Polarization Handedness Convention

When light is elliptically polarized, the electric field (E field) vector rotates with respect to a Cartesian coordinate system as it propagates. The PAX1000 and PAX Polarimeter System software plot the E field vector with respect to a right-handed coordinate system with x, y, and z axes. The light wave propagates in the +z direction, and the (virtual) observer looks along the -z direction towards the light source. The handedness of the elliptically polarized light describes the direction of rotation of the E field vector as seen by this observer. This convention is common in the technical literature.

Each state of polarization can be split into two linearly polarized orthogonal components, in which one is oriented in the x direction and one in the y direction. If both components have equal magnitudes and the phase shift of the y component relative to the x component is $+\pi/2$ or $-\pi/2$, the light is circularly polarized. The sign of the phase difference determines the handedness of the rotation. A clockwise rotation corresponds to a right-hand circular polarization state and a phase shift of $-\pi/2$, while a counterclockwise rotation refers to left-hand circular polarization state and a phase shift of $+\pi/2$.

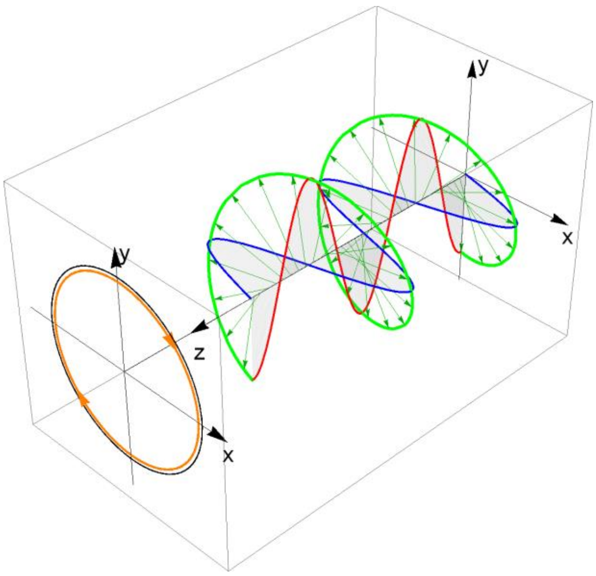

Figure 1 Right-Circularly Polarized Light

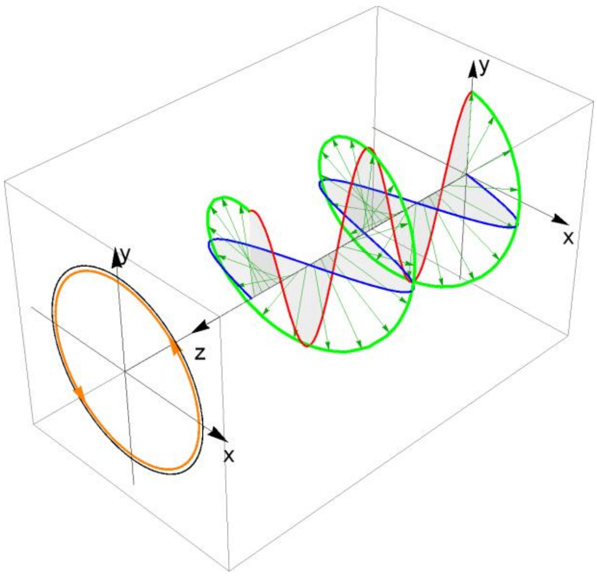

Figure 2 Left-Circularly Polarized Light

In **Figure 1** and **Figure 2**, the projection of the rotating E field vector on a virtual screen generates a circle over time as the E field vector rotates clockwise (counterclockwise) representing right (left) circular polarization.

Generation of Circularly Polarized Light

A circular polarization state can be generated using linearly polarized light and a quarter waveplate. When the E field vector of the incident linearly polarized light is oriented at a 45° angle to the slow and fast axes of the quarter waveplate, the output light is circularly polarized. It is convenient to mathematically describe this transformation using matrix algebra, with Jones vectors representing the polarized light and a Jones matrix representing the quarter waveplate.

