Department of Physics
The University of Hong Kong
PHYS3851 Atomic and Nuclear Physics
PHYS3851-2 Gamma-Gamma Coincidence Techniques Laboratory Manual

A. AIMS

1. To learn the coincidence technique to study the gamma decay of ^{60}Co by using two NaI(Tl) scintillation detectors
2. To learn about the angular correlation concept of gamma-gamma emission.

B. INTRODUCTION

There are two parts to this experiment:

(1) A gamma-gamma coincidence experiment that will be performed to show that the two gammas from ^{60}Co are in coincidence and
(2) Measurement of the angular correlation of these two gammas and determination of the anisotropy and the coefficients of the correlation function. The decay scheme for ^{60}Co is shown in Figure 1.

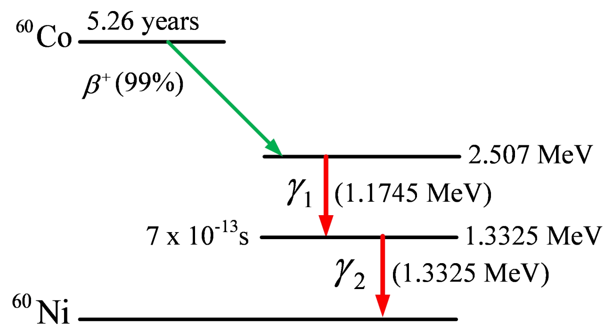

Figure 1. Decay Scheme for 60Co

Note in Figure 1 that the ^{60}Co beta decays to the 2.507-MeV level of ^{60}Ni and this de-excites by a gamma cascade through the 1.3325-MeV state. Since the lifetime of the 1.3325-MeV state is only 0.7 ps, the two gammas will appear to be in coincidence experimentally. Figure 2 shows an Nal(TI) spectrum of ^{60}Co.

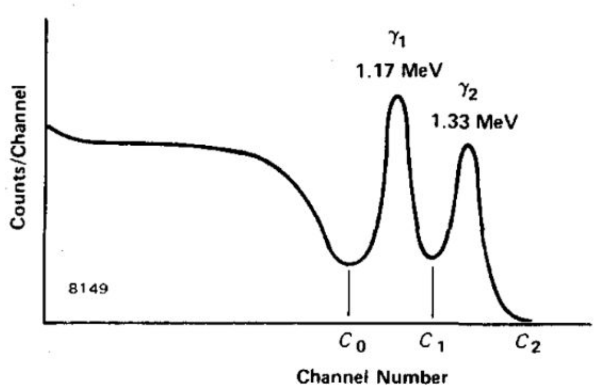

Figure 2. Nal(TI) spectrum of ^{60}Co.

Figure 3 shows the geometrical setup that will be used for both the γ_1-γ_2 coincidence verification and the angular correlation measurement in this experiment.

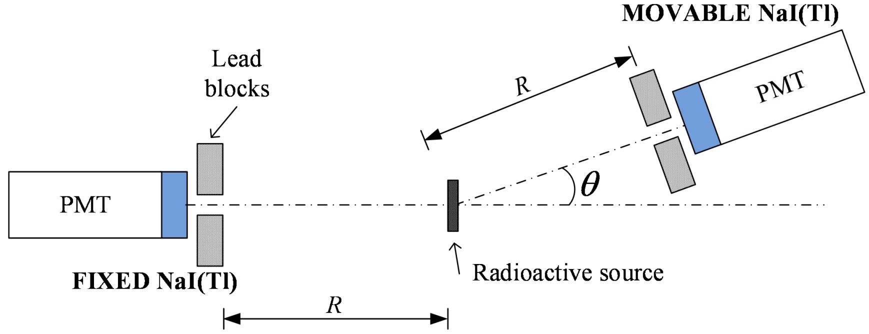

Figure 3. Experimental Arrangement for Coincidence and Angular Correlation Measurements

Since the angular correlation of γ_1-γ_2 is nearly isotropic, the angle θ in Figure 3 can be set at any value for the coincidence verification. Usually the most convenient angle is 180°. A typical electronics setup for the measurement is Figure 4.

