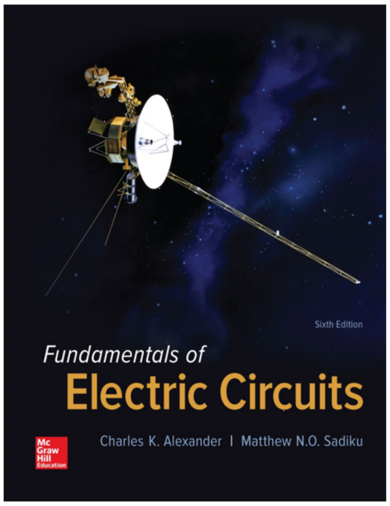


Chapter 6 - Capacitors and Inductors

- **This chapter will introduce two new linear circuit elements:**
- **The capacitor**
- **The inductor**
- **Unlike resistors, these elements do not dissipate energy**
- **They instead store energy**
- **We will also look at how to analyze them in a circuit**

Ch 6, p1

6.1 Introduction

Resistors, studied in the previous chapters, are passive linear circuit elements that dissipate energy. Capacitors and inductors are, likewise, passive elements but they act to store energy rather than dissipate it.

6.2 Capacitors

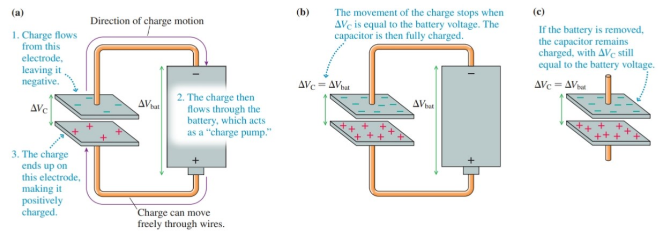
Two conductors with equal but opposite charges form a capacitor, a device that is widely used in electronic circuits since they have the ability to store charge.

The two conductors are known as the *electrodes* or *plates* and are typically made of aluminum foil.

Capacitors can come in many shapes and all that is needed is to have two electrodes separated by a region of an insulating material (known as a *dielectric*), which can be air, ceramic, paper, plastic, or mica.

The simplest way to "charge" a capacitor is to connect the plates of a capacitor to a source of potential difference, and the simplest source of potential difference is a battery.

Ch 6, p2



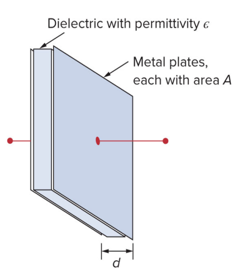
PhET "Capacitor Lab - basics"?

- When a voltage source v is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other.
- The charges will be equal in magnitude
- The amount of charge is proportional to the voltage: $q = Cv$.

where C, the constant of proportionality, is known as the capacitance of the capacitor, and the unit of capacitance is the farad (F) in honor of Michael Faraday. 1 F = 1 Coulomb / Volt. Most capacitors are rated in pico-farad or micro-farad.

Capacitance, therefore, is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads. Alternatively, capacitance is the amount of charge stored per plate for a unit voltage difference in a capacitor.

Even though capacitance is the ratio of charge per plate to the applied voltage, capacitance does not depend on either and, instead, depends on the physical characteristics of the capacitor.



For a parallel plate capacitor, capacitance is given by:

$$C = \frac{\epsilon_0 A}{d}$$

Capacitance of a parallel-plate capacitor with plate area A and separation d

Although this equation applies only to parallel-plate capacitors, some inferences concerning capacitance can be drawn:

1. the larger the surface area of the plates, the larger the capacitance.
2. the smaller the spacing between the plates, the larger the capacitance.
3. the higher the permittivity of the dielectric (separating) material, the greater the capacitance.

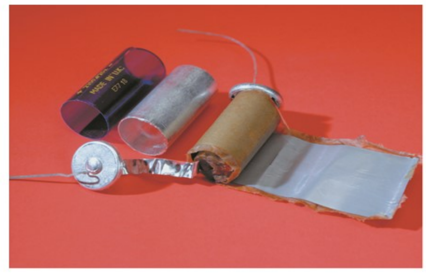
Capacitors are commercially available in different values and types, are described by the dielectric material they are made with, and by whether they are fixed or variable.

