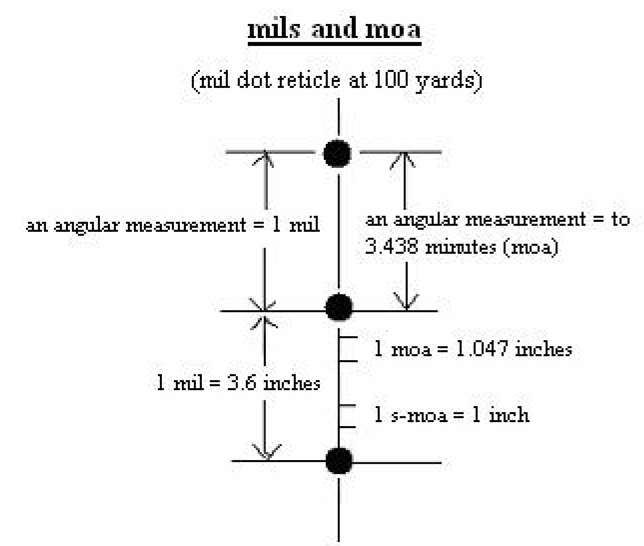
MILS and MOA

A Shooters Guide to Understanding:

- Mils
- Minute of Angle (moa)
- The Range Estimation Equations

By Robert J. Simeone



$$\frac{\text{Height of Target (yards)}}{\text{mils}} \times 1000 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{mils}} \times 27.78 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{moa}} \times 100 = \text{Distance to Target (yards)}$$

Shooters use the following equations to determine the range to a target depending on what type of reticle they have:

$$\frac{\text{Height of Target (yards)}}{\text{mils}} \times 1000 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{mils}} \times 27.78 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{moa}} \times 100 = \text{Distance to Target (yards)}$$

What really is a "mil" or a "moa", and where did these equations come from? In this paper I will attempt to explain, in simple terms, what **mils** and **moa** are, and I will **derive the distance equations** involving them. I will try to keep it simple, methodical and easy to understand. <u>The reason I wanted to do this is because I believe that if you know where something comes from, as well as how it works, you get a better understanding and appreciation of it, as well as its uses and limitations.</u>

What is a Mil?

A "mil" in "shooters" terminology is short for "**milliradian"**, a **real trigonometric unit of angular measurement** of a circle. Every circle has 6283.2 milliradians, which are small angles, in it (we'll discuss how they got this number later).

A mil is finer in measurement than degrees, thus more precise (6,283.2 mils in a circle vs. 360 degrees in a circle). In shooting, we use mils to find the distance to targets, which we need to know, in order to adjust our shots. It's also used to adjust shots for winds and/or the movements of targets.

(Note: The actual techniques of how to use mils for shot adjustments are beyond the scope of this paper since this paper mainly deals with the math behind mils, moa, and their distance equations.)

There is some controversy out there about what type of "mils" American military and tactical shooters use. Some think they use a mil that is based on a circle that has **6400** mils in it instead of 6283.2. This has widely been circulated, written about and even taught, including in the military. But this is not the case. While it may be true that some Artillery and other military units do use this type of mil (the one based on 6400), **American military snipers and tactical shooters use scopes that are based on and calibrated using "real mathematical milliradians", which are 6283.2 milliradians (mils) per circle**. (Note: Some other countries, like Russia for example, do use different types of mils for their scopes, but for American shooters, our "mil" is 6283.2 milliradians per circle).

MOA

What is a **minute of angle (moa)**? Like the "mil", it is an actual "true" unit of angular measurement, and is used quite often in shooting (**I will abbreviate this "true" moa angular measurement as just "moa"**). It is even more precise than a mil (21,600 minutes in a circle vs. 6283.2 mils). Many scopes use reticles etched in mils to find the range to the target, but have their knob adjustments in "minutes of angle" (moa). Many times we also talk about our shot "groupings" in moa. Furthermore, some scopes like mine have their reticles etched in minutes of angle (moa) rather than in "mils".

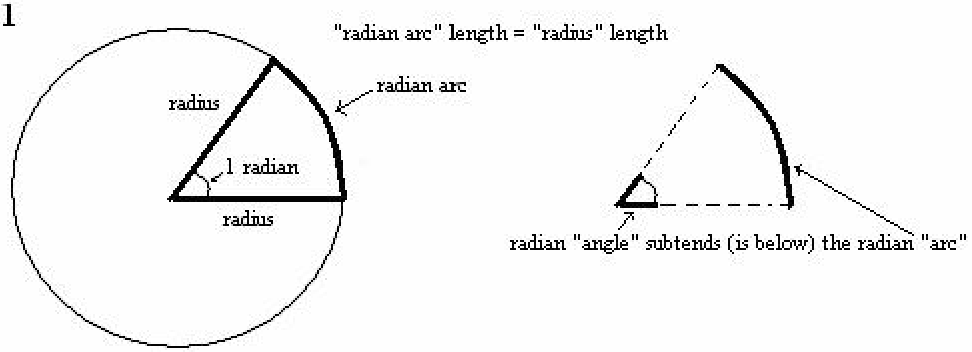
There is another slightly different "moa" that we will discuss that some shooters use and that some rifle scopes are calibrated in. It is close to the actual "true" moa but not equal to it. It is referred to as **"inch per 100 yards" (IPHY)** or **"shooters moa" (s-moa)**. (Because "shooters moa" (s-moa) is easier to say than "IPHY", I will mostly use that terminology in this paper).

Before we can proceed, we need to define a few terms and establish a few relations. So just follow along, hang in there, and you will see why we will need these later.

Radians

What is a radian? (Warning: you might have to read this paragraph a few times). A "radian" is a unit of angular measurement. Officially, **one radian subtends an arc equal in length to the radius (r) of a circle**. (Yeah that helps). How about this? A radian associates an arc length, called a **"radian arc", which is equal in length to the radius** of the circle, with an angle at the center of the circle. The **angle the arc creates is called a "radian"**. Or simply, think of it as a piece of apple pie, where the two sides of the pie (the radii) are each equal in length to the curvature part of the pie (the arc). The angle created by the three sides at the center of the circle equals 1 radian (Fig.1).

Fig. 1



To find out how many "radians" are in a circle, first note that **there are the same number of "radian angles" in a circle as there are "radian arcs"**. Therefore, if we find out how many "radian arcs" are in a circle, we'll then know the number of "radian angles", or "radians", there are.