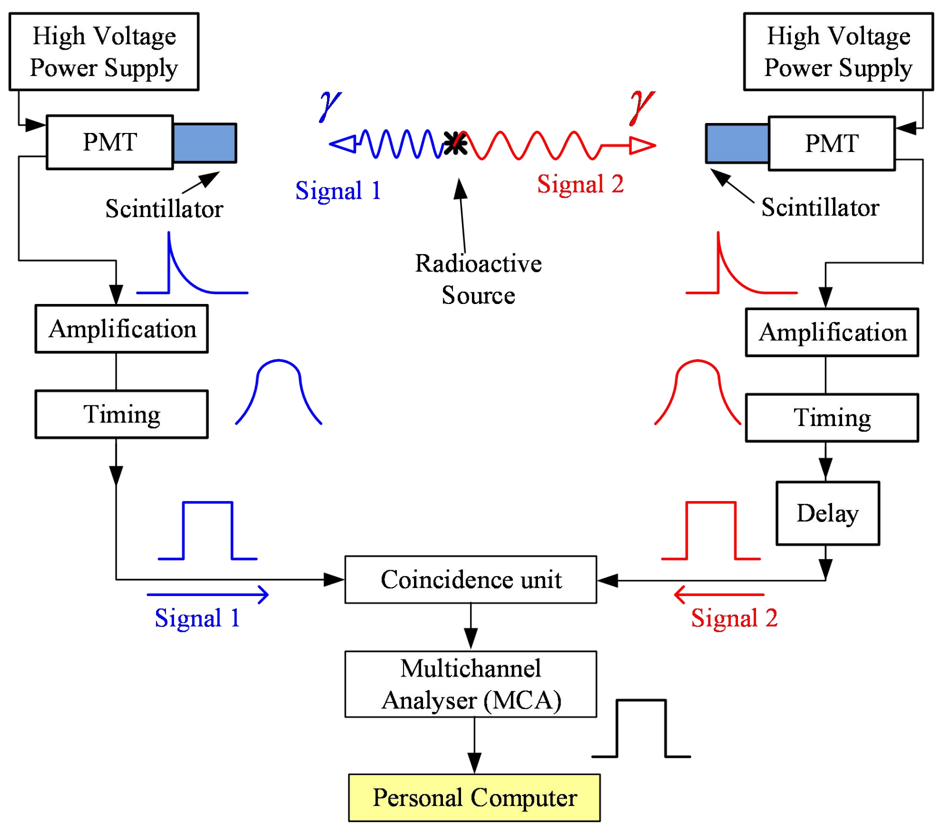

Figure 4. General Schematic Diagram for Gamma-Gamma Coincidence Experiment

C. PRE-LAB READING MATERIAL: Theory - Background Information

(a) Unipolar and biopolar pulses:

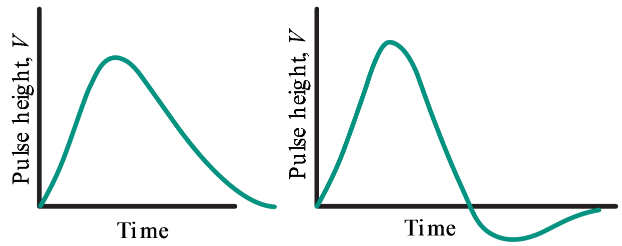

Figure 6. The Pulse at the Output of the Amplifier: Unipolar Pulse (Left) and Bipolar Pulse (Right).

(b) Coincidence Signal:

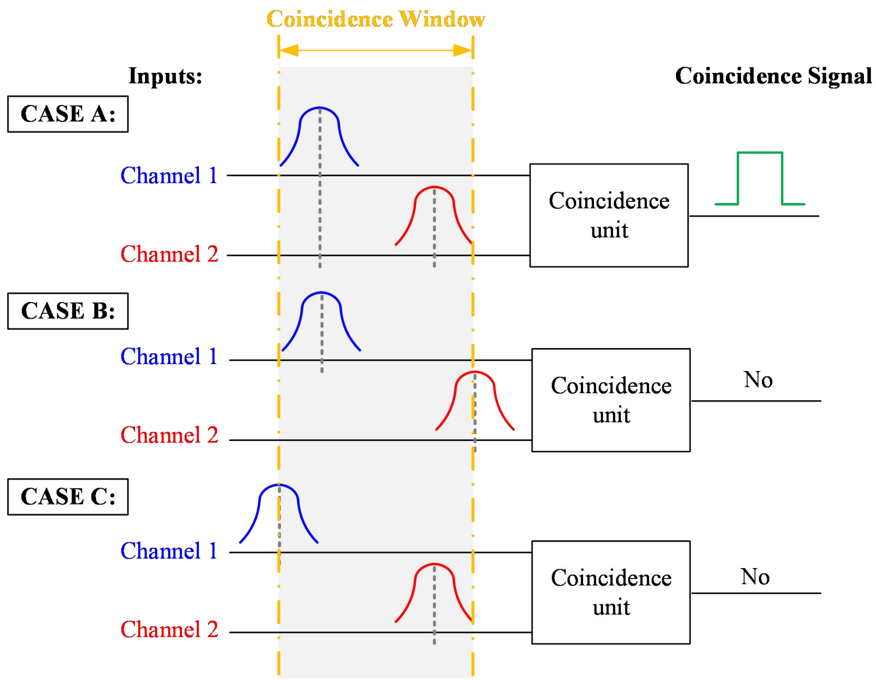

Figure 7. Illustration of the Working Principle of Coincidence Unit.

(c) Accidental coincidences

By making coincidence measurements we have to consider the possibility of "accidental coincidences" which can occur from uncorrelated background events in the detector.

