$Y_t \equiv \left[\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} = C_t. \tag{18}$$

In this economy, a *flexible-price equilibrium* is the one that obtains when all prices are flexible, so that (17) holds for each i, with aggregate output Y_t given by (18). The corresponding level of aggregate output is what is sometimes referred to as the *natural* level of output, Y_t^n. Because we abstract from real shocks in our simple economy, natural output is constant: $Y_t^n = \overline{Y} = \frac{\theta-1}{\theta}$.

To derive a relation between the frictionless optimal price and the output gap, $Y_t/\overline{Y}$, simply rearrange (16) to obtain:

$$\begin{aligned} P_t^* &= P_t \frac{\theta}{\theta - 1} Y_t \\ &= P_t \frac{Y_t}{\overline{Y}}. \end{aligned}$$

Rewriting the above expression in terms of log-deviations from the deterministic steady state yields equation (1) in the main text:

$$p_t^* = p_t + y_t, \tag{19}$$

where $p_t^* = \log\left(P_t^*/\overline{P}\right)$, $p_t = \log\left(P_t/\overline{P}\right)$, $y_t \equiv \log\left(Y_t/\overline{Y}\right)$, and $\overline{P}$ denotes the aggregate price level in the zero-inflation steady state.

Appendix B

Formally, the pricing problem of a firm may be written as:[25]

$$\widetilde{V}\left(s_{t_0}\right)=\max_{\left\{\left(t_j,X_{t_j}\right)\right\}_{j=1}^{\infty}} E_{t_0}\sum_{j=0}^{\infty}e^{-\rho\left(t_j-t_0\right)}\left\{E_{t_j}\left[\int_{t_j}^{t_{j+1}}e^{-\rho r}\Pi\left(\frac{X_{t_j}}{P_r},Y_r\right)dr\right]-e^{-\rho\left(t_{j+1}-t_j\right)}\widehat{F}\right\},$$

so that $\widetilde{V}\left(s_{t_0}\right)$ denotes the expected present value of real profits Π, net of pricing costs $\widehat{F}$, when the state of the economy is s_{t_0} and $\left\{\left(t_j,X_{t_j}\right)\right\}_{j=1}^{\infty}$ denotes the sequence of pricing dates and nominal prices set on each of those dates.

Let $V^*\left(s_{t_0}\right)$ denote the expected present value of profits of a hypothetical identical firm in the same economy that does not face any pricing cost. Then,

$$V^*\left(s_{t_0}\right)=E_{t_0}\left[\int_{t_0}^{\infty}e^{-\rho r}\Pi\left(\frac{P_r^*}{P_r},Y_r\right)dr\right],$$

where P_r^* is the individual price that maximizes real profits at time r, i.e. the frictionless optimal price of the firm. With this auxiliary value function, $\widehat{V}\left(s_{t_0}\right)\equiv V^*\left(s_{t_0}\right)-\widetilde{V}\left(s_{t_0}\right)$ is the minimized expected present value of the real profit losses due to the existence of pricing costs, and our problem can be stated equivalently as one of minimizing the expected present value of such losses:

$$\widehat{V}\left(s_{t_0}\right)=\min_{\left\{\left(t_j,X_{t_j}\right)\right\}_{j=0}^{\infty}} E_{t_0}\sum_{j=0}^{\infty}e^{-\rho\left(t_j-t_0\right)}\left\{E_{t_j}\int_{t_j}^{t_{j+1}}e^{-\rho r}\left[\begin{array}{c}\Pi\left(\frac{P_r^*}{P_r},Y_r\right)\\-\Pi\left(\frac{X_{t_j}}{P_r},Y_r\right)\end{array}\right]dr+e^{-\rho\left(t_{j+1}-t_j\right)}\widehat{F}\right\}.$$

Defining $\widehat{L}\left(\frac{P^*}{P},\frac{P_i}{P},Y\right)\equiv\Pi\left(\frac{P^*}{P},Y\right)-\Pi\left(\frac{P_i}{P},Y\right)$ to be the instantaneous real profit loss due to a "suboptimal" price P_i, we can rewrite $\widehat{V}$ as:

$$\widehat{V}\left(s_{t_0}\right)=\min_{\left\{\left(t_j,X_{t_j}\right)\right\}_{j=0}^{\infty}} E_{t_0}\sum_{j=0}^{\infty}e^{-\rho\left(t_j-t_0\right)}\left\{E_{t_j}\int_{t_j}^{t_{j+1}}e^{-\rho r}\widehat{L}\left(\frac{P_r^*}{P_r},\frac{X_{t_j}}{P_r},Y_r\right)dr+e^{-\rho\left(t_{j+1}-t_j\right)}\widehat{F}\right\}.$$

A recursive formulation to this minimization problem is given by the following Bellman equation:

$$\widehat{V}\left(s_t\right)=\min_{X,\tau}E_t\left[\int_0^{\tau}e^{-\rho r}\widehat{L}\left(\frac{P_{t+r}^*}{P_{t+r}},\frac{X}{P_{t+r}},Y_{t+r}\right)dr+e^{-\rho\tau}\left(\widehat{F}+\widehat{V}\left(s_{t+\tau}\right)\right)\right].$$

[25] Initially we drop the i subscripts in order to simplify the notation.

