Mass dependence of the heavy quark potential and its effects on quarkonium states

Alexander Laschka
Norbert Kaiser Wolfram Weise

Physik Department
Technische Universität München

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Heavy quark-antiquark potential

- History: phenomenological potential models
  - Fitted to low lying charmonium and bottomonium states
  - Typical shape: "Coulomb-plus-linear"

- Today: heavy quark-antiquark potential from QCD
  - Characteristic scales of non-relativistic bound states

m	heavy quark mass	**hard scale**
mv	heavy quark momentum	**soft scale**
mv^2	heavy quark energy	**ultrasoft scale**

  - Effective field theory (EFT) methods
    QCD $\Rightarrow$ non-relativistic QCD (NRQCD, pNRQCD, vNRQCD)

Topics: Extended range of validity of perturbative potential

Spectroscopy at order $1/m$

Detailed analysis of the role of quark masses

Outline

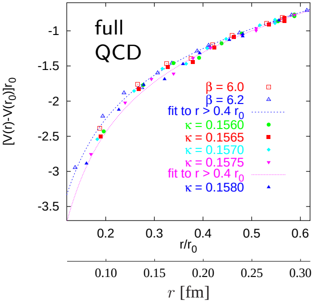
1. **Static quark-antiquark potential**
2. Heavy quark potential at order $1/m$

The static potential

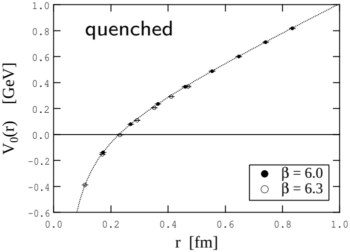
Non-perturbative sector: lattice studies of quenched and full QCD

- Static QCD potential (from static Wilson loop)

G.S. Bali et al., Phys.Rev.D62 (2000)



Y. & M. Koma, Nucl.Phys.B769 (2007)



- Sea quark effects important at small distances

Fourier transform of the static potential

Perturbative sector: static potential is known at three-loop order

M. Peter, Phys.Rev.Lett.78 (1997), Y. Schröder, Phys.Lett.B447 (1999)
Three-loop: C. Anzai, Y. Kiyo, Y. Sumino, Phys.Rev.Lett.104 (2010),
A. & V. Smirnov, M. Steinhauser, Phys.Rev.Lett.104 (2010)

- Momentum space

$$\tilde{V}^{(0)}(|\vec{q}\,|) = -\frac{4\pi C_F\,\alpha_s(|\vec{q}\,|)}{\vec{q}^{\,2}}\left[1 + \frac{\alpha_s(|\vec{q}\,|)}{4\pi}\,a_1 + \left(\frac{\alpha_s(|\vec{q}\,|)}{4\pi}\right)^2 a_2 + \left(\frac{\alpha_s(|\vec{q}\,|)}{4\pi}\right)^3\left(a_3 + 8\pi^2 C_A^3 \ln\frac{\mu_{\mathrm{IR}}^2}{\vec{q}^{\,2}}\right) + \ldots\right]$$

where $C_F = 4/3$, $C_A = 3$,
$a_1 = 7$, $a_2 \approx 268.8$, $a_3 \approx 5199.8$ $(n_f = 3)$

- At $\mathrm{N^3LO}$ (three-loop order):
infrared divergences (μ_{IR}^2) from ultrasoft gluons

- Avoid expansion of $\alpha_s(|\vec{q}\,|)$ about a fixed scale μ

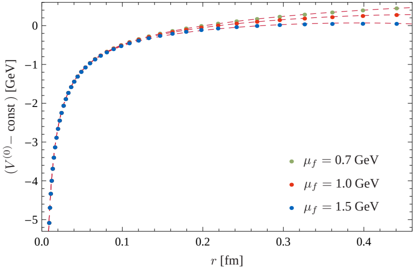
- Reliable potential from extremely small distances up to $r \approx 0.15$ fm needed

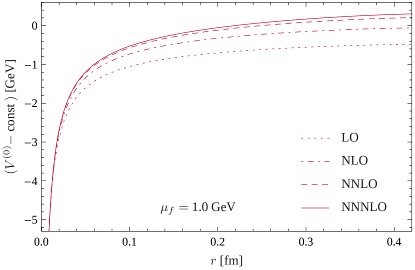
Potential subtracted (PS) scheme

PS scheme with numerical Fourier transform

- Evaluate numerically (with a low-momentum cutoff μ_f)

$$V^{(0)}(\vec{r},\mu_f) = -4\pi C_F \int_{|\vec{q}|>\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}}\, \frac{\alpha_s(|\vec{q}|)}{\vec{q}^2} \left[\underset{\Downarrow\ \text{LO}}{1} + \underset{\Downarrow\ \text{NLO}}{\frac{\alpha_s(|\vec{q}|)}{4\pi}} a_1 + \underset{\Downarrow\ \text{NNLO}}{\left(\frac{\alpha_s(|\vec{q}|)}{4\pi}\right)^2} a_2 + \ldots \right]$$



