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Modulation transfer function of the Stokes lens and its computer-controlled application for clinical optometry of astigmatism

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Astigmatism is a highly prevalent cylindrical refractive vision defect. The capability of the Stokes lens to compensate for astigmatism of a human eye was examined with a primary interest towards clinical optometry. A prototype of a computer-controlled apparatus, based on the Stokes lens principle, was designed and built with an objective to simplify clinical subjective refraction of astigmatism. Physical properties of the employed optical system were studied using modulation transfer function (MTF) as a metric. It was shown that the apparatus is capable of both compensating an arbitrary astigmatic defect and measuring its optical characteristics. The accuracy of the optometric measurements was evaluated. The proposed apparatus enables a novel method of optometry of astigmatism which relies on a patient's feedback obtained through intuitive manipulation of a computer human interface device, such as a mouse, in a 2-dimensional X-Y plane, rather than verbal interaction. This two-dimensional input is then translated into a virtual 2-dimensional space of optical power and the orientation angle of the cylinder axis and provides a measurement of astigmatism of the eye.

1 Introduction

Astigmatism is a refractive vision defect that affects approximately one-third of the world population [1]. The magnitude and the orientation of the astigmatic defect are usually determined by both objective and subjective refraction methods. In current optometric procedures, the objective refraction methods usually serve as a starting point for the subsequent subjective refraction procedure, which defines the final prescription lenses. Subjective refraction is a necessary step in the optometry of astigmatism, as prescribing from objective findings alone achieves limited patient satisfaction and occasionally yields erroneous prescriptions. Furthermore, subjective refraction allows for the evaluation of the functionality of a patient's entire vision system (including the retina and the brain), which is presently unachievable with objective methods. [2]

The widely used flip-cylinder procedure for clinical subjective refraction of astigmatism was developed in the 19th century [3] and has since become the method of choice for subjective refraction of astigmatism. Since the incorporation of Jackson cross-cylinder into phoropters in 1930s, the method has not been significantly modified, although the mechanical implementation received a few improvements. The method relies on the use of a set of fixed-power cylindrical lenses, usually combined into a phoropter apparatus, supplemented by a Jackson cross-cylinder lens. In this method, the refractive status of the eye is indirectly determined by testing the vision acuity in the presence of various corrective lenses. It involves iterative lens changes and rotations in front of the patient's eye, as well as the flipping of the Jackson cross-cylinder lens, and relies on verbal feedback from the patient. One of the significant disadvantages of this method is the necessity for the practitioner administering the test to rely on the accuracy of patient responses while making multiple iterative lens changes. The procedure can be difficult to execute when there are language barriers, communication issues or when there are uncontrolled factors affecting the patient's subjective perception of the test target or the conditions of the test. Additionally, as the lenses of the utilized set have inherently discrete powers, the accuracy of the measurement is limited by the magnitude of the step of the optical power of the available lens set. The measurement error cannot be made smaller than 1/2 of the step magnitude, which is typically 0.25 D. Thus, the ideal theoretical measurement resolution of a phoropter utilizing such a lens set is 0.125 D at best. However, in practice, the measurement error is still greater due to a number of other objective and subjective contributing factors. [2]