A large electrode area and small spacing are needed in order to get a reasonable value of capacitance.

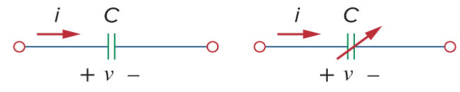
In practice, this is frequently accomplished by using large foil electrodes that are separated by a very thin layer of insulation and then rolled up. This allows for large electrodes to be squeezed into small packages.

Even though the electrodes are no longer flat plates/planes, the parallel-plate capacitor equation still predicts the capacitance reasonably well if the separation (d) is much smaller than

- the width of the plates,
- the length of the spirals, and
- the radius of curvature of the spiral.



Circuit symbols for capacitors: (a) fixed capacitor (b) variable capacitor.

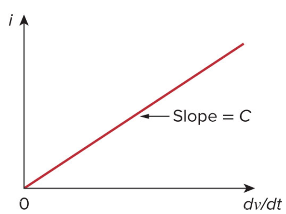


Ch 6, p5

Using the formula for the charge stored in a capacitor, the current-voltage relationship of a capacitor is found by taking the first derivative with respect to time:

$$i = C\frac{dv}{dt}$$

Capacitors that satisfy this relationship are said to be linear. Although some capacitors are non-linear, most are linear.



The voltage-current relationship is found be integrating both sides of the current-voltage equation:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

where $v(t_0)/C$ is the charge across the capacitor at time t_0.

The instantaneous power delivered to the capacitor is given by:

$$p = vi = Cv\frac{dv}{dt}$$

The energy stored* in the electric field between the plates of a capacitor is given by:

$$w = \frac{1}{2}Cv^2 \quad \text{or} \quad w = \frac{q^2}{2C}$$

* In fact, the word capacitor is derived from this element's capacity to store energy in an electric field.

Ch 6, p6

Capacitors all have these characteristics:

- when the voltage is not changing, the current through the capacitor is zero. This means that no current will flow with DC applied to the terminals and a capacitor is an open circuit to DC.
- the voltage on a capacitor must be continuous. The voltage on the capacitor's plates cannot change instantaneously or abruptly since an abrupt change in voltage would require an infinite current which is impossible.

  This means that, if the voltage on the capacitor does not equal the applied voltage, charge will flow and the voltage will eventually reach the applied voltage.

  Conversely, the current through a capacitor can change instantaneously.
- an ideal capacitor does not dissipate energy, meaning that it takes power/energy from the circuit when storing energy in its field and returns that energy when delivering power to the circuit.
- a real (non-ideal) capacitor, however, has a parallel-model leakage resistance, which leads to a slow loss of the internally stored energy.

  This resistance is typically very high, on the order of 100 MΩ, and can be ignored for many circuit applications.

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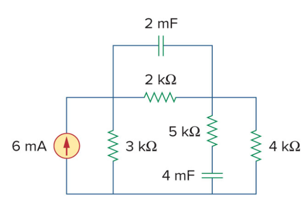
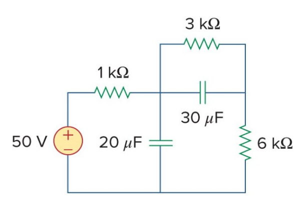
Example and Practice Problems (Cont.)

E6.5 Determine the energy stored in each capacitor of the circuit shown below under DC conditions.



P6.5 Determine the energy stored in the capacitors in the circuit shown below under DC conditions.



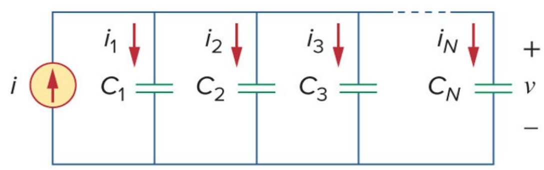
Ch 6, p9

6.3 Series and Parallel Capacitors

With resistors, applying the equivalent series and parallel combinations can simply many circuits. That technique can also be extended to capacitors.