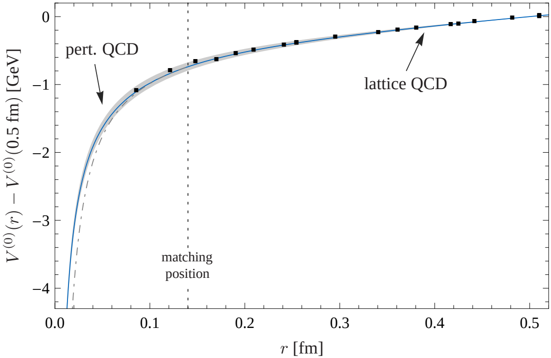


- No free scale parameter μ
- Unknown constant is moved into the definition of m_{PS}:

$$2m_{\mathsf{pole}} + V^{(0)}(r) = 2m_{\mathsf{PS}}(\mu_f) + V^{(0)}(r,\mu_f)$$

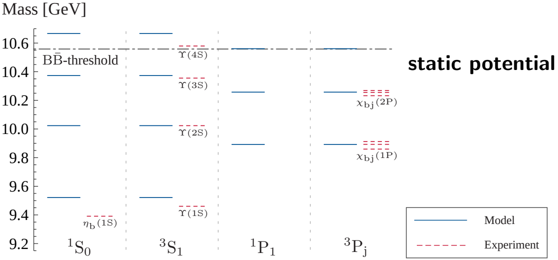
Matching and uncertainty estimate

- Perturbative potential (here NNLO) and lattice potential matched



- Differentiable quark-antiquark potential for distances up to ∼1 fm
- Matching at 0.14 fm gives $\mu_f = 0.9^{+0.3}_{-0.2}$ GeV
  (for charmonium and bottomonium)
- Grey band: uncertainty of lattice calculation and uncertainty of α_s
- Dot-dashed curve: continuation of the "Coulomb-plus-linear" fit

Solve the Schrödinger equation with this matched potential



- Single parameter $m_{\text{PS}}(0.908\text{ GeV}) = 4.78\text{ GeV}$
- Can be converted to the $\overline{\text{MS}}$ scheme

$\overline{\text{MS}}$ masses [GeV]	$m_{\overline{\text{MS}}}$	PDG 2010
bottom quark	4.20±0.04	$4.19^{+0.18}_{-0.06}$
charm quark	1.23±0.04	$1.27^{+0.07}_{-0.09}$

Outline

1. Static quark-antiquark potential

2. **Heavy quark potential at order $1/m$**

Quark-antiquark potential at order $1/m$

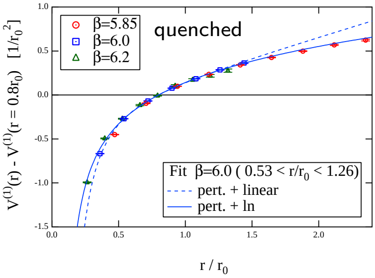
- Expansion in inverse powers of the heavy quark mass m

$$V(r) = V^{(0)}(r) + \frac{V^{(1)}(r)}{m/2} + \frac{V^{(2)}(r)}{(m/2)^2} + \dots$$

- Non-perturbative expression for $1/m$ potential is known
  N. Brambilla et al., Phys.Rev.D63 (2001) 014023

- **Lattice simulations**

> Efficient method from M. & Y. Koma and H. Wittig
> Quenched simulation, renormalization issues ($\approx 15\%$ error estimated)



Contains a non-perturbative contribution

Fit function

$$V^{(1)}_{\ln}(r) = -\frac{A_2}{r^2} + B_2 \ln r + C_2$$

Effective string theory suggests logarithmic shape: $V^{(1)} \propto \ln r + C$
G. Perez-Nadal, J. Soto, Phys.Rev.D79 (2009)

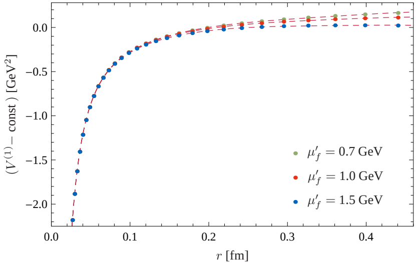
Quark-antiquark potential at order $1/m$

- Perturbative potential at order $1/m$ ($C_F = \frac{4}{3}$, $C_A = 3$)

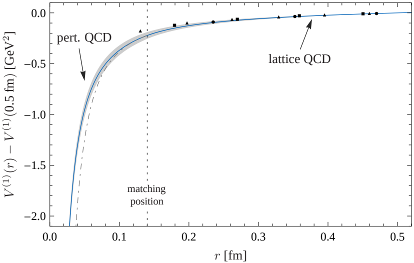
$$\tilde{V}^{(1)}(|\vec{q}|) = \frac{C_F\,\pi^2\,\alpha_s^2(|\vec{q}|)}{2\,|\vec{q}|}\left[(-C_A) + \mathcal{O}(\alpha_s)\right]$$

