RAMA UNIVERSITY

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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 38

Topic: Theories of Failure II

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Theories of Failure

The generally accepted theories are

Ductile materials (yield criteria)

- Maximum shear stress (MSS)
- Distortion energy (DE)
- **Ductile Coulomb-Mohr (DCM)**

Brittle materials (fracture criteria)

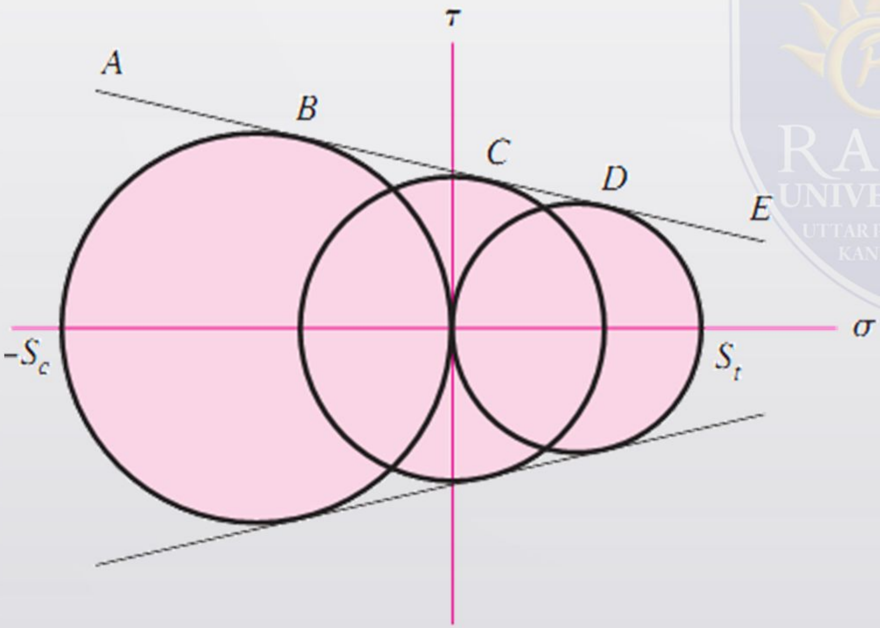
- **Maximum normal stress (MNS)**
- **Brittle Coulomb-Mohr (BCM)**
- **Modified Mohr (MM)**

Coulomb-Mohr Theory for Ductile Materials

- ✓ Not all materials have compressive strengths equal to their corresponding tensile values.
- ✓ For example, the yield strength of magnesium alloys in compression may be as little as 50 percent of their yield strength in tension. The ultimate strength of gray cast irons in compression varies from 3 to 4 times greater than the ultimate tensile strength.
- ✓ The idea of Mohr is based on three "simple" tests: tension, compression, and shear, to yielding if the material can yield, or to rupture. It is easier to define shear yield strength as $\mathbf{S_{sy}}$ than it is to test for it.

Coulomb-Mohr Theory for Ductile Materials

- ✓ Mohr's hypothesis was to use the results of tensile, compressive, and torsional shear tests to construct the three circles defining a failure envelope, depicted as line ABCDE in the figure, above the σ axis. The failure envelope need not be straight



Three Mohr circles, one for the uniaxial compression test, one for the test in pure shear, and one for the uniaxial tension test, are used to define failure by the Mohr hypothesis. The strengths S_c and S_t are the compressive and tensile strengths, respectively; they can be used for yield or ultimate strength.

Coulomb-Mohr Theory for Ductile Materials

- ✓ A variation of Mohr's theory, called the Coulomb-Mohr theory or the internal-friction theory, assumes that the boundary BCD is straight

$$\frac{B_2C_2 - B_1C_1}{OC_2 - OC_1} = \frac{B_3C_3 - B_1C_1}{OC_3 - OC_1}$$

$$\frac{\dfrac{\sigma_1 - \sigma_3}{2} - \dfrac{S_t}{2}}{\dfrac{S_t}{2} - \dfrac{\sigma_1 + \sigma_3}{2}} = \frac{\dfrac{S_c}{2} - \dfrac{S_t}{2}}{\dfrac{S_c}{2} + \dfrac{S_t}{2}}$$