To do this, we'll use the circumference formula of a circle, which is $C = 2\pi r$. Take $2\pi r$ and divide by "r". We divide by "r" because that will give us the number of radii in the circumference of a circle, remembering that the radius length equals the radian "arc" length. Since the number of radian "arcs" equals the number of radian "angles", we'll then know how many "radians" are in a circle. (Note: $\pi = 3.14159\ldots$).

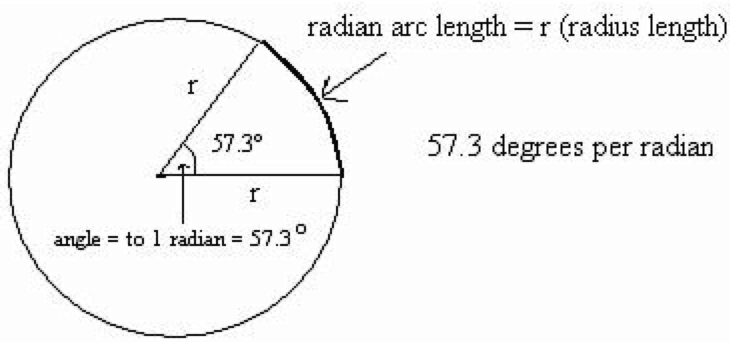
$$\frac{2\pi r}{r} = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi = 2 \times 3.14159 = 6.2832.$$

Therefore, there are 6.2832 radians in a circle (or, 6.2832 radian arc's that go around the circumference of a circle).

No matter how long the radius "r" is, there will always be 6.2832 radians in any circle (because the "r" always gets cancelled out in the math and all you're left with is 2π).

How many degrees are in a radian (or how big is the angle created by the radian arc)? Since there are 360 degrees in a circle and there are 6.2832 radians in a circle, then there are: 360 / 6.2832 = **57.3 degrees per radian** in all circles (no matter how long "r" is). (Fig.2 below).

Fig. 2



- (A) <u>Minutes in a Circle (moa)</u>

(<u>Note: "Minute" is interchangeable with and is the same thing as "minute(s) of angle", or "moa"</u>). There are 360 degrees in a circle, and each **one degree is composed of 60 minutes (**or **1/60th of a degree = 1 minute** or **1 moa)**. Therefore:

$$360 \left(\cancel{\text{degrees}}\right) \times 60 \frac{(\text{minutes})}{(\cancel{\text{degree}})}$$ = **21,600 minutes in a circle** (or 21,600 moa in a circle).

- (B) <u>Milliradians (mils)</u>

What is a milliradian? The prefix "mil" is defined as "one one-thousandth". Therefore, a **milliradian is 1/1000th of a radian**. Take each of the radians that go around a circle and chop it up into a thousand pieces. Since there are 6.2832 radians in a circle,

and each radian is chopped up into a thousand pieces, then there are:
6.2832 x 1000 = **6283.2 milliradians in a circle**. Recall, milliradians is usually just shortened to **“mils”**.

- (C) <u>Degrees in a Milliradian</u> (or degrees per mil)

I also need to find out how many degrees are in each milliradian. A circle has 360 degrees, and/or 6,283.2 milliradians (**B**, previous page and above) in it. Therefore:

$$\frac{\text{360 degrees in a circle}}{\text{6,283.2 mils in a circle}} = .0573 \text{ degrees/mil}$$

There are .0573 degrees per mil (degrees/mil)

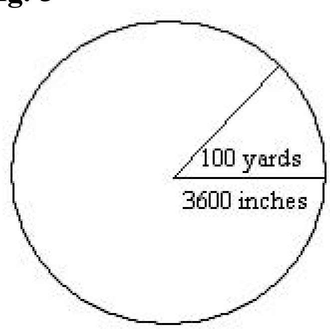
- (D) <u>Minutes in a Milliradian (or minutes per mil)</u>

I also need to find out how many minutes (or “minutes of angle”) there are in each mil. Let’s review. We have 21,600 minutes in a circle (**A,** page 4). And we have 6,283.2 mils in a circle (**B**, pages 4 & 5). Take 21,600 minutes and divide that by 6,283.2 mils and you get: $\frac{\text{21600 minutes}}{\text{6283.2 mils}}$ = 3.4377 minutes/mil. Let’s round that to just: **3.438 minutes/mil**

- (E) <u>Inches per Mil at 100 yards.</u>

Look at the circle in Fig.3 below. Make the radius 100 yards, or 3,600 inches.

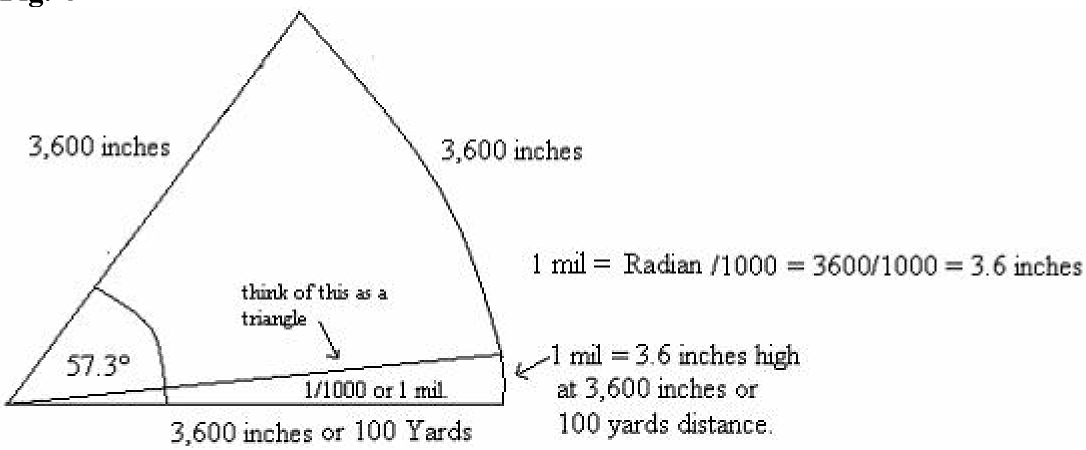
Fig. 3



Recall earlier from Fig.1 (page 3) that all the sides of the “piece of pie” are equal. Therefore, if the radius is 3,600 inches, then all the sides of the pie are also 3,600 inches (see Fig.4, next page). So what is 1/1000th of any of those sides, which would also be 1/1000th of the radian arc, which would also be 1/1000th of the “radian” angle? Essentially, what is 1 mil equal to (remember, 1 mil is defined as 1/1000th of a radian)? 3,600 inches / 1000 = 3.6 inches.