Let $\overline{\Pi}$ be the steady-state level of real profits in a frictionless economy:

$$\overline{\Pi} \equiv \Pi\left(\frac{P_t^*}{P_t}, \overline{Y}\right) = \Pi\left(1, \overline{Y}\right).$$

We can renormalize the pricing problem by $\overline{\Pi}$ and rewrite it as:

$$\overline{V}(s_t) = \min_{X,\tau} E_t\left[\int_0^{\tau} e^{-\rho r}\overline{L}\left(\frac{P_{t+r}^*}{P_{t+r}}, \frac{X}{P_{t+r}}, Y_{t+r}\right) dr + e^{-\rho\tau}\left(\overline{F} + \overline{V}(s_{t+\tau})\right)\right], \tag{20}$$

where $\overline{V}(s_t) \equiv \frac{\widehat{V}(s_t)}{\overline{\Pi}}$, $\overline{L}\left(\frac{P^*}{P}, \frac{X}{P}, Y\right) \equiv \frac{\widehat{L}\left(\frac{P^*}{P}, \frac{X}{P}, Y\right)}{\overline{\Pi}}$, $\overline{F} \equiv \frac{\widehat{F}}{\overline{\Pi}}$.

Given the primitives for preferences and technology, the expression for flow real profits can be written as:[26]

$$\left(\frac{P_i}{P}\right)^{1-\theta} Y - \frac{W}{P} Y \left(\frac{P_i}{P}\right)^{-\theta},$$

where P_i is the price *charged* by firm i. We can use the labor supply equation for this economy to express the real wage as a function of aggregate output ($\frac{W}{P} = Y$), and rewrite the expression for flow real profits as:

$$\Pi\left(\frac{P_i}{P}, Y\right) = \left(\frac{P_i}{P}\right)^{1-\theta} Y - Y^2 \left(\frac{P_i}{P}\right)^{-\theta}.$$

We will later want to approximate the loss function $\overline{L}$ in (20). Observe that:

$$\begin{aligned}
\overline{L}\left(\frac{P^*}{P}, \frac{P_i}{P}, Y\right) &= \frac{\Pi\left(\frac{P^*}{P}, Y\right) - \Pi\left(\frac{P_i}{P}, Y\right)}{\overline{\Pi}} \\
&= \frac{\Pi\left(\frac{P^*}{P}, Y\right) - \Pi\left(\frac{P_i}{P}, Y\right)}{\Pi\left(\frac{P^*}{P}, Y\right)} \frac{\Pi\left(\frac{P^*}{P}, Y\right)}{\overline{\Pi}}.
\end{aligned} \tag{21}$$

The second ratio can be written as

$$\begin{aligned}
\frac{\Pi\left(\frac{P^*}{P}, Y\right)}{\overline{\Pi}} &= \frac{\left(\frac{P^*}{P}\right)^{1-\theta} Y - Y^2 \left(\frac{P^*}{P}\right)^{-\theta}}{(1)^{1-\theta}\overline{Y} - \overline{Y}^2 (1)^{-\theta}} \\
&= \frac{\left(\frac{P^*}{P}\right)^{1-\theta} Y - \frac{\theta-1}{\theta} Y \left(\frac{P^*}{P}\right)^{1-\theta}}{\overline{Y} - \frac{\theta-1}{\theta}\overline{Y}} \\
&= \frac{Y}{\overline{\overline{Y}}}\left(\frac{P^*}{P}\right)^{1-\theta} \\
&= \left(\frac{Y}{\overline{\overline{Y}}}\right)^{2-\theta},
\end{aligned} \tag{22}$$

[26] Now we reintroduce the i subscript.

where in the second equality we use the facts that:

$$\begin{aligned}\frac{P^*}{P} &= \frac{\theta}{\theta-1}Y,\\ \overline{Y} &= \frac{\theta-1}{\theta}.\end{aligned}$$

The first ratio in (21) is the proportional profit loss (relative to the level of profits that would obtain if the firm had flexible prices) due to the "suboptimal" price. It is convenient to rewrite it as:

$$\frac{\Pi\left(\frac{P^*}{P},Y\right)-\Pi\left(\frac{P_i}{P},Y\right)}{\Pi\left(\frac{P^*}{P},Y\right)} = 1-\frac{\Pi\left(\frac{P_i}{P},Y\right)}{\Pi\left(\frac{P^*}{P},Y\right)}.$$