The rate of accidental coincidences can be calculated from the singles rates of each scintillator with the given formula:

$$\text{Rate of accidental coincidences } N_{accidental} = 2\tau N_1 N_2$$

where

$\tau =$ resolving time of the coincidence unit (i.e. the time period in which two signals are considered as coincident) ,

$N_1 =$ counting rate in the counter for the fixed detector

$N_2 =$ counting rate in the counter for the movable detector

D. SETUP

Experimental Apparatus:

- Two NaI(TI) Crystal,
- Phototube Assembly
- Photomultiplier Tube Base
- Preamplifiers NIM
- Multi-Channel Analyser
- High Voltage Power Supply
- Two Amplifier NIM
- Two Timing Single-Channel Analyzer NIM
- Universal Coincidence Counter (or Scaler) NIM
- NIM Bin and Power Supply
- Oscilloscope
- Co-60 source
- Computer

E. EXPERIMENT 1: A Simple Slow Coincidence System Investigated via a Pulse Generator

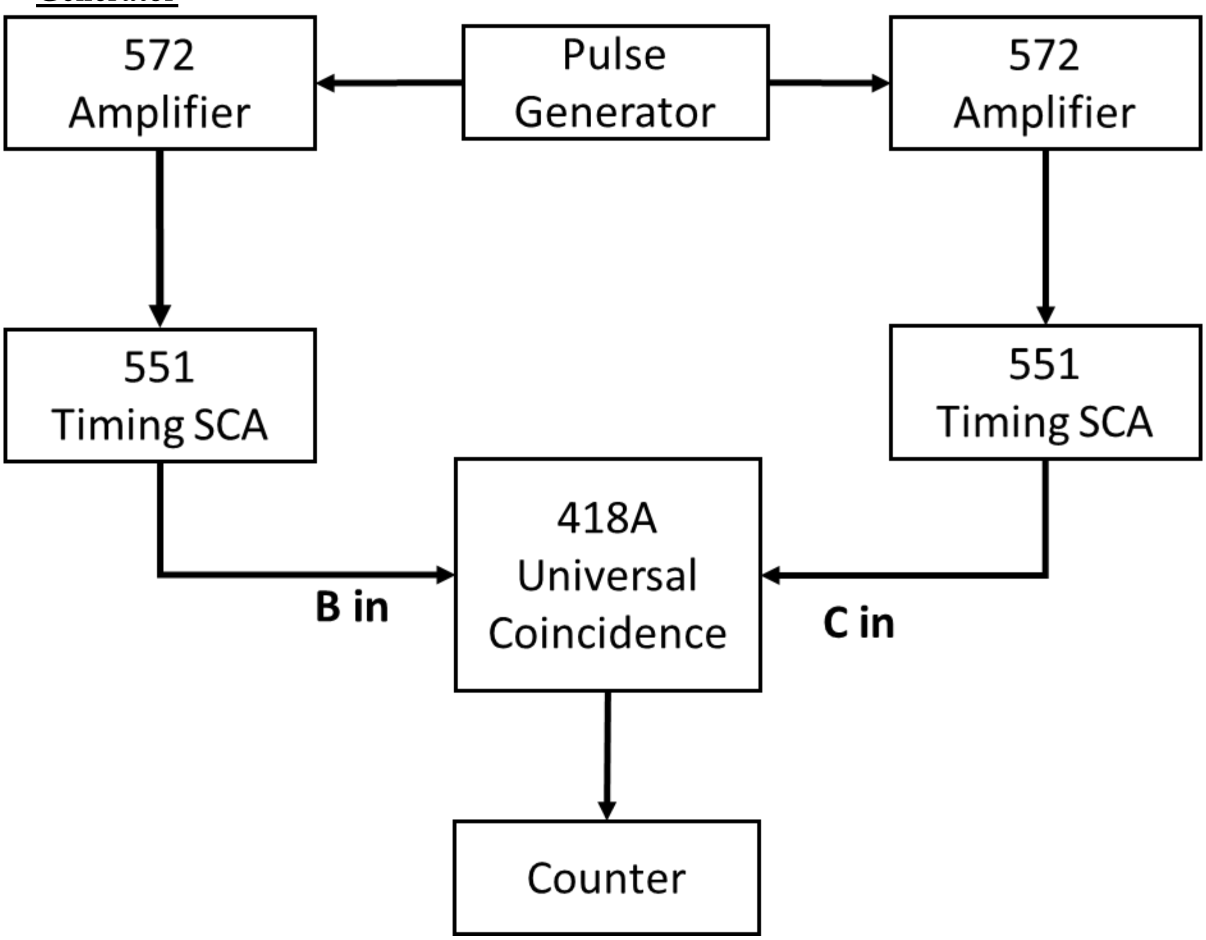

Figure 9. Electronics for Slow Coincidence System Investigated via a Pulse Generator

Procedure

1. Turn on the Pulse Generator, set the pulse frequency 10kHz, pulse with 50µs and the amplitude 500mV
2. Set the amplifier course gain 50, fine gain 1.5
3. In SCA, set the lower level 10mV, upper level 10V and Delay 5µs.
4. In Universal Coincidence, tune the resolving time to 100ns(minimum) and Set the COINCIDENCE REQUIREMENTS to 2. Set the B and C channel to Coinc and remain other channels Off.
5. For the SCA feeding the C input on the 418A Universal Coincidence NIM, increase the **DELAY dial** in 100 ns steps and record the number of counts accumulated on the Counter in 10 seconds for each setting. Continue increasing the DELAY in steps of 100 ns until the coincidence counting rate drops to zero.
6. Repeat step 5 for decreasing DELAY settings in steps of 100 ns.

