Misallocative Growth

Niklas Engbom*
New York University

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Abstract

Exploiting variation across Swedish local labor markets between 1986 and 2018, I estimate that individuals are less likely to start new firms and switch employers in an older labor market. To account for these patterns, I propose an equilibrium theory of growth with frictional labor markets. On the one hand, workforce aging raises the level of output by increasing the share of people who have found a good match with existing production technologies. On the other hand, the higher opportunity cost of switching to new technologies discourages their introduction. The offsetting level and growth effects result in high growth through the 1990s, even though the rate at which new technologies are introduced declines monotonically since the 1970s. I estimate that it will be suppressed for the next 30 years. The lower growth rate in the older economy lowers welfare for labor market entrants, but raises the value of the high-productive jobs typically held by older individuals.

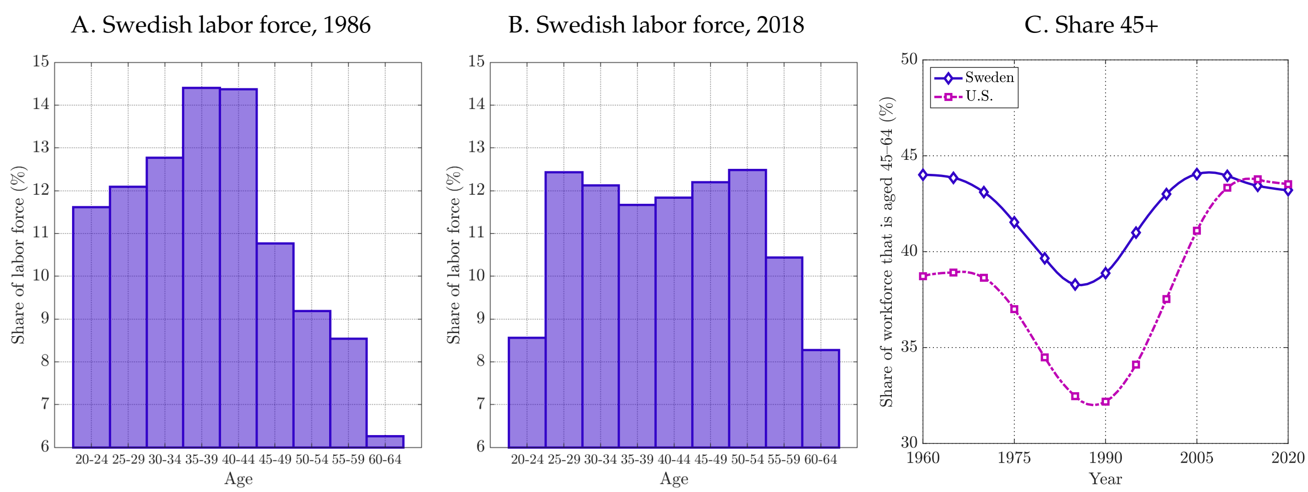
Keywords: Aggregate Labor Productivity; Demographic Trends; Technological Change
JEL Codes: E24; J11; O33

*New York University, CEPR, IFAU, NBER and UCLS. Email: nengbom@stern.nyu.edu. I thank Harun Alp (discussant), Adrien Bilal, Jonathan Goodman, Jonathan Heathcote, Hugo Hopenhayn, Alisdair McKay, Patrick Kehoe, Pete Klenow, Chris Moser, Fabrizio Perri, Michael Peters, Richard Rogerson, Aysegul Sahin, Tom Sargent, Robert Shimer, Venky Venkateswaran, Gianluca Violante and Abigail Wozniak for their generous advice. I am grateful to Uppsala University for their hospitality in hosting me and Statistics Sweden for granting data access. Adam Gill and Madeleine Lindblad provided excellent research assistance. I gratefully acknowledge financial support from the NBER Economics of Working Longer and an Aging Workforce and the Center for Global Economy and Business at New York University. I thank the Federal Reserve Bank of Minneapolis and the Heller-Hurwicz Economics Institute at the University of Minnesota for their generous hospitality during a share of the period of work on this project.

1 Introduction

The rapid aging of the labor force over the past 35 years is one of the most profound issues facing advanced economies. Across the OECD, high fertility rates after World War II resulted in the entry to the labor force of the large baby boomer generation between 1965 and 1985. As this inflow subsequently came to an abrupt end, the fraction of the workforce aged 45 years and older experienced a sharp reversal, as illustrated by Figure 1. For instance, in the country at the focus of this study, Sweden, the share of the workforce that is aged 45 years and older rose by six percentage points between 1986 and 2010, while it increased by 11 percentage points in the U.S. The purpose of this paper is to assess the impact of these dramatic demographic shifts on the performance of the labor market, growth and welfare.

FIGURE 1. CHANGES IN THE AGE COMPOSITION OF THE WORKFORCE/LABOR FORCE



Panels A–B plot the share of the Swedish labor force aged 15–64 that is in each 5 year age bin in 1986 and 2018, respectively. Panel C plots the share of all individuals aged 20–64 that is aged 45–64. *Source:* OECD.

To that end, I proceed in three steps. First, I provide reduced-form evidence of the impact of aging on the performance of the labor market, exploiting variation in the timing and magnitude of aging across 68 Swedish local labor markets between 1986 and 2018 (Shimer, 2001; Skans, 2005). I estimate that a one percent increase in the share of workforce participants younger than 45 raises firm creation by 1.21 percent and worker relocation by 0.99 percent. These estimates would predict (out of sample) that aging reduced firm creation by 12 percent and worker relocation by 10 percent at the national level between 1986 and 2018, relative to 25 and 18 percent declines in the data, respectively.

One possibility is that young individuals move to areas that experience increases in entry and worker relocation. Using lagged births to instrument for the current age composition, however, leads to a similar conclusion, suggesting that migration is not driving the results. A second hypothesis is that aging is correlated with changes in the growth rate of labor supply, and that the estimates reflect the causal effect of the latter (Karahan et al., 2022; Hopenhayn et al., 2022; Peters and Walsh, 2022). Indeed, consistent with the earlier literature, I confirm a role for labor supply growth in driving firm entry. Yet controlling for it does not change the estimated impact of aging. A third conjecture is that aging impacts the labor market through a demand channel (Bornstein, 2021). Consistent with this view, I estimate a smaller effect of aging in the tradable sector, yet it remains both statistically and economically significant.

I proceed to show that older individuals are less likely to start firms and switch employers, although the probability of the former rises early in careers. Given these pronounced life-cycle differences, one may expect aging to impact the aggregate firm creation and worker relocation rates by shifting the weight placed on subpopulations that differ in their probabilities of starting firms and switching employers. Over and above this composition effect, I estimate that individuals of all ages who live in older labor markets are less likely to start firms and switch employers. Indeed, such age-conditional declines account for the majority of the aggregate fall in entry and worker mobility.

Holding her own age fixed, why would a person be less likely to start a firm and switch employer when people around her age? Moreover, what are the aggregate and distributional implications of these patterns? What is the impact of commonly proposed policies to counter the effects of an aging workforce? The second step of this paper is to develop a theory that can address these questions.[1] In the model, a unique consumption good is produced by entrepreneurs, who differ in the idiosyncratic productivities of their blueprints for transforming labor into output. Following Stokey (1988), production features constant returns to scale and I abstract from physical capital, so that the only source of long-run growth is better ideas. Entrants imitate and improve blueprints of existing firms (Luttmer, 2007). As they do so, they bid up prices of factors of production, gradually displacing incumbent entrepreneurs.

The process of creative destruction requires the continual relocation of workers across heterogeneous firms, whose rank in the productivity distribution deteriorates as they gradually become obsolete. Due to labor market frictions, such relocation takes time (Diamond, 1982; Mortensen and Pissarides, 1994). In contrast to much of the existing literature on growth and labor market dynamics, however, in the current framework relocation need not involve a spell of unemployment (Aghion and Howitt, 1994; Mortensen and Pissarides, 1998; Hornstein et al., 2007). Instead, I emphasize the role of direct *job-to-job (JJ)* mobility from one employer to another in facilitating such relocation (Lentz and Mortensen, 2012).

Whereas semi-endogenous growth theory assigns a key role to the growth rate of the workforce in determining economic growth (Jones, 2022a,b), I highlight the importance of its *age composition*. Workforce aging increases the share of individuals who have had more time to confront labor market frictions

[1] The theory expands on Engbom (2019) in several key dimensions, including the modeling of knowledge spillovers.

and hence tend to be better matched to existing technologies. Consequently, an older workforce brings better static allocative efficiency, which I define as the ability to produce using existing blueprints. This contributes a positive *level effect* to aggregate output. The economy's better ability to produce using existing technologies, however, is achieved at the cost of raising the opportunity cost of switching to new technologies, discouraging their introduction. In equilibrium, growth falls—a negative *growth effect*. As the rate of obsolescence declines, individuals are provided more time to relocate across existing technologies before these technologies become obsolete, leading them to be better matched to existing blueprints also given their own age. As a result, age-specific mobility rates are lower in an older economy.

The decline in growth induced by aging has important distributional effects. On the one hand, the lower arrival rate of new, better technologies inhibits growth in individuals' outside options. On the other hand, the lower rate of creative destruction shields jobs from destruction. The former force is more important for labor market entrants, whose welfare falls with aging. The latter force is more important for the high-productivity jobs typically held by older individuals, whose values rise.

The effects of aging highlighted by the theory arise because better matches with existing technologies raise the opportunity cost of switching to new technologies. A better match with existing technologies may, however, also facilitate an individual's ability to come up with better new ideas. Indeed in the data, those working for more productive firms are less likely to start a new firm, consistent with a higher opportunity cost of doing so. Conditional on starting one, however, it tends to be more productive. To speak to this pattern, I allow new entrepreneurs to build on the blueprints of their previous employer.

The third step of the paper is to quantify the impact of aging. To that end, I infer parameters of the model from a set of moments in 2014–2018 on firm, worker and entrepreneur dynamics that are uniquely available in the Swedish matched employer-employee-entrepreneur data. The theory matches well the joint life-cycle dynamics of workers and firms, including the facts that older firms are more productive and more likely to hire (and employ) older individuals (Ouimet and Zarutskie, 2014). The latter pattern arises in equilibrium because older individuals tend to be better matched to existing technologies. Consequently, they are less inclined to accept an outside offer from a new firm.

With the estimated model in hand, I change the growth rate of labor supply to match the evolution of the age composition of the Swedish labor force between 1960 and 2060 (based on official projections). Across Balanced Growth Paths (BGPs), aging of the magnitude experienced by Sweden since 1986 reduces firm creation by 13 percent, relative to a 25 percent decline in the data. It lowers worker relocation by 14 percent, relative to an 18 percent decline in the data. These structural estimates are similar in magnitude to the evidence across local labor markets.

Consistent with the data, the lower entry rate in the older economy is mostly accounted for by a smaller age-specific entry rate, as opposed to a shift in composition toward age groups with a lower probability of entry. Age-specific entry primarily falls due to a higher opportunity cost of entry, since prospective entrepreneurs are better matched to incumbent firms as wage employees. They are better matched because the lower rate of creative destruction in the older economy affords them more time to relocate across existing technologies before they are displaced. This prediction is consistent with the view of Salgado (2020) and Jiang and Sohail (2021) that a higher return to wage employment relative to self-employment has reduced entrepreneurship in the U.S.[2] The direct, composition effect accounts for just over half of the aggregate decline in worker relocation, with the remainder due to an age-specific decline. As for firm creation, the main reason why workers of all ages in the older, slower growing economy are less likely to switch employer is that they are better matched to existing technologies.

To address the issue of a shrinking workforce, several countries are contemplating raising the retirement age. Such a policy has opposing effects on firm creation and worker mobility. On the one hand, it implies a lower effective discount rate, since individuals expect to remain in the market for longer. *Ceteris parisbus*, the value of jobs and firms rises, encouraging job creation and entry. On the other hand, by shifting workforce composition toward those who are well matched with existing technologies, it discourages new technologies. On net, the latter force is stronger, so that an increase in the retirement age from 65 to 67 years reduces the annual growth rate of output per worker by 0.06 percentage points.

Growth is not monotone over the transition path between 1970 and 2020, despite the fact that the rate at which new technologies are introduced declines monotonically. Instead, growth is low in the 1970s and early 1980s, as a large number of poorly matched young individuals enter the labor market. It subsequently rises until the mid-1990s, as the big baby boomer generation gradually finds a good match. After that, it falls again, matching well the Swedish experience. My estimates imply that growth will continue to be depressed for another 30 years. Relative to a counter-factual scenario without demographic change, a young worker is 0.8 percent worse off entering in 2018 relative to in 1986, whereas the value of the typically high-productive jobs held by older individuals is 0.3 percent higher.

Related literature. This paper makes three contributions relative to two influential recent papers that study the impact of a decline in labor supply growth on business dynamism in the U.S. (Karahan et al., 2022; Hopenhayn et al., 2022).[3] First, these papers are motivated by the observation that firm exit and size changed little conditional on firm age over the past 40 years in the U.S., which I confirm is also the

[2]These authors also show that the decline in entrepreneurship in the U.S. was more pronounced among the high-skilled. Although I abstract from skill heterogeneity, the effect of aging depends on the rate at which employed workers move up the job ladder. To the extent that low-skilled workers move up the job ladder less, they would be differentially impacted by aging.

[3]For an overview of trends in U.S. business dynamism, see Davis and Haltiwanger (2014) and Akcigit and Ates (2021, 2022).

case in Sweden. At the same time, both Sweden and the U.S. experienced a pronounced decline in job reallocation conditional on firm age, which the labor supply channel in these papers cannot account for. This observation calls for an assessment also of other potential channels.

Second, I focus on changes in the age composition as distinct from labor supply growth. Whereas a vast literature emphasizes the role of labor supply growth in driving economic growth—see, for instance, Peters and Walsh (2022) for an assessment of the effects of changes in labor supply growth on U.S. economic growth—less is known theoretically about the impact of the age composition.[4] The distinction, however, is highly policy relevant. For instance, policies that encourage women to stay in the labor force or an increase in the retirement age would increase labor supply (relative to counterfactual), without necessarily making the labor force younger. Consistent with a role for the age composition, I present the evidence that aging lowers firm creation and worker relocation, controlling for labor supply growth.

Third, I analyze the effect of demographic change on growth and welfare. Maestas et al. (2022) find that more rapidly aging U.S. states experience relative declines in growth, while Acemoglu and Restrepo (2017) estimate a null effect across the OECD. The offsetting level and growth effects that I highlight may reconcile these contrasting findings.[5] My assessment of the distributional impact of growth complements Aghion et al. (2016)'s study of the effect of growth on average welfare.

This paper also contributes to a theoretical literature on growth and labor market dynamics by identifying a new channel through which higher growth incentivizes job and firm creation. It arises due to the introduction of on-the-job search.[6] This channel resolves the tension highlighted by Pissarides and Vallanti (2007) that higher growth only raises job creation if growth is almost entirely disembodied, in contrast to much evidence (Greenwood et al., 1997). Lentz and Mortensen (2012) micro-found worker relocation in a Klette and Kortum (2004) model of firm-level innovation and show that it is consistent with micro data on wages, productivity and worker flows. Martellini and Menzio (2020) quantify the contribution of improvements in the search technology toward aggregate growth. Bilal et al. (2021) study how a decline in the economy's ability to come up with great ideas impacts labor market dynamics.

Finally, this paper relates to a literature on vintage capital (Chari and Hopenhayn, 1991). In particular, Atkeson and Kehoe (2007) argue that agents are more likely to adopt new technologies when the pace of technological change is fast, because it implies that they have accumulated less knowledge about existing technologies. Jovanovic and Nyarko (1996) derive a similar insight within the context of a Bayesian

[4] Aksoy et al. (2019) show that aging lowers incentives to innovate by reducing the marginal product of capital, while Acemoglu and Restrepo (2022) presents a theory that highlights that aging incentivizes adoption of industrial robots.

[5] Skans (2008) estimates that a larger share of older workers raises productivity levels across Swedish local labor markets.

[6] Michau (2013) allows for on-the-job search in a model of exogenous embodied growth, but two simplifying assumptions shut down the channel highlighted here. First, jobs enter at the technological frontier, such that workers always accept the job regardless of employment status. Second, employed workers bargain with firms as though they are unemployed.

model of learning. In a related spirit, I show that new blueprints are more valuable when growth is high, because it implies that individuals have had less time to become well-matched with existing blueprints. This effect is also related to Violante (2002)'s finding that higher growth increases inequality.

This paper is organized as follows. Second 2 introduces the data and overviews recent Swedish trends. Section 3 presents reduced-form evidence on the impact of aging on firm creation and worker relocation. Section 4 provides a qualitative analysis of the impact of aging on labor market dynamics, growth and welfare, while Section 5 offers a quantitative assessment. Section 6 concludes.

2 Swedish labor market trends

This section introduces the data and provides a brief overview of Swedish labor market trends.

2.1 Data

My analysis relies mainly on three administrative data sources, linked via individual and firm identifiers into a matched employer-employee-entrepreneur data set covering all Swedish individuals and firms. I discuss these three data sources briefly below, and relegate a more detailed discussion to Appendix A.1.

The *Jobbregistret* (JOBB) contains all employment spells of all individuals aged 16–74 between 1985 and 2021. These data are collected annually, with information on total annual gross pay, type of employment, as well as start and end month of the employment spell.[7] Employment spells are classified as wage employment, self-employment in unincorporated firms, or self-employment in incorporated firms. Prior to 1993, the self-employed in incorporated firms were instead classified as wage employees, making it impossible to identify incorporated owner-operators prior to 1993. As I discuss in further detail in Appendix A.1, self-employed in incorporated firms are only identified if firm ownership is sufficiently concentrated. The age thresholds for who is included in JOBB have changed slightly over time, particularly at the upper age threshold. For this reason, I restrict my analysis throughout to individuals aged 20–64. As I discuss further below, the average age of entry into the labor force has been stable at around 22–23 years and the average retirement age has been stable at just below 65 over the 1986–2018 period.

The *Longitudinell integrationsdatabas för sjukförsäkrings- och arbetsmarknadsstudier* (LISA) contains demographic information for all Swedish individuals aged 16 and above starting in 1990. I standardize gender, year of birth, and highest obtained educational degree to the modal value across all years of

[7]Since 2019, gross pay has instead been reported on a monthly basis for each spell that is active in that month. Although this switch in reporting is arguably an improvement, it introduces a significant time-series break, leading me to stop my analysis in 2018. Because I also require one prior year to correctly construct firm entry rates, I start my analysis in 1986.

available data. I use LISA to assign these outcomes also in 1986–1989 to individuals in JOBB. As a result, I drop from my analysis any individual aged 20–64 in 1985–1989 if they either emigrated from Sweden or died before the first year of LISA data in 1990. I do not believe that this is a major source of error.

The *Företagens ekonomi* (FEK) contains annual income and balance sheet data on all private sector firms in Sweden starting in 1997, with only a few limited exceptions. It forms the basis for the Swedish national accounts. Although some information is also provided at the establishment level, the firm-level data are more extensive. For this reason, I use outcomes at the firm-level throughout my analysis.

TABLE 1. SUMMARY STATISTICS

A. Overall population aged 20–64 (monthly flows)						
Years	Non-empl. (%)	Self empl. (%)	Wage empl. (%)	Public (%)	Entry to public (%)	Exit from public (%)
177,032,547	20.97	10.76	41.13	27.14	0.400	0.812
B. Excluding public sector (monthly flows)						
Male (%)	College (%)	St.d. earnings	JJ (%)	EN (%)	NE (%)	Entry (%)
58.71	28.39	0.577	2.259	1.078	3.788	0.087
C. Private sector firms (annual flows)						
Years	Firm size	St.d. va.p.w.	Firms, 250+ (%)	Empl., 250+ (%)	Firms, age 11+ (%)	Empl., age 11+ (%)
22,562,643	4.927	0.763	0.169	30.33	38.86	60.46
Entry (%)	JC (%)	JD (%)	JC, inc. (%)	JD, inc. (%)	Hires from E (%)	Hires from N (%)
14.60	14.25	13.31	7.73	7.45	27.58	13.53

Table 1 provides summary statistics on all individuals aged 20–64 between 1986–2018 (panel A), all individuals aged 20–64 between 1986–2018 who are not public sector employees (panel B), and private sector firms between 1997–2018 (panel C). Share college includes those with a bachelor's degree or higher. Earnings are log average monthly real earnings. The JJ rate is the fraction of private sector workers in month t who have a different main employer in month $t+1$ (including switching to a public sector employer). The EN rate is the fraction of private sector workers in month t who are not employed in month $t+1$. The NE rate is the fraction of non-employed individuals in month t who are employed in month $t+1$ (including in the public sector). The entry rate is the fraction of not self-employed individuals in month t who are the owner-operator of a new firm in month $t+1$, where a new firm is a firm founded in the current year with at most 10 identified founders. St.d. of va.p.w. is the employment-unweighted standard deviation of log value added per worker. Firms, 250+ (age 11+), is the share of firms with 250 or more employees (aged 11 or older). Empl, 250+ (age 11+), is the share of workers employed by firms with 250 or more employees (aged 11 or older). Firm entry is the share of all firms with positive employment in year t that had zero employment in $t-1$. JC (JD) is the sum of employment gains (losses) across expanding (contracting) firms in the year divided by average employment. JC (JD), inc., is the sum of employment gains (losses) of expanding (contracting) incumbents in the year divided by average employment of all firms. Hires from E (N), is the sum of hires from employment/non-employment in the year divided by average employment. All moments are constructed by first computing means by year, and subsequently computing means across all years, giving equal weight to each year. Whenever applicable, the moments are computed at the firm level. *Source:* FEK, JOBB, LISA.

2.2 Sample selection

Table 1 summarizes the data. Panel A includes all individuals aged 20–64, of which 21 percent are not employed, 11 percent are self-employed, 41 percent are private sector wage employees and 27 percent work in the public sector. It is not possible to separate not in the labor force from unemployment in the administrative data. Flows between the public and private sector are small, i.e. workers tend to remain in their respective sectors. Panel B summarizes the population of non-public sector individuals, which is my core population (see Appendix A.2 for additional outcomes). The monthly JJ rate is 2.3 percent and the employment-to-nonemployment (EN) rate is 1.1 percent. The low nonemployment-to-employment (NE) rate of 3.8 percent is likely due to the fact that the nonemployed include also those not in the labor force. The monthly firm creation rate is 0.1 percent, i.e. an order of magnitude lower than worker flows.

Panel C summarizes the firm-level data, which cover roughly 23 million firm-years in the private sector. The low average firm size of just over four is due to the fact that the data include also small, non-employer businesses.[8] The firm size distribution is highly skewed, with the 0.15 percent largest firms accounting for 30 percent of employment. The annual job reallocation rate—the sum of jobs created and destroyed in a year relative to employment—is roughly 28 percent, which is comparable to the U.S. (see Appendix A.3 for a further discussion). A majority of job creation and destruction is done by incumbent firms as opposed to entrant and exiting firms. Worker flows are almost three times as high as job flows, and two thirds of hires are poached directly from other firms, as opposed to hired from non-employment.

2.3 Swedish labor market trends

Figure 12 summarizes Swedish labor market trends over the past 35 years (see Appendix A.3 for a comparison with the U.S.). The entry rate fell by 31 percent since 1986 (panel A). The decline is similarly large if I include only firms that subsequently grew by at least 25, 50 or 100 percent over the first five years of operation (but note the large differences in levels). Job reallocation also declined, regardless of whether I measure it at the firm or establishment level (panel B).[9] Panel C illustrates further the declines by showing trends relative to 1986. Both job reallocation of entrant and exiting firms and that of

[8] In the U.S. *Business Dynamic Statistics* (BDS) data, average firm size is around 20, but this number excludes non-employer businesses. According to the U.S. Census Bureau, there were roughly 26.5 million firms in the U.S. in 2018 but only 6.1 million employer firms. Assuming that each non-employer firm consists of only one self-employed, average firm size in the U.S. including non-employer firms is approximately $(20 * 6.1 + 20.4)/26.5 = 5.4$, i.e. broadly consistent with the Swedish data.

[9] Heyman et al. (2019) document only minor changes in firm dynamics in Sweden over the 1993–2013 period. Three reasons lead me to a different conclusion. First, Heyman et al. (2019) focus on the distribution of overall job creation, employment and firms across firms by firm age, finding relatively small changes in such shares over time. Although these shares are related to outcomes such as the aggregate job reallocation rate, they are not identical. Second, consistent with their findings as well as U.S. evidence (Karahan et al., 2022), I find that job reallocation fell by less conditional on firm age (see Appendix A.7). Third, Heyman et al. (2019) rely on firm and establishment identifiers that are constructed by Sweden's national statistical office—*Statistiska centralbyrån* (SCB)—to be consistent over time, available via a data source known as *Företagens och arbetsställenas dynamik* (FAD) (the firm and establishment identifiers in this data set are referred to as *FAD-F-ID* and *FAD-A-ID*, respectively). FAD abstracts, however, from all firms which no individual has as their "main" employer in November of the year. This concerns almost 40 percent of firms in JOBB, which covers the universe of firms which at least one individual works for at some point in the year (including as self-employed). Although such firms tend to be small, some of them later grow to be large, so it is not clear that one wants to discard such observations. Moreover, the mapping between the firm and establishment identifiers in JOBB (*PeOrgNr* and *Cfar-Nr*, respectively) and *FAD-F-ID* and *FAD-A-ID* in a given year is not unique across vintages of FAD. Possibly for this reason, measures of firm and establishment dynamics based on *FAD-F-ID* and *FAD-A-ID* are very high, more than twice the corresponding numbers in the U.S. (this observation is consistent with Figure 5 in Heyman et al., 2019, which shows that entrant firms based on *FAD-F-ID* create more than 40 percent of all jobs in Sweden or more than twice the corresponding share in the U.S. BDS). Although JOBB lacks a consistent firm identifier—*PeOrgNr* changes whenever the firm changes ownership, organizational form, etc—*Cfar-Nr* only changes when at least two of the following three change for the establishment: location, sector, or ownership. Hence *Cfar-Nr* should be a reasonably consistent establishment identifier. Indeed, measures of establishment dynamics based on *Cfar-Nr* broadly align with the U.S., i.e. they are much lower than those implied by *FAD-A-ID*. In fact, job reallocation rate based on *PeOrgNr* is only modestly lower than that based on *Cfar-Nr*, suggesting that these issues may be of less concern. Appendix A.4 provides a further discussion.

incumbents declined. The worker relocation rate—the sum of hires and separations in a year divided by average employment—is more volatile over the business cycle, but also displays a trend decline. In contrast to the secular declines in firm creation, job reallocation and worker relocation, the aggregate growth rate rose from the mid-1970s through the 1990s, and only subsequently declined (panel D).[10]

One recently emphasized factor behind the decline in firm entry in the U.S. is a fall in the growth rate of labor supply (Karahan et al., 2022; Hopenhayn et al., 2022; Peters and Walsh, 2022). Sweden, however, experienced no pronounced change in the growth rate of the working age population over the past 60 years (panel E). In fact, it modestly *rose* since the mid-1970s. Growth in the number of labor force participants aged 16 or older did decline until the mid-1990s (data on the labor force start in 1968). Since then, however, labor force growth trended up (the divergence in the growth rate of the workforce and labor force in the late 1980s and 1990s coincides with an economic boom followed by a large contraction).

Over the past 60 years, Sweden saw large shifts in the age composition of the workforce (panel F). For instance, the share of the workforce that is 45 years and older first fell by six percentage points from the late 1950s until the mid-1980s, and subsequently rose by six percentage points. Although the levels differ, the trends in the age composition of the workforce and labor force are similar, with the latter rising by nine percentage points since the mid-1980s. Appendix A.5 discusses the difference between the labor and work force further, highlighting no pronounced trends in labor force participation rates by age over this period.[11] This finding is consistent with official publications showing an increase in the average age of labor force entry during the financial crisis of the early 1990s, but a subsequent stabilization around 22–23 years old, as well as only a marginal change in the average age of retirement, with most Swedes retiring at age 65.[12] A substantial share of these changes in the age composition are accounted for by past fertility, as illustrated by the "lagged birth" series. Specifically, it plots the ratio of the sum of births 45–64 years earlier to the sum of births 20–64 years earlier, i.e. the share of the workforce that would be older solely based on past fertility, abstracting from immigration, emigration and mortality.

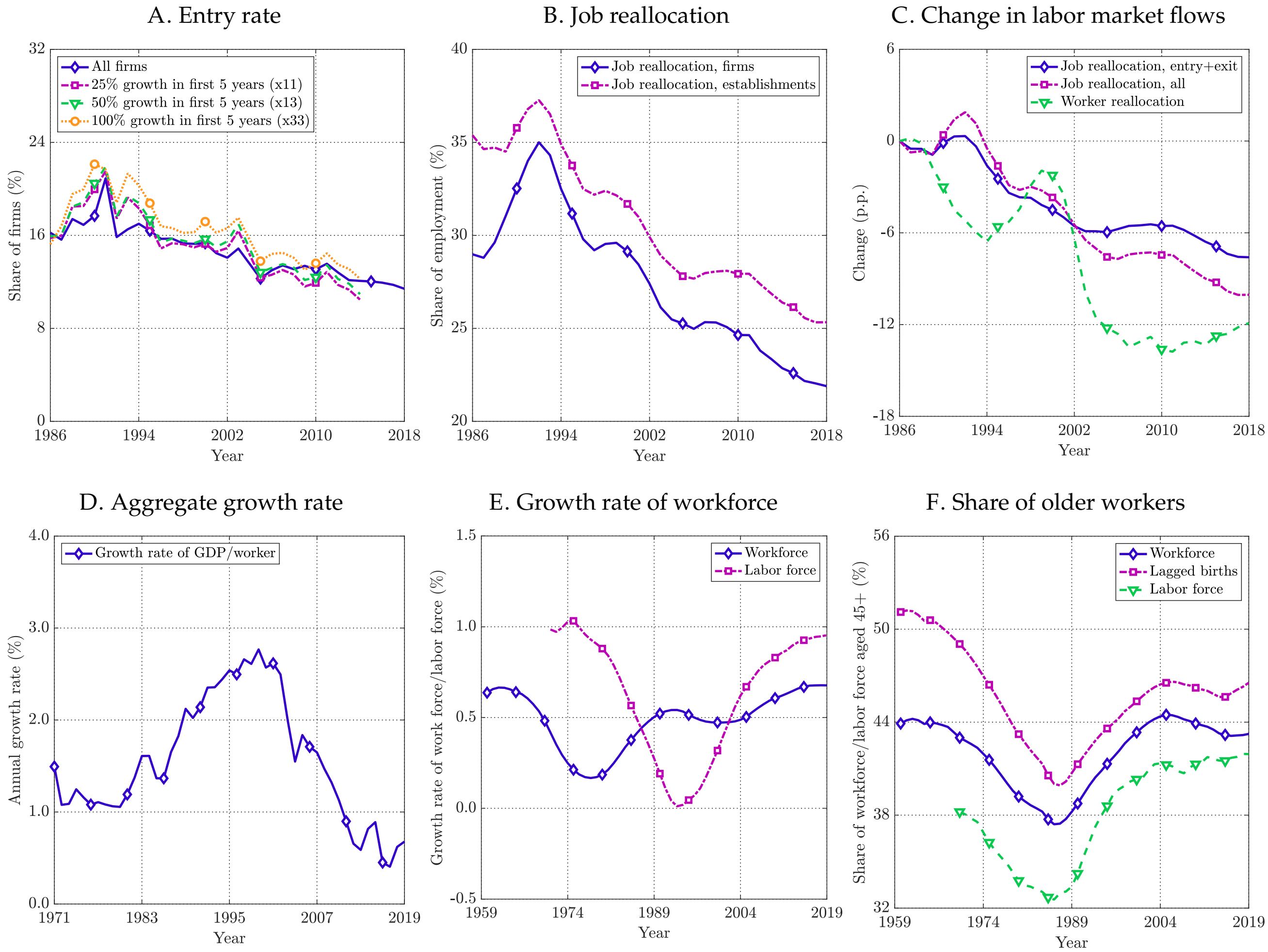
Appendix A.2 establishes a series of additional facts, which I briefly summarize here. The firm exit rate fell, average firm size trended up modestly, the ratio of workforce participants to firms increased by even less, productivity and wage dispersion rose, while the volatility of productivity changes remained constant. Entrant firms are less productive than all firms, but more productive than exiting firms. In a relative sense, the productivity of entrant firms remained roughly constant, while exiting firms became

[10] I plot the 11-year centered moving average of the growth rate of real GDP per worker, available since 1971 from the OECD.

[11] The main exception is an increasing participation rate of women during the 1970s and early 1980s. Since the early 1980s, women's participation rates have remained high and steady in Sweden, with the exception of women aged 55-64, who experienced a secular increase in labor force participation rates over the entire 1968–2019 period.

[12] Figure 3.6 and Table 3.1 in https://www.regeringen.se/4a67b3/contentassets/c1063c03c89247b694cb895aae28741d/hojda-aldersgranser-i-pensionssystemet-och-i-andra-trygghetssystem_ds-2019_2.pdf (Swedish only).

FIGURE 2. SWEDISH LABOR MARKET TRENDS



Panel A plots the entry rate of firms, either unconditionally or conditional on a minimum growth rate over the first five years of a firm's existence. The entry rates are spliced in 2004 due to a data break; see Appendix A.6 for details. Panel B plots the sum of jobs created and jobs destroyed across firms/establishments in a year divided by average employment in the year. Job creation (destruction) is the sum of employment gains (losses) across all expanding (contracting) firms/establishments. Panel C plots the sum of jobs created and destroyed by entrant/exiting firms and incumbent firms, in both cases divided by total employment in the year. The worker relocation rate is the sum of hires and separations divided by total employment in the year. All series are in percentage point deviations from 1986. Panels A–C include private sector firms and workers aged 20–64. Panel D plots the 11-year centered moving average of annual growth in real GDP per worker. Panel E plots the annual growth rate of the workforce (population aged 20–64) or labor force (all labor force participants older than 15 years). Panel F plots the share of the workforce or labor force who is 45 years and older. The lagged birth series is the sum of births 45–64 years earlier divided by the sum of births 20–64 years earlier. *Source:* FEK, JOBB, LISA, OECD.

less productive. Finally, the distributions of both employment and firms shifted toward older firms.

Appendix A.7 documents that conditional on firm age, exit rates and firm size declined modestly, while the fall in job reallocation remains pronounced. Appendix A.7 also shows that these patterns are similar in the U.S., where exit rates fell only modestly and average firm size remained constant conditional on firm age (Karahan et al., 2022; Hopenhayn et al., 2022), but job reallocation declined substantially. Appendix A.7 also shows that job reallocation fell across firm size classes. Appendix A.8 highlights that economic activity shifted from less dynamic sectors such as manufacturing toward more dynamic sectors such as services. Consequently, the within-sector decline in job reallocation is *larger* than the aggregate decline. Appendix A.8 finds that job reallocation fell in all major sectors.

3 Evidence from a descriptive statistical model

Several forces are likely behind the observed changes over time in firm and worker transitions in Sweden, including advancements in IT technology (Lashkari et al., 2021), increasing use of intangibles (de Ridder, 2021), falling overhead costs (Aghion et al., 2022), and changes in the nature of knowledge spillovers (Akcigit and Ates, 2022; Olmstead-Rumsey, 2022). This section employs a descriptive statistical model to provide evidence on the impact of aging on firm and worker dynamics.

3.1 Methodology

The Swedish national statistical office, *Statistiska centralbyrån* (SCB), aggregates Sweden's roughly 290 municipalities into local labor markets—*lokala arbetsmarknader* (LA)—based on commuting flows. The number of LAs declined over time, so to be consistent I use the 2018 delineation, providing 69 LAs. Because *Årjäng* LA forms part of Oslo's local labor market, I drop it from my analysis (but similar results hold including it).[13] I obtain from publicly available data the share of individuals aged 20–64 that are aged 20–44 at the LA-year level,[14] and from the administrative micro data moments on labor market dynamics as well as the share male, the share with a college degree or higher, the share immigrant and the share of immigrants aged 20–64 that are aged 20–44 at the LA-year level for 1986–2018.

I regress labor market outcomes such as the firm creation rate and worker relocation rate in LA i in year t on the share of the local workforce that is 20–44—henceforth the share young—always controlling for LA (ψ_i) and year (ξ_t) fixed effects, and potentially also other time-varying covariates ($X_{i,t}$),

$$\log y_{i,t} \quad = \quad \alpha \log young_{i,t} + \psi_i + \xi_t + X_{i,t}\beta + \varepsilon_{i,t} \tag{1}$$

The coefficient α captures the extent to which labor market outcome y changes as a local labor market ages. Because all variables are in logs, it can be interpreted as an elasticity.[15] Motivated by evidence in Appendix B.1 of a serial correlation within LAs and a spatial correlation across LAs in the dependent

[13] Although the Swedish and Finnish parts of a northern border area known as *Tornedalen* see significant cross-border commuting, SCB has not yet been able to determine whether the three border local labor markets that this concerns—*Haparanda*, *Övertorneå* and *Kiruna*—form part of larger Swedish-Finnish local labor markets (see the Swedish discussion in https://www.scb.se/contentassets/c2d754bcaf964bcca33ac7cc2510c765/lokala-arbetsmarknader-over-riksgrans-2003.pdf). Dropping these three local labor markets makes little difference to the estimates below.

[14] https://www.statistikdatabasen.scb.se/pxweb/sv/ssd/START__BE__BE0101__BE0101A/BefolkningNy/.

[15] I prefer logs as I find it easier to interpret an elasticity, but estimating (1) in levels instead yields similar results.

and independent variables, I two-way cluster standard errors at the LA and year level (Cameron et al., 2011).[16] Regression (1) leverages the fact that Swedish LAs have aged differently, both in terms of overall magnitude and timing. In particular, although the inclusion of the LA and year fixed effects shrinks the standard deviation of the log share of young from 0.088 to 0.025, significant variation remains.

A concern with estimation of (1) via OLS is that individuals may move across LAs in response to labor market performance. In particular, results may be biased if young people differentially move to areas that display *temporarily* high labor market dynamics. I address such concerns by instrumenting for the current age composition using lagged births 20–44 years earlier.

To that end, I collect data on births at the LA-year level from 1940 to 2018. Since 1968, annual data on births are available online at the municipality-year level.[17] From 1958–1967, I digitalize statistical abstracts at the municipality level, which I map into the 69 LAs.[18] Prior to 1958, births are only available at the level of Swedish *län*, separately reported for its urban and rural components.[19] *Län* are a higher level of local administration than municipalities, with the number of *län* having fluctuated around 25. Births are also reported separately for the roughly 110 largest cities in Sweden. I construct cross-walks between the larger cities, on the one hand, and the LAs and the *län*, on the other. Subsequently, I impute births in an LA by summing births in the larger cities that form part of the LA as well as "residual births" in the LA. To obtain the latter, I first sum rural births in the *län* and the difference between reported urban births in the *län* and the sum of births in the larger cities that belong to the *län*. I apportion this sum across the LAs that form part of a *län* based on each LAs population share of the *län* in 1958.

I use the log of the sum of births 20–44 years earlier in the LA as an instrument for the current log share of young, conditional on LA and year fixed effects. The identifying assumption is that fertility 20–44 years earlier is not based on contemporaneous labor market dynamics. Lagged births predict well the current share young.[20] A one percent increase in lagged births is associated with a 0.127 percent increase in the current share young, conditional on LA and year fixed effects, with a standard error of 0.023. Lagged births account for 26 percent of the residual variation in the current share young after

[16] Appendix B.1 finds no evidence of a correlation between the contemporaneous residual in a LA i and the residuals in neighboring LAs in other years, after controlling for the lagged residuals in LA i and the contemporaneous residuals in neighboring LAs. Nevertheless, also accounting for aggregate serial correlation for up to three years following Driscoll and Kraay (1998) does not meaningfully change the standard errors reported below.

[17] `https://www.statistikdatabasen.scb.se/pxweb/sv/ssd/START__BE__BE0101__BE0101H/FoddaK/`.

[18] Available via SCB's *Äldre statistik* at `https://www.scb.se/hitta-statistik/sok/?query=&tab=older` in Table 4 (for 1958–1959), Table 5 (for 1960) and Table 6 (for 1961) of the yearly publications of *Folkmängden inom administrativa områden*; in Table 1 of the *folkmängdsförändringar kommunvis* as part of *Statistiska meddelanden (SM)* (publications B 1963:16 for 1962, B 1964:9 for 1963, and B 1965:5 for 1964, Be 1966:5 for 1965 and Be 1967:7 for 1966); and Table 1 of the *Befolkningsförändringar del 1* (for 1967).

[19] Available in Tables 3–4 of *Befolkningsrörelsen* (`https://www.scb.se/hitta-statistik/sok/?query=&tab=older`).

[20] A closely related alternative instrument would be the birth *rate*. Although it also predicts well the current share young, it provides a weaker first stage than the lagged sum of births, conditional on LA and year fixed effects.

accounting for the fixed effects. Jointly, lagged births and the fixed effects account for 94 percent of the variation in the current share young. An F-test of overall statistical significance is 27 (with standard errors two-way clustered at the LA and year level). Evidently, while some Swedes do move across LAs, many remain where they are born. Appendix B.2 visualizes the correlation.

Appendix B.3 summarizes key worker and firm level outcomes across Swedish local labor markets, including the size of the workforce, average wages, net wealth, productivity and demographics. Appendix B.4 illustrates some of the identifying variation that drives the regression results below.

3.2 Results

According to the OLS estimate in column 1 panel A of Table 2, a one percent increase in the share of young is associated with a 1.21 percent increase in firm creation.[21] Variation in the residual share of young accounts for 6.0 percent of the residual variation in firm creation, conditional on LA and year fixed effects. A one standard deviation change in the residual log share young is associated with roughly a quarter of a standard deviation change in the residual log firm creation rate. At the national level, the share of young fell by roughly 12 percent between 1986 and 2018. Hence, abstracting from national level equilibrium effects, the estimate would imply that aging has reduced firm creation by about 14 percent over this period, corresponding to roughly 40 percent of the aggregate decline.

The IV estimate in column 2 is larger than the OLS estimate, indicating that a one percent increase in the predictable share of young leads to a 2.20 percent increase in firm creation. The fact that the IV estimate is larger than the OLS estimate is a common finding in related work on the impact of aging in U.S.—see, for instance, the discussion in Shimer (2001) or Karahan et al. (2022). At the same time, the underlying variation in the predictable share young is smaller than that in the realized share young. As a result, similar to the OLS estimate, a one standard deviation increase in the predictable share young leads to roughly a quarter of a standard deviation rise in residual firm creation.

Columns 3–4 repeat the analysis for the manufacturing sector only, which constitutes about 30 percent of employment in Sweden (Appendix A.2 shows that this share has declined over time). The estimates are smaller than for the overall economy. This observation is consistent with changes in demand induced by aging being behind some of the patterns, which Bornstein (2021) shows is important in the U.S. context. At the same time, the point estimates indicate that aging also reduces firm creation in the tradable sector in Sweden, suggesting that also a supply-based channel is at work.

[21] I do not weigh LAs by their population size. Weighing LAs by their population size in 1986 results in an even more pronounced point estimate, with the OLS specification indicating that a one percent increase in the share of young is associated with a 1.91 percent increase in firm creation and the IV estimate a 2.55 percent increase.

Recent work shows that a decline in labor supply growth accounts for some of the decline in firm creation in the U.S. (Karahan et al., 2022; Hopenhayn et al., 2022; Peters and Walsh, 2022). Column 5 instead projects outcomes on the growth rate of the workforce (all individuals aged 20–64), confirming previous findings of a positive impact of labor supply growth on entry. Column 6 adds as instrument for the growth rate of the workforce the log number of births in the LA 20 years earlier. The point estimate turns negative, but the first stage is too weak to be able to draw any robust inference.[22]

Columns 7–8 include both the share of young and the growth rate of the workforce, suggesting a role for both (although the point estimate on labor supply growth turns statistically insignificant, it remains economically substantial—see Karahan et al., 2022, for complementary evidence to this point across U.S. states). The first stage is too weak to allow any robust inference in the IV specification.

Finally, columns 9–10 include the share of private sector employment that is male, has a college degree or higher and is immigrant, as well as the size of the workforce aged 20–64 and its growth rate. Controlling for changes in these dimensions reduces the point estimate in the OLS specification, but raises it in the IV specification. In the IV specification, however, the first stage weakens and the standard errors grow, such that the point estimate is only weakly statistically significant (p-value of 0.073).

Panel B shows that aging reduces also worker mobility. The OLS estimate in column 1 indicates that a one percent increase in the share of young is associated with a 0.99 percent increase in worker relocation. Variation in the residual share of young accounts for 5.1 percent of the residual variation in worker relocation, conditional on LA and year fixed effects. A one standard deviation change in the residual log share young is associated with about a quarter of a standard deviation change in the residual log worker relocation rate. As for firm creation, the IV estimate in column 2 is larger than the OLS estimate.

The point estimates for the manufacturing sector only in columns 3–4 are smaller than the aggregate economy. Yet the estimates in the tradable sector remain statistically and economically significant, suggesting that aging also impacts labor market dynamics through a supply channel. Controlling for labor supply growth (columns 7–8) or for changes in the gender, educational and immigrant composition of the workforce as well as its size and growth rate (columns 9–10) does not change the conclusion.

A remaining concern is that these estimates reflect a third factor that is correlated with fertility 20–44 years earlier and contemporaneous labor market dynamics. For instance, urban areas may have disproportionately benefitted from forces such as globalization, and they may have experienced smaller declines in fertility. Appendix B.5 contains a battery of additional robustness exercises. For instance, the point estimates are only marginally affected by adding separate linear time trends interacted with the

[22] I have alternatively used the log of the birth rate 20 years earlier, which leads to an even weaker first stage. The point estimate on the growth rate of the workforce in the second stage continues to be negative.

TABLE 2. THE IMPACT OF AGING ON LABOR MARKET DYNAMICS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Baseline		Manufacturing		Supply growth		Joint		Controls	
	OLS	IV	OLS	IV	OLS	IV	OLS	IV	OLS	IV
Panel A. Firm creation rate										
Share 20–44	1.207***	2.196**	1.012***	1.767**			1.200***	2.397***	0.798**	2.810*
	(0.358)	(0.820)	(0.305)	(0.679)			(0.359)	(0.839)	(0.371)	(1.516)
Δ 20–64					0.623**	–16.552	0.378	–14.550	0.426	0.236
					(0.269)	(23.131)	(0.310)	(18.511)	(0.287)	(0.396)
Obs.	2,244	2,244	2,241	2,241	2,244	2,244	2,244	2,244	2,244	2,244
R-squared	0.691		0.491		0.673		0.692		0.716	
within	0.059		0.012		0.002		0.060		0.134	
F-stat		27.1		27		2.3		1.2		12.2
Panel B. Worker relocation rate										
Share 20–44	0.988***	2.594***	0.768**	2.375***			0.981***	2.756***	0.578**	4.013**
	(0.237)	(0.754)	(0.316)	(0.789)			(0.237)	(0.763)	(0.246)	(1.618)
Δ 20–64					0.571	–14.069	0.370	–11.767	0.462	0.137
					(0.443)	(17.213)	(0.360)	(13.269)	(0.354)	(0.410)
Obs.	2,244	2,244	2,244	2,244	2,244	2,244	2,244	2,244	2,244	2,244
R-squared	0.791		0.619		0.780		0.791		0.796	
within	0.051		0.011		0.002		0.051		0.074	
F-stat		27.1		27.1		2.3		1.2		12.2

Table 2 presents OLS and IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. The outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. Columns 5–6 regress outcomes on the annual log difference in the number of workforce participants (all individuals aged 20–64). The instrument is the log number of births 20 years earlier in the LA. Columns 7–8 include both the share of young and the growth rate of the workforce, potentially instrumented for using the log sum of births 20–44 years earlier and the 20-year lagged number of births. Columns 9–10 adds to the independent variables in columns 7–8 the share of private sector employment aged 20–64 that is male, has a college degree or higher and is immigrant, as well as the number of individuals aged 20–64. The share of young is potentially instrumented for using the log sum of births 20–44 years earlier. Panel A shows results for the firm creation rate as the dependent variable, defined as the share of firms with positive employment in the current year that had zero employment in the previous year. Panel B shows results for the worker relocation rate as the dependent variable, defined as the sum of hires and separations in a year divided by average employment in the year. *Source:* JOBB, LISA, SCB.

initial share of private sector employment that is male, has a college degree or more, or is immigrant, as well as the initial size of the workforce and initial average net wealth, or separate linear time trends interacted with initial value added per worker, the initial share of manufacturing firms and initial investment per worker. In Appendix B.6, I establish that aging also leads to declines in the entry rate of high growth firms, measured by growth of at least 25 or 50 percent in the first five years of a firm's existence; has no statistically significant effect on average firm size or investment per worker; and lowers job reallocation. The effects of aging hold controlling for sector (Appendix B.7) and firm age (Appendix B.8).

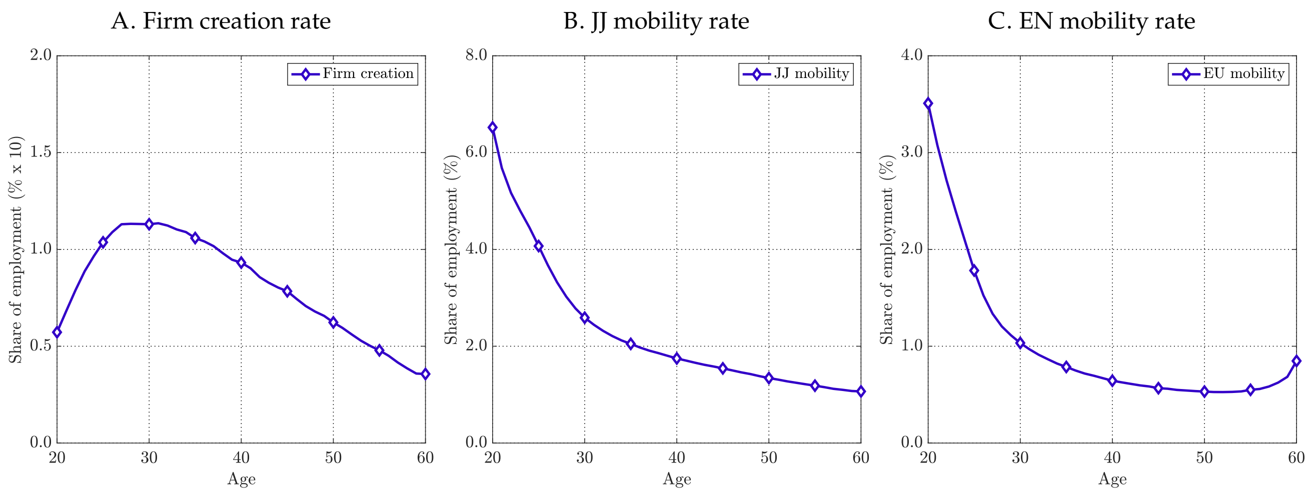
3.3 Age-specific declines in entry and worker mobility

Firm creation and worker relocation display distinct life-cycle patterns (Figure 3). For instance, the probability of starting a firm rises up to age 30, then declines, particularly after around age 40 (panel A).[23]

[23] Azoulay et al. (2018) find that in the population of new firms in the U.S., the modal age of founders is about 38 years. Apart from the fact that Sweden and the U.S. are different countries, several factors could be behind the difference. First, they study the age distribution of founders, as opposed to the entry rate by age. These two objects are only the same if the underlying population age distribution is uniform (which is neither the case in the U.S. nor Sweden). Second, due to data limitations,

The probability of making a JJ (panel B) or EN (panel C) transition decline monotonically with age.[24]

FIGURE 3. COMPOSITION EFFECT OF AGING



Panel A plots the share of individuals in month m who are the owner-operator of a newly founded firm in month $m+1$ which they had never previously been the owner-operator for. A newly founded firm is one which was founded in the current year t, and includes both unincorporated and incorporated firms (as long as it a *fåmansföretag*—see Appendix A.1 for details). Panel B plots the fraction of individuals in month m who have a different main employer in month $m+1$. Panel C plots the fraction of individuals in month m who are non-employed in month $m+1$. All panels: Non-public sector firms and individuals aged 20–64 pooling all years 1993–2017. *Source:* FEK, JOBB, LISA.

Given these distinct life-cycle patterns, aging has a mechanical effect on aggregate firm creation and worker relocation, holding fixed age-specific mobility. Additionally, aging may also impact age-specific mobility rates. To assess the importance of the composition effect versus indirect equilibrium forces, I compute the firm creation and worker relocation rates in LA-year-age bins. Because some LAs are small and firm creation is rare, I use three age groups: 20–29, 30–44 and 45–64 (similar results hold within more disaggregated groups). I define the firm creation rate at the individual level as the fraction of employment whose first month as self-employed (either unincorporated or incorporated) in firm f was month m and firm f was founded in the current year.[25] The worker relocation rate is the fraction of employment in month m that was either employed by a different employer or non-employed in $m-1$.

I project these outcomes at the LA-year-age level on the share of young in the LA-year, controlling for LA, year and age fixed effects and two-way clustering standard errors at the LA and year level

$$\log y_{i,t,a} = \alpha \log young_{i,t} + \psi_i + \xi_t + \phi_a + \varepsilon_{i,t,a} \quad (2)$$

they are forced to impute the founders of a large share of new firms. Third, their data cover primarily firms founded in the Great Recession and its immediate aftermath. It is possible that older, more experienced founders fared relatively better during this turbulent period. In any case, the peak entry age to entrepreneurship in Sweden is consistent with evidence from other countries (Liang et al., 2018). Moreover, Section 5 estimates a relatively small *direct, composition* effect of aging on firm creation, with most of the estimated fall being accounted for by a lower probability of entry conditional on age in the older economy. For this reason, I believe that it less critical for the structural estimates where exactly the peak in entry takes place.

[24]The firm-level worker relocation rate in Table 2 is the sum of twice the JJ rate, the EN and the hiring rate from non-employment (which in steady-state equals the EN rate plus labor supply growth).

[25]This measure corresponds to the employment-weighted entry rate, whereas that in Table 2 is employment-unweighted.

I weigh regressions such that each age group receives a weight commensurate with its employment share in the LA-year, and each LA-year receives the same aggregate weight.

TABLE 3. AGING AND AGE-SPECIFIC ENTRY AND WORKER MOBILITY ACROSS SPACE

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	All ages		Age controls		Ages 20–29		Ages 30–44		Ages 45–64	
	OLS	IV	OLS	IV	OLS	IV	OLS	IV	OLS	IV
	Panel A. Firm creation rate									
Share 20–44	1.175***	3.685***	0.863***	3.223***	1.276***	3.819***	0.384	1.939**	0.916**	3.639***
	(0.292)	(1.117)	(0.283)	(1.082)	(0.363)	(1.345)	(0.290)	(0.840)	(0.401)	(1.371)
Obs.	6,711	6,711	6,711	6,711	2,230	2,230	2,243	2,243	2,238	2,238
R-squared	0.327		0.582		0.362		0.564		0.469	
within	0.005		0.005		0.009		0.001		0.005	
F-stat		30.1		30.0		32.6		31.7		27.3
	Panel B. Worker relocation rate									
Share 20–44	1.112***	2.714***	0.808***	2.227***	0.355	1.676***	0.716***	1.811***	1.021***	2.671**
	(0.264)	(0.847)	(0.250)	(0.767)	(0.219)	(0.553)	(0.241)	(0.596)	(0.340)	(1.083)
Obs.	6,732	6,732	6,732	6,732	2,244	2,244	2,244	2,244	2,244	2,244
R-squared	0.166		0.919		0.810		0.769		0.679	
within	0.003		0.018		0.008		0.022		0.028	
F-stat		30.2		30.2		32.3		31.8		27.6

Table 3 presents OLS and IV estimates based on regression (2) using annual data from 68 LA from 1986 to 2018 for three age groups (20–29, 30–44 and 45–64). The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year-age group level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year level. Columns 1–2 show results pooling all age groups without age controls. Columns 3–4 add age controls. Columns 5–10 estimate (2) separately by age groups. Panel A presents results for the firm creation rate, defined as the fraction of employment in month m whose first month working in firm f as either an unincorporated or incorporated self-employed was month m, and firm f was founded in the current year. Panel B presents results for the worker relocation rate, defined as the fraction of employment in month m that was either employed by a different employer or non-employed in month $m-1$. *Source:* FEK, JOBB, LISA, SCB.