The Jones matrix, $M_{\lambda/4}$, describing a quarter waveplate with its slow axis oriented along the x (horizontal) axis is :

$$M_{\lambda/4} = e^{i\pi/4}\begin{bmatrix}1 & 0\\0 & -i\end{bmatrix}$$

The phase factor $e^{i\pi/4}$ can be omitted in almost all cases.

The Jones vector, $\vec{J}_{+45}$,of a linear polarization state oriented at + 45° is written as:

$$\vec{J}_{+45} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

When this linearly polarized light travels through a quarter waveplate, the Jones vector, $\vec{J}_{RHC}$, of the output light can be calculated as follows:

$$\vec{J}_{RHC} = M_{\lambda/4}\vec{J}_{+45} = \begin{bmatrix}1 & 0\\0 & -i\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$$

The output polarization is right-hand circular.

This is illustrated by **Figure 3**, in which the slow and fast axes of the quarter waveplate are aligned with the x- and y-axes, respectively, of the coordinate system. The purple vector at the origin indicates the orientation of the incident 45° linearly polarized light, while the red and blue vectors represent this E field decomposed into its orthogonal components. These two components are in phase. The x-axis component, which is represented by the blue vector, is aligned with the slow axis of the waveplate and travels at a slower velocity through the waveplate than the y-axis component, which is aligned with the fast axis and represented by the red vector. Propagating through the waveplate retards the phase of the x-axis component with respect to that of the y-axis component. The amount of the retardation is determined by the thickness of the waveplate, and quarter waveplates are designed to give a phase shift of -π/2. This retardation produces right-hand circularly polarized output light, whose E field vector rotates clockwise as it propagates along the z-axis.