EXPERIMENT 2: Verification of the Gamma-Gamma Coincidence of ^{60}Co Procedure

1. Set up the electronics as shown in Figure 10. Adjust the High Voltage Power Supplies NIM to the values recommended for the NaI(Tl) Detectors. Adjust the gain of each Amplifier so that the 1.33-MeV gamma pulses at the output are ~6 V in amplitude. Figure 2 shows a typical spectrum that could be obtained from either amplifier output with an MCA.
2. Set the Timing SCA NIM for the Window mode and adjust its Lower-Level and Upper-Level controls to bracket the 1.17-MeVphotopeak pulses. In Figure 2, this is the region between C_0 and C_1. Use a 100-ns delay.
3. Adjust the gate width on the Gate Generator NIM and the delay setting of the Delay amplifier NIM so that the pulses out of the linear gate are similar to Figure 11 in the oscilloscope.
4. Accumulate a spectrum in the MCA. This spectrum should include only the 1.33-MeV peak and its Compton edge. The 1.17-MeV peak of Figure 2 will be virtually eliminated. These results will show that the 1.17- and 1.33-MeV gammas are in coincidence because a 1.17-MeV gamma was required in the SCA in order to pass each 1.33-MeV pulse that was contributed into the spectrum.
5. Repeat the experiment with the Timing SCA NIM set to bracket the 1.33-MeV peak. Under these conditions, only the 1.17-MeV peak and its Compton should appear in the MCA spectrum. These two measurements verify that γ_1 and γ_2 in Figure 1 are prompt cascade gammas.

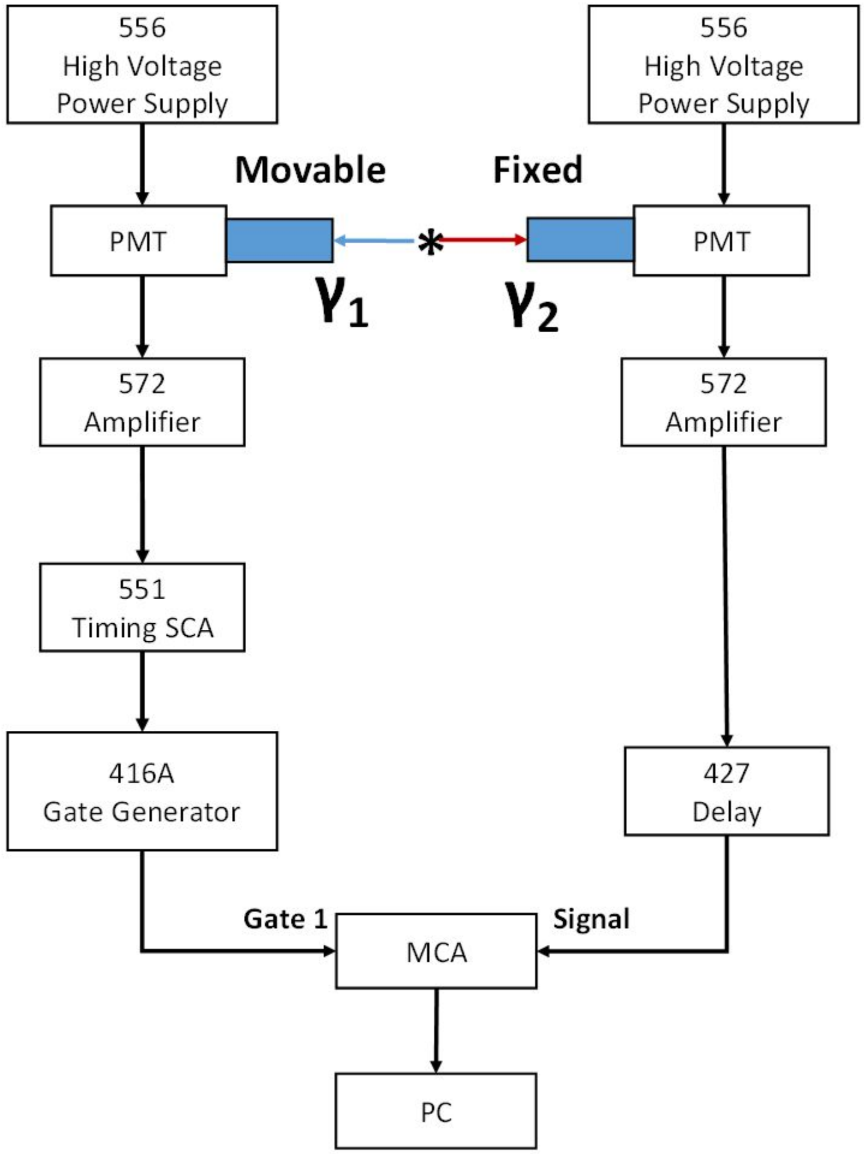

Figure 10. Electronics for ^{60}Co Coincidence Experiment (The exact NIM unit can be changed due to availability)

