Panel 1

Hw 5 p 1

Find Fourier Series Coeff. for

(a) ① $x(t) = -1 + \cos(\overset{3t}{2t}) + 3\cos(\overset{7t}{4t} + \pi/4) \rightarrow \omega_0 = 2$ $\longrightarrow \omega_0 = 1$

$$C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$x(t) = -1 + \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) + \frac{3}{2}\left(e^{j(4t+\pi/4)} + e^{-j(4t+\pi/4)}\right)$$

② $x(t) = C_0 + \sum_{k=1}^{\infty} 2|C_k| \cos(k\omega_0 t + \theta_k)$

$2|C_1| = 1$ $\quad \theta_1 = 0$ $\quad C_1 = \frac{1}{2}e^{j0}$

$2|C_2| = 3$ $\quad \theta_2 = \pi/4$ $\quad C_2 = \frac{3}{2}e^{j\pi/4}$

$C_{-1} = \frac{1}{2}e^{-j0} = \frac{1}{2}$

$C_0 = -1$ $\quad C_{-2} = \frac{3}{2}e^{-j\pi/4}$

1

Panel 2

1b)

$$x(t) = \cos^2(2t)$$

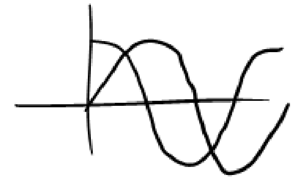
$$= \left[\frac{e^{j2t} + e^{-j2t}}{2}\right]^2$$

$$= \frac{1}{2}\left[1 + \cos(4t)\right]$$

2

Panel 3

$$(c)\quad x(t) = 2\cos(3t) + 4\sin\left(6t - \pi/3\right)$$



$$= 2\cos(3t) + 4\cos\left(6t - \pi/3 - \pi/2\right)$$

$$= 2\cos(\overset{k\omega_0 t}{3t}) + 4\cos\left(6t - 5\pi/6\right)$$

$$\omega_0 = 3$$

$$C_0 = 0$$

$$C_1 = 1e^{j0} \qquad C_{-1} = 1e^{j0}$$

$$C_2 = 2e^{-j5\pi/6} \qquad C_{-2} = 2e^{+j5\pi/6}$$

Panel 4

$$e^{j6t} - e^{-j2t} = e^{j\frac{a+b}{2}t}\left[e^{j\frac{a-b}{2}t} - e^{-j\frac{a-b}{2}t}\right]$$

$$e^{ja} - e^{jb}$$

$$= e^{j\frac{4}{2}t}\left[e^{j\frac{8}{2}t} - e^{-j\frac{8}{2}t}\right]$$

$$= e^{j2t}\left[e^{j4t} - e^{-j4t}\right]$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$= \frac{2j}{2j} e^{j2t}\left[e^{j4t} - e^{-j4t}\right]$$

$$= 2j\, e^{j2t} \sin(4t)$$

$$= 2j\, e^{j2t} \sin\left(\frac{4t}{\pi}\pi\right)$$

4

Panel 5

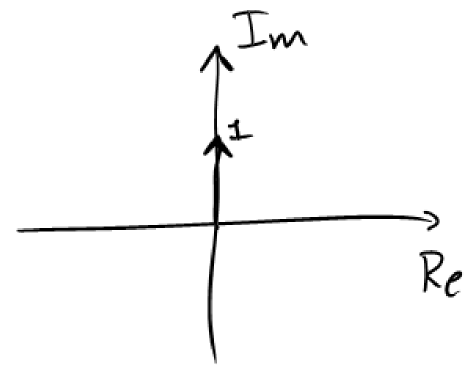
$$= 2j\,e^{j2t}\,\sin\left(\frac{4t}{\pi}\pi\right)$$

$$= \frac{\frac{4\pi t}{\pi}}{\frac{4\pi t}{\pi}}\,2j\,e^{j2t}\,\sin\left(\frac{4\pi t}{\pi}\right)$$

$$= \frac{4\pi t}{\pi}\,2j\,e^{j2t}\,\mathrm{sinc}\left(\frac{4t}{\pi}\right)$$

$$= 8t\,e^{j\pi/2}\,e^{j2t}\,\mathrm{sinc}\left(\frac{4t}{\pi}\right)$$

$$= 8t\,e^{j(2t+\pi/2)}\,\mathrm{sinc}\left(\frac{4t}{\pi}\right)$$



$$e^{j\frac{\pi}{2}} = \cancelto{0}{\cos\left(\frac{\pi}{2}\right)} + j\sin\left(\frac{\pi}{2}\right)$$