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

Coulomb-Mohr Theory for Ductile Materials

- ✓ For plane stress, when the two nonzero principal stresses are $\sigma_A \geq \sigma_B$, we have three cases

Case 1: $\sigma_A \geq \sigma_B \geq 0$. For this case, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$. Equation reduces to a yield condition of

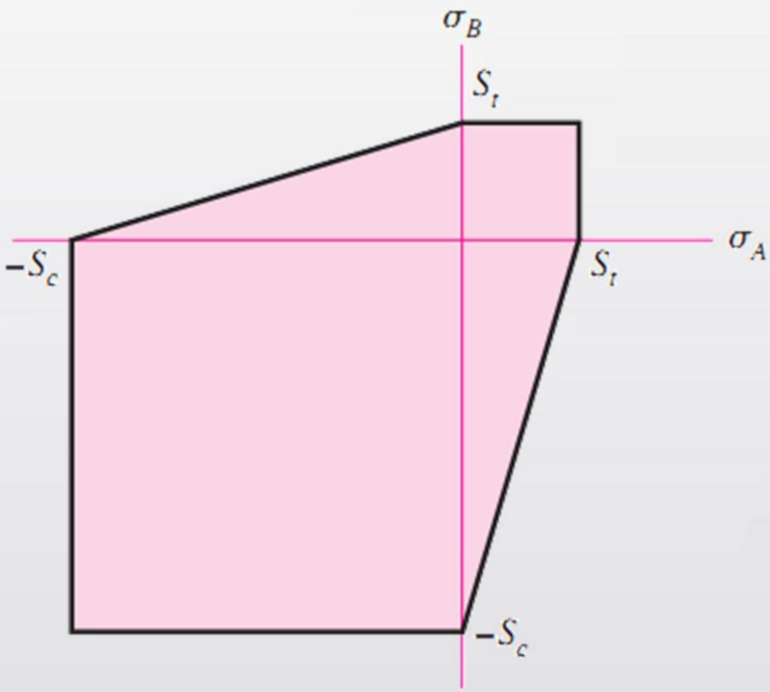
$$\sigma_A \geq S_t$$

Case 2: $\sigma_A \geq 0 \geq \sigma_B$. Here, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$,

$$(\sigma_A / S_t) - (\sigma_B / S_c) \geq S_y$$

Case 3: $0 \geq \sigma_A \geq \sigma_B$. For this case, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$

$$\sigma_B \leq -S_c$$



Maximum-Normal-Stress Theory for Brittle Materials

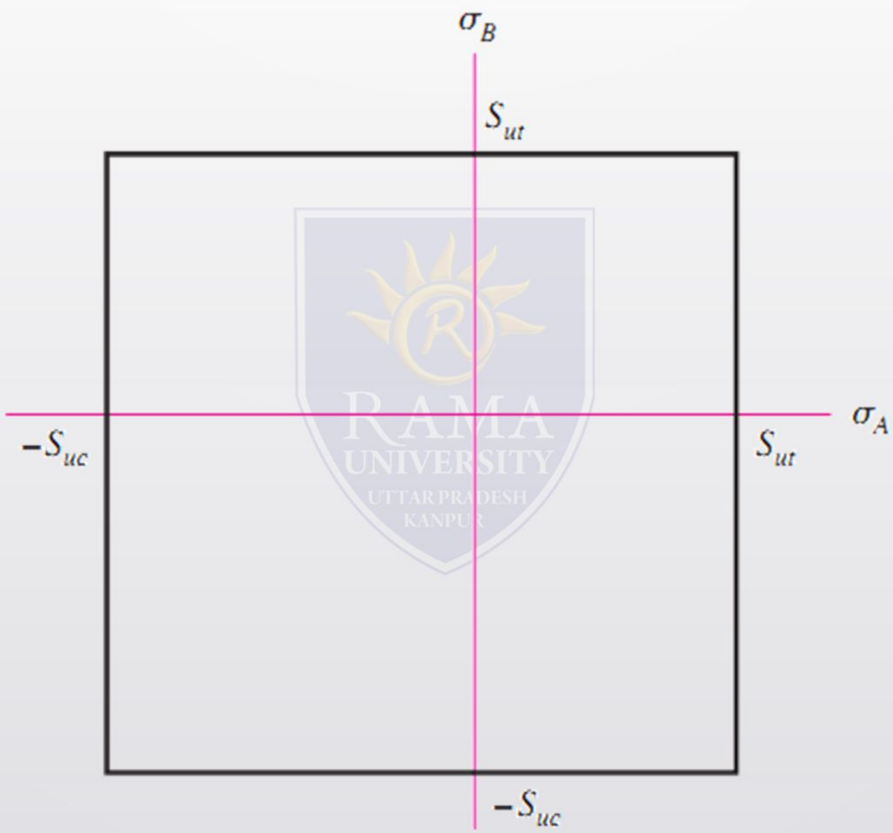
- The maximum-normal-stress (MNS) theory states that failure occurs whenever one of the three principal stresses equals or exceeds the strength. This theory then predicts that failure occurs whenever