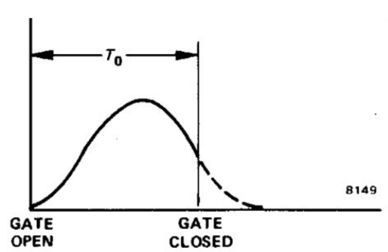

Figure 11. Output of the Liner Gate NIM

F. EXPERIMENT 3: Angular Correlation of ^{60}Co

In the case of gamma-gamma angular correlation, an experimental arrangement similar to Figure 3 is used. The fixed detector is set to measure only γ_1, and the movable detector observes γ_2. The number of coincidences between γ_1 and γ_2 is then determined as a function of θ (the angle between the two detectors). A plot of the number of coincidence events per unit time as a function of the angle, θ , is called the measured angular correlation. The measurement of γ_1 in a fixed direction determines nuclei which have an angular distribution of the resulting radiation, γ_2, which is nonisotropic. This is a result of the nonisotropic distribution of spin orientations in ^{60}Co. Figure 1 shows that ^{60}Co beta decays to the 2.507-MeV, (4+), state which gamma branches through the 1.3325-MeV, (2+), state to the ground state, (0+), of ^{60}Ni.

These angular momenta determine the shape of the correlation function of the isotope. A complete discussion of the theoretical arguments associated with the angular correlation measurements is presented in references 1 and 2. The theoretical correlation function, $w(\theta)$, for ^{60}Co is given by

$$w(\theta) = a_0 + a_2 \cos^2\theta + a_4 \cos^4\theta \qquad \text{Eq.1}$$

$$\text{where } a_0 = 1,\ a_2 = \frac{1}{8} \text{ and } a_4 = \frac{1}{24}$$

Table 1 below shows the calculated values for $w(\theta)$ for angles between 90° and 180° in 10° increments for ^{60}Co. It can be seen from Table 1 that the correlation function, $w(\theta)$, changes by only 17% from 90° to 180°. Therefore, counting statistics of ~1% should be obtained when the experiment is performed.

Table 1 Angular Correlation Function $w(\theta)$ for ^{60}Co

θ (deg)	$w(\theta)$
90	1.00000
100	1.00381
110	1.01519
120	1.03385
130	1.05876
140	1.08770
150	1.11719
160	1.14287
170	1.16042
180	1.16667

The anisotropy factor, A, associated with an angular correlation measurement is defined as

$$A = \frac{w(180°) - w(90°)}{w(90°)} \qquad \text{Eq.2}$$

A comparison of the experimental anisotropy with the theoretical value will reveal that angular correlation measurements are capable of rather high precision. In this experiment the experimental angular correlation, $w(\theta)$, will be compared to Eq. (1) and the values shown in Table 1, and the calculated and measured anisotropy will be compared to Eq. (2).

Procedure

1. Set up the electronics as shown in Figure 12.

2. Set the resolving time of the Universal Coincidence NIM at 100 ns. Adjust the delays in the outputs of both Timing SCAs NIM to minimum.

3. Set both Timing SCAs NIM for the Window mode and adjust the window widths. Set the window for pulses from the fixed detector so that it spans the 1.17-MeV peak. Set the window for the movable detector so that it spans the 1.33-MeV peak.

4. Set the angle, θ, carefully at 180° (Figure 3). The distance between the detectors and the sources should be the same, which is 8cm. Accumulate a total count rate, N_{Total} for 30 minutes. Also measure the number of counts in the counters for the side channels. To determine the actual time coincidence rate a correction must be made for the number of accidental coincidence counts. Determine the accidental rate, $N_{accidental}$, from the formula

$$N_{accidental} = 2\tau N_1 N_2 \qquad \text{Eq.3}$$

where
$\tau =$ resolving time of the coincidence unit,
$N_1 =$ counting rate in the counter for the fixed detector
$N_2 =$ counting rate in the counter for the movable detector

The angular correlation function, $w(\theta)$, for 180° is then

$$w(\theta) = N_{Total} - N_{accidental} \qquad \text{Eq.4}$$

5. Repeat the measurements in step 4 and determine $w(\theta)$ for the other angles(θ=90º,135º and 180º). It is a good practice to repeat the measurement at 90° several times during the course of the experiment to ensure proper alignment of the system.