An alternative method of subjective refraction of astigmatism is based on the use of the astigmatic Stokes lens. This device is comprised of two cylindrical lenses of the same absolute value of optical power, but opposite signs, aligned on the same optical axis. These two lenses can be independently rotated about their common optical axis to achieve a combined cylindrical optical power that can be continuously varied between zero and twice the optical power of either lens, and any angle of the cylinder's axis orientation [4]. Alternative configurations comprising two identical optical elements were presented in [5] and [6]. The Stokes lens can be used for subjective refraction of astigmatism by allowing the patient to rotate the lenses until the position of the most acute vision is found. While the Stokes lens and the respective method of optometry was an advanced invention for its time, it was later replaced by the Jackson cross-cylinder method, which continues to be in use to this day. The factors that contributed to the demise of the Stokes lens were the difficulty of operation by the patient and the absence of an easy way to obtain the readings of the parameters of the required corrective optics [3]. Recently, with the increased availability of motorized and computerized controls, the interest towards continuously variable optics for application in optometry has been revived, thus making the use of the Stokes lens attractive again [7]. Computerized control can eliminate the problems of both the ease of use and the readout, thus paving the way for the use of the many advantages that Stokes lens has to offer. Besides the continuous variability of the optical parameters, a major advantage of the Stokes lens is its mode of operation, wherein the patient manipulates it on one's own, without any significant involvement of the optometry practitioner. The practitioner no longer needs to give the patient discrete options of optical parameters to choose from, and the patient does not need to tell which of the offered options is better. Instead, the patient oneself continuously varies optical parameters and finds the best combination of cylindrical optical power and the orientation angle. The Stokes lens is not the only continuously-variable optics solution that attracts attention of the optometry community. Alternative designs of continuously variable cylinder power refractors include fluidic lenses [8], elastomeric lenses [9], and transversally movable lenses using the Alvarez principle [10]. At the same time, the application of the Stokes lens is not limited just to the optometry. Areas of its application include characterization and compensation of a wide range of optical systems [5, 11, 12, 13]. An analysis and characterization of the optical power and aberrations of the Stokes lens were performed in [7, 11].

In the presented work, we examine the capability of the Stokes lens to compensate for a cylindrical refractive imperfection of an optical system and to provide a means of measurement of such an imperfection, primarily in application to a human eye with astigmatism. Unlike previously published research, we performed our characterization using an MTF approach. We further introduce a computer-controlled apparatus based on a Stokes lens principle that is designed to simplify the clinical subjective refraction of astigmatism.

2 Method

Experiments were conducted using a specially engineered rotatable skew-axis cylindrical lens optical system (SACLOS), which is a development of the Stokes lens with capabilities for motorized computer-controlled operation of lens rotation and data acquisition (Fig. 1). For the optical characterization of the system, we built a manually-operated laboratory prototype. In the experiments presented here, the Stokes lens was comprised of two cylindrical lenses of +0.25 D and -0.25 D optical power. A Siemens star was used as the test object, and a photo camera was used for image capturing. The SACLOS and an optional distorting cylindrical lens of a chosen optical power and orientation were placed on an optical bench in front of the camera lens. The distorting cylindrical lens was used to simulate an astigmatic vision defect of a person, and the SACLOS was used to correct the defect. Thus, the camera captured images of the test object as it would be seen by a patient having the vision defect. MTF50 was used as a metric of the simulated vision acuity. The MTF50 is defined as the spatial frequency magnitude, measured in deg^{-1} (or cycles per degree), yielding a 50% modulation transfer function of the optical system. Higher MTF50 values correspond to higher image sharpness and better vision acuity. The angles of orientation of the cylindrical lenses within SACLOS were varied, and resulting photographs of the test object were generated. Subsequently, the photographs were analyzed using image analysis software to obtain the MTF50 value for each

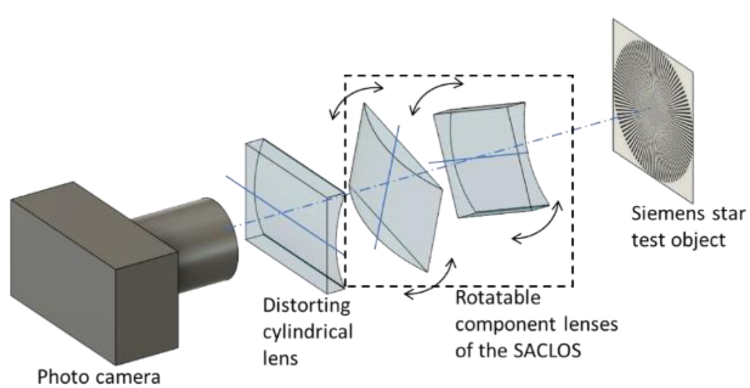

Fig. 1. Schematic diagram of the optical system used in the experiments.

photograph. Thus, dependencies of the MTF50 of the system on the optical parameters were obtained.