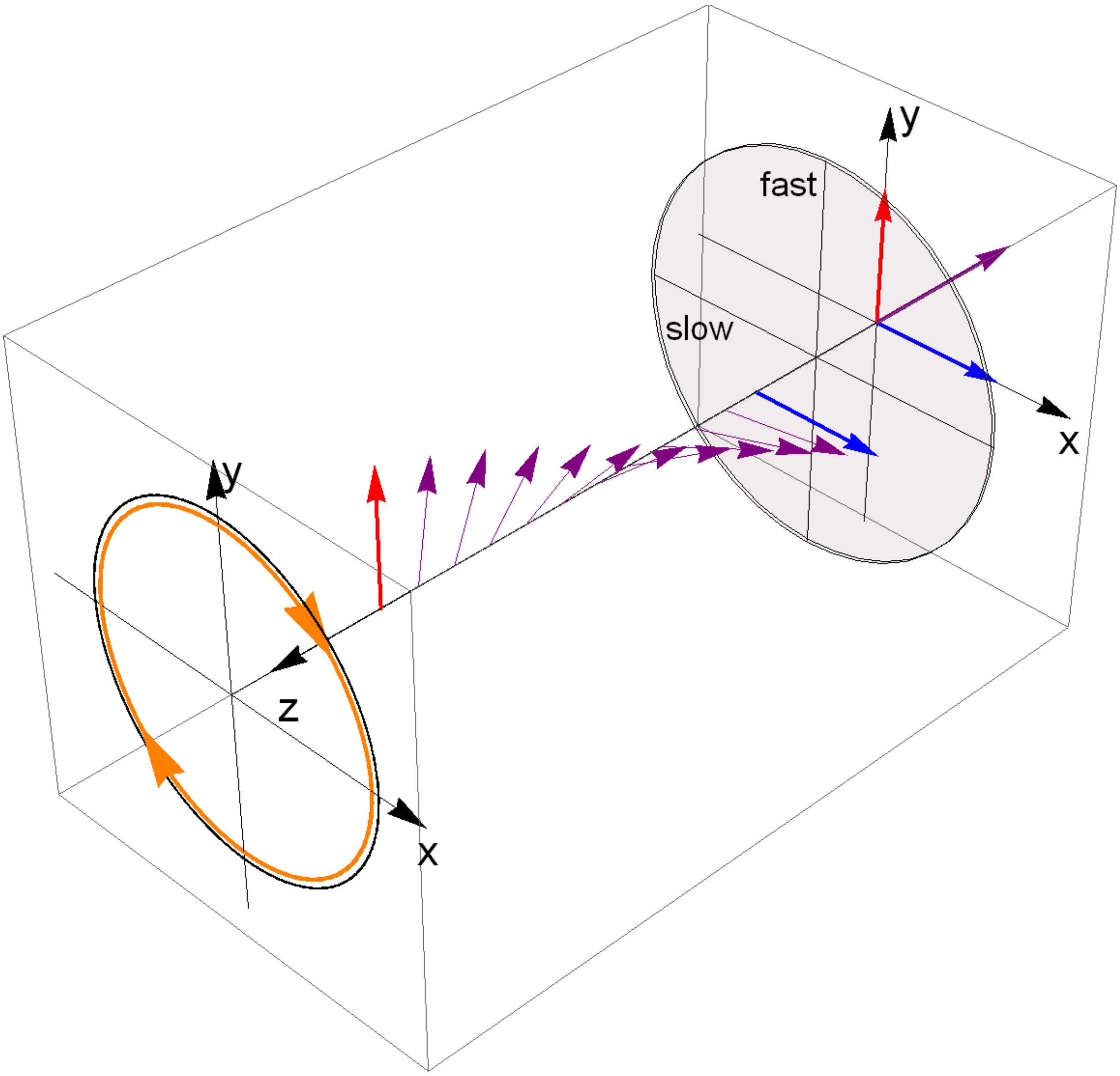

Figure 3 The input polarization is +45° linear and a right - hand circular polarization state is generated. The slow axis of the quarter waveplate is oriented along the x – axis.

When the incident beam has a -45° linear polarization, its Jones vector, $\vec{J}_{-45}$, is:

$$\vec{J}_{-45} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The output polarization state, $\vec{J}_{LHC}$, is then expressed:

$$\vec{J}_{LHC} = M_{\lambda/4}\vec{J}_{-45} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

The output light is left-hand circularly polarized.

Figure 4 illustrates this case. The waveplate is oriented as in Figure 3, and the purple vector at the origin again represents the orientation of the incident linearly polarized light. As this is linearly polarized light, there is no phase difference between the x- and y-axis components of the E field. The -45° orientation of the polarization vector means means it is azimuthally rotated. The red vector points in the +y direction and the blue vector points in the –x direction. Propagating through the waveplate causes the component aligned along the slow axis to add a phase shift of $+\pi/2$ with respect to the component aligned along the fast axis. The output light is left-hand circularly polarized and has an E field vector that rotates counterclockwise as it propagates along the z-axis.