Therefore, at 100 yards, 1 mil = 3.6 inches.

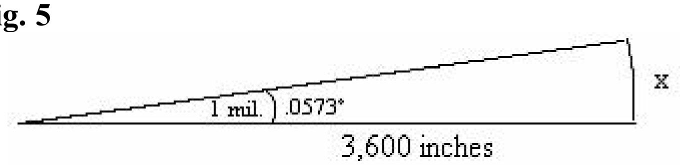
Fig. 4



For you math majors out there, another way to find the answer is to look at the bottom of Fig.4 as a triangle (enlarged in Fig.5 below). We want to find the value of "x" in Fig.5. The way to do this is to use the **tangent** function of trigonometry.

We know the length of one side of the triangle (3600 inches) and the angle (1 mil). But to use the tangent function, we need to convert the angle that is expressed in "mils" into an angle that is expressed in "degrees". Recall from **(C)** on page (5), **1 mil = .0573 degrees**. Now we can solve for "x":

Fig. 5



1) $\text{Tan}\ \theta = \dfrac{\text{opposite}}{\text{adjacent}}$ **2)** $\text{Tan}\ (\underset{1\ \text{mil}\uparrow}{.0573°}) = \dfrac{x}{3{,}600\ \text{in.}}$ **3)** $3{,}600\ (\text{Tan}\ .0573) = x$

(Note: using your calculator, the tangent of .0573 is .001)

4) $3{,}600\ (.001) = x$ **5)** $x = 3.6$ inches

- (E) Therefore, 1 mil at 100 yards equals 3.6 inches in height.

Note: Even though the opposite side of the triangle, "x" in Fig.5 above, is really not a straight line but a curve because it is actually a part of a circle, it is a very small curve. At this distance and at this small of an angle, for all practicable purposes we can consider it a straight line and its effects on the math are negligible.

- (F) Inches per minute (moa) at 100 yards

This next part could be confusing, so follow the units here. Remember from **(D)** on page (5), there are 3.438 minutes per mil. Also, from **(E)** on page (6), there are 3.6 inches per mil at 100 yards. The question here is, how many "inches per minute" (or "inches per moa") are there at 100 yards?

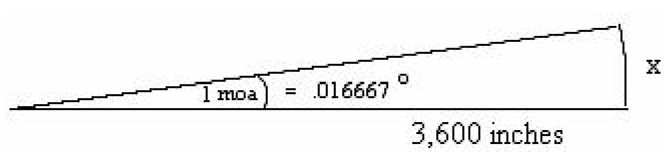
1) $\frac{\text{3.6 inches per mil (at 100 yards)}}{\text{3.438 minutes per mil.}}$ **2)** $\frac{\text{3.6 inches } \cancel{\text{per mil}} \text{ (at 100 yards).}}{\text{3.438 minutes } \cancel{\text{per mil.}}}$

3) $\frac{\text{3.6 inches (at 100 yards)}}{\text{3.438 minutes}}$ = 1.047 inches/minute (at 100 yards)

There are 1.047 inches per "minute of angle" (1.047 inches/moa) **at 100 yards**.

Another way to figure this out: (Reference Fig.6 below)

Fig. 6



Recall from **(A)** on page (4) that 1 minute (or 1 moa) equals 1/60th of a degree. $\frac{1}{60°}$ = .016667°. Therefore, **1 moa = .016667 °**. Just like the example on page (6):

1) $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ **2)** $\tan(.01667°) = \frac{x}{3{,}600 \text{ in.}}$ (1 moa ↑) **3)** 3,600 (Tan .016667) = x

* Note: On the calculator, the tangent of **.016667 = .00029089** (remember this for later).

4) 3,600 (.00029089) = x **5)** x = 1.047 inches (at 100 yards)

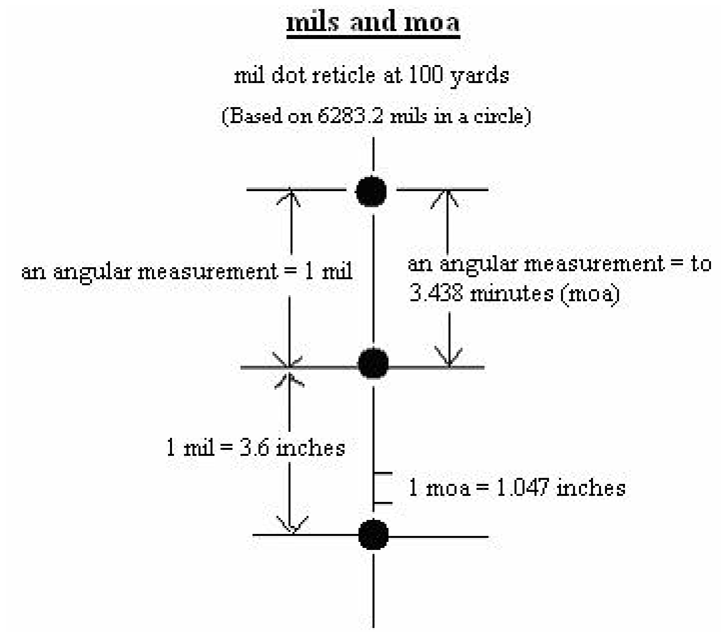
Or, one other way to figure this out (I promise, this will be the last way):

There are 3,600 inches in the radius of this circle. Therefore, the circumference of this circle would be 2π 3,600 = 22,619.467 inches. Recall from **(A)** on page (4), there are 21,600 minutes in a circle. Therefore, $\frac{\text{22,619.467 inches}}{\text{21,600 minutes}}$ = 1.047 inches per minute at 100 yards.

Talk about beating a dead horse! (I just wanted to show that you can come up with the same answer a few different ways).

So let's review. This is what we have while looking through a mil dot rifle scope at 100 yards (Fig.7, next page). Each dot is separated by 1 mil:

Fig. 7



Keep in mind that when we talk about "**mils**", we're talking about the angular measurement of a circle in "**milliradians**" or "**mils**". And when we talk about "**moa**", we're talking about the angular measurement of a circle in **"minutes"**, which are a subset of **degrees**. They're two different angular measurements, but measuring the same thing, which in our case are small sections of a circle that look like triangles (see Figures 4, 5 and 6 on the previous pages).

(See **Appendix A** (page 20) for a comprehensive picture of what we've talked about so far).

In the past when I've read about mils, somewhere around here in their explanation is where they usually say **without ever explaining it**:

"…and here is the equation that you use to find the distance to the target using mils."