MTF contour plots of the Siemens star were also generated. This test object has a pattern of varying spatial frequency – highest at the center of the target and lowering towards the periphery. As the Siemens star is comprised of alternating black and white segments converging towards the center, the brightness gradient at any point of it is oriented perpendicularly to the rays of the star, tangentially to the circles concentric with the center of the star. Since all possible angles of the brightness gradient orientation are thus represented, this object allows to measure and graphically characterize the degree of the image blur depending on the brightness gradient orientation with respect to the orientation of the cylinder axis. Therefore, not only is it possible to simulate the overall vision acuity of a patient under given test conditions, but also to see how it depends on the relative orientation of the object contrast features with respect to the axis of the astigmatic defect.

3 Results and Discussion

As an initial reference point, the MTF50 of the system in the best focus condition without the distorting lens or the SACLOS was measured to be 55.4±0.3 deg^{-1}. Additionally, the MTF contour plot of the Siemens star object was obtained (Fig. 2a). It is predominantly rotationally symmetric with insignificant deviations. After a +0.50 D cylindrical lens was introduced into the system as a simulation of an astigmatic distortion, the MTF map of the test object transformed and showed a significant anisotropy that is coaxial with the axis of the cylinder (Fig. 2b). The MTF50 value ranged

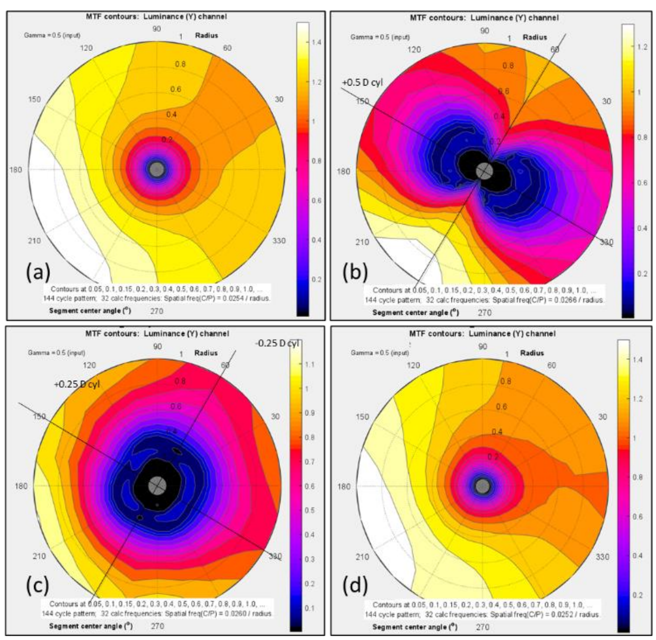

Fig. 2. MTF map of the Siemens star test object: (a) without distorting lens or Stokes lens; (b) with +0.50 D cylindrical distorting lens; (c) with +0.50 D cylindrical and -0.25 D spherical distorting lens; (d) with cylindrical and spherical distorting lens and Stokes lens adjusted to neutralize the effect of the distorting lens.

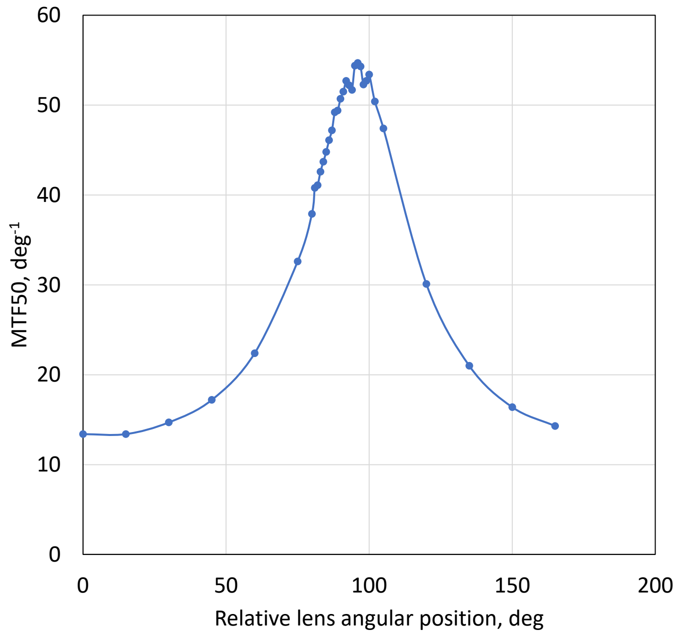

Fig. 3. A plot of the MTF50 dependence on the position angle of one element of the SACLOS, with the other element's angle being fixed.