$$= j$$

5

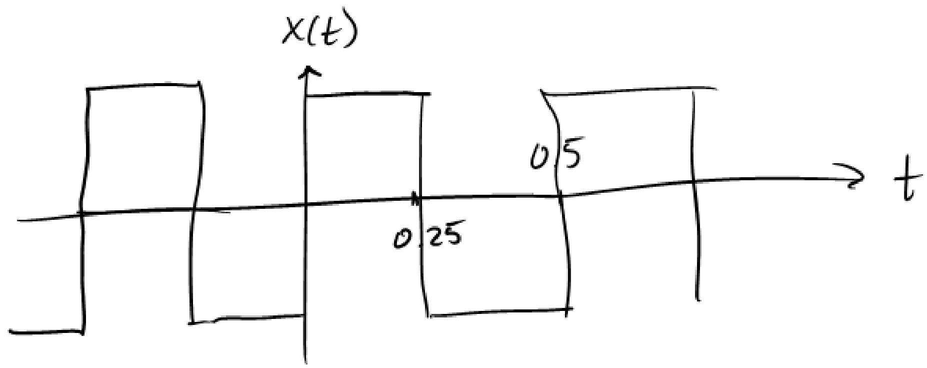
Panel 6

Hw 5 p 3

$T_0 = 0.5$

$$x(t) = \begin{cases} 1 & 0 \le t < 0.25 \\ -1 & 0.25 \le t < 0.5 \end{cases}$$

$$c_k = \begin{cases} \frac{-2j}{k\pi} & k \text{ odd} \quad \frac{2}{jk\pi} \\ 0 & k \text{ even} \end{cases}$$



6

Panel 7

$$C_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$T_0 = 0.5$

$\omega_0 = \frac{2\pi}{T_0} = 4\pi$

$$= \frac{1}{0.5}\left[\int_0^{0.25} e^{-jk\omega_0 t}\, dt + \int_{0.25}^{0.5} -e^{-jk\omega_0 t}\, dt\right]$$

$$= \frac{1}{0.5}\left[\left[\frac{1}{-jk\omega_0} e^{-jk\omega_0 t}\right]\Bigg|_0^{0.25} - \frac{1}{-jk\omega_0}\left[e^{-jk\omega_0 t}\right]\Bigg|_{0.25}^{0.5}\right]$$

$$= \frac{1}{0.5}\left[\frac{1}{-jk\omega_0}\left[e^{-jk\frac{4\pi}{4}} - e^{-jk4\pi 0}\right] + \frac{1}{jk\omega_0}\left[e^{-jk\frac{4\pi}{2}} - e^{-jk\frac{4\pi}{4}}\right]\right]$$

7

Panel 8

$$= \frac{1}{0.5}\left\{\frac{1}{-jk\omega_0}\left[e^{-jk\frac{4\pi}{4}} - e^{-jk4\pi 0}\right] + \frac{1}{jk\omega_0}\left[e^{-jk\frac{4\pi}{2}} - e^{-jk\frac{4\pi}{4}}\right]\right.$$

$$= \frac{1}{0.5}\left[e^{-jk\pi}\left[-\frac{1}{jk\omega_0} - \frac{1}{jk\omega_0}\right] + \frac{1}{jk\omega_0} + \frac{1}{jk\omega_0}e^{-jk2\pi}\right]$$

$$= \frac{1}{0.5}\,\frac{1}{jk\omega_0}\left[-2e^{-jk\pi} + 1 + e^{-jk2\pi}\right]$$

$$= \frac{1}{0.5}\,\frac{1}{jk\omega_0}\left[1 + \overset{1}{\cancel{\cos(2\pi k)}} - j\,\overset{0}{\cancel{\sin(2\pi k)}} - 2\cos(k\pi) + 2j\,\overset{0}{\cancel{\sin(k\pi)}}\right]$$

$$= \frac{1}{0.5}\,\frac{1}{jk\omega_0}\left[1 + 1 - 2\cos(k\pi)\right] = \frac{2}{jk4\pi}\left[2 - 2\cos(k\pi)\right]$$

8

Panel 9

Hw 6 p 4

$$x(t) = \sum_k X_k e^{jk\omega_0 t} \qquad y(t) = \sum_k y_k e^{jk\omega_0 t}$$

(a) $y(t) = b\,x(t-a)$

$$= b \sum_k X_k e^{jk\omega_0 (t-a)} = b \sum_k X_k e^{jk\omega_0 t} e^{-jk\omega_0 a}$$

$$= \sum_k \underbrace{b X_k e^{-jk\omega_0 a}}_{Y_k} e^{jk\omega_0 t}$$

$$H(j\omega) = \frac{Y_k}{X_k} = \frac{b X_k e^{-jk\omega_0 a}}{X_k} = b e^{-jk\omega_0 a}$$