Columns 1–2 of Table 3 show results without age controls, confirming that aging leads to a relative decline also in employment-weighted firm creation.[26] Controlling for a prospective founder's own age reduces the point estimate (columns 3–4). That is, part of the impact of aging on firm creation is accounted for by the fact that older individuals are less likely to start firms, and aging increases their population share. In addition, when a local labor market ages, individuals of all ages become less likely to start new firms. According to columns 5–10, aggregate aging particularly reduces firm creation among young and older potential founders, with a smaller impact on middle-aged potential founders.

Panel B confirms the findings from Table 2 that aging leads to declines in worker relocation (if the dependent variable had been in levels instead of logs, the point estimates in columns 1–2 would coincide with those in Table 2 panel B columns 1–2). As expected given Figure 3, controlling for the direct effect of age reduces the impact of aging (columns 3–4). Nevertheless, aggregate aging also reduces age-conditional worker mobility. That is, an individual is less mobile in an older labor market, conditional on her own age. According to columns 5–10, the effect is particularly pronounced among older individuals.

[26]The number of observations differs slightly across specifications because the entry rate is zero in a few LA-year-age groups cells. Consequently, these observations are dropped when taking logs. Similar results hold in levels including these cells.

4 A model of labor market dynamics, growth and aging

Why is a person less likely to start a firm and switch employer when people around her are older? Moreover, what are the aggregate and distributional implications of these empirical patterns? What are the effects of policies often proposed to counter the effects of an aging workforce? To answer these questions, this section proposes an equilibrium theory of growth with labor market frictions.

4.1 Demographics, preferences and technology

Time $t \geq 0$ is continuous and the horizon is infinite, and there are no aggregate shocks. I focus first on an economy on a long-run BGP, and later turn to its transitional dynamics.

A mass $\xi e^{\lambda t}$ of entrepreneurs and $e^{\lambda t}$ of workers constitute the labor force at time t. Individuals have one child with probability ω and $1+\nu$ children with probability $1-\omega$, so that the labor force grows at rate $\lambda = \kappa(1-\omega)\nu$. They permanently exit the labor force at rate κ, at which point their children enter. Entrepreneurs' children take over parents' firms, while workers' children enter as unemployed.[27]

Intertemporal utility of an individual at time t is the expected discounted value of flow consumption

$$\mathscr{U}(t) \quad = \quad \mathbb{E}_t \int_t^\infty e^{-(\rho+\kappa)(\tau-t)} \widehat{C}(\tau) d\tau$$

where $\widehat{C}(\tau)$ is flow consumption of a unique final good at time τ and ρ the discount rate. Unemployed individuals enjoy a consumption-equivalent flow value of leisure $\widehat{b}(t)$ and entrepreneurs enjoy a flow value of being their own boss $\widehat{k}(t)$, in addition to any profits made (Hurst and Pugsley, 2011).

A mass $\widehat{L}(t)$ of heterogeneous firms produce the unique, numeraire final good. To operate, a firm requires an entrepreneur with a blueprint for converting labor into output. Let $\widehat{Z}(i)$ denote the idiosyncratic productivity of firm i's blueprint, which is drawn at entry and remains fixed thereafter. The production technology features constant returns to scale in labor as the only factor of production. To recruit workers, firms advertise $\widehat{v}(i,t)$ jobs subject to iso-elastic flow recruiting cost $\frac{c_v}{1+\eta_v}\widehat{Z}(i)\widehat{v}(i,t)^{1+\eta_v}$, where $\eta_v > 0$. It scales in firm productivity, $\widehat{Z}(i)$, expressing that recruiting requires incumbent workers' time, at the cost of foregone goods production (Klenow and Li, 2022). Hence, if an entrepreneur with productivity $\widehat{Z}(i)$ currently employs $\widehat{n}(i,t)$ workers and advertises $\widehat{v}(i,t)$ jobs, the firm makes flow profits

$$\Pi\Big(\widehat{Z}(i), \widehat{n}(i,t), \widehat{v}(i,t)\Big) \quad = \quad \underbrace{\Big(\widehat{Z}(i) - \widehat{c}(t)\Big)}_{\text{value added per worker}} \widehat{n}(i,t) - \underbrace{\int_{j\in i} \widehat{w}(j,t)dj}_{\text{labor cost}} - \underbrace{\frac{c_v}{1+\eta_v}\widehat{Z}(i)\widehat{v}(i,t)^{1+\eta_v}}_{\text{recruiting cost}} - \underbrace{\widehat{r}(t)}_{\text{fixed cost}}$$

[27] If an entrepreneur has multiple children, I assume that the other children may start spin-off firms based on the original blueprint, i.e. knowledge of business ideas transfers perfectly across generations of business owners.

where $\widehat{c}(t) \geq 0$ are costs of intermediate inputs, $\widehat{w}(j,t)$ is the wage paid to worker j working for firm i at time t, and $\widehat{r}(t)$ is a fixed cost of operation. As in Hopenhayn (1992), the presence of the fixed cost ensures that entrepreneurs at some point want to exit. If the fixed cost is not paid, the blueprint is permanently lost, the entrepreneur has to look for a new business idea, and the firm's workers become unemployed.

4.2 Markets

The final good is traded in a competitive market. As noted above, it serves as the numeraire.

Firms and workers contact each other at random in a common, frictional labor market. Both unemployed and employed workers search for jobs, in the latter case with relative search efficiency $\phi \in [0, \infty)$. Let $\widehat{V}(t)$ denote aggregate vacancies and $\widehat{S}(t) = \widehat{u}(t) + \phi\widehat{e}(t)$ aggregate search intensity, where $\widehat{u}(t)$ is the number of unemployed and $\widehat{e}(t)$ the number of employed. The rate at which firms contact workers, $\widehat{q}(t)$, and the rate at which workers contact firms, $\widehat{p}(t)$, are given by

$$\widehat{q}(t) = \frac{\chi \widehat{V}(t)^{\theta} \widehat{S}(t)^{1-\theta}}{\widehat{V}(t)}, \qquad \text{and} \qquad \widehat{p}(t) = \frac{\chi \widehat{V}(t)^{\theta} \widehat{S}(t)^{1-\theta}}{\widehat{S}(t)} \tag{3}$$

where $\chi > 0$ is matching efficiency and $\theta \in [0, 1]$ the elasticity of matches with respect to vacancies.

Wages are determined via bargaining over marginal surplus following Bilal et al. (2022). In the interest of space, I describe only the outcome of the game here. When a worker and firm meet, the firm makes the worker a take-leave offer to purchase the job for a price equal to the value of the worker's current job. In equilibrium, employed workers accept the offer if they currently work for a less productive firm. A worker has the right to the job's proceeds as well as the option of when to terminate it, with one qualification. The entrepreneur, who has to bear the fixed cost of operation, can always shut down the firm and send its workers into unemployed. To prevent such an outcome, workers can make the entrepreneur a take-leave offer to keep the firm alive for another instant.

Following Aghion and Howitt (1994), I interpret the fixed cost $\widehat{r}(t)$ as the price of a fixed factor, whose supply grows at the rate of the labor force, $le^{\lambda t}$. It is traded in a competitive, frictionless market such that

$$\widehat{L}(t) \leq le^{\lambda t}$$

with equality if $\widehat{r}(t) > 0$. The assumption of an inelastic supply of the fixed factor facilitates tractability by reducing the model to a system of two equations in two unknowns, as I demonstrate below. At a conceptual level, the model hence focuses on improvements in a fixed number of blueprints per worker, as opposed to an expansion in the number of blueprints per worker. In their rich estimation exercise, Garcia-Macia et al. (2019) find that the former is a much more important source of aggregate growth.

4.3 The value of an incumbent firm

Let m be the BGP growth rate of output per worker and $\widehat{\underline{Z}}(t)$ the least productive firm in operation at time t, which also grows at the rate m on a BGP. To ensure existence of a BGP, I adopt that

Assumption 1. *The flow value of leisure $\widehat{b}(t) = b\widehat{\underline{Z}}(t)$, the cost of intermediates $\widehat{c}(t) = c\widehat{\underline{Z}}(t)$ and the flow value of being one's own boss $\widehat{k}(t) = k\widehat{\underline{Z}}(t)$ all grow at the rate of the economy m, with $b > 0$, $c > 0$, $c + b = 1$ and k being "sufficiently" high (as made more precise below).*[28]

It is convenient to analyze a scaled version of the growing economy. To that end, let $\widehat{z}(i) \equiv \ln \widehat{Z}(i)$ and $\widehat{\underline{z}}(t) \equiv \ln \widehat{\underline{Z}}(t)$, and define the *relative productivity* of firm i at time t as $z(i,t) \equiv \widehat{z}(i) - \widehat{\underline{z}}(t)$. Then

$$dz(i,t) \quad = \quad -mdt$$

Productivity effectively falls behind the market at the *rate of obsolescence m*, as new firms bid up prices of labor and the fixed factor, but incumbent firms' productivity does not improve.

Under the stipulated bargaining protocol, Bilal et al. (2022) show that an equilibrium allocation can be computed by focusing on the joint value of a firm to its entrepreneur owner and workers, $\widehat{\mathbf{W}}(\widehat{z}, n, t)$

Lemma 1. *The scaled stationary joint value of a firm, $\mathbf{W}(z,n) = e^{-\widehat{\underline{z}}(t)}\widehat{\mathbf{W}}(\widehat{z}, n, t)$, satisfies*[29]

$$\begin{aligned}(\rho - m)\mathbf{W}(z,n) \quad = \quad \max_{v \geq 0} \Bigg\{ & \underbrace{(e^z - c)\, n + k - r}_{\text{net output}} - \underbrace{\kappa\Big(n\mathbf{W}_n(z,n) + \mathbf{E}(z,n)\Big)}_{\text{retirement}} - \underbrace{m\mathbf{W}_z(z,n)}_{\text{obsolescence}} - \underbrace{\frac{c_v e^z v^{1+\eta_v}}{1+\eta_v}}_{\text{cost of advertising jobs}} \\ & + \underbrace{qv\left(\frac{u}{S}\Big(\mathbf{W}_n(z,n) - U\Big)^+ + \frac{\phi(1-u)}{S}\int \Big(\mathbf{W}_n(z,n) - \mathbf{W}_n(\tilde{z},\tilde{n})\Big)^+ d\mathbf{G}(\tilde{z},\tilde{n})\right)}_{\text{return to advertising jobs}} \Bigg\} \end{aligned} \tag{4}$$

where $x^+ = \max\{x, 0\}$, for (z,n) interior to separation and exit boundaries defined by

$$\mathbf{W}_n(z,n) \quad \geq \quad U, \qquad \qquad \text{and} \qquad \qquad \mathbf{W}(z,n) \quad \geq \quad nU + U^f \tag{5}$$

[28]The assumption that these objects scale in the least productive firm is innocuous. For instance, if costs instead had scaled in average productivity, a change of variables would recover assumption 1, since the gap between the average and least productive firm remains fixed on a BGP (in fact, as I demonstrate below this gap is fixed also as the age composition changes).

[29]I assume that households own firms and receive any profits they make as lump-sum dividends. With linear utility, such lump-sum transfers do not affect any decisions, so for simplicity I abstract from them when formulating the value functions.

where $q = \widehat{q}(t)$, $\mathbf{E}(z,n)$ is the scaled value of an entrepreneur with productivity z and workers n, $S = \widehat{S}(t)e^{-\lambda t}$ is aggregate search intensity per worker, $V = \widehat{V}(t)e^{-\lambda t}$ is aggregate vacancies per worker, $u = \widehat{u}(t)e^{-\lambda t}$ is the unemployment rate, $\mathbf{G}(z,n)$ is the distribution of employment over firms, U^f is the scaled value of a prospective entrepreneur, and $U = e^{-\widehat{\underline{z}}(t)}\widehat{U}(t)$ is the scaled value of unemployment, which solves $(\rho - m)U = b - \kappa U$.

Proof. See Appendix C.1. □

Bilal et al. (2022) derive the Hamilton-Jacobi-Bellman (HJB) equation (4) as the solution to a HJB variational inequality. The first term is flow output net of the cost of intermediates and the fixed cost. Second, each of the firm's n workers retires at rate κ, reducing the joint value by the marginal value of the worker, and the entrepreneur retires at rate κ. In the latter case, incumbent matches are preserved since the entrepreneur's offspring takes over the firm, but the joint value falls by the value of the entrepreneur since individuals do not value their offspring. At most one such event can take place at a point in time. Third, the joint value falls as the firm's blueprint gradually becomes obsolete. Finally, the current coalition gains from hiring new workers into the firm. Workers separate to unemployment if their marginal value in the firm falls below the value of unemployment, as indicated by the first boundary condition in (5). A firm exits if its joint value falls below the value of unemployment to its workers and the value to the entrepreneur of searching for new ideas, as highlighted by the second boundary condition.

4.4 Entry

At a point in time, an entrepreneur may either have a blueprint or exert effort looking for one. In the latter case, the entrepreneur chooses the arrival rate πs of ideas subject to iso-elastic flow cost $\frac{c_e}{1+\eta_e}\widehat{b}(t)s^{1+\eta_e}$, where $\eta_e > 0$. It scales in the flow value of leisure, $\widehat{b}(t)$, since search comes at the cost of foregone leisure.

Entrants can imitate and improve upon the blueprints of incumbent firms. Specifically, following Luttmer (2012), I assume that an entrant draws an idiosyncratic, proportional improvement ε over the least productive firm at time t from an innovation distribution $\Gamma(\varepsilon)$ that is exponential with rate ζ

$$\widehat{z}(i) \quad = \quad \widehat{\underline{z}}(t) + \varepsilon(i), \qquad\qquad \Gamma(\varepsilon) \quad = \quad 1 - e^{-\zeta\varepsilon} \tag{6}$$

I later provide a richer model of entrepreneurial choice and knowledge spillovers.

Lemma 2. *The scaled stationary value of a prospective entrepreneur, $U^f = e^{-\widehat{\underline{z}}(t)}\widehat{U}^f(t)$, solves*

$$\Big(\rho - m\Big)U^f \quad = \quad b - \kappa U^f + \max_s \left\{ s\pi \int \Big(\mathbf{W}(z,0) - U^f\Big)^+ d\Gamma(z) - \frac{c_e s^{1+\eta_e}}{1+\eta_e} \right\} \tag{7}$$

Proof. See Appendix C.1. □

4.5 Reducing the state-space

Although technology is linear in labor, the presence of the fixed cost r implies increasing returns to scale. As a result, it is necessary to keep the size of a firm as a state. Moreover, a multilateral bargaining protocol is required to resolve situations in which the entrepreneur wants to shut down the firm to avoid the fixed cost, but there are joint gains from keeping the firm alive. Under assumption 1, we have

Proposition 1. *The joint surplus of the firm, $\mathbf{S}(z,n) = \mathbf{W}(z,n) - nU - U^f$, satisfies for $z \geq \underline{z}$*

$$\mathbf{S}(z,n) = n \underbrace{J(z)}_{\text{match surplus}} + \underbrace{\frac{\iota}{m}\int_0^z e^{-\frac{\rho+\kappa-m}{m}(z-\tilde{z})-\frac{1}{\eta_v}\tilde{z}}\left(q\left(\frac{u}{S}J(\tilde{z})+\frac{\phi(1-u)}{S}\int_0^{\tilde{z}} J'(\hat{z})G(\hat{z})d\hat{z}\right)\right)^{\frac{1+\eta_v}{\eta_v}} d\tilde{z}}_{\text{discounted cumulative net return to hiring new workers into firm}}$$

while for $z < \underline{z}$, $\mathbf{S}(z,n) = 0$, where $\iota \equiv \frac{\eta_v}{1+\eta_v}\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}}$ and $\underline{z} = 0$. The surplus of a match, $J(z)$, satisfies for $z \geq \underline{z}^w$

$$J(z) = \frac{1}{\rho+\kappa-m}\left(\frac{m}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z} - 1\right) + \frac{1}{\rho+\kappa}e^z \tag{8}$$

with $J(z) = 0$ for $z < \underline{z}^w$, and the reservation threshold of workers is $\underline{z}^w = 0$. The value of unemployment U is

$$U = (\rho+\kappa-m)^{-1}b \tag{9}$$

Optimal job creation $v(z)$ by incumbent firms equates the marginal cost of a vacancy with its expected return

$$\underbrace{c_v e^z v(z)^{\eta_v}}_{\text{marginal cost of a vacancy}} = \underbrace{q}_{\text{worker finding rate}} \left(\underbrace{\frac{u}{S}J(z)}_{\text{return from contacting unemployed}} + \underbrace{\frac{\phi(1-u)}{S}\int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z}}_{\text{return from contacting employed}} \right) \tag{10}$$

Optimal search by prospective entrepreneurs s equates its marginal cost with the expected value of a blueprint

$$c_e s^{\eta_e} = \pi\frac{\iota}{m}\int_0^\infty\int_0^{\tilde{z}} e^{-\frac{\rho+\kappa-m}{m}(\tilde{z}-z)-\frac{1}{\eta_v}z}\left(q\left(\frac{u}{S}J(z)+\frac{\phi(1-u)}{S}\int_0^z J'(\hat{z})G(\hat{z})d\hat{z}\right)\right)^{\frac{1+\eta_v}{\eta_v}} dz d\Gamma(\tilde{z}) \tag{11}$$

The number of active firms per worker is $L = \widehat{L}(t)e^{-\lambda t} = l$, and the equilibrium fixed cost is

$$r = k - b - \frac{\eta_e}{1+\eta_e}c_e s^{1+\eta_e} \tag{12}$$

Proof. See Appendix C.2. □

Note that, as expected, the surplus of a match $J(z)$ is strictly increasing in relative productivity, z.

Assumption 1 ensures that workers quit to unemployment before the entrepreneur wants to exit, so that the exit decision is inconsequential for workers. As a result, there is no need for a multilateral bargaining protocol; a firm's size does not impact its constituent matches; and the job ladder reduces to a one-dimensional ranking of firms in terms of productivity, z. Despite this tractability, the framework features rich worker and firm dynamics, including endogenous exit and JJ mobility.

4.6 The distribution of entrepreneurs and workers

Let $\widehat{x}(z,t)$ be the number of entrepreneurs who own a blueprint with relative productivity z at time t, with $x(z)$ its associated probability density function (pdf), and $\widehat{y}(t) = ye^{\lambda t}$ the number of new blueprints created at time t, with y the per worker entry rate. The evolution of the number of entrepreneurs with productivity $z \geq 0$ for $t > 0$ is given by the Fokker-Planck (or Kolmogorov Forward) equation

$$\widehat{x}_t(z,t) = \underbrace{m\widehat{x}_z(z,t)}_{\text{technological obsolescence}} - \underbrace{\kappa\widehat{x}(z,t)}_{\text{retirement of founder}} + \underbrace{\kappa(1+(1-\omega)\nu)\widehat{x}(z,t)}_{\text{inheritance of children}} + \underbrace{\widehat{y}(t)\gamma(z)}_{\text{entry of new firms}} \tag{13}$$

subject to $\widehat{x}(z,0) = \widehat{x}_0(z)$ for $z \geq 0$; $\widehat{x}(z,t) < \infty$ for $z \geq 0, t > 0$; and $\lim_{z\to\infty} \int^z \widehat{x}(\tilde{z},t)d\tilde{z} = Le^{\lambda t}$ for $t > 0$.

On a BGP, the number of entrepreneurs at each point in the distribution must grow at the rate of labor supply. Appendix C.3 uses this fact to show that the BGP distribution of entrepreneurs over relative productivity is for $z \geq 0$ given by the linear second-order ordinary differential equation (ODE)

$$0 = mx'(z) + \frac{y}{L}\gamma(z) \tag{14}$$

subject to the boundary conditions $X(0) = 0$ and $\lim_{z\to\infty} X(z) = 1$, where $X(z) = \int^z x(\tilde{z})d\tilde{z} = 1$.

The distribution of workers over productivity solves the second-order ODE (see Appendix C.4)

$$\Big(\lambda+\kappa\Big)g(z) = \underbrace{mg'(z)}_{\text{obsolescence}} - \underbrace{\phi p\Big(1-F(z)\Big)g(z)}_{\text{separations up the job ladder}} + \underbrace{pf(z)\frac{u}{1-u}}_{\text{hires from unemployment}} + \underbrace{pf(z)\phi G(z)}_{\text{hires from below in the job ladder}} \tag{15}$$

subject to $G(0) = 0$ and $\lim_{z\to\infty} G(z) = 1$, where $G(z) = \int^z g(\tilde{z})d\tilde{z}$ and the unemployment rate u is

$$u = \frac{\kappa+\lambda+mg(0)}{\kappa+\lambda+mg(0)+p} \tag{16}$$

The offer distribution $F(z)$ and aggregate vacancies V are given by

$$F(z) = \frac{L}{V}\int_0^z v(\tilde{z})x(\tilde{z})d\tilde{z}, \qquad \text{and} \qquad V = L\int_0^\infty v(z)x(z)dz \tag{17}$$

4.7 Equilibrium

An equilibrium can be characterized as a system of two equations—an *exit curve* and an *entry curve*—in two endogenous variables—the entry/exit rate and the growth rate. The exit curve $\tilde{y}(m)$ gives the optimal exit rate, $\tilde{y}$, of incumbent entrepreneurs given growth rate m

Proposition 2. *There exists a unique stationary distribution of entrepreneurs over relative productivity given by*

$$x(z) = \zeta e^{-\zeta z} \tag{18}$$

and the optimally chosen exit rate associated with a particular growth rate is given by the exit curve

$$\tilde{y}(m) = \zeta L m \tag{19}$$

Proof. See Appendix C.5. □

When growth is higher, incumbent firms fall behind the market faster. Consequently, a larger share drifts below the endogenous exit threshold at any point in time. Note that the distribution of incumbent entrepreneurs, $x(z)$, is independent of the rate of obsolescence, m.

The entry curve $y(m)$ shows how many individuals want to enter entrepreneurship given a rate of obsolescence m. Specifically, I show in Appendix C.6 that the aggregate entry rate is

$$y(m) = \Omega\left(\frac{1}{m}\int_0^\infty\int_0^{\tilde{z}} e^{-\frac{\rho+\kappa-m}{m}(\tilde{z}-z)-\frac{1}{\eta_v}z}\left(q\left(\frac{u}{S}J(z)+\frac{\phi(1-u)}{S}\int_0^z J'(\hat{z})G(\hat{z})d\hat{z}\right)\right)^{\frac{1+\eta_v}{\eta_v}} dz d\Gamma(\tilde{z})\right)^{\frac{1}{\eta_e}} \tag{20}$$

where $\Omega = (\xi - L)\pi\left(\frac{\pi\iota}{c_e}\right)^{\frac{1}{\eta_e}} > 0$ is a positive parameter and $\iota = \frac{\eta_v}{1+\eta_v}\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}} > 0$ as defined above.

Definition 1. *A stationary equilibrium is a match surplus $J(z;m)$ and reservation threshold $\underline{z}^w(m)$; an incumbent vacancy policy $v(z;m)$ and reservation threshold $\underline{z}(m)$; a search intensity of prospective entrepreneurs $s(m)$; a number of active firms $L(m)$; a fixed cost $r(m)$; finding rates $p(m)$ and $q(m)$; aggregate vacancies $V(m)$ and search intensity $S(m)$; a distribution of firms $x(z;m)$; an offer distribution $F(z;m)$; a distribution of employment $G(z;m)$ and unemployment rate $u(m)$; an exit rate $\tilde{y}(m)$ and entry rate $y(m)$; and a growth rate $m \in (0, \rho+\kappa)$*

such that: (i) $J(z;m)$ and $\underline{z}^w(m)$ is given by (8); (ii) $v(z;m)$ satisfies (10) and $\underline{z}(m) = 0$; (iii) $s(m)$ is given by (11); (iv) $L(m) = l$ and $r(m)$ is given by (12); (v) $p(m)$ and $q(m)$ are given by (3); (vi) V is given by (17) and $S(m) = u(m) + \phi(1 - u(m))$; (vii) $x(z;m)$ is given by (18); (viii) $F(z;m)$ is given by (17); (ix) $G(z;m)$ and $u(m)$ solve (15)–(16); (x) $\tilde{y}(m)$ is given by (19); (xi) $y(m)$ is given by (20); and (xii) m is such that $\tilde{y} = y$.

To make further progress analytically, I assume

Assumption 2. *Vacancy creation is inelastic, $\eta_v \to \infty$, and the elasticity of the job finding rate with respect to vacancies is zero, $\theta \to 0$.*

Under assumption 2, all firms create exactly one vacancy, $v(z) \equiv 1$, so that $V = L$, $p = \chi$, $q = \chi S/L$, and $F(z) = 1 - e^{-\zeta z}$. Hence, key choices reduce to incumbent firms' decision of when to exit and prospective entrepreneurs' choice of how hard to search for new business ideas. Section 5 shows quantitatively that the insights below continue to hold when $\eta_v < \infty$ and $\theta > 0$.

Proposition 3. *There exists a unique stationary distribution of workers over relative productivity given by*

$$G(z;m) = \frac{\kappa+\lambda}{m} \int_0^z e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}\left(e^{-\zeta\tilde{z}} - e^{-\zeta z}\right) + z - \tilde{z}\right)} \left(\frac{\int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta\hat{z}} - \hat{z}\right)} \left(e^{-\zeta\tilde{z}} - e^{-\zeta\hat{z}}\right) d\hat{z}}{\int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta\hat{z}} - \hat{z}\right) - \zeta\hat{z}} d\hat{z}} \right) d\tilde{z} \tag{21}$$

where $\beta \equiv \frac{\phi\chi}{\kappa+\lambda}$ and the unemployment rate is given by

$$u(m) = \left(\int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta z} - z\right)} \left(\frac{\beta}{\phi} e^{-\zeta z} + 1 \right) dz \right)^{-1} \int_0^\infty e^{\frac{\kappa+\lambda}{m}\left(\frac{\beta}{\zeta}e^{-\zeta z} - z\right)} dz \tag{22}$$

Proof. See Appendix C.7. □

A stationary equilibrium exists if the exit and entry curves cross at least once for $m \in (0, \rho + \kappa)$. A negative growth rate would imply negative exit, which is not sensible, while the growth rate must be below the rate of effective discount, $\rho + \kappa$, or the values of a searching entrepreneur (7) and of being unemployed (9) explode. The following is sufficient for the existence of a unique equilibrium

Proposition 4. *If the arrival rate of ideas per unit of search intensity (π) or the share of entrepreneurs (ξ) is sufficiently small, there exists at least one stationary equilibrium with $m \in (0, \rho + \kappa)$. If in addition the curvature in the cost of searching for ideas (η_e) is sufficiently high, the stationary equilibrium is unique.*

Proof. See Appendix C.8. □

The exit rate (19) is clearly increasing in the growth rate m. How does a change in growth affect incentives to start new firms, i.e. what is the slope of the entry curve (20)? To analyze this, I focus on

Assumption 3. *Growth is small relative to the sum of the labor force exit rate and labor supply growth, $m \ll \kappa + \lambda$, as well as relative to the sum of the labor force exit rate and the discount rate, $m \ll \kappa + \rho$.*

Higher growth affects the value of a new blueprint and hence incentives to enter through three channels. First, a higher growth rate implies that jobs and firms are expected to last shorter, reducing the value of a blueprint through an *obsolescence effect*. Second, a higher growth rate increases the capitalized value of jobs and firms, raising the value of a blueprint via a *capitalization effect*. These two forces, which were highlighted by Aghion and Howitt (1994), are here resolved in favor of the obsolescence effect

Lemma 3. *Holding fixed the composition of the labor force—$u(m)$ and $G(z;m)$—an increase in growth unambiguously reduces the value of a new blueprint and hence discourages entry* (20)*.*

Proof. See Appendix C.9. □

The current framework, however, highlights also a third, new channel through which higher growth affects incentives to enter. It works by making the labor market more *misallocated* (in a first best sense)

Lemma 4. *Higher growth raises unemployment and shifts employment toward relatively less productive firms.*

Proof. See Appendix C.10. □

When new technologies are introduced at a rapid pace, workers are afforded little time to relocate across these technologies before they are replaced. As a result, workers are poorly matched to existing technologies, reducing the opportunity cost of switching to new technologies.

What governs the relative strength of the obsolescence, capitalization and new *misallocation effect*? The slope of the entry curve is inverse-U shaped in the severity of labor market frictions

Proposition 5. *There exists a sufficiently low matching efficiency $\chi_1 \in (0, \infty)$ such that for $\chi < \chi_1$, entry falls with growth, $y'(m) < 0$. There exists a sufficiently high matching efficiency $\chi_2 \in (0, \infty)$ such that for $\chi > \chi_2$, entry falls with growth, $y'(m) < 0$. For moderate levels of labor market frictions, $\chi \in (0, \infty)$ and $\phi \in (0, 1]$, there exists a sufficiently high discount rate $\rho' \in (0, \infty)$ such that for $\rho > \rho'$, entry rises with growth, $y'(m) > 0$.*

Proof. See Appendix C.11. □

To understand proposition 5, note that the unemployment rate converges to one and the employment distribution to the distribution of firms as labor market frictions become severe, $\chi \to 0$. In this case, as shown in panel A of Figure 4, growth does not affect the composition of the labor force. Absent a misallocation effect, the value of a new blueprint and hence entry fall with growth (lemma 3).

Consider next the case in which workers very quickly take full advantage of new technologies, $\chi \to \infty$. With an unbounded productivity distribution and constant returns to scale in production, all workers would instantly move to an infinitely productive firm and enjoy unbounded utility, which is not sensible. The intuition, however, carries over to the case in which the offer distribution $F(z)$ has finite upper support $\bar{z} < \infty$. In this case, the economy converges to a situation in which all workers work for the most productive firm $\bar{z}$, as illustrated by panel B. Growth has no effect on the composition of the labor market, again rendering the misallocation effect inoperative so that entry falls with growth.

In contrast, for moderate levels of labor market frictions, workers gradually become better at using existing technologies by relocating across them. High growth affords workers less time to relocate across existing technologies before these technologies are displaced. As a result, as shown by panel C workers are worse matched to existing technologies, raising the value of a new blueprint.

FIGURE 4. THE EFFECT OF GROWTH ON THE EMPLOYMENT DISTRIBUTION

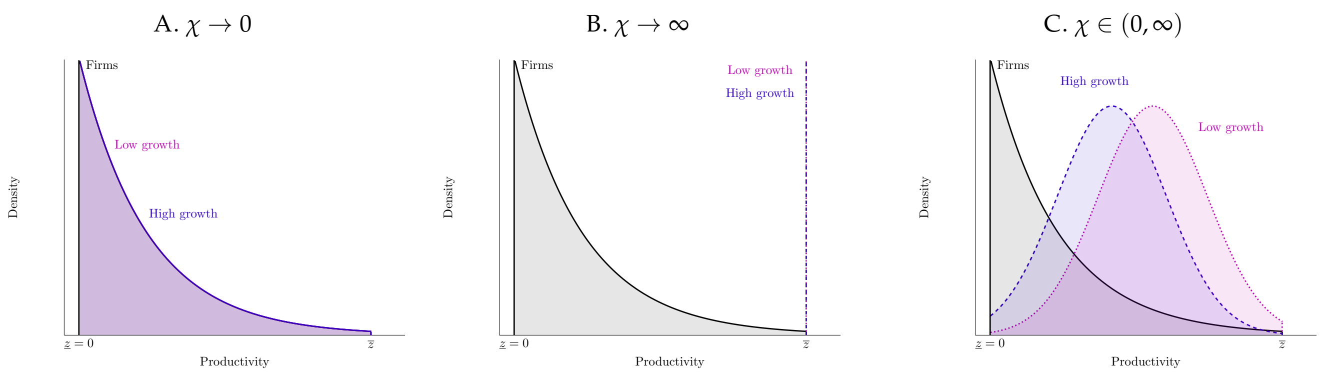


Figure 4 plots the stationary distribution of employment, $g(z)$, for a high and low growth rate, m. Panel A shows the case of severe labor market frictions, $\chi \to 0$. Panel B shows the case of vanishing labor market frictions, $\chi \to \infty$. Panel C shows the case of moderate labor market frictions, $\chi \in (0, \infty)$. *Source:* Model.

4.8 Aging

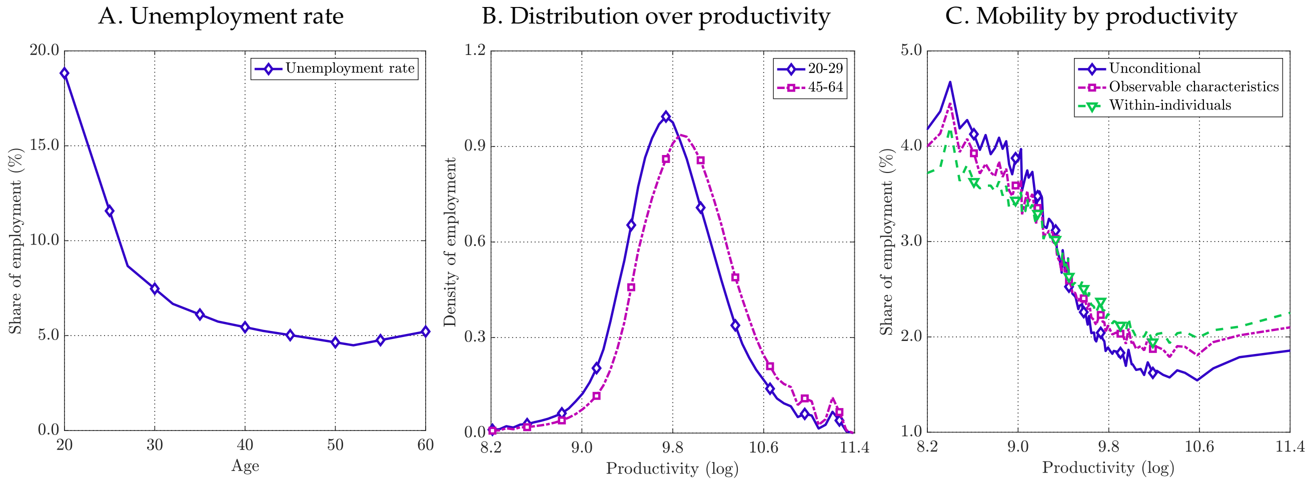
I now turn to an analysis of the comparative static effect of aging on the BGP equilibrium. To that end, it is useful to track an individual's age a, even though it is not a state

Lemma 5. *In the limit $m \to 0$, older individuals are less likely to be unemployed, $\widehat{u}'(a) < 0$, and they tend to be better matched to incumbent technologies, $\frac{\partial \widehat{G}(z|a)}{\partial a} < 0$, so that they are less likely to make a JJ move, $\widehat{JJ}'(a) < 0$.*

Proof. See Appendix C.12. □

Consistent with these assertions, Figure 5 shows that older individuals are less likely to be unemployed (panel A) and tend to work for more productive firms (panel B). According to panel C, individuals working for more productive firms are less likely to switch employer.

FIGURE 5. DISTRIBUTION OF INDIVIDUALS BY AGE AND MOBILITY BY PRODUCTIVITY



Panel A plots the unemployment rate by age based on the standard ILO definition. Panel B plots the distribution of individuals by age over log value added per worker. Panel C plots the share of workers in month t who have a different main employer in month $t+1$, where the main employer in a month is that paying the most in the year, by log value added per worker. "Observable characteristics" controls flexibly for gender, education and age. "Within-individuals" controls for individual-fixed effects. *Source:* EU-LFS, FEK, JOBB, LISA.

In contrast to its success in accounting for the cross-sectional patterns of JJ mobility over the life-cycle, the theory cannot match the inverse-U shaped pattern of entry by age in the data (recall Figure 3)

Lemma 6. *The entry rate is independent of age,* $\widehat{y}'(a) = 0$.

Proof. See Appendix C.13. □

Consequently, the theory does not feature a direct, composition effect of aging on entry. Although I stress that such composition effects are less important than age-specific declines in accounting for both the aggregate fall in entry over time as well as the estimated impact of aging on entry across local labor markets, I provide below an extension that can also speak to the empirical life-cycle profile of entry.

Lemma 7. *Lower labor supply growth reduces the share of the workforce younger than a,* $-\frac{\partial \Lambda(a;\lambda)}{\partial \lambda} < 0$.

Proof. See Appendix C.14. □

Because a change in labor supply growth only impacts the economy through the resulting change in the composition of the workforce, I henceforth refer to a decline in labor supply growth as aging.

Proposition 6. *Holding fixed growth, aging lowers aggregate unemployment and improves the economy's ability to produce using existing technologies—it shifts employment up the job ladder*

$$-\left.\frac{\partial u(m,\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} \approx -\frac{\beta}{(1+\beta)^2}\frac{1}{\kappa+\lambda} < 0$$

$$-\left.\frac{\partial G(z;m,\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} \approx -\frac{\beta e^{-\zeta z}}{(1+\beta e^{-\zeta z})^2}\frac{1}{\kappa+\lambda}\left(1-e^{-\zeta z}\right) < 0$$

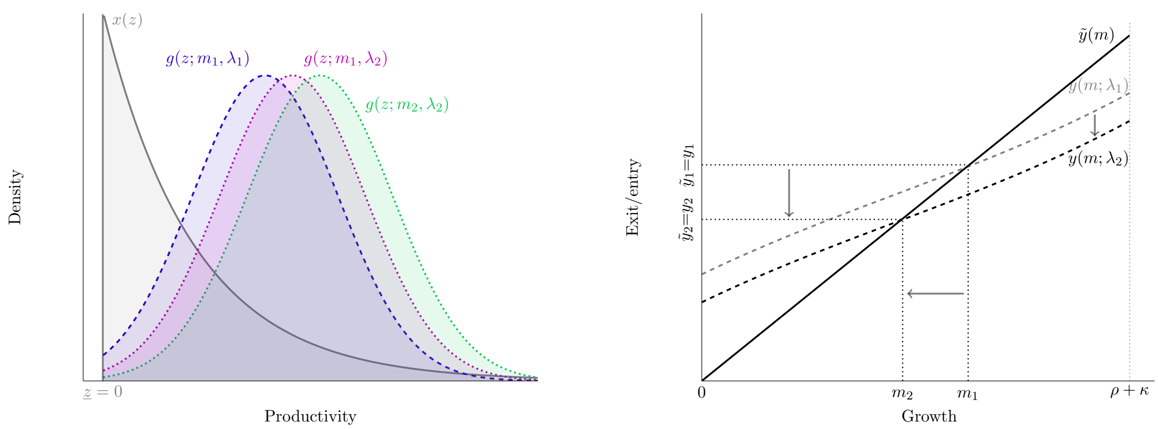
Moreover, it has no effect on the exit curve (19)*. If $\phi \in (0, 1]$, aging reduces the value of a new blueprint and shifts the entry curve* (20) *down, in equilibrium reducing the growth rate, $-m'(\lambda) < 0$.*

Proof. See Appendix C.15. □

By increasing the employment share of well-matched individuals, aging shifts employment toward more productive firms, holding fixed the growth rate, as the pink line in panel A of Figure 6 illustrates. Consequently, it improves static allocative efficiency, which I define as the economy's ability to produce using the blueprints currently in existence. The better match with existing technologies reduces the value of new blueprints, shifting down the entry curve as in panel B of Figure 6.[30]

FIGURE 6. THE IMPACT OF AGING ON LABOR MISALLOCATION AND GROWTH

A. The employment distribution B. Entry, exit & growth



Panel A plots the stationary distribution of employment in the young economy $g(z; m_1, \lambda_1)$, the counterfactual distribution in an older economy holding the growth rate fixed at its value in the young economy, $g(z; m_1, \lambda_2$, and that in the older economy, $g(z; m_2, \lambda_2)$. Panel B plots the exit curve (19) and entry curve (20) for a high and low rate of labor supply growth, λ. *Source:* Model.

Appendix C.16 provides reduced-form support for these predictions. I estimate that aging shifts employment toward high-productive firms, raising aggregate productivity as well as pay in wage employment. Moreover, it reduces poaching and, conditional on a poach taking place, increases the productivity of the poached worker's previous employer relative to that of the new employer. Finally, it reduces profits in self-employment (although with a not statistically significant p-value of 0.26).

Aging affects JJ mobility through both a direct and an indirect channel

Proposition 7. *Suppose $\phi \in (0, 1]$. Aging reduces the aggregate JJ mobility by shifting composition toward less*

[30]If $\phi > 1$, older individuals search more efficiently, since they are more likely to be employed. By raising aggregate search intensity and hence the worker finding rate, aging may raise the value of recruiting, even though it makes the labor market better matched. My estimates in the next section indicate that this force is not strong enough to result in this outcome.

mobile individuals holding fixed growth, and by lowering the growth rate, $-m'(\lambda) < 0$,

$$-JJ'(\lambda) \quad = \quad -\phi p \int_0^\infty f(z) \left(\underbrace{\left.\frac{\partial G(z; m, \lambda)}{\partial \lambda}\right|_{m \text{ fixed}}}_{\text{direct effect, }>0} + \underbrace{m'(\lambda) \frac{\partial G(z; m, \lambda)}{\partial m}}_{\text{indirect effect, }>0} \right) dz$$

The resulting fall in growth leaves JJ mobility of young workers unaffected but reduces that of older individuals

$$-\lim_{a \to 0} \frac{\partial \widehat{JJ}(a; \lambda)}{\partial \lambda} \quad = \quad -\phi p \int_0^\infty f(z) \lim_{a \to 0} \left(\underbrace{\left.\frac{\partial \widehat{G}(z|a; m, \lambda)}{\partial \lambda}\right|_{m \text{ fixed}}}_{\text{direct effect, }\equiv 0} + m'(\lambda) \underbrace{\frac{\partial \widehat{G}(z|a; m, \lambda)}{\partial m}}_{\text{indirect effect, }=0} \right) dz \quad = \quad 0$$

$$-\lim_{a \to \infty} \frac{\partial \widehat{JJ}(a; \lambda)}{\partial \lambda} \quad = \quad -\phi p \int_0^\infty f(z) \lim_{a \to \infty} \left(\underbrace{\left.\frac{\partial \widehat{G}(z|a; m, \lambda)}{\partial \lambda}\right|_{m \text{ fixed}}}_{\text{direct effect, }\equiv 0} + m'(\lambda) \underbrace{\frac{\partial \widehat{G}(z|a; m, \lambda)}{\partial m}}_{\text{indirect effect, }>0} \right) dz \quad < \quad 0$$

Proof. See Appendix C.17. □

Slower growth in the older economy affords individuals more time to relocate across existing technologies before these technologies are replaced. Consequently, people of all ages are better matched to existing technologies in the older economy, i.e. the equilibrium employment distribution is shifted further up the job ladder as illustrated by the green line in panel A of Figure 6. Their better match with existing blueprints reduces the chance that they accept an outside job offer also conditional on age, as shown by panel A of Figure 7. The decline is particularly pronounced among older individuals, because the slowdown in growth significantly improves their match with existing technologies. These theoretical predictions are exactly as indicated by the reduced-form evidence in Section 3.[31]

Since entry does not vary with age, workforce aging has no direct, composition effect on entry, holding fixed age-specific entry rates. It does, however, affect entry through two equilibrium channels

Proposition 8. *Suppose $\phi \in (0, 1]$. Holding fixed growth, aging (proportionally) reduces age-specific entry rates*

$$-\left.\frac{\partial \widehat{y}(a; \lambda)}{\partial \lambda}\right|_{m \text{ fixed}} \quad = \quad -\left.\frac{\partial y(m, \lambda)}{\partial \lambda}\right|_{m \text{ fixed}} \quad < \quad 0$$

[31] The next section relaxes the assumption that $\eta_v \to \infty$, such that the job finding rate p changes with aging. In this case, aging tends to also reduce JJ mobility of labor market entrants, but by less than for older individuals, consistent with the data.

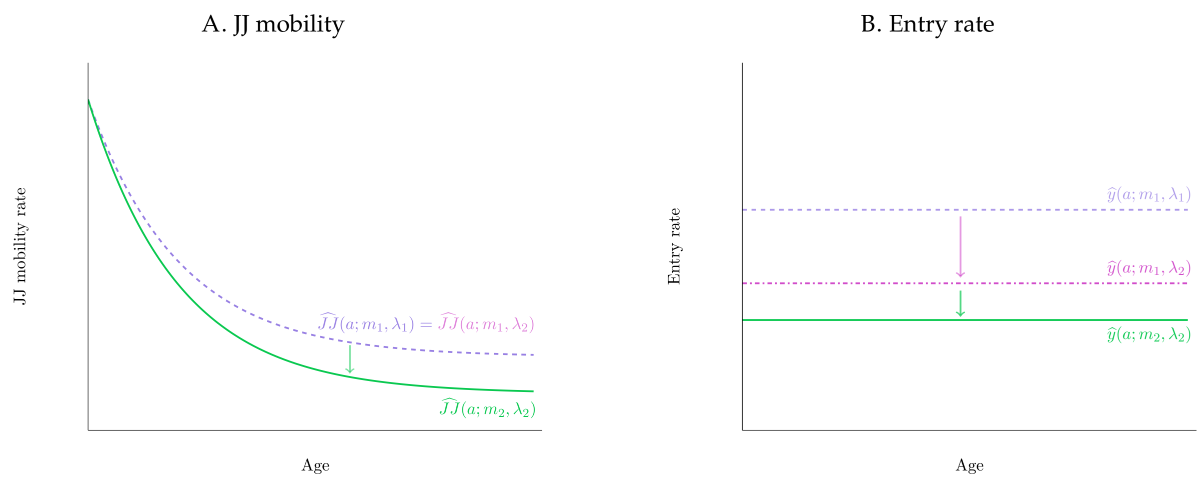
The resulting fall in growth in the older economy may (proportionally) decrease entry further or on net (proportionally) increase it, depending on the relative strength of the obsolescence, capitalization and misallocation effects

$$-\frac{\partial \widehat{y}(a;\lambda)}{\partial \lambda} \quad = \quad -\left(\frac{\partial y(m,\lambda)}{\partial \lambda}\bigg|_{m \text{ fixed}} + m'(\lambda)\frac{\partial y(m,\lambda)}{\partial m} \right) \quad \lesseqgtr \quad -\frac{\partial \widehat{y}(a;\lambda)}{\partial \lambda}\bigg|_{m \text{ fixed}}$$

Proof. See Appendix C.18. □

Anticipating a harder recruiting environment, prospective entrepreneurs are discouraged from entering in the older economy. As highlighted by proposition 5, if labor market frictions are moderate, the resulting fall in growth discourages entry further in equilibrium. Panel B of Figure 7 illustrates this case. The proportional decline in entry is broadly consistent with the reduced-form patterns in Section 3.

FIGURE 7. THE IMPACT OF AGING ON AGE-SPECIFIC LABOR MARKET DYNAMICS



Panel A plots the age-specific JJ mobility rate in a young (λ_1) and old ($\lambda_2 < \lambda_1$) economy. $\widehat{JJ}(a;\lambda_2,m_1)$ lets the age composition adjust, but holds the growth rate fixed at its value in the young economy. Panel B plots the age-specific entry rate in a young (λ_1) and old ($\lambda_2 < \lambda_1$) economy. $\widehat{y}(a;\lambda_2,m_1)$ lets the age composition adjust, but holds the growth rate fixed at its value in the young economy. *Source:* Model.

Who benefits and who loses from the decline in growth induced by aging?

Proposition 9. *Suppose $\phi \in (0,1]$. Aging reduces the value of labor market entrants*

$$-U'(\lambda) \quad = \quad -m'(\lambda)\frac{b}{(\rho+\kappa-m)^2} \quad < \quad 0$$

and the joint value to the worker and firm of low-productive jobs, but raises the joint value of high-productive jobs

$$-\lim_{z\to 0}\frac{\partial(J(z;\lambda)+U(\lambda))}{\partial\lambda} \quad < \quad 0, \qquad \textit{but} \qquad -\lim_{z\to\infty}\frac{\partial(J(z;\lambda)+U(\lambda))}{\partial\lambda} \quad > \quad 0$$

Proof. See Appendix C.19. □

The lower growth rate in the older economy implies that new, better job opportunities arrive at a slower pace. As a result, the value of unemployment and of low-productive jobs fall. At the same time, jobs last longer, *ceteris paribus* raising their value. This destructive aspect of growth is more important for high-productive jobs, such that their value rises with a fall in growth. Because older individuals tend to be employed in more productive jobs (lemma 5), workforce aging raises the value of their jobs.

5 A quantitative assessment of the impact of aging

To quantify the impact of aging, I calibrate an extended version of the model to a set of micro moments in 2014–2018 that are uniquely available in the Swedish matched employer-employee-entrepreneurship data.[32] Subsequently, I change labor supply growth λ, holding all other parameters fixed at their estimated values, either across BGPs or over the transition path.

5.1 Empirical extensions

To better speak to the unique micro data, I first enrich the model in a few dimensions.

Entrepreneurial choice. The theory accounts for the facts that older individuals are less likely to make a JJ move and that aging reduces age-specific JJ mobility and entry rates, but it cannot match entry by age in Figure 3. To speak to this pattern, I allow individuals to move between wage and self employment.[33]

Panel A of Figure 8 shows that employees in more productive firms are less likely to start new firms. Motivated by this pattern, I assume that workers choose how hard to search for ideas subject to cost $\frac{c_e}{1+\eta_e} e^z s^{1+\eta_e}$, where z is the productivity of the current employer, with $z = 0$ for the unemployed. Because older individuals tend to be better matched, their opportunity cost of entry is higher, so entry falls with age. An exiting entrepreneur becomes unemployed, at which point she looks for new jobs or ideas.

Knowledge spillovers. The effect of aging arises because JJ mobility facilitates people's ability to produce using existing technologies, without similarly improving their innovative capacity. A better match with existing technologies may, however, also improve a person's ability to come up with new ideas. Consistent with this view, panel B shows that workers in more productive firms tend to themselves start

[32] As illustrated by Figure 1, aging of the workforce was largely completed by the mid-2000s. When I later consider a transition experiment, I find that labor market dynamics converge to steady-state relatively quickly. Hence, labor market flows in 2014–2018 are not far off those in the older economy steady-state, motivating the choice to target 2014–2018 averages.

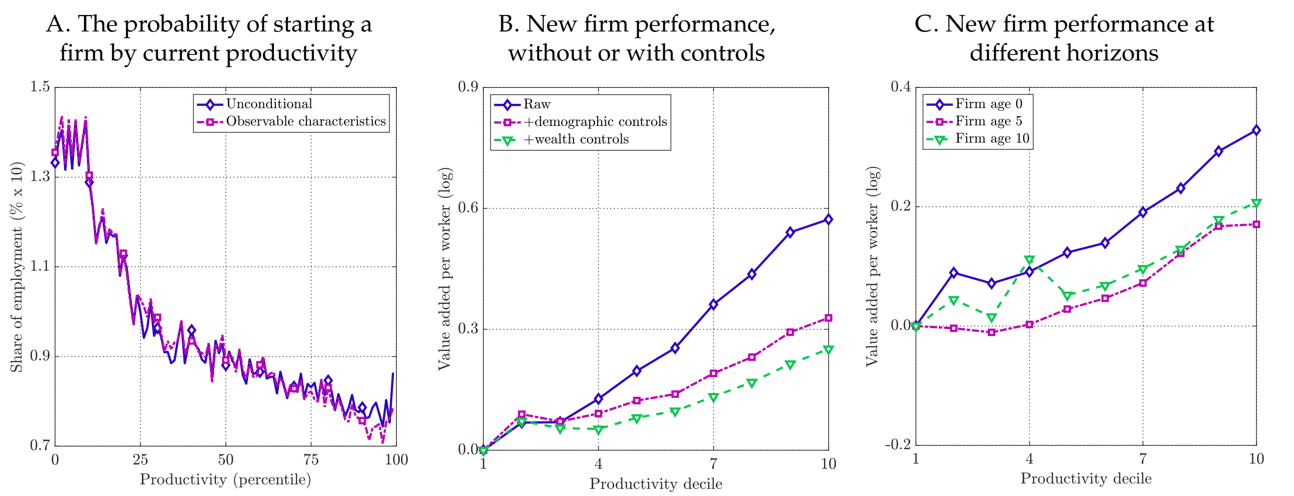
[33] The vast majority of founders of new firms at worked at some earlier point as wage employees (Appendix D.1).

more productive firms. Although the differences moderate somewhat over time, they remain substantial 10 years after the new firm was founded (panel C). To speak to these patterns, I assume that entrants draw a productivity that depends on that of their current employer

$$\Gamma(z|\tilde{z}) \quad \sim \quad \mathcal{N}\Big(\alpha_0 + (1-\alpha_1)\overline{z} + \alpha_1 \tilde{z} \, , \, \zeta \Big) \tag{23}$$

where $\overline{z}$ is average relative productivity, α_0 governs the extent of *general knowledge spillovers*, α_1 captures the degree of *specific knowledge spillovers* from the prior employer $\tilde{z}$, with $\tilde{z} = 0$ for the unemployed. If specific knowledge spillovers are sufficiently strong, entry rises with age. If they are moderate, entry may first rise with age, then decline as the higher opportunity cost becomes more important.

FIGURE 8. FIRM CREATION AND SUBSEQUENT FIRM PERFORMANCE BY FIRM PRODUCTIVITY



Panel A plots the share of non-entrepreneurs in month t who are the owner-operator of a new firm in month $t+1$ by the percentile of value added per worker of the individual's current employer, where a new firm is a firm started in the current year and which has at most 10 identified founders in its first year of operation. Residual controls flexibly for gender and education. Panel B plots value added per worker of new firms by decile of the productivity of the founder's prior employer. Demographic controls controls for sector and year of foundation of the new firm, as well as education, gender and age of the founder at time of foundation fully interacted with age of the new firm. Wealth controls also for the net wealth of the founder. Panel C plots value added per worker of new firms by decile of the productivity of the founder's prior employer, controlling for sector and year of foundation of the new firm, as well as education, gender and age of the founder at time of foundation fully interacted with age of the new firm, at firm age 0, 5 and 10. All panels. Private sector firms and individuals aged 20–64 between years 1997–2018. *Source:* FEK, JOBB, LISA.

A horizon effect. I allow for a direct role for age to avoid forcing the estimation to load all empirical life-cycle patterns on the job ladder mechanism emphasized here. Individuals enter the labor market at age $a = 0$ and remain with certainty up to age $\overline{A}$, after which they exit at rate κ. Consequently, older individuals are less likely to start firms both because they are better matched to existing technologies and because they have a shorter horizon remaining to recoup the entry cost. The flow value of leisure, $b(a)$, and of being one's own boss, $k(a)$, depend flexibly on age, subject to $\underline{z}^w(a) \geq \underline{z}(a) = 0$ for all a. Because entrepreneurs stay in business at least up to the point where workers prefer to quit and entrepreneurs' children take over firms, the age of the entrepreneur is irrelevant for workers.

At labor market entry, the arrival rate of business ideas is π_0 per unit of search intensity, which increases to $\pi_1 \geq \pi_0$ at age $\overline{A}^{\pi}$. This assumption captures in reduced form the idea that some minimum amount of work experience facilitates coming up with successful business ideas (Liang et al., 2018).

Exogenous separations. Firms also exit at exogenous rate d, which allows the model to match the exit rate of high-productive firms in the data. Workers also separate to unemployment at exogenous rate $\delta(z)$, which allows the model to match a decline in the EN rate with productivity in the data.

Productivity shocks and incumbent innovation. Let μ be a deterministic drift capturing incumbent innovation, σ an intensity of idiosyncratic shocks, and $W(t)$ the standard Wiener process. In logs $\widehat{z}(i) = \ln \widehat{Z}(i)$, the idiosyncratic productivity of an incumbent firm evolves according to the Brownian motion $d\widehat{z}(i,t) = \mu dt + \sigma dW(t)$. The inclusion of the drift allows the data to flexibly determine the relative importance of entry/exit versus incumbent innovation toward overall economic growth. Incumbent innovation is assumed to be unaffected by aging, motivated by the empirical observation that aging does not impact investment per worker (Appendix B.6).[34] Appendix D.2 shows that a fall in entry raises productivity dispersion, consistent with Swedish trends over this period.