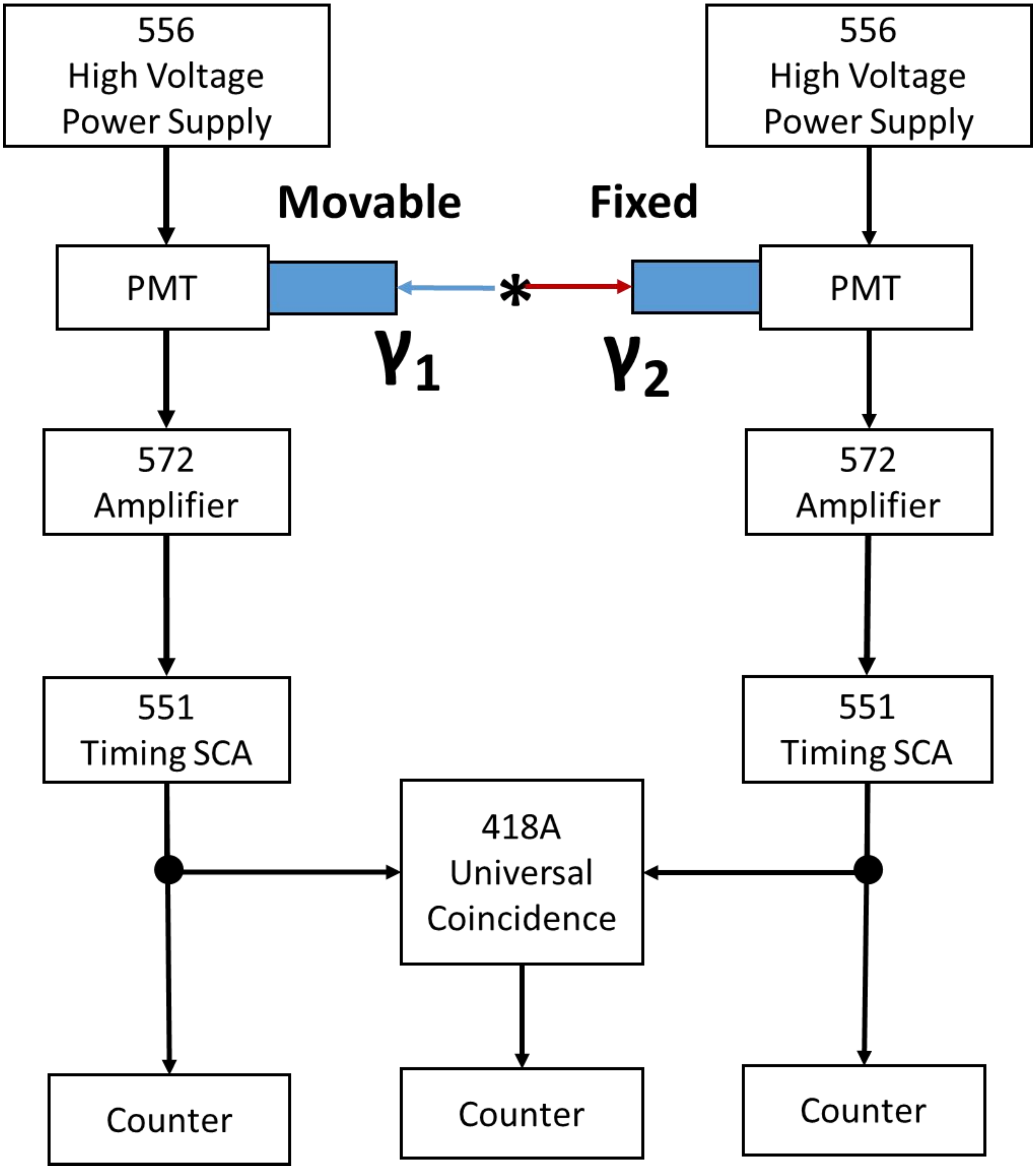

Figure 12. Electronics for $\gamma - \gamma$ Angular Correlation Measurements

G. ANALYSIS:

Experiment 1: A Simple Slow Coincidence System Investigated via a Pulse Generator

1. Plot the data taken in steps 24 and 25 with Counts on the vertical scale and Delay on the horizontal scale. The result should look like Figure 13, except the total width, 2τ, should be about 1 μs.

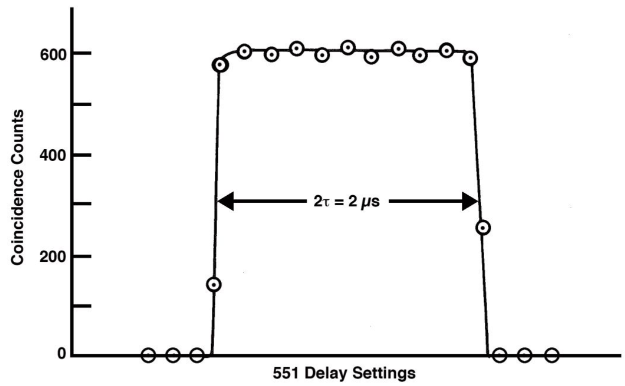

Figure 13. A Typical Time Coincidence Graph for an Overlap Coincidence Circuit

Experiment 2: Verification of the Gamma-Gamma Coincidence of ^{60}Co and Experiment 3: Angular Correlation of ^{60}Co

2. Assume that the 2+ state lifetime of 60Co is in order of a few seconds not $7\text{x}10^{-13}$s, does our setup still measure gamma-gamma coincidence of 60Co? Explain why or why not and give a solution.