The profit ratio in the above expression can be written as:

$$\begin{aligned}\frac{\Pi\left(\frac{P_i}{P},Y\right)}{\Pi\left(\frac{P^*}{P},Y\right)} &= \frac{\left(\frac{P_i}{P}\right)^{1-\theta}Y-Y^2\left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P^*}{P}\right)^{1-\theta}Y-Y^2\left(\frac{P^*}{P}\right)^{-\theta}}\\ &= \frac{\left(\frac{P_i}{P}\right)^{1-\theta}-\frac{\theta-1}{\theta}\frac{P^*}{P}\left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P^*}{P}\right)^{1-\theta}-\frac{\theta-1}{\theta}\frac{P^*}{P}\left(\frac{P^*}{P}\right)^{-\theta}}\\ &= \theta\frac{\left(\frac{P_i}{P}\right)^{1-\theta}-\frac{\theta-1}{\theta}\frac{P^*}{P}\left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P^*}{P}\right)^{1-\theta}}\\ &= \theta\left(\frac{P^*}{P_i}\right)^{\theta-1}-(\theta-1)\left(\frac{P^*}{P_i}\right)^{\theta},\end{aligned}$$

so that:

$$\frac{\Pi\left(\frac{P^*}{P},Y\right)-\Pi\left(\frac{P_i}{P},Y\right)}{\Pi\left(\frac{P^*}{P},Y\right)} = 1-\theta\left(\frac{P^*}{P_i}\right)^{\theta-1}+(\theta-1)\left(\frac{P^*}{P_i}\right)^{\theta}. \tag{23}$$

Combining (22) and (23) and keeping only the relevant arguments, we obtain:

$$\overline{L}\left(\frac{P_i}{P^*},Y\right) = \left(\frac{Y}{\overline{\overline{Y}}}\right)^{2-\theta}\left[1-\theta\left(\frac{P_i}{P^*}\right)^{1-\theta}+(\theta-1)\left(\frac{P_i}{P^*}\right)^{-\theta}\right].$$

We can rewrite the loss function $\overline{L}$ in terms of log-deviations from the deterministic zero-inflation steady state:

$$G\left(p_i-p^*,y\right) = e^{(2-\theta)y}\left[\left(1-\theta e^{(1-\theta)(p_i-p^*)}\right)+(\theta-1)\,e^{-\theta(p_i-p^*)}\right].$$

This allows us to rewrite the optimal pricing problem as:

$$\overline{V}(s_t) = \min_{x,\tau} E_t \left[\begin{array}{c} \int_0^\tau e^{-\rho r} e^{(2-\theta) y_{t+r}} \left[\left(1 - \theta e^{(1-\theta)\left(x - p^*_{t+r}\right)}\right) + (\theta - 1) e^{-\theta\left(x - p^*_{t+r}\right)} \right] dr \\ + e^{-\rho\tau} \left(\overline{F} + \overline{V}(s_{t+\tau}) \right) \end{array} \right].$$

The presence of aggregate output in the loss function implies that solving for the optimal pricing rule involves a fixed-point problem, even in the absence of strategic complementarity or substitutability in price setting. To make the optimal pricing problem more tractable, we eliminate the effect of aggregate output by assuming $\theta = 2$ (as in Danziger 1999). Then, we take a second-order Taylor expansion of flow profit losses around the path for the frictionless optimal price in order to obtain an approximate dynamic pricing problem:

$$\overline{V}_{app}(s_t) = \min_{x,\tau} E_t \left[\int_0^\tau 2e^{-\rho r} \left(x - p^*_{t+r}\right)^2 dr + e^{-\rho\tau} \left(\overline{F} + \overline{V}_{app}(s_{t+\tau}) \right) \right].$$

Finally, defining $V(s_t) \equiv \frac{\overline{V}_{app}(s_t)}{2}$, $F \equiv \frac{\overline{F}}{2}$, and using the discrepancy $z \equiv x - p^*_t$, we arrive at the pricing problem analyzed in the main text:

$$V(s_t) = \min_{z,\tau} E_t \left[\int_0^\tau e^{-\rho r} \left[z - \left(p^*_{t+r} - p^*_t\right) \right]^2 dr + e^{-\rho\tau} \left(F + V(s_{t+\tau}) \right) \right].$$

Appendix C

Here we present the set of equations to which we refer in the main text, but which were omitted for expositional clarity.