Appendix D.3 presents the value functions in the extended model and Appendix D.4 defines an equilibrium. Appendix D.5 discusses the algorithm I use to solve the model.

5.2 Calibration

I parameterize the model at a monthly frequency in three steps, as summarized by Table 4.

Pre set. I pre-set or normalize five parameters (panel A), including the equivalent of an annual discount rate (ρ) of four percent and an elasticity of the job finding rate to vacancies of $\alpha = 0.5$.

Offline. I calibrate offline 10 parameters (panel B). I fix the overall growth rate to two percent annually. The supply of the fixed factor (l) targets a ratio of workers to firms of 4.62 and the exogenous firm exit rate targets the exit rate of high-productive firms ($d = 0.002$).

I construct a grid for age and assume that people enter the labor market at its lowest point. Individuals' recorded "calendar age" at entry is drawn from a bounded exponential distribution over ages

[34] Although I do not find an impact of aging on incumbent investment, it would nevertheless be interesting to extend the framework to study how aging impacts also incumbent innovation in future work.

18 to 35. I set the rate parameter to 0.029 to match the labor force participation rate by age in the data. The arrival rate of business ideas increases at $\overline{A}^{\pi} = 120$ (10 years into careers). Individuals start to retire at $\overline{A} = 480$ (40 years into careers) at a rate calibrated to match the labor force participation rate late in careers ($\kappa = 0.022$). Finally, I find the growth rate of labor supply (λ) such that the model matches the share of the labor force that is 45 years and older. Appendix D.6 shows that the model matches well the empirical labor force participation rate by age and the age composition of the labor force.

Internal. I determine 13 parameters so as to minimize the sum of squared percent deviations between 13 moments in the model and data. For each potential vector of these 13 parameters, I normalize the flow value of being one's own boss $k(a)$ such that entrepreneurs of all ages are indifferent between keeping their firm alive and exiting at the lowest point on the discretized grid for productivity, under a normalized relative fixed cost of $r = 1$. The implied flow value of being one's own boss corresponds to about 20 percent of average flow output per worker (see Appendix D.6).

Panel D summarizes the internally calibrated parameters. Although the estimation is joint, some moments particularly inform some parameters, as I now discuss heuristically (Appendix D.7 offers further support for this argument). I set matching efficiency (χ) to target the NE rate.[35] Relative search efficiency of the employed (ϕ) is informed by the JJ rate. A higher ϕ raises the JJ rate, holding χ fixed. The estimated $\phi \approx 2.8$ is high, consistent with recent evidence that the employed are more efficient at searching for jobs (Faberman et al., 2020). Mechanically, the primary reason for the high estimated ϕ is the low NE rate, which implies a low matching efficiency, χ. Hence, ϕ needs to be high to match the JJ rate.

I assume that workers' reservation threshold is a linear function of their age $\underline{z}^{w}(a) = b_a a$. Although the reservation threshold is an endogenous equilibrium object, the flow value of leisure $b(a)$ is allowed to vary freely. Appendix D.5 shows how it can be recovered to rationalize any reservation threshold. I set the slope b_a to target the JJ rate at age 50 relative to age 30. If older individuals are less likely to accept low-productive jobs with a high subsequent JJ rate, the JJ rate falls more with age. Although I prefer to target the life-cycle behavior of the JJ rate given its central role in the analysis, the model matches well also the decline in the NE rate with age in the data. The implied flow value of leisure corresponds to between 5–25 percent of average flow output per worker (see Appendix D.6).

I parameterize the separation rate as $\delta(z) = \delta_0 e^{\delta_1 z}$ and inform δ_0 and δ_1 by the overall EN rate as well as the EN rate in the third quintile of firm productivity relative to the first quintile. If δ_0 is higher, the

[35]The non-employed in the administrative data likely contains some individuals who never participate in the labor market, which the theory abstracts from. I have alternatively targeted an imputed NE rate based on the steady-state flow balance identity that $n = en/(en + ne)$ and a measure of the unemployed plus marginally attached from a separate labor force survey, *Arbetskraftsundersökningarna* (AKU).[36] AKU indicates a stock of unemployed plus marginally attached of 10.94 percent, for an imputed NE rate of 7.87 percent. The estimated impact of aging changes little under this alternative calibration.

overall EN rate is higher. If δ_1 is more negative, the EN rate falls more with productivity.

I target for the arrival rate of business ideas (π_0, π_1) the overall entry rate as well as the entry rate at age 20 relative to age 30. If π_0 and π_1 are both higher, the entry rate is higher, whereas if π_1 is higher holding π_0 fixed, entry rises more early in careers. The curvature of the cost of searching for ideas ($\eta_e \approx 2.2$) is set to match the entry rate at age 50 relative to age 30. In the estimated model, the net gain from entry declines with age, because older individuals are better matched and hence have a higher opportunity cost of entry as well as a shorter expected time to recoup the entry cost. The lower is η_e, the more entry responds to the fall in the net gain, so that entry declines more with age.

General knowledge spillovers (α_0) target growth in residual log value added per worker between firm age one and 11. If α_0 is more negative, productivity grows more with firm age. Specific knowledge spillovers (α_1) target the difference in residual value added per worker at firm age five between founders who were previously employed in a firm in the third quintile of residual value added per worker relative to the first quintile. The shape of the entry distribution (ζ) targets the annual autocovariance of residual log value added per worker among firms aged one. I target the annual autocovariance, assuming that measured productivity is the sum of a permanent "true" component and i.i.d. measurement error.

The dispersion of productivity shocks (σ) targets the increase in the autocovariance of residual log value added per worker between firm age one and five. More pronounced shocks lead productivity dispersion to rise more with firm age.

I target for the curvature of the vacancy cost (η_v) the vacancy share of the 40 percent highest value added per worker firms (employment-unweighted). If η_v is higher, it is costlier for firms to scale up hiring, such that high-productive firms create fewer jobs. The very high estimated curvature, $\eta_v \approx 18$, is intriguing. At a mechanical level, it is driven by the relatively low vacancy share of high-productive firms in the data. Although this moment may be prone to error, the model is broadly consistent with average firm size and job reallocation by firm age. In any case, the impact of aging is insensitive to η_v.

5.3 Life-cycle worker and firm dynamics

The model matches well life-cycle worker and firm dynamics (Figure 9). Entry initially rises with age and then declines (panel A). The JJ rate declines with age (panel B), since older individuals are better matched. In addition, they are more selective in terms of what jobs they accept from unemployment ($b_a > 0$), such that they have a lower subsequent JJ mobility. Appendix D.8 discusses further the role of different forces in driving these life-cycle patterns.

New firms do not enter at the "technological frontier", in the sense that they are on average low-

TABLE 4. PARAMETER VALUES AND TARGETED MOMENTS

Parameter		Value	Moment	Data	Model
			A. Externally set or externally normalized		
ρ	Discount rate	0.003	5% annual real interest rate		
θ	Elasticity of matches w.r.t. vacancies	0.5	Moscarini and Postel-Vinay (2018)		
c_v	Scalar in vacancy cost	1	Normalization		
c_e	Scalar in search cost	1	Normalization		
c	Cost of intermediates	0	Normalization		
			B. Calibrated offline		
$m+\mu$	Aggregate growth rate	0.002	Annual growth rate of 2%	0.020	0.020
l	Share of managers	0.216	Worker-to-firm ratio	4.619	4.619
d	Exogenous exit rate	0.002	Exit rate, top productivity quintile	0.002	0.002
	Calendar age at entry, shape	0.029	Labor force participation rate by age	See Appendix D.6	
	Calendar age at entry, minimum	18	Labor force participation rate by age	See Appendix D.6	
	Calendar age at entry, maximum	35	Labor force participation rate by age	See Appendix D.6	
$\overline{A}^{\pi}$	Age of high arrival rate	120	Peak entry age	30	30
$\overline{A}$	Retirement age	480	Labor force participation rate by age	See Appendix D.6	
κ	Retirement rate	0.022	Labor force participation rate by age	See Appendix D.6	
λ	Labor force growth rate	0.000	Age distribution of labor force	See Appendix D.6	
			C. Internally normalized		
$k(a)$	Flow value of being own boss	See Appendix D.6	Indifference at 1st grid point btw unempl. and entrepreneurship		
			D. Minimum distance routine		
χ	Matching efficiency	0.343	NE rate	0.036	0.033
ϕ	Relative search efficiency	2.800	JJ rate	0.022	0.022
b_a	Reservation productivity, slope	0.723	JJ rate, age 50	0.560	0.589
δ_0	Separation rate, intercept	0.020	EN rate	0.010	0.010
δ_1	Separation rate, slope	-0.417	EN rate, productivity quintile 3 to 1	0.579	0.590
π_0	Arrival rate, initial	0.001	Entry rate (x100)	0.118	0.093
π_1	Arrival rate, later (relative to initial)	2.318	Entry rate, age 20 (x100)	0.580	0.591
η_e	Curvature of search cost	2.223	Entry rate, age 50	0.613	0.608
α_0	Diffusion, general	-0.323	Va.p.w., firm age 11 to age 1	0.082	0.090
α_1	Diffusion, specific	0.083	Va.p.w. by va.p.w. of founder at firm age 5	0.072	0.061
ζ	Dispersion in entry productivity	0.548	Autocovariance of va.p.w., firm age 1	0.248	0.255
σ	St.d. of productivity shocks	0.046	Autocovariance of va.p.w., firm age 5	0.015	0.016
η_v	Curvature of vacancy cost	17.540	Vacancy share, 40% most productive firms	0.705	0.723

Table 4 summarizes the targeted moments in the data and model as well as the estimated parameter values that minimize the (equally weighted) sum of squared percentage deviations between the moments in the data and model. Worker relocation rates are monthly. In the model, they are constructed based on the exact continuous time solution of the model, time aggregated to a monthly frequency. Firm reallocation rates are annual. In the model, they are constructed based on a simulated discrete-time monthly approximation of the underlying continuous-time model for 500,000 firms, and subsequently aggregated to an annual frequency in an identical manner as the data. The empirical moments are for private sector firms and individuals aged 20–64 between years 2014–2018. *Source:* FEK, JOBB, LISA, model.

productive (panel C). The current model of imitation and gradual improvements at the bottom/middle of the productivity distribution matches this empirical feature well. Only through a fortunate sequence of shocks does an entrant eventually become high-productive. One explanation for the initial jump in productivity—which the model cannot account for—is that new business ideas are partly an experience good. In both the model and data, firm size grows over firms' first 10 years, as productivity rises and firms gradually grow toward their optimal size conditional on productivity (panel D).[37]

Young firms have a high job creation rate (panel E) and job destruction rate (panel F), although the decline in the latter with firm age is less pronounced. Young firms have a high poaching rate (panel

[37] The curvature in the hiring cost implies a well-defined optimal size, where optimal hiring equals separations.

FIGURE 9. TARGETED AND NON-TARGETED MOMENTS

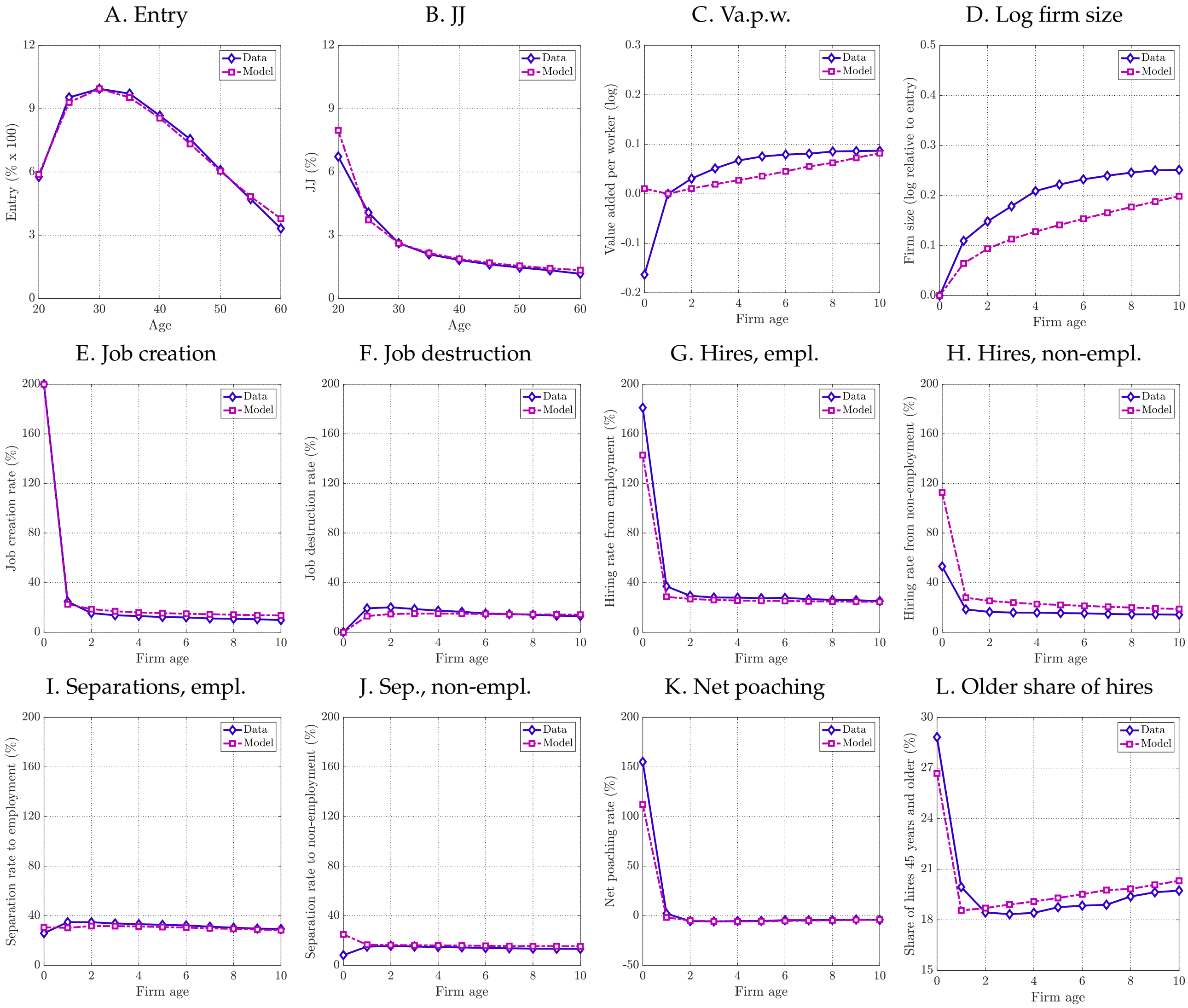


Figure 9 contrasts worker and firm life-cycles outcomes in the model and data. Worker outcomes in panels A–B are monthly. In the model, they are constructed based on the exact continuous time solution, time-aggregated to a monthly frequency. Firm outcomes in panels C–L are annual. In the model, they are constructed based on a simulated discrete-time monthly approximation of the underlying continuous-time model for 500,000 firms, and subsequently aggregated to the annual frequency in an identical manner as the actually data. The empirical moments are for private sector firms and individuals aged 20–64 between years 2014–2018. The model moment for the share of hires that are 45 years or older is normalized such that the mean equals that in the data. *Source:* FEK, JOBB, LISA, model.

G) and hiring rate from non-employment (panel H), as well as high separation rates to other employers (panel I) and non-employment (panel J). Net poaching—the difference between poached hires and poached separations—at first declines sharply with firm age (panel K). The high net poaching rate of new firms is despite the fact that young firms are low productive (panel C) and hence low in the job ladder. Consequently, they have a low vacancy fill rate and create few vacancies, in an absolute sense. Yet because they are small, their vacancy rate is high and they hire many workers relative to their size.

Young firms disproportionately hire (and employ) young workers (panel L). The reason is that they are low-productive (panel C) and hence low in the job ladder. Consequently, they cannot compete for well-matched, older workers. This pattern is consistent with the view that new firms depend on young workers, who have not yet found a good match with existing technologies. The initial decline is because I include in hires also the founder, both in the model and data, who tends to be older than hires.

TABLE 5. EFFECT OF AGING ACROSS BGPS

	Panel A. Individual outcomes (monthly)							
	Entry rate		JJ rate		NE rate		EN rate	
	Data	Model	Data	Model	Data	Model	Data	Model
Early period	0.16%	0.11%	2.67%	2.53%	4.47%	3.34%	0.90%	1.04%
Late period	0.12%	0.09%	2.19%	2.17%	3.57%	3.34%	0.97%	0.96%
Change	-24.52%	-13.18%	-17.84%	-14.10%	-20.19%	0.20%	7.17%	-7.45%
	Panel B. Firm outcomes (annual)							
	JC rate		JD rate		Covariance, va.p.w.		Empl., firms 11+	
	Data	Model	Data	Model	Data	Model	Data	Model
Early period	15.51%	12.32%	14.11%	10.88%	0.229	0.391	54.9%	76.2%
Late period	11.59%	10.00%	10.58%	9.74%	0.279	0.457	65.8%	85.6%
Change	-25.23%	-18.81%	-25.04%	-10.48%	0.049	0.066	10.9 p.p.	9.4 p.p.

Table 5 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply λ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. The early period refers to 1993–1997 for the entry rate, 1986–1990 for the JJ, NE and EN rates as well as the JC and JD rates, and 1997–2000 for the covariance and employment share of firms aged 11 and older. These are the earliest years for which each series can be consistently constructed. The late period refers to 2014–2018 for all moments. *Source:* FEK, JOBB, LISA, model.

5.4 The impact of aging across BGPs

I start by considering a comparative static exercise. To that end, I change the growth rate of labor supply, λ, to generate a decrease in the share of the labor force that is aged 45–64 from 39.7 percent in 2014–2018 to 34.2 percent in 1986–1990.[38] This requires λ to rise from 0.0003 to 0.0013. Appendix D.10 shows that the model matches well the age composition of the labor force in 1986–1990 (it matches that in 2014–2018 very well, see Appendix D.6). All other parameters are held fixed at their estimated values.

Table 5 contrasts the impact of aging with Swedish trends. The entry rate falls by 13 percent, relative to a 25 percent decline in the data, and the JJ rate by 14 percent, relative to an 18 percent decline in the data. I discuss further below the mechanisms behind these declines. Aging accounts for little of the decline in the NE rate and generates a counter-factual fall in the EN rate of seven percent, as the fall in the rate of obsolescence implies that matches last longer.

The job creation rate falls by 19 percent, relative to a 25 percent decline in the data. Because the total

[38]In estimation, I target the share that is 45–74 of the overall labor force age 15–74. Historical data are only available for the share of the labor force that is aged 45–64 of the overall labor force aged 16–64.

number of jobs must to grow at the same rate as labor supply on the BGP, the fall in job creation is partly mechanical. In addition, jobs reallocate less in the older, slower growing economy, leading to a fall also in the job destruction rate of 10 percent, relative to a 25 percent fall in the data.

Productivity dispersion rises, consistent with the data. The reason is that technological obsolescence restrains some firms from becoming in relative sense very productive (Appendix D.2). The fall in obsolescence lengthens firms' life-cycle, leading to a shift of employment toward older firms, consistent with Swedish trends Appendix D.11 shows how employment shifts toward high-productive firms.

5.5 Why is age-specific mobility lower in an older labor market?

The aggregate firm creation and JJ mobility rates can be written as the weighted sum of age-specific rates

$$y(m,\lambda) = \int_0^\infty \underbrace{\pi(a)\left(\frac{\widehat{u}(a;m,\lambda)s_u(a;m,\lambda) + \widehat{e}(a;m,\lambda)\int_0^\infty s(z,a;m,\lambda)d\widehat{G}(z|a;m,\lambda)}{\widehat{u}(a;m,\lambda)+\widehat{e}(a;m,\lambda)}\right)}_{\text{age-specific entry rate}} \underbrace{\tilde{w}(a;m,\lambda)}_{\text{population share}} da$$

$$JJ(m,\lambda) = \int_0^\infty \underbrace{p(m,\lambda)\phi\int_0^\infty \Big(1-F(z;m,\lambda)\Big)d\widehat{G}(z|a;m,\lambda)}_{\text{age-specific JJ mobility rate}} \underbrace{\frac{\tilde{e}(a;m,\lambda)}{e(m,\lambda)}}_{\text{employment share}} da$$

where $\widehat{u}(a;m,\lambda)$ is the age-specific unemployment rate, $\widehat{e}(a;m,\lambda)$ the age-specific employment rate, $\tilde{w}(a;m,\lambda)$ the number of non-entrepreneurial individuals who are of age a, $\tilde{e}(a;m,\lambda)$ the number of employed individuals of age a and $e(m,\lambda)$ the total number of employed.

Relative to the qualitative analysis in Section 4, the change in composition now also has a mechanical effect on firm creation, since the likelihood of starting a firm varies over the life-cycle. To isolate the importance of this direct effect, I change the composition of the labor force, but hold all equilibrium objects fixed at their values in the estimated economy. According to Table 6, the direct effect contributes to an 1.5 percent decline in entry and a 7.8 percent fall in JJ mobility.[39]

In addition to the direct effect, age-specific entry and JJ mobility also change, for two reasons. First, holding fixed growth, the probability that an individual enters entrepreneurship conditional on the productivity of her current job falls in the older economy. The reason is that potential entrepreneurs are discouraged by the harder recruiting environment. This channel contributes to a further 1.5 percent fall in entry.[40] Moreover, because incumbent firms are dissuaded from creating jobs, the probability that an

[39] A shift-share analysis finds that most of the decline in entry is accounted for by an age-specific decline in both the model and data, whereas shifts in composition are more important in accounting for the fall in worker relocation (Appendix D.9). Appendix D.10 shows that the structural estimates match well the evidence across Swedish local labor markets.

[40] To reduce clutter, I focus on the youngest age group, but similar insights hold for other age groups.

TABLE 6. DECOMPOSITION OF EFFECT OF AGING ACROSS BGPS

	Entry	JJ
Composition effect: fixed policies & growth rate	-1.52%	-7.81%
Change in age-specific entry & mobility rates	-11.66%	-6.29%
Fixed growth rate	*-3.02%*	*-0.78%*
Growth rate adjusts	*-8.64%*	*-5.51%*
Total effect	**-13.18%**	**-14.10%**

Table 6 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply λ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. The composition effect changes the growth rate of labor supply λ, holding fixed all policies and the growth rate at their levels in the estimated model. The "fixed growth rate" effect resolves the model for the new λ, counterfactually holding the growth rate fixed at its level in the estimated model. The "growth rate adjusts" effect is computed as the residual between the total change and the sum of the composition and "fixed growth rate" effects. *Source:* Model.

individual gets poached conditional on the productivity of her current job falls. This channel generates an 0.8 percent decline in JJ mobility. Appendix D.12 illustrates these channels.

Second, the decline in growth shifts age-specific employment toward more productive firms. The reason is that workers have more time to relocate across incumbent technologies before these technologies are displaced. The higher opportunity cost of entry and job switching generates a further 8.6 and 5.5 percent fall in entry and JJ mobility, respectively. The finding that a higher relative return to wage employment is an important contributor to the decline in entry is consistent with evidence in Salgado (2020) and Jiang and Sohail (2021) for the U.S. Appendix C.16 estimates that aging raises the return to wage employment relative to self-employment across Swedish local labor markets.

5.6 An increase in the retirement age

To address the issue of a shrinking workforce, several countries are contemplating raising the retirement age. An increase in the retirement age has two opposing effects on firm creation and worker relocation. On the one hand, it implies an effectively lower discount rate, since individuals expect to remain in the market for longer. *Ceteris parisbus*, this horizon effect raises the value of matches and firms, encouraging job and firm creation. On the other hand, by increasing the share of well-matched individuals, it makes it harder for firms to grow. This misallocation effect discourages job and firm creation.

To quantify the relative importance of these two channels, I change the rate at which the oldest age group retires (κ) to generate an increase in the average age of retirement by two years. On net, my estimates imply that the second force is stronger. Consequently, the increase in the retirement age from 65 to 67 years reduces the annual growth rate of output per worker by 0.06 percentage points. At the same time, it raises the level of output per worker by eight percent.

5.7 The aggregate and distributional implications of aging

I end with an assessment of the impact of aging on growth and welfare over the transition. Starting from a BGP in 1930, I assume that individuals realize that labor supply growth, $\lambda(t)$, will evolve so as to match the evolution of the share of older labor force participants between 1960 and 2060 (based on official projections after 2020). After that, it will gradually return to its original level. All other parameters are held fixed. Appendix D.13 characterizes the perfect foresight equilibrium as well as the algorithm I use to solve the model. Appendix D.14 contrasts the time path for the share of older in the model and data.

Over the transition, two forces impact the growth rate of output per worker. First, the rate at which new technologies are introduced changes, i.e. the growth rate of the least productive firm varies. I refer to this channel as the *growth effect*. Second, individuals' match with existing technologies varies, i.e. the employment distribution relative to the least productive firm changes. I refer to the growth rate of average employment-weighted output per worker relative to the least productive firm as the *level effect*.

Panel A of Figure 10 plots these two effects (I express the growth effect as deviations from the BGP growth rate). The growth effect declines since 1970 and contributes to below-trend growth after around 2000 (panel A). In contrast, the level effect generates depressed growth in the 1970s and 1980s, as a large number of poorly matched young individuals enter the labor force. As these individuals gradually relocate up the job ladder, the level effect contributes positively toward aggregate growth.

Panel B plots the resulting annual, demeaned growth rate of output per worker as well as the growth rate of real GDP per worker in the data (an 11 year moving average). Note that the former is scaled by a factor of 10. The level and growth effects combine to generate a hump-shaped aggregate growth rate, which rises up to the early 1990s and then falls. The correlation between the raw data and the estimated impact of aging is high. That being said, aging accounts for less then one-tenth of the more than one percentage point increase in growth from the early 1970s to mid-1990s. Evidently, other important forces were at work too over this period. The drag on growth from aging is expected to last another 30 years.

Panel C plots the value of a young worker who just entered the labor market, as well as the joint value of a match between a firm and an older worker (aged 58+). Both are expressed relative to their corresponding values in a counterfactual economy without demographic change. Relative to this counterfactual, a young worker is 0.8 percent worse off entering in 2018 relative to entering in 1986. In contrast, the average joint value of a match for an older worker is 0.3 percent higher in 2018 relative to 1986. The value of these typically high-productive matches rises as the growth rate falls, since the lower rate of creative destruction implies that they are likely to last longer.

FIGURE 10. EFFECT OF AGING OVER THE TRANSITION PATH

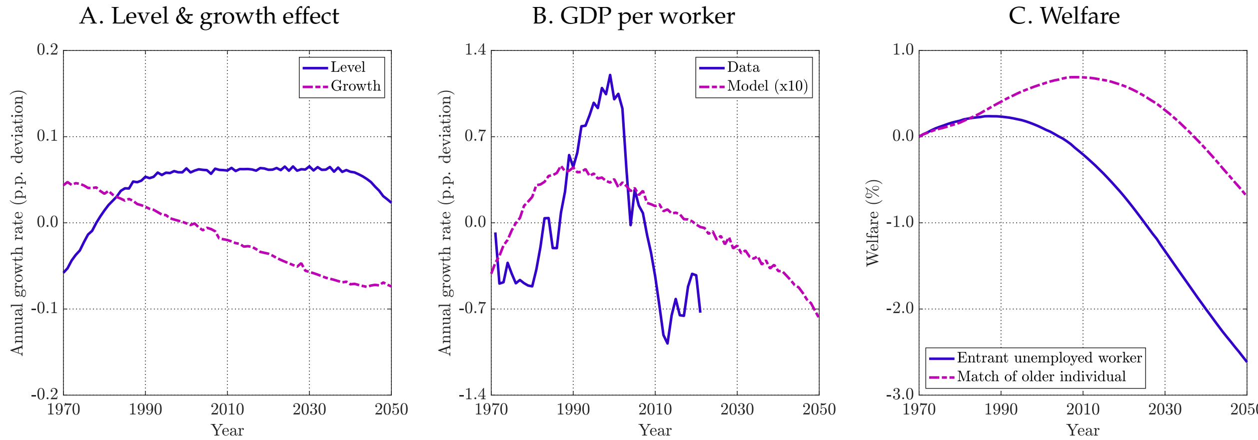


Figure 10 shows the effect of, starting from a BGP in 1930, letting the growth rate of labor supply, $\lambda(t)$, vary to match the evolution of the share of the labor force that is aged 45–64 over the 1960–2060 period, using official projections for the future. All other parameters are held fixed. The growth effect in panel A is the annual rate of obsolescence, normalized to its steady-state value. The level effect is the annual growth rate of average gross output per worker relative to the least productive firm. The data series in panel B is the annual real growth rate of output per worker, smoothed using an 11 year moving average. Welfare at time t in panel C is the life-time discounted value of a labor market entrant at time t as well as the average discounted joint value of a match for older workers (aged 58+). Both are expressed relative to their corresponding values in a counter-factual economy without demographic change. *Source:* SCB, model.

6 Conclusion

Based on variation across 68 Swedish local labor markets between 1986–2018, I estimate that individuals of all ages are less likely to start firms and switch employers when people around them are older. I develop a theory of endogenous growth with labor market frictions, which highlights that older individuals have had more time to find a good match with existing technologies. By shifting workforce composition toward such better matched subpopulations, workforce aging on the one hand increases the level of output. On the other hand, the economy's better ability to produce using existing technologies discourages new technologies. The resulting lower growth rate in the older economy reduces welfare for labor market entrants. At the same time, the lower rate of creative destruction shields the high-productive jobs typically held by older individuals from destruction, raising their value.

The rich, tractable framework presented in this paper could be extended in several directions to study further how labor market dynamics and growth interact. For instance, it would be interesting to study optimal policy when growth causes some workers to become displaced. To properly answer this question may require adding risk aversion and imperfect financial markets. It would also be interesting to endogenize innovation of incumbent firms in order to study how aging—or more broadly measures of the quality of the workforce—affects incumbent firms' incentives to innovate.

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A Swedish labor market trends

This section contains further details on the data sources and variable definitions (Appendix A.1); additional Swedish secular labor market trends (Appendix A.2); a comparison with secular trends in U.S. firm dynamics (Appendix A.3); a discussion of the *FAD* data base (Appendix A.4); trends in labor force participation rates (Appendix A.5); important time series breaks in the data sources and how I adjust the raw data for them (Appendix A.6); labor market trends by firm age and size (Appendix A.7); and labor market trends by sector (Appendix A.8).

A.1 Data sources and variable definitions

I discuss in this section the data sources, sample selection and variable construction in further detail.

JOBB. Data on wage employees come from information provided by employers via the *Kontrolluppgift* (KU) tax form *KU10*, an English version of which is shown in Figure 11. Prior to 2019, employers provided this information for all employment spells that were active at any point during the previous year. This had to be reported to *Skatteverket* by January 31 of the following year. As I discuss further in Appendix A.6, since 2019 employers have instead been required to submit payroll data on a monthly basis for all ongoing employment spells as part of the *Arbetsgivardeklaration på individnivå* (AGI). Employers report gross salary (box 011), inclusive of taxes leveled on employees, bonus payments, exercised stock options, etc, but exclusive of social security taxes leveled on employers. Such taxes are non-trivial in Sweden, adding roughly 30 percent to reported gross pay. Given that this project does not rely heavily on the wage data and the fact that—with only very limited exceptions—social security taxes are proportional, I do not adjust the reported gross pay to reflect social security payments by the employer. Employers must also report in kind payments (in box 012).

The KU data also include a work site number (box 060), information on whether the payee is a partner in a close company (box 061), and the start (box 008) and end (box 009) month of the employment relationship. The work site number allows the linking of employees to establishments within firms. The information in box 061 allows the identification of incorporated self-employed in so called *fåmansföretag* starting in 1993. A *fåmansföretag* is an incorporated, not publicly listed company in which at most four owner-groups collectively control more than 50 percent of voting shares. An owner-group consists of one or more individuals who are linked through family ties, including spouses, parents, grandparents, children (including spouses' children, foster children and children's spouses), siblings and siblings' spouses and children, and non-married couples who live together who were either previously married or have or have had children together.[41] Individuals who are not connected through family ties but are actively working together in a company are also counted as one owner-group (for instance law firms with multiple partners).[42] For this reason, the number of identified founders can be significantly larger than four. Indeed, the rules are designed exactly so as to classify the great majority of small Swedish companies as *fåmansföretag* in order to limit owners' ability to reclassify labor income as capital income.[43] I stress that because I link firms to founders in the first year of a firm's operation, I am able to identify the founders also of firms that subsequently acquire more owners, for instance through a public listing.

The data on unincorporated self-employed in *JOBB* come from a different underlying data source, the *Inkomst- och taxeringsregistret* (IoT) prior to 2004 and the *Standardiserade räkenskapsutdrag* (SKU) starting in 2004. These data contain information on annual earnings of the non-incorporated self-employed, but lack information on start and end dates of employment spells. Lacking such data, I assume that all non-incorporated self-employment spells last the entire year.

Based on the *JOBB* data, I construct monthly individual and firm level data sets for 1985–2019. For each month, I determine an individual's main employer in the following way. I start by classifying an individual as self-employed if they have an ongoing self-employment spell—either unincorporated or incorporated—in that month. In case they have multiple active self-employment spells, I determine their main firm as that with the highest earnings in the year. If an individual has no active self-employment spells in the month, but an active wage employment spell, I classify them as wage employed. In case they have multiple active wage employment spells, their main firm is that with the highest earnings in the year. If an individual neither has any active self or wage employment spell, they are coded as non-employed.

[41] The rules governing what is classified as a *fåmansföretag* are extensive and complex, designed with the goal of preventing tax evasion (known as the *3:12 rules* in Sweden after the paragraph in the law that outlines them). The law also prevents owners from artificially pooling several independent companies under one parent company, with the goal of avoiding a classification as a *fåmansföretag*. In this case, Swedish tax authorities have broad latitude in classifying each subsidiary as a *fåmansföretag*.

[42] For instance, in court ruling RÅ1993 ref. 99, the *Högsta förvaltningsdomstolen* (the Swedish high court) ruled that a company with 150 owners who were also full-time employed in the company was to be classified as a *fåmansföretag*.

[43] Since a major tax reform in 1990, Sweden has implemented a dual tax system, whereby labor income is taxed at an almost 70 percent effective marginal rate at high incomes, but dividend income is subject to a proportional effective tax rate of about 45 percent. Consequently, many owners have an incentive to reclassify labor income as capital income in order to lower their tax rate. Swedish tax authorities are notoriously zealous in clamping down on such attempts to avoid taxes.

FIGURE 11. TAX FORM KU10

Swedish Tax Agency

Information is available in Swedish in the brochure SKV 304 ("Kontrolluppgifter - lön, förmåner, m.m.").
Amounts should be stated as whole numbers.

Income statement from employers etc.

KU10
Income year
2015

Samråd enligt SFS 1982:668 har skett med Näringslivets Regelnämnd.

570 Specification number	

This income statement shall: 210 correct a previously submitted income statement ☐ 205 remove a previously submitted income statement ☐

Payer/Employer

201 Personal/corporate identity number	
Name	

Tax

001 Tax deducted	

Salary and other cash payments

011 Gross salary etc.	
025 Remunerations for which the employee pays individual social security contributions	
031 Remunerations for which social security contributions are not paid	

093 ☐ Social security agreement exists

Benefits in kind etc.

012 Taxable benefits exclusive of employer-provided car and free fuel in connection with employer-provided car	

041 ☐ Free housing 1- or 2-family house 042 ☐ Free meals 043 ☐ Free housing, other than code 041
044 ☐ Interest 045 ☐ Free parking 047 ☐ Other benefits
048 ☐ Benefit has been adjusted 049 ☐ Benefit as pension

065 Specification of other benefits at code 047	
013 Taxable benefit of employer-provided car exclusive of fuel	
018 Free fuel in connection with employer-provided car	
014 SKV-code of employer-provided car	
015 Number of months with employer-provided car	
016 Number of kilometers with car allowance for employer-provided car	
017 Employee's payment for employer-provided car	

Payee/Employee

215 Personal/corporate identity number	
Name	
Street address	
Postal number	Postal address

061 ☐ Partner etc. in a close company

	From 008	Up to 009
Employment time (e.g. 04-12)		
060 Work site number allocated by the Central Bureau of Statistics (SCB)		

Compensation for expenses

According to fixed standard rates: 050 ☐ Car allowance 051 ☐ Per diem, Sweden 052 ☐ Per diem, other countries
Equivalent to actual costs etc. for: 055 ☐ Business travel expenses 056 ☐ Accomodation, business travels
Business trip lasting more than three months: 053 ☐ Within Sweden 054 ☐ Other countries

020 Compensation for expenses not ticked in boxes by codes 050-056	

Occupational pension, other remunerations, certain deductions

030 Occupational pension	
032 Remunerations for which social security contributions are not paid and which are not entitled to special job deduction	
037 Certain deductions	
070 Specification of amount at code 037	
035 Not taxable remunerations to foreign key persons according to a decision from the Swedish Forskarskattenämnden	

Capital

039 Rent	

Tax reduction for "rut-/rot-work"

021 Basis for tax reduction for "rut-work"	
022 Basis for tax reduction for "rot-work"	

SKV-2300-1-25-en-KU10-2015

SKV 2300 25 en web 01

Note: Tax form that employers are required to submit on behalf of all employees during the year. *Source:* Skatteverket.

FEK. The FEK form the basis of Swedish national accounts (it was referred to as the *FÃ¶retagsstatistik* from 1997–2002). Given its importance, these data are subject to extensive checks by SCB in order to limit measurement error. Among others, SCB cross-checks the collected data with previous years, other data sources and follow-ups with firms in order to achieve as precise a measure as possible of, ultimately, aggregate GDP.

The collection of firm financial data has a long history in Sweden. Since 1950, the *Finansstatistik för företag* has surveyed medium and large manufacturing and trading firms with questions on revenues and costs on an annual basis. Coverage was gradually expanded to other sectors and firm size classes, and contains since 1965 also information on balance sheet items. The *Industristatistik* started in 1913 with questions on quantities produced, prices, costs, employment, investments, energy consumption, etc. It covers medium and large manufacturing firms. The *Industristatistik* in turn builds on earlier firm surveys that stretch as far back as 1830. Given that this survey is still ongoing—now called the *Industrins varuproduktion*—it is one of the longest continuous data sources on firm outcomes in the world. Unfortunately, the historical data prior to 1997 are in long-term storage at *Riksarkivet* (the Swedish National Archives), and not easily accessible.

In 1997, SCB switched to collecting income and balance sheet data from *SKU*, which covers all firms with the exception of holding companies and similar. For the largest roughly 450 companies, however, data are collected via a survey, the so called *fullständiga blanketten* (the "complete survey"). Although the tax registry data from *SKU* are considered of high quality, the purpose of surveying the largest firms is to further reduce measurement error (these firms account for roughly a third of Swedish GDP). The primary object of *FEK* is firms, but some information is also reported at the establishment level.

Even though the tax data are already fairly detailed, SCB additionally surveys a sample of firms every year with more detailed questionnaires. Two such surveys obtain more detailed information on types of investment than what is available via *SKU*: *Investeringsenkäten* and *SpecI*. *Investeringsenkäten* samples about 2,000 firms (in 2020), with complete coverage of the largest firms and (stratified) random sampling of smaller firms (the smallest firms are completely excluded from the survey). It is conducted at the level of establishments (covering roughly 7,500 establishments). *SpecI* samples roughly 5,000 firms (in 2020). A third survey obtains more detailed information on revenues and costs, *SpecRR*, covering roughly 16,500 companies (in 2020).

A.2 Additional time series trends

Figure 12 presents additional facts on Swedish labor market trends over the past 35 years. Firm exit declined over this period (panel A), while average firm size trended up modestly (panel B).[44] Because average firm size can be decomposed as $\frac{N_w}{N_f} = (1-u)\frac{N_{wf}}{N_f}$, where N_w (N_f) is the number of workers (firms), u the non-employment rate, and N_{wf} the size of the workforce, the trend in average firm size can be decomposed into changes in the non-employment rate and the ratio of the size of the workforce to the number of firms. Panel B finds that the ratio of the size of the workforce to the number of firms remained roughly constant. Hence, the increase in average firm size is accounted for by a fall in non-employment.

At the same time as Sweden saw a constant ratio of firms to workforce participants, firms grew older and employment shifted toward older firms (panel C). Because older firms are less dynamic, this shift partly accounts for the aggregate declines in firm dynamics in Sweden, similar to the U.S. (**?**Karahan et al., 2022).

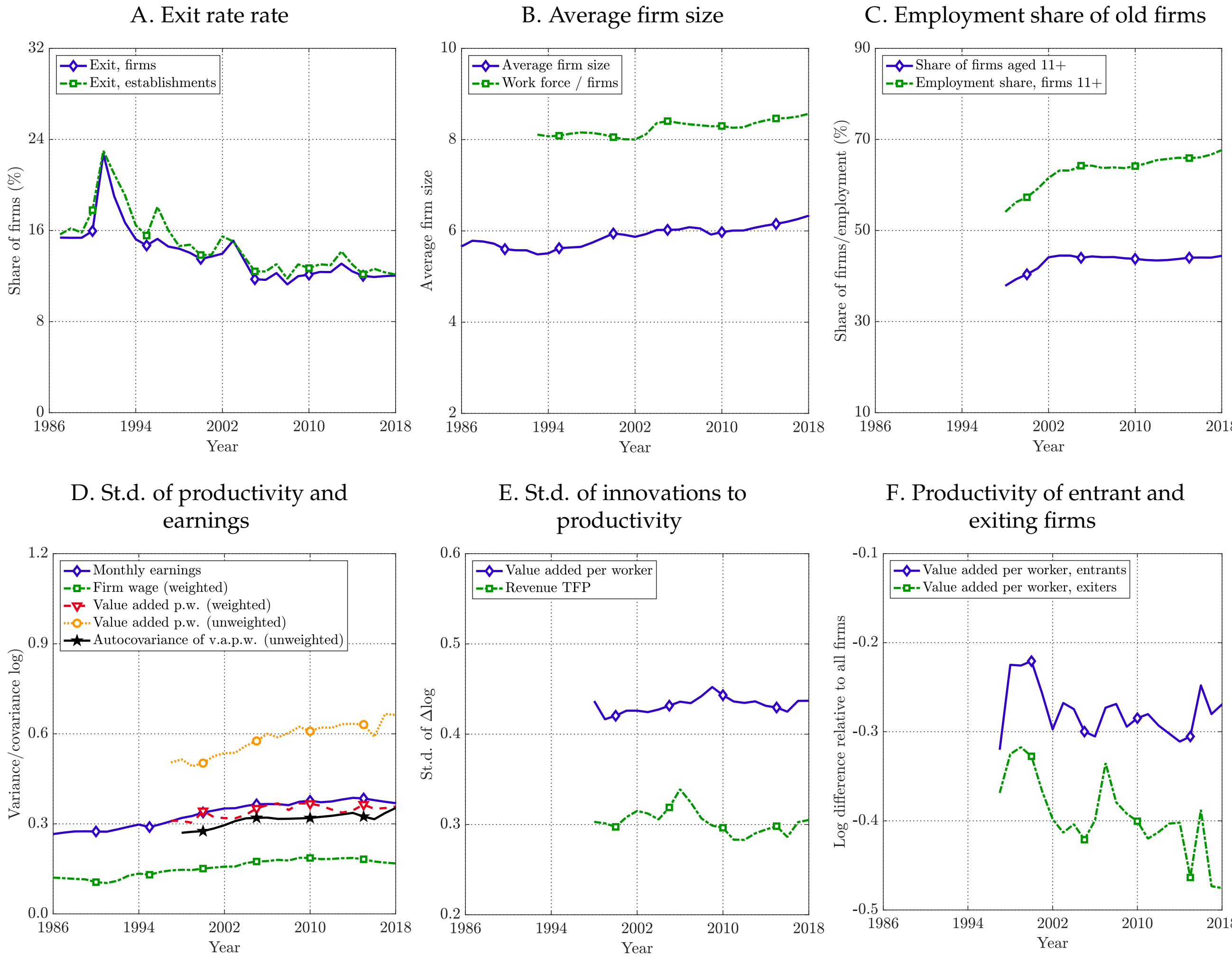
Earnings dispersion widened in Sweden over this period (panel D), mirroring well-known patterns in the U.S. (Barth et al., 2016). In particular, the standard deviation of residual log monthly earnings rose from 0.53 to 0.64. Productivity dispersion also increased, although consistent productivity data are not available prior to 1997. The rise in productivity dispersion is evident in the standard deviation of log value added per worker, as well as firm revenue TFP and the first autocovariance of log value added per worker. The latter suggests that a large share of the increasing dispersion is permanent in nature.

Firm productivity is subject to large idiosyncratic changes, but the volatility of such innovations to firm productivity has not changed much over time (panel E). I compute these innovations as the residual of a regression of productivity on its own lag.

Panel F plots labor productivity of entrant and existing firms relative to all firms. Three observations stand out. First, new firms do not appear to enter at the "technological frontier," in the sense that they are less productive than incumbent firms. This observation motives me to model entry and innovation at the bottom/middle of the productivity distribution, following Perla and Tonetti (2014). Second, there is little evidence of entrants falling increasingly behind incumbent firms over this period. Third, although entrants are not particularly productive, exiting firms are even less productive. Consequently, in an accounting sense replacing an exiting firm with an entrant contributes to growth.

[44] Average firm size is significantly lower in Sweden than in the U.S., likely due to the fact that the Swedish data cover also non-employer firms. These firms are almost exclusively one-person firms. Nevertheless, because job and worker reallocation are employment-weighted outcomes, the inclusion of such firms is likely of less relevance for these outcomes. Indeed, I show below that also large firms experienced declines in job reallocation over time in Sweden.

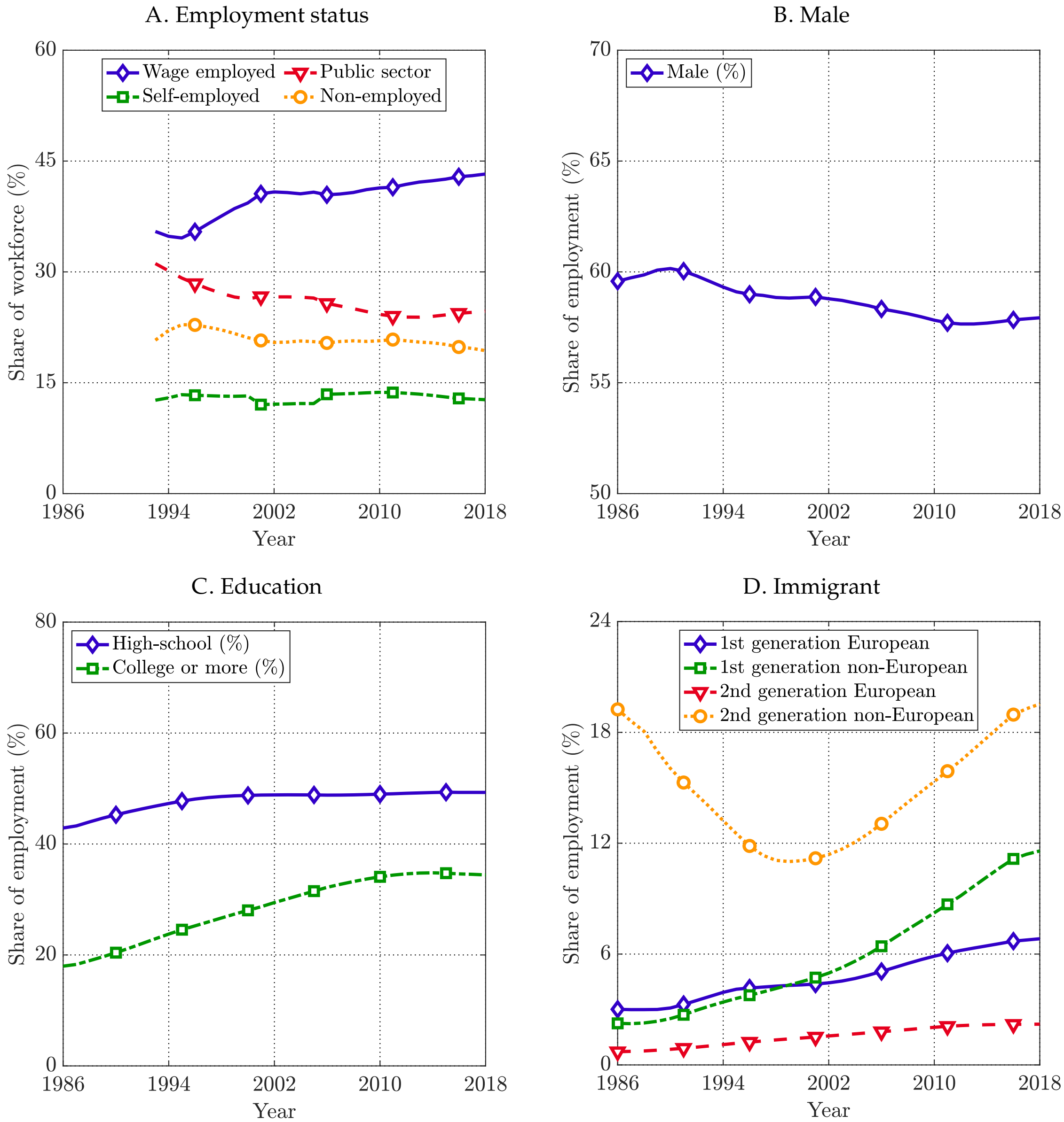
FIGURE 12. SWEDISH LABOR MARKET TRENDS



Note: Panel A. Annual firm and establishment exit rate. Panel B. Workforce/firms is the total non-private sector workforce aged 20–64 divided by the total number of private sector firms. The data are spliced in 2004; see Appendix A.6 for details. Panel C. Share of firms that are 11 years and older as well as the are of employment that works for firms that are 11 years and older. The data are spliced in 2004; see Appendix A.6 for details. Panel D. Firm pay is the log of average monthly earnings at the firm. Autocovariance is the first annual autocovariance. Panel F. Average log productivity of entrants/exiting firms in the year minus average log productivity of all firms (all employment-*un*weighted). All panels. Private sector firms and workers aged 20–64. *Source:* FEK, JOBB, LISA.

Figure 13 summarizes other changes in the demographic composition of the Swedish workforce over this period apart from in the age dimension. The employment share of the public sector has declined, as has the non-employment rate (panel A). The wage employment rate has risen, while the self-employment rate has remained roughly constant. Note that the ability to identify self-employed individuals is incomplete prior to 1993, since those running incorporated firms are classified as wage employees prior to 1993. The share of men in the non-public workforce has trended down modestly over this period (panel B), while the share with a college degree or more has risen (panel C). The share of immigrants has gradually risen, to stand at roughly 20 percent of the non-public workforce today (panel D).

FIGURE 13. COMPOSITION OF NON-PUBLIC WORKFORCE

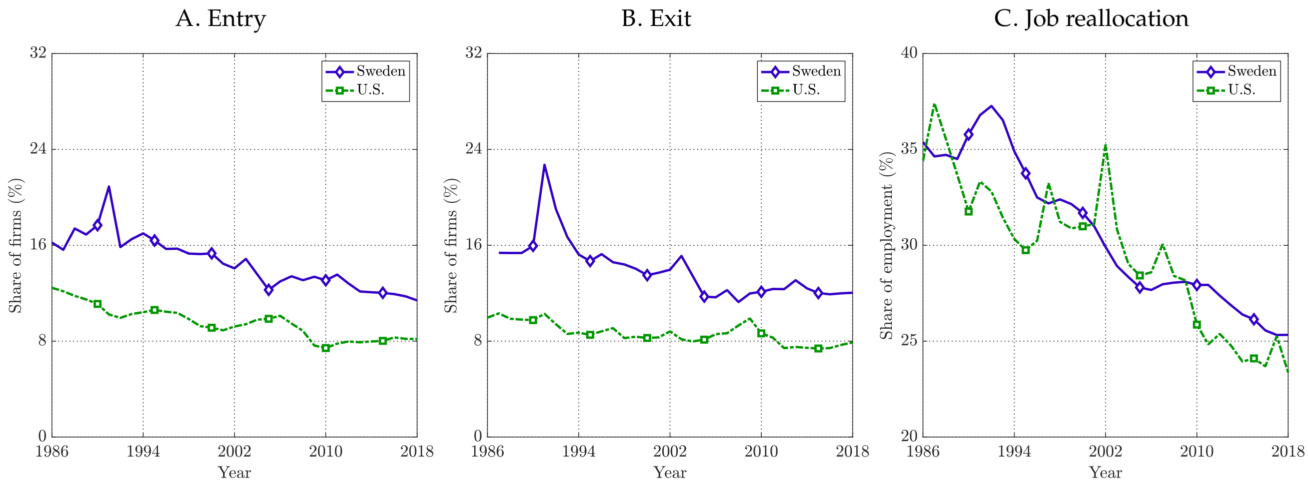


Note: Panel A. Share of all individuals aged 20–64 who are private sector wage employees, private sector self-employed (including both unincorporated and incorporated owner-operator of *fåmansaktiebolag*), public sector employees and non-employed. Panel B. Share of non-public sector individuals aged 20–64 who are men. Panel C. Share of non-public sector individuals aged 20–64 who have a college degree or more or a high-school degree. The excluded third education group consists of those with less than a high-school degree. Panel D. Share of non-public sector individuals aged 20–64 who are a first or second generation immigrant by source country. An individual is classified as a second-generation immigrant if they have at least one parent who is born abroad (and a non-European second generation immigrant if at least one parent was born outside Europe). *Source:* JOBB, LISA.

A.3 Benchmarking against U.S. labor market trends

Figure 14 plots the annual firm entry rate (panel A), firm exit rate (panel B) and job reallocation rate (panel C) in the U.S. The level of entry and exit is higher in Sweden, likely due to the fact that the Swedish data contain also non-employer firms. These firms are typically small and likely have high entry and exit rates. Yet in a relative sense, the fall in entry and exit is very similar in the two countries. Both the level and the decline in job reallocation are of a similar magnitude in the U.S. and Sweden.

FIGURE 14. SWEDISH AND U.S. FIRM DYNAMICS



Note: Panel A. Share of all firms that started in the current year. Panel **??**. Share of all firms that exit in the current year. Panel C. Sum of jobs created and destroyed across establishments divided by average employment in the year. *Source:* BDS, JOBB, LISA.

A.4 The firm and establishment identifiers and FAD data

A general issue with administrative, tax based data sets such as those used in this paper is that firm and establishment identifiers change for reasons such as ownership changes etc. In the Swedish data, this is particularly a concern with the firm identifier, known as the *PeOrgNr*. It is less of a concern with the establishment identifier (*CfarNr*), since it is designed to remain the same when, for instance, ownership changes.

To address such concerns, SCB constructs a data set, the *Företagens och arbetsställenas dynamik* (FAD), which contains firm IDs (*FAD-F-ID*) and establishment IDs (*FAD-A-ID*) that are designed to be unaffected by changes in the underlying firm (*PeOrgNr*) and establishment (*CfarNr*) identifiers in the tax data that are not due to "true" economic events. To this end, SCB exploits the nature of worker flows to link firms over time. In particular, if a firm in the underlying data changes ID across two years, but a majority of its workforce remains the same, *FAD* classifies this as the same firm (with a few exceptions for small employers). Although this adjustment seems reasonable, there are several important issues with *FAD*. First, *FAD* contains only firms and establishments which at least one individual has as their main employer in November of the year. The reason is that it is based on the *Registerbaserad arbetsmarknadsstatistik* (*RAMS*), which is a consolidated version of *JOBB* with information only on the main employer in November of the year (SCB classifies the main employer in November primarily based on highest annual income, with some adjustments such as inflating self-employment income). As a result, *FAD* does not assign a *FAD-F-ID* and *FAD-A-ID* for close to 40 percent of all *PeOrgNr*'s and *CfarNr*'s IDs in *JOBB*. Although these firms are small—typically secondary employment for small independent business owners—it is not clear that one wants to drop these firms, in particular as some of them subsequently grow.