from 10 to 37 deg $^{-1}$, depending on the orientation of the brightness gradient, -- the minimum being noticeably lower than the initial reference value. This behavior is expected since the target has the maximum contrast in the tangential direction, but practically no contrast in the radial direction. The contrast of the resulting image in the direction of the distorting lens cylinder axis is virtually unaffected, while the contrast in the perpendicular direction is reduced due to defocusing with the optical power of the cylindrical lens. This condition is representative of the vision of a patient with an astigmatic vision defect. However, in most cases, a human eye will attempt to accommodate to achieve the circle of least confusion, i.e. the same degree of blur of the object contrast features of all directions. In the experiments, this condition was simulated by an additional introduction of a -0.25 D spherical lens into the system, which resulted in the circle of least confusion through spherical focusing. As a result, in this case, the MTF50 of the system becomes 13.1±1.0 deg $^{-1}$, while the anisotropy of the MTF map is virtually eliminated (Fig. 2c). This value is still significantly lower than the initial reference value.

Similarly, the MTF50 values were measured for the SACLOS alone, in the absence of a distorting cylindrical and focusing spherical lenses. Those values were obtained as a function of the rotational positions of two cylindrical elements of the Stokes lens. A plot of MTF50 dependence on the position angle of one lens, with the other lens angle being fixed, is shown in Fig. 3. It has prominent peaks where cylinder axes of the two lenses are aligned, and their optical powers neutralize each other. These peaks correspond to the condition of the highest contrast of the image produced by the system. Rotation of the other lens shifts the curve horizontally, but its shape remains unchanged.

Finally, all three components were introduced into the system, i.e., the distorting +0.50 D cylindrical lens, -0.25 D focusing spherical lens, and the SACLOS. MTF50 values of the resulting combination were mapped over the 2-dimensional virtual space of orientation angles of the Stokes lens rotatable elements. The dependence is shown in Fig. 4. It has a pronounced peak, which corresponds to the condition of the best compensation of the distorting cylindrical lens by the SACLOS. A contour plot of MTF50 in the vicinity of the peak is shown in Fig. 5, and a corresponding MTF map of the test object is shown in Fig. 2d. Similarly to the map without an introduced cylindrical distortion, it is close

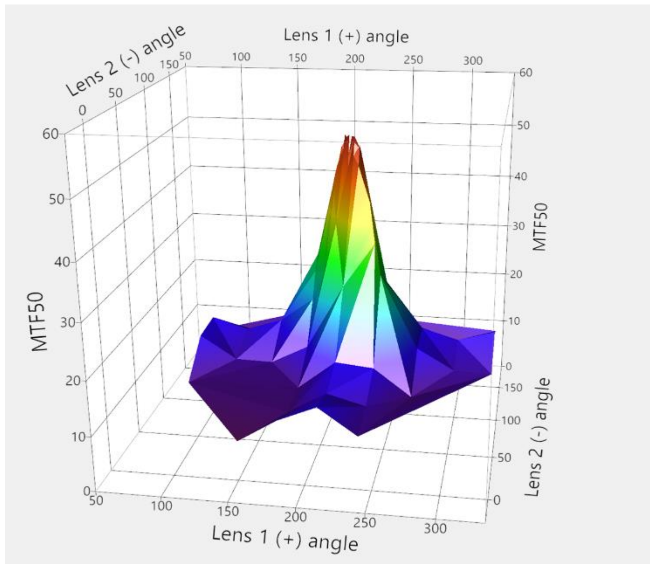

Fig. 4. The MTF50 of the optical system including a +0.50 D distorting cylindrical lens as a function of the position angles of the elements of the SACLOS.