- Restricted numerical Fourier transform

$V^{(1)}(r)$



$V^{(1)}(r)$



- Differentiable quark-antiquark potential for distances up to ∼1 fm
- Matching at 0.14 fm gives $\mu'_f = 1.6^{+0.5}_{-0.8}$ GeV (for charmonium)
  $\mu'_f = 1.9^{+0.4}_{-0.6}$ GeV (for bottomonium)
- Grey band: uncertainty of lattice calculation and uncertainty of α_s

Heavy quark masses at order $1/m$

PS mass needs redefinition $m_{\mathsf{PS}}(\mu_f) \to m_{\widehat{\mathsf{PS}}}(\mu_f, \mu'_f)$

- $m_{\widehat{\mathsf{PS}}}(\mu_f, \mu'_f) \equiv m_{\mathsf{PS}}(\mu_f) - \frac{1}{8m} C_F C_A \alpha_s^2 \, \mu'^2_f$
- Quark masses from comparison with empirical quarkonium states

$\overline{\mathsf{MS}}$ masses [GeV]	static	static $+ 1/m$	PDG 2010
bottom quark	4.20±0.04	$4.18^{+0.05}_{-0.04}$	$4.19^{+0.18}_{-0.06}$
charm quark	1.23±0.04	$1.28^{+0.07}_{-0.06}$	$1.27^{+0.07}_{-0.09}$

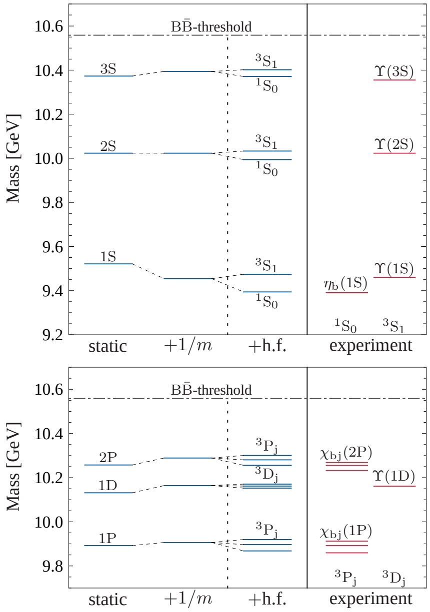
- Error estimates include:
  - uncertainties in the potentials (static and order $1/m$)
  - uncertainties from matching to experimental spectra

Spectroscopy

Bottomonium spectrum

- Tightly bound $\eta_b(1S)$ and $\Upsilon(1S)$ states are most sensitive to $1/m$-effects
- Hyperfine effects (h.f.) added phenomenologically (one-gluon exchange) with $\alpha_s^{\text{eff}} = 0.3$

  ... (work in progress)
  to be substituted by the full $1/m^2$ potential
- String tension $\sigma = 1.01$ GeV/fm
- Different strategies needed above $B\overline{B}$ threshold

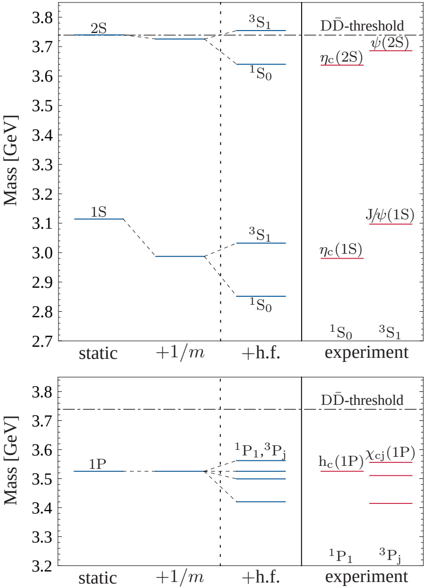


Spectroscopy

Charmonium spectrum

- Downward shift from $V^{(1)}$ in the 1S states (η_c and J/ψ) to large
- $1/m^2$ effects significant
- Hyperfine effects (h.f.) added phenomenologically (one-gluon exchange) with $\alpha_s^{\text{eff}} = 0.3$

  …(work in progress)
  to be substituted by the full $1/m^2$ potential
- String tension $\sigma = 1.01$ GeV/fm
- Different strategies needed above $D\overline{D}$ threshold



Summary

- Heavy quark-antiquark potential from QCD (perturbative QCD $\leftrightarrow$ lattice QCD)
- Excellent matching in r-space up to order $1/m$
- Spectroscopy at order $1/m$
  - Works well for bottomonium
  - Less successful for charmonium ($1/m^2$ effects sizeable: work in progress)
- Quark masses can be extracted

$\overline{\text{MS}}$ masses [GeV]	static	static + $1/m$
charm quark	1.23±0.04	$1.28^{+0.07}_{-0.06}$
bottom quark	4.20±0.04	$4.18^{+0.05}_{-0.04}$

See for details: A. Laschka, N. Kaiser, W. Weise, Phys.Rev.D83 (2011) 094002

Thank you for your attention!