(G)

$$\frac{\text{Height of Target (yards)}}{\text{mils}} \times 1000 = \text{Distance to target (yards)}$$

How did they come up with this equation?

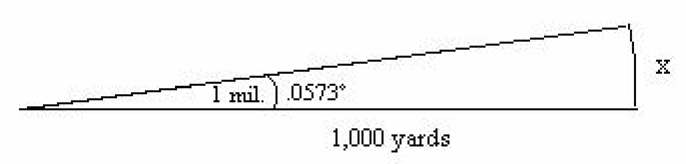
The Derivation of the "Mil" Range Estimation Equation

To help derive equation **(G)** (above) and keep it simple, I'm going to enlarge the "triangles" that we have been using (see Fig.5, page 6 for example). I'm doing this so I can have the height of the target and the distance to it in yards, which is the form that the most simplified mil equation is usually in (see **G** above).

If 1 mil = 3.6 inches at 100 yards (**E**, page 5), then at 10 times that distance, or at 1,000 yards, 1 mil = 36 inches, or 1 yard. Here's why. Let's do our trigonometry again.

Still using 1 mil, but looking out at a range of 1000 yards: (Note: 1 mil = .0573°, **C,** page 5)

Fig. 8



1) $\text{Tan } \theta = \frac{\text{opposite}}{\text{adjacent}}$ **2)** $\text{Tan } (.0573°) = \frac{x}{1000 \text{ yards}}$ **3)** 1,000 (Tan .0573) = x

(Note: The tangent of .0573 is .001)

4) 1000 (.001) = x **5)** x = 1 yard

- (H) Therefore, 1 mil at 1000 yards equals 1 yard (or 36 inches).

<u>The "tangent" function is where the distance equation comes from</u>, so let's look at the tangent function without any actual numbers in it for a moment.

1) $\text{Tan } (\theta \text{ in mils}) = \frac{\text{opposite (height of target)}}{\text{adjacent (distance to target)}}$

or in another easier to read form:

2) $\text{Tan } (\theta \text{ in mils}) = \frac{\text{height of target}}{\text{distance to target}}$

(To make it easier in the following math, let's use "H" equals the "height of the target" and "D" equals the "distance to the target". Also, remember that as of right now, the angle "θ" is still measured in "mils").

I want to cross multiply to solve for "D", the distance to the target, which is our goal.

3) $\text{Tan } (\theta) = \frac{H}{D}$

4) D (Tan θ) = H

5) $D = \frac{H}{\text{Tan } (\theta)}$ or (with "D" on the right hand side like the equation is usually written) **(I)** $\frac{H}{\text{Tan } \theta \text{ (in mils)}} = D$

The Tangent function works easier for things written in degrees, therefore we need to change the angle "θ" from **mils** to **degrees** (which we'll do later, but for now, just assume we already converted it). Now equation **(I)** (previous page) looks like this:

$$\frac{H}{\text{Tan } \theta \ (\textbf{in degrees})} = D$$

Let's plug in some numbers and watch what happens. Let's say the height of an object is 1 yard, and we're reading 1 mil on our scope. What is the distance to the target?

Recall from **(C)** on page (5), 1 mil is equal to .0573 degrees. Therefore, equation **(I)** on page (9) that starts out as:

1. $\frac{1 \text{ yard}}{\text{Tan (1 mil)}} = D$ **can be rewritten in degrees as:** $\frac{1}{\text{Tan (.0573°)}} = D$

2. On the calculator, the tangent of .0573 is equal to .001, which can be rewritten as 1/1000.

3. Now the equation becomes:

$$\frac{1}{\frac{1}{1000}} = D$$

Simplifying algebraically, this now can be rewritten as:

$$\frac{1 \times 1000}{1} = D, \quad D = 1000 \text{ yards}$$

Therefore, the distance to this target that has a height of 1 yard, measured at 1 mil on my scope, is 1000 yards, just like we saw in **(H)** on page (9).

Here is where it gets interesting. Guess what would happen if I measured 2 mils on my scope. 2 mils converted to degrees is 2 x (**.0573° per mil** (C page 5)) = .1146. The Tan (.1146) = .002, or 2/1000. I can now rewrite equation **(I)** page (9) as:

$$\frac{1}{\text{Tan (2 mils)}} = D = \frac{1}{\frac{2}{1000}} = D \text{ which equals } \frac{1 \times 1000}{2} = D: \quad D = 500 \text{ yards to the target}$$

So if I measured my 1 yard high target at 2 mils, the target would be 500 yards away.

Guess what the Tangent of 3 mils equals? Answer: 3/**1000**. i.e. (3 x .0573°) = .1719. The Tan (.1719) =.003, or 3/1000. Are you starting to see a pattern here?

Even the Tangent of fractions of mils are always divided by exactly 1000.
i.e. 3.25 mils = (3.25 x .0573°) = .1862. = Tan (.1862) = .00325 = 3.25/1000).

And it goes on and on like this:

Tan (4 mils) = Tan (4 x .0573°) = Tan (.2292) = .004 = 4/1000

Tan (5 mils) = 5/1000

Tan (6 mils) = 6/1000

•

•

Tan (25 mils) = 25/1000 etc.

Let's examine what's going on. As we've shown in the examples above, **the tangent of any "mil" number "x" always turns out to be that number "x" over 1000**. For example: Tan (7 mils) = $\frac{7}{1000}$. Remember, this is in the denominator of equation (**I**).

Therefore, equation **(I)** on page (9), after **converting the "mil" angle to degrees** (1), then **taking the tangent of that angle** (2), and **putting it in the denominator** of the equation where it belongs (3), is going to look like this:

$$\frac{H}{\frac{x \text{ mils}}{1000}} = D$$

After simplifying algebraically, it will look like this:

$$\frac{H \;\; 1000}{x \text{ mils}} = D \quad \text{(where x = the number of mils).}$$

Again, equation **(I)** on page (9) that was in the form:

$$\frac{H}{\text{Tan } \theta \text{ (in "x" mils)}} = D$$

First goes to this:

$$\frac{H}{\text{Tan } \theta \text{ (in "x" mils converted to degrees)}} = D$$

Then this after taking the tangent of the "degree number":

$$\frac{H}{\frac{\text{"x" mils}}{1000}} = D$$

And then this after simplifying:

(J)

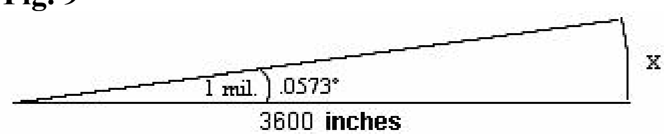
$$\frac{\mathbf{H\ 1000}}{\mathbf{x\ mils}} = \mathbf{D}$$

Which is the **Distance Equation using “Mils”**
(Recall that this is the same as **(G)** on page 8).