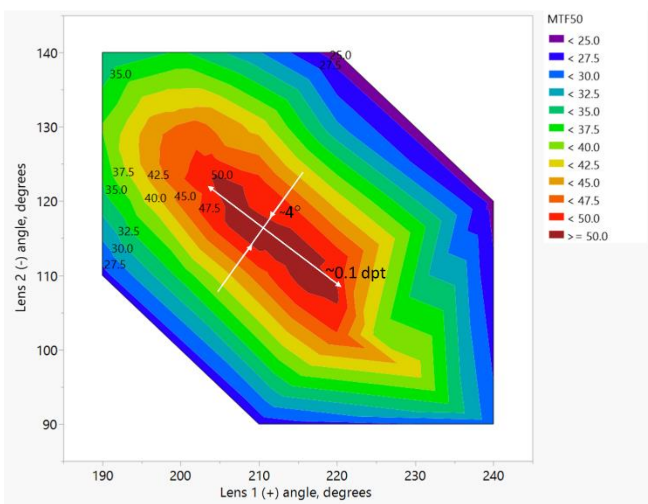

Fig. 5. Contour plot of the MTF50 of the optical system including a +0.50 D distorting cylindrical lens as a function of the position angles of the elements of the SACLOS. The optical power and angle bars are shown to illustrate the scale and do not represent a specific measurement.

to rotationally symmetrical. The MTF50 value is 51.5±3.0 deg^{-1}, which is just slightly below the initial reference value. Thus, a Stokes lens has the capability to almost fully compensate a cylindrical distortion of an optical system with an insignificant degradation of MTF. Moreover, the exact values of the orientation angles of the rotatable cylindrical lenses of the SACLOS can be readily measured, and the resulting cylindrical optical power and cylinder axis angle can be calculated.

This approach was used to measure the optical power values and cylinder axis orientation of known distorting reference lenses and to compare the measured values against the known values. It should be noted that both parameters were measured in conjunction, without presetting one of them to measure the other. The measured values closely matched the known parameters of the distorting reference lenses. The magnitude of the measurement error is summarized in Table 1. The mean measurement error was below 3% of the value, and was consistently less than the admissible discrepancy value of 0.13 D recommended by the ANSI Z80.1-2005 standard [14] for single vision lenses with nominal power below 6.5 D. The measured orientation angles also showed a good agreement with the actual values, with the absolute measurement error of the angle of cylinder orientation being 2.1±1.7 degrees. This shows that a device utilizing the Stokes lens principle is capable of measuring the parameters of a distorting cylindrical lens with an accuracy sufficient for many practical applications, including the optometry of astigmatism.

Table 1. Summary of measurements of distorting lens parameters performed using the SACLOS

Nominal cylindrical optical power of the reference lens, D	Mean measured cylindrical optical power of the reference lens, D	Standard deviation of the measured optical power, D	Mean measurement error of the angle of orientation, deg	Standard deviation of the measurement error of the angle of orientation, deg
0.5	0.52	0.04	-0.2	2.7
1.0	1.03	0.06	-3.1	2.3
1.5	1.54	0.05	0.2	0.9

4 Application

Based on the above study, we for the first time introduce the concept of the SACLOS as a development of Stokes lens with motorized computer-controlled operation of the lens rotation and data acquisition of the lens angular positions. The device is operated using a controller, such as a computer mouse, trackball, joystick, or other similar computer human interface device (HID), which receives and processes the input in the form of coordinates in a 2-dimensional physical X-Y space. The X-Y coordinate input coming from the controller is then electronically converted to set the angular positions of the lenses of the SACLOS in real time using stepper or server motors. That, in turn, determines the cylindrical optical power and the cylinder angle of orientation of the system. The dependence of the SACLOS lens angular positions on HID X and Y coordinates can be linear or follow other mathematically continuous and monotonic function. Thus, these two optical parameters are simultaneously, independently, and predictably set and controlled by a HID. This control can be performed in real time with just an insignificant delay, which is determined primarily by the mechanical responsiveness of the system.