Second, SCB constructs annual vintages of *FAD*, where a given vintage t contains an underlying $PeOrgNr_1$, $CfarNr_1$, $PeOrgNr_2$, $CfarNr_2$, *FAD-F-ID* and *FAD-A-ID*. The $PeOrgNr_1$ ($CfarNr_1$) refers to the *PeOrgNr* (*CfarNr*) in year $t-1$ that maps into the *FAD-F-ID* (*FAD-A-ID*), while the $PeOrgNr_2$ ($CfarNr_2$) refers to the *PeOrgNr* (*CfarNr*) in year t that maps into the unique *FAD-F-ID* (*FAD-A-ID*). Hence in theory, a *PeOrgNr* in *JOBB* in year t can be mapped to a unique *FAD-F-ID* based on either $PeOrgNr_2$ in FAD vintage t or $PeOrgNr_1$ in FAD vintage $t+1$. The issue is that the mapping provided by different vintages of *FAD* is not unique in the same year. In particular, roughly four percent of *PeOrgNr* map to a different *FAD-F-ID* depending on whether $PeOrgNr_1$ in FAD vintage $t+1$ or $PeOrgNr_2$ in FAD vintage t is used. In discussions with the data experts at SCB, we have confirmed that this is an issue, but it remains unclear what causes it (because the underlying code used to generate *FAD* is confidential, I have not been granted the chance to inspect the code that generates *FAD*). The lack of insight into what causes this issue leaves me uncomfortable using *FAD*.

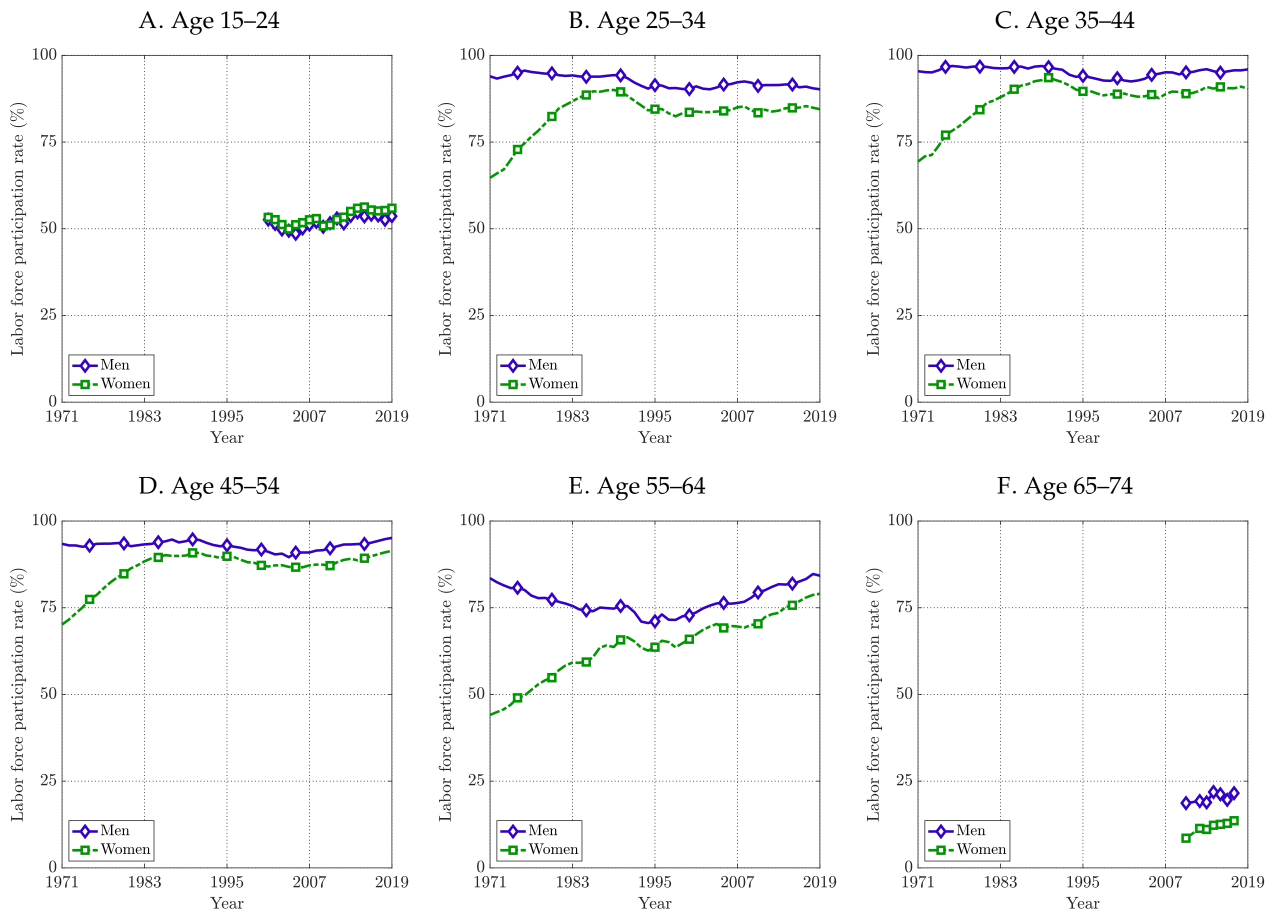
Finally, labor market flows based on the firm and establishment IDs in *FAD* appear to be very high. For instance, the job reallocation rate based on *FAD-A-ID* is more than twice the corresponding rate in the U.S. BDS data. Moreover, the job reallocation rate based on *FAD-A-ID* is more than twice as high as that based on *CfarNr*, even though the latter is supposed to be consistent over time (whereas the firm ID in the tax data, *PeOrgNr*, arguably is not). I view these differences as implausibly large.

For these reasons, I prefer to use the underlying *CfarNr* and *PeOrgNr* in my analysis. That being said, the aggregate annual job reallocation rate based on *FAD-F-ID* fell from 88.5 percent in 1986 to 58.1 percent in 2018, or a 34 percent decline. Although the level of job reallocation is much higher based on the *FAD* identifiers, the relative decline is of a similar magnitude as that using *PeOrgNr* and *CfarNr*.

A.5 Participation rates by age over time

Figure 15 plots labor force participation rates by age groups over time. Participation rates rise rapidly between age 20 and 25 and fall rapidly after age 65. Between ages 30–55 they are, to a first order, flat in age. Men have not seen any major changes over time in their age-conditional participation rates. The most prominent change was a decline in male participation rates until the mid-1990s, and a subsequent recovery over the past 25 years. Women, on the other hand, experienced significant gains in participation rates throughout the 1970s and early 1980s. After that, female participation rates have largely stabilized at a level somewhat below that of men. The main exception are women aged 55–64, who experienced continuous gains in participation over this period. My reading of these trends is that from the mid-1980s, labor force participation rates have been broadly stable, with the main exception being women aged 55–64.

FIGURE 15. LABOR FORCE PARTICIPATION RATES BY AGE AND TIME



Note: Ratio of the sum of employed and unemployed to total population. Data for the youngest age group, 15–24, is not available prior to 2001. Data for the oldest age group, 65–74, is not available prior to 2010 and after 2017. *Source:* SCB.

A.6 Important breaks in the data

The underlying data sources used in this project have been subject to several adjustments to the way the data are collected and processed, which may carry implications for some of the time series trends documented in this paper. I discuss in more detail below some of the most important breaks. I also caution, however, that slow-moving forces have also been at work over this period. For instance, *Skatteverket* has increasingly allowed employers to submit tax information electronically, which may have impacted the magnitude of measurement error in earnings, start and end dates of employment spells, etc. In fact, one could even imagine that such changes could be correlated with aging across space, to the extent that young individuals are more likely to adopt such new ways of reporting information. As little can be done to assess the implications of such changes, I take to simply note such valid concerns.

The 2019 switch to AGI. In 2019, *Skatteverket* started requiring employers to report payroll data on all active employment spells in a month on a monthly basis, as opposed to once a year. Although employers prior to 2019 reported start and end dates of employment spells, this change likely reduced measurement error in the start and end months of spells. Indeed, hiring and separation rates show an increase in the 2018–2010 break. While the increase in the poaching rate is relatively modest of less than 10 percent, the hiring from and separation rates to non-employment show a more pronounced increase of almost 40 percent. This is consistent with some measurement error in start and end dates leading to a general understatement of both poaching flows and flows through non-employment, together with some short non-employment spells being incorrectly classified as poaching flows. Consequently, all flows rise with the change in reporting, but in particular flows through non-employment.

To limit the impact of this break, I end my analysis in 2018. I note that although measurement error in the start and end months may contribute to deflated flows prior to 2019, under the assumption that such measurement error has not changed over time, the time series trends would still be consistent.

The 2004 switch to SKU. Starting in 2004, data on the unincorporated self-employed come from the *SKU* instead of the *IoT*. This switch led to the inclusion of also unincorporated self-employed with a negative profit in the year, which were previously excluded since they are not required to pay tax. The change led to a permanent increase in the number of self-employed in 2004 and a temporary spike in the measured entry rate in 2004, as a substantial number of firms that likely were already in existence prior to 2004 started to be recorded in the data. It also led to a discrete drop in average firm size and the share of firms that are old.

Given that these are unincorporated firms with negative profits, they are almost exclusively small and likely of limited macroeconomic importance. Nevertheless, in order to create a consistent time series for high growth entry, the stock of self and wage employed, average firm size, and the share of employment in large and old firms, I splice these aggregate data series in the break.[45] I do so by pooling data for the outcome of interest plus/minus 15 years around the 2004 gap, dropping 2004 for the entry rate due to a large spike in measured entry in this year. I subsequently project the outcome of interest on a constant, a linear time trend and an indicator for whether the year is after 2004. Finally, I adjust the series by adding the estimated coefficient on the post 2004 dummy to the outcome of interest to all years after 2004. Admittedly, this approach is somewhat ad hoc. In any case, because the spatial regressions in Section **??** include a year fixed effect, the data break and the adjustment I do to the series are inconsequential for any of the results in this paper.

Information on incorporated self-employment. Data on incorporated self-employment are only available since 1993, when a major tax reform led to the creation of the current Swedish dual income taxation system (these reforms, which were initiated in 1990, are referred to as the "tax reform of a century" in Sweden). As a result, *Skatteverket* started collecting data on owner-operators of *fåmansföretag*. Prior to 1993, the incorporated self-employed are coded as regular wage employees. For this reason, my analysis linking founders to firms starts in 1993. To avoid any mechanical bias in the measured rates of worker relocation over time, I include in my measures of hires and separations also the self-employed. That is, for instance the JJ rate includes also self-employed individuals who switch firm. I note, however, that flows into and out of self-employment are an order of magnitude lower than flows of wage employees across firms. Hence in practice, it makes little difference whether the self-employed are included in the measured worker relocation rates.

Although *Skatteverket* has recorded information on owner-operators of *fåmansföretag* since 1993, this information is mysteriously missing from the *JOBB* data base prior to 2004, even though the information is available in other data products at SCB such as *LISA*. It is not clear what exactly has caused this information to be missing from JOBB, but together with SCB we have designed a workaround that uses information on the top three employers in a year from *LISA* to import an indicator for whether what looks like a wage employment spell in *JOBB* 1993–2003 is in fact an incorporated self-employment spell. Unfortunately, *LISA* only contains information on up to three employers, so this imputation fails to reclassify a wage employee as an incorporated self-employed in 1993–2003 if this employment spell was not one of the top three sources of income for an individual in that year. Because few individuals have

[45] In theory, the overall entry rate would also be affected by the change in methodology. In practice, however, I find that the impact of the change in data collection is too minor to make any notable difference to the aggregate entry rate. For this reason, I do not splice this.

more than three employment spells in a year, I do not think that this is a major source of error.

A.7 Firm dynamics by firm age and size

Figure 16 plots key firm reallocation rates by firm age groups. In contrast to the modest increase in average firm size in the aggregate, firm size conditional on firm age has not changed much over this period (panel A). The jumps in some of these series are driven by changes in the thresholds for when firms are included in the underlying tax level data. I have tried as well as possible to splice such changes, but the influence of these changes remain a concern. That being said, because these changes exclusively affected low value added, typically small firms, I believe that they are of little concern for employment-weighted measured of reallocation. The exit rate has declined also conditional on firm age, particularly among younger firms (panel B). In contrast, the oldest firms in fact saw a modest increase in their exit rate. The job reallocation rate displays a pronounced secular decline also conditional on firm age (panel C), even among the oldest firms which employ most workers.

FIGURE 16. DYNAMICS BY FIRM AGE

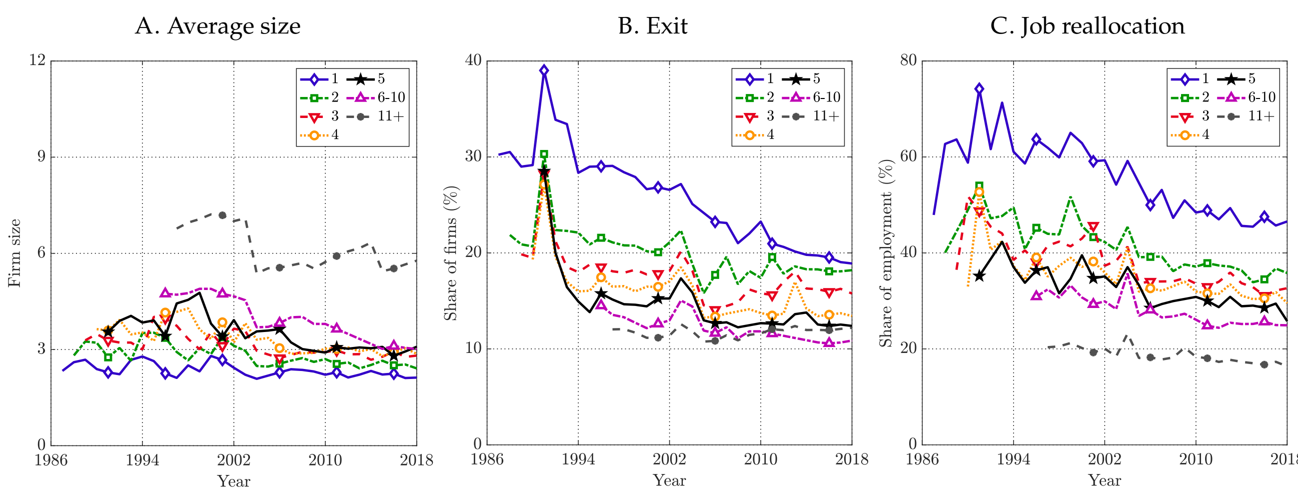


Figure 16 plots average firm size (panel A), the fraction of firms with positive employment in year t that have zero employment in year $t+1$ (panel B), and the job reallocation rate (C), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* JOBB, LISA.

Figure 17 shows that the Swedish trends in firm dynamics by firm age are similar to those in the U.S. over the same period. In particular, average firm size conditional on firm age shows little change over the past 35 years in the U.S. (panel A), while firm exit fell among young firms but did not change by much for older firms (Karahan et al., 2022) (panel B). In contrast, also the U.S. displays a pronounced secular decline in job reallocation conditional on firm age over the past 35 years (panel C).

Note also the large differences in average firm size between the U.S. and Sweden. As I discuss in

Section 2, this difference is likely largely due to the fact that the U.S. BDS includes only employer firms, whereas the Swedish data include all firms. The U.S. Census Bureau reports that there were roughly 26.5 million firms in the U.S. in 2018, but only 6.1 million employer firms. Since non-employer firms are mostly small, average firm size of all firms is presumably much lower than that of employer firms only.

FIGURE 17. DYNAMICS BY FIRM AGE IN THE U.S.

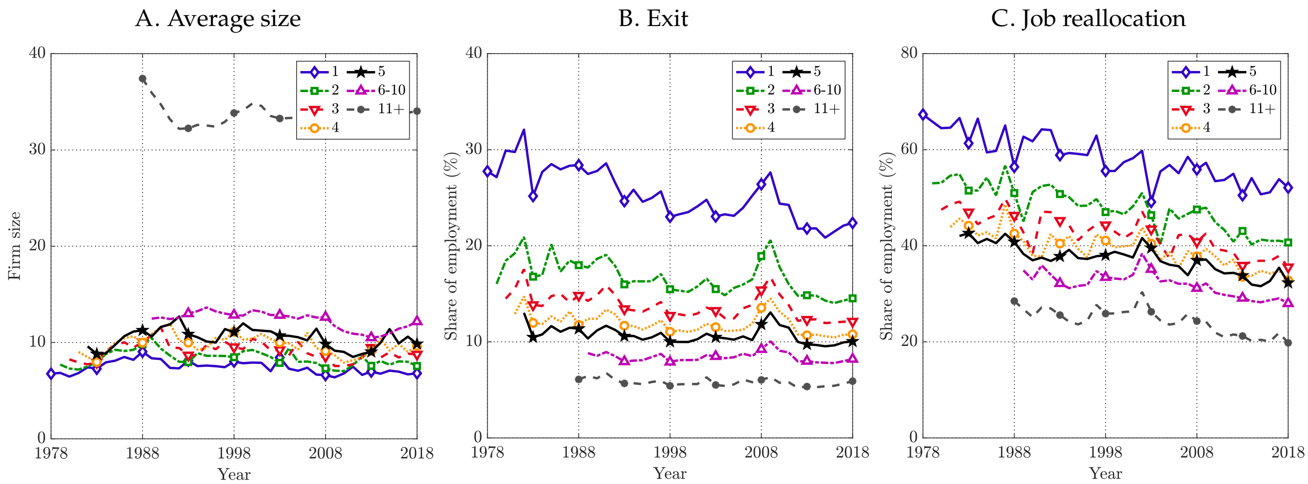


Figure 17 plots average firm size (panel A), the employment-unweighted exit rate of firms (panel B), and the job reallocation rate (C), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* BDS.

Figure 18 shows that firm exit (panel A) and job reallocation (panel B) fell within firm size groups over the past 35 years in Sweden.

FIGURE 18. DYNAMICS BY FIRM SIZE GROUPS

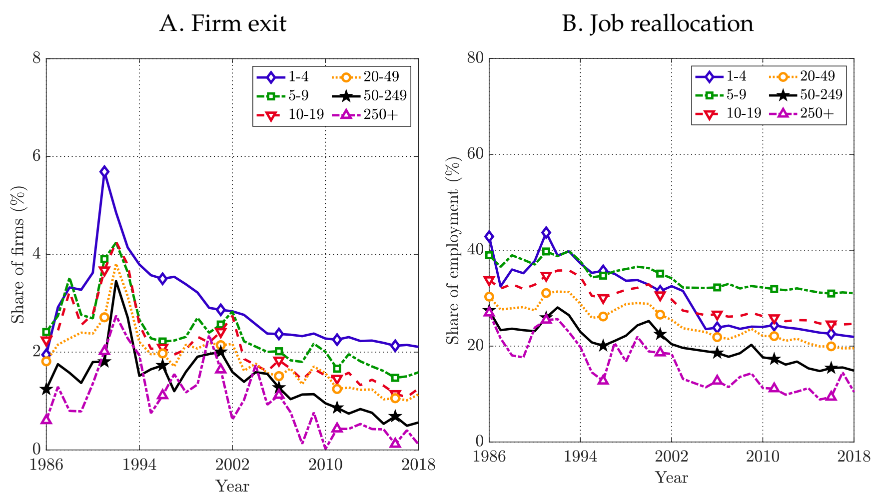


Figure 18 plots the share of firms with positive employment in year t that have zero employment in year $t+1$ (panel A) and the job reallocation rate by firm size groups (panel B), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* JOBB, LISA.

A.8 Trends within one-digit sectors

Figure 19 plots the share of firms (panel A), share of employment (panel B) and job reallocation rate (panel C) by one digit sector. As in many advanced countries over this period, employment has shifted out of manufacturing and into services in Sweden. Because the latter sectors tend to have higher reallocation rates, this shift tends to, *ceteris paribus*, lead to higher reallocation rates. Consequently, the within-sector decline in job reallocation is larger than the aggregate decline. While there are some differences in the timing and magnitude of the fall in job reallocation across sectors, there is also substantial similarity in that all major sectors experienced declining job reallocation over this period.

FIGURE 19. DYNAMICS BY ONE DIGIT SECTOR

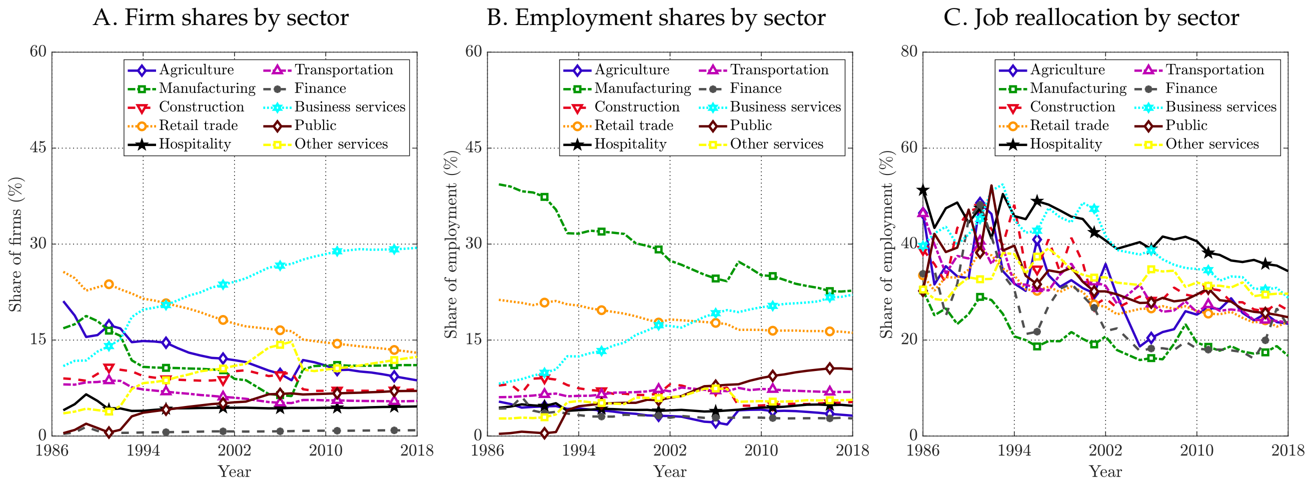


Figure 19 plots the share of firms (panel A) and the share of employment (panel B) by one-digit sector, as well as the job reallocation rate by one-digit sector (panel C), defined as the sum of jobs created and destroyed in a year divided by average employment in the year. *Source:* JOBB, LISA.

B Descriptive statistical analysis

This section contains an assessment of the correlation structure of the dependent and independent variables (Appendix B.1); further details on the relationship between past births and aging across Swedish LAs (Appendix B.2); an illustration of key characteristics of Swedish local labor markets (Appendix B.3); a graphical illustration of long run changes in aging and key labor market outcomes (Appendix B.4); additional robustness specifications (Appendix B.5); additional results on the relationship between aging and labor market outcomes (Appendix B.6); the relationship between aging and labor market dynamics conditional on sector (Appendix B.7); as well as the relationship between aging and firm age conditional outcomes (Appendix B.8).

B.1 On the spatial and time series correlation

To assess the prevalence of a time series of errors, I first obtain the residuals $\hat{\varepsilon}_{it}$ from a projection of the dependent and independent variables on LA and time fixed effects. Subsequently, I first project the contemporaneous residuals on up to seven years of their own lags

$$\hat{\varepsilon}_{it} \quad = \quad \sum_{\tau=1}^{7} \beta_\tau \hat{\varepsilon}_{it-\tau} + \gamma_{it} \tag{24}$$

Figure 20 plots the point estimates $\hat{\beta}_i$ from regression (24) for the share of young (panel A), firm creation (panel B) and worker relocation (panel C). When the share young, firm creation and worker relocation were above trend in the previous year, it is above trend in the current year. That is, the residuals are autocorrelated, with evidence suggesting that an AR1 correction is not sufficient to capture the structure of residuals. Consequently, I cluster standard errors by LA.

To assess the prevalence of a spatial structure of errors, I project the contemporaneous residuals $\hat{\varepsilon}_{it}$ on the contemporaneous residuals $\hat{\varepsilon}_{jt}$ in all other labor markets interacted with the distance d_{ij} to labor market i (binned into deciles), as well as two years of lags of the residuals in labor market i and two years of lagged residuals from all other labor markets interacted with their distance to labor market i,

$$\hat{\varepsilon}_{it} \quad = \quad \sum_{\tau=1}^{2} \alpha_\tau \hat{\varepsilon}_{it-\tau} + \sum_{\tau=0}^{2} \sum_{j \neq i} \beta_\tau^d \hat{\varepsilon}_{jt-\tau} d_{ij} + \gamma_{it} \tag{25}$$

Figure 21 plots the point estimates $\hat{\beta}_i$ from regression (25) for the share of young (panel A), firm creation (panel B) and worker relocation (panel C). When the share young, firm creation and worker relocation are above trend in neighboring markets, they are also above trend in market i. This evidence leads

FIGURE 20. AUTOCORRELATION OF RESIDUALS

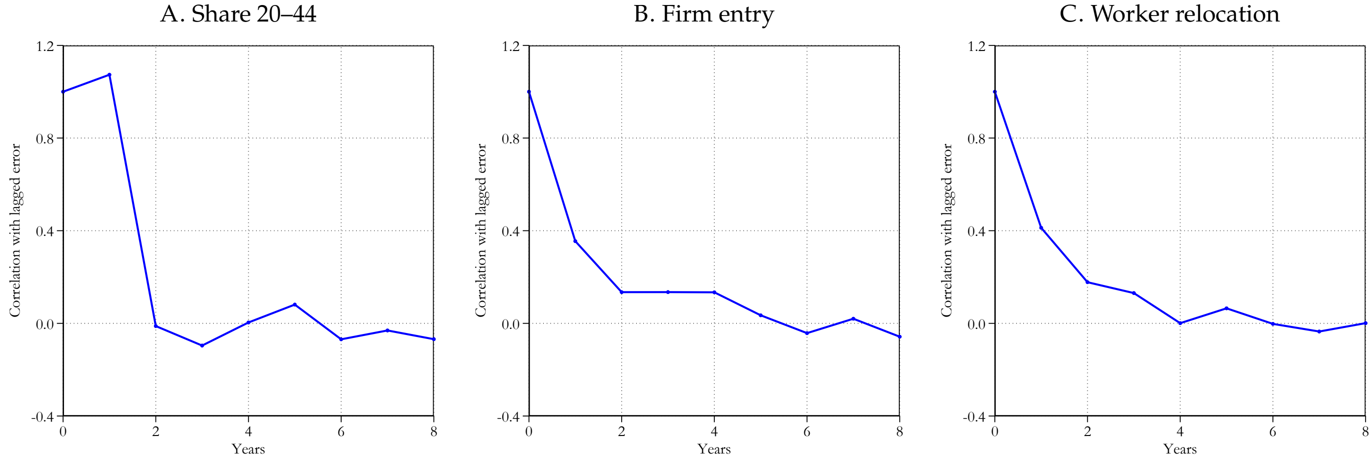


Figure 20 plots the point estimates from regression (24) for the share young (panel A), firm creation (panel B) and worker relocation (panel C).
Source: JOBB, LISA, SCB.

me to also cluster standard errors by year. I note, however, that controlling for the contemporaneous residual in neighboring markets as well as lagged residuals in market i, the contemporaneous residual in market i is not systematically correlated to the lagged residuals in neighboring markets. Nevertheless, also allowing for an aggregate autocorrelation for up to two years following Driscoll and Kraay (1998) does not change the main results in this paper.

FIGURE 21. SPATIAL CORRELATION OF RESIDUALS

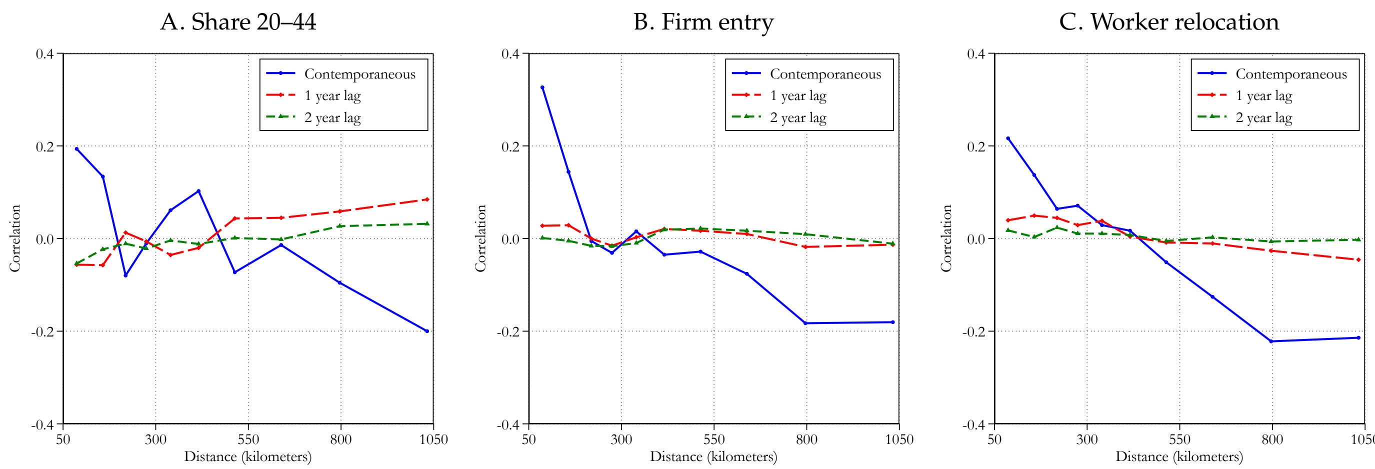


Figure 21 plots the point estimates from regression (25) for the share young (panel A), firm creation (panel B) and worker relocation (panel C).
Source: JOBB, LISA, SCB.

B.2 The relationship between past births and aging across Swedish LAs

Figure 22 illustrates the relationship between past births and aging. Panel A plots the long-run cumulative change between 1986 and 2018 in the residual log share of young against the long-run change in the residual log sum of lagged births. As expected, LAs that experienced larger relative declines in lagged births have aged more. Panel B plots the residual share young against the residual log lagged births pooling all years 1986–2018. Both are constructed as the residual conditional on LA and year fixed effects. As expected, when a LA has a larger number of residual lagged births, it is younger.

FIGURE 22. RESIDUAL SHARE YOUNG AND RESIDUAL LAGGED BIRTHS, 1986–2018

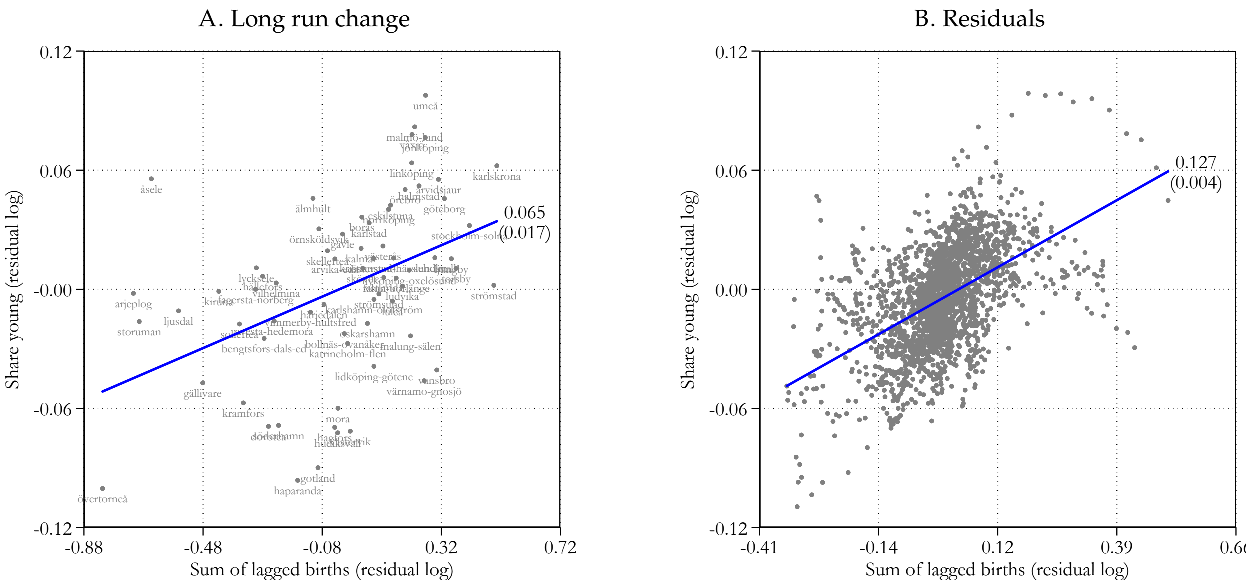


Figure 22 plots the within-LA long difference between 2018 and 1986 in the residual share young (share of all individuals aged 20–64 who are aged 20–44) against the within-LA long difference in the residual log sum of lagged births 20–44 years earlier (panel A) and the residual share young against the residual log lagged births (panel B). Residuals are conditional on LA and year fixed effects. *Source:* SCB.

B.3 Characteristics of Swedish local labor markets

Figure 23 illustrates characteristics of Swedish LAs based on average outcomes over the 1986–2018 period (1999–2007 for net wealth). The largest areas are Stockholm, Gothenburg and Malmö (panel A). These areas also experienced the greatest growth in their workforces over the past 35 years (panel B), and they are the youngest (panel C).

Figure 24 further illustrates differences across Swedish LAs. Stockholm, *Årjäng* close to Oslo, and areas in the southern Swedish region of *Småland* are the richest in terms of net wealth (panel A). The large metropolitan areas are the highest paying, although pay is also high in several less populous areas (panel B). Public sector employment is more prevalent in the more rural inland areas (panel C).

Figure 25 illustrates employment outcomes across Swedish local labor markets based on average outcomes over the 1986–2018 period. The likely reason for the high measured non-employment rate in the LAs bordering Finland in the north is cross-border commuting—employment abroad is not recorded in the Swedish administrative data (panel A). The same reason is likely behind the high measured non-employment in the areas across the border from Oslo. Although there is cross-border commuting to Denmark from the southernmost region of *Skåne*, such flows are unlikely to be a major factor behind its high non-employment rate, because available data indicate that such flows are a small share of this area's large population. Private sector wage employment is more common in southern Sweden (panel B), while self-employment is more common in the northern inland (panel C).

FIGURE 23. CHARACTERISTICS OF SWEDISH LAs

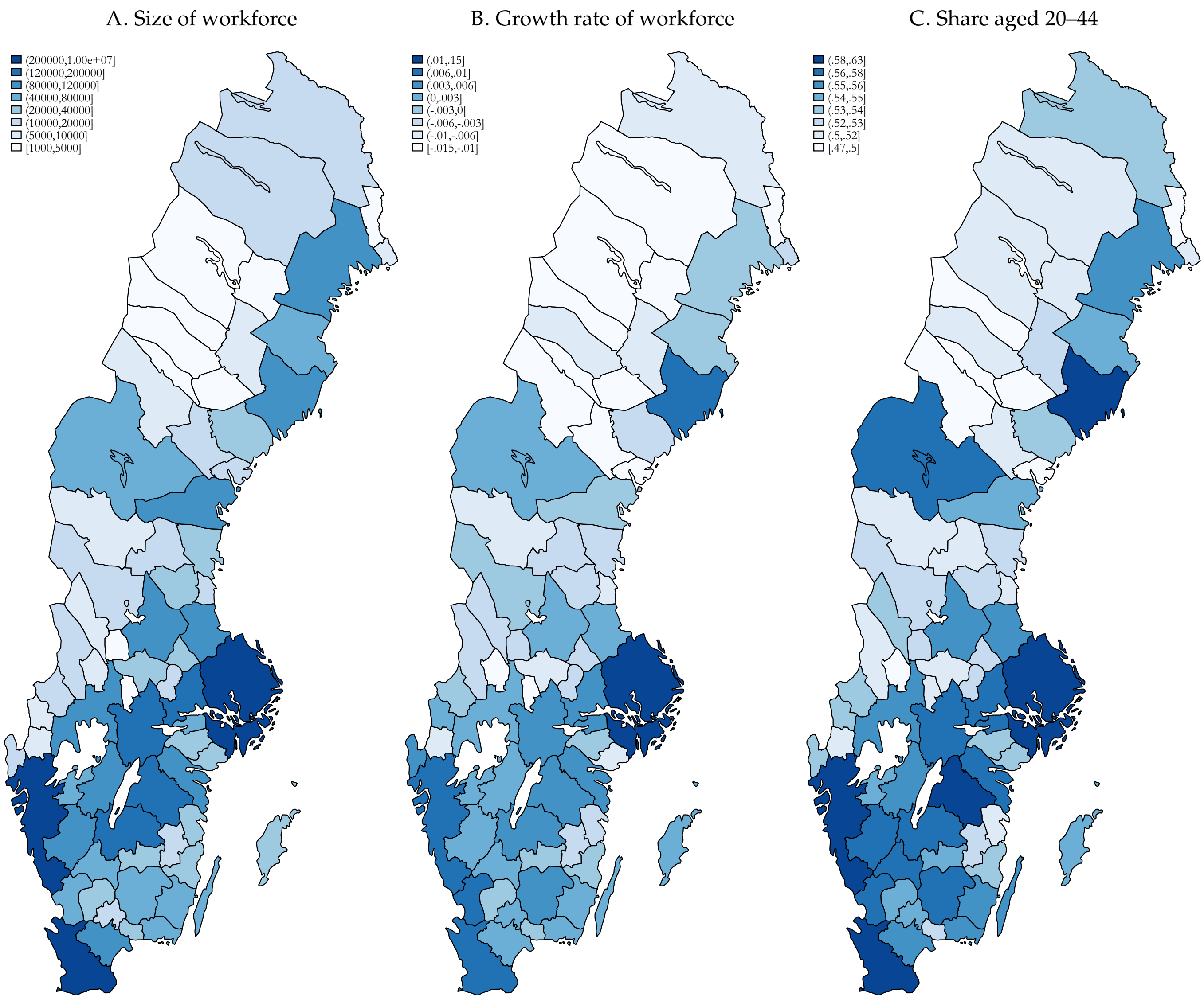


Figure 23 plots characteristics of Swedish LAs based on average outcomes over the 1986–2018 period. Panel A plots the number of individuals aged 20–64. Panel B plots the growth rate of the workforce (all individuals aged 20–64). Panel C plots the share of all individuals aged 20–64 who are aged 20–44. *Source:* JOBB, LISA, SCB.

Figure 26 provides further worker-level outcomes across Swedish LAs based on average outcomes over the 1986–2018 period. The most educated areas include the three largest cities of Stockholm, Gothenburg and Malmö, but also, for instance, the northern area around *Umeå*, which is a major university town (panel A). Women constitute a larger share of private sector employment in the more populous coastal areas, likely due to a smaller role for the public sector in these areas (panel B). Immigrants as concentrated in southern Sweden and in particular Stockholm and Malmö, with the exception of the LAs bordering Finland to the north which has a large number of Finnish immigrants (panel C).

FIGURE 24. CHARACTERISTICS OF SWEDISH LAS, CONTINUED

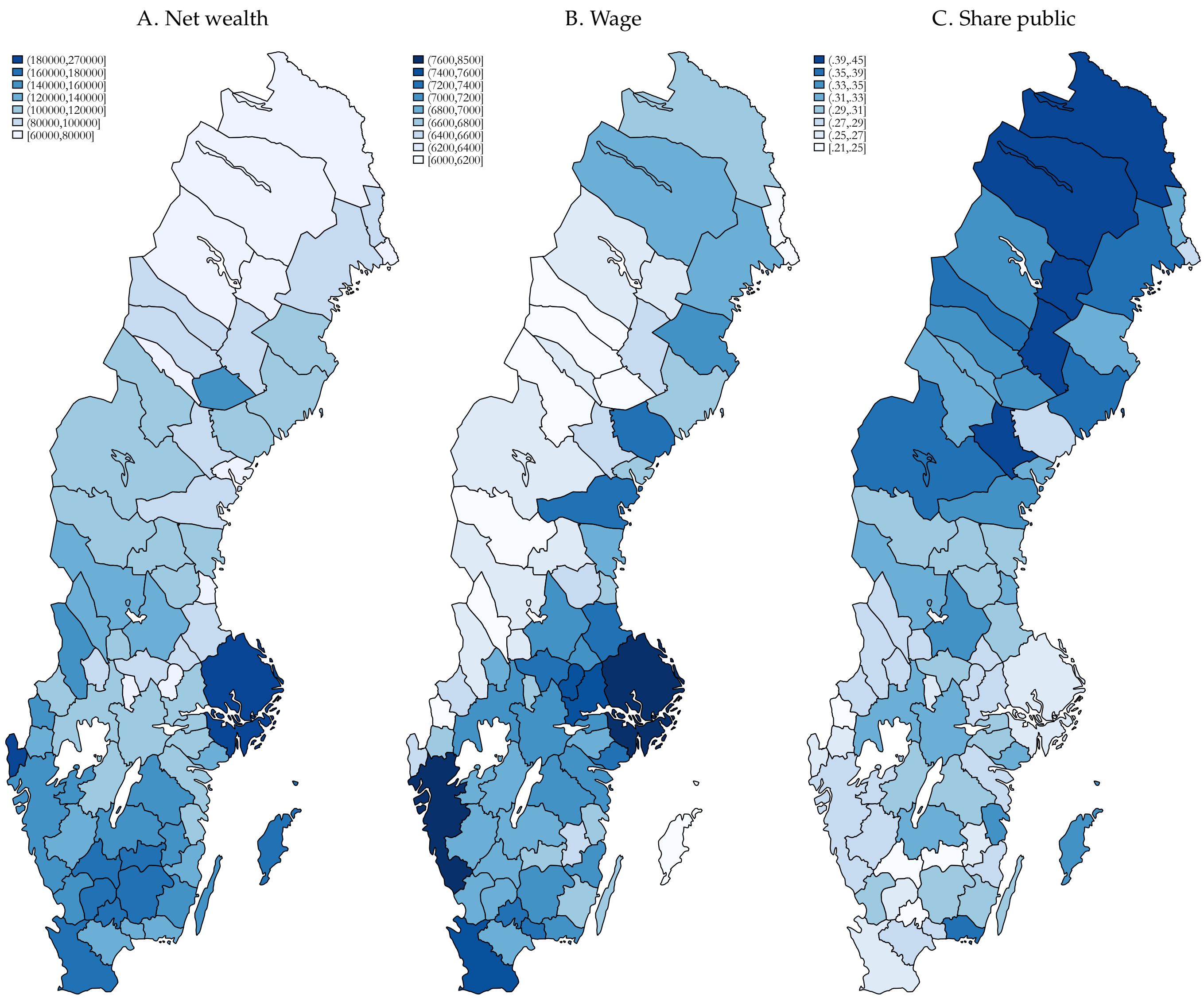


Figure 24 plots characteristics of Swedish LAs based on average outcomes over the 1986–2018 period (1999–2007 for net wealth). Panel A plots the average net wealth in real 1980 Swedish kronor. Panel B plots the average monthly wage of individuals aged 20–64 working in the private sector in real 1980 Swedish kronor. Panel C plots the number of individuals aged 20–64. *Source:* JOBB, LISA, SCB.

Figure 27 illustrates additional firm-level characteristics of Swedish LAs based on on average outcomes over the 1986–2018 period (1997–2018 for value added). Manufacturing is concentrated in the

Swedish inland (panel A). The most productive LAs are generally those which have a large share of manufacturing, as well as the metropolitan areas of Stockholm and Gothenburg (panel B). One exception is the south-eastern area of *Oskarshamn*, which is home to a nuclear power plant that produces about 10 percent of Sweden's electricity. Average firm size is highest in Stockholm (panel C).

FIGURE 25. EMPLOYMENT OUTCOMES ACROSS SWEDISH LAS

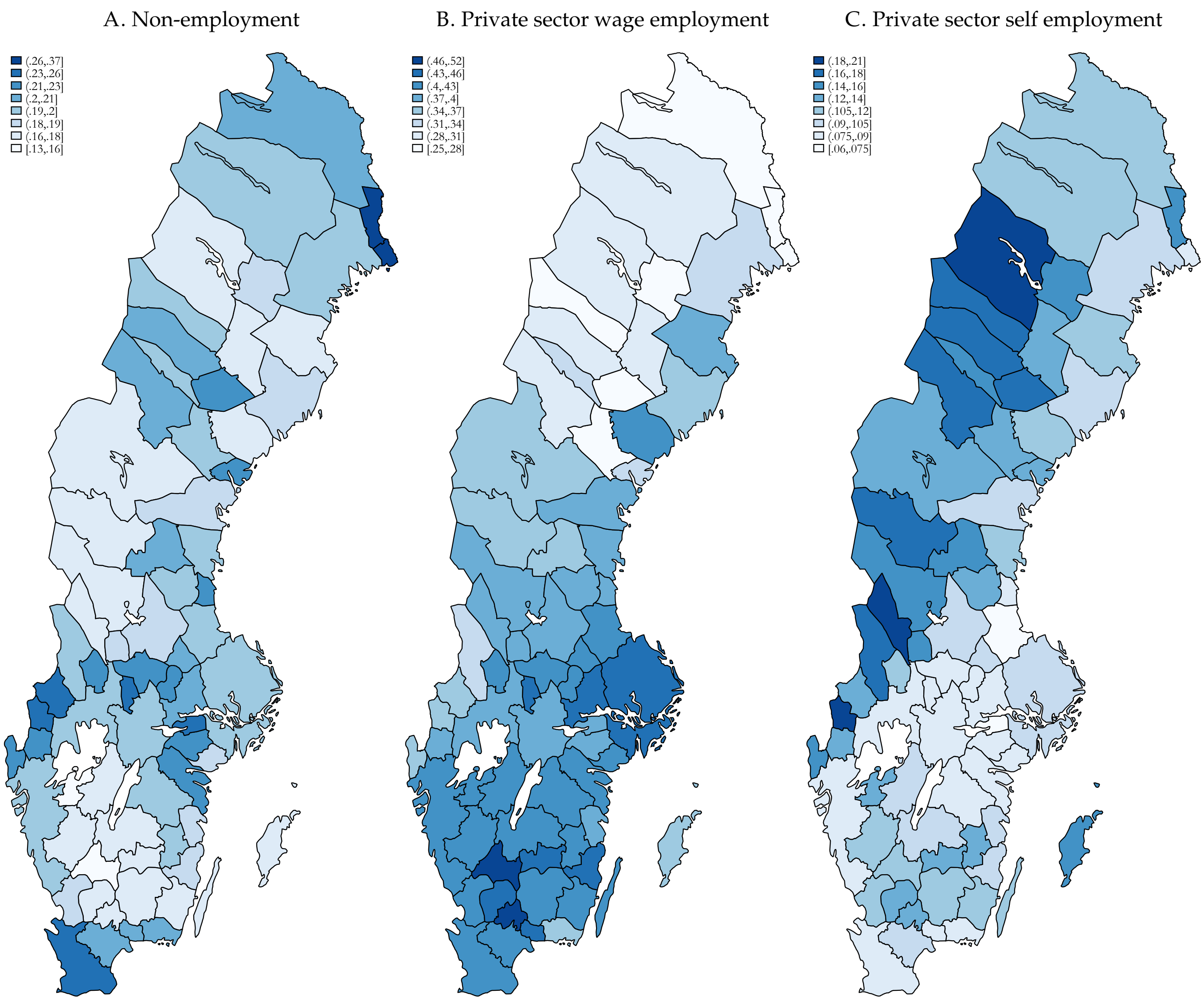


Figure 25 plots employment outcomes of Swedish LAs based on average outcomes over the 1986–2018 period. Panel A shows the share of non-employed individuals aged 20–64. Panel B shows the share of private sector wage employed individuals aged 20–64. Panel C shows the share of self employed individuals aged 20–64. *Source:* JOBB, LISA, SCB.

Figure 28 plots the predicted share of young, firm entry and worker relocation across Swedish LAs based on averages over the 1986–2018 period. The predicted youngest areas based on past fertility are the three largest cities, Stockholm, Gothenburg and Malmö (panel A). The highest firm entry rates are found in the three largest cities (panel B), while worker relocation is the highest in the north (panel C). One likely factor behind the latter is seasonal employment associated with winter tourism. Consistent with this view, the highest worker relocation rates are found in *Kiruna*, home to the *Riksgränsen*, *Björkliden* and *Abisko* ski resorts as well as the *Jukkasjärvi Icehotel*; *Storuman*, which contains the *Hemavan* and *Tärnaby* ski resorts; *Härjedalen*, with several large ski resorts; and *Malung-Sälen*, with the *Sälen* skiing complex.

FIGURE 26. CHARACTERISTICS OF PRIVATE SECTOR EMPLOYMENT

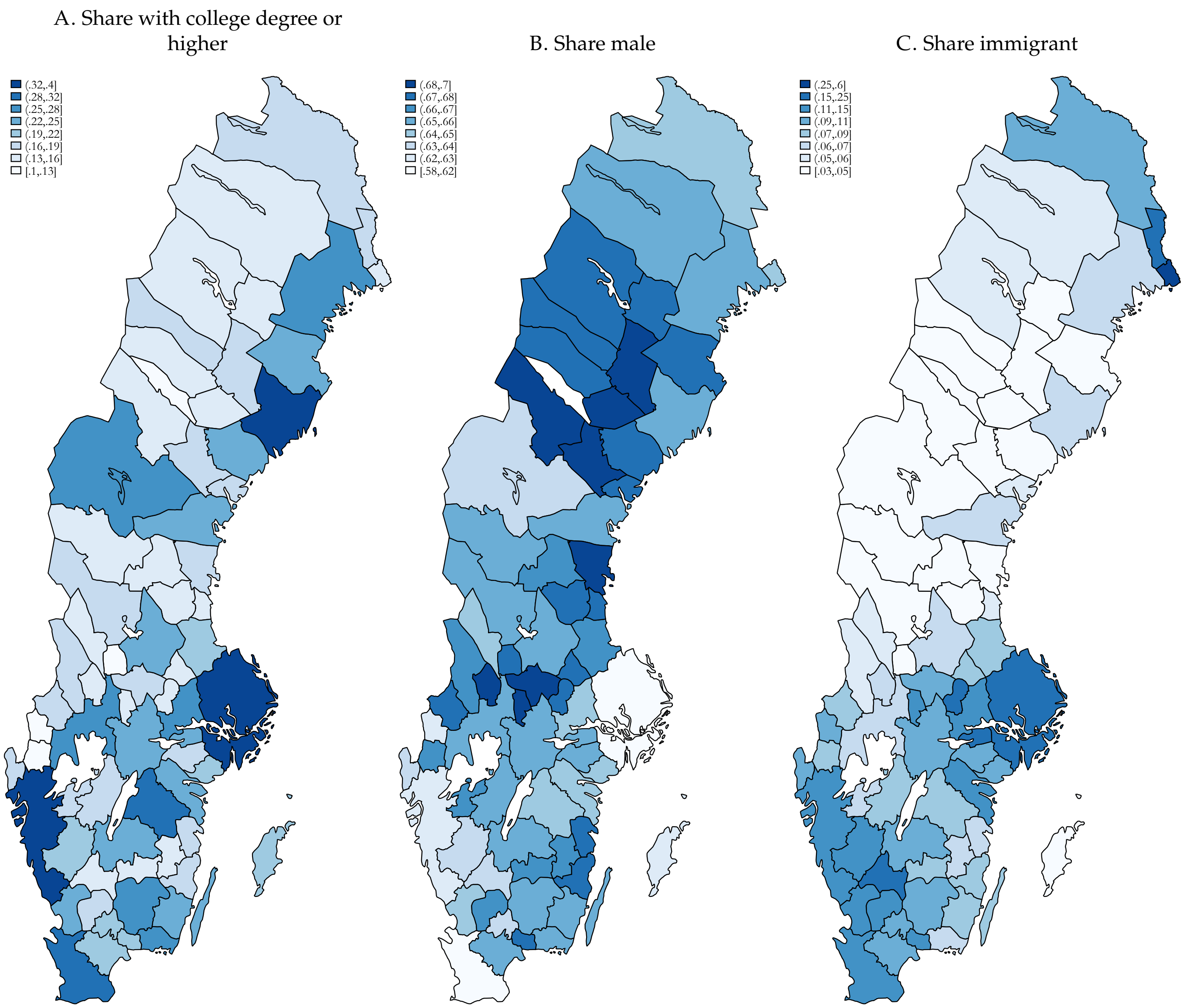


Figure 26 plots characteristics of Swedish LAs based on average outcomes over the 1986–2018 period. Panel A plots the share of private sector employment aged 20–64 with a college degree or higher. Panel B plots the share of private sector employment aged 20–64 that is male. Panel C plots the share of private sector employment aged 20–64 that was born abroad. *Source:* JOBB, LISA, SCB.

FIGURE 27. FIRM CHARACTERISTICS

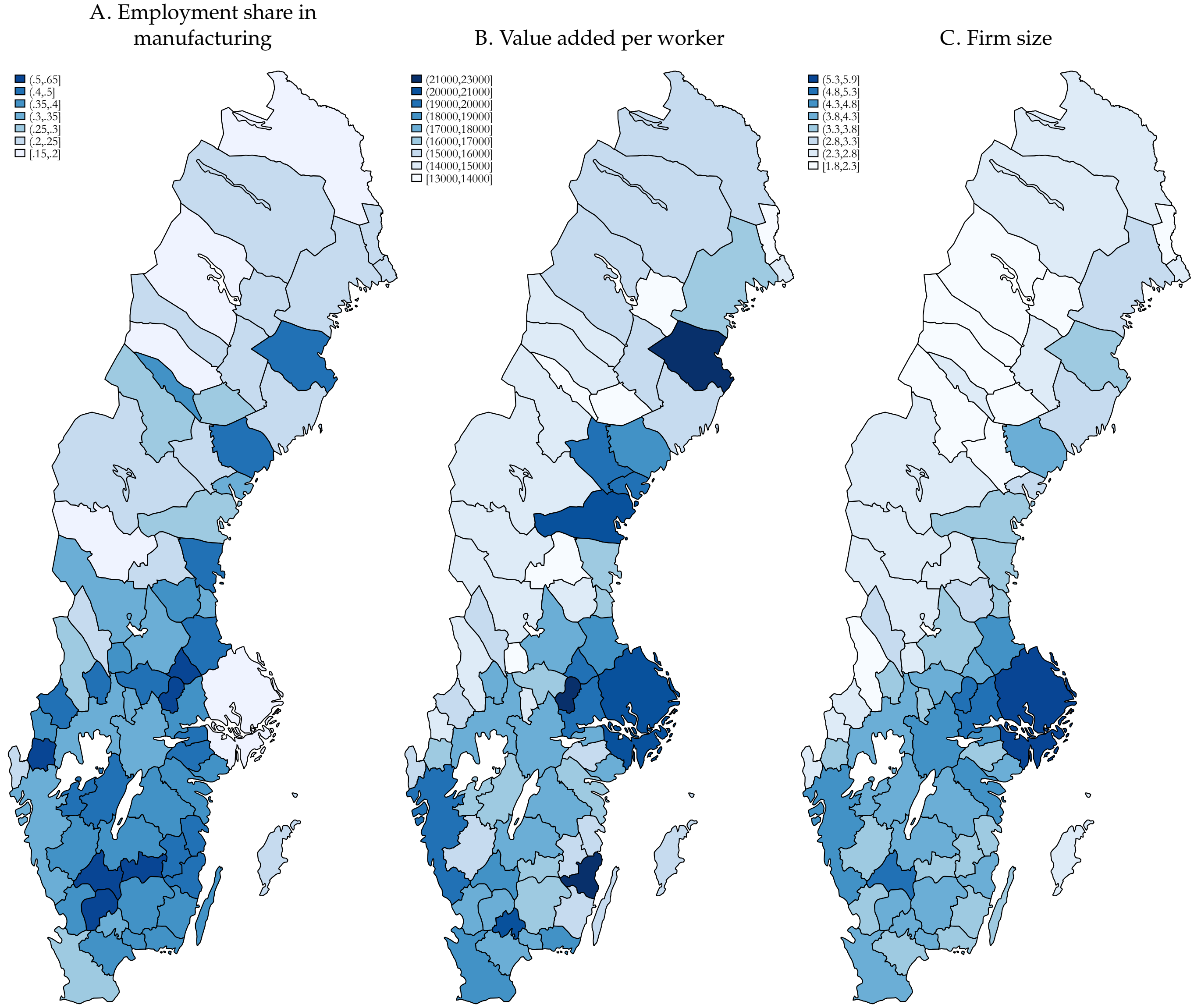


Figure 27 plots characteristics of Swedish local labor markets based on average outcomes over the 1986–2018 period (1997–2018 for value added). Panel A plots the fraction of private sector employment aged 20–64 that is in manufacturing. Panel B plots average value added per worker of private sector employment aged 20–64 in real 1980 Swedish kronor. Panel C plots average firm size of private sector firms (including all employees aged 20–64). *Source:* FEK, JOBB, LISA, SCB.