(Where **H** = the **height** of the target in **yards**; “**x**” mils = the **number of mils** read on the scope; and **D** = the **distance** to the target in **yards**).

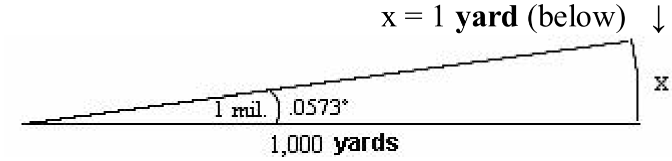
Something interesting needs to be mentioned here. It might not have seemed obvious at the time, but remember the triangles we used earlier in Figures (5) and (8) (pages 6 and 9)? Whatever units we used for the distance part of the triangle (the adjacent side), the height part of the triangle “x” (the opposite side) came out in the same units (see Fig.9, below).

Fig. 9



x = 3.6 **inches** (above) ↑

x = 1 **yard** (below) ↓



The opposite also is true. Whatever units you are measuring the height “H” in (x in the picture above), the distance to the target “D” will also be in the same units.

Example: You measure the height in **meters**. Then: $\frac{\text{H (\textbf{meters}) 1000}}{\text{mils}} = \text{D (\textbf{meters})}$

Therefore, **whatever units you measure the “height” of an object in, the “distance” to it will be in the same units.**

Now, knowing the height of an object in inches may be reasonable, and in fact preferable, but getting the distance to it also in inches might not be.

For example: a 29 inch tire measured at 2 mils is by our equation:

$$\frac{\text{29 in. x 1000}}{\text{2 mils}} = \text{14,500 inches to the target.}$$

How the heck far is that?

Well, 14,500 inches / 36 inches per yard = 402 yards. 402 yards is much more comprehendible than 14,500 inches.

So let's make the equation more workable for us with units that we might be more comfortable with. It is sometimes easier and more practicable to **know the height of an object in inches**, like a wheel or a head, but it's not that practicable to get the distance to it also in inch's, which our present equation does. So let's have the equation let us use **inches for the height** of an object but have it give us the **distance to it in yards**.

We need to convert the "H" part of equation **(J)** (page 12) **from inches to yards**. That way, even though we are **using inches for the height**, we'll get the **distance in yards**. The way we do that, just like above with the 14,500 inches, is by **dividing by 36**. Why? Well, take a 36 inch object and divide by 36. What do you get? 1, or more accurately, 1 yard. How about a 72 inch object, like a persons height: 72/36 = 2 yards. Get it? **H/36 converts inches to yards**. So our mil distance equation **(J)** (on page 12) now becomes (with "**H" still measured in inches**):

$$\frac{\frac{H_{\text{(inches)}}1000}{36}}{x \text{ mils}} = D, = \frac{\frac{H \quad 1000}{36}}{x \text{ mils}} = D, = \frac{H_{\text{(inches)}}\ 27.78}{x \text{ mils}} = D_{\text{(yards)}}$$

Note: The left-hand sides of the equations in the steps above are in **yards** even though we are using **inches** for the height of the object (because we divided "H" by 36, which converts inches to yards). Like we talked about before (page 12), the right hand sides of the equations, "D", will also be in the same units, which are **yards** in this case.

Therefore, the **mil distance equation** in a more user friendly version is:

(K)

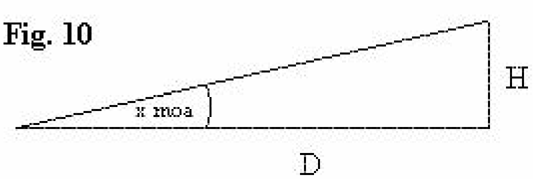
$$\frac{\mathbf{H}_{\text{(inches)}}\ \mathbf{27.78}}{\mathbf{Mils}} = \mathbf{Dist}_{\text{(yards)}}$$

Example: An object 36 inches high, measured at 1 mil = (36 x 27.78) / 1 =1000 yards, just like it should (see **(H)** on page 9).

How to Derive the "True" MOA Distance Equation

Look at Fig.10 below. Let's analyze the tangent equation for this triangle that has a "moa" angle instead of a "mil" angle and solve for "D", the distance to the target:

Fig. 10



The tangent equation for Fig.10 (previous page) is: $\text{Tan (x moa)} = \frac{H}{D}$, **(L)** $D = \frac{H}{\text{Tan (x moa)}}$.

Let's concentrate on the denominator of equation **(L)** (above) for a moment. Make it equal to 1 moa. Remember from **(A)** on page (4), 1 moa = 1/60th of a degree. Therefore: Tan (1 moa) = Tan (1/60°) = Tan (.016667) = **.0002908882 = Tan (1 moa)**. Let's start looking for a pattern. What if the angle in the picture above was 2, 3 or 4 moa?

Tan (2 moa) = Tan (2/60°) = Tan (.03333333) = .0005817765
Tan (3 moa) = Tan (3/60°) = Tan (.05000000) = .0008726648
Tan (4 moa) = Tan (4/60°) = Tan (.06666667) = .0011635533

At first it might not be obvious, but each of the tangents of the moa's above (2, 3, and 4) is really **just a multiple of .0002908882, which is the tangent of 1 moa**. That is, the tangent of 2 moa can be thought of as 2 x Tan (1 moa) which equals 2 x .0002908882 = .0005817765. The tangent of 3 moa can be thought of as 3 x .0002908882 = .000876647 etc. Therefore, we can rewrite equation **(L)** above as:

$$D = \frac{H}{\text{x (Tan 1 moa)}} \quad \text{(where x = some moa value).}$$

For example, if the moa value is 2, then:

$$D = \frac{H}{\text{Tan (}\mathbf{2}\text{ moa)}} \text{ can be rewritten as; } D = \frac{H}{\text{2 (Tan }\mathbf{1}\text{ moa)}} = D = \frac{H}{2\,(.0002908882)}.$$

Therefore, it follows that equation **(L)** (above) can now be rewritten as:

(M)

$$D = \frac{H}{x\,(.0002908882)} \quad \text{(where "x" is some moa value, and .0002908882 = Tan 1 moa)}$$

Note: .0002908882 can be rewritten as and is equal to $\frac{2.908882}{10{,}000}$. Therefore, equation

(M) above can be rewritten as: $D = \frac{H}{x\left(\frac{2.908882}{10{,}000}\right)} = \frac{H\ \ 10{,}000}{x\,(2.908882)} = \frac{H\ \ 3437.75}{x} =$

$$\textbf{(N)} \quad \frac{H\ \ 3437.75}{\text{x (in moa)}} = D.$$

Therefore, the distance to a target using **moa** values is the **height** of the target multiplied by the constant **3437.75** divided by the moa value "**x**".