Starting with *N* parallel capacitors, one can note that the voltages on all the capacitors are the same

Applying KCL: $i = i_1 + i_2 + i_3 + ... + i_N$



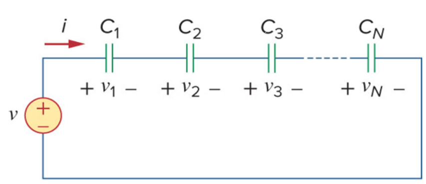
Since $i = C\frac{dv}{dt}$ and considering the current-voltage relationship for each capacitor:

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

where: $C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$

The equivalent capacitance of *N* parallel-connected capacitors is the sum of the individual capacitances. Parallel capacitors combine in the same manner as series resistors.



Turning to a series arrangement of capacitors, each capacitor shares the same current. Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

Applying the voltage-current relationship for each capacitor:

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + v_2(t_0) + \frac{1}{C_3}\int_{t_0}^{t} i(\tau)d\tau + v_3(t_0) + ... + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + v_2(t_0) + v_3(t_0) + \cdots + v_N(t_0)$$

The factor ahead of the integral can be rewritten as:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

The equivalent capacitance of *N* series-connected is the reciprocal of the sum of the reciprocals of the individual capacitances. Capacitors in series combine in the same way as resistors in parallel.

For the special case of two capacitors in series:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

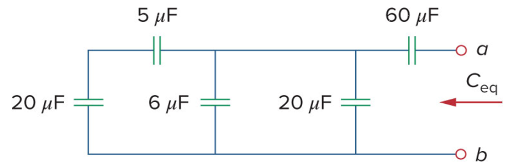
Another way to think about the combinations of capacitors is this:

Combining capacitors in parallel is equivalent to increasing the surface area of the capacitors, and this would lead to an increased overall capacitance (as is observed)

A series combination can be seen as increasing the total plate separation, which would result in a decrease in capacitance (as is observed)

Example and Practice Problems

E6.6 Find the equivalent capacitance seen between terminals a and b:



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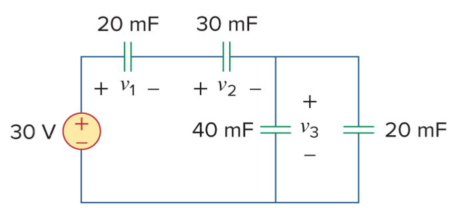
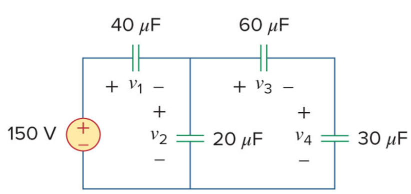
Example and Practice Problems (Cont.)

E6.7 For the circuit shown, find the voltage across each capacitor.



P6.7 Find the voltage across each of the capacitors in the figure shown below.



Ch 6, p13

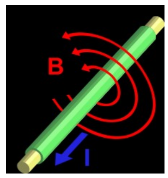
6.4 Inductors

An inductor is a passive element that stores energy in its magnetic field, and inductors have applications in power supplies, transformers, radios, TVs, radars, and electric motors.

During a demonstration as part of a physics lecture, the Danish physicist Hans Christian Oersted (1777-1851) noticed that an electric current flowing through a wire created an magnetic field. Oersted had accidentally discovered that electricity and magnetism, two forces thought to be totally different, were actually related to one another.

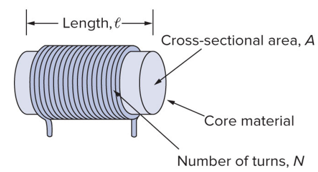
An electric current through a straight wire creates a magnetic field around the wire that is oriented according to the right-hand rule for fields:

1. Point your **right** thumb in *I* the direction of the current.
2. Wrap your fingers around the wire to indicate a circle.
3. Your fingers curl in the direction of the magnetic field lines around the wire.



Any electrical conductor has inductive properties and may be thought of as an inductor, but the effect is typically enhanced by coiling the wire up.

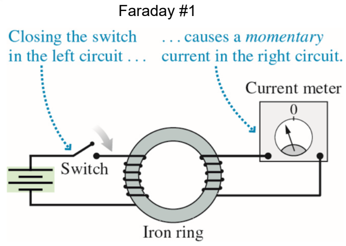
For most practical purposes, therefore,an inductor consists of a coil of conducting wire.