B.4 Long run changes in predicted aging and key labor market dynamics outcomes

Figure 29 plots aging as predicted by lagged fertility (panel A), the change in firm creation (panel B) and worker relocation (panel C) between 1986 and 2018 after taking out local labor market and year fixed effects. There is some evidence of spatial correlation in the extent of aging, but important variation also remains within Swedish regions. For instance, Sweden is divided into three broad regions: a southern region (*Götaland*), a middle region (*Svealand*) and a northern region (*Norrland*). Adding region-year fixed effects to the projection of the log share of young on LA and year fixed effects shrinks the standard deviation of the residual log share of young only marginally from 0.025 to 0.023.

FIGURE 28. SHARE YOUNG, FIRM CREATION AND WORKER RELOCATION

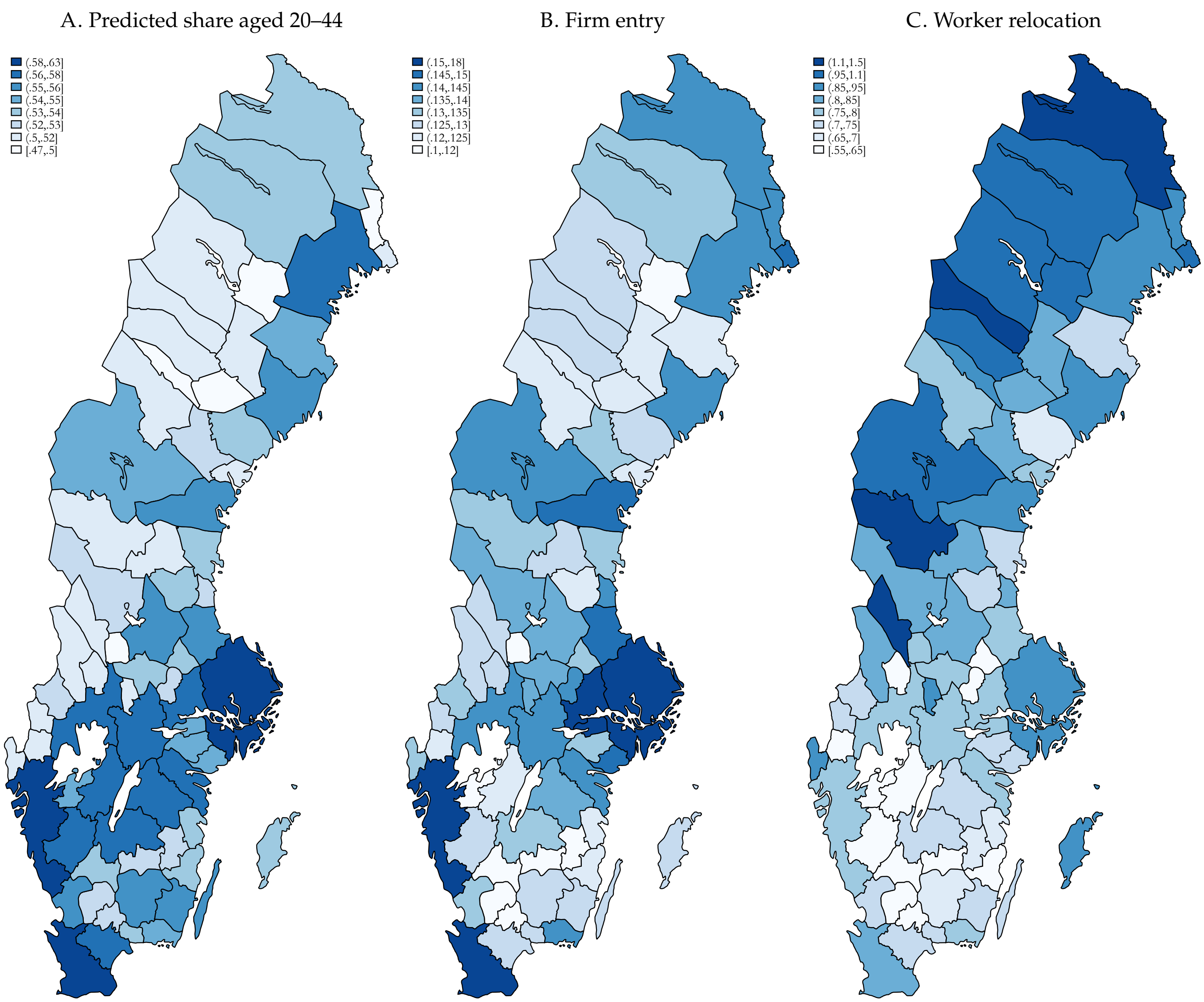


Figure 28 plots the age composition, firm creation and worker relocation across Swedish local labor markets based on average outcomes over the 1986–2018 period. Panel A plots the number of individuals aged 20–64 who are aged 20–44. Panel B plots the fraction of all private sector firms with positive employment in a year that had zero employment in the previous year. Panel C plots the fraction of private sector individuals aged 20–64 who were either hired or separated in the current year. *Source:* JOBB, LISA, SCB.

Figure 30 illustrates some of the patterns that drive the main results. Panel A plots the change in the residual firm entry rate between 1986 and 2018 against the change in the predicted residual log share of young based on lagged births (in both cases conditional on LA and year fixed effects). Panel B shows the change in the residual worker relocation rate relative to the change in the predicted share young. Predicted aging based on changes in lagged births is associated with a statistically significant relative decline in firm entry and worker relocation. Although a useful first way to illustrate the variation, it is important to note that regression (1) also exploits differential timing of aging over these 35 years.

FIGURE 29. CHANGE IN INDEPENDENT AND DEPENDENT VARIABLES BETWEEN 1986–2018

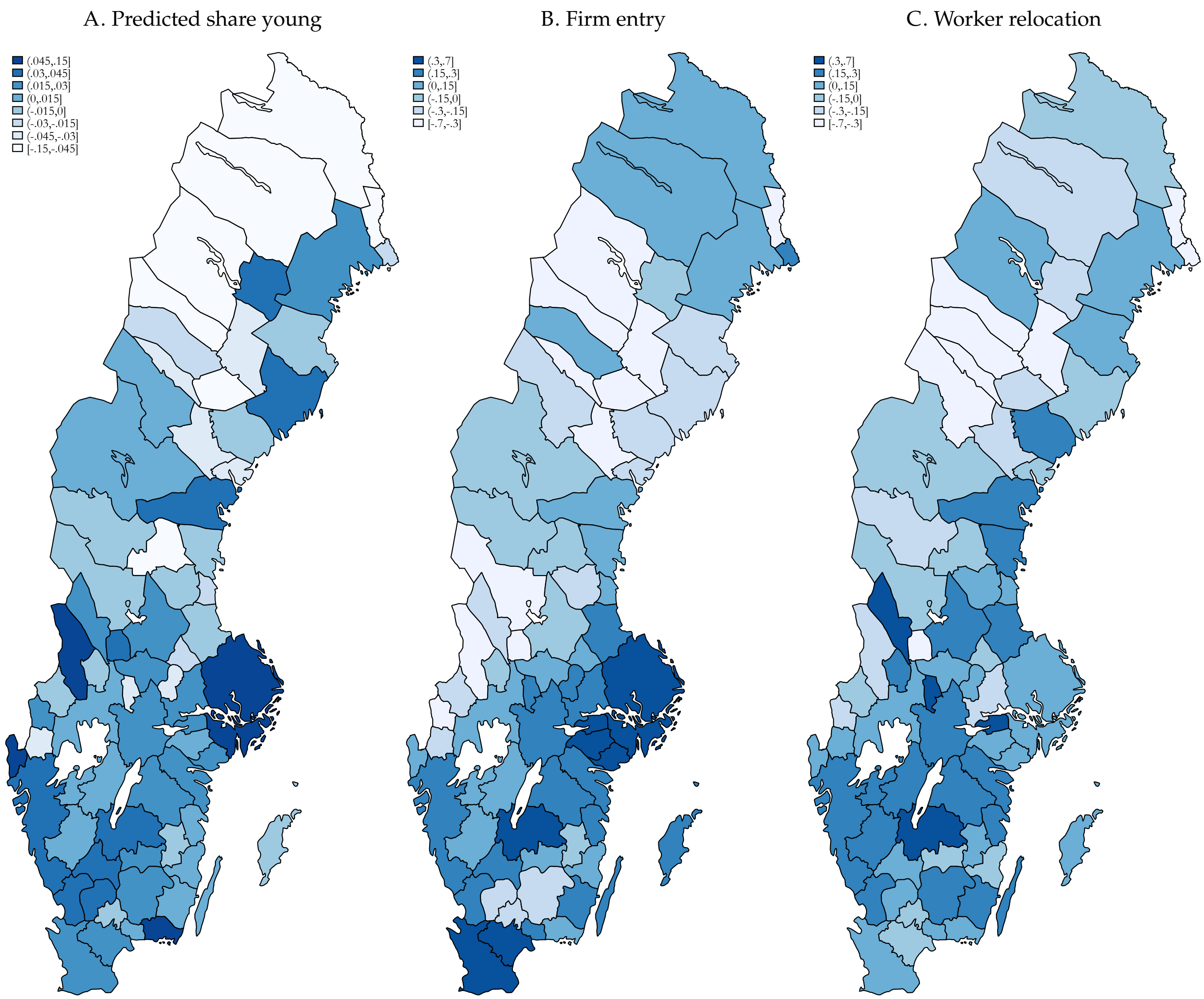


Figure **??** plots the long difference between 2018 and 1986 in the predicted share of young (panel A), firm creation (panel **??**).) and worker relocation (panel C). Panel A first computes the predicted share young by projecting the actual log share on the log sum of lagged births 20–44 years earlier, LA and year fixed effects, obtaining the predicted values based on the log sum of lagged births, and subsequently computing the long difference between 2018 and 1986. Panels **??**–C residualize each outcome by projecting it on LA and year fixed effects, and subsequently compute the long difference between 2018 and 1986. *Source:* SCB.

B.5 Robustness specifications

Table 7 presents a series of robustness specifications. Columns 3–4 add separate linear time trends interacted with the initial share of private sector employment that is male, has a college degree or more, or is immigrant, as well as the initial size of the workforce and initial average net wealth. It changes the point estimates by little, but the standard errors widen significantly in the IV specification such that the estimated impact of aging on firm creation is no longer statistically significant at conventional levels. The weaker first stage and wider standard errors are due to the inclusion of the interaction of time with the initial share with a college degree, which on its own is highly statistically significant (p-value 0.806). Dropping this statistically insignificant control, the point estimate again becomes statistically significant.

FIGURE 30. LONG-RUN CHANGE IN KEY LABOR MARKET OUTCOMES AGAINST THE CHANGE IN THE PREDICTED SHARE OF YOUNG

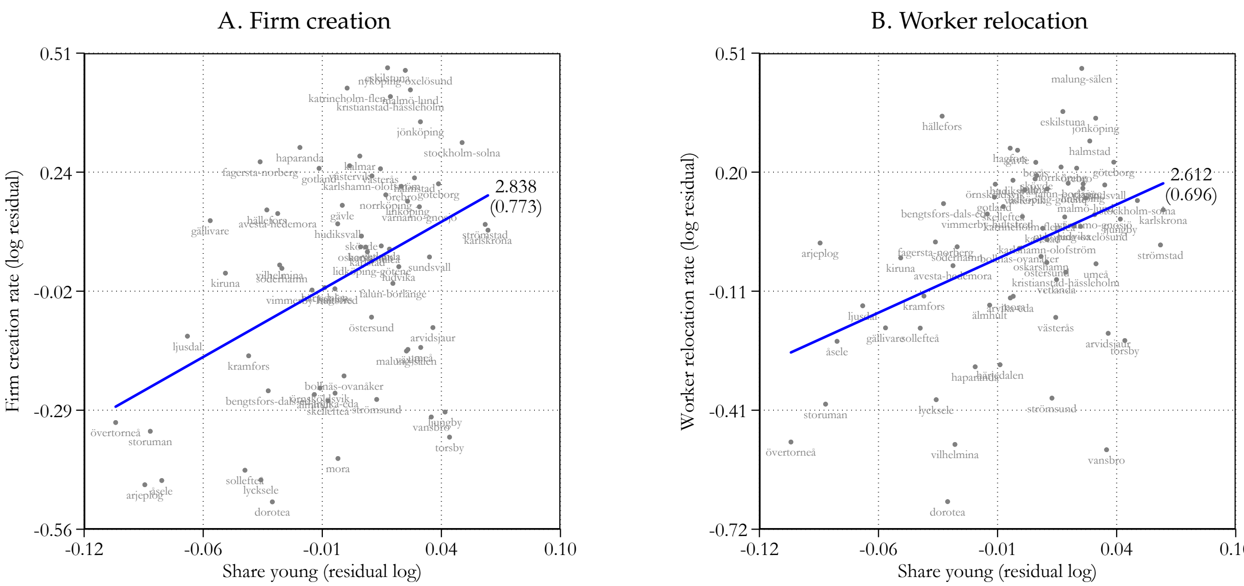


Figure 30 plots the difference between residual outcome in 2018 and 1986. Panel A shows residual log firm creation, based on a projection of actual log firm creation on LA and year fixed effects, against the predicted log share of young, based on a projection of the actual log share of young on the residual log sum of lagged births (conditional on LA and year fixed effects). Panel B shows residual log worker relocation, based on a projection of actual log worker relocation on LA and year fixed effects, against the predicted log share of young, based on a projection of the actual log share of young on the residual log sum of lagged births (conditional on LA and year fixed effects). *Source:* FEK, JOBB, LISA, SCB.

Columns 5–6 include separate linear time trends interacted with initial value added per worker, the initial share of manufacturing firms and initial investment per worker. Results are robust. Moreover, results are little changed if I drop the three largest LAs Stockholm, Gothenburg and Malmö, which jointly account for about half of Sweden's population (columns 7–8). Finally, the standard errors rise if I add LA times decade fixed effects, to the point where the IV estimate for firm creation and the OLS estimate for worker relocation are no longer statistically significant at conventional levels (p-values of 0.117 and 0.138, respectively, under two-way clustered standard errors).

B.6 The impact of aging: additional outcomes

Table 8 summarizes the impact of aging on a range of additional outcomes. Aging reduces the entry rate also of high growth firms, measured by either at least 25 or 50 percent growth in employment during the

TABLE 7. THE IMPACT OF AGING ON LABOR MARKET DYNAMICS, ROBUSTNESS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Baseline		Controls-time trend		Size-time trend		Drop large cities		LA-decade FEs	
	OLS	IV	OLS	IV	OLS	IV	OLS	IV	OLS	IV
	Panel A. Firm creation rate									
Share 20–44	1.207***	2.196**	0.757**	2.106	1.232***	1.916**	1.029***	2.054**	1.044**	2.536
	(0.358)	(0.820)	(0.338)	(1.262)	(0.320)	(0.822)	(0.361)	(0.876)	(0.427)	(1.574)
P-value	0.002	0.012	0.032	0.105	0.001	0.026	0.008	0.025	0.020	0.117
Obs.	2,244	2,244	2,244	2,244	2,244	2,244	2,145	2,145	2,244	2,244
R-squared	0.691		0.732		0.694		0.684		0.808	
within	0.059		0.184		0.066		0.041		0.013	
F-stat		27.1		16.3		24.2		23.4		17
	Panel B. Worker relocation rate									
Share 20–44	0.988***	2.594***	0.646**	3.051**	0.938***	2.490***	1.022***	2.772***	0.471	3.251**
	(0.237)	(0.754)	(0.245)	(1.300)	(0.246)	(0.851)	(0.241)	(0.829)	(0.310)	(1.347)
P-value	0.000	0.002	0.013	0.025	0.001	0.006	0.000	0.002	0.138	0.022
Obs.	2,244	2,244	2,244	2,244	2,244	2,244	2,145	2,145	2,244	2,244
R-squared	0.791		0.804		0.793		0.791		0.886	
within	0.051		0.108		0.059		0.050		0.004	
F-stat		27.1		16.3		24.2		23.4		17

Table 7 presents OLS and IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. Columns 1–2 reproduce the baseline results from Table 2. Columns 3–4 include separate linear time trends interacted with the share college, male, immigrant and population size, all measured in 1986. Columns 5–6 include separate linear time trends interacted with average value added per worker, the share of firms in manufacturing, and average investment per worker, all measured in the earliest year of data (1997 for value added and investment, 1986 for the share in manufacturing). Columns 7–8 exclude the largest three metro areas—Stockholm, Gothenburg and Malmö—which constitute roughly half of the Swedish population. Columns 9–10 add LA-decade fixed effects. Panel A shows results for the firm creation rate as the dependent variable, defined as the share of firms with positive employment in the current year that had zero employment in the previous year. Panel B shows results for the worker relocation rate as the dependent variable, defined as the sum of hires and separations in a year divided by average employment in the year. *Source:* FEK, JOBB, LISA, SCB.

first five years of a firm's operation. It has no statistically significant effect on firm size or investment per worker. Finally, it lowers job reallocation.

B.7 Aging and within-sector outcomes

The impact of aging on labor market dynamics could partly arise through a shift of sectoral activity if aging shifts economic activity toward intrinsically less dynamic sectors. To assess the importance of such shifts, I collect outcome variables at the LA-year-sector level and project them on the share of young in the LA-year, controlling for LA, year and sector fixed effects,

$$\log y_{i,t,s} \quad = \quad \alpha \log young_{i,t} + \psi_i + \xi_t + \phi_s + \varepsilon_{i,t,a} \tag{26}$$

I weigh (27) such that each sector gets a weight corresponding to its share of firms (firm entry) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight.

Table 9 presents results from (26). Aging reduces firm entry, job and worker reallocation conditional on sector. This is broadly consistent with the time series evidence, which also shows that the majority of

TABLE 8. THE IMPACT OF AGING ON ADDITIONAL LABOR MARKET DYNAMICS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Entry, 25%		Entry, 50%		Firm size		Investment p.w.		JR	
	OLS	IV	OLS	IV	OLS	IV	OLS	IV	OLS	IV
Share 20–44	0.998**	2.057**	0.835*	1.970*	0.281	−0.157	0.453	1.042	0.526**	1.372*
	(0.390)	(0.925)	(0.435)	(1.040)	(0.291)	(0.661)	(0.561)	(1.552)	(0.238)	(0.687)
p value	0.016	0.034	0.065	0.069	0.342	0.813	0.428	0.509	0.034	0.054
Obs.	1,963	1,963	1,958	1,958	2,244	2,244	1,496	1,496	2,244	2,244
R-squared	0.510		0.466		0.862		0.501		0.639	
within	0.009		0.005		0.004		0.001		0.006	
F-stat		30.4		29.9		27.1		14.8		27.1

Table 8 presents OLS and IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. The outcome variables are for private sector firms and individuals aged 20–64, averaged in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. *Source:* JOBB, LISA, SCB.

the decline in labor market dynamics has taken place within sectors (Appendix A.8).

TABLE 9. AGING AND SECTOR-CONDITIONAL OUTCOMES ACROSS SPACE

	(1)	(2)	(3)	(4)	(9)	(10)
	Firm entry		WR		JR	
	OLS	IV	OLS	IV	OLS	IV
Share 20–44	1.038***	1.901**	0.791***	2.079***	0.467**	1.263*
	(0.346)	(0.784)	(0.213)	(0.728)	(0.226)	(0.674)
Obs.	21,260	21,260	22,018	22,018	21,988	21,988
Clusters	68	68	68	68	68	68
R-squared	0.536		0.679		0.447	
within	0.007		0.006		0.001	
F-stat		30.5		30.2		30.2

Table 9 presents OLS and IV estimates based on regression (27) using annual data from 68 LA between 1997–2018. Share 20–44 is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64. All dependent variables are first averaged in levels at the LA-year-firm age level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Regressions are weighed such that each firm age bin gets a weight corresponding to its share of firms (firm exit and firm size) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight. Standard errors are clustered at the LA level. *Source:* FEK, JOBB, LISA, SCB.

B.8 Aging and firm age conditional outcomes

Appendix B.6 finds that aging leads to an increase in the share of firms that are 11 years and older. Since older firms are on average less dynamic, this shift may hence account for some or all of the overall impact of aging on labor market dynamics. To assess the importance of this shift toward older firms, I project labor market dynamics outcomes at the LA-year-firm age level on the share of young in the LA-year, controlling for LA, year and firm age fixed effects,

$$\log y_{i,t,a} = \alpha \log young_{i,t} + \psi_i + \xi_t + \phi_a + \varepsilon_{i,t,a} \tag{27}$$

I weigh (27) such that each firm age bin gets a weight corresponding to its share of firms (firm exit and firm size) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight. Due to the left-censoring of the data, I focus on the 1997–2018 period.

Table 10 presents results from (27). Aging reduces worker and job reallocation as well as firm exit conditional on firm age, but has no statistically significant effect on average size. These findings are consistent with the time series evidence, which also shows only a small change in firm size conditional on firm age, but a substantial decline in job reallocation conditional on firm age (Appendix A.7).

TABLE 10. AGING AND FIRM AGE CONDITIONAL OUTCOMES ACROSS SPACE

	(1)	(2)	(3)	(4)	(9)	(10)	(11)	(12)
	WR		JR		Exit		Size	
	OLS	IV	OLS	IV	OLS	IV	OLS	IV
Share 20–44	1.130***	3.441***	0.734***	1.533*	1.073***	2.111**	-0.337	-2.905
	(0.270)	(1.077)	(0.246)	(0.830)	(0.399)	(0.945)	(1.493)	(3.473)
Obs.	19,498	19,498	19,491	19,491	19,041	19,041	19,513	19,513
Clusters	68	68	68	68	68	68	68	68
R-squared	0.546		0.625		0.661		0.426	
within	0.008		0.002		0.005		0.000	
F-stat		23		23		23.373		23

Table 10 presents OLS and IV estimates based on regression (27) using annual data from 68 LA between 1997–2018. Share 20–44 is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. Outcome variables are for private sector firms and individuals aged 20–64. All dependent variables are first averaged in levels at the LA-year-firm age level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Regressions are weighed such that each firm age bin gets a weight corresponding to its share of firms (firm exit and firm size) or employment (worker and job reallocation) in the LA-year, and each LA-year gets the same aggregate weight. Standard errors are clustered at the LA level. *Source:* FEK, JOBB, LISA, SCB.

C Qualitative analysis

This section contains a proof of Lemmas 1–2 (Appendix C.1); a proof of Proposition 1 (Appendix C.2); a derivation of the FP equation for the distribution of entrepreneurs (Appendix C.3); a derivation of the FP equation for the distribution of employment (Appendix C.4); a proof of Proposition 2 (Appendix C.5); a derivation of the entry curve (20) (Appendix C.6); a proof of Proposition 3 (Appendix C.7); a proof of Proposition 4 (Appendix C.8); a proof of Proposition 3 (Appendix C.9); a proof of Proposition 4 (Appendix C.10); a proof of Proposition 5 (Appendix C.11); a proof of Lemma 5 (Appendix C.12); a proof of Lemma 6 (Appendix C.13); a proof of Lemma 7 (Appendix C.14); a proof of Proposition 6 (Appendix C.15); reduced-form support for some of the key predictions of the theory (Appendix C.16); a proof of Proposition 7 (Appendix C.17); a proof of Proposition 8 (Appendix C.18); and a proof of Proposition 9 (Appendix C.19).

C.1 Lemmas 1–2: Stationary transformation

The value of unemployment $\widehat{U}(t)$ at time t is given by the HJB equation

$$\rho\widehat{U}(t) = \underbrace{be^{\widehat{z}(t)}}_{\text{flow value of leisure}} + \underbrace{\dot{\widehat{U}}(t)}_{\text{time drift}} - \underbrace{\kappa\widehat{U}(t)}_{\text{retirement of worker}} \tag{28}$$

The joint value of a coalition between an entrepreneur with productivity $\widehat{z}$ and their n workers at time t, $\widehat{\mathbf{W}}(\widehat{z},n,t)$, is given by

$$\begin{aligned}
\rho\widehat{\mathbf{W}}(\widehat{z},n,t) &= \underbrace{\left(e^{\widehat{z}} - e^{\widehat{z}(t)}c\right)n + e^{\widehat{z}(t)}k - \widehat{r}(t)}_{\text{net flow output}} + \underbrace{\widehat{\mathbf{W}}_t(\widehat{z},n,t)}_{\text{time drift}} - \underbrace{\kappa n\widehat{\mathbf{W}}_n(\widehat{z},n,t)}_{\text{retirement of a worker}} - \underbrace{\kappa\widehat{\mathbf{E}}(\widehat{z},\widehat{n},t)}_{\text{retirement of entrepreneur}} \\
&+ \max_v \Bigg\{ -\underbrace{\frac{c_v}{1+\eta_v}e^{\widehat{z}}v^{1+\eta_v}}_{\text{cost of recruiting}} + \widehat{q}(t)\Bigg(\underbrace{\frac{\widehat{u}(t)}{\widehat{S}(t)}\left(\widehat{\mathbf{W}}_n(\widehat{z},n,t) - \widehat{U}(t)\right)^+}_{\text{contact with unemployed potential hire}} \\
&+ \underbrace{\frac{\phi\widehat{e}(t)}{\widehat{S}(t)}\int_0^\infty\int_0^\infty \left(\widehat{\mathbf{W}}_n(\widehat{z},n,t) - \widehat{\mathbf{W}}_n(\widehat{z}',n',t)\right)^+ \tilde{\mathbf{g}}(\widehat{z}',n',t)d\widehat{z}'dn'}_{\text{contact with employed potential hire}} \Bigg)\Bigg\}
\end{aligned} \tag{29}$$

subject to

$$\widehat{\mathbf{W}}(\widehat{z},n,t) \;\geq\; n\widehat{U}(t)+\widehat{U}^f(t), \quad \text{and} \quad \widehat{\mathbf{W}}_n(\widehat{z},n,t) \;\geq\; \widehat{U}(t) \tag{30}$$

where $\widehat{\mathbf{W}}_i(\cdot) = \partial\widehat{\mathbf{W}}(\cdot)/\partial i$ is short-hand for the derivative of $\widehat{\mathbf{W}}$ with respect to i, and $\widehat{\mathbf{E}}(\widehat{z},\widehat{n},t)$ is the value to the entrepreneur of recruiting more workers into the firm.

The value of a non-producing entrepreneur is

$$\begin{aligned}\rho\widehat{U}^f(t) \;=\;& \underbrace{be^{\widehat{\underline{z}}(t)}}_{\text{flow value of leisure}} + \underbrace{\dot{\widehat{U}}^f(t)}_{\text{time drift}} - \underbrace{\kappa\widehat{U}^f(t)}_{\text{retirement}} \\ &+ \underbrace{\max_s \left\{ s\pi \int_0^\infty \left(\widehat{\mathbf{E}}(\widehat{\underline{z}}(t)+z,0,t) - \widehat{U}^f(t)\right)^+ d\Gamma(z) - c_e e^{\widehat{\underline{z}}(t)} \frac{s^{1+\eta_e}}{1+\eta_e} \right\}}_{\text{net expected return to search}}\end{aligned} \tag{31}$$

Guess that $\widehat{\mathbf{W}}(\widehat{z},n,t) = \mathbf{W}(z,n)e^{\widehat{\underline{z}}(t)}$, $\widehat{\mathbf{E}}(\widehat{z},n,t) = \mathbf{E}(z,n)e^{\widehat{\underline{z}}(t)}$, $\widehat{U}(t) = Ue^{\widehat{\underline{z}}(t)}$, and $\widehat{U}^f(t) = U^f e^{\widehat{\underline{z}}(t)}$, where $\widehat{z} = \widehat{\underline{z}}(t)+z$ and $\widehat{\underline{z}}(t) = \widehat{\underline{z}}(0)+mt$. Hence, $\widehat{\mathbf{W}}(\widehat{z},n,t) = \mathbf{W}(\widehat{z}-mt,n)e^{\widehat{\underline{z}}(t)}$. Differentiating

$$\begin{aligned}\widehat{\mathbf{W}}_{\widehat{z}}(\widehat{z},n,t) &= \mathbf{W}_z(z,n)e^{\widehat{\underline{z}}(t)} \\ \widehat{\mathbf{W}}_n(\widehat{z},n,t) &= \mathbf{W}_n(z,n)e^{\widehat{\underline{z}}(t)} \\ \widehat{\mathbf{W}}_t(\widehat{z},n,t) &= -\mathbf{W}_z(z,n)e^{\widehat{\underline{z}}(t)}m + \mathbf{W}(z,n)e^{\widehat{\underline{z}}(t)}m\end{aligned}$$

Since $z = \widehat{z} - \widehat{\underline{z}}(t)$, it follows from a change of variables that

$$\mathbf{g}(z,n) \;=\; \widehat{\mathbf{g}}(\widehat{z},n,t)$$

Substituting these observations into the value function (29), and imposing the BGP condition that $q = \widehat{q}(t)$, $u = \widehat{u}(t)$, $e = \widehat{e}(t)$ and $\widehat{r}(t) = e^{\widehat{\underline{z}}(t)}r$

$$\begin{aligned}\rho e^{\widehat{\underline{z}}(t)}\mathbf{W}(z,n) \;=\;& e^{\widehat{\underline{z}}(t)}\left(e^z - c\right)n + e^{\widehat{\underline{z}}(t)}k - e^{\widehat{\underline{z}}(t)}r - \mathbf{W}_z(z,n)e^{\widehat{\underline{z}}(t)}m + \mathbf{W}(z,n)e^{\widehat{\underline{z}}(t)}m \\ &- n\kappa e^{\widehat{\underline{z}}(t)}\mathbf{W}_n(z,n) - \kappa e^{\widehat{\underline{z}}(t)}\mathbf{E}(z,n) \\ &+ \max_v \left\{ -\frac{c_v}{1+\eta_v} e^z v^{1+\eta_v} + q\left(\frac{u}{S}\left(\mathbf{W}_n(z,n)e^{\widehat{\underline{z}}(t)} - Ue^{\widehat{\underline{z}}(t)}\right)^+ \right.\right. \\ &+ \left.\left. \frac{\phi e}{S}\int_0^\infty\int_0^\infty \left(\mathbf{W}_n(z,n)e^{\widehat{\underline{z}}(t)} - \mathbf{W}_n(z',n')e^{\widehat{\underline{z}}(t)}\right)^+ \mathbf{g}(z',n')dz'dn'\right)\right\}\end{aligned}$$

subject to

$$\mathbf{W}(z,n)e^{\widehat{\underline{z}}(t)} \;\geq\; nUe^{\widehat{\underline{z}}(t)} + U^f e^{\widehat{\underline{z}}(t)}, \quad \text{and} \quad \mathbf{W}_n(z,n)e^{\widehat{\underline{z}}(t)} \;\geq\; Ue^{\widehat{\underline{z}}(t)}$$

Cancelling the $e^{\widehat{\underline{z}}(t)}$ term on all sides and moving the $\mathbf{W}(z,n)m$ term to the other side gives (4)–(5). A straightforward exercise along the same lines yields the recursion characterizing the value of unemployment, $(\rho - m)U = b - \kappa U$, as well as the value of a prospective entrepreneur (7).

C.2 Proposition 1: Constant returns to scale

Guess that the joint value $\mathbf{W}(z,n)$ given by (8) can be written as

$$\mathbf{W}(z,n) \;=\; n(J(z) + U) + O(z) + U^f \tag{32}$$

where the surplus of a match $J(z)$ solves the stopping time problem

$$(\rho - m)\,J(z) \;=\; e^z - c - b - mJ'(z) - \kappa J(z) \tag{33}$$

subject to $J(\underline{z}^w) = 0$ and $J'(\underline{z}^w) = 0$, and the surplus of an entrepreneur $O(z)$ solves

$$\begin{aligned}(\rho - m)\,O(z) \;&=\; k - b - r - mO'(z) - \kappa O(z) - \max_s \left\{ s\pi \int_0^\infty O(\tilde{z})^+ d\Gamma(\tilde{z}) - \frac{c_e}{1+\eta_e} s^{1+\eta_e} \right\} \\ &+ \; \max_v \left\{ qv \left(\frac{u}{S} J(z) + \phi \frac{e}{S} \int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z} \right) - \frac{c_v}{1+\eta_v} e^z v^{1+\eta_v} \right\}\end{aligned} \tag{34}$$

subject to $O(\underline{z}) = 0$ and $O'(\underline{z}) = 0$, where $\underline{z} = 0$

Under the guess (32)

$$\begin{aligned}\mathbf{W}_n(z,n) \;&=\; J(z) + U \\ \mathbf{W}_z(z,n) \;&=\; nJ'(z) + O'(z) \\ \mathbf{E}(z,n) \;&=\; \mathbf{W}(z,0) \;=\; O(z) + U^f\end{aligned}$$

Using the guess in the value of a coalition (4)

$$
\begin{aligned}
(\rho-m)\mathbf{W}(z,n) &= \max_{v\geq 0}\Big\{(e^{z}-c)\,n+k-r-\kappa n\mathbf{W}_{n}(z,n)-\kappa\mathbf{E}(z,n)-m\mathbf{W}_{z}(z,n) \\
&+ qv\left(\frac{u}{S}\Big(\mathbf{W}_{n}(z,n)-U\Big)^{+}+\frac{e\phi}{S}\int\Big(\mathbf{W}_{n}(z,n)-\mathbf{W}_{n}(\tilde{z},\tilde{n})\Big)^{+}d\mathbf{G}(\tilde{z},\tilde{n})\right)-\frac{c_{v}e^{z}v^{1+\eta_{v}}}{1+\eta_{v}}\Big\} \\
(\rho-m)\mathbf{W}(z,n) &= \max_{v\geq 0}\Big\{(e^{z}-c)\,n+k-r-\kappa n\,(J(z)+U)-\kappa\left(O(z)+U^{f}\right)-m\left(nJ'(z)+O'(z)\right) \\
&+ qv\left(\frac{u}{S}\Big(J(z)+U-U\Big)^{+}+\frac{e\phi}{S}\int\Big(J(z)+U-J(\tilde{z})-U\Big)^{+}dG(\tilde{z})\right)-\frac{c_{v}e^{z}v^{1+\eta_{v}}}{1+\eta_{v}}\Big\} \\
(\rho-m)\mathbf{W}(z,n) &= \max_{v\geq 0}\Bigg\{\Bigg(\underbrace{e^{z}-c-b-\kappa J(z)-mJ'(z)}_{=(\rho-m)J(z)}\Bigg)n+\Bigg(\underbrace{b-\kappa U}_{=(\rho-m)U}\Bigg)n \\
&+ \underbrace{k-b-r-\kappa O(z)-mO'(z)+qv\left(\frac{u}{S}J(z)+\frac{e\phi}{S}\int^{z}J(z)-J(\tilde{z})dG(\tilde{z})\right)-\frac{c_{v}e^{z}v^{1+\eta_{v}}}{1+\eta_{v}}}_{=(\rho-m)O(z)}\Bigg\} \\
&+ \underbrace{b-\kappa U^{f}}_{=(\rho-m)U^{f}}
\end{aligned}
$$

which confirms the guess (32).

The surplus of a match is given by (33) for $z>\underline{z}^{w}$ and is $J(z)=0$ otherwise. Hence, for $z>\underline{z}^{w}$

$$
\mathbf{W}_{n}(z,n) \;=\; J(z)+U \;>\; U
$$

since $J(z)>0$, while for $z\leq\underline{z}^{w}$

$$
\mathbf{W}_{n}(z,n) \;=\; J(z)+U \;=\; U
$$

since $J(z)=0$. Hence the separation boundaries of the coalition and the match coincide.

The exit decision is the potentially problematic decision. Because the fixed cost can be split over many workers, size may matter for exit, thus requiring keeping track of size and stipulating a multilateral bargaining protocol. Under the assumption that $c+b=1$, however, matches want to separate to unemployment at the same time as the entrepreneur wants to exit, $\underline{z}^{w}=\underline{z}=0$, as I demonstrate below. Consequently, the exit threshold chosen by the entrepreneur trivially coincides with that which maximizes the joint value of a firm, since the firm consists of only the entrepreneur at the point of exit. The surplus of an entrepreneur is given by (34) for $z > 0$ and $O(z) = 0$ otherwise. Hence, for $z > 0$

$$\mathbf{W}(z,n) \;=\; n\Big(J(z)+U\Big)+O(z)+U^f \;>\; nU+U^f$$

since $J(z) > 0$ and $O(z) > 0$, while for $z \leq 0$

$$\mathbf{W}(z,n) \;=\; n\Big(J(z)+U\Big)+O(z)+U^f \;=\; nU+U^f$$

since $J(z) = 0$ and $O(z) = 0$. Hence, individual matches and the entrepreneur behave in a way that also maximizes coalition value, verifying that the joint surplus can be written as (32).

The value of unemployment U solves $(\rho - m)U = b - \kappa U$, which gives

$$U \;=\; \frac{b}{\rho+\kappa-m}$$

The surplus of a match $J(z)$ and the boundary $\underline{z}^w$ solve the stopping time problem (33)

$$\frac{dJ}{dz}+\frac{(\rho+\kappa-m)}{m}J \;=\; \frac{e^z-c-b}{m}$$

Multiply through by the integrating factor $\mu(z)$ which satisfies

$$\mu(z)\frac{(\rho+\kappa-m)}{m} \;=\; \mu'(z)$$

$$\begin{aligned}
\mu(z)\frac{dJ}{dz}+\mu(z)\frac{(\rho+\kappa-m)}{m}J &= \mu(z)\frac{e^z-c-b}{m} \\
\mu(z)\frac{dJ}{dz}+\mu'(z)J &= \mu(z)\frac{e^z-c-b}{m} \\
(\mu(z)J)' &= \mu(z)\frac{e^z-c-b}{m} \\
J(z) &= \frac{\int \mu(z)\frac{e^z-c-b}{m}dz+c}{\mu(z)}
\end{aligned}$$

Since

$$\begin{aligned}
\mu(z)\frac{(\rho+\kappa-m)}{m} &= \mu'(z) \\
\frac{\rho+\kappa-m}{m}dz &= \frac{1}{\mu}d\mu \\
\ln a + \frac{\rho+\kappa-m}{m}z &= \ln\mu(z) \\
\mu(z) &= Ae^{\frac{\rho+\kappa-m}{m}z}
\end{aligned}$$

Hence

$$\begin{aligned}
J(z) &= \frac{\int Ae^{\frac{\rho+\kappa-m}{m}z}\frac{e^z-c-b}{m}dz + a}{Ae^{\frac{\rho+\kappa-m}{m}z}} \\
&= \frac{1}{m}e^{-\frac{\rho+\kappa-m}{m}z}\int e^{\frac{\rho+\kappa}{m}z}dz - \frac{c+b}{m}e^{-\frac{\rho+\kappa-m}{m}z}\int e^{\frac{\rho+\kappa-m}{m}z}dz + \frac{a}{A}e^{-\frac{\rho+\kappa-m}{m}z} \\
&= \frac{1}{\rho+\kappa}e^z - \frac{c+b}{\rho+\kappa-m} + ae^{-\frac{\rho+\kappa-m}{m}z}
\end{aligned}$$

where with abuse of terminology, I redefined $a = \frac{a}{A}$. The derivative is

$$J'(z) = \frac{1}{\rho+\kappa}e^z - \frac{\rho+\kappa-m}{m}ae^{-\frac{\rho+\kappa-m}{m}z}$$

The optimal boundary $\underline{z}^w$ satisfies the value matching and smooth pasting conditions

$$\begin{aligned}
0 &= \frac{1}{\rho+\kappa}e^{\underline{z}^w} - \frac{c+b}{\rho+\kappa-m} + ae^{-\frac{\rho+\kappa-m}{m}\underline{z}^w} \\
0 &= \frac{1}{\rho+\kappa}e^{\underline{z}^w} - \frac{\rho+\kappa-m}{m}ae^{-\frac{\rho+\kappa-m}{m}\underline{z}^w} \\
-\frac{1}{\rho+\kappa}e^{\underline{z}^w} + \frac{c+b}{\rho+\kappa-m} &= ae^{-\frac{\rho+\kappa-m}{m}\underline{z}^w} \\
ae^{-\frac{\rho+\kappa-m}{m}z} &= \frac{m}{\rho+\kappa-m}\frac{1}{\rho+\kappa}e^{\underline{z}^w} \\
-\frac{1}{\rho+\kappa}e^{\underline{z}^w} + \frac{c+b}{\rho+\kappa-m} &= \frac{m}{\rho+\kappa-m}\frac{1}{\rho+\kappa}e^{\underline{z}^w} \\
\underline{z}^w &= \ln(c+b) = 0
\end{aligned}$$

Since by assumption 1, $c+b=1$. Then a is given by

$$a = \frac{1}{\rho+\kappa}\frac{m}{\rho+\kappa-m}$$

Hence

$$
\begin{aligned}
J(z) &= \frac{1}{\rho+\kappa}e^{z} - \frac{c+b}{\rho+\kappa-m} + \frac{1}{\rho+\kappa}\frac{m}{\rho+\kappa-m}e^{-\frac{\rho+\kappa-m}{m}z} \\
&= \frac{1}{\rho+\kappa-m}\left(\frac{m}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z}-1\right)+\frac{1}{\rho+\kappa}e^{z}
\end{aligned}
$$

Hence, the surplus of a match (8) together with $\underline{z}^{w}=0$ solve the stopping time problem of a match.

The first-order condition for optimal vacancy creation in (34) is

$$
\begin{aligned}
c_v e^{z} v(z)^{\eta_v} &= q\left(\frac{u}{S}J(z)+\phi\frac{e}{S}\int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z}\right) \\
v(z) &= \left(\frac{R(z)}{c_v e^{z}}\right)^{\frac{1}{\eta_v}} \qquad (35)
\end{aligned}
$$

where

$$
R(z) = q\left(\frac{u}{S}J(z)+\phi\frac{e}{S}\int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z}\right)
$$

Since $v(z)=0$ for $z\leq 0$, imposing the boundary conditions $O(\underline{z})=0$ and $O'(\underline{z})=0$ in (34)

$$
0 = k-b-r-\max_s\left\{s\pi\int_0^\infty O(\tilde{z})^{+}d\Gamma(\tilde{z})-\frac{c_e}{1+\eta_e}s^{1+\eta_e}\right\} \qquad (36)
$$

Substituting (35) and (36) into (34)

$$
\begin{aligned}
(\rho-m)\,O(z) &= -mO'(z)-\kappa O(z)+\left(\frac{R(z)}{c_v e^{z}}\right)^{\frac{1}{\eta_v}}R(z)-\frac{c_v}{1+\eta_v}e^{z}\left(\frac{R(z)}{c_v e^{z}}\right)^{\frac{1+\eta_v}{\eta_v}} \\
(\rho+\kappa-m)\,O(z) &= -mO'(z)+\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}}e^{-z\frac{1}{\eta_v}}R(z)^{\frac{1+\eta_v}{\eta_v}}-\frac{1}{1+\eta_v}R(z)^{\frac{1+\eta_v}{\eta_v}}\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}}e^{-\frac{1}{\eta_v}} \\
&= -mO'(z)+\frac{\eta_v}{1+\eta_v}\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}}e^{-z\frac{1}{\eta_v}}R(z)^{\frac{1+\eta_v}{\eta_v}} \qquad (37)
\end{aligned}
$$

Proceeding in an identical fashion as above to find the integrating factor, the solution that satisfies the initial value conditions $O(0)=0$ and $O'(0)=0$ is

$$
O(z) = \frac{\eta_v}{1+\eta_v}\left(\frac{1}{c_v}\right)^{\frac{1}{\eta_v}}\frac{1}{m}e^{-\frac{\rho+\kappa-m}{m}z}\int_0^z e^{\frac{\rho+\kappa-m}{m}\tilde{z}-\frac{1}{\eta_v}\tilde{z}}R(\tilde{z})^{\frac{1+\eta_v}{\eta_v}}d\tilde{z}
$$

with $\underline{z}=0$.

The optimal search intensity is given by the first-order condition

$$s = \left(\frac{\pi \int_0^\infty O(\tilde{z}) d\Gamma(\tilde{z})}{c_e} \right)^{\frac{1}{\eta_e}} \tag{38}$$

An entrepreneur with productivity $\underline{z} = 0$ cannot gain from attempting to recruit. Consequently, $v(0) = 0$. Moreover, by construction such an entrepreneur is indifferent between keeping their firm alive and exiting, $O(0) = 0$. Imposing these observations in the surplus of an entrepreneur (34)

$$0 = k - b - r - \frac{\eta_e}{1+\eta_e} c_e s^{1+\eta_e}$$

As long as the equilibrium fixed cost is positive, providers of it prefer to provide it, such that $L = l$.

C.3 Derivation of the FP equation for entrepreneurs

The number of entrepreneurs at each point in the distribution, $\widehat{x}(z,t)$, is the share of entrepreneurs with relative productivity z, $x(z)$—which is constant on the BGP—times the overall number of entrepreneurs $\hat{L}(t) = le^{\lambda t}$, which must grow at the rate of labor supply on the BGB. Hence

$$\widehat{x}(z,t) = x(z)\widehat{L}(t) = x(z)le^{\lambda t}$$

Hence

$$\begin{aligned} \widehat{x}_t(z,t) &= x(z)l\lambda e^{\lambda t} \\ \widehat{x}_z(z,t) &= x'(z)le^{\lambda t} \end{aligned}$$

Substituting these observations as well as the fact that $\widehat{y}(t) = ye^{\lambda t}$ on the BGP into (13)

$$\begin{aligned} \widehat{x}_t(z,t) &= m\widehat{x}_z(z,t) - \kappa\widehat{x}(z,t) + \kappa(1+(1-\omega)\nu)\widehat{x}(z,t) + \widehat{y}(t)\gamma(z) \\ x(z)l\lambda e^{\lambda t} &= mx'(z)le^{\lambda t} - \kappa x(z)le^{\lambda t} + \kappa(1+(1-\omega)\nu)x(z)le^{\lambda t} + ye^{\lambda t}\gamma(z) \\ x(z)l\lambda &= mx'(z)l - \kappa x(z)l + \kappa(1+(1-\omega)\nu)x(z)l + y\gamma(z) \end{aligned}$$

Recall that labor supply growth equals $\lambda = \kappa(1-\omega)\nu$. Imposing this

$$\begin{aligned} x(z)l\lambda &= mx'(z)l - \kappa x(z)l + (\kappa+\lambda)x(z)l + y\gamma(z) \\ 0 &= mx'(z) + \frac{y}{l}\gamma(z) \end{aligned}$$

C.4 Derivation of the FP equation for employment

Let $\widehat{g}(z,t)$ denote the number of workers employed in firms with relative productivity z at time t (suppressing the dependence on m and λ to reduce clutter). Its evolution is characterized by the FP equation

$$\begin{aligned} \widehat{g}_t(z,t) = {} & \underbrace{m\widehat{g}_z(z,t)}_{\text{technological obsolescence}} - \underbrace{\kappa\widehat{g}(z,t)}_{\text{retirement of worker}} - \underbrace{\phi\widehat{p}(t)\left(1-\widehat{F}(z,t)\right)\widehat{g}(z,t)}_{\text{separations up the job ladder}} \\ & + \widehat{p}(t)\widehat{f}(z,t)\left(\underbrace{\widehat{u}(t)}_{\text{hires from unemployment}} + \underbrace{\phi\widehat{G}(z,t)}_{\text{hires from below in the job ladder}}\right) \end{aligned} \tag{39}$$

where $\widehat{f}(z,t)$ is the distribution of recruiting firms at time t

$$\widehat{f}(z,t) = \frac{1}{\widehat{V}(t)}\widehat{v}(z,t)\widehat{x}(z,t), \qquad \widehat{V}(t) = \int_0^\infty \widehat{v}(z,t)\widehat{x}(z,t)dz$$

On the BGP, $\widehat{v}(z,t) = v(z)$ and $\widehat{x}(z,t) = x(z)L(t)$ such that $\widehat{f}(z,t) = f(z)$ is stationary. Moreover, the job finding rate is constant,

$$\widehat{p}(t) = \chi\widehat{S}(t)^{1-\theta}\widehat{V}(t)^{\theta-1} = \chi\left(Se^{\lambda t}\right)^{1-\theta}\left(Ve^{\lambda t}\right)^{\theta-1} = \chi S^{1-\theta}V^{\theta-1}$$

where S is aggregate search efficiency per worker and V is aggregate vacancies per worker. Finally, the number of unemployed workers is $\widehat{u}(t) = ue^{\lambda t}$, the number of workers in firms with relative productivity z is $\widehat{g}(z,t) = g(z)(1-u)e^{\lambda t}$, and the number of workers in firms with at most relative productivity z is $\widehat{G}(z,t) = G(z)(1-u)e^{\lambda t}$. Hence

$$\begin{aligned} \widehat{g}_t(z,t) &= g(z)(1-u)\lambda e^{\lambda t} \\ \widehat{g}_z(z,t) &= g'(z)(1-u)e^{\lambda t} \end{aligned}$$

Imposing these conditions in (39)

$$
\begin{aligned}
g(z)(1-u)\lambda e^{\lambda t} &= mg'(z)(1-u)e^{\lambda t} - \kappa g(z)(1-u)e^{\lambda t} \\
&- \phi p\Big(1-F(z)\Big)g(z)(1-u)e^{\lambda t} + pf(z)\Big(ue^{\lambda t} + \phi G(z)(1-u)e^{\lambda t}\Big) \\
\Big(\lambda+\kappa\Big)g(z) &= mg'(z) - \phi p\Big(1-F(z)\Big)g(z) + pf(z)\left(\frac{u}{1-u} + \phi G(z)\right)
\end{aligned}
$$

Note first that $\lim_{z\to\infty} G(z) = 1$ implies $\lim_{z\to\infty} g(z) = 0$. Integrating (15) from zero to infinity

$$
\kappa+\lambda = -mg(0) - \phi p\int_0^\infty \Big(1-F(z)\Big)g(z)dz + p\frac{u}{1-u} + \phi p\int_0^\infty f(z)G(z)dz
$$

Integrating the third term by parts and cancelling terms

$$
\begin{aligned}
\kappa+\lambda &= -mg(0) - \phi p\Big(1-\int_0^\infty F(z)g(z)dz\Big) + p\frac{u}{1-u} \\
&+ \phi p\left(1-\int_0^\infty F(z)g(z)dz\right) \\
\kappa+\lambda &= -mg(0) + p\frac{u}{1-u}
\end{aligned}
$$

Solving this for u gives

$$
u = \frac{\kappa+\lambda+mg(0)}{p+\kappa+\lambda+mg(0)}
$$

Integrating the second-order ODE (15) from 0 to z and using the boundary condition that $G(0)=0$

$$
\Big(\lambda+\kappa\Big)G(z) = m\Big(g(z)-g(0)\Big) - \phi p\int_0^z \Big(1-F(\tilde{z})\Big)g(\tilde{z})d\tilde{z} + pF(z)\frac{u}{1-u} + \phi p\int_0^z f(\tilde{z})G(\tilde{z})d\tilde{z}
$$

Integrating by parts, again using the fact that $G(0)=0$

$$
\begin{aligned}
\Big(\lambda+\kappa\Big)G(z) &= m\Big(g(z)-g(0)\Big) + pF(z)\frac{u}{1-u} \\
&- \phi p\left(\Big(1-F(z)\Big)G(z) + \int_0^z f(\tilde{z})G(\tilde{z})d\tilde{z} - \int_0^z f(\tilde{z})G(\tilde{z})d\tilde{z}\right) \\
\Big(\lambda+\kappa+\phi p(1-F(z))\Big)G(z) &= mg(z) - mg(0) + pF(z)\frac{u}{1-u}
\end{aligned}
$$

Finally, using (16) to substitute for $u/(1-u)$ gives the linear first-order ODE

$$
\frac{m}{\kappa+\lambda}g(z) - \Big(1+\beta(1-F(z))\Big)G(z) = \frac{m}{\kappa+\lambda}g(0)(1-F(z)) - F(z) \tag{40}
$$

subject to $G(0) = 0$, where $g(0)$ is such that $\lim_{z\to\infty} G(z) = 1$ and

$$\beta \equiv \frac{\phi p}{\kappa + \lambda}$$

C.5 Proposition 2: The exit curve

Recall the boundary value problem (14)

$$0 = mx'(z) + \frac{y}{l}\zeta e^{-\zeta z}$$

subject to the boundary values

$$\begin{aligned} X(0) &= 0 \\ \lim_{z\to\infty} X(z) &= 1 \end{aligned}$$

Integrating the ODE (14) from 0 to z

$$x(z) = \frac{y}{ml}e^{-\zeta z} + c_1 \tag{41}$$

Integrating this again

$$X(z) = c_1 z + c_2 - \frac{y}{ml\zeta}e^{-\zeta z}$$

The first boundary value requires that

$$\begin{aligned} X(0) &= c_2 - \frac{y}{ml\zeta} = 0 \\ c_2 &= \frac{y}{ml\zeta} \end{aligned}$$

The second boundary value requires that

$$\begin{aligned} \lim_{z\to\infty} X(z) &= \lim_{z\to\infty}\left(c_1 z + \frac{y}{ml\zeta}\left(1 - e^{-\zeta z}\right)\right) = 1 \\ 1 &= \lim_{z\to\infty}\left(c_1 z + \frac{y}{ml\zeta}\right) \\ c_1 &= 0 \\ y &= ml\zeta \end{aligned}$$

Combining these insights

$$
\begin{aligned}
X(z) &= \frac{y}{\zeta m l}\left(1 - e^{-\zeta z}\right) \\
X(z) &= 1 - e^{-\zeta z} \\
x(z) &= \zeta e^{-\zeta z}
\end{aligned}
$$

C.6 Derivation of the entry curve (20)

The aggregate entry rate is the product of the arrival rate of business ideas per unit of search intensity (π), the number of prospective entrepreneurs ($\xi - L$), and the rate at which prospective entrepreneurs search for ideas

$$
y(m) = \pi(\xi - L)s
$$

Substituting the optimally chosen search intensity (11) and simplifying gives (20).

