STT 351-203 GRADED HOMEWORK ASSIGNMENTS #5 (6.1, 7.1-7.4) 07/31/2019

SOLUTIONS

Due Monday, August 5, 2019, 12:45 PM

<u>Instruction:</u> *Print this file. Write the answer on a dotted line …………… if provided. Show your work in the space below a question.* ***Not justified answers earn no points (even if they are correct)****. Attach extra pages if needed. Staple everything together. You may use any calculator and any computer software to solve the problems. Number of points for each problem is in square brackets* [ ]. *There are no bonus points in this homework.*

1. [4] Assume that the helium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with true standard deviation $\sigma = .75$.
   a. Compute a 99% CI for the true average porosity of a certain seam if the average porosity for 20 specimens from the seam was 4.85.

      99% CI: $4.85 \pm 2.576 \cdot \frac{0.75}{\sqrt{20}} = 4.85 \pm 0.43$ $\boxed{= (4.42, 5.28)}$

   b. What sample size is necessary to estimate true average porosity to within .2 with 99% confidence?

      $2.576 \frac{0.75}{\sqrt{n}} = 0.2 \implies n = \left(\frac{2.576 \cdot 0.75}{0.2}\right)^2 = 93.3$

      $\boxed{n \geq 94}$

2. [4] The article "Gas Cooking, Kitchen Ventilation, and Exposure to Combustion Products" (Indoor Air, 2006: 65–73) reported that for a sample of 50 kitchens with gas cooking appliances monitored during a one-week period, the sample mean CO_2 level (ppm) was 654.16, and the sample standard deviation was 164.43.
   a. Calculate and interpret a 90% (two-sided) confidence interval for true average CO_2 level in the population of all homes from which the sample was selected.

      $s = 164.43$ but n is large $\implies$ z-interval

      $\alpha/2 = 0.05$

      $z_{0.05} = 1.645$

      90% CI: $654.16 \pm 1.645 \frac{164.43}{\sqrt{50}} = 654.16 \pm 38.25$

      $= \boxed{(615.91, 692.41)}$

      It is also OK to use T-interval

   b. Suppose the investigators had made a rough guess of 175 for the value of σ before collecting data. What sample size would be necessary to obtain an interval width of 50 ppm for a confidence level of 90%?

      $1.645 \cdot \frac{175}{\sqrt{n}} = 25 \implies n = \left(\frac{1.645 \cdot 175}{25}\right)^2 = 132.6$

      $\boxed{n \geq 133}$

3. [4] The article "Limited Yield Estimation for Visual Defect Sources" (IEEE Trans. on Semiconductor Manuf., 1997: 17–23) reported that, in a study of a particular wafer inspection process, 356 dies were examined by an inspection probe and 201 of these passed the probe.

a. Calculate a 95% approximate confidence interval for the proportion of all dies that pass the probe.

$\alpha/2 = 0.025$

$z_{0.025} = 1.96$

$\hat{p} = \frac{201}{356} = 0.564$

95% CI: $0.564 \pm 1.96\sqrt{\frac{0.564(1-0.564)}{356}}$

$= 0.564 \pm 0.052 =$

$= \boxed{(0.513,\ 0.616)}$

b. What sample size would be required for the width of a 95% CI to be at most .05 irrespective of the value of $\hat{p}$?

$$1.96\sqrt{\frac{0.5\cdot 0.5}{n}} = 0.025 \implies n = \left(\frac{1.96\cdot 0.5}{0.025}\right)^2 = 1536.64$$

$\boxed{n \geq 1537}$

4. [8] A study of the ability of individuals to walk in a straight line ("Can We Really Walk Straight?" Amer. J. of Physical Anthro., 1992: 19–27) reported the accompanying data on cadence (strides per second) for a sample of $n = 13$ randomly selected healthy men.

.95	.78	.85	.92	.95	.93	.86	1.00	.92	.85	.81	.93	.93

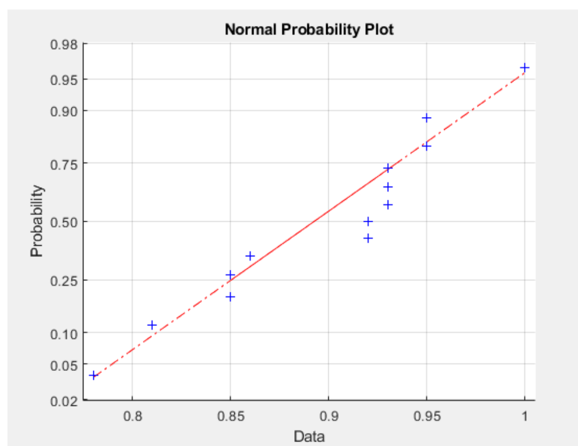
Assume that the population distribution of cadence is approximately normal.

a. Calculate the sample mean $\bar{x}$ and the sample standard deviation s.

ANSWERS: $\bar{x}$ **= 0.898,** s **= 0.063**

b. Make a normal probability plot to confirm the assumption of `normality (if you use a software or calculator copy the graph)

I used normplot(x) function of MATLAB.



c. Calculate a 95% confidence interval for population mean cadence.

n-small, normal population, σ-not known ⟹ T-interval

$n = 13$ $\quad \alpha/2 = 0.025$

$\bar{x} = 0.898$ $\quad \nu = 12$

$s = 0.063$ $\quad t_{0.025,12} = 2.179$

95% C.I.: $0.898 \pm 2.179\frac{0.063}{\sqrt{13}}$

$= 0.898 \pm 0.038$

$\boxed{= (0.860,\ 0.936)}$

d. Calculate a 95% confidence interval for the standard deviation of the population cadence.

$\chi^2_{0.025,12} = 23.337$

$\chi^2_{0.975,12} = 4.404$

$\frac{12\cdot 0.063^2}{23.337} \leq \sigma^2 \leq \frac{12\cdot 0.063^2}{4.404}$

$0.00204 \leq \sigma^2 \leq 0.01080$

$\boxed{0.045 \leq \sigma \leq 0.104}$