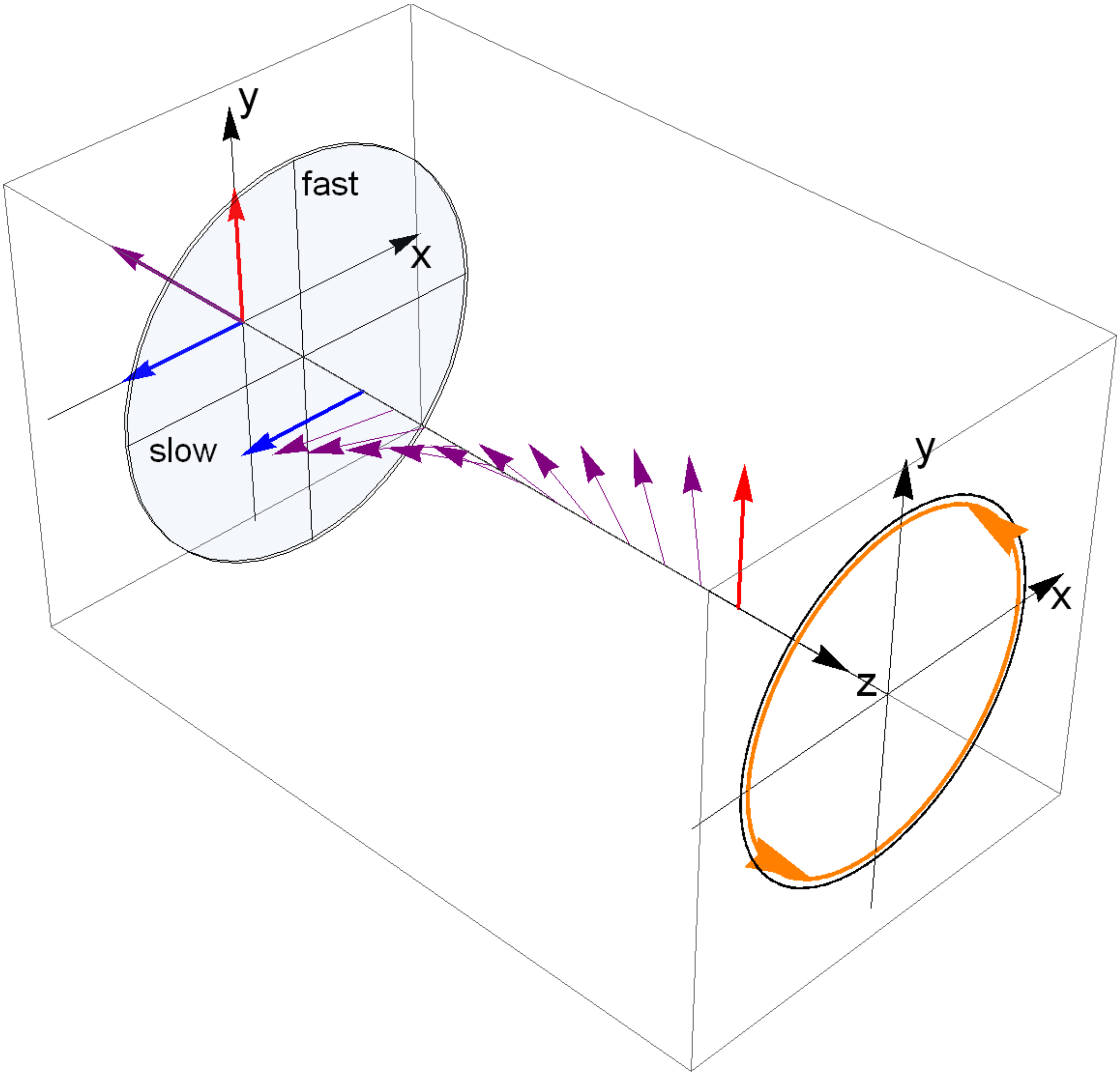

Figure 4 The input polarization is -45° linear and a left - hand circular polarization state is generated. The slow axis of the quarter waveplate is oriented along the x – axis.

Jones Vectors and Matrices

The following tables contain the Jones vectors for selected polarizations of light and the Jones matrices of standard optical elements, respectively.

State of Polarization	Jones Vector
Horizontal Linear Polarization	$\begin{bmatrix}1\\0\end{bmatrix}$
Vertical Linear Polarization	$\begin{bmatrix}0\\1\end{bmatrix}$
Linear Polarization at +45°	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$
Linear Polarization at -45°	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$
Right Circular Polarization	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$
Left Circular Polarization	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$

Table 1 Jones Vector for Selected Polarization States

Linear optical Element	Jones Matrix
Horizontal Linear Polarizer	$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$
Vertical Linear Polarizer	$\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$
Linear Polarizer at +45°	$\frac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$
Linear Polarizer at -45°	$\frac{1}{2}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$
Quarter Wave Plate, Slow Axis Horizontal	$e^{i\pi/4}\begin{bmatrix}1 & 0\\0 & -i\end{bmatrix}$
Quarter Wave Plate, Slow Axis Vertical	$e^{i\pi/4}\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$

Table 2 Jones Matrices for Standard Optical Elements

Further Reading

1. Serge Huard, *Polarization of Light*, Wiley, New York, 1997.
2. Eugene Hecht, *Optics*, Addison Wesley, New York, 2016.
3. Christian Brosseau, *Fundamentals of Polarized Light: A Statistical Optics Approach*, Wiley, New York, 1998.