3. In order to easily compare the experimental values with the theoretical values given by Eq. (1), it is more convenient to plot $G(\theta)$ vs θ, where $G(\theta)$ is calculated by

$$G(\theta) = \frac{w(\theta)}{w(90°)} \qquad \text{Eq.5}$$

4. Compare and describe the difference between experimental results and calculation results from Eq.(1) in θ=90º,135º and 180º.

5. What kinds of ways do we need in order to improve the measurement?

H. APPENDIX: Guidelines for Source-to-Detector Distances, Counting Rates and Counting Times.

The following tables provide guidance on adjusting the source-to-collimator distances, *R*, to achieve reasonable counting rates and viable counting times with a 100 μCi ^{60}Co source. The distance *R* should be the same for both the fixed and the movable detector.

The predicted singles counting rate (counts per second) is computed from equation (6).

$$\mathrm{CPS}_{\mathrm{singles}} = A\eta C\left(\varepsilon_{T_{1.17}} + \varepsilon_{T_{1.33}}\right)\frac{\Delta\Omega}{4\pi} \qquad \text{Eq.6}$$

Where
A is the source activity in μCi,
η = 3.7 x 104 is the number of disintegrations per second per μCi,
C = 0.999 is the fraction of the decays that result in a 1.17-MeV/1.33-MeV cascade of gamma rays,
$\varepsilon_{T_{1.17}}$ is the total intrinsic detection efficiency for the 1.17 MeV gamma-ray,
$\varepsilon_{T_{1.33}}$ is the total intrinsic detection efficiency for the 1.33 MeV gamma-ray,
$\Delta\Omega$ is the solid angle subtended by the collimator aperture at the point source, and
4π is the total solid angle in the sphere of radius R surrounding the point source.

The fractional solid angle can be expressed as\

$$\frac{\Delta\Omega}{4\pi} = \frac{\pi\frac{\left(\frac{d}{2}\right)^2}{R^2}}{4\pi} = \frac{\left(\frac{d}{2}\right)^2}{4R^2} \qquad \text{Eq.7}$$

where d is the diameter of the collimator aperture.

The true coincidence counting rate (counts per second) for the 1.17 MeV and 1.33 MeV photopeaks is calculated from equation (8).

$$\mathrm{CPS}_{\mathrm{Coincidence}} = A\eta C\varepsilon_{T_{1.17}}\varepsilon_{T_{1.33}}\left(\frac{\Delta\Omega}{4\pi}\right)^2 \qquad \text{Eq.8}$$

where
$\varepsilon_{T_{1.17}}$ is the photopeak intrinsic detection efficiency for the 1.17 MeV gamma ray, and
$\varepsilon_{T_{1.33}}$ is the photopeak intrinsic detection efficiency for the 1.33 MeV gamma ray.

Equation (8) assumes the angular correlation of the two gamma rays is isotropic. That is an adequate approximation for estimating the counting rates for set-up purposes, because the actual angular correlation varies by only 17% over the range of possible angles. For true coincidences, both detected gamma rays come from the same atom. Thus, true coincidences are also known as correlated coincidences.

For practical counting rates, there is always a finite probability that the 1.17 MeV gamma ray will be detected from the decay of one atom, while the 1.33 MeV gamma-ray is detected at the same time from the decay of a different atom.
Such events are labeled accidental, chance or uncorrelated coincidences. The 414A Fast Coincidence NIM measures the total coincidences, i.e., the sum of the true and accidental coincidences. Therefore, the accidental coincidence rate must be calculated in the experiment from the measured singles rate and the known coincidence resolving time. This deduced accidental coincidence rate is subtracted from the total coincidence rate to determine the measured true coincidence rate.
For guidance in planning the experiment, the accidental coincidence rate, $CPS_{accident}$, can be forecast from equation (9).

$$\mathrm{CPS}_{\mathrm{Accident}} = 2\tau\left[A\eta C\varepsilon_{T_{1.17}}\left(\frac{\Delta\Omega}{4\pi}\right)\right]\left[A\eta C\varepsilon_{T_{1.33}}\left(\frac{\Delta\Omega}{4\pi}\right)\right] \qquad \text{Eq.9}$$

By dividing equation (8) into equation (9), it can be seen from equation (10) that the ratio of the accidental
coincidence counting rate to the true coincidence counting rate depends primarily on the activity of the source, *A*, and the coincidence resolving time, 2τ. That is why the larger resolving time in Experiment 2 requires a lower source activity.

$$\frac{\mathrm{CPS}_{\mathrm{Accident}}}{\mathrm{CPS}_{\mathrm{Coincidence}}} = 2\tau A\eta C \qquad \text{Eq.10}$$

As can be seen in Table 2, the accidental coincidence rate is about 37% of the true coincidence rate in Experiment 3, with a 100 ns resolving time and the 100 μCi source activity. For Experiment 2, the accidental coincidence rate for the photopeak is approximately 3.7% of the true photopeak coincidence rate, because the source activity is 1 μCi, and the coincidence resolving time is 1 μs.