$$\sigma_1 \geq S_{ut} \qquad \text{or} \qquad \sigma_3 \leq -S_{uc}$$

- where $\mathbf{S_{ut}}$ and $\mathbf{S_{uc}}$ are the ultimate tensile and compressive strengths, respectively, given as positive quantities.
- For plane stress, with $\sigma_A \geq \sigma_B$, we can write

$$\sigma_A \geq S_{ut} \qquad \text{or} \qquad \sigma_B \leq -S_{uc}$$

Maximum-Normal-Stress Theory for Brittle Materials



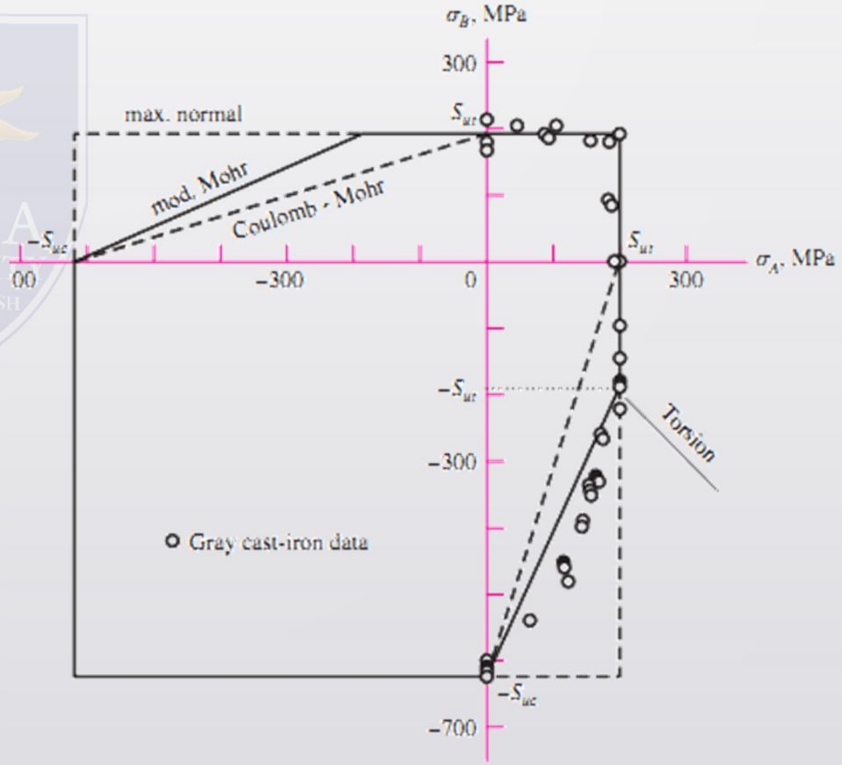
Modifications of the Mohr Theory for Brittle Materials

- Brittle-Coulomb-Mohr
  - On the basis of observed data for the fourth quadrant, the modified Mohr theory expands the fourth quadrant

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$



Modifications of the Mohr Theory for Brittle Materials

✓ Modified Mohr

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

$$\frac{(S_{uc} - S_{ut})\,\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B \quad \text{and} \quad \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

Selection of Failure Criteria