Recall from before (pages 12 and 13) that the distance will be in the same units as the height of the target, i.e. if the height is in inches, then the distance to it will also be in inches; if the height is in yards, then the distance to it will also be in yards.

So let's make this equation more user friendly as we did earlier with the mil distance equation on pages 12 and 13. In equation **(N)** (previous page), let's use **inches** for the height "H" of the target, and let's get the distance "D" to the target in **yards**. Remember from page (13), **H/36 converts inches to yards**. Therefore, equation **(N)** (previous page) now becomes:

$$D = \frac{\frac{H}{36}\ 3437.75}{x} = \frac{H\ \frac{3437.75}{36}}{x} = \frac{H\ 95.5}{x} = D$$

(Where "H" is in inches and "D" is in yards).

Therefore, the distance equation using **"true" moa** is (with labels back on):

$$\textbf{(O)} \qquad \frac{H\ (\text{inches}) \times 95.5}{\text{moa}} = D\ (\text{yards})$$

Some simplify it to: $\frac{H\ (\text{inches}) \times 100}{\text{moa}} = D\ (\text{yards})$, which makes it real easy to use.

Let's see if it works. Recall from **(F)** on page (7), there are 1.047 inches per minute of angle (moa) at 100 yards. If I draw a line on a target at a known distance of 100 yards that is 1.047 inches long, then looking through my "moa" scope, I should see that 1.047 inch line exactly between my "1" moa tics on my reticle. Plugging those numbers in the equation, I get:

$$\frac{1.047\ \text{inches (line I drew)} \times 95.5}{1\ \text{moa (on my scope)}} = 99.99\ \text{yards (or 100 yards rounded)}$$

Exactly what I should be getting. In the example above, if I used the constant 100 instead of 95.5, then I would have gotten 104.7 yards. Close enough. We can all multiply by 100 in our head, but how many can multiply by 95.5. So a lot of shooters just use:

$$\textbf{(P)} \qquad \frac{H\ (\text{inches}) \times 100}{\text{moa}} = D\ (\text{yards})$$

Note: Be careful though, at greater distances, the discrepancy between 95.5 and 100 as the constant could change the result enough to be critical to the accuracy of your shot.

IPHY or "Shooters" moa

What is "inch per hundred yards" (IPHY), or more commonly known as and easier to say, **"shooters" moa** (**s-moa**)? A "shooters" moa is close to but not exactly equal to a “true” moa. Recall that 1 “true” moa will give us **1.047** inches at **100** yards (**F**, page 7). **Shooters like to keep things simple**, so they wanted an angle that will give them **exactly “1” inch at 100 yards**, hence the name **“shooters” moa** (I’m only guessing on that last statement, but it sounds reasonable). This “s-moa” will also give them 2 inches at 200 yards, 3 inches at 300 yards etc. (Note: Be aware, since “Shooters” moa and “true” moa are very close in value, they are frequently confused with and/or interchanged with each other). So, what is the “s-moa” distance equation?

“Shooters” MOA Distance Equation

The “Shooters” moa distance equation is solved initially by doing the opposite of what we’ve done so far. Instead of first defining an angle (i.e. 1 mil or 1 moa etc.) and then getting certain measurements from that angle (i.e. 3.6 inches at 100 yards or 1.047 inches at 100 yards), we’re first going to define what measurement we want (exactly 1 inch at 100 yards), then find out what angle will give us that measurement.

The question again is; **what angle will give us exactly 1 inch at 100 yards**? Recall from earlier (look at the examples on pages 6, 7 or 9), we knew the angle “θ” in degrees, but didn’t know one of the sides length (the opposite side “x” in those cases). We then used our calculator to get the tangent of that angle and then, using a little algebra, solved for the other side’s length (the opposite side “x” in those examples).

This time, we know both sides’ lengths, but don’t know the angle. What we’ll do now is use something called the **“inverse” tangent**. The “inverse tangent” will **get us an angle** from the two sides’ lengths in the tangent equation instead of **using the angle** to get one of the side’s length. (You might have to read that last sentence again).

For example, what if on page (7) under Fig.6, we knew the two sides’ lengths but didn’t know the angle? What would we do? The tangent equation would be:

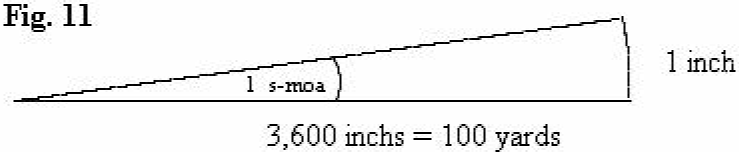
$$\text{Tan}\ \theta = \frac{\text{opp}}{\text{adj}} = \frac{1.047}{3600} = .00029089; \qquad \text{Tan}\ \theta = .00029089$$

What we do is take the **inverse tangent** of **.00029089** = **.016667°**, just like you would expect (see the “Note” on page 7 next to the *).

(Note: On your calculator, you usually have to type in the **number**, then press the **Inv** then **Tan** buttons, in that order, to get the Inverse Tangent).

Therefore, let’s do the same thing for the tangent equation with 1 inch for the opposite side (the height) and 3600 inches for the adjacent side (the distance) which is what we want the two sides to be if we want exactly 1 inch at 100 yards. For example, look at Fig.11 below.

Fig. 11



We know the distance (3600 inches), and we know the height (1 inch). What angle will give us this triangle? The equation is: $\text{Tan}(\theta) = \frac{1}{3600}$; $\text{Tan}(\theta) = .000277778$. Take the **inverse tangent** of .000277778 and you get the angle .0159154°. Therefore, **1 shooters moa = .0159154°** (compare that to 1 true moa = .016667°). Now that we know this, we can now come up with the distance equation just like we did starting on page (13) with "true moa".