9

Panel 10

(d) $$\ddot{y}(t) + \frac{2\xi}{\omega_n}\dot{y}(t) + \frac{1}{\omega_n^2}y(t) = K\,x(t)$$

$$\dot{y}(t) = \sum_k (jk\omega_0)\,Y_k\,e^{jk\omega_0 t}$$

$$\frac{d}{dt}e^u = e^u\frac{du}{dt}$$

$$\ddot{y}(t) = \sum_k -(k\omega_0)^2\,Y_k\,e^{jk\omega_0 t}$$

$$\sum_k -(k\omega_0)^2 Y_k e^{jk\omega_0 t} + \frac{2\xi}{\omega_n}\sum_k jk\omega_0 Y_k e^{jk\omega_0 t} + \frac{1}{\omega_n^2}\sum_k Y_k e^{jk\omega_0 t} = K\sum_k X_k e^{jk\omega_0 t}$$

$$y(t) = \sum_k Y_k e^{jk\omega_0 t}$$

10

Panel 11

$$\sum_k Y_k \left(-(k\omega_0)^2 + jk\omega_0 \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n^2} \right) e^{jk\omega_0 t} = \sum_k X_k K e^{jk\omega_0 t}$$

$$\sum_k Y_k e^{jk\omega_0 t}$$

$$Y_k = X_k \frac{K}{\left(\qquad \right)}$$

Panel 12

(c) $y(t) = b\, x(t) \cos(\omega_0 t)$

$= A(t)\, x(t)$

12

Panel 13

EXAM 1 p. 7

	Linear	TI	Mless	Causal
$\dot{y}(t) + t^2 y^2(t) = x^2(t+1)$	yes	No	No	No

$$a\dot{y}_1 + t^2 (a y_1)^2 = (a x_1)^2$$

$$b\dot{y}_2 + t^2 (b y_2)^2 = (b x_2)^2$$

$$a\dot{y}_1 + b\dot{y}_2 + t^2\left((a y_1)^2 + (b y_2)^2\right) = (a x_1)^2 + (b x_2)^2 \longrightarrow \dot{Y} + t^2 Y^2 = X^2$$

$$Y = a y_1 + b y_2$$

$$X = a x_1 + b x_2$$

13

Panel 14

	Linear	MLess	TI	Causal
$y(t) = x\left(1-\frac{t}{2}\right)$	yes	No	No	No

$y = x$

$y(1) = x\left(\frac{1}{2}\right)$

$y(-1) = x\left(\frac{3}{2}\right)$

	Linear	MLess	TI	Causal
$y(t) = 2$	No	yes	yes	No
$y(t) = x(2t)$	yes	No	No	No

$y(1) = x(2)$

Panel 15

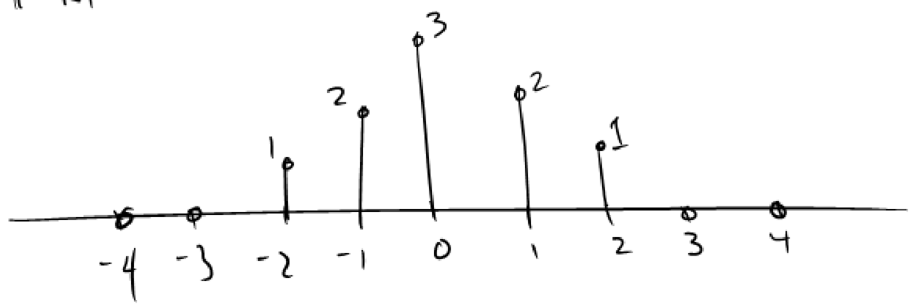
HW 5 p 6

$$P_{ave} = \sum_{k=-\infty}^{\infty} |c_k|^2$$

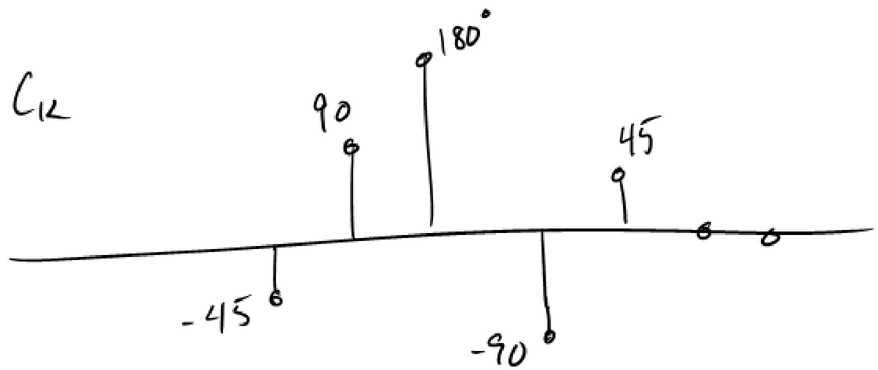
$$= |c_0|^2 + \sum_{k=1}^{\infty} 2|c_k|^2$$

$$= 3^2 + 2(2)^2 + 2(1)^2$$

$|c_k|$



$\angle c_k$



$$P_{ave} = \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\,dt$$

$$c_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt$$

15

Panel 16

$$2|C_k|\cos(\qquad)$$

$$= \frac{2|C_k|}{2}\left[e^{j(k\omega_0 t+\theta_k)} + e^{-j(k\omega_0 t+\theta_k)}\right]$$

$$= |C_k| e^{j(k\omega_0 t+\theta_k)} + |C_k| e^{-j(k\omega_0 t+\theta_k)}$$

$$|C_{-k}| e^{-jk\omega_0 t - \theta_{-k}}$$

16

Panel 17

17