If The *d* = 2.54 cm apertures in the lead collimators in front of each 2 inch x 2 inch NaI(Tl) detector improve the ratio of the counts in the photopeak to the counts in the total spectrum by eliminating the Compton scattering interactions that would otherwise take place near the outer diameter of the scintillator. This eliminates many of the Compton scattered photons that can escape the scintillator without terminating in a photoelectric absorption. The total linear absorption coefficient can be used in equation (11) to calculate the total intrinsic efficiency of the collimated NaI(Tl) detector.

$$\varepsilon_{Total} = \left(1 - e^{-\mu T_{\mathrm{scintillator}}}\right) \qquad \text{Eq.11}$$

Where μ is the total linear absorption coefficient for NaI at the photon energy, and $T_{\mathrm{Scintillator}}$ is the thickness of the scintillator.

The collimation improves the peak-to-total ratio for the collimated 2 inch x 2 inch NaI(Tl) detector to approach that of an uncollimated 3 inch x 3 inch detector. Consequently, the total intrinsic detection efficiency from equation (11 can be multiplied by the tabulated peak-to-total ratios for a 3 inch x 3 inch detector to estimate the intrinsic photopeak efficiency.

Alternatively, the peak-to-total ratio can be measured on the actual collimated detectors and multiplied by the total intrinsic efficiency to determine the intrinsic photopeak efficiency. The former method was used for the estimates in the following tables. However, the latter method is more accurate.

Table 2 estimates the angular resolution, the singles counting rate, and the coincidence counting rate of the collimated detectors as a function of the source-to-collimator distance, *R*. It also includes the counting time required to achieve 1% counting statistics in the coincidence mode of Experiment 3. To avoid gain shifting with counting rate, it is generally advisable to keep the counting rate below 10,000 counts/second with a NaI(Tl) detector. Consequently, a source-to-collimator distance greater than 13 cm is necessary for Experiment 3.

For Experiment 3, smaller values of *R* yield 1% counting statistics in a shorter time period. However, the angular resolution degrades at lower values of *R*. The *R* = 20 cm distance highlighted in green in Table 2 is a reasonable compromise between angular resolution and counting time for the angular correlation measurement.
The parameters used for computing the results in Table 2 are summarized in Table 3.

Table 2. Angular Resolution and Counting Rates Versus Source-to-Collimator Distances.

Source-to-Collimator Distance, *R* (cm)	**Angular Resolution (Degrees)**	**Singles Counting Rate (cps)**	**True Coincidence Counting Rate,** $CPS_{Coincidence}$ **(counts/s)**	**Coincidence Counting Time for 1% Std. Dev. (hours)**	**Accidental Coincidence Counting Rate,** $CPS_{Accident}$ **(counts/s)**
30	4.8	2020	0.024	115.9	0.0089
28	5.2	2319	0.032	88.0	0.0117
26	5.6	2690	0.042	65.4	0.0157
24	6.1	3157	0.058	47.5	0.0216
22	6.6	3757	0.083	33.5	0.0306
20	7.3	4546	0.121	22.9	0.0448
18	8.1	5612	0.185	15.0	0.0683
16	9.1	7103	0.296	9.4	0.1095
14	10.4	9277	0.505	5.5	0.1868
12	21.1	12627	0.936	3.0	0.3460

Table 3. Source, Geometry and Detector Parameters for Table 3

Source	100 μCi = ^{60}Co source activity 3.7×10^4 = disintegrations per second per μCi 0.999 = fractions of disintegrations yielding 1.17 and 1.33-MeV gamma cascade
Geometry	*R* = source to collimator distance 2.54 cm = diameter of collimator aperture
Detector: 2-inch x 2-inch NaI(Tl)	Collimated to 2.54-cm Diameter 0.62 = total intrinsic efficiency for 1.17-MeV gamma ray 0.60 = total intrinsic efficiency for 1.33-MeV gamma ray

	0.31 = peak to total ratio for 1.17 MeV 0.28 = peak to total ratio for 1.13 MeV 0.19 = peak intrinsic efficiency for 1.17-MeV gamma ray 0.17 = peak intrinsic efficiency for 1.13-MeV gamm ray 10000 = maximum permissible counts per second to avoid spectrum shifting
Coincidence:	$1.00 \times 10^{-7} = 2\,\tau$ resolving time (sec.)

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