First-order conditions for the optimal pricing rule in Subsection 2.3:

$$z_h^* = \frac{\rho}{1-e^{-\rho\tau_h^*}} \int_0^{\tau_h^*} \left[\pi' r + (\pi - \pi')\left(r - \frac{1-e^{-hr}}{h}\right)\right] e^{-\rho r} dr, \tag{24}$$

$$\begin{aligned}&\left(z_h^* - \pi'\tau_h^*\right)^2 + \sigma^2\tau_h^* + he^{-h\tau_h^*}\left(V_\pi - V_h(0)\right) - \rho F - \rho\left(e^{-h\tau_h^*}V_h(0) + \left(1-e^{-h\tau_h^*}\right)V_\pi\right) \\ &+ \int_0^{\tau_h^*}\left((\pi'-\pi)(\tau_h^*-r)\right)^2 he^{-hr}dr + 2(\pi'-\pi)\left(z_h^*-\pi'\tau_h^*\right)\int_0^{\tau_h^*}(\tau_h^*-r)he^{-hr}dr = 0.\end{aligned} \tag{25}$$

First-order conditions for the optimal pricing rule in Subsection 4.1:

$$z_t^* = \frac{\rho}{1-e^{-\rho\tau_t^*}} \int_0^{\tau_t^*} \left[\pi' r + (\pi-\pi')\left(r - \left(\mu_t r + (1-\mu_t)\left(\frac{1-e^{-hr}}{h}\right)\right)\right)\right] e^{-\rho r} dr, \tag{26}$$

$$\begin{aligned}&\left(z_t^* - \pi'\tau_t^*\right)^2 + \sigma^2\tau_t^* + (1-\mu_t)\,he^{-h\tau_t^*}\left(V_\pi - V\left(0, \mu_{t+\tau_t^*}\right)\right) \\ &+ \left(\mu_t + (1-\mu_t)\,e^{-h\tau_t^*}\right)\frac{\partial V\left(0,\mu_{t+\tau_t^*}\right)}{\partial t} \\ &-\rho\left[F + V_\pi + \left(\mu_t + (1-\mu_t)\,e^{-h\tau_t^*}\right)\left(V\left(0,\mu_{t+\tau_t^*}\right) - V_\pi\right)\right] \\ &+ \int_0^{\tau_t^*}\left((\pi'-\pi)(\tau_t^*-r)\right)^2(1-\mu_t)\,he^{-hr}dr \\ &+2(\pi'-\pi)\left(z_t^*-\pi'\tau_t^*\right)\int_0^{\tau_t^*}(\tau_t^*-r)(1-\mu_t)\,he^{-hr}dr = 0.\end{aligned} \tag{27}$$

To avoid numerical derivatives, instead of using (27), we use (26), and (13) to find τ_t^* with a grid search.

Appendix D

This appendix formalizes the aggregation procedure described in Subsection 3.1, based on Bonomo and Carvalho (2004). Let $g(\cdot)$ be the function which gives the next pricing date: $g(t) = t + \tau_t^*$, where τ_t^* denotes the optimal spell chosen at time t.[27] In order to calculate the aggregate price level at an arbitrary time after the disinflation announcement, we use the function g to relate the measure of firms which set their prices on a specific pricing date u to the measure of firms at times before u that would have chosen u as their next pricing date (those times are $g^{-1}(u)$). For that purpose, let $\Gamma(t)$ be the correspondence that assigns to t the set of pricing dates when the current prices were chosen:

$$\Gamma(t) = \{t' : t' \leq t \text{ and } g(t') > t\}. \tag{28}$$

Let $g^{-1}(S)$ be the inverse-image of the set S under g. Then, $g^{-1}(\Gamma(t))$ is the set of pricing dates for which the next pricing date would be in $\Gamma(t)$. To evaluate the average price at t we need to know the probability measure v of the firms which last adjusted at subsets of $\Gamma(t)$. We can easily relate this measure to the measure φ of subsets of $g^{-1}(\Gamma(t))$, since v is the image-measure of φ under g. Then, we have:

$$p_t = \int_{\Gamma(t)} x_r v(dr) = \int_{g^{-1}(\Gamma(t))} x_{g(r)} \varphi(dr), \tag{29}$$

where x_r is the average price of firms which set prices at time r. We apply (29) recursively by relating distributions and pricing dates during disinflation to preceding times. We proceed in this way until we arrive at a set $\Gamma^{-n}(t) \equiv g^{-n}(\Gamma(t))$ such that the measure of firms adjusting at the subset of pricing dates of $\Gamma^{-n}(t)$ corresponds to the uniform distribution of the initial inflationary steady state.