In application for the clinical optometry of astigmatism, just like using the Stokes lens, an astigmatic patient can view a test target, e.g., a Snellen chart, through the SACLOS while directly manipulating the optical parameters – cylindrical optical power and the cylinder angle of orientation. However, unlike the classic Stokes lens, the manipulation is performed using an intuitive 2-dimensional computer HID. The patient would use a single parameter - vision acuity - as the criterion of optimization in the continuous space with two degrees of freedom. Our MTF data shows that such an optimum point, the maximum of MTF50, exists and is unique within a 180° range of cylinder orientation angles and the entire range of optical power (Fig. 4). The values of MTF50 function are monotonically increasing along any direction leading to the point of its maximum. Thus, by manipulating the HUD and preferentially following the direction of the gradient of MTF50, i.e., intuitively preferring the direction that improves the vision acuity, a patient will be able to find the location of the maximum of MTF50, which is also the maximum of vison acuity. The fundamental distinguishing feature of SACLOS is its capability to effortlessly move in any direction within the virtual D-θ space. In particular, this enables the movement in the direction of the vision acuity gradient. In contrast, the classic Stokes lens only allows the movement in two discrete directions determined by the rotation of either one of the two component lenses. Simultaneous rotation of both lenses, although possible, is unintuitive. Thus, the movement in the direction of the vision acuity gradient could only be accomplished iteratively, not continuously, which makes it difficult to perform. It is important that there are no additional local maxima that could mislead and inadvertently lock the search to a wrong location. Thus, the SACLOS allows a patient to locate the point of one's highest vision acuity without the need for back-and-forth verbal interaction with the test-administering optometry practitioner. It is important that the variation of the optical power of the compensating lens is continuous and is performed in real time with HUD manipulation. It should be noted that the approach described above is impossible to implement within the framework of the state-of-the-art refractors because of their discrete-optics design.

The SACLOS device control emphasizes the subjective perception and intuitive actions of the patient. Once the condition of the highest vision acuity is achieved, the readings of the SACLOS lens angles can be electronically acquired and recalculated into the optical power and orientation angle of the optimum corrective lens. Being operated by a computer, the device can be further enhanced by providing assistive algorithms aiding the patient. Moreover, it can also be used to execute the Jackson cross-cylinder flip-lens protocol under the direct intuitive control of the patient alone using just a computer mouse, without the involvement of an optometry practitioner into the lens manipulation. The device can also be combined with either discrete or continuously-variable spherical lenses to compensate and measure the patient's spherical refractive defect.

Additional experiments involving human subjects will be required to evaluate the practical aspects of the application and the accuracy of subjective tuning of the vision acuity. While such experiments remain outside of the scope of this work, the instrumentational capabilities of the employed optical system presented here suggest that an improved subjective refraction of astigmatism using SACLOS is plausible.

The proposed device has the potential to fundamentally transform the roles of the patient and the optometry practitioner in the procedure of the subjective refraction, resulting in a more streamlined protocol and a reduction of cost. It is envisioned that the subjective refraction could be simplified to a point where self-service kiosks for a quick vision check might become feasible and commonly used.

5 Conclusion

In summary, a skew-axis cylindrical lens optical system (SACLOS) has been proposed as a development of Stokes lens with motorized computer-controlled operation of the lens rotation and data acquisition for the subjective optometry of astigmatism. The optical characterization of the system using MTF analysis demonstrated the capability to compensate for, and measure, the optical characteristics of astigmatic refractive defects of a human eye with an accuracy sufficient for clinical application. The dependence of MTF50 on the optical power and cylinder axis orientation has no local maxima, which is essential for the unambiguous location of the point of highest vision acuity. The system opens a possibility for the new approach to subjective refraction of astigmatism that emphasizes the subjective perception and intuitive actions of the patient.

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Data availability. Data underlying the results presented in this paper are available in Ref. [15]

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