The Tangent equation for this problem, with no numbers in it yet, looks like this:

(Q)

$$\text{Tan}(x \text{ s-moa}) = \frac{H}{D}, \qquad D = \frac{H}{\text{Tan}(x \text{ s-moa})}$$ (where "x" is some s-moa value)

Let's concentrate on the denominator of equation **(Q)** (above) for a moment. Make "x" equal to **1 shooters moa** . Therefore: Tan (1 s-moa) = Tan (.0159154°) = **.000277778**. Let's start looking for a pattern. What if the angle in the picture above was 2, 3 or 4 moa? Just like on page (14), each of the tangents of the moa's above (2, 3, and 4) is really **just a multiple of .000277778, which is the tangent of 1 s-moa**. That is, the tangent of 2 s-moa can be thought of as 2 x Tan (1 s-moa) which equals 2 x .000277778 = .00055555. The tangent of 3 s-moa can be thought of as 3 x .000277778 = .00083333 etc.

Therefore, we can rewrite equation **(Q)** above as:

$$D = \frac{H}{x\,(\text{Tan } 1 \text{ s-moa})}$$ (where x = some s-moa value).

For example, if the s-moa value is 2, then:

$$D = \frac{H}{\text{Tan}(\mathbf{2} \text{ s-moa})}$$ can be rewritten as $$D = \frac{H}{2\,(\text{Tan } \mathbf{1} \text{ s-moa})} = D = \frac{H}{2\,(.000277778)}.$$

Therefore, it follows that equation **(Q)** (above) can now be rewritten as:

(R)

$$D = \frac{H}{x\,(.000277778)}$$ (where "x" is some s-moa value, and .000277778 = Tan 1 s-moa)

Note: .000277778 can be rewritten as and is equal to $\frac{2.777778}{10,000}$. Therefore, equation **(R)** (above) can be rewritten as:

$$D = \frac{H}{x\left(\frac{2.777778}{10,000}\right)} = \frac{H\ 10,000}{x\,(2.777778)} = \frac{H\ 3600}{x} = D$$

(continued next page)

$$\textbf{(S)} \qquad \frac{H \cdot 3600}{x \text{ (in s-moa)}} = D.$$

Therefore, the distance to a target using **s-moa** values is the **height** of the target multiplied by the constant **3600** divided by the s-moa value "**x**".

Remember from earlier (pages 12 and 13) that the distance will be in the same units as the height of the target, i.e. if the height is in inches, then the distance to it will also be in inches; if the height is in yards, then the distance to it will also be in yards.

So let's make this equation more user friendly as we did earlier with the mil and moa distance equations (pages 12,13 & 15). In equation **(S)** (above), let's use **inches** for the height "H" of the target, and let's get the distance "D" to the target in **yards**. Remember from page (13), **H/36 converts inches to yards**.

Therefore, equation **(S)** (above) now becomes:

$$D = \frac{\frac{H}{36} \; 3600}{x} = \frac{\frac{H \; 3600}{36}}{x} = \frac{H \; 100}{x} = D$$

(Where "H" is in inches, "D" is in yards, and x is in s-moa).

Therefore, the distance equation using **"shooters moa"** is (with labels back on):

$$\textbf{(T)} \qquad \frac{\mathbf{H} \text{ (inches) } \mathbf{x\ 100}}{\textbf{s-moa}} = \mathbf{D} \text{ (yards)}$$

Let's see if it works. If I draw a line on a target at a known distance of 100 yards that is "1" inch long, then looking through my "shooters moa" scope, I should see that "1" inch line exactly between my "1" s-moa tics on my reticle. Plugging those numbers in equation **(T)** (above), I get:

$$\frac{\mathbf{1} \text{ inch (line I drew) x } \mathbf{100}}{\mathbf{1} \text{ moa (on my scope)}} = 100 \text{ yards.}$$

Just like I should get (see the definition of. s-moa starting on the bottom of page 15).

As you can see, the two "moa" distance equations, **(O)** on page (15) and **(T)** above, are nearly the same. Therefore, no matter what system you're using (moa or s-moa), most use equation **(T)** above (which is the same as **P** on page 15) because it works and is a lot easier to use. **Be cautious though if using "true" moa**. As my note on page (15) states:

(Note: Be careful though, at greater distances, the discrepancy between 95.5 and 100 as the constant could change the result enough to be critical to the accuracy of your shot).

In summary: Mathematically, we've found out exactly what **mils**, **moa** and **s-moa** are and where they come from. We also discussed the two "moa" systems (**true** and **shooters**). We also **derived the mil, moa, and s-moa distance equations**.

(Note: See page 22 for other distance equations not discussed that you might find useful depending on your combination of scope, reticle and units you prefer).

Why did we do all this? Because I think there's a lot of confusion and misconceptions out there on mils, moa, and s-moa, and I'm hoping that this paper will erase some of that confusion. Also, like I said at the beginning of this paper; "....*I believe that if you know where something comes from, as well as how it works, you get a better understanding and appreciation of it, as well as its uses and limitations*".

This knowledge will help you better understand how your reticle works and how to better utilize it. It will also help you with your windage and elevation adjustments. Do they adjust for "true" moa or "shooters" moa every time you move them? Or do they adjust in "mils"? Frequently, scope manuals don't accurately specify this or go into much detail about their scopes in general. I've seen and read many different combinations of reticles and turret adjustments for many different scopes. You might have to make some phone calls to find out exactly what you have.

Phew! Finally done!

At this point, you can now brain dump most of this math and definition stuff, and just use mils, moa and s-moa, and their equations, like they've always been used.....but hopefully with a little better knowledge. Thank you.