We implement the aggregation algorithm computationally, as follows. We discretize time so that one year has 1000 possible pricing dates. The optimal interval between pricing dates obtained in the previous section is rounded accordingly, so that both the domain and image of g coincide with the time grid. Given g, we move forward in time to find the subset of dates in which some firms actually make pricing decisions. For each such pricing date, we construct the set defined in (28), and aggregate firms' prices according to (29). Between pricing dates, the aggregate price level remains constant. We continue long enough for the transition to the new steady state to be completed.

[27] In the disinflation of Subsection 2.3, with fixed beliefs, τ_t^* is constant and equal to τ_h^*. However, this is no longer the case under learning, in Section 4. Therefore we explain the aggregation method using a notation that can be applied to both cases.

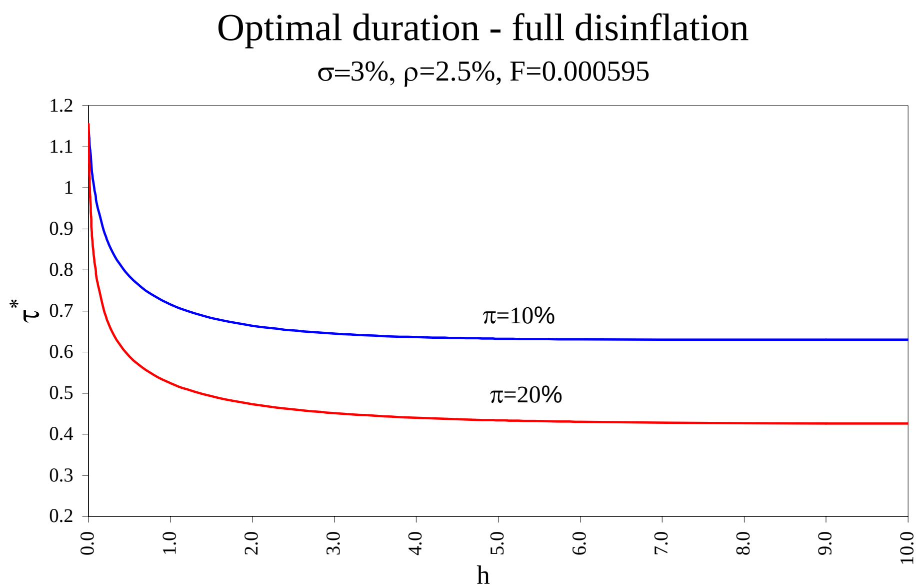


Figure 1

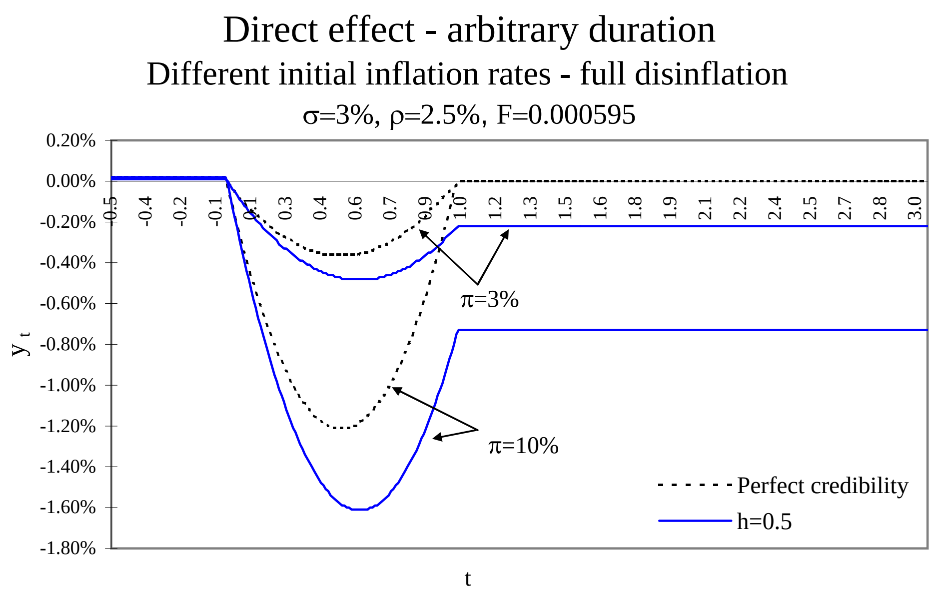


Figure 2

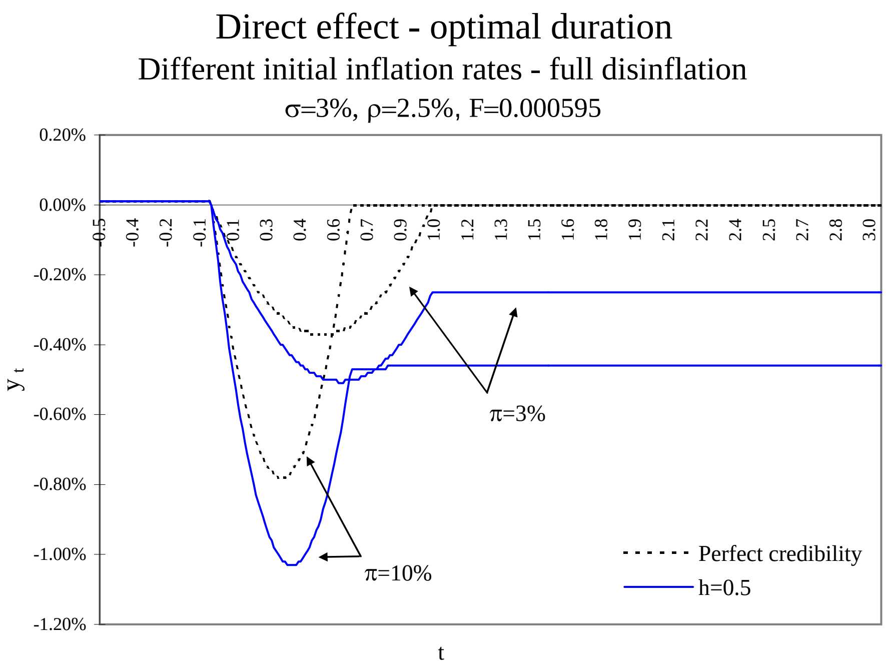


Figure 3

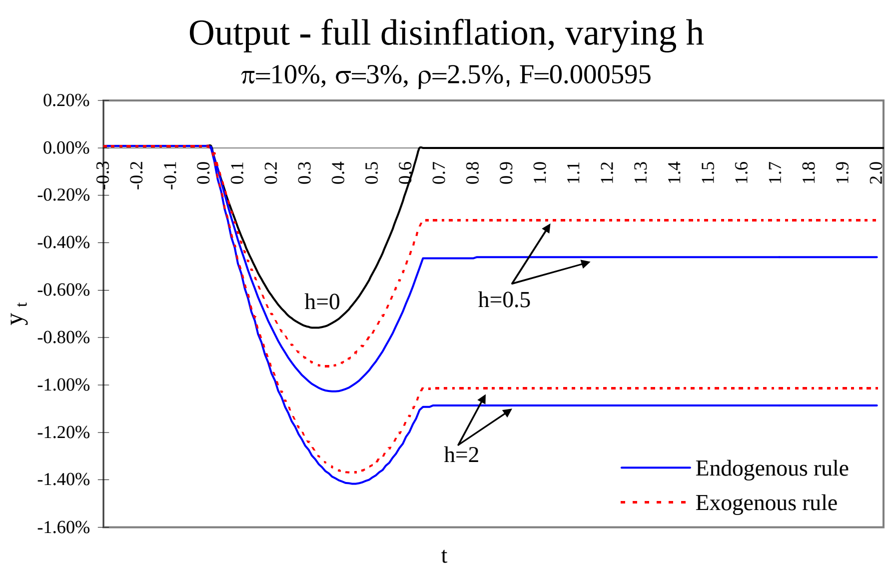
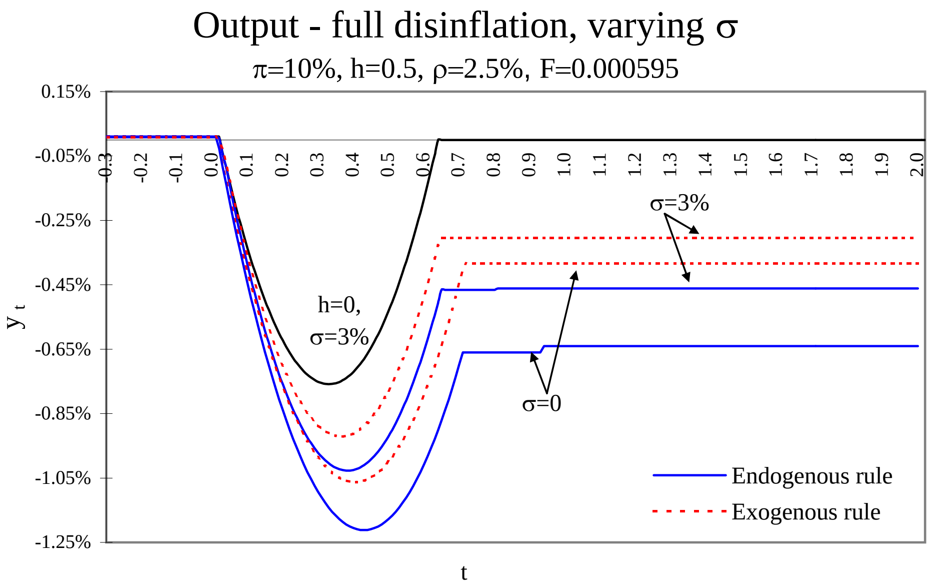