C.7 Proposition 3: Employment distribution

When the offer distribution is exponential, (40) becomes (omitting the dependence on m to reduce clutter)

$$
g(z) \underbrace{- \frac{\kappa + \lambda + \phi p e^{-\zeta z}}{m}}_{\equiv x(z)} G(z) = \underbrace{- \frac{\kappa + \lambda - \frac{p u e^{-\zeta z}}{1-u}}{m}}_{\equiv y(x)}
$$

Solve first the homogenous equation

$$
\begin{aligned}
g(z) + x(z)G(z) &= 0 \\
\frac{g(z)}{G(z)} &= -x(z) \\
\log G(z) &= \log C + \int_0^z -x(z)dz \\
G(z) &= C \underbrace{e^{-\int_0^z x(\tilde{z})d\tilde{z}}}_{\equiv h(z)} \\
h'(z) &= C e^{-\int_0^z x(\tilde{z})d\tilde{z}}(-x(z)) \\
&= -h(z)x(z)
\end{aligned}
$$

For the non-homogenous equation, apply variation of parameters

$$\begin{aligned}
\tilde{G}(z) &= v(z)h(z) \\
\tilde{g}(z) &= v'(z)h(z) + v(z)h'(z) \\
\tilde{g}(z) &= v'(z)h(z) - v(z)h(z)x(z) \\
\tilde{g}(z) &= v'(z)h(z) - \tilde{G}(z)x(z) \\
\tilde{g}(z) + x(z)\tilde{G}(z) &= v'(z)h(z)
\end{aligned}$$

For $\tilde{G}(z)$ to solve the non-homogenous equation, we need

$$\begin{aligned}
\tilde{g}(z) + x(z)\tilde{G}(z) &= y(x) \\
v'(z)h(z) &= y(x) \\
v'(z) &= \frac{y(x)}{h(z)} \\
v(z) &= K + \int_0^z \frac{y(\tilde{z})}{h(\tilde{z})} d\tilde{z}
\end{aligned}$$

Hence, the general solution is

$$G(z) = \left(K + \int_0^z \frac{y(\tilde{z})}{h(\tilde{z})} d\tilde{z} \right) e^{\frac{\kappa+\lambda}{m} z + \frac{\phi p}{m\zeta}\left(1-e^{-\zeta z}\right)}$$

Since the initial value condition $G(0) = 0$ requires $K = 0$

$$\begin{aligned}
G(z) &= e^{\frac{\kappa+\lambda}{m} z + \frac{\phi p}{m\zeta}\left(1-e^{-\zeta z}\right)} \int_0^z \frac{-\frac{\kappa+\lambda-\frac{pue^{-\zeta\tilde{z}}}{1-u}}{m}}{e^{\frac{\kappa+\lambda}{m}\tilde{z} + \frac{\phi p}{m\zeta}(1-e^{-\zeta\tilde{z}})}} d\tilde{z} \\
&= \frac{1}{m} \int_0^z e^{\frac{\phi p}{m\zeta} e^{-\zeta\tilde{z}} - \frac{\kappa+\lambda}{m}\tilde{z} - \left(\frac{\phi p}{m\zeta} e^{-\zeta z} - \frac{\kappa+\lambda}{m} z\right)} \left(\frac{pue^{-\zeta\tilde{z}}}{1-u} - (\kappa+\lambda) \right) d\tilde{z}
\end{aligned}$$

where u is such that

$$\begin{aligned}
\lim_{z\to\infty} G(z) &= 1 \\
\frac{1}{m} \lim_{z\to\infty} e^{-\frac{\phi p e^{-\zeta z}}{m\zeta} + \frac{\kappa+\lambda}{m} z} \int_0^z e^{\frac{\phi p e^{-\zeta\tilde{z}}}{m\zeta} - \frac{\kappa+\lambda}{m}\tilde{z} - \zeta\tilde{z}} \left(\frac{pu}{1-u} - (\kappa+\lambda)e^{\zeta\tilde{z}} \right) d\tilde{z} &= 1 \qquad (42)
\end{aligned}$$

Is there always a $\frac{u}{1-u}$ such that (42) holds? Consider the limit

$$\frac{1}{m}\lim_{z\to\infty}\underbrace{e^{-\frac{\phi pe^{-\zeta z}}{m\zeta}+\frac{\kappa+\lambda}{m}z}}_{\equiv h(z)}\underbrace{\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z}}_{\equiv A(z)}$$

Since $\lim_{z\to\infty}h(z)=\infty$, such a u must at the very least ensure that

$$\begin{aligned}
\lim_{z\to\infty}A(z) &= 0\\
\lim_{z\to\infty}\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z} &= 0\\
\frac{pu}{1-u}\lim_{z\to\infty}\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}d\tilde{z} &= (\kappa+\lambda)\lim_{z\to\infty}\int_0^z e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}}d\tilde{z}\\
\frac{pu}{1-u} &= (\kappa+\lambda)\frac{\int_0^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}}d\tilde{z}}{\int_0^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}d\tilde{z}}
\end{aligned}$$

Not only, however, does u have to be such that $\lim_{z\to\infty}A(z)=0$, it must be such that $A(z)$ goes to zero as exactly the rate that $h(z)$ goes to infinity as $z\to\infty$, such that the two forces offset. Moreover, the product must equal exactly m, such that $\lim_{z\to\infty}\frac{1}{m}h(z)A(z)=1$. To see that this choice of u also guarantees that these more stringent conditions hold, note that with some abuse of terminology, we can write

$$A(z) = A(\infty)-\underbrace{\int_z^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z}}_{\equiv a(z)}$$

where under our particular choice of u

$$A(\infty) = 0$$

Consider the behavior of

$$a(z) = \int_z^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}-\zeta\tilde{z}}\left(\frac{pu}{1-u}-(\kappa+\lambda)e^{\zeta\tilde{z}}\right)d\tilde{z}$$

as $z\to\infty$. For sufficiently large z, the second term in the parenthesis dominates the first, such that $a(z)$ is well-approximated as

$$a(z) \approx -(\kappa+\lambda)\int_z^\infty e^{\frac{\phi pe^{-\zeta\tilde{z}}}{m\zeta}-\frac{\kappa+\lambda}{m}\tilde{z}}d\tilde{z}$$

Moreover, for sufficiently large z, the contribution of the $\frac{\phi p e^{-\zeta \tilde{z}}}{m\zeta}$ term becomes small, so that $a(z)$ can be well-approximated as

$$a(z) \approx -(\kappa+\lambda)\int_z^\infty e^{-\frac{\kappa+\lambda}{m}\tilde{z}}d\tilde{z} = -(\kappa+\lambda)\left[-\frac{m}{\kappa+\lambda}e^{-\frac{\kappa+\lambda}{m}\tilde{z}}\right]_{\tilde{z}=z}^{\infty} = -me^{-\frac{\kappa+\lambda}{m}z}$$

Combining these insights, the product $\lim_{z\to\infty}\frac{1}{m}h(z)A(z)$ is hence

$$\frac{1}{m}\lim_{z\to\infty}h(z)\left(\underbrace{A(\infty)}_{\equiv 0}-A(z)\right) = \frac{1}{m}\lim_{z\to\infty}e^{-\frac{\phi p e^{-\zeta z}}{m\zeta}+\frac{\kappa+\lambda}{m}z}me^{-\frac{\kappa+\lambda}{m}z} = 1$$

Hence, the choice of u (42) ensures that $\lim_{z\to\infty}G(z)=1$.

C.8 Proposition 4: Existence and uniqueness of the equilibrium

The analysis above has established that for a given growth rate $m \in (0, \rho+\kappa)$, there exists a unique surplus of a match, distribution of entrepreneurs and distribution of workers. Hence, to show that there exists at least one equilibrium, it suffices to show that the exit curve (19) and entry curve (20) cross at least once for $m \in (0, \rho+\kappa)$.

The limits of the exit curve (19) are

$$\lim_{m\to 0}\tilde{y}(m) = 0$$
$$\lim_{m\to\rho+\kappa}\tilde{y}(m) = \zeta L(\rho+\kappa) > 0$$

When $\eta_v \to \infty$ and $\theta \to 0$, the return to hiring (45) simplifies to

$$R(z;m) = \frac{\chi}{L}\left(u(m)J(z;m)+\phi\Big(1-u(m)\Big)\int_0^z J'(\tilde{z};m)G(\tilde{z};m)d\tilde{z}\right)$$

Since $J(z;m)$ and $J'(z;m)$ are both strictly positive and bounded for $z\in(0,\infty)$ and $m\in[0,\rho+\kappa]$, $u(m)\in(0,1)$ and $G(z;m)\in[0,1]$, it follows that $R(z;m)$ is strictly positive and bounded for $m\in[0,\rho+\kappa]$ and $z\in(0,\infty)$.

When $\eta_v\to\infty$, the entry curve (20) reduces to

$$y(m) = \pi(\xi-L)\left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}}\left(\frac{1}{m}\int_0^\infty\int_0^{\tilde{z}}e^{-\frac{\rho+\kappa-m}{m}(\tilde{z}-z)}R(z;m)dzd\Gamma(\tilde{z})\right)^{\frac{1}{\eta_e}}$$

Since $R(z;m)$ is strictly positive and bounded for $z\in(0,\infty)$ and by assumption the tail of the innovation distribution is sufficiently thin to ensure that expectations are finite, the entry rate is strictly positive and bounded for $m \in (0, \rho + \kappa)$. In particular,

$$\begin{aligned} \lim_{m \to 0} y(m) &= \pi(\xi - L) \left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}} \left(\frac{1}{\rho + \kappa} \int_0^\infty \lim_{m \to 0} R(z; m) d\Gamma(z)\right)^{\frac{1}{\eta_e}} \in (0, \infty) \\ \lim_{m \to \rho + \kappa} y(m) &= \pi(\xi - L) \left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}} \left(\frac{1}{\rho + \kappa} \int_0^\infty \int_0^z \lim_{m \to \rho + \kappa} R(\tilde{z}; m) d\tilde{z} d\Gamma(z)\right)^{\frac{1}{\eta_e}} \in (0, \infty) \end{aligned}$$

As $\pi \to 0$ and/or $\xi \to L$, $\lim_{m \to \rho + \kappa} y(m)$ becomes arbitrarily small (but strictly positive).

It follows from the observations above that $\lim_{m \to 0} y(m) > \lim_{m \to 0} \tilde{y}(m) = 0$ and that, for sufficiently small π and/or ξ sufficiently close to L, $\lim_{m \to \rho + \kappa} y(m) < \lim_{m \to \rho + \kappa} \tilde{y}(m) = \zeta L(\rho + \kappa)$. It follows from continuity that the exit and entry curves intersect at least once in $m \in (0, \rho + \kappa)$.

In the limit $\lim_{\eta_e \to \infty} y(m) = \pi(\xi - L)$ for all $m \in (0, \rho + \kappa)$. That is, the entry curve (20) becomes arbitrarily flat at a strictly positive number that is independent of any equilibrium objects. As long as

$$\lim_{m \to 0} \tilde{y}(m) = 0 < \pi(\xi - L) < \lim_{m \to \rho + \kappa} \tilde{y}(m) = \zeta L(\rho + \kappa)$$

there is a unique intersection between the exit and entry curves. It follows from continuity of the entry curve in η_e that a unique intersection exists for sufficiently high η_e.

C.9 Lemma 3: Obsolescence vs capitalization effects

Differentiating the surplus of a match (8) with respect to m, for all $m \in [0, \rho + \kappa)$, gives

$$\begin{aligned} \frac{\partial J(z; m)}{\partial m} &= -\frac{e^{-\frac{\rho + \kappa - m}{m} z}}{(\rho + \kappa - m)^2} \left(e^{\frac{\rho + \kappa - m}{m} z} - \left(1 + z\frac{\rho + \kappa - m}{m}\right)\right) \\ &= -\frac{e^{-\frac{\rho + \kappa - m}{m} z}}{(\rho + \kappa - m)^2} \left(1 + \frac{\rho + \kappa - m}{m} z + \frac{\left(\frac{\rho + \kappa - m}{m} z\right)^2}{2!} + \cdots - \left(1 + z\frac{\rho + \kappa - m}{m}\right)\right) \\ &= -\frac{e^{-\frac{\rho + \kappa - m}{m} z}}{(\rho + \kappa - m)^2} \left(\frac{\left(\frac{\rho + \kappa - m}{m} z\right)^2}{2!} + \frac{\left(\frac{\rho + \kappa - m}{m} z\right)^3}{3!} + \cdots\right) < 0 \end{aligned}$$

For utility to be finite requires $\rho + \kappa > m$, which I assume holds. Hence, $\partial J(z; m)/\partial m < 0$ for all $z > 0$.

Differentiating the surplus of a match (8), the marginal surplus of a match writes

$$J'(z; m) = \frac{1}{\rho + \kappa} e^z \left(1 - e^{-\frac{\rho + \kappa}{m} z}\right) \tag{43}$$

Differentiating the marginal surplus of a match (43) with respect to m

$$\frac{\partial J'(z;m)}{\partial m} = -\frac{1}{m^2}e^{-\frac{\rho+\kappa-m}{m}z}z \tag{44}$$

which clearly is negative for all $z > 0$.

Define the return to hiring as

$$R(z;m) = q\left(\frac{u(m)}{S(m)}J(z;m) + \phi\frac{1-u(m)}{S(m)}\int_0^z J'(\tilde{z};m)G(\tilde{z};m)d\tilde{z}\right)$$

$$R(z;m) = \frac{\chi}{L}\left(u(m)J(z;m) + \phi\Big(1-u(m)\Big)\int_0^z J'(\tilde{z};m)G(\tilde{z};m)d\tilde{z}\right) \tag{45}$$

since the worker finding rate is $q = \chi S(m)/L$ under the assumption that $\eta_v \to \infty$ and $\theta \to 0$. Differentiating with respect to the growth rate m holding fixed the composition of the labor force

$$\left.\frac{\partial R(z;m)}{\partial m}\right|_{u,G(z)\text{ fixed}} = \frac{\chi}{L}\left(u(m)\frac{\partial J(z;m)}{\partial m} + \phi\Big(1-u(m)\Big)\int_0^z \frac{\partial J'(\tilde{z};m)}{\partial m}G(\tilde{z};m)d\tilde{z}\right) \tag{46}$$

For $m \ll \kappa+\lambda$ and $m \ll \kappa+\rho$, to a first order

$$J(z;m) \approx \frac{1}{\rho+\kappa}e^z \tag{47}$$

$$J'(z;m) \approx \frac{1}{\rho+\kappa}e^z \tag{48}$$

$$\frac{\partial J(z;m)}{\partial m} \approx -\frac{1}{(\rho+\kappa)^2} \tag{49}$$

$$\frac{\partial J'(z;m)}{\partial m} \approx 0 \tag{50}$$

$$u(m) \approx \frac{\kappa+\lambda}{\kappa+\lambda+p} \tag{51}$$

$$G(z;m) \approx \frac{(\kappa+\lambda)\left(1-e^{-\zeta z}\right)}{\kappa+\lambda+\phi p e^{-\zeta z}} \tag{52}$$

$$g(z;m) \approx \frac{(\kappa+\lambda+\phi p)(\kappa+\lambda)}{\left(\kappa+\lambda+\phi p e^{-\zeta z}\right)^2}\zeta e^{-\zeta z} \tag{53}$$

Using (49)–(52) in (46)

$$\left.\frac{\partial R(z;m)}{\partial m}\right|_{u,G(z)\text{ fixed}} \approx -\frac{\chi}{L}\frac{\kappa+\lambda}{\kappa+\lambda+p}\frac{1}{(\rho+\kappa)^2} < 0 \tag{54}$$

Imposing $\eta_v \to \infty$ in the ODE characterizing the surplus of an entrepreneur (37)

$$(\rho + \kappa - m)\, O(z) \quad = \quad -mO'(z) + R(z; m) \tag{55}$$

For $m \ll \kappa + \lambda$ and $m \ll \kappa + \rho$, guess that $O(z; m)$ can be well-approximated as

$$O(z; m) \quad = \quad O_0(z; m) + mO_1(z; m) + O(m^2)$$

where $O(m^2)$ are second and higher order terms in m. Then

$$O'(z; m) \quad = \quad O_0'(z; m) + mO_1'(z; m) + O(m^2)$$

Substituting this into the ODE (55) and collecting terms verifies the guess, with

$$\begin{aligned} O_0(z; m) \quad &= \quad \frac{R(z; m)}{\rho + \kappa} \\ O_1(z; m) \quad &= \quad \frac{1}{\rho + \kappa} \left(\frac{R(z; m)}{\rho + \kappa} - \frac{R'(z; m)}{\rho + \kappa} \right) \end{aligned}$$

Hence for $m \ll \kappa + \lambda$ and $m \ll \kappa + \rho$, the surplus of an entrepreneur (55) can be approximated to a first-order as

$$O(z; m) \quad \approx \quad \frac{R(z; m)}{\rho + \kappa} + \frac{m}{\rho + \kappa} \left(\frac{R(z; m)}{\rho + \kappa} - \frac{R'(z; m)}{\rho + \kappa} \right)$$

Differentiating with respect to m (and dropping the dependence on m to reduce clutter)

$$\frac{\partial O(z)}{\partial m} \quad = \quad \frac{1}{\rho + \kappa} \left(\frac{1}{\rho + \kappa} \left(R(z) - R'(z) \right) + \frac{\partial R(z)}{\partial m} \right) \tag{56}$$

The derivative of the return to hiring (45) is

$$R'(z) \quad = \quad \frac{\chi}{L} \Big(uJ'(z) + \phi(1 - u) J'(z) G(z) \Big) \tag{57}$$

Consequently,

$$R(z) - R'(z) \quad = \quad \frac{\chi}{L} \left(uJ(z) + \phi(1 - u) \int_0^z J'(\tilde{z}) G(\tilde{z}) d\tilde{z} - uJ'(z) - \phi(1 - u) J'(z) G(z) \right)$$

Using (47)–(48) to substitute for the surplus and marginal surplus of a match, and simplifying

$$R(z) - R'(z) \quad = \quad \frac{\chi}{L}\phi(1-u)\left(\int_0^z J'(\tilde{z})G(\tilde{z})d\tilde{z} - J'(z)G(z)\right)$$

Integrating by parts

$$R(z) - R'(z) \quad = \quad \frac{\chi}{L}\phi(1-u)\left(J(z)G(z) - \int_0^z J(\tilde{z})g(\tilde{z})d\tilde{z} - J'(z)G(z)\right)$$

and again using (47)–(48) as well as (51) and (53)

$$\begin{aligned} R(z) - R'(z) &= -\frac{\chi}{L}\frac{\phi p}{\kappa+\lambda+p}\int_0^z \frac{e^{\tilde{z}}}{\rho+\kappa}\frac{(\kappa+\lambda+\phi p)(\kappa+\lambda)}{(\kappa+\lambda+\phi p e^{-\zeta\tilde{z}})^2}\zeta e^{-\zeta\tilde{z}}d\tilde{z} \\ &= -\frac{\chi}{L}\frac{\beta(1+\beta)}{1+\frac{p}{\kappa+\lambda}}\frac{\zeta}{\rho+\kappa}\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z} \;<\; 0 \end{aligned}$$

Substituting this into (56)

$$\frac{\partial O(z)}{\partial m} \quad = \quad \frac{1}{\rho+\kappa}\left(-\frac{\zeta}{(\rho+\kappa)^2}\frac{\chi}{L}\frac{\beta(1+\beta)}{1+\frac{p}{\kappa+\lambda}}\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z} + \frac{\partial R(z)}{\partial m}\right) \tag{58}$$

Holding composition fixed and using (54), we hence have

$$\begin{aligned} \left.\frac{\partial O(z)}{\partial m}\right|_{u,S,G(z)\text{ fixed}} &= \frac{1}{\rho+\kappa}\left(-\frac{\zeta}{(\rho+\kappa)^2}\frac{\chi}{L}\frac{\beta(1+\beta)}{1+\frac{p}{\kappa+\lambda}}\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z} - \frac{\chi}{L}\frac{1}{1+\frac{p}{\kappa+\lambda}}\frac{1}{(\rho+\kappa)^2}\right) \\ &= -\frac{1}{(\rho+\kappa)^3}\frac{\chi}{L}\frac{1}{1+\frac{p}{\kappa+\lambda}}\left(\zeta\beta(1+\beta)\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z} + 1\right) \\ &< 0 \end{aligned}$$

The slope of the entry curve is

$$y'(m)\big|_{u,S,G(z)\text{ fixed}} \quad = \quad \pi(\xi - l)\left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}}\frac{1}{\eta_e}\left(\int_0^\infty O(z;m)d\Gamma(z)\right)^{\frac{1}{\eta_e}-1}\int_0^\infty \left.\frac{\partial O(z;m)}{\partial m}\right|_{u,S,G(z)\text{ fixed}} d\Gamma(z) \;<\; 0$$

C.10 Lemma 4: The misallocation effect

Suppose that for small $\frac{m}{\kappa+\lambda}$, $G(z)$ can be well-approximated as

$$G(z) = \frac{1}{1+\beta e^{-\zeta z}}\left(1-e^{-\zeta z}\right)+\frac{m}{\kappa+\lambda}\frac{1+\beta}{1+\beta e^{-\zeta z}}\zeta e^{-\zeta z}\left(\frac{1}{(1+\beta e^{-\zeta z})^2}-\frac{1}{(1+\beta)^2}\right) + O\left(\left(\frac{m}{\kappa+\lambda}\right)^2\right) \tag{59}$$

where $O\left(\left(\frac{m}{\kappa+\lambda}\right)^2\right)$ are second and higher-order terms in $\frac{m}{\kappa+\lambda}$. Then

$$g(z) = \frac{1+\beta}{(1+\beta e^{-\zeta z})^2}\zeta e^{-\zeta z}+O\left(\left(\frac{m}{\kappa+\lambda}\right)^1\right)$$

where $O\left(\left(\frac{m}{\kappa+\lambda}\right)^1\right)$ are first and higher-order terms in m. Under the guess

$$\begin{aligned} g(z)-g(0)e^{-\zeta z} &= \frac{1+\beta}{(1+\beta e^{-\zeta z})^2}\zeta e^{-\zeta z}+O\left(\left(\frac{m}{\kappa+\lambda}\right)^1\right)-e^{-\zeta z}\left(\frac{1+\beta}{(1+\beta)^2}\zeta+O\left(\left(\frac{m}{\kappa+\lambda}\right)^1\right)\right) \\ &= (1+\beta)\,\zeta e^{-\zeta z}\left(\frac{1}{(1+\beta e^{-\zeta z})^2}-\frac{1}{(1+\beta)^2}\right)+O\left(\left(\frac{m}{\kappa+\lambda}\right)^1\right) \end{aligned}$$

Furthermore

$$\begin{aligned} &(\kappa+\lambda)\left(1+\beta e^{-\zeta z}\right)G(z) \\ = \ &(\kappa+\lambda)\left(1-e^{-\zeta z}+\frac{m}{\kappa+\lambda}(1+\beta)\,\zeta e^{-\zeta z}\left(\frac{1}{(1+\beta e^{-\zeta z})^2}-\frac{1}{(1+\beta)^2}\right)\right)+O\left(\left(\frac{m}{\kappa+\lambda}\right)^2\right) \end{aligned}$$

Substituting this into (40)

$$\begin{aligned} m\left(\zeta e^{-\zeta z}\left(\frac{1+\beta}{(1+\beta e^{-\zeta z})^2}-\frac{1+\beta}{(1+\beta)^2}\right)\right)+O\left(\left(\frac{m}{\kappa+\lambda}\right)^2\right) &= (\kappa+\lambda)\left(1-e^{-\zeta z}\right) \\ &+ m\zeta e^{-\zeta z}\left(\frac{1+\beta}{(1+\beta e^{-\zeta z})^2}-\frac{1+\beta}{(1+\beta)^2}\right) \\ &- (\kappa+\lambda)\left(1-e^{-\zeta z}\right)+O\left(\left(\frac{m}{\kappa+\lambda}\right)^2\right) \end{aligned}$$

verifying the guess. Note that the guess also satisfies the boundary condition $G(0)=0$ as well as $\lim_{z\to\infty}G(z)=1$. Hence for small $\frac{m}{\kappa+\lambda}$, the employment distribution (21) can be approximated to a first-order as

$$G(z) \approx \frac{1}{1+\beta e^{-\zeta z}}\left(1-e^{-\zeta z}+\frac{m}{\kappa+\lambda}\left(1+\beta\right)\zeta e^{-\zeta z}\left(\frac{1}{\left(1+\beta e^{-\zeta z}\right)^2}-\frac{1}{\left(1+\beta\right)^2}\right)\right)$$

Differentiating (59) with respect to m and evaluating it for $m \ll \kappa+\lambda$ small

$$\begin{aligned}
\frac{\partial G(z)}{\partial m} &\approx \frac{1}{1+\beta e^{-\zeta z}}\frac{1}{\kappa+\lambda}\left(1+\beta\right)\zeta e^{-\zeta z}\left(\frac{1}{\left(1+\beta e^{-\zeta z}\right)^2}-\frac{1}{\left(1+\beta\right)^2}\right) \\
&\approx \frac{1}{1+\beta e^{-\zeta z}}\frac{1}{\kappa+\lambda}\left(1+\beta\right)\zeta e^{-\zeta z}\left(\frac{\left(1+\beta\right)^2-\left(1+\beta e^{-\zeta z}\right)^2}{\left(1+\beta e^{-\zeta z}\right)^2\left(1+\beta\right)^2}\right) \\
&\approx \frac{1}{1+\beta e^{-\zeta z}}\frac{1}{\kappa+\lambda}\zeta e^{-\zeta z}\left(\frac{1+2\beta+\beta^2-1-2\beta e^{-\zeta z}-\beta^2 e^{-2\zeta z}}{\left(1+\beta e^{-\zeta z}\right)^2\left(1+\beta\right)}\right) \\
&\approx \frac{1}{1+\beta e^{-\zeta z}}\frac{1}{\kappa+\lambda}\zeta e^{-\zeta z}\beta\left(\frac{2(1-e^{-\zeta z})+\beta(1-e^{-2\zeta z})}{\left(1+\beta e^{-\zeta z}\right)^2\left(1+\beta\right)}\right) \\
&\approx \frac{1}{\kappa+\lambda}\zeta e^{-\zeta z}\frac{\beta}{1+\beta}\frac{2(1-e^{-\zeta z})+\beta(1-e^{-2\zeta z})}{\left(1+\beta e^{-\zeta z}\right)^3} \\
&\approx \frac{1}{\kappa+\lambda}\zeta e^{-\zeta z}\frac{\beta}{1+\beta}\frac{2(1-e^{-\zeta z})+\beta(1+e^{-\zeta z})(1-e^{-\zeta z})}{\left(1+\beta e^{-\zeta z}\right)^3} \\
&\approx \frac{1}{\kappa+\lambda}\frac{\beta}{1+\beta}\frac{2+\beta(1+e^{-\zeta z})}{\left(1+\beta e^{-\zeta z}\right)^3}\zeta e^{-\zeta z}(1-e^{-\zeta z})
\end{aligned} \tag{60}$$

Differentiating the unemployment rate (16) with respect to m

$$u'(m) = \frac{g(0)+m\frac{\partial g(0;m)}{\partial m}}{\kappa+\lambda+mg(0;m)+p}-\frac{\kappa+\lambda+mg(0;m)}{\left(\kappa+\lambda+mg(0;m)+p\right)^2}\left(g(0;m)+m\frac{\partial g(0;m)}{\partial m}\right)$$

Using (53) to evaluate this for $m \ll \kappa+\lambda$ small

$$\begin{aligned}
u'(m) &\approx \frac{g(0)}{\kappa+\lambda+p}-\frac{\kappa+\lambda}{\left(\kappa+\lambda+p\right)^2}g(0) \\
&\approx \frac{g(0)p}{\left(\kappa+\lambda+p\right)^2} \\
&\approx \frac{\frac{\kappa+\lambda}{\kappa+\lambda+\phi p}\zeta p}{\left(\kappa+\lambda+p\right)^2} \\
&\approx \frac{p\zeta}{\left(\kappa+\lambda+p\right)^2\left(1+\beta\right)}
\end{aligned} \tag{61}$$

C.11 Proposition 5: The entry curve

Recall the return to hiring (45)

$$R(z;m) \quad = \quad \frac{\chi}{L}\Big(u(m)J(z;m) + \phi\Big(1-u(m)\Big)\int_0^z J'(\tilde{z};m)G(\tilde{z};m)d\tilde{z}\Big)$$

Differentiating with respect to the growth rate m

$$\begin{aligned}
\frac{\partial R(z;m)}{\partial m} &= \frac{\chi}{L}u'(m)\left(J(z;m) - \phi\int_0^z J'(\tilde{z};m)G(\tilde{z};m)d\tilde{z}\right) \\
&+ \frac{\chi}{L}\left(u(m)\frac{\partial J(z;m)}{\partial m} + \phi\left(1-u(m)\right)\int_0^z\left(\frac{\partial J'(\tilde{z};m)}{\partial m}G(\tilde{z};m) + J'(\tilde{z};m)\frac{\partial G(\tilde{z};m)}{\partial m}\right)d\tilde{z}\right) \\
&= \frac{\chi}{L}\left(u'(m)\left(J(z;m)(1-\phi G(z;m)) + \phi\int_0^z J(\tilde{z};m)g(\tilde{z};m)d\tilde{z}\right)\right. \\
&+ \left. u(m)\frac{\partial J(z;m)}{\partial m} + \phi\left(1-u(m)\right)\int_0^z\left(\frac{\partial J'(\tilde{z};m)}{\partial m}G(\tilde{z};m) + J'(\tilde{z};m)\frac{\partial G(\tilde{z};m)}{\partial m}\right)d\tilde{z}\right)
\end{aligned} \tag{62}$$

where the second equality follows from integration by parts. Using (47)–(53) as well as (61)–(60), to a first order

$$\begin{aligned}
\frac{\partial R(z;m)}{\partial m} &\approx \frac{\chi}{L}\left(\frac{p\zeta}{(\kappa+\lambda+p)^2(1+\beta)}\frac{1}{\rho+\kappa}e^z\left(1-\phi\frac{(\kappa+\lambda)\left(1-e^{-\zeta z}\right)}{\kappa+\lambda+\phi p e^{-\zeta z}}\right)\right. \\
&+ \frac{p\phi\zeta}{(\kappa+\lambda+p)^2(1+\beta)}\int_0^z\frac{1}{\rho+\kappa}e^{\tilde{z}}\frac{(\kappa+\lambda+\phi p)(\kappa+\lambda)}{(\kappa+\lambda+\phi p e^{-\zeta\tilde{z}})^2}\zeta e^{-\zeta\tilde{z}}d\tilde{z} \\
&- \frac{\kappa+\lambda}{\kappa+\lambda+p}\frac{1}{(\rho+\kappa)^2} \\
&+ \left.\frac{\phi p}{\kappa+\lambda+p}\int_0^z\frac{1}{\rho+\kappa}e^{\tilde{z}}\frac{1}{\kappa+\lambda}\frac{\beta}{1+\beta}\frac{2+\beta(1+e^{-\zeta\tilde{z}})}{(1+\beta e^{-\zeta\tilde{z}})^3}\zeta e^{-\zeta\tilde{z}}(1-e^{-\zeta\tilde{z}})d\tilde{z}\right) \\
&\approx \frac{\chi}{L}\frac{1}{\rho+\kappa}\frac{1}{\kappa+\lambda+p}\left(\frac{p\zeta}{(\kappa+\lambda+p)(1+\beta)}e^z\left(1-\phi\frac{1-e^{-\zeta z}}{1+\beta e^{-\zeta z}}\right)\right. \\
&+ \frac{p\phi\zeta^2}{\kappa+\lambda+p}\int_0^z\frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z} \\
&- \frac{\kappa+\lambda}{\rho+\kappa} \\
&+ \left.\zeta\frac{\beta^2}{1+\beta}\int_0^z\frac{2+\beta(1+e^{-\zeta\tilde{z}})}{(1+\beta e^{-\zeta\tilde{z}})^3}e^{(1-\zeta)\tilde{z}}(1-e^{-\zeta\tilde{z}})d\tilde{z}\right)
\end{aligned}$$

Substituting this into (58)

$$
\begin{aligned}
\frac{\partial O(z;m)}{\partial m} \approx\ & \frac{1}{\rho+\kappa}\Bigg(-\frac{\zeta}{(\rho+\kappa)^2}\frac{\chi}{L}\frac{\beta\,(1+\beta)}{1+\frac{p}{\kappa+\lambda}}\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z} \\
&+ \frac{\chi}{L}\frac{1}{\rho+\kappa}\frac{1}{\kappa+\lambda+p}\Bigg(\frac{p\zeta}{(\kappa+\lambda+p)(1+\beta)}e^z\left(1-\phi\frac{1-e^{-\zeta z}}{1+\beta e^{-\zeta z}}\right) \\
&+ \frac{p\phi\zeta^2}{\kappa+\lambda+p}\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}d\tilde{z} \\
&- \frac{\kappa+\lambda}{\rho+\kappa} \\
&+ \zeta\frac{\beta^2}{1+\beta}\int_0^z \frac{2+\beta(1+e^{-\zeta\tilde{z}})}{(1+\beta e^{-\zeta\tilde{z}})^3}e^{(1-\zeta)\tilde{z}}(1-e^{-\zeta\tilde{z}})d\tilde{z}\Bigg)\Bigg)
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
\frac{\partial O(z;m)}{\partial m} \approx\ & \frac{1}{(\rho+\kappa)^2}\frac{\chi}{L}\Bigg(-\underbrace{\frac{\kappa+\lambda}{\rho+\kappa}}_{\equiv A_1(\rho)} - \underbrace{\frac{\kappa+\lambda}{\rho+\kappa}\zeta\frac{\beta(\chi)\,(1+\beta(\chi))}{\kappa+\lambda+\chi}\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta(\chi)e^{-\zeta\tilde{z}})^2}d\tilde{z}}_{\equiv A_2(\rho,\chi)} + \frac{\zeta}{\kappa+\lambda+\chi} \\
&\times \Bigg(\underbrace{\frac{\chi}{(\kappa+\lambda+\chi)(1+\beta(\chi))}e^z\left(1-\phi\frac{1-e^{-\zeta z}}{1+\beta(\chi)e^{-\zeta z}}\right)}_{\equiv B_1(\chi)} \\
&+ \underbrace{\frac{\chi\phi\zeta}{\kappa+\lambda+\chi}\int_0^z \frac{e^{(1-\zeta)\tilde{z}}}{(1+\beta(\chi)e^{-\zeta\tilde{z}})^2}d\tilde{z}}_{\equiv B_2(\chi)} \\
&+ \underbrace{\frac{\beta(\chi)^2}{1+\beta(\chi)}\int_0^z \frac{2+\beta(\chi)(1+e^{-\zeta\tilde{z}})}{(1+\beta(\chi)e^{-\zeta\tilde{z}})^3}e^{(1-\zeta)\tilde{z}}(1-e^{-\zeta\tilde{z}})d\tilde{z}}_{\equiv B_3(\chi)}\Bigg)\Bigg)
\end{aligned}
$$

Note that as $\chi \to 0$, $\beta(\chi) \to 0$. Note also that

$$\begin{aligned}
\lim_{\chi \to 0} A_1(\rho) &> 0 \\
\lim_{\chi \to 0} A_2(\rho, \chi) &= 0 \\
\lim_{\chi \to 0} B_1(\rho, \chi) &= 0 \\
\lim_{\chi \to 0} B_2(\rho, \chi) &= 0 \\
\lim_{\chi \to 0} B_3(\rho, \chi) &= 0
\end{aligned}$$

It follows from continuity that for sufficiently small χ, $\frac{\partial O(z;m)}{\partial m} < 0$.

Note next that as $\chi \to \infty$, $\beta(\chi) \to \infty$. Note also that

$$\begin{aligned}
\lim_{\chi \to \infty} A_1(\rho) &> 0 \\
\lim_{\chi \to \infty} A_2(\rho, \chi) &= 0 \\
\lim_{\chi \to \infty} B_1(\rho, \chi) &= 0 \\
\lim_{\chi \to \infty} B_2(\rho, \chi) &= 0 \\
\lim_{\chi \to \infty} B_3(\rho, \chi) &= 0
\end{aligned}$$

It follows from continuity that for sufficiently large χ, $\frac{\partial O(z;m)}{\partial m} < 0$.

Finally, note that

$$\begin{aligned}
\lim_{\rho \to \infty} A_1(\rho) &= 0 \\
\lim_{\rho \to \infty} A_2(\rho, \chi) &= 0
\end{aligned}$$

Moreover

$$\begin{aligned}
\lim_{\rho \to \infty} B_2(\rho, \chi) &> 0 \\
\lim_{\rho \to \infty} B_3(\rho, \chi) &> 0
\end{aligned}$$

Finally, when $\phi \leq 1$

$$\lim_{\rho \to \infty} B_1(\rho, \chi) \geq 0$$

It follows from continuity that for sufficiently large ρ, $\frac{\partial O(z;m)}{\partial m} > 0$.

For $m \ll \kappa + \lambda$ and $m \ll \kappa + \rho$, the slope of the entry curve is

$$y'(m) = \pi(\xi - l)\left(\frac{\pi}{c_e}\right)^{\frac{1}{\eta_e}} \frac{1}{\eta_e}\left(\int_0^\infty O(z;m)d\Gamma(z)\right)^{\frac{1}{\eta_e}-1} \int_0^\infty \frac{\partial O(z;m)}{\partial m} d\Gamma(z)$$

Since the first terms are strictly positive, the sign depends on the sign of $\int_0^\infty \frac{\partial O(z;m)}{\partial m} d\Gamma(z)$.

C.12 Lemma 5: Age-conditional mobility

Let $\tilde{g}(z,a)$ denote the share of individuals that are of age a and who work in a firm with productivity z on the BGP. It is given by the FP equation

$$(\lambda + \kappa)\,\tilde{g}(z,a) = m\tilde{g}_z(z,a) - \tilde{g}_a(z,a) - \phi p(1 - F(z))\tilde{g}(z,a) + pf(z)\tilde{u}(a) + \phi p f(z) \int_0^z \tilde{g}(\tilde{z},a)d\tilde{z}$$

where the share of individuals who are aged a and unemployed are

$$(\lambda + \kappa)\tilde{u}(a) = m\tilde{g}(0,a) - \tilde{u}'(a) - p\tilde{u}(a)$$

subject to the additional boundary conditions that $\lim_{a\to 0} \tilde{g}(z,a) = 0$ for all z since nobody enters the labor market as employed, and $\tilde{u}(0) = \lambda + \kappa$ since all individuals whose parent was not an entrepreneur enter as unemployed.

Introduce the change of variables

$$\begin{aligned}
\tilde{u}(a) &= (\lambda + \kappa)e^{-(\lambda+\kappa)a}\widehat{u}(a) \\
\tilde{g}(z,a) &= (1 - \widehat{u}(a))(\lambda + \kappa)e^{-(\lambda+\kappa)a}\widehat{g}(z|a)
\end{aligned}$$

where $\widehat{u}(a)$ is the age-conditional unemployment rate and $\widehat{g}(z|a)$ is the distribution of individuals over the job ladder conditional on being employed and of age a. Differentiating

$$\begin{aligned}
\tilde{u}'(a) &= -(\lambda + \kappa)^2 e^{-(\lambda+\kappa)a}\widehat{u}(a) + (\lambda + \kappa)e^{-(\lambda+\kappa)a}\widehat{u}'(a) \\
\tilde{g}_z(z,a) &= (1 - \widehat{u}(a))(\lambda + \kappa)e^{-(\lambda+\kappa)a}\frac{\partial \widehat{g}(z|a)}{\partial z} \\
\tilde{g}_a(z,a) &= -\widehat{u}'(a)(\lambda + \kappa)e^{-(\lambda+\kappa)a}\widehat{g}(z|a) - (1 - \widehat{u}(a))(\lambda + \kappa)^2 e^{-(\lambda+\kappa)a}\widehat{g}(z|a) \\
&+ (1 - \widehat{u}(a))(\lambda + \kappa)e^{-(\lambda+\kappa)a}\frac{\partial \widehat{g}(z|a)}{\partial a}
\end{aligned}$$

Substituting this into the evolution of the unemployed

$$\begin{aligned}
(\lambda+\kappa+p)(\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{u}(a) &= m(1-\widehat{u}(a))(\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{g}(0|a) \\
&- \left(-(\lambda+\kappa)^2 e^{-(\lambda+\kappa)a}\widehat{u}(a) + (\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{u}'(a)\right) \\
(\lambda+\kappa+p)\widehat{u}(a) &= m(1-\widehat{u}(a))\widehat{g}(0|a) + (\lambda+\kappa)\widehat{u}(a) - \widehat{u}'(a) \\
\widehat{u}'(a) &= m\widehat{g}(0|a) - (m\widehat{g}(0|a)+p)\,\widehat{u}(a) \qquad (63)
\end{aligned}$$

Substituting (65) into the evolution of the employed

$$\begin{aligned}
(\lambda+\kappa)\,(1-\widehat{u}(a))(\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{g}(z|a) &= m(1-\widehat{u}(a))(\lambda+\kappa)e^{-(\lambda+\kappa)a}\frac{\partial\widehat{g}(z|a)}{\partial z} \\
&+ \widehat{u}'(a)(\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{g}(z|a) \\
&+ (1-\widehat{u}(a))(\lambda+\kappa)^2 e^{-(\lambda+\kappa)a}\widehat{g}(z|a) \\
&- (1-\widehat{u}(a))(\lambda+\kappa)e^{-(\lambda+\kappa)a}\frac{\partial\widehat{g}(z|a)}{\partial a} \\
&- \phi p(1-F(z))(1-\widehat{u}(a))(\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{g}(z|a) \\
&+ pf(z)(\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{u}(a) \\
&+ \phi pf(z)(1-\widehat{u}(a))(\lambda+\kappa)e^{-(\lambda+\kappa)a}\widehat{G}(z|a)
\end{aligned}$$

Cancelling terms

$$0 = m\frac{\partial\widehat{g}(z|a)}{\partial z} + \frac{\widehat{u}'(a)}{1-\widehat{u}(a)}\widehat{g}(z|a) - \frac{\partial\widehat{g}(z|a)}{\partial a} - \phi p(1-F(z))\widehat{g}(z|a) + pf(z)\frac{\widehat{u}(a)}{1-\widehat{u}(a)} + \phi pf(z)\widehat{G}(z|a)$$

Integrating from 0 to z (using integration by parts)

$$0 = m\left(\widehat{g}(z|a)-\widehat{g}(0|a)\right) + \frac{\widehat{u}'(a)}{1-\widehat{u}(a)}\widehat{G}(z|a) - \frac{\partial\widehat{G}(z|a)}{\partial a} - \phi p(1-F(z))\widehat{G}(z|a) + pF(z)\frac{\widehat{u}(a)}{1-\widehat{u}(a)} \qquad (64)$$

Note that (63) can be alternatively written as

$$\frac{\widehat{u}'(a)}{1-\widehat{u}(a)} = m\widehat{g}(0|a) - p\frac{\widehat{u}(a)}{1-\widehat{u}(a)} \qquad (65)$$

Substituting (65) into (64)

$$\frac{\partial\widehat{G}(z|a)}{\partial a} = m\widehat{g}(z|a) - m\widehat{g}(0|a)(1-\widehat{G}(z|a)) - \phi p(1-F(z))\widehat{G}(z|a) + p(F(z)-\widehat{G}(z|a))\frac{\widehat{u}(a)}{1-\widehat{u}(a)} \qquad (66)$$

subject to the boundary conditions that $\lim_{z\to\infty} \widehat{G}(z|a) = 1$ and $\lim_{a\to 0} \widehat{G}(z|a) = 1 - e^{-\zeta z}$. Solving the ODE (63) subject to initial value $\widehat{u}(0) = 1$

$$\begin{aligned}
\widehat{u}(a) &= \frac{m\widehat{g}(0|a)}{p + m\widehat{g}(0|a)} + e^{-(p+m\widehat{g}(0|a))a}\frac{p}{p + m\widehat{g}(0|a)} \\
1 - \widehat{u}(a) &= \frac{p}{p + m\widehat{g}(0|a)}\left(1 - e^{-(p+m\widehat{g}(0|a))a}\right) \\
\frac{\widehat{u}(a)}{1 - \widehat{u}(a)} &= \frac{m\widehat{g}(0|a) + pe^{-(p+m\widehat{g}(0|a))a}}{p\left(1 - e^{-(p+m\widehat{g}(0|a))a}\right)}
\end{aligned}$$

Substituting this back into (66)

$$\begin{aligned}
\frac{\partial \widehat{G}(z|a)}{\partial a} &= m\widehat{g}(z|a) - m\widehat{g}(0|a)\left(1 - \widehat{G}(z|a)\right) - \phi p\left(1 - F(z)\right)\widehat{G}(z|a) \\
&+ \left(F(z) - \widehat{G}(z|a)\right)\frac{m\widehat{g}(0|a) + pe^{-(p+m\widehat{g}(0|a))a}}{1 - e^{-(p+m\widehat{g}(0|a))a}}
\end{aligned} \tag{67}$$

subject to $\lim_{a\to 0} \widehat{G}(z|a) = 1 - e^{-\zeta z}$. Note that this does not contain λ, and hence $\widehat{G}(z|a)$ does not depend directly on λ, and neither does $\widehat{u}(a)$.

Imposing $m = 0$ in (65) and solving the resulting ODE, the age-specific unemployment rate is

$$\widehat{u}(a) = e^{-pa} \tag{68}$$

Imposing this as well as $m = 0$ and the fact that the offer distribution is exponential in (67), $\widehat{G}(z|a)$ solves the partial differential equation

$$\frac{\partial \widehat{G}(z|a)}{\partial a} + \left(\frac{p}{e^{pa} - 1} + \phi p e^{-\zeta z}\right)\widehat{G}(z|a) = \frac{p\left(1 - e^{-\zeta z}\right)}{e^{pa} - 1}$$

subject to the boundary condition that $\lim_{a\to 0} \widehat{G}(z|a) = 1 - e^{-\zeta z}$. One can verify that a solution is

$$\widehat{G}(z|a) = \frac{1 - e^{-\zeta z}}{(e^{ap} - 1)\left(1 - \phi e^{-\zeta z}\right)}\left(e^{ap\left(1 - \phi e^{-\zeta z}\right)} - 1\right) \tag{69}$$

and that it satisfies the boundary condition (using, for instance, L'Hôpital's rule).

Differentiating (69) with respect to a

$$
\begin{aligned}
\frac{\partial \widehat{G}(z|a)}{\partial a} &= -\frac{pe^{ap}}{(e^{ap}-1)^2}\frac{1-e^{-\zeta z}}{1-\phi e^{-\zeta z}}\left(e^{ap\left(1-\phi e^{-\zeta z}\right)}-1\right)+\frac{1}{e^{ap}-1}\frac{1-e^{-\zeta z}}{1-\phi e^{-\zeta z}}e^{ap\left(1-\phi e^{-\zeta z}\right)}p\left(1-\phi e^{-\zeta z}\right) \\
&= -\frac{1-e^{-\zeta z}}{1-\phi e^{-\zeta z}}\frac{pe^{ap}}{(e^{ap}-1)^2}\left(e^{ap\left(1-\phi e^{-\zeta z}\right)}-1-(e^{ap}-1)\,e^{-ap\phi e^{-\zeta z}}\left(1-\phi e^{-\zeta z}\right)\right) \\
&= -\frac{1-e^{-\zeta z}}{1-\phi e^{-\zeta z}}\frac{pe^{ap}}{(e^{ap}-1)^2}\left(e^{ap\left(1-\phi e^{-\zeta z}\right)}-1-e^{ap\left(1-\phi e^{-\zeta z}\right)}\left(1-\phi e^{-\zeta z}\right)+e^{-ap\phi e^{-\zeta z}}\left(1-\phi e^{-\zeta z}\right)\right) \\
&= -\frac{1-e^{-\zeta z}}{1-\phi e^{-\zeta z}}\frac{pe^{ap}}{(e^{ap}-1)^2}\left(-1+e^{ap\left(1-\phi e^{-\zeta z}\right)}\phi e^{-\zeta z}+e^{-ap\phi e^{-\zeta z}}\left(1-\phi e^{-\zeta z}\right)\right) \\
&= -\frac{1-e^{-\zeta z}}{1-\phi e^{-\zeta z}}\frac{pe^{ap}}{(e^{ap}-1)^2}\left(-1+e^{-ap\phi e^{-\zeta z}}+(e^{ap}-1)\,e^{-ap\phi e^{-\zeta z}}\phi e^{-\zeta z}\right) \qquad (70)
\end{aligned}
$$

Consider first the case in which $\phi e^{-\zeta z} < 1$. Then $\partial \widehat{G}(z|a)/\partial a < 0$ if the last term in parenthesis in (70) is positive. When is this the case?

$$
\begin{aligned}
-1+e^{-ap\phi e^{-\zeta z}}+(e^{ap}-1)\,e^{-ap\phi e^{-\zeta z}}\phi e^{-\zeta z} &\geq 0 \\
e^{-ap\phi e^{-\zeta z}}+(e^{ap}-1)\,e^{-ap\phi e^{-\zeta z}}\phi e^{-\zeta z} &\geq 1 \\
1+(e^{ap}-1)\,\phi e^{-\zeta z} &\geq e^{ap\phi e^{-\zeta z}} \\
1 &\geq e^{ap\phi e^{-\zeta z}}-(e^{ap}-1)\,\phi e^{-\zeta z} \qquad (71)
\end{aligned}
$$

Differentiate the right-hand size of (71) with respect to a

$$
e^{ap\phi e^{-\zeta z}}p\phi e^{-\zeta z}-pe^{ap}\phi e^{-\zeta z}
$$

If this is negative, the right hand of (71) falls in a, such that its maximum is reached for $a=0$. Hence, if the inequality (71) holds for $a=0$, it holds for all a. Is the derivative negative?

$$
\begin{aligned}
e^{ap\phi e^{-\zeta z}}p\phi e^{-\zeta z}-pe^{ap}\phi e^{-\zeta z} &\leq 0 \\
e^{ap\phi e^{-\zeta z}} &\leq e^{ap} \\
\phi e^{-\zeta z} &\leq 1
\end{aligned}
$$

which is true by assumption. Since

$$
1 \;\geq\; e^{0p\phi e^{-\zeta z}}-\left(e^{0p}-1\right)\phi e^{-\zeta z} \;=\; 1
$$

it follows that

$$\frac{\partial \widehat{G}(z|a)}{\partial a} \leq 0$$

Consider next the case in which $\phi e^{-\zeta z} > 1$. Then $\partial \widehat{G}(z|a)/\partial a < 0$ if the last term in parenthesis in (70) is negative. When is this the case?

$$\begin{aligned}
-1 + e^{-ap\phi e^{-\zeta z}} + \left(e^{ap} - 1\right) e^{-ap\phi e^{-\zeta z}} \phi e^{-\zeta z} &\leq 0 \\
e^{-ap\phi e^{-\zeta z}} + \left(e^{ap} - 1\right) e^{-ap\phi e^{-\zeta z}} \phi e^{-\zeta z} &\leq 1 \\
1 + \left(e^{ap} - 1\right) \phi e^{-\zeta z} &\leq e^{ap\phi e^{-\zeta z}} \\
1 &\leq e^{ap\phi e^{-\zeta z}} - \left(e^{ap} - 1\right) \phi e^{-\zeta z} \qquad (72)
\end{aligned}$$

Differentiate the right-hand size of (72) with respect to a

$$e^{ap\phi e^{-\zeta z}} p\phi e^{-\zeta z} - pe^{ap}\phi e^{-\zeta z}$$

If this is positive, the right hand of (72) rises in a, such that its minimum is reached for $a = 0$. Hence, if the inequality (72) holds for $a = 0$, it holds for all a. Is the derivative positive?

$$\begin{aligned}
e^{ap\phi e^{-\zeta z}} p\phi e^{-\zeta z} - pe^{ap}\phi e^{-\zeta z} &\geq 0 \\
e^{ap\phi e^{-\zeta z}} &\geq e^{ap} \\
\phi e^{-\zeta z} &\geq 1
\end{aligned}$$

which is true by assumption. Since

$$1 \geq e^{0p\phi e^{-\zeta z}} - \left(e^{0p} - 1\right) \phi e^{-\zeta z} = 1$$

it follows that

$$\frac{\partial \widehat{G}(z|a)}{\partial a} \leq 0$$

also in this case. Since the two cases for $\phi e^{-\zeta z}$ are exhaustive, it follows that $\frac{\partial \widehat{G}(z|a)}{\partial a} \leq 0$.

The age-conditional JJ mobility rate is

$$\widehat{JJ}(a) = \phi p \int_0^\infty (1 - F(z)) d\widehat{G}(z|a) = \phi p \int_0^\infty f(z)\widehat{G}(z|a) dz$$

where the second equality follows from an integration by parts and the fact that $\lim_{z\to\infty} 1 - F(z) = 0$ and $\widehat{G}(0|a) = 0$.

C.13 Lemma 6: Age conditional entry

Let $\widehat{x}(z|a)$ denote the age-conditional distribution of producing entrepreneurs over relative productivity z, and $\widehat{u}^f(a)$ the age-specific share of entrepreneurs who currently are not producing. Following the same steps as in lemma 5, one can show that it evolves according to the FP equation

$$\frac{\partial \widehat{x}(z|a)}{\partial a} = m \frac{\partial \widehat{x}(z|a)}{\partial z} + \pi s \gamma(z) \frac{\widehat{u}^f(a)}{1 - \widehat{u}^f(a)}$$

subject to $\lim_{a\to 0} \widehat{x}(z|a) = x(z)$, where $x(z)$ is the aggregate distribution of entrepreneurs. This boundary condition must hold due to the perpetual youth assumption, which implies that offspring inherit a randomly drawn existing firm. Note that the optimal search intensity of non-producing entrepreneurs, s, is independent of age.

The share of non-producing entrepreneurs evolves according to

$$\frac{d\widehat{u}^f(a)}{da} = m\widehat{x}(0|a) - \pi s \frac{\widehat{u}^f(a)}{1 - \widehat{u}^f(a)}$$

subject to $\widehat{u}^f(0) = u^f$, where u^f is the aggregate share of non-producing entrepreneurs (again due to the perpetual youth assumption).

Guess that $\widehat{x}(z|a) = \zeta e^{-\zeta z}$ and $\widehat{u}^f(a) = u^f$ for all a. Then

$$\frac{d\widehat{u}^f(a)}{da} = m\zeta - \pi s \frac{u^f}{1 - u^f} = 0$$

since aggregate exit $m\zeta(1 - u^f)$ must equal aggregate entry $\pi s u^f$. Imposing this and the innovation distribution $\gamma(z) = \zeta e^{-\zeta z}$ in the law of motion for the age-specific distribution of producing entrepreneurs

$$\begin{aligned} \frac{\partial \widehat{x}(z|a)}{\partial a} &= m\widehat{x}'(z|a) + \gamma(z) m\widehat{x}(0|a) \\ &= -m\zeta^2 e^{-\zeta z} + \zeta e^{-\zeta z} m\zeta \\ &= 0 \end{aligned}$$

Since the guess also satisfies the boundary conditions, this verifies it.

Since both the optimally chosen search intensity s and the share of non-producing entrepreneurs is independent of age, so is the age-specific entry rate.

C.14 Lemma 7: Labor supply and aging

The share of workforce participants aged a, $\Lambda'(a;\lambda)$, evolves according to

$$\lambda\Lambda'(a) = -\Lambda''(a) - \kappa\Lambda'(a)$$

subject to the initial value condition $\Lambda'(0) = \lambda + \kappa$. It is straightforward to establish that the unique solution is

$$\Lambda(a;\lambda) = 1 - e^{-(\lambda+\kappa)a}$$

Differentiating this with respect to λ gives the lemma.

C.15 Proposition 6: The impact of aging

For small $\frac{m}{\kappa+\lambda}$

$$u(m,\lambda) \approx \frac{\kappa+\lambda}{\kappa+\lambda+p}$$

and hence

$$\left.\frac{\partial u(m,\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} \approx \frac{p}{(\kappa+\lambda+p)^2}$$

Recall that for small $\frac{m}{\kappa+\lambda}$, the employment distribution is to a first-order given by (59). Differentiating this with respect to λ holding fixed m

$$\left.\frac{\partial G(z;m,\lambda)}{\partial\lambda}\right|_{m\text{ fixed}} \approx \frac{\beta e^{-\zeta z}}{(1+\beta e^{-\zeta z})^2}\frac{1}{\kappa+\lambda}\left(1-e^{-\zeta z}\right)$$

Recall that under the assumption that $\eta_v \to \infty$ and $\theta \to 0$, $q = \chi S/L$ such that the return to hiring (45) is

$$R(z;m,\lambda) = \frac{\chi}{L}\left(u(m,\lambda)J(z;m) + \phi\Big(1-u(m,\lambda)\Big)\int_0^z J'(\tilde{z};m)G(\tilde{z};m,\lambda)d\tilde{z}\right)$$

Differentiating with respect to λ holding fixed the growth rate

$$\begin{aligned}\left.\frac{\partial R(z;m,\lambda)}{\partial\lambda}\right|_{m\text{ fixed}} &= \frac{\chi}{L}\left(\left.\frac{\partial u(m,\lambda)}{\partial\lambda}\right|_{m\text{ fixed}}\left(J(z;m)(1-\phi G(z;m,\lambda)) + \phi\int_0^z J(\tilde{z};m)g(\tilde{z};m,\lambda)d\tilde{z}\right)\right.\\ &+ \left.\phi\,(1-u(m,\lambda))\int_0^z J'(\tilde{z};m)\left.\frac{\partial G(\tilde{z};m,\lambda)}{\partial\lambda}\right|_{m\text{ fixed}}d\tilde{z}\right)\end{aligned}$$

Taking m to be small

$$\begin{aligned}\left.\frac{\partial R(z;m,\lambda)}{\partial\lambda}\right|_{m\text{ fixed}} &= \frac{\chi}{L}\left(\frac{p}{(\kappa+\lambda+p)^2}\frac{1}{\rho+\kappa}e^z\left(1-\phi\frac{(\kappa+\lambda)\left(1-e^{-\zeta z}\right)}{\kappa+\lambda+\phi p e^{-\zeta z}}\right)\right.\\ &+ \frac{p}{(\kappa+\lambda+p)^2}\phi\int_0^z\frac{1}{\rho+\kappa}e^{\tilde{z}}\frac{(\kappa+\lambda+\phi p)(\kappa+\lambda)}{(\kappa+\lambda+\phi p e^{-\zeta\tilde{z}})^2}\zeta e^{-\zeta\tilde{z}}d\tilde{z}\\ &+ \left.\frac{\phi p}{\kappa+\lambda+p}\int_0^z\frac{1}{\rho+\kappa}e^{\tilde{z}}\frac{\beta e^{-\zeta\tilde{z}}}{(1+\beta e^{-\zeta\tilde{z}})^2}\frac{1}{\kappa+\lambda}\left(1-e^{-\zeta\tilde{z}}\right)d\tilde{z}\right)\end{aligned}$$

which is positive for $\phi \leq 1$.