Good Shooting,

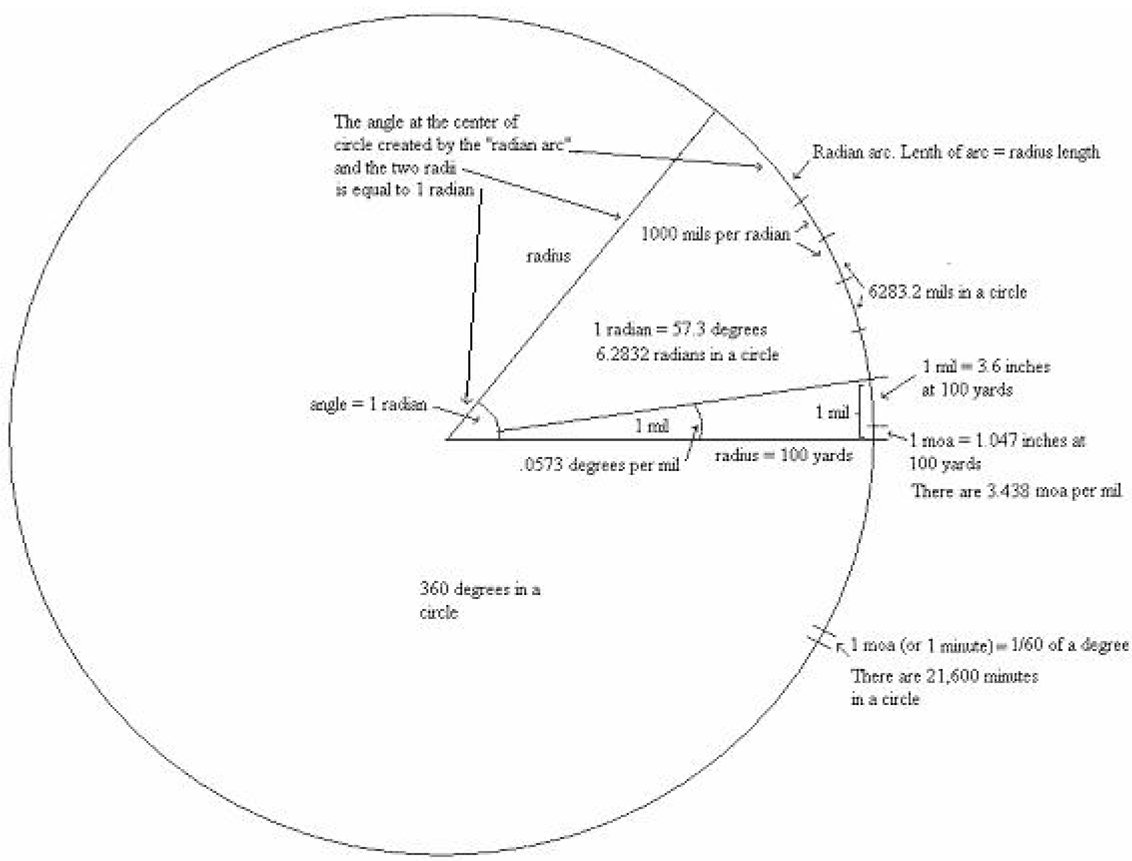
Robert J. Simeone

Updated: 02/10/08

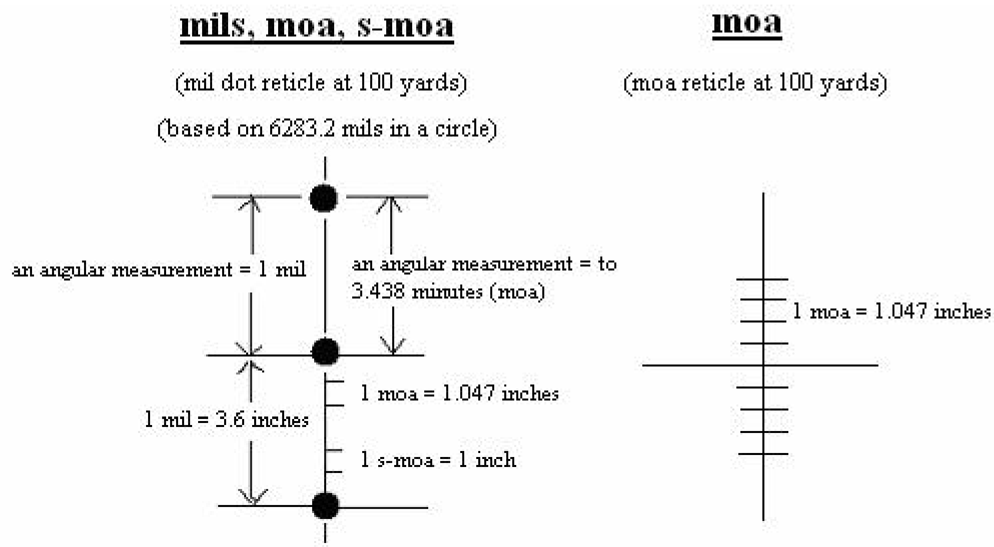
Appendix A

A comprehensive look at **Mils** (the actual mathematical milliradian), **Moa** ("true moa"), and how they relate to a circle and each other **at 100 yards**.

(Note: The figure below is not to scale).



Quick Reference Chart



$$\frac{\text{Height of Target (yards)}}{\text{mils}} \times 1000 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{mils}} \times 27.78 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{moa}} \times 100 = \text{Distance to Target (yards)}$$

- 6.2832 radians in a circle: page (4)

- 57.3° per radian: page (4)

- 21,600 minutes (moa) in a circle: **A**, page (4)

* - 6283.2 milliradians in a circle: **B**, pages (4) & (5)

- (.0573°) per mil: **C,** page (5)

* - 3.438 minutes (moa) per mil: **D**, page (5)

* - 3.6 inches per mil at 100 yards: **E,** page (5)

* - 1.047 inches per “true” moa at 100 yards: **F,** page (7)

- (.016667°) per “true” moa: page (7)

* - 1 “shooters” moa = 1 inch at 100 yards: page (16)

- (.0159154°) per “shooters” moa: page (17)

* = good numbers to remember

Other Equations

Below are the distance equations for various combinations of reticles, units of height, and units of distance to the target. Pick the one that you have a reticle for, that you feel comfortable with, and that is easy for you to use and that you will use all the time.

I won't go through all the derivations for these, but I derived these the same way I did earlier. I used the base equations **(J)** on page (12), **(N)** on page (14), and **(S)** on page (18), and the methods on pages (13), (15) and (18).

$$\frac{\text{Height of Target (yards)}}{\text{mils}} \times 1000 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{mils}} \times 27.78 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{mils}} \times 25.4 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (meters)}}{\text{mils}} \times 1000 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (cm)}}{\text{mils}} \times 10 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (inches)}}{\text{moa}} \times 95.5 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{moa}} \times 87.32 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (meters)}}{\text{moa}} \times 3437.75 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (cm)}}{\text{moa}} \times 34.37 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (inches)}}{\text{s- moa}} \times 100 = \text{Distance to Target (yards)}$$

$$\frac{\text{Height of Target (inches)}}{\text{s- moa}} \times 91.44 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (meters)}}{\text{s- moa}} \times 3600 = \text{Distance to Target (meters)}$$

$$\frac{\text{Height of Target (cm)}}{\text{s- moa}} \times 36 = \text{Distance to Target (meters)}$$