Once Oersted's discovery showed that magnetic effects arise from electricity, other scientists, particularly British physicist Michael Farady (1791-1867), were quick to show that electrical effects arise from magnetism.

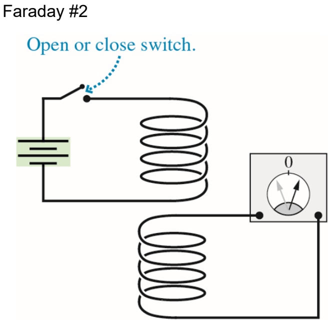
- Michael Faraday experimented with two coils of wire wrapped around an iron ring in an attempt to generate a current from a magnetic field.
- Faraday's experiment did not generate a steady current; however, in the instant he closed the switch in the circuit, there was a brief indication of a current.
- He realized that a current was generated only if the magnetic field was **changing** as it passed through the coil.
- Faraday then set up a series of experiments to test this hypothesis.

Faraday #1



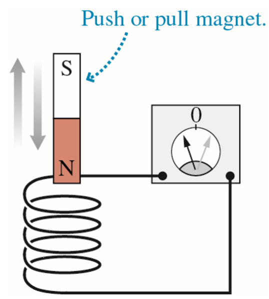
Faraday #2

- Faraday placed one coil directly above the other, without the iron ring.
- There was no current in the lower circuit while the switch was in the closed position, but a momentary current appeared whenever the switch was opened or closed.



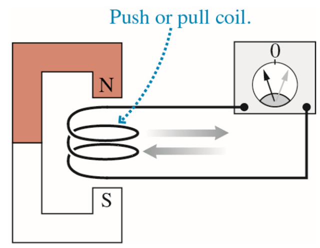
Faraday #3

- Faraday pushed a bar magnet into a coil of wire. This action caused a momentary deflection of the needle in the current meter, although **holding** the magnet inside the coil had no effect.
- A quick withdrawal of the magnet deflected the needle in the other direction.



Faraday #4

- Faraday created a momentary current by rapidly pulling a coil of wire out of a magnetic field. There was no current if the coil was stationary in the magnetic field.
- Pushing the coil **into** the magnet caused the needle to deflect in the opposite direction.



PhET "Faraday's Electromagnetic Lab"??

Faraday found that there is a current in a coil of wire if and only if the magnetic field passing through the coil is changing.

The current in a circuit due to a changing magnetic field is known as an *induced current*, and its creation is an example of what is known as *electromagnetic induction*.

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If a current is passed through an inductor, the voltage across the inductor is directly proportional to the time rate of change in current:

$$v = L\frac{di}{dt}$$

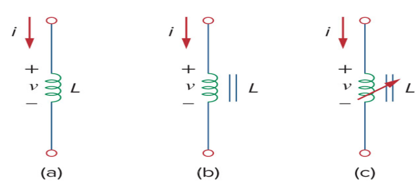
where *L*, the constant of proportionality, is called the inductance of the inductor, and the unit of inductance is the henry (H) in honor of Joseph Henry. 1 H = 1 volt-second / ampere.

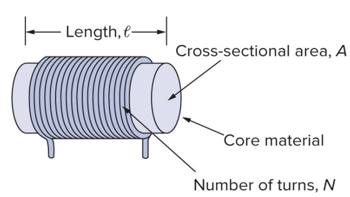
The inductance (*L*) of an inductor depends on its physical dimensions and construction. For an inductor of the type shown below (also known as a solenoid):

$$L = \frac{N^2 \mu A}{l}$$

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it.

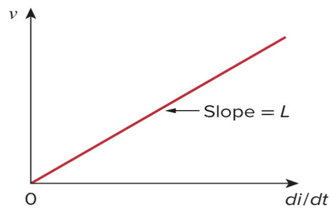
Circuit symbols for inductors: (a) air-core inductor (b) iron-core inductor, and (c) variable iron-core inductor.





Ch 6, p17

Inductors that satisfy the voltage-current relationship ($v = L\frac{di}{dt}$) are said to be linear.