Figure 4



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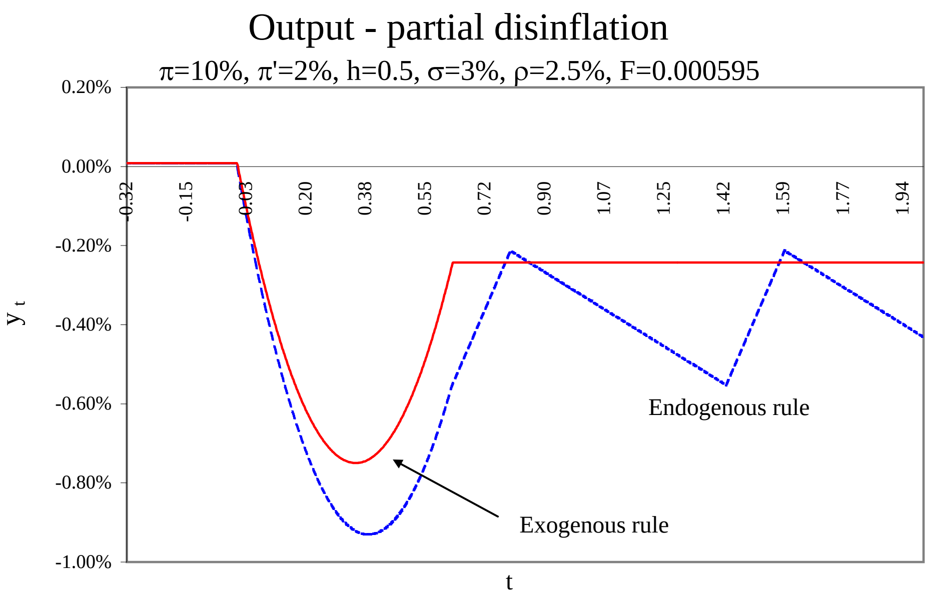


Figure 6

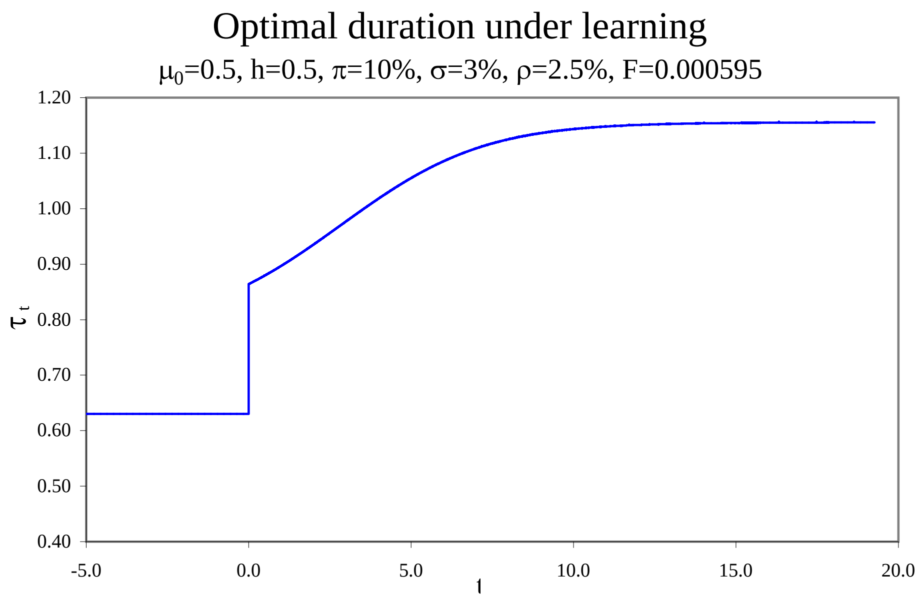
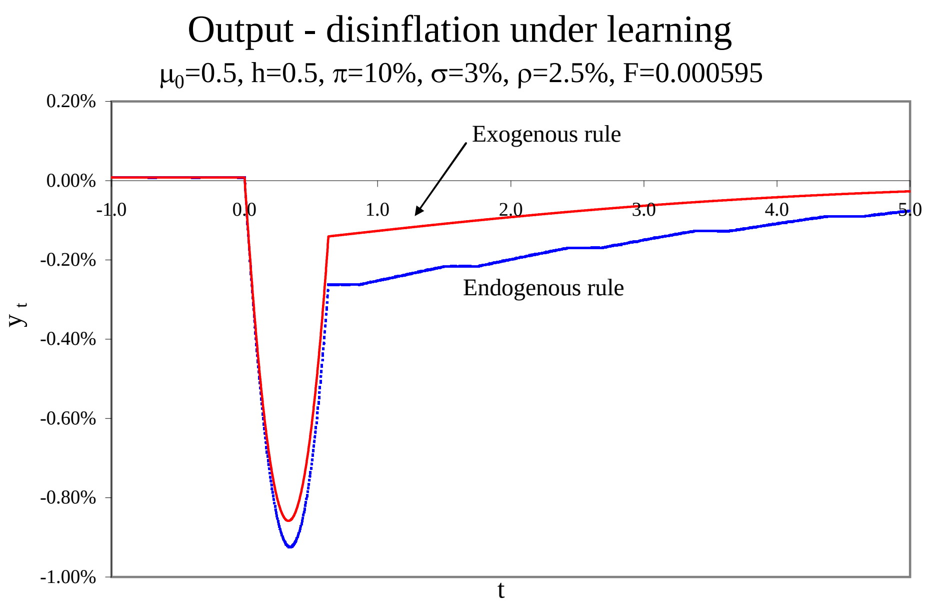


Figure 7



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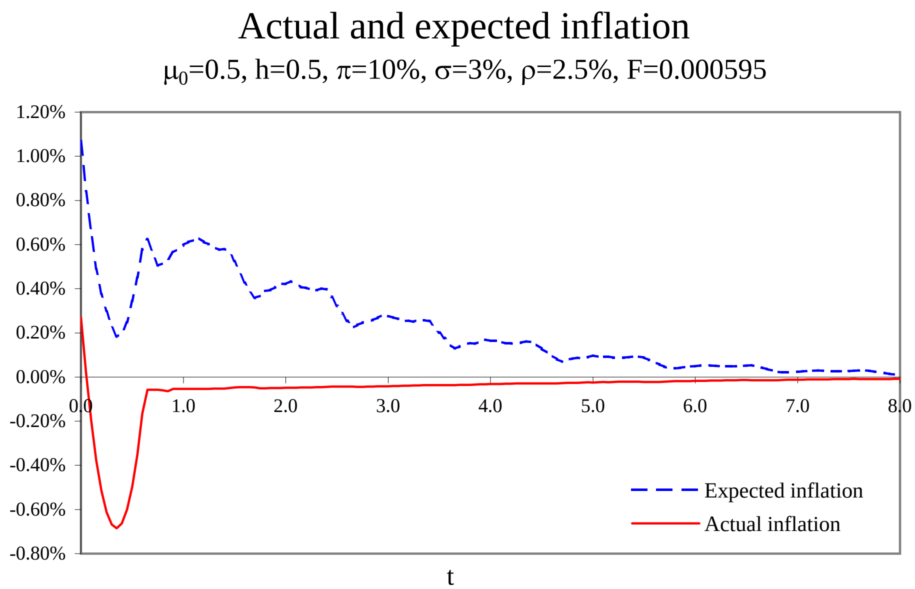

Figure 9

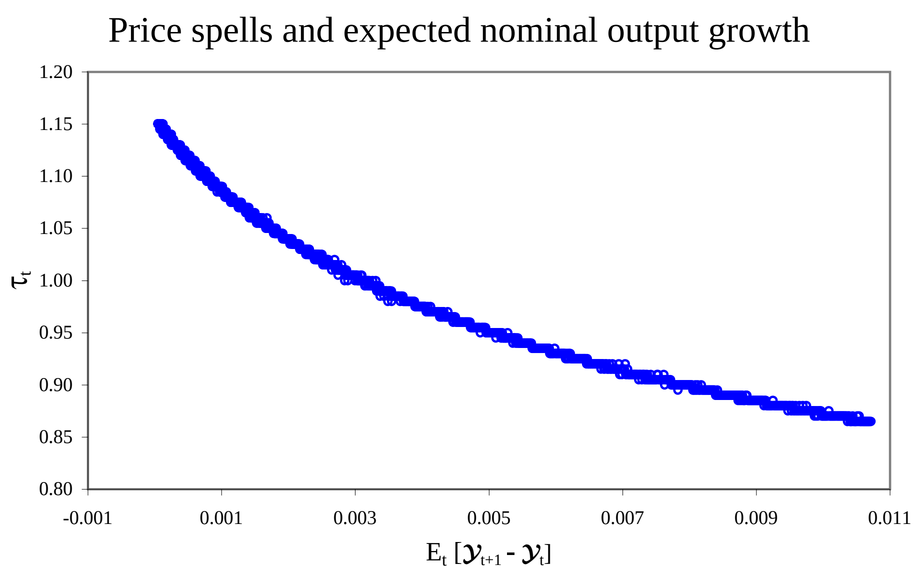

Figure 10