λ only enters the entry curve (20) via the return to hiring (45). In particular

$$\left.\frac{\partial y(m;\lambda)}{\partial\lambda}\right|_{m\text{ fixed}} = \left(\frac{\Omega}{y(m;\lambda)}\right)^{\eta_e}\frac{y(m;\lambda)}{\eta_e}\frac{\iota}{m}\int_0^\infty\int_0^{\tilde{z}} e^{-\frac{\rho+\kappa-m}{m}(\tilde{z}-z)}\left.\frac{\partial R(z;m)}{\partial\lambda}\right|_{m\text{ fixed}}dz\,d\Gamma(\tilde{z})$$

Since $\left.\frac{\partial R(z;m)}{\partial\lambda}\right|_{m\text{ fixed}} > 0$, it follows that $\left.\frac{\partial y(m;\lambda)}{\partial\lambda}\right|_{m\text{ fixed}} > 0$, i.e. aging shifts the entry curve down. λ does not independently enter the exit curve (19), and hence only affects exit through the growth rate m. When the economy displays a unique equilibrium, the entry curve cuts the exit curve from above at that equilibrium (see proposition 4). It follows that a downward shift in the entry curve results in a fall in the equilibrium growth rate, $-m'(\lambda) < 0$.

C.16 Validating key predictions of the theory

Table 11 provides reduced-form support for some of the key predictions of the theory. In the interest of space, I focus on the IV specification. Aging reduces firm exit (column 1) and shifts the population of firms toward old firms (column 2), consistent with the hypothesis that it reduces the rate of obsolescence.

Aging has no statistically significant effect on employment-unweighted value added per worker (column 3). It raises employment-unweighted pay (column 4), consistent with the view that firms need to pay workers more in an older, better matched labor market. In contrast to the null effect on unweighted productivity, aging raises employment-weighted value added per worker (column 5) as well as firm pay (column 6), consistent with aging shifting employment toward more productive, higher paying firms.[46] More directly, column 7 shows that aging shifts employment toward more productive firms.

Aging reduces poaching (column 8) and raises the productivity of poached hires' previous employer (column 9), consistent with the hypothesis that poaching is harder in the older labor market.

Aging raises pay in wage employment (column 10), for two reasons. First, it induces firms to pay better (column 4). Second, it relocates employment toward higher-paying firms (column 6). On the other hand, aging reduces income in self-employment (column 11), although the point estimate is not statistically significant at conventional levels (p-value 0.261). These patterns are consistent with the hypothesis that aging raises the opportunity cost of entry by improving potential entrepreneurs' match in the labor market. Moreover, the return to entry is lower, since entrepreneurs have to pay labor more.

TABLE 11. THE IMPACT OF AGING ON LABOR MARKET DYNAMICS

	(1) Exit	(2) 11+	(3) $\frac{y}{n}$ (UW)	(4) $\frac{w}{n}$ (UW)	(5) $\frac{y}{n}$ (W)	(6) $\frac{w}{n}$ (W)	(7) Empl. 10%	(8) Hire (e)	(9) Hire $\frac{y}{n}$	(10) y, wage	(11) y, self
Share 20–44	1.915**	–1.152***	–0.629	–0.569*	–1.792*	–1.203**	–3.215*	2.846***	–0.762**	–1.412***	1.216
	(0.744)	(0.370)	(0.666)	(0.296)	(0.937)	(0.510)	(1.751)	(0.910)	(0.323)	(0.446)	(1.062)
P-value	0.015	0.005	0.356	0.068	0.069	0.028	0.080	0.004	0.028	0.003	0.261
Obs.	2,244	1,564	1,496	1,496	1,496	1,496	1,496	2,244	1,496	2,244	2,244
F-stat	27.1	16.8	14.8	14.8	14.8	14.8	14.8	27.1	14.8	27.1	27.1

Table 11 presents IV estimates based on regression (1) using annual data from 68 LA between years 1986–2018. The independent variable is the log share of all individuals aged 20–64 that are aged 20–44 in the LA in that year. The outcome variables are for private sector firms and individuals aged 20–64. Income of the wage/self employed and the value added per worker of the previous employer are aggregated in logs at the LA-year level; other outcomes are aggregated in levels at the LA-year level and subsequently logged. The instrument is the sum of births 20–44 years earlier in the LA, and subsequently logged. Standard errors are two-way clustered at the LA and year levels. Columns 1 shows the firm exit rate, defined as the fraction of firms with positive employment in year t which have zero employment in year $t+1$. Column 2 shows the share of firms that are 11 years or older. Column 3 shows employment-unweighted value added per worker. Column 4 shows employment-unweighted firm level average wages. Column 5 shows employment-weighted value added per worker. Column 6 shows employment-weighted firm level average wages. Column 7 shows the share of employment of the top decile of firms ranked by employment-unweighted value added per worker. Column 8 shows the fraction of employment that was hired directly from another employer in the year. Column 9 shows the value added per worker of a poached hire's previous employer relative to the value added per worker of the current employer. Column 10 shows annual income of wage employed individuals. Column 11 shows annual income of self- employed individuals. *Source:* FEK, JOBB, LISA, SCB.

[46] Due to the shift toward more productive firms, which tend to have a lower labor share, aging does not raise the aggregate labor share, despite the fact that it raises pay conditional on productivity. This insight is also born out in the model.

C.17 Proposition 7: The impact of aging on worker mobility

The age-conditional JJ mobility rate is

$$\begin{aligned}\widehat{JJ}(a;m,\lambda) &= \phi p \int_0^\infty \Big(1 - F(z)\Big) d\widehat{G}(z|a,m,\lambda) \\ &= \phi p \left(\Big[\Big(1 - F(z)\Big) \widehat{G}(z|a,m,\lambda) \Big]_{z=0}^\infty + \int_0^\infty f(z)\widehat{G}(z|a,m,\lambda)dz \right) \\ &= \phi p \int_0^\infty f(z)\widehat{G}(z|a,m,\lambda)dz\end{aligned}$$

where the second equality follows from an integration by parts and the third from $\lim_{z\to 0} \widehat{G}(z|a,m,\lambda) = 0$ and $\lim_{z\to\infty} 1 - F(z) = 0$. Differentiating with respect to λ holding growth fixed

$$\left.\frac{\partial \widehat{JJ}(a;m,\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} = \phi p \int_0^\infty f(z) \left.\frac{\partial \widehat{G}(z|a,m,\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} dz = 0$$

since the age-conditional distribution of employment (69) does not depend directly on λ.

The aggregate JJ mobility rate is the rate at which a worker is poached integrated across all employed individuals in the economy

$$\begin{aligned}JJ(m,\lambda) &= \phi p \int_0^\infty \Big(1 - F(z)\Big) dG(z) \\ &= \phi p \left(\Big[\Big(1 - F(z)\Big) G(z) \Big]_{z=0}^\infty + \int_0^\infty f(z)G(z)dz \right) \\ &= \phi p \int_0^\infty f(z)G(z)dz\end{aligned}$$

where the second equality follows from an integration by parts and the third from $\lim_{z\to 0} G(z) = 0$ and $\lim_{z\to\infty} 1 - F(z) = 0$. The proposition follows from proposition 6 that

$$-\left.\frac{\partial G(z;m,\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} < 0$$

the fact that aging reduces the growth rate (proposition 6), and proposition **??** that

$$\frac{\partial G(z;m,\lambda)}{\partial m} > 0$$

Recall that aging only affects age-specific mobility rates through the resulting fall in growth, and that aging reduces the growth rate. Recall also the evolution of the age-conditional distribution of employment (67)

$$\begin{aligned}\frac{\partial \widehat{G}(z|a)}{\partial a} &= m\widehat{g}(z|a) - m\widehat{g}(0|a)\Big(1-\widehat{G}(z|a)\Big) - \phi p\Big(1-F(z)\Big)\widehat{G}(z|a) \\ &+ \Big(F(z)-\widehat{G}(z|a)\Big)\frac{m\widehat{g}(0|a)+pe^{-(p+m\widehat{g}(0|a))a}}{1-e^{-(p+m\widehat{g}(0|a))a}}\end{aligned}$$

subject to $\lim_{a\to 0}\widehat{G}(z|a) = 1-e^{-\zeta z}$. The boundary condition stems from the fact that at a very young age Δ, the employment distribution consists exclusively of those who just found a job (drawn from the offer distribution $F(z) = 1-e^{-\zeta z}$). It hence follows that

$$\lim_{a\to 0}\frac{\widehat{JJ}(a)}{\partial m} = p\phi\int_0^\infty f(z)\lim_{a\to 0}\frac{\partial \widehat{G}(z|a)}{\partial m}dz = 0$$

Consider next the limit as $a\to\infty$, at which point for $m>0$ the employment distribution converges to its stationary distribution

$$0 = \widehat{g}(z|\infty) - \widehat{g}(0|\infty)e^{-\zeta z} - \frac{\phi p}{m}e^{-\zeta z}\widehat{G}(z|\infty)$$

subject to $\widehat{G}(0|\infty) = 0$ and $\lim_{z\to\infty}\widehat{G}(z|\infty) = 1$. The general solution to this ODE is

$$\widehat{G}(z|\infty) = -\frac{m\widehat{g}(0|\infty)}{\phi p} + e^{-\frac{\phi p}{m\zeta}e^{-\zeta z}}c$$

The boundary conditions require that

$$\begin{aligned}\widehat{G}(0|\infty) &= -\frac{m\widehat{g}(0|\infty)}{\phi p} + e^{-\frac{\phi p}{m\zeta}}c = 0 \\ \lim_{z\to\infty}\widehat{G}(z|\infty) &= -\frac{m\widehat{g}(0|\infty)}{\phi p} + c = 1\end{aligned}$$

Solving these two equations for c and $\widehat{g}(0|\infty)$ gives the particular solution

$$\widehat{G}(z|\infty) = \frac{e^{\frac{\phi p}{m\zeta}\left(1-e^{-\zeta z}\right)}-1}{e^{\frac{\phi p}{m\zeta}}-1}$$

The derivative with respect to m is

$$\frac{\partial \widehat{G}(z|\infty)}{\partial m} = \frac{\frac{\phi p}{\zeta m^2}e^{\frac{\phi p}{\zeta m}}}{\left(e^{\frac{\phi p}{\zeta m}}-1\right)^2}\left(\underbrace{e^{-\frac{\phi p}{\zeta m}e^{-\zeta z}}\left(\left(e^{\frac{\phi p}{m\zeta}}-1\right)e^{-\zeta z}+1\right)-1}_{\equiv C(z,m)}\right)$$

Since the first term is strictly positive, the sign is determined by the sign of $C(z,m)$. Note first that for any $m \in (0,\infty)$

$$C(0,m) \;=\; e^{-\frac{\phi p}{\zeta m}}\left(e^{\frac{\phi p}{m\zeta}} - 1 + 1\right) - 1 \;=\; 0$$

Note next that for any $m \in (0,\infty)$

$$\lim_{z\to\infty} C(z,m) \;=\; 1 - 1 \;=\; 0$$

Finally, for any $z \in (0,\infty)$, consider the limit

$$\lim_{m\to 0} C(z,m) \;=\; \lim_{m\to 0}\left(e^{\frac{\phi p}{m\zeta}} - 1\right)e^{-\zeta z} \;=\; \infty$$

By continuity, for sufficiently low m, $C(z,m) > 0$ and hence $\frac{\partial \widehat{G}(z|\infty)}{\partial m} > 0$.

C.18 Proposition 8: The impact of aging on entry

By lemma 6, the age-specific entry rate is independent of age, $\widehat{y}(a) = y(m,\lambda)$. It follows from proposition 6 that, holding fixed growth, aging reduces the age-specific entry rate proportionally,

$$-\left.\frac{\partial \widehat{y}(a;\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} \;=\; -\left.\frac{\partial y(m,\lambda)}{\partial \lambda}\right|_{m \text{ fixed}} \;<\; 0$$

It follows from proposition 5 that the resulting fall in growth further reduces (proportionally) age-specific entry if the employed search with positive but moderate efficiency, and the discount rate is not too low

$$\frac{\partial \widehat{y}(a;m)}{\partial m} \;=\; y'(m) \;>\; 0$$

If not, the equilibrium fall in growth on net stimulates age-specific entry (proportionally)

$$\frac{\partial \widehat{y}(a;m)}{\partial m} \;=\; y'(m) \;<\; 0$$

C.19 Proposition 9: The unequal effect of aging

The first claim follows directly from a differentiation of the value of unemployment (9).

The joint value of a match is

$$V(z;m) \quad = \quad J(z;m) + U(m) \quad = \quad \frac{1}{\rho+\kappa-m}\left(\frac{m}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z} - c\right) + \frac{1}{\rho+\kappa}e^{z}$$

Differentiating this with respect to m

$$\begin{aligned}
\frac{\partial V(z;m)}{\partial m} &= \frac{1}{(\rho+\kappa-m)^2}\left(\frac{m}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z} - c\right) \\
&+ \frac{1}{\rho+\kappa-m}\left(\frac{1}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z} + \frac{m}{\rho+\kappa}e^{-\frac{\rho+\kappa-m}{m}z}\frac{\rho+\kappa}{m^2}z\right) \\
&= \frac{1}{(\rho+\kappa-m)^2}\left(e^{-\frac{\rho+\kappa-m}{m}z}\left(1+\frac{\rho+\kappa-m}{m}z\right) - c\right)
\end{aligned}$$

Since by assumption, $c, b > 0$ and $c + b = 1$

$$\begin{aligned}
\lim_{z\to 0} e^{-\frac{\rho+\kappa-m}{m}z}\left(1+\frac{\rho+\kappa-m}{m}z\right) - c &= 1-c \quad > \quad 0 \\
\lim_{z\to \infty} e^{-\frac{\rho+\kappa-m}{m}z}\left(1+\frac{\rho+\kappa-m}{m}z\right) - c &= -c \quad < \quad 0
\end{aligned}$$

Differentiating the derivative of the joint value with respect to m in turn with respect to z

$$\begin{aligned}
\frac{\partial^2 V(z;m)}{\partial m \partial z} &= \frac{1}{(\rho+\kappa-m)^2}\left(\left(-\frac{\rho+\kappa-m}{m}\right)e^{-\frac{\rho+\kappa-m}{m}z}\left(1+\frac{\rho+\kappa-m}{m}z\right) + e^{-\frac{\rho+\kappa-m}{m}z}\frac{\rho+\kappa-m}{m}\right) \\
&= \frac{1}{(\rho+\kappa-m)^2}e^{-\frac{\rho+\kappa-m}{m}z}\left(\left(-\frac{\rho+\kappa-m}{m}\right)\left(1+\frac{\rho+\kappa-m}{m}z\right) + \frac{\rho+\kappa-m}{m}\right) \\
&= -\frac{1}{m^2}e^{-\frac{\rho+\kappa-m}{m}z}z \quad < \quad 0
\end{aligned}$$

D Quantitative analysis

This section contains a reduced-form assessment of the joint dynamics of entrepreneurship and search in the data (Appendix D.1); the law of motion for relative productivity as well as an illustration of the impact of entry on inequality (Appendix D.2); the value functions in the full model (Appendix D.3); an equilibrium definition for the full model (Appendix D.4); an outline of the algorithm used to solve the model (Appendix D.5); additional targeted moments and parameters (Appendix D.6); details on how the model is estimated and identification (Appendix D.7); a decomposition of life-cycle dynamics in the estimated model (Appendix D.8); a shift-share analysis of the effects of aging on entry and JJ mobility (Appendix D.9); a comparison of the reduced-form estimated effect of aging with the structural estimates (Appendix D.10); an analysis of the impact of aging on firms (Appendix D.11); a discussion of the algorithm used to solve the transition experiment (Appendix D.13); and an illustration of aging in the model and data over the transition path (Appendix D.14).

D.1 The joint dynamics of entrepreneurship and search

To motivate the focus on h joint theory of labor market mobility and entrepreneurship, Table 12 shows the distribution of individuals born between 1953 and 1973 across number of jobs held and number of new firms started between 1993 and 2017.[47] Because attrition is limited—only if an individual moves to or from Sweden or dies does she leave the sample—I have 24 years of close to complete data for the 1953–1973 cohorts. The vast majority of individuals who founded h firm over this period also at some point worked as a wage employee. For instance, only 0.9 percent of all individuals started at least one firm over this 24 year period without ever holding a wage employment job. For comparison, 24.9 percent of individuals started at least one firm. In fact, even among the less than 0.1 percent of individuals who started five or more firms, most of them held at least one wage employment job over h 24 year period.

D.2 Incorporating productivity shocks

With incumbent innovation at rate μ and stochastic shocks to productivity at intensity σ, relative productivity $z(i,t) = \widehat{z}(i) - \widehat{\underline{z}}(t)$ evolves according to

$$dz \;=\; -mdt \;+\; \sigma dW(t)$$

[47] Because an individual could later become a wage employee in their own firm—for instance through h public listing—I standardize employment status within an individual-firm match to that when the match was first formed. I drop 2004 due to h time series break in the underlying data. Specifically, Statistics Sweden switched to h different source of data to identify the self-employed—the *Standardiserade rr̈kenskapsutdrag*—and they started recording also companies with negative profits. This lead to h one time jump (consisting of almost exclusively small firms) in the measured entry rate.

TABLE 12. DISTRIBUTION OF EMPLOYMENT AND FIRM CREATION SPELLS

	# jobs held											
# firms created	0	1	2	3	4	5	6	7	8	9	10+	**Marginal**
0	8.37	14.36	11.75	9.93	8.02	6.27	4.75	3.51	2.54	1.76	3.83	**75.09**
1	0.72	3.36	3.02	2.60	2.15	1.70	1.29	0.96	0.70	0.49	1.04	**18.01**
2	0.13	0.73	0.86	0.78	0.67	0.52	0.41	0.30	0.22	0.15	0.31	**5.08**
3	0.03	0.17	0.22	0.22	0.19	0.16	0.12	0.09	0.06	0.04	0.08	**1.39**
4	0.01	0.04	0.05	0.06	0.05	0.04	0.03	0.02	0.02	0.01	0.02	**0.34**
5+	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	**0.08**
Marginal	**9.26**	**18.66**	**15.91**	**13.60**	**11.08**	**8.70**	**6.62**	**4.89**	**3.53**	**2.46**	**5.28**	**100.00**

Table 12 shows the distribution of individuals born between 1953 and 1973 across number of different jobs and firms started between 1993–2017. The data exclude year 2004 due to h break in the sample. All individuals aged 20–64. *Source:* FEK, JOBB, LISA.

and the overall growth rate is $m + \mu$. Apart from the fact that the economy now grows at rate $m + \mu$, the HJB and FP equations remain essentially the same as above with the addition of the standard term for productivity shocks. In particular, the evolution of the distribution of entrepreneurs is now

$$0 = mx'(z) + \frac{\sigma^2}{2}x''(z) + \frac{y}{L}\gamma(z)$$

subject to $X(0) = 0$, $\lim_{z\to\infty} X(z) = 1$ and[48]

$$\frac{y}{l} = \frac{\sigma^2}{2}x'(0)$$

Given an aggregate entry rate, this second-order ODE determines the distribution, x, and the rate of obsolescence, m. The solution is[49]

$$x(z) = \frac{\frac{y}{l}}{-m + \frac{\sigma^2}{2}\zeta}\left(e^{\frac{2(-m)}{\sigma^2}z} - e^{-\zeta z}\right)$$

and the exit curve continues to be given by

$$\widehat{y}(m) = l\zeta m$$

Define log productivity z as following a power law if constants $a, \zeta^* > 0$ exist such that $Pr(Z > z) = ae^{-\zeta^* z}$. Productivity follows an asymptotic power law if constants $a, \zeta^* > 0$ exist such that $Pr(Z > z) \sim ae^{-\zeta^* z}$ as $z \to \infty$, where for any f, g, $f(z) \sim g(z)$ means $\lim_{z\to\infty} f(z)/g(z) = 1$. Then the endogenous

[48]This follows from the fact that $0 = m\int_0^\infty x'(z)dz + \frac{\sigma^2}{2}\int_0^\infty x''(z)dz + \frac{y}{L}\int_{\underline{z}}^\infty \gamma(z)dz = -\frac{\sigma^2}{2}x'(0) + y$ since in order for the density to integrate to one, $\lim_{z\to\infty} x(z) = 0$ and $\lim_{z\to\infty} x'(z) = 0$, and $x(0) = 0$.

[49]Parameter restrictions have to be imposed to ensure that h BGP equilibrium exists. As before, the resulting growth rate must be below $\rho + \kappa$ or values explode. Moreover with productivity shocks, it cannot be too small, because then a stationary distribution of firms may not exist. I assume throughout that these parameter restrictions hold.

stationary distribution of productivity follows an asymptotic power law with tail parameter ζ^* given by

$$\zeta^* = \begin{cases} \zeta & \text{if } \frac{(\sigma\zeta)^2}{2} \leq \frac{y}{l} \\ \frac{2\frac{y}{l}}{\sigma^2\zeta} & \text{if } \frac{(\sigma\zeta)^2}{2} > \frac{y}{l} \end{cases}$$

Hence, if the initial entry rate is sufficiently low, $\frac{(\sigma\zeta)^2}{2} > \frac{y}{l}$—the empirically relevant case—a decline in entry increases inequality. Creative destruction is a force that holds some firms back from becoming in a relative sense very productive. If the entry rate is high, $\frac{y}{l} > (\sigma\zeta)^2/2$, this force is so strong that the right tail of the productivity distribution is entirely driven by the tail of the entry distribution. It is as though firms fall behind the market so fast that no firm manages to move into the right tail of the distribution after entry. If, on the other hand, the entry rate is lower than this (the empirically relevant case), the tail of the productivity distribution is driven by endogenous productivity dynamics. In this case, lower entry is associated with greater tail dispersion in productivity across firms.

D.3 Value functions in the full model

In the extended model, the value of unemployment writes

$$\begin{aligned} (\rho - m)U(a) &= \underbrace{b(a)}_{\text{flow value of leisure}} \\ &+ \underbrace{U'(a)}_{\text{aging}} \\ &- \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa U(a)}_{\text{labor force exit}} \\ &+ \underbrace{\frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} s_u(a)^{1+\eta_e}}_{\text{net return to search for business ideas}} \end{aligned} \tag{73}$$

where $\mathbb{1}\{x \geq X\}$ is an indicator taking value one if $x \geq X$ and zero otherwise, and optimal search is

$$s_u(a) = \left(\frac{\pi(a)}{c_e e^{\underline{z}^w(a)}} \int_0^\infty \Big(V^e(\tilde{z}, a) - U(a) \Big) d\Gamma(\tilde{z}|\underline{z}^w(a)) \right)^{\frac{1}{\eta_e}} \tag{74}$$

where

$$\pi(a) = \begin{cases} \pi^l & \textit{if } a \leq \overline{A}^\pi \\ \pi^h & \textit{if } a \geq \overline{A}^\pi \end{cases}$$

Note that I assume that the cost of search scales in the reservation threshold, $e^{\underline{z}^w(a)}$, and not the flow value of leisure, $b(a)$, as in the analytical model in Section **??**. The reason for this minor change is that it allows me to fix a separation threshold $\underline{z}^w(a)$, solve for the equilibrium allocation, and ex post set the flow value of leisure such that the pre-set $\underline{z}^w(a)$ is the optimal choice of workers. This approach substantially speeds up the solution of the model, since it avoids solving for the endogenous separation threshold $\underline{z}^w(a)$ in estimation.

Let $V^w(z, a)$ be the value of a match, which now depends on both productivity, z, and the age of the worker, a. Importantly, however, it does not depend on the age of the founder, as long as $b(a)$ is high enough that all workers want to terminate a match before an entrepreneur of each age wants to shut down the firm. This property simplifies the solution of the problem substantially by reducing the need to keep track of the entrepreneur's age when solving the problem of the match. The value of a match solves for $z \geq \underline{z}^w(a)$ the stopping time problem

$$\begin{aligned}
(\rho - m)V^w(z,a) \quad = \quad & \underbrace{e^z - c}_{\text{net flow output}} \\
+ \quad & \underbrace{V_a^w(z,a)}_{\text{aging}} \\
- \quad & \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa V^w(z,a)}_{\text{labor force exit}} \\
- \quad & \underbrace{mV_z^w(z,a)}_{\text{technological obsolescence}} \\
+ \quad & \underbrace{\frac{\sigma^2}{2}V_{zz}^w(z,a)}_{\text{productivity shocks}} \\
+ \quad & \underbrace{\frac{\eta_e}{1+\eta_e}c_e e^z s(z,a)^{1+\eta_e}}_{\text{net return to search for business ideas}} \\
+ \quad & \underbrace{\delta(z)\Big(U(a) - V^w(z,a)\Big)}_{\text{exogenous match separation}} \\
+ \quad & \underbrace{d\Big(U(a) - V^w(z,a)\Big)}_{\text{exogenous firm exit}}
\end{aligned} \tag{75}$$

where $X_i = \partial X / \partial i$ is short hand for the partial derivative of X with respect to i, subject to the value matching and smooth pasting conditions $V^w(\underline{z}^w(a), a) = U(a)$ and $V_z^w(\underline{z}^w(a), a) = 0$, where optimal search is

$$s(z,a) = \left(\frac{\pi(a)}{c_e e^z}\int_0^\infty \Big(V^e(\tilde{z},a) - V^w(z,a)\Big) d\Gamma(\tilde{z}|z)\right)^{\frac{1}{\eta_e}} \tag{76}$$

Subtracting the value of unemployment (73) from the value of a match (75); evaluating the difference at $z = \underline{z}^w(a)$; using the fact that $V^w(\underline{z}^w(a),a) = U(a)$ for all a implies that $V^w_a(\underline{z}^w(a),a) = U'(a)$; noting that by (74)–(76), $s_u(a) = s(\underline{z}^w(a),a)$; and imposing the optimal stopping time conditions $V^w(\underline{z}^w(a),a) = U(a)$ and $V^w_z(\underline{z}^w(a),a) = 0$ for all a gives that the reservation threshold is characterized by

$$e^{\underline{z}^w(a)} - c = b(a) - \frac{\sigma^2}{2} V^w_{zz}\Big(\underline{z}^w(a),a\Big) \tag{77}$$

The value of entrepreneurship $V^e(z,a)$ solves for $z \geq \underline{z}(a)$ the stopping time problem

$$\begin{aligned}(\rho - m)V^e(z,a) = \; & \underbrace{k(a)}_{\text{flow value of being one's own boss}} \\ - \; & \underbrace{r}_{\text{fixed cost}} \\ + \; & \underbrace{V^e_a(z,a)}_{\text{aging}} \\ - \; & \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa V^e(z,a)}_{\text{labor force exit}} \\ - \; & \underbrace{mV^e_z(z,a)}_{\text{technological obsolescence}} \\ + \; & \underbrace{\frac{\sigma^2}{2}V^e_{zz}(z,a)}_{\text{productivity shocks}} \\ + \; & \underbrace{d\Big(U(a) - V^e(z,a)\Big)}_{\text{exogenous firm exit}} \\ + \; & \underbrace{\frac{\eta_v}{1+\eta_v} c_v e^z v(z,a)^{1+\eta_v}}_{\text{net return from hiring}}\end{aligned} \tag{78}$$

subject to $V^e(\underline{z}(a),a) = U(a)$ and $V^e_z(\underline{z}(a),a) = 0$, where optimal vacancy creation is

$$v(z,a) = \left(\frac{q}{c_v e^z}\int_0^\infty \left(\frac{u(\tilde{a})}{S}\Big(V^w(z,\tilde{a}) - U(\tilde{a})\Big)^+ + \frac{\phi e}{S}\int_0^\infty \Big(V^w(z,\tilde{a}) - V^w(\tilde{z},\tilde{a})\Big)^+ g(\tilde{z},\tilde{a})d\tilde{z}\right) d\tilde{a}\right)^{\frac{1}{\eta_v}} \tag{79}$$

where $\widehat{u}(a)$ is the number of age a unemployed and e the total number of employed.

Subtracting the value of unemployment (73) from that of entrepreneurship (78), and imposing the optimal stopping time conditions $V^e(\underline{z}(a), a) = U(a)$ and $V^e_z(\underline{z}(a), a) = 0$ for all a implies that

$$b(a) + \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} s_u(a)^{1+\eta_e} \quad = \quad k(a) - r + \frac{\sigma^2}{2} V^e_{zz}\Big(\underline{z}(a), a\Big)$$

Using (77), the fixed cost is

$$r \quad = \quad k(a) + \frac{\sigma^2}{2} V^e_{zz}\Big(\underline{z}(a), a\Big) - e^{\underline{z}^w(a)} + c - \frac{\sigma^2}{2} V^w_{zz}\Big(\underline{z}^w(a), a\Big) - \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} s_u(a)^{1+\eta_e} \tag{80}$$

where since the economy is normalized to the least productive firm,

$$\min_a \underline{z}(a) \quad = \quad 0 \tag{81}$$

D.4 Equilibrium in the full model

The distribution of workers over productivity and age, $g(z,a)$, is given by the FP equation

$$
\begin{aligned}
\underbrace{\lambda g(z,a)}_{\text{labor supply growth}} &= -\underbrace{g_a(z,a)}_{\text{aging}} \\
&- \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa g(z,a)}_{\text{labor force exit}} \\
&+ \underbrace{m g_z(z,a)}_{\text{technological obsolescence at rate -}m} \\
&+ \underbrace{\frac{\sigma^2}{2} g_{zz}(z,a)}_{\text{productivity shocks}} \\
&- \underbrace{\delta(z) g(z,a)}_{\text{exogenous match separation}} \\
&- \underbrace{d g(z,a)}_{\text{exogenous firm exit}} \\
&- \underbrace{\pi(a) s(z,a) g(z,a)}_{\text{exit to entrepreneurship}} \\
&- \underbrace{\phi p \Big(1 - F(z)\Big) g(z,a)}_{\text{separations to employment}} \\
&+ \underbrace{f(z) p \frac{\widehat{u}(a)}{e}}_{\text{hires from unemployment}} \\
&+ \underbrace{\phi p f(z) \int_{\underline{z}^w(a)}^{z} g(\tilde{z},a) d\tilde{z}}_{\text{hires from employment}}
\end{aligned}
\tag{82}
$$

where $e = 1 - l - \int_0^\infty \widehat{u}(a)da$, subject to the boundary conditions

$$
g(\underline{z}^w(a), a) = 0 \tag{83}
$$

$$
\int_0^\infty \int_0^\infty g(z,a) dz da = 1 \tag{84}
$$

The first boundary condition (83) imposes that the density is zero at the boundary, which holds in the presence of shocks, $\sigma > 0$. The intuition is that at $\underline{z}^w(a)$, the outflow due to the shocks is an order of magnitude larger than the inflow, because there is no inflow of workers from below in the distribution (since workers exit whenever their productivity falls below $\underline{z}^w(a)$). The second boundary condition (84) is implied by the fact that $g(z,a)$ is a density.

The number of unemployed of age a are given by

$$\begin{aligned}
\underbrace{\lambda\widehat{u}(a)}_{\text{labor supply growth}} &= -\underbrace{\widehat{u}'(a)}_{\text{aging}} && (85)\\
&- \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa\widehat{u}(a)}_{\text{labor force exit}}\\
&- \underbrace{p\widehat{u}(a)}_{\text{outflow to employment}}\\
&- \underbrace{\pi(a)s_u(a)\widehat{u}(a)}_{\text{outflow to entrepreneurship}}\\
&+ \underbrace{\frac{\sigma^2}{2}g_z\left(\underline{z}^w(a),a\right)e}_{\text{endogenous separation (workers)}}\\
&+ \underbrace{e\int_{\underline{z}^w(a)}^{\infty}\delta(z)g(z,a)dz}_{\text{exogenous match separation}}\\
&+ \underbrace{ed\int_{\underline{z}^w(a)}^{\infty}g(z,a)dz}_{\text{exogenous firm exit (workers)}}\\
&+ \underbrace{\frac{\sigma^2}{2}x_z\left(\underline{z}(a),a\right)l}_{\text{endogenous separation (entrepreneurs)}}\\
&+ \underbrace{ld\int_{\underline{z}(a)}^{\infty}x(z,a)dz}_{\text{exogenous firm exit (entrepreneurs)}}
\end{aligned}$$

where

$$\widehat{u}(0) = \kappa(1+(1-\omega)\nu)\int_{\overline{A}}^{\infty}\left(\widehat{u}(a)+(1-l)\int_{\underline{z}^w(a)}^{\infty}g(z,a)dz\right)da \qquad (86)$$

The evolution of the share of entrepreneurs with productivity z and age a is given by

$$\begin{aligned}
\lambda x(z,a) \quad = \quad & -\underbrace{x_a(z,a)}_{\text{aging}} \\
& - \underbrace{\mathbb{1}\{a \geq \overline{A}\}\kappa x(z,a)}_{\text{labor force exit}} \\
& + \underbrace{m x_z(z,a)}_{\text{technological obsolescence at rate -}m} \\
& + \underbrace{\frac{\sigma^2}{2} x_{zz}(z,a)}_{\text{productivity shocks}} \\
& + \underbrace{\frac{\pi(a)}{l}\widehat{u}(a)\gamma\left(z|\underline{z}^w(a)\right)s_u(a)}_{\text{entry from unemployment}} \\
& + \underbrace{\frac{\pi(a)}{l} e \int_{\underline{z}^w(a)}^{\infty} \gamma(z|\tilde{z})s(\tilde{z},a)g(\tilde{z},a)d\tilde{z}}_{\text{entry from employment}}
\end{aligned} \tag{87}$$

subject to the boundary conditions

$$x(\underline{z}(a),a) \quad = \quad 0 \tag{88}$$

$$\int_0^\infty \int_0^\infty x(z,a)dzda \quad = \quad 1 \tag{89}$$

$$x(z,0) \quad = \quad \kappa(1+(1-\omega)\nu)\int_{\overline{A}}^{\infty} x(z,a)da \tag{90}$$

Definition 2. *A stationary equilibrium consists of value functions $U(a)$, $V^w(z,a)$ and $V^e(z,a)$; reservation thresholds $\underline{z}^w(a)$ and $\underline{z}(a)$; search policies $s_u(a)$ and $s(z,a)$; a vacancy policy $v(z,a)$; a fixed cost, r; finding rates p and q; a distribution of workers $g(z,a)$, number of employed and unemployed $\{e(a), \widehat{u}(a)\}$, and a distribution of entrepreneurs $x(z,a)$; an offer distribution $f(z)$, an aggregate mass of vacancies V, and aggregate search intensity S; an aggregate exit rate $\widehat{y}(m)$; an aggregate entry rate $y(m)$; and a rate of obsolescence, m, such that:*

1. *The value function $U(a)$ is given by (73) and the optimal search intensity $s_u(a)$ by (74);*

2. *The value function $V^w(z,a)$ is given by (75), the optimal search intensity $s(z,a)$ by (76), and the optimal reservation threshold $\underline{z}^w(a)$ by (77);*

3. *The value function $V^e(z,a)$ is given by (78), the optimal vacancy policy by (79), and the optimal reservation policy and fixed cost by the system (80)–(81);*

4. *The finding rates p and q are given by (3) given aggregate vacancies V and an aggregate search intensity S;*

5. *The distribution of workers, $g(z,a)$, the number of unemployed, $\widehat{u}(a)$, and the distribution of entrepreneurs, $x(z,a)$, are given by (82)–(90).*

6. *The vacancy-weighted distribution of firms $f(z)$ is given by*

$$f(z) \quad = \quad \frac{l}{V}\int_0^\infty v(z,a)x(z,a)da$$

where the aggregate number of vacancies V are given by

$$V \quad = \quad l\int_0^\infty \int_0^\infty v(z,a)x(z,a)dzda$$

and aggregate search intensity is $S = u + \phi e$, where $u = \int_0^\infty \widehat{u}(a)da$ and $e = 1 - u$;

7. *The aggregate exit rate is*

$$\widehat{y}(m) \quad = \quad l\left(d + \frac{\sigma^2}{2}\int_0^\infty x_z(\underline{z}(a),a)da\right)$$

8. *The aggregate entry rate is*

$$y(m) \quad = \quad \int_0^\infty \pi(a)\left(s_u(a)\widehat{u}(a) + e\int_{\underline{z}^w(a)}^\infty s(z,a)g(z,a)dz\right)da$$

9. *The aggregate exit rate equals the aggregate entry rate, $\widehat{y}(m) = y(m)$.*

D.5 Algorithm

Equation (77) can be rewritten as

$$b(a) \quad = \quad e^{\underline{z}^w(a)} - c + \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w(a),a)$$

while equation (80) can be rearranged as

$$k(a) - r \quad = \quad e^{\underline{z}^w(a)} - c - \frac{\sigma^2}{2}V^e_{zz}(\underline{z}(a),a) + \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w(a),a) + \frac{\eta_e}{1+\eta_e}c_e e^{\underline{z}^w(a)}s_u(a)^{1+\eta_e}$$

Substituting for $b(a)$ and $k(a) - r$ using these equations, optimal search intensity in unemployment using (74) and employment using (76), and optimal vacancy creation using (79), the value functions can be written as (suppressing the arguments z and a whenever possible to simplify the notation)

$$\begin{aligned}
(\rho+\kappa-m)U &= e^{\underline{z}^w}-c \\
&+ \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w,a) \\
&+ U_a \\
&+ \pi(a)s_u(a)\frac{\eta_e}{1+\eta_e}\int_0^\infty \Big(V^e(\tilde{z},a)-U\Big)\gamma(\tilde{z}|\underline{z}^w)d\tilde{z} \\
(\rho+\kappa-m)V^w &= e^{z}-c \\
&+ V^w_a \\
&- mV^w_z \\
&+ \frac{\sigma^2}{2}V^w_{zz} \\
&+ \delta(z)\Big(U-V^w\Big) \\
&+ d\Big(U-V^w\Big) \\
&+ \pi(a)s(z,a)\frac{\eta_e}{1+\eta_e}\int_0^\infty \Big(V^e(\tilde{z},a)-V^w\Big)\gamma(\tilde{z}|z)d\tilde{z} \\
(\rho+\kappa-m)V^e &= e^{\underline{z}^w}-c \\
&+ \frac{\sigma^2}{2}V^w_{zz}(\underline{z}^w,a)-\frac{\sigma^2}{2}V^e_{zz}(\underline{z},a)+\pi(a)s_u(a)\frac{\eta_e}{1+\eta_e}\int_0^\infty \Big(V^e(\tilde{z},a)-U\Big)\gamma(\tilde{z}|\underline{z}^w)d\tilde{z} \\
&+ V^e_a \\
&- mV^e_z \\
&+ \frac{\sigma^2}{2}V^e_{zz} \\
&+ d(U-V^e) \\
&+ qv(z,a)\frac{\eta_v}{1+\eta_v}\int_0^\infty \left(\frac{u(\tilde{a})}{S}\Big(V^w-U\Big)^+ + \frac{\phi e}{S}\int_{\underline{z}^w}^\infty \Big(V^w(z,\tilde{a})-V^w(\tilde{z},\tilde{a})\Big)^+ g(\tilde{z},\tilde{a})d\tilde{z}\right)d\tilde{a}
\end{aligned} \tag{91}$$

In estimation, I normalize $\underline{z}(a)=0$ for all a and $\underline{z}^w(a)=\underline{z}^w$ for some constant $\underline{z}^w \geq 0$—in practice, I set it to the second point on the grid for productivity. Under the normalization, I solve the value functions based on (91). Having solved for the equilibrium allocation, I recover the flow values of leisure and of being one's own boss based on (77) and (80).

As the economy ages, the reservation threshold $\underline{z}^w(a)$ will in general change, which imposes an additional computational burden. I note based on (77), however, that the only reason $\underline{z}^w(a)$ changes in response to aging is through changes in the second order term, $\frac{\sigma^2}{2}V^w_{zz}$. Since the value function turns out to not be very concave, to simplify the numerical solution, I assume that $b(a)$ adjusts in response to aging such that $\underline{z}^w(a)$ remains fixed, i.e. I set updated flow values of leisure based on

$$\widehat{b}(a) \quad = \quad e^{\underline{z}^w(a)} - c + \frac{\sigma^2}{2}\widehat{V}^w_{zz}(\underline{z}^w(a), a)$$

where $\widehat{V}^w(z,a)$ is the value of a match under a different age composition. In practice given the estimated volatility of productivity, the required changes in $b(a)$ in response to the empirically relevant amount of aging are small.

In response to aging, the reservation threshold $\underline{z}(a)$ will also in general change. Although I set $k(a)$ such that $\underline{z}(a) = 0$ for all a in estimation and even though the fixed cost r adjusts in response to aging based on (81) such that $\min_a \underline{z}(a) = 0$, it may be that $\underline{z}(a) > 0$ for some a as the economy ages. I simplify by assuming that also $k(a)$ adjusts based on (80) such that $\underline{z}(a) = 0$ for all a. That is, I first compute the new fixed cost based on

$$\widehat{r} \quad = \quad \max_a \left\{ k(a) + \frac{\sigma^2}{2}\widehat{V}^e_{zz}(\underline{z}(a), a) - e^{\underline{z}^w(a)} + c - \frac{\sigma^2}{2}\widehat{V}^w_{zz}(\underline{z}^w(a), a) - \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} \widehat{s}_u(a)^{1+\eta_e} \right\}$$

Setting the fixed cost to the highest valuation of land across age groups ensures that all age groups want to exit at $\underline{z}(a) \geq 0$. Given a new fixed cost $\widehat{r}$, I recover updated flow values of being one's own boss based on

$$\widehat{k}(a) \quad = \quad \widehat{r} - \frac{\sigma^2}{2}\widehat{V}^e_{zz}(\underline{z}(a), a) + e^{\underline{z}^w(a)} - c + \frac{\sigma^2}{2}\widehat{V}^w_{zz}(\underline{z}^w(a), a) + \frac{\eta_e}{1+\eta_e} c_e e^{\underline{z}^w(a)} \widehat{s}_u(a)^{1+\eta_e}$$

In practice, the required adjustments to $k(a)$ are small.

These two simplifying assumptions imply that the system (91) can be used to also solve for the value functions across BGPs and over the transition path.

Define the stacked vector of values

$$W = \begin{bmatrix} U(a_1) \\ V^w(z_1, a_1) \\ \vdots \\ V^w(z_{N_z}, a_1) \\ V^e(z_1, a_1) \\ \vdots \\ V^e(z_{N_z}, a_1) \\ \vdots \\ U(a_{N_a}) \\ V^w(z_1, a_{N_a}) \\ \vdots \\ V^w(z_{N_z}, a_{N_a}) \\ V^e(z_1, a_{N_a}) \\ \vdots \\ V^e(z_{N_z}, a_{N_a}) \end{bmatrix}$$

and the flow payoff vector

$$Y = \mathbb{1}_{N_a \times 1} \otimes \begin{bmatrix} e^{z_1} - c \\ e^{z_1} - c \\ e^{z_2} - c \\ \vdots \\ e^{z_{N_z}} - c \\ e^{z_1} - c \\ \vdots \\ e^{z_1} - c \end{bmatrix}$$

where $\mathbb{1}_{n \times 1}$ is an $n \times 1$ vector of ones. Note that I set the flow value of leisure such that the reservation threshold of workers is the first point on the productivity grid.

Define the stacked distribution of individuals

$$g = \begin{bmatrix} u(a_1) \\ e * g(z_1, a_1) \\ \vdots \\ e * g(z_{N_z}, a_1) \\ l * x(z_1, a_1) \\ \vdots \\ l * x(z_{N_z}, a_1) \\ \vdots \\ u(a_{N_a}) \\ e * g(z_1, a_{N_a}) \\ \vdots \\ e * g(z_{N_z}, a_{N_a}) \\ l * x(z_1, a_{N_a}) \\ \vdots \\ l * x(z_{N_z}, a_{N_a}) \end{bmatrix}$$

and the entry vector

$$B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Exogenous matrices. Define the following matrices which do not change with decisions:

1. The aging matrix

$$\mathbf{A} = \frac{1}{da}\texttt{spdiags}\left(\begin{bmatrix} -\mathbb{1}_{((N_a-1)(2N_z+1))\times 1} & 0_{(2N_z+1)\times 1} \\ 0_{(2N_z+1)\times 1} & \mathbb{1}_{((N_a-1)(2N_z+1))\times 1} \end{bmatrix}, [0, 2N_z+1], N_a(2N_z+1), N_a(2N_z+1) \right)$$

2. The exit matrices

$$\begin{aligned}
\mathbf{X}^v &= -\kappa \times \texttt{spdiags}\left(\left[\begin{array}{c} 0_{(N_a-1)(2N_z+1)\times 1} \\ \mathbb{1}_{(2N_z+1)\times 1} \end{array}\right], 0, N_a(2N_z+1), N_a(2N_z+1)\right) \\
\mathbf{X}_1 &= -\kappa \times \texttt{spdiags}\left(\left[\begin{array}{c} 0_{(N_z+1)\times 1} \\ \mathbb{1}_{N_z\times 1} \end{array}\right], 0, 2N_z+1, 2N_z+1\right) \\
\mathbf{X}_2 &= \texttt{spdiags}\Big(N_a, 1, 1, N_a, N_a\Big) \\
\mathbf{X}^d &= \mathbf{X}^v + \mathbf{X}_2 \otimes \mathbf{X}_1
\end{aligned}$$

3. The labor supply matrix

$$\begin{aligned}
\mathbf{L}_1 &= -\lambda \times \texttt{speye}(N_a) \otimes \texttt{spdiags}\left(\left[\begin{array}{c} \mathbb{1}_{(N_z+1)\times 1} \\ 0_{N_z\times 1} \end{array}\right], 0, 2Nz+1, 2Nz+1\right) \\
\mathbf{L}_2 &= -\lambda \times \texttt{speye}(N_a) \otimes \texttt{spdiags}\left(\left[\begin{array}{c} 0_{(N_z+1)\times 1} \\ \mathbb{1}_{N_z\times 1} \end{array}\right], 0, 2Nz+1, 2Nz+1\right) \\
\mathbf{L}_2 &= \mathbf{L}_2 - \texttt{sum}\Big(\mathbf{L}_2, 2\Big) \\
\mathbf{L} &= \mathbf{L}_1 + \mathbf{L}_2
\end{aligned}$$

4. The drift matrix

$$\mathbf{D} = \frac{1}{dz}\texttt{speye}(N_a) \otimes \texttt{spdiags}\left(\left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -\mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 \\ 1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -\mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 \end{array}\right], [-1, 0], 2N_z+1, 2N_z+1\right)$$

5. The shock matrix

$$
\begin{aligned}
\mathbf{S} &= \frac{1}{dz^2}\texttt{speye}(N_a) \\
&\otimes \ \texttt{spdiags}\left(\begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -2_{(N_z-2)\times 1} & \mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 & 1 \\ 1 & -1 & 0 \\ \mathbb{1}_{(N_z-2)\times 1} & -2_{(N_z-2)\times 1} & \mathbb{1}_{(N_z-2)\times 1} \\ 0 & -1 & 1 \end{bmatrix}, -1:1, 2N_z+1, 2N_z+1\right)
\end{aligned}
$$

6. The exogenous separation matrix

$$
\mathbf{Q}_1 = \texttt{speye}(N_a) \otimes \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 & \\ \delta(z_1) & -\delta(z_1) & 0 & \cdots & \vdots & \\ \vdots & 0 & \ddots & \ddots & \vdots & 0_{(N_z+1)\times N_z} \\ \vdots & & \ddots & \ddots & 0 & \\ \delta(z_{N_z}) & 0 & \cdots & 0 & -\delta(z_{N_z}) & \\ & & 0_{(N_z)\times(2N_z+1)} & & & \end{bmatrix}
$$

7. The exogenous exit matrix

$$
\mathbf{E}_1 = d \times \texttt{speye}(N_a) \otimes \begin{bmatrix} 0 & & 0_{1\times 2N_z} & & \\ 1 & -1 & & 0_{1\times(2N_z-1)} & \\ 1 & 0 & -1 & 0_{1\times(2N_z-2)} & \\ \vdots & \vdots & & \ddots & \ddots \\ 1 & 0 & \ldots & 0 & -1 \end{bmatrix}
$$

I combine these matrices into the exogenous transition matrices

$$
\begin{aligned}
\mathbf{T}^v &= \mathbf{A} + \mathbf{X}^v + m\mathbf{D} + \frac{\sigma^2}{2}\mathbf{S} + \mathbf{Q}_1 + \mathbf{E}_1 \\
\mathbf{T}^d &= \mathbf{A} + \mathbf{X}^d + \mathbf{L} + m\mathbf{D} + \frac{\sigma^2}{2}\mathbf{S} + \mathbf{Q}_1 + \mathbf{E}_1
\end{aligned}
$$

Note that $\mathbf{T}^v$ and $\mathbf{T}^d$ can be constructed without solving for individuals' optimal behavior, i.e. they do not have to be updated.

Next, define the flow value of leisure matrix

$$\mathbf{B} = \frac{\sigma^2}{2dz^2}\texttt{speye}(N_a) \otimes \begin{bmatrix} 0 & 1 & -2 & 1 & 0_{1\times 2N_z-3} \\ & & 0_{N_z\times(2N_z+1)} & & \\ 0 & 1 & -2 & 1 & 0_{1\times 2N_z-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & -2 & 1 & 0_{1\times 2N_z-3} \end{bmatrix}$$

and the flow value of being one's own boss matrix

$$\mathbf{K} = -\frac{\sigma^2}{2dz^2}\texttt{speye}(N_a) \otimes \begin{bmatrix} & & 0_{N_z+1\times(2N_z+1)} & & \\ 0_{1\times N_z+1} & 1 & -2 & 1 & 0_{1\times N_z-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1\times N_z+1} & 1 & -2 & 1 & 0_{1\times N_z-3} \end{bmatrix}$$

These matrices can also be constructed without solving for individuals' optimal behavior.

Endogenous matrices. It remains to construct the matrices that change across iterations as individuals' optimal behavior is updated. To that end, define first the following useful vectors

1. The arrival of ideas vector

$$\Pi = \begin{bmatrix} \pi(a_1) \\ \vdots \\ \pi(a_{N_a}) \end{bmatrix} \otimes \mathbb{1}_{(2N_z+1)\times 1}$$

where $\pi(a_1) = \pi^l$ if $a_1 < \overline{A}$ and π^h otherwise.

2. The cost of search vectors

$$Z = \mathbb{1}_{N_a \times 1} \otimes \begin{bmatrix} e^{-z_1} \\ e^{-z_1} \\ \vdots \\ e^{-z_{N_z}} \\ e^{-z_1} \\ \vdots \\ e^{-z_{N_z}} \end{bmatrix}$$

$$ZE = \mathbb{1}_{N_a \times 1} \otimes \begin{bmatrix} e^{-z_1} \\ e^{-z_1} \\ \vdots \\ e^{-z_1} \\ e^{-z_1} \\ \vdots \\ e^{-z_1} \end{bmatrix}$$

Then define the following matrices

1. The endogenous separation matrix

$$\mathbf{Q}_2 = v \times \texttt{speye}(N_a) \otimes \begin{bmatrix} 0 & & 0_{1\times 2N_z} & \\ 1 & -1 & & 0_{1\times(2N_z-1)} \\ 1 & 0 & -1 & 0_{1\times(2N_z-2)} \\ & & 0_{(2N_z-2)\times(2N_z+1)} & \end{bmatrix}$$

where v is a large number.

2. The endogenous exit matrix

$$\mathbf{E}_2 = v \times \texttt{speye}(N_a) \otimes \begin{bmatrix} & & 0_{(N_z+1)\times(2N_z+1)} & \\ 1 & 0_{1\times N_z} & -1 & 0_{1\times(N_z-1)} \\ & & 0_{(N_z-1)\times(2N_z+1)} & \end{bmatrix}$$

where v is a large number.