Rearranging the equation for voltage across an inductor, the current-voltage relationship of an inductor is given by:

$$di = \frac{1}{L} v dt$$

Integrating both sides of the current-voltage equation:

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

An inductor is designed to store energy in its magnetic field, and the power delivered to the capacitor is given by:

$$p = vi = Li\frac{di}{dt}$$

The energy stored in the magnetic field of an inductor is given by:

$$w = \int_{-\infty}^{t} p(\tau)d\tau = L\int_{-\infty}^{t} \frac{di}{d\tau} i d\tau$$

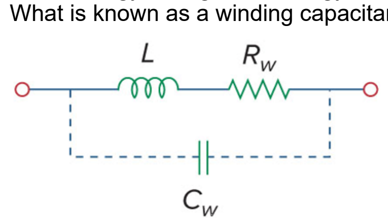
$$= L\int_{-\infty}^{t} i di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

Since $i(-\infty) = 0$: $w = \frac{1}{2}Li^2$

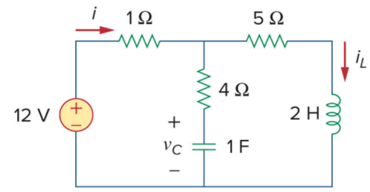
Some important properties of inductors to note:

- when the current is not changing, the voltage across an inductor is zero. This means that an inductor acts as a short circuit to DC.
- an inductor's opposition to the change in current flowing through it means that the current through an inductor cannot change instantaneously since a discontinuous change in current would require an infinite voltage, which is not physically possible.

  Conversely, the voltage across an inductor can change abruptly.
- an ideal inductor does not dissipate energy, meaning that it takes power/energy from the circuit when storing energy in its field and returns that energy when delivering power to the circuit.
- a real (non-ideal) inductor, however, has a significant resistive component due to the fact that the inductor is made of a conducting wire which has some resistance. This resistance is called the winding resistance (R_w), which makes real inductors both an energy storage and energy dissipation device. R_w is typically very small and can be ignored. What is known as a winding capacitance (C_w) also exists and can also be ignored in most cases.



Examples and Sample Problems



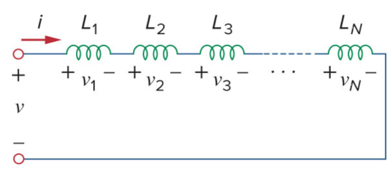
E6.10 Under DC conditions and considering this circuit:

(a) find i, $v_{C,}$ and i_L and

(b) the energy stored in the capacitor and inductor.

Ch 6, p20

6.5 Series and Parallel Inductors



Consider a series combination of inductors.

Applying KVL to the loop: $v = v_1 + v_2 + v_3 + \ldots + v_N$

Factoring in the voltage-current relationship and simplifying:

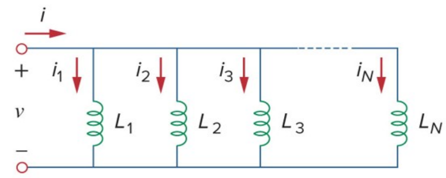
$$v = \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt} \qquad \text{where} \quad L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N$$

The equivalent inductance of series-connected inductors is the sum of the individual conductances.

Inductors in series are combined in exactly the same way as resistors in series.

Ch 6, p21

Consider a combination of inductors in parallel.



Applying KCL to the loop: $i = i_1 + i_2 + i_3 + ... + i_N$

When the current voltage relationship is considered:

$$i = \left(\sum_{k=1}^{N} \frac{1}{L_{eq}} \right) \int_{t_0}^{t} v dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^{t} v dt + i(t_0) \quad \text{where: } \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + ... \frac{1}{L_N}$$

The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

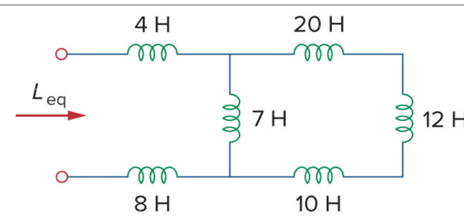
<u>Inductors in parallel are combined in the same way as resistors in parallel.</u>

For the special case of two inductors in parallel: $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

One final note - the Delta-Wye transformation can also be applied to inductors and capacitors in a similar manner, as long as all elements are the same type.

Examples and Practice Problems

E6.11 Find the equivalent inductance of this circuit.



Here is a (useful) summary of the important characteristics of the three, basic (passive) circuit elements.

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Ch 6, p24