3. The innovation matrix

$$\mathbf{\Gamma} \quad = \quad \texttt{speye}(N_a) \otimes \begin{bmatrix} -1 & 0 & \cdots & 0 & \gamma(z_1|z_2) & \cdots & \gamma(z_{N_z}|z_2) \\ 0 & \ddots & \ddots & \vdots & \gamma(z_1|z_1) & \cdots & \gamma(z_{N_z}|z_1) \\ \vdots & \ddots & \ddots & 0 & & & \\ 0 & \cdots & 0 & -1 & \gamma(z_1|z_{N_z}) & \cdots & \gamma(z_{N_z}|z_{N_z}) \\ & & & 0_{N_z \times (2N_z+1)} & & & \end{bmatrix}$$

and

$$\mathbf{\Gamma}_e \quad = \quad \texttt{speye}(N_a) \otimes \begin{bmatrix} & & 0_{(N_z+1)\times(2N_z+1)} & & \\ -1 & 0_{1\times 2N} & \gamma(z_1|z_2) & \cdots & \gamma(z_{N_z}|z_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & 0_{1\times 2N} & \gamma(z_1|z_2) & \cdots & \gamma(z_{N_z}|z_2) \end{bmatrix}$$

These objects need to be updated in each iteration, because they depend on average productivity of incumbents through general knowledge spillovers. The mobility into entrepreneurship vector is

$$P \quad = \quad \Pi .* \left(\frac{1}{c_e} Z .* \Pi .* (\mathbf{\Gamma} * \mathbf{W}) \right)^{\frac{1}{\eta_e}}$$

where the power is element by element, and the forgone option of search vector is

$$P_e \quad = \quad \Pi .* \left(\frac{1}{c_e e^{z_2}} .* \Pi .* (\mathbf{\Gamma}_e * \mathbf{W}) \right)^{\frac{1}{\eta_e}}$$

Define the matrices

$$\begin{aligned} \mathbf{P}_1^v &= \frac{\eta_e}{1+\eta_e} * P .* \mathbf{\Gamma} \\ \mathbf{P}_2^v &= \frac{\eta_e}{1+\eta_e} * P_e .* \mathbf{\Gamma}_e \\ \mathbf{P}^d &= P .* \mathbf{\Gamma} \end{aligned}$$

4. The search-weighted distribution of hires matrix needs to be defined iteratively, looping over age.

To that end, let $\mathbf{g} = \texttt{reshape}(g, 2N_z+1, N_a)$. Then for $i = 1, \cdots, N_a$, define

$$\mathbf{G}_i = \begin{bmatrix} & & 0_{(N_z+1)\times(N_z+1)} & & & \\ -\mathbf{g}(u,i) & \mathbf{g}(u,i) & 0 & \cdots & 0 & \\ -\mathbf{g}(u,i) & -\phi * \mathbf{g}(z_1,i) & \mathbf{g}(u,i) + \phi \sum_{j=1}^{1} \mathbf{g}(z_j,i) & \ddots & \vdots & \\ -\mathbf{g}(u,i) & -\phi * \mathbf{g}(z_1,i) & \ddots & \ddots & \vdots & 0_{(2N_z+1)\times N_z} \\ \vdots & \vdots & \ddots & \ddots & 0 & \\ -\mathbf{g}(u,i) & -\phi * \mathbf{g}(z_1,i) & \cdots & \cdots & \mathbf{g}(u,i) + \phi \sum_{j=1}^{N_z-1} \mathbf{g}(z_j,i) & \end{bmatrix}$$

then the stacked matrix

$$\mathbf{G} = \frac{1}{S} \mathbb{1}_{N_a \times 1} \otimes \left[\mathbf{G}_1, \ldots, \mathbf{G}_{N_z} \right]$$

and the return to vacancy creation vector

$$R = \frac{1}{c_v} Z. * (\mathbf{G} * \mathbf{W})$$

Then

$$\begin{aligned}
q &= \chi V^{\theta-1} S^{1-\theta} \\
q &= \chi \left(l \int_0^\infty \int_0^\infty v(z,a) x(z,a) dz da \right)^{\theta-1} S^{1-\theta} \\
q &= \chi \left(l \int_0^\infty \int_0^\infty (qR(z,a))^{\frac{1}{\eta v}} x(z,a) dz da \right)^{\theta-1} S^{1-\theta} \\
q^{\frac{1}{\theta-1}} &= \frac{\chi^{\frac{1}{\theta-1}}}{S} l q^{\frac{1}{\eta v}} \int_0^\infty \int_0^\infty R(z,a)^{\frac{1}{\eta v}} x(z,a) dz da \\
q^{\frac{1}{\theta-1} - \frac{1}{\eta v}} &= \frac{\chi^{\frac{1}{\theta-1}}}{S} l \Big(\left(R^{\frac{1}{\eta v}} \right)^T * \left(\left(\mathbb{1}_{N_a \times 1} \otimes \begin{bmatrix} 0_{(N_z+1)\times 1} \\ \mathbb{1}_{N_z \times 1} \end{bmatrix} \right) . * g \right) \qquad (92)
\end{aligned}$$

Once q is pinned down by (92), aggregate vacancies V, the job finding rate p and firm vacancies are

$$\begin{aligned}
V &= \left(\frac{q}{\chi} \right)^{\frac{1}{\theta-1}} S \\
p &= \chi \left(\frac{V}{S} \right)^{\theta} \\
v &= \left(qR \right)^{\frac{1}{\eta v}}
\end{aligned}$$

The hiring matrix is

$$\mathbf{V}^v = qv\frac{\eta_v}{1+\eta_v}.*\mathbf{G}$$

Define $\mathbf{v} = \texttt{reshape}\,(v, 2N_z+1, N_a)$ and the vacancy shares

$$f = \texttt{sum}\Big(\mathbf{v}(N_z+2:\texttt{end},:).*\mathbf{g}(N_z+2:\texttt{end},:),2\Big)/V$$

Then the mobility matrix is

$$\mathbf{M} = p\begin{bmatrix} \begin{matrix} 0 & f_1 & f_2 & \cdots & f_{N_z} \\ 0 & 0 & \phi f_2 & \cdots & \phi f_{N_z} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \phi f_{N_z} \\ 0 & \cdots & \cdots & \cdots & 0 \end{matrix} & 0_{N_z+1,N_z} \\ \multicolumn{2}{c}{0_{N_z\times(2N_z+1)}} \end{bmatrix}$$

$$\mathbf{V}^d = \texttt{speye}(N_a)\otimes\Big(\mathbf{M} - \texttt{spdiags}\ \Big(\texttt{sum}\Big(\mathbf{M},2\Big),0,2N_z+1,2N+1\Big)\Big)$$

Using these matrices, the value function is updated according to

$$\mathbf{W}' = \Big(\Big(\rho+\kappa-m+1/dt\Big)\texttt{speye}(N_a(2Nz+1)) - \Big(\mathbf{T}+\mathbf{B}+\mathbf{K}+\mathbf{P}^v+\mathbf{V}^v\Big)\Big)\setminus\Big(F+1/dt\mathbf{W}\Big)$$

and the distribution of individuals is updated according to

$$g = -\Big(\Big(\mathbf{T}+\mathbf{P}^d+\mathbf{V}^d+\mathbf{L}\Big)\Big)^T\setminus B$$

D.6 Additional moments and parameters

Panel A of Figure 31 contrasts the labor force participation rate by age groups in the model with the data. Panel B plots the share of the labor force by age in 2014–2018. The model matches these outcomes well. Panel C plots the estimated flow values of leisure, $b(a)$, and of being one's own boss, $k(a)$, relative to average age-conditional match output. On average, the flow value of leisure corresponds to 5–25 percent of match output, while the flow value of being one's own boss is about 20 percent of average match output.

FIGURE 31. ADDITIONAL MOMENTS AND PARAMETERS

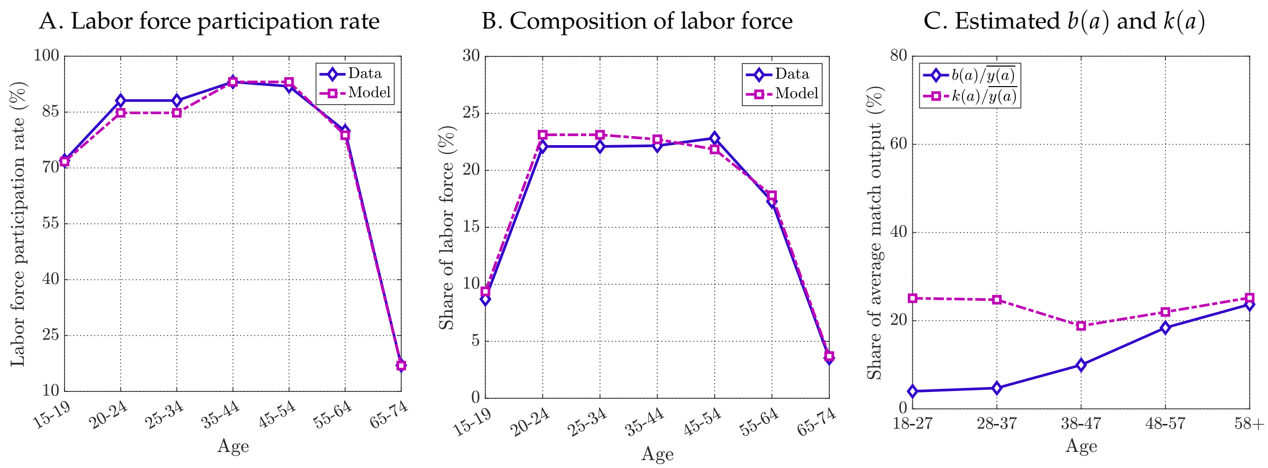


Figure 31 shows additional moments and parameters in the model and data. Panel A plots the share of the population that participates in the labor market by age groups. Panel B plots each age group's share of the labor force (all labor force participants aged 15–74). Panel C plots the estimated flow values of leisure, $b(a)$, and of being one's own boss, $k(a)$, relative to average flow output per employed worker of that age group, $\overline{y}(a)$. All data moments refer to 2014–2018 averages. *Source:* AKU, model.

D.7 Identification

To determine the internally estimated parameters, I use a two-step algorithm. First, I search for an approximate global solution for the 13 internally estimated parameters. To that end, I sample parameter vectors randomly across a wide grid of potential values using Sobol sequences. This approach is beneficial because it avoids getting stuck when a particular parameter vector is not associated with an equilibrium. For each draw of a parameter vector, I determine internally the flow value of leisure such that workers' reservation threshold equals the reduced form specification $\underline{z}^w(a) = \beta a$ and the flow value of being one's own boss such that entrepreneurs of each age are indifferent between keeping their firm in business and exiting to unemployment at the lowest grid point for productivity. I solve the model, compute a set of moments, and store these. Subsequently, I pick the parameter vector that minimizes the sum of squared percentage deviations between the 13 targeted moments in the model and data.

In the second step, I perfect the global solution by constructing a new grid for each of the 13 parameters that spans a smaller ball around the approximate global solution. I again solve the model a large number of times for parameter vectors in this more narrow space, and record a set of moments. I pick as the estimated parameter vector that which minimizes the objective function.

Although the estimation is joint in the sense that all moments inform all parameters, some moments are more informative of some parameters. To highlight the heuristic identification argument made in Section 5, Figure 32 plots how each parameter affects its designated moment as it varies around its estimated value, holding all other parameters fixed at their estimated values. Each parameter induces a distinct movement in its chosen targeted moment, in the expected direction. That is, a greater efficiency of the matching function (χ) raises the NE rate. A higher relative search intensity in employment (ϕ) raises JJ mobility, although the impact is surprisingly modest. A larger rise in the reservation threshold with age (b_a) leads to a smaller ratio of JJ mobility at age 50 to age 30. The reason is that it implies that older workers are pickier in terms of what offers they accept out of non-employment, such that their subsequent probability of accepting an outside offer is smaller.

A higher exogenous separation rate (δ_0) raises the EN rate. A more negative slope with productivity (δ_1)—going to the right in the graph—is associated with a larger fall in the EN rate with productivity. A higher arrival rate of business ideas per unit of search intensity (π_0) is associated with a higher entry rate. A larger increase in the arrival rate with experience (π_1) is associated with a lower entry rate at age 20 to age 30. A higher curvature of the cost of searching for ideas (η_e) leads to a smaller decline in entry with productivity.

Smaller general knowledge spillovers (α_0)—going to the right in the graph since the estimated α_0 is negative—is associated with a larger productivity gap between firms of age 1 and 10. A higher α_1 leads to relatively higher productivity of firms started by individuals who were previously employed in better jobs. Larger dispersion in the innovation distribution (ζ) is associated with higher productivity dispersion among entrant firms. Higher dispersion of subsequent shocks (σ) leads to greater productivity dispersion among older firms. A higher curvature of the vacancy cost (η_v) is associated with a smaller vacancy share of the most productive firms.

FIGURE 32. CHANGE IN TARGETED MOMENT IN RESPONSE TO EACH PARAMETER

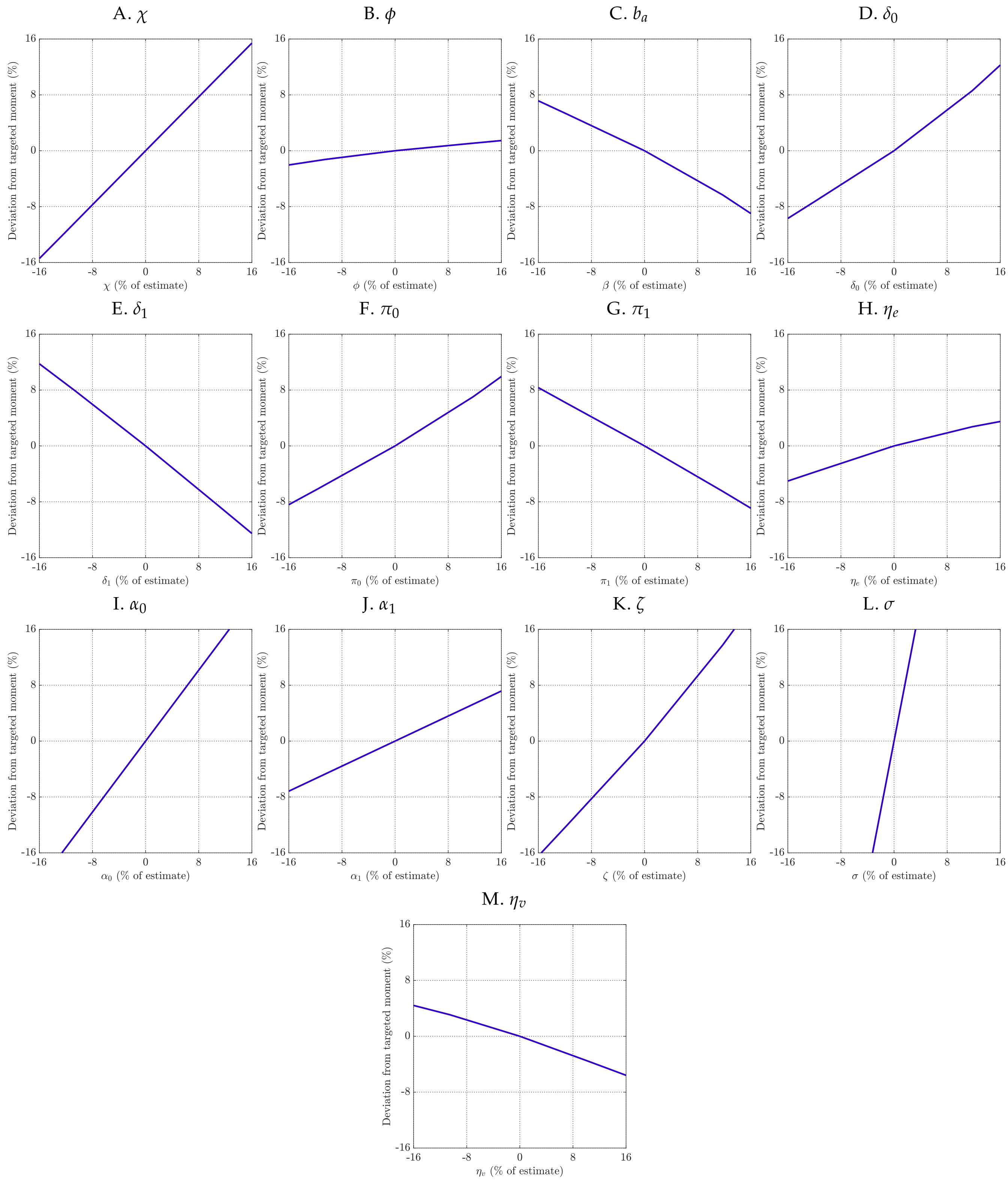


Figure 32 plots how each moment (in percent deviations from that at the estimated parameter vector) varies as each highlighted parameter varies, holding all other parameters fixed at their estimated values. *Source:* Model.

Figure 33 instead plots how the overall minimum distance moves with each parameter, holding all other parameters fixed at their estimated values. The parameters appear to be well-informed by the targeted moments, with one main caveat. The model wants η_v to be at the very end of the pre-set global grid. Because in practice the estimated $\eta_v \approx 18$ implies that job creation of incumbents is close to completely inelastic, I opt to terminate the estimation at this high value. I have, however, confirmed that letting η_v take even higher values makes practically no difference to the results (because η_v hits the end of the grid, it is not meaningful to include the minimum distance plot for this parameter).

FIGURE 33. CHANGE IN MINIMUM DISTANCE IN RESPONSE TO EACH PARAMETER

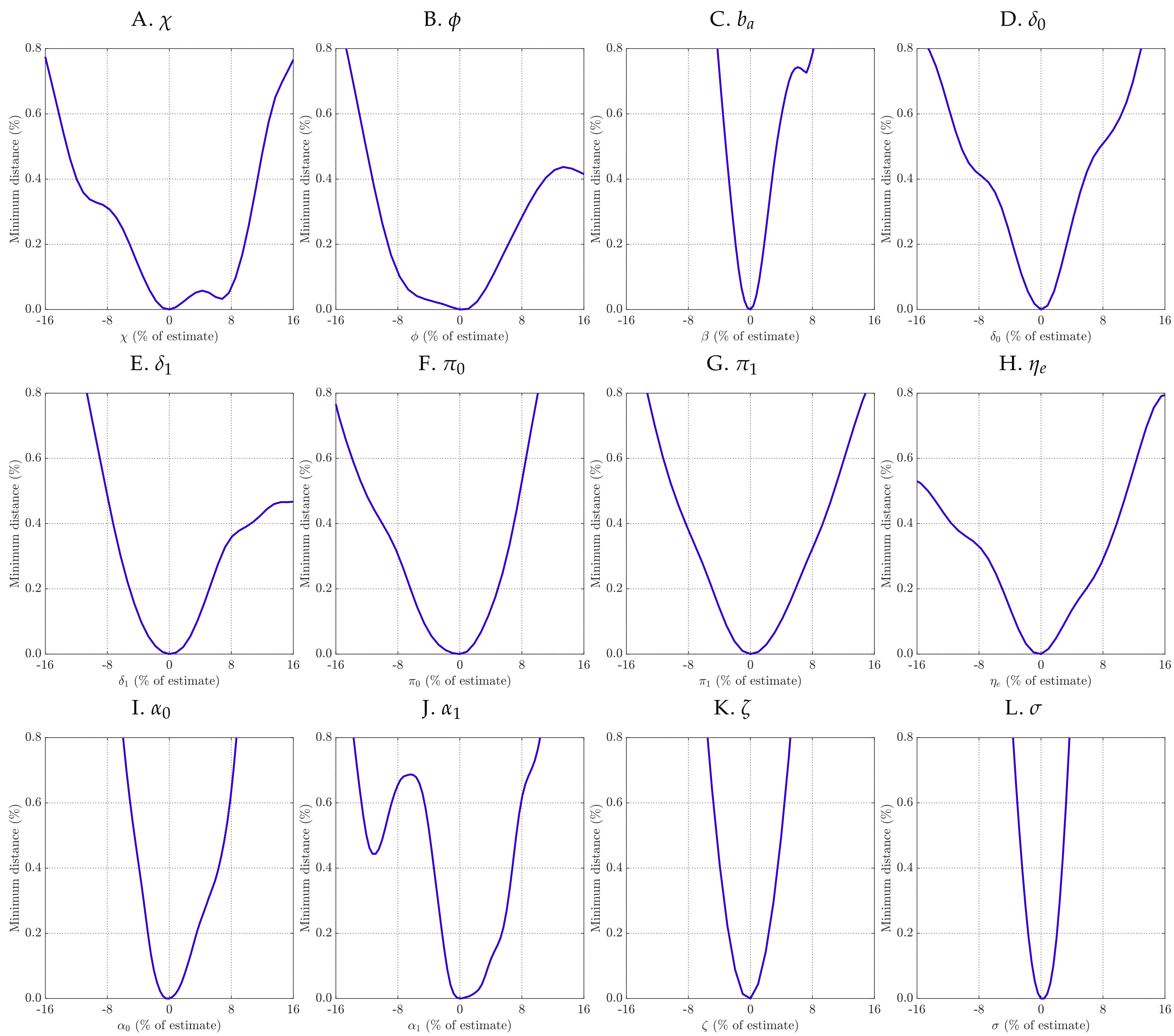


Figure 33 plots how the minimum distance criterium varies as each highlighted parameter varies around its estimated value, holding all other parameters fixed at their estimated values. *Source:* Model.

D.8 A decomposition of life-cycle dynamics

Figure 34 provides a decomposition of life-cycle dynamics in the estimated model into the role of participating in the labor market, climbing the job ladder, and aging. According to panel A, the initial increase in firm creation is accounted for by two forces. First, individuals enter the labor market at a random age such that the model matches the labor force participation rate by age in the data, and by assumption individuals cannot start firms before they have entered the labor market. Second, the arrival rate of ideas is estimated to increase during the first 10 years of careers. The subsequent decline in firm creation with age is accounted for by three forces. First, at some point individuals start exiting the labor force, and by assumption individuals cannot create firms when they do not participate in the labor market. Second, older individuals have a shorter expected time remaining in the market, and are hence less likely to enter. Third, older individuals are better matched and hence have a higher opportunity cost of entry.

Most of the decline in JJ mobility with age is accounted for by individuals moving up the job ladder, with some role also for a higher reservation threshold for older individuals (panel B).

FIGURE 34. LIFE-CYCLE DECOMPOSITION

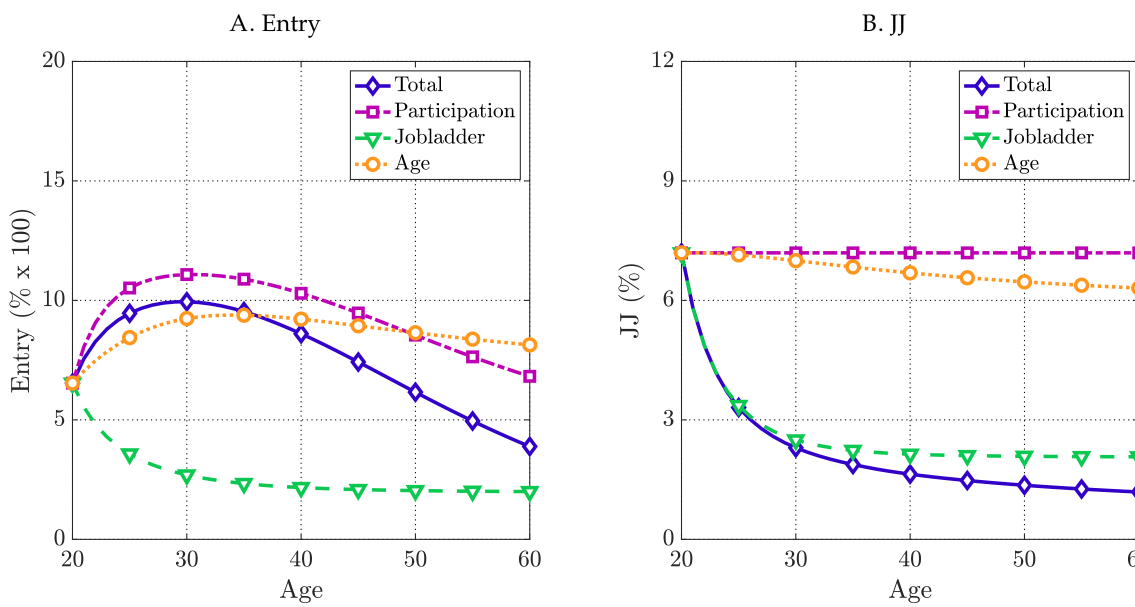


Figure 34 provides a decomposition of life-cycle dynamics in the estimated model. The participation effect in panel A arises because individuals enter the labor market at an age calibrated such that the model matches the empirical labor force participation rate by age, and by assumption individuals cannot start firms before they have entered the labor market. The age effect is due to the fact the arrival rate of business ideas jumps with some minimum labor market experience, and later in life incentives to create firms are lower because individuals have a shorter expected time remaining in the market. The age effect in panel B arises because older individuals are estimated to have a higher reservation threshold ($\beta > 0$). Consequently, they are less likely to accept a low productive job with a high subsequent mobility rate. *Source:* Model.

D.9 Shift-share analysis of the effect of aging

Table 13 provides a shift-share analysis of the impact of aging in the data and model across BGPs.[50] Holding fixed age-specific mobility, shifts in the age composition generates a five percent fall in entry in the data and a four percent decline in the model. Mechanically, shifts in the age composition play a relatively minor role in accounting for the aggregate decline in firm creation, because firm creation is non-monotone in age. Hence, most of the aggregate decline in firm creation is accounted for by an age-specific decline in the probability of entry. Shifts in composition generates an eight percent fall in JJ mobility in the data versus a nine percent fall in the model. Changes in age-specific mobility accounts for another eight percent fall in JJ mobility in the data and a five percent decline in the model. Hence, both shifts in composition and age-specific declines in mobility play an important role in accounting for the declines in JJ mobility. Appendix D.9 provides a further discussion of these effects.

TABLE 13. SHIFT-SHARE ANALYSIS OF EFFECT OF AGING ACROSS BGPS

	Entry				JJ			
	Data		Model		Data		Model	
	p.p.	%	p.p.	%	p.p.	%	p.p.	%
Composition	-0.005	-5.1%	-0.003	-3.5%	-0.227	-8.3%	-0.242	-8.8%
Return	-0.021	-22.0%	-0.009	-11.2%	-0.229	-8.3%	-0.124	-4.5%
Total change	-0.025	-25.4%	-0.013	-14.8%	-0.446	-16.2%	-0.360	-13.1%

Table 5 shows the effect of an increase in the share of the overall labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs, driven by a change in the growth rate of labor supply λ from 0.0013 to 0.0003 percent. All other parameters are held fixed at their estimated values. The composition effect constructs age conditional mobility rates in age bins 16-24, 25-34, 35-44, 45-54 and 55-64, holds these fixed at their level in the early period (1993–1997 for entry and 1986–1990 for JJ mobility), and shifts only share of each age group to match the change in their share of the labor force. In the data, the latter is constructed as the change between 1986–1990 and 2014–2018. In the model, it is constructed as the change in response to a change in λ from 0.0013 to 0.0003 percent. The return effect instead holds each age group's share of the labor force fixed, and shifts age conditional mobility rates. In the data, the latter is constructed as the change between 1993–1997 (entry) and 1986–1990 (JJ), and 2014–2018. In the model, it is constructed as the change in response to a change in λ from 0.0013 to 0.0003 percent. *Source:* AKU, FEK, JOBB, LISA, model.

D.10 Structural versus reduced-form estimates of age-specific impact

Panel B of Figure 35 shows that aging reduces the age-conditional probability of starting a firm, consistent with Swedish trends over this period. The estimated effect of aging on the age-conditional entry rate also matches well the cross-sectional evidence in Section 3. In particular, I plot in Figure 35 the change in age-conditional mobility implied by the OLS and IV estimates in Table 3 in response to the observed change in the share of older labor force participants. Panel C shows that the structural estimate is also consistent with both the time series trend in JJ by age in Sweden as well as the cross-sectional estimates in Section 3.

[50] Table 13 aggregates ages to 10 year age bins in both the model and data, which accounts for the slight difference to Table 5.

FIGURE 35. EFFECT OF AGING ACROSS BGPS

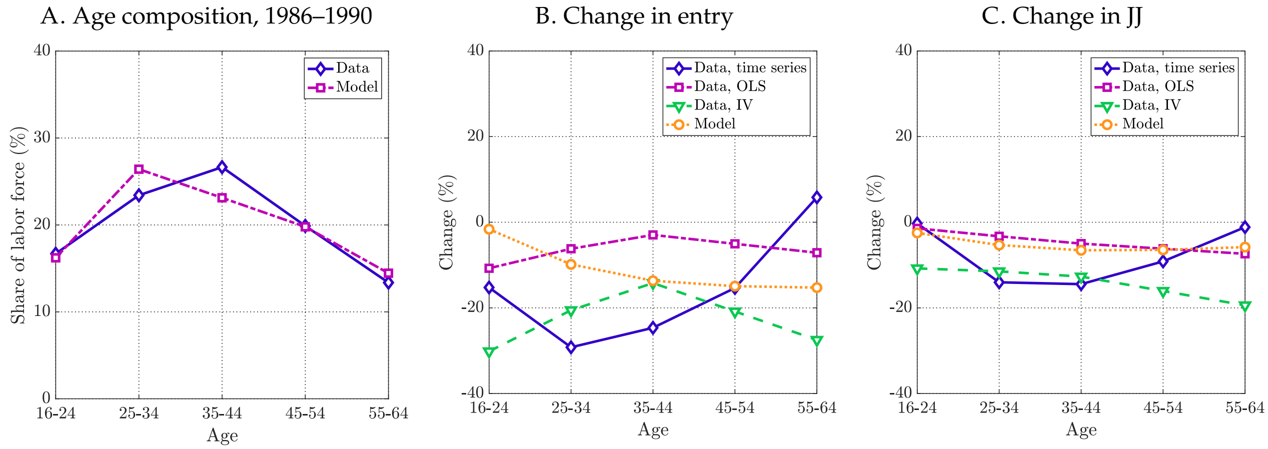


Figure 35 shows the effect of an increase in the share of the overall labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs, driven by a change in the growth rate of labor supply λ from 0.0013 to 0.0003 percent. All other parameters are held fixed at their estimated values. Panel A plots each age group's share of the overall labor force aged 16–64. The time series change in panels B–C plots the change in the entry and JJ rate by age groups between years 1993–1997 (entry) and 1986–1990 (JJ rate) and 2014–2018. The entry rate can only be constructed at the individual level since 1993. The cross-sectional change is that predicted by the cross-sectional OLS estimate by age in Section 3 in response to a 6.6 log point change in the share of older individuals. I linearly interpolate between the three aggregate age groups in Table 3. *Source:* AKU, FEK, JOBB, LISA, model.

D.11 Aging shifts employment toward high-productive firms

Aging impacts the distribution of employment over firm productivity through three channels. First, older individuals tend to be employed by more productive firms, since they have had more time to relocate in the labor market. Consequently, employment shifts toward more productive firm through a composition effect, holding equilibrium objects fixed. The dashed-green line in panel A of Figure 36 illustrates the impact of this composition effect on the average size of firms conditional on productivity.

Second, holding fixed growth, firms create fewer jobs conditional on productivity in the older economy, since they anticipate a harder time filling them (the dash-dotted pink line in panel B). This force is particularly important for low-productive firms, because they depend heavily on poorly matched, typically young workers to fill their jobs, and there are fewer of them in the older economy.[51] The smaller share of job creation by low-productive firms would, *ceteris paribus*, generate a shift of employment toward high-productive firms. At the same time, however, low-productive firms on net lose workers through poaching, and there is less poaching in the older economy. *Ceteris paribus*, the decline in poaching shifts employment toward low-productive firms. On net, these two forces roughly offset, leaving the distribution of employment largely unaffected (the dash-dotted pink line in panel A). Because individuals are discouraged from entering entrepreneurship by the harder hiring environment, however, wage employment rises and self-employment falls, generating a rise in firm size across the board.

[51] This finding is reminiscent of the argument in Moscarini and Postel-Vinay (2018) that low-productive, small firms are better able to compete for workers during economic troughs.

Third, the lower rate of obsolescence in the older economy implies that high-productive firms remain so for longer before they are displaced, and in the process grow larger. Moreover, low-productive firms disproportionately hurt from the harder recruiting environment in the slower growing, better matched economy, such that they cut vacancy creation disproportionately (the solid blue line in panel B). For both reasons, employment shifts further up the job ladder (the solid blue line in panel A).

FIGURE 36. IMPACT OF AGING ON FIRMS ACROSS BGPS

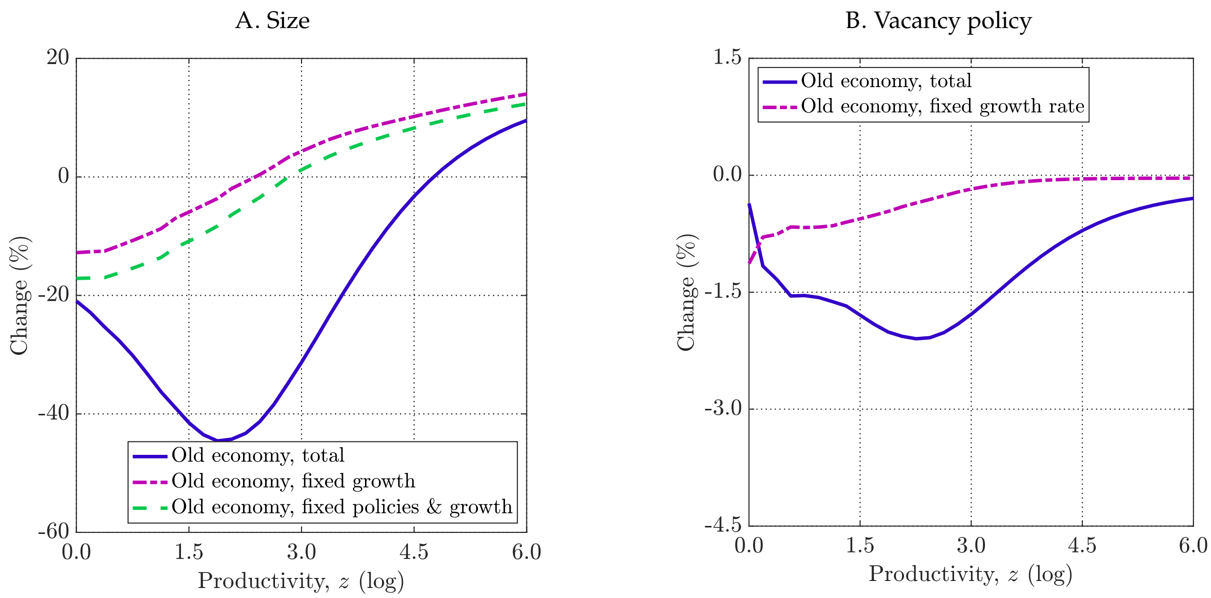


Figure 36 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply λ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. Panel A plots the change in size of firms conditional on productivity. The fixed growth rate line shows the effect of letting policies adjust to the old economy, but counter-factually holding the growth rate fixed at that in the young economy. Panel B plots the size of firms conditional on productivity. *Source:* Model.

D.12 Illustration of the impact of aging

Figure **??** shows how aging impacts the optimal search decision for ideas (panel A) as well as the probability of making a JJ transition (panel B). Panel C shows the distribution of employment over relative productivity. I focus on the youngest age group to reduce clutter, but similar results hold for the other age groups. The fixed growth rate line shows a counterfactual economy in which the growth rate is held fixed at that in the young economy.

D.13 Solving the transition path

I illustrate how to solve for the transition path using the baseline model in Section 4—the extended quantitative model follows the same logic, but at the cost of much additional notation.

FIGURE 37. DISSECTING THE EFFECTS OF AGING ACROSS BGPS

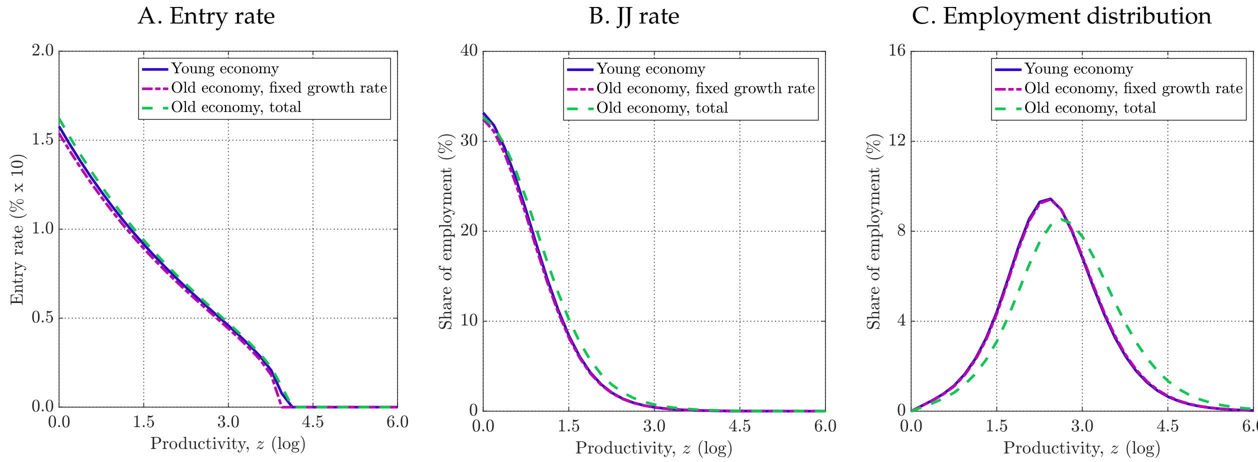


Figure 37 shows the effect of an increase in the share of the labor force aged 16–64 that is aged 45–64 from 34.2 to 39.7 percent across BGPs. It requires a change in the growth rate of labor supply λ from 0.0013 to 0.0003 percent (at a monthly frequency). All other parameters are held fixed at their estimated values. Results are for the youngest age group, $a = 1$. Panel A shows the probability of entering entrepreneurship conditional on the productivity of a potential entrepreneur's current employer. Panel B shows the probability of switching employer conditional on the productivity of a worker's current employer. Panel C plots the distribution of employment over firms by productivity. *Source:* Model.

Over the transition path, the value of unemployment is

$$(\rho + \kappa)\,\hat{U}(t) = \hat{b}(t) + \dot{\hat{U}}(t) \tag{93}$$

subject to the boundary condition that eventually the value grows at the rate of the economy

$$\lim_{t\to\infty}\left(\frac{\dot{\hat{U}}(t)}{\hat{U}(t)}\right) = m \tag{94}$$

where m is the growth rate on the terminal BGP.[52]

The value of a match is given by

$$(\rho + \kappa)\,\hat{V}(\hat{z}, t) = e^{\hat{z}} - \hat{c}(t) + \hat{V}_t(\hat{z}, t)$$

subject to the boundary conditions

$$\begin{aligned} \hat{V}(\underline{\hat{z}}^w(t), t) &= \hat{U}(t) \\ \hat{V}_{\hat{z}}(\underline{\hat{z}}^w(t), t) &= 0 \\ \lim_{t\to\infty}\left(\frac{\hat{V}_t(\hat{z},t)}{\hat{V}(\hat{z},t)}\right) &= 0 \end{aligned}$$

[52]More generally, the value of unemployment (93) as well as the surplus of a match (95) and the surplus of an entrepreneur (100) below have to satisfy *transversality conditions* of the form $\lim_{t\to\infty} e^{-(\rho+\kappa)t}\hat{U}(t) = 0$.

Let $m(t)$ be the growth rate of the economy at time t and define the transformed values

$$\begin{aligned}
\hat{z} &= \underline{\hat{z}}(t) + z \\
\underline{\hat{z}}(t) &= \int_0^t m(\tau) d\tau \\
\hat{U}(t) &= e^{\underline{\hat{z}}(t)} U(t) \\
\hat{V}(\hat{z}, t) &= e^{\underline{\hat{z}}(t)} V(z, t)
\end{aligned}$$

Moreover, assume that the flow value of leisure and the cost of intermediates are proportional to the least productive firm at all times. Then

$$\begin{aligned}
\dot{\hat{U}}(t) &= m(t) e^{\underline{\hat{z}}(t)} U(t) + e^{\underline{\hat{z}}(t)} \dot{U}(t) \\
\hat{V}_{\hat{z}}(\hat{z}, t) &= e^{\underline{\hat{z}}(t)} V_z(z, t) \\
\hat{V}_t(\hat{z}, t) &= m(t) e^{\underline{\hat{z}}(t)} V(z, t) - e^{\underline{\hat{z}}(t)} V_z(z, t) m(t) + e^{\underline{\hat{z}}(t)} V_t(z, t)
\end{aligned}$$

The transformed value of unemployment hence writes

$$\begin{aligned}
(\rho + \kappa) e^{\underline{\hat{z}}(t)} U(t) &= e^{\underline{\hat{z}}(t)} b + m(t) e^{\underline{\hat{z}}(t)} U(t) + e^{\underline{\hat{z}}(t)} \dot{U}(t) \\
(\rho + \kappa - m(t)) U(t) &= b + \dot{U}(t)
\end{aligned}$$

subject to the boundary condition that $\lim_{t \to \infty} \dot{U}(t) = 0$.

The transformed joint value of a match is given by the partial differential equation (PDE)

$$\begin{aligned}
(\rho + \kappa) e^{\underline{\hat{z}}(t)} V(z, t) &= e^{\underline{\hat{z}}(t)} (e^z - c) + m(t) e^{\underline{\hat{z}}(t)} V(z, t) - e^{\underline{\hat{z}}(t)} V_z(z, t) m(t) + e^{\underline{\hat{z}}(t)} V_t(z, t) \\
(\rho + \kappa - m(t)) V(z, t) &= e^z - c - m(t) V_z(z, t) + V_t(z, t)
\end{aligned}$$

subject to the boundary conditions that

$$\begin{aligned}
V(\underline{z}^w(t), t) &= U(t) \\
V_z(\underline{z}^w(t), t) &= 0 \\
V_t(z, t) &= 0
\end{aligned}$$

Define $\hat{J}(\hat{z}, t) = \hat{V}(\hat{z}, t) - \hat{U}(t)$ as the joint surplus of a match at time t and $J(z, t) = V(z, t) - U(t)$ as the transformed joint surplus of a match at time t. The surplus of a match and the optimal threshold $\underline{z}^w(t)$

solve the stopping time problem

$$(\rho+\kappa-m(t))\,J(z,t) \;=\; e^{z}-c-b-m(t)J_z(z,t)+J_t(z,t) \tag{95}$$

subject to

$$J(\underline{z}^{w}(t),t) \;=\; 0 \tag{96}$$

$$J_z(\underline{z}^{w}(t),t) \;=\; 0 \tag{97}$$

$$\lim_{t\to\infty} J_t(z,t) \;=\; 0 \tag{98}$$

Let $\hat{U}^f(t)$ denote the value of an inactive entrepreneur at time t, given by

$$\Big(\rho+\kappa\Big)\hat{U}^f(t) \;=\; \hat{b}(t)+\max_s\left\{s\pi\int_0^{\infty}\Big(\hat{V}^f(\underline{\hat{z}}(t)+\hat{z},t)-\hat{U}^f(t)\Big)^{+}d\Gamma(\hat{z})-\frac{\hat{c}_e(t)s^{1+\eta_e}}{1+\eta_e}\right\}+\dot{\hat{U}}^f(t)$$

subject to $\lim_{t\to\infty}\dot{\hat{U}}^f(t) = 0$, where $\hat{V}^f(\hat{z},t)$ is the value to an entrepreneur of a business idea with productivity $\hat{z}$ at time t, given by

$$\begin{aligned}(\rho+\kappa)\,\hat{V}^f(\hat{z},t) \;&=\; \hat{k}(t)-\hat{r}(t)+\hat{V}^f_t(\hat{z},t)\\ &+\;\max_v\left\{q(t)v\left(\frac{u(t)}{S(t)}\hat{J}(\hat{z},t)+\phi\frac{e(t)}{S(t)}\int_0^{\hat{z}}\hat{J}_{\hat{z}}(\hat{z}',t)G(\hat{z}'|t)d\hat{z}'\right)-\frac{c_v}{1+\eta_v}e^{\hat{z}}v^{1+\eta_v}\right\}\end{aligned}$$

subject to the boundary conditions

$$\hat{V}^f(\underline{\hat{z}}(t),t) \;=\; \hat{U}^f(t)$$

$$\hat{V}^f_{\hat{z}}(\underline{\hat{z}}(t),t) \;=\; 0$$

$$\lim_{t\to\infty}\left(\frac{\hat{V}^f_t(\hat{z},t)}{\hat{V}^f(\hat{z},t)}\right) \;=\; m$$

Define the transformed values $U^f(t)=e^{-\underline{\hat{z}}(t)}\hat{U}^f(t)$ and $V^f(z,t)=e^{-\underline{\hat{z}}(t)}\hat{V}^f(\hat{z},t)$, and proceed analogously to above to get

$$\Big(\rho+\kappa-m(t)\Big)U^f(t) \;=\; b+\max_s\left\{s\pi\int_0^{\infty}\Big(V^f(z,t)-U^f(t)\Big)^{+}d\Gamma(z)-\frac{c_e s^{1+\eta_e}}{1+\eta_e}\right\}+\dot{U}^f(t)$$

subject to $\lim_{t\to\infty}\dot{U}^f(t)=0$ and

$$\begin{aligned}(\rho+\kappa-m(t))\,V^f(z,t) &= k-r(t)-mV_z^f(z,t)+V_t^f(z,t)\\ &+ \max_v\left\{q(t)v\left(\frac{u(t)}{S(t)}J(z,t)+\phi\frac{e(t)}{S(t)}\int_0^z J_z(\tilde{z},t)G(\tilde{z}|t)d\tilde{z}\right)-\frac{c_v}{1+\eta_v}e^z v^{1+\eta_v}\right\}\end{aligned}$$

where $G(\tilde{z}|t)$ is the cdf of employment at time t, subject to

$$\begin{aligned}V^f(\underline{z}(t),t) &= U^f(t)\\ V_z^f(\underline{z}(t),t) &= 0\\ \lim_{t\to\infty}V_t^f(z,t) &= 0\end{aligned}$$

Defining the surplus of an entrepreneur $O(z,t)=V^f(z,t)-U^f(t)$

$$\begin{aligned}(\rho+\kappa-m(t))\,O(z,t) &= k-b-r(t)+O_t(z,t)+\max_v\left\{vR(z,t)-\frac{c_v}{1+\eta_v}e^z v^{1+\eta_v}\right\}\\ &- \max_s\left\{s\pi\int_0^\infty O(z,t)d\Gamma(z)-\frac{c_e s^{1+\eta_e}}{1+\eta_e}\right\}\end{aligned}$$

where the return to hiring at time z is

$$R(z,t) = q(t)\left(\frac{u(t)}{S(t)}J(z,t)+\phi\frac{e(t)}{S(t)}\int_0^z J_z(\tilde{z},t)G(\tilde{z}|t)d\tilde{z}\right)$$

Imposing the assumption that $\underline{z}(t)\le\underline{z}^w(t)$ as well as the boundary conditions

$$r(t) = k-b-\max_s\left\{s\pi\int_0^\infty O(z,t)d\Gamma(z)-\frac{c_e s^{1+\eta_e}}{1+\eta_e}\right\} \tag{99}$$

Substituting this back into the surplus of an entrepreneur

$$(\rho+\kappa-m(t))\,O(z,t) = O_t(z,t)+\max_v\left\{vR(z,t)-\frac{c_v}{1+\eta_v}e^z v^{1+\eta_v}\right\}$$

Optimal vacancy creation is given by the first-order condition

$$v(z,t) = \left(\frac{e^{-z}}{c_v}R(z,t)\right)^{\frac{1}{\eta_v}}$$

Substituting this back into the surplus and simplifying, the surplus of an entrepreneur and the exit threshold $\underline{z}(t)$ solve the stopping time problem

$$(\rho + \kappa - m(t))\, O(z,t) = O_t(z,t) + \frac{\eta_v}{1+\eta_v} \left(\frac{e^{-z}}{c_v} \right)^{\frac{1}{\eta_v}} R(z,t)^{\frac{1+\eta_v}{\eta_v}} \tag{100}$$

subject to the boundary conditions

$$O(\underline{z}(t), t) = 0 \tag{101}$$

$$O_z(\underline{z}(t), t) = 0 \tag{102}$$

$$\lim_{t \to \infty} O_t(z,t) = 0 \tag{103}$$

Optimal search intensity for business ideas is given by the first-order condition

$$s(t) = \left(\frac{\pi}{c_e} \int_0^\infty O(z,t) d\Gamma(z) \right)^{\frac{1}{\eta_e}} \tag{104}$$

and the aggregate entry rate is

$$y(t) = \pi(\xi - l) \left(\frac{\pi}{c_e} \int_0^\infty O(z,t) d\Gamma(z) \right)^{\frac{1}{\eta_e}} \tag{105}$$

The number of entrepreneurs with relative productivity z at time t, $\hat{x}(z,t)$, is given by the PDE

$$\hat{x}_t(z,t) = m(t)\hat{x}_z(z,t) - \kappa \hat{x}(z,t) + \kappa(1 + (1-\omega)\nu)\hat{x}(z,t) + \hat{y}(t)\gamma(z)$$

Define the share of entrepreneurs with productivity z at time t

$$\hat{x}(z,t) = l e^{\lambda t} x(z,t)$$

Then

$$\hat{x}_t(z,t) = l\lambda e^{\lambda t} x(z,t) + l e^{\lambda t} x_t(z,t)$$

$$\hat{x}_z(z,t) = l e^{\lambda t} x_z(z,t)$$

Hence, the share of entrepreneurs evolves according to

$$
\begin{aligned}
l\lambda e^{\lambda t}x(z,t) + le^{\lambda t}x_t(z,t) &= m(t)le^{\lambda t}x_z(z,t) - \kappa le^{\lambda t}x(z,t) + \kappa(1+(1-\omega)\nu)le^{\lambda t}x(z,t) + y(t)le^{\lambda t}\gamma(z) \\
x_t(z,t) &= m(t)x_z(z,t) + y(t)\gamma(z) \qquad (106)
\end{aligned}
$$

where $y(t) = \frac{\hat{y}(t)}{le^{\lambda t}}$ is the entry rate, subject to the boundary conditions

$$
\begin{aligned}
\lim_{z\to 0}\int_0^z x(\tilde{z},t)d\tilde{z} &= 0 \qquad (107) \\
\lim_{z\to \infty}\int_0^z x(\tilde{z},t)d\tilde{z} &= 1 \qquad (108) \\
x(z,0) &= x^{BGP}(z) \qquad (109)
\end{aligned}
$$

where x^{BGP} is the distribution of firms on the initial BGP. The exit rate of firms at time t is given by

$$
x(0,t) = \frac{\tilde{y}(t)}{lm(t)} \qquad (110)
$$

The number of workers employed in a firm with relative productivity z at time t, $\hat{g}(z,t)$, is given by the PDE

$$
\begin{aligned}
\hat{g}_t(z,t) &= m\hat{g}_z(z,t) - \kappa\hat{g}(z,t) - \phi p(t)\Big(1-F(z,t)\Big)\hat{g}(z,t) \\
&+ p(t)f(z,t)\left(\hat{u}(t) + \phi\int_0^z \hat{g}(\tilde{z},t)d\tilde{z}\right)
\end{aligned}
$$

where

$$
\dot{\hat{u}}(t) = -\kappa\hat{u}(t) - p(t)\hat{u}(t) + (\kappa+\lambda)\,e^{\lambda t} + m(t)\hat{g}(\underline{z}^w(t),t)
$$

Define the share of employed workers working for a firm with productivity z at time t, $(1-u(t))g(z,t) = e^{-\lambda t}\hat{g}(z,t)$, where the share of workers who are unemployed at time t is $u(t) = e^{-\lambda t}\hat{u}(t)$. Then

$$
\begin{aligned}
\hat{g}_z(z,t) &= (1-u(t))e^{\lambda t}g_z(z,t) \\
\hat{g}_t(z,t) &= -\dot{u}(t)e^{\lambda t}g(z,t) + (1-u(t))\lambda e^{\lambda t}g(z,t) + (1-u(t))e^{\lambda t}g_t(z,t) \\
\dot{\hat{u}}(t) &= \lambda e^{\lambda t}u(t) + e^{\lambda t}\dot{u}(t)
\end{aligned}
$$

Substituting this into the evolution of the number of workers, cancelling the $e^{\lambda t}$ term from both sides and dividing both sides by $1 - u(t)$

$$
\begin{aligned}
g_t(z,t) &= m(t) g_z(z,t) - \Big(\kappa + \lambda + \phi p(t)\Big(1 - F(z,t)\Big)\Big) g(z,t) \\
&+ p(t) f(z,t) \left(\frac{u(t)}{1-u(t)} + \phi G(z,t)\right) + \frac{\dot{u}(t)}{1-u(t)} g(z,t) \qquad (111)
\end{aligned}
$$

subject to the boundary conditions

$$
\begin{aligned}
\lim_{z \to \underline{z}^w(t)} \int_{\underline{z}^w(t)}^{z} g(\tilde{z},t) d\tilde{z} &= 0 \qquad (112) \\
\lim_{z \to \infty} \int_{\underline{z}^w(t)}^{z} g(\tilde{z},t) d\tilde{z} &= 1 \qquad (113) \\
g(z,0) &= g^{BGP}(z) \qquad (114)
\end{aligned}
$$

where $g^{BGP}(z)$ is the distribution of workers on the initial BGP, and

$$
\begin{aligned}
\lambda e^{\lambda t} u(t) + e^{\lambda t} \dot{u}(t) &= -\kappa e^{\lambda t} u(t) - p(t) e^{\lambda t} u(t) + (\kappa + \lambda) e^{\lambda t} + m(t) e^{\lambda t} (1 - u(t)) g(\underline{z}^w(t), t) \\
\dot{u}(t) &= -\Big(\kappa + \lambda + p(t) + m(t) g(\underline{z}^w(t), t)\Big) u(t) + \kappa + \lambda + m(t) g(\underline{z}^w(t), t) \qquad (115)
\end{aligned}
$$

subject to the boundary condition

$$
u(0) = u^{BGP} \qquad (116)
$$

where u^{BGP} is the unemployment rate on the initial BGP.

Finally, the offer distribution is given by

$$
F(z,t) = \frac{l}{V(t)} \int_0^z v(\tilde{z},t) x(\tilde{z},t) d\tilde{z} \qquad (117)
$$

where aggregate vacancies are

$$
V(t) = l \int_0^\infty v(z,t) x(z,t) dz \qquad (118)
$$

Definition 3. *A perfect foresight equilibrium is* $J(z,t)$, $\underline{z}^w(t)$, $O(z,t)$, $v(z,t)$, $\underline{z}(t)$, $s(t)$, $r(t)$, $L(t)$, $p(t)$, $q(t)$, $V(t)$, $S(t)$, $x(z,t)$, $F(z,t)$, $G(z,t)$, $u(t)$, $\tilde{y}(t)$, $y(t)$ *and* $m(t)$ *such that:*

1. *The surplus and reservation threshold of a match solve the stopping time problem* (95) *subject to* (96)–(98)*;*

2. *The surplus, vacancy policy and reservation threshold of an entrepreneur solve the stopping time problem*

(100) *subject to* (101)–(103), *with* $\underline{z}(t) \equiv 0$ *for all* t;

3. *The search intensity* $s(t)$ *solves the problem of non-producing entrepreneurs* (104);

4. *The fixed cost of a manager given by* (99) *is such that* $r(t) \geq b$ *for all* t *such that demand for managerial services equals supply,* $L(t) = l$ *for all* t;

5. *The finding rates are consistent with aggregate vacancy creation and search intensity*

$$q(t) = \chi V(t)^{\theta-1} S(t)^{1-\theta}, \qquad p(t) = \chi V(t)^{\theta} S(t)^{-\theta}$$

6. *Aggregate vacancies are given by* (118) *and aggregate search intensity is given by*

$$S(t) = u(t) + \phi(1 - u(t))$$

7. *The distribution of entrepreneurs solves the ODE* (106) *subject to* (107)–(109);

8. *The offer distribution is given by* (117);

9. *The distribution of workers and the unemployment rate solve the system of ODEs* (111)–(116);

10. *The exit rate* $\tilde{y}(t)$ *is consistent with incumbent firms' optimal behavior* (110);

11. *The entry rate* $y(t)$ *is consistent with non-producing entrepreneurs' optimal behavior* (105); *and*

12. *The growth rate* $m(t) \in (0, \rho + \kappa)$ *is such that exit equals entry,* $\tilde{y}(t) = y(t)$ *for all* t.

The extension of the discussion above to the extended model is straightforward, but involves a lot of additional notation and hence omitted in the interest of space.

I solve the transition path in terms of transformed variables, and later recover the non-transformed values over the transition from

$$\begin{aligned}
\hat{\underline{z}}(t) &= \int_0^t m(\tau) d\tau \\
\hat{U}(t) &= e^{\hat{\underline{z}}(t)} U(t) \\
\hat{V}(z,t) &= e^{\hat{\underline{z}}(t)} V(z,t) \\
\hat{U}^f(t) &= e^{\hat{\underline{z}}(t)} U^f(t) \\
\hat{V}^f(z,t) &= e^{\hat{\underline{z}}(t)} V^f(z,t)
\end{aligned}$$

To solve the transition path, I guess a path for optimal behavior of entrepreneurs and workers over a discretized grid for productivity and time. Subsequently, I iterate on

1. Given a path for behavior, update the evolution of the distribution of entrepreneurs and workers forward in time;
2. Given a path for the distribution of entrepreneurs and workers, update optimal behavior of entrepreneurs and workers backwards in time;
3. If the updated path for the distributions and behavior are close enough to the original path, stop. Otherwise return to 1.

D.14 Aging over the transition

Figure 38 plots the share of older individuals in the labor force over time in the data and model over the transition. The model is able to match pretty well the evolution of the age composition of the labor force.

FIGURE 38. AGING OVER THE TRANSITION PATH

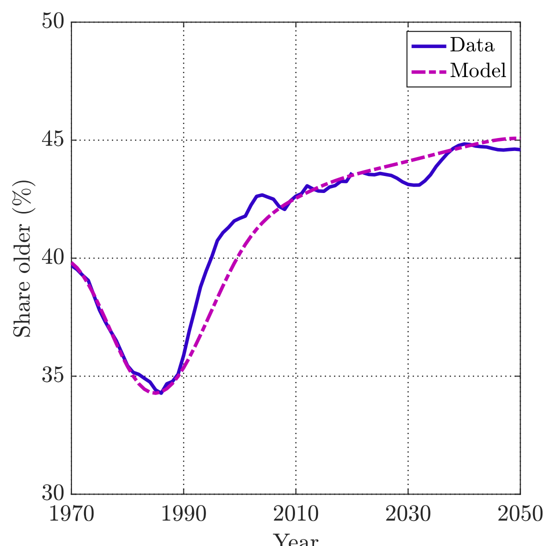


Figure 38 shows the targeted evolution of the share of the labor force that is aged 45–64 over the 1960–2060 period, using official projections for the future. *